

SMO - First round

Bellinzona, Lausanne, Zürich - 16th january 2016

Duration: 3 hours

Each problem is worth 7 points.

- Two circles k_1 and k_2 intersect at points A and C . Let B be the second point of intersection of k_1 and the tangent line to k_2 through A , and let D be the second point of intersection of k_2 and the tangent line to k_1 through C . Prove that the lines AD and BC are parallel.
- Quirin has n blocks, with height 1 to n , and he wants to place them in a line in such a way that his cat can go through them from left to right. The cat can jump from one block to the next one if either the block from which it jumps is higher, or if the difference in height is at most 1. At the beginning his cat is on the leftmost block.
In how many ways can Quirin place his blocks in such a line?

Remark: for $n = 5$, $3 - 4 - 5 - 1 - 2$ is possible, but $1 - 3 - 4 - 5 - 2$ is not.

- Find all natural numbers n such that for every positive divisor d of n we have:

$$d + 1 \mid n + 1.$$

- 22 mathematical competitions are organised, and for each competition five contestants are given an award. After the end of all the competitions, the organizers remark that for any two competitions exactly one contestant has received an award in both competitions. Prove that one of the contestants has received an award in all the competitions.
- Let ABC be a triangle with $AB < AC$. The angle bisector of $\angle BAC$ intersects the side BC at D . Let k be the circle going through D which is tangent to the sides AC and AB at the points E respectively F . Let G be the second point of intersection of k and BC and let S be the point of intersection of the segments EG and DF . Prove that AD and BS are perpendicular.

Good Luck!