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Mock exam 2020

Mock exam

Duration: 4 hours
Difficulty: The problems are orderd by difficulty.
Points: Each problem is worth 7 points.

1. Let ABC be a triangle with $\angle CAB = 90°$. Let D, E be points on AC, AB respectively such that $BCDE$ is cyclic. Let Ω_1, Ω_2 be the circles through A with centres E, D respectively. Denote by P the second intersection of Ω_1 and Ω_2. Prove that the line AP bisects the side BC.

2. Prove that for any power of 2, no permutations of its digits gives rise to another power of 2.

Taking 128 *for example, none of the numbers* 182, 218, 281, 812, 821 *is a power of* 2.

3. Let a, b, c be the lengths of the sides of a triangle. Prove that

$$\sqrt{6}\sqrt{a+b+c} \leq \sum_{cyc} \frac{a+b}{\sqrt{a+c}} < 2\sqrt{2(a+b+c)}.$$

4. In a $2 \times n$ grid we have positive real numbers such that the sum of the two numbers in each of the n columns is 1. Show that we can select one number in each column such that the sum of the selected numbers in both rows is at most $\frac{n+1}{4}$.

Good Luck!