IMO - Selection 2018

Second exam - 13 May 2018

Duration: 4.5 hours
Difficulty: The problems are orderd by difficulty.
Points: Each problem is worth 7 points.

4. Let n be an even positive integer. We partition the numbers $1, 2, \ldots, n^2$ into two sets A and B with the same size such that all of the n^2 numbers belong to exactly one of the two sets. Let S_A and S_B be the sum of all the elements in A respectively B. Determine all n such that there is a partition with

$$\frac{S_A}{S_B} = \frac{39}{64}.$$

5. Let n be a positive integer. We consider an $n \times n$ grid. We colour k squares in black, such that given any three columns, there exists at most one row that intersects the three columns at a black square. Prove that

$$\frac{2k}{n} \leq \sqrt{8n-7} + 1.$$

6. Let A, B, C and D be four points on a circle in this order. Assume that there is a point K on the segment AB such that BD bisects KC and AC bisects KD. Determine the minimal value that $\left|\frac{AB}{CD}\right|$ can take.

Good Luck!