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Exam 2020

Mathematical Olympiad, Round 1

Informations

- Questions: 35
- Time: 75 Minuten
- Auxiliaries: You may use any auxiliary means (calculator, internet, etc.) however, you must take the exam on your own and without help of other people.

Question types

- Multiple-Choice (MC): Each question has exactly one correct answer. If you choose the wrong answer, points will be deducted to not reward guessing.
- Integer questions (INT): Each question has as an answer a non-negative integer from 0 to 99999. No points are deducted for wrong answers.
- Multiple True/False (T/F): Each question has four statements which each can be true or false. Points are deducted for wrong answers.

Points

There are three difficulty levels. Harder questions are worth more points. It can be a good idea to skip questions if no progress is made.

Level 1

MC:	+8 for the correct answer,	−2 for a wrong answer,	0 for unanswered
T/F:	+2 for each correct answer,	−2 for each wrong answer,	0 for unanswered
NUM:	+8 for the correct answer,	0 for wrong or unanswered	

Question 1 (MC):

The sum of five consecutive integers equals the sum of the three next bigger integers. What is the biggest of these eight integers?

A: 4
B: 8
C: 9
D: 11
E: 12

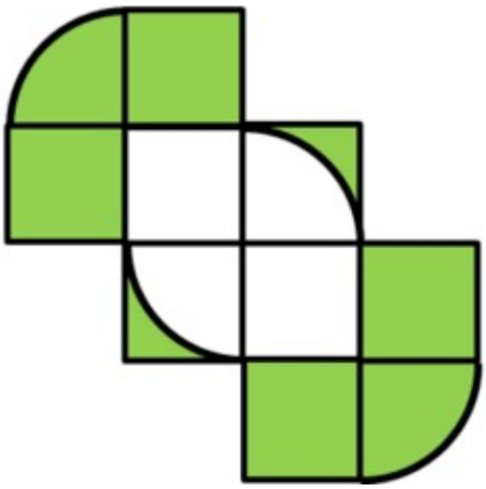
Question 2 (MC):

On a cube, we label each of the faces and each of the vertices with 1 and each edge with -1. What is the total sum of the labels?

A: 0
B: 2
C: 4
D: 6
E: 8

Question 3 (MC):

Using her compass, Rada drew the figure below on a grid paper. If the side length of one small square is 2, what is the area in green?



A: 12
B: 16
C: $16 + 2\pi$
D: 24
E: $24 + 2\pi$

Question 4 (MC):

Tim forgot the last five letters of his password. He only remembers that each letter is one of I, M or O. How many possibilities does he have to take into consideration?

A: 15
B: 20
C: 120
D: 125
E: 243

Question 5 (INT):

What is the smallest integer greater than 1 which is both a square and a cube?

Question 6 (INT):

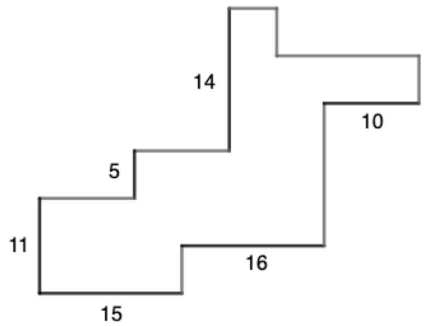
What is the smallest positive odd integer that is not a prime and not a square?

Question 7 (INT):

How many integers are there between 10 and 1000 that stay the same when the order of their digits is reversed?

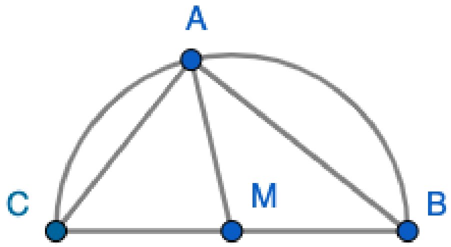
Question 8 (INT):

What is the perimeter of the shape below?



Question 9 (MTF):

Which statements about the following configuration must certainly hold, assuming that M is the center of the semicircle?



A: $\angle CAM = \angle ABC$

B: $\angle AMC = 2\angle ABC$

C: $\angle ACB + \angle ABC = 90°$

D: $\text{Area}(ACM) = \text{Area}(AMB)$

Question 10 (MTF):

The number 323 is...

A: a prime.

B: a square.

C: the difference of two primes.

D: the difference of two squares.

Level 2

MC:	+12 for the correct answer,	–3 for a wrong answer,	0 for unanswered
T/F:	+3 for each correct answer,	–3 for each wrong answer,	0 for unanswered
NUM:	+12 for the correct answer,	0 for wrong or unanswered	

Question 11 (MC):

What is the remainder of $2^2 \cdot 3^3 \cdot 5^5 \cdot 7^7$ when divided by 8?

A: 2

B: 3

C: 4

D: 5

E: 7

Question 12 (MC):

Given a square $ABCD$ in the plane, how many squares are there that share exactly two vertices with $ABCD$?

A: 4

B: 6

C: 8

D: 12

E: 16

Question 13 (MC):

Julia and Florian independently think of an integer between 1 and 10 (inclusively 1 and 10). Julia says: "*No matter what number you chose: If we compute the product of our two numbers, it will not contain the digit* 6." Florian says: "*Alright, then the sum of our numbers must be* 14." What is Florian's number?

A: 4

B: 5

C: 6

D: 8

E: 9

Question 14 (MC):

Which is the smallest integer $n > 2$ such that $(2^2 - 1) \cdot (3^2 - 1) \cdot \ldots \cdot (n^2 - 1)$ is a square number?

A: 7

B: 8

C: 9

D: 11

E: 12

Question 15 (MC):

Tanish thinks of an integer $1 \leq n \leq 100$ and Marco wants to guess the number. After each guess, Tanish tells him whether his guess was correct, too small or too big. What is the smallest number of guesses Marco needs to figure out Tanish's number, assuming he has a good guessing strategy?

A: 4

B: 5

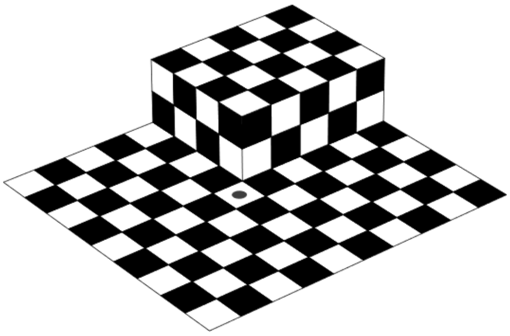
C: 6

D: 7

E: 8

Question 16 (MC):

An ant is initially on the square marked with the black dot. The ant moves across an edge from one square to an adjacent square four times and then stops. How many of the possible finishing squares are black?



A: 6

B: 8

C: 10

D: 12

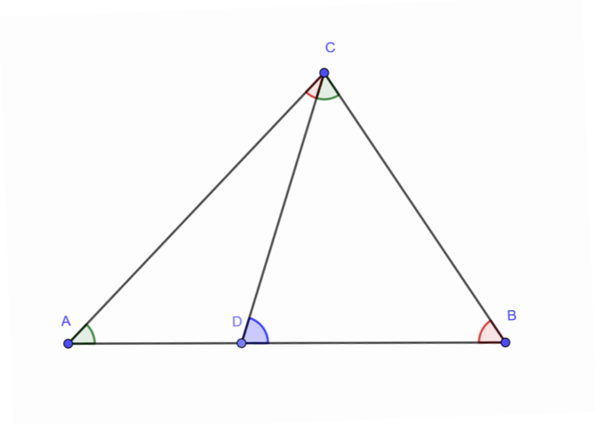
E: 14

Question 17 (INT):

If the diagonal of square A is 16 times larger than the perimeter of square B, how many times larger is the area of square A than the area of square B?

Question 18 (INT):

Suppose that, in the diagram below, the angles $\angle DAC$ and $\angle DCB$ (both highlighted green) are 45 degrees and that the angle $\angle CBD$ is twice as large as the angle $\angle ACD$ (both highlighted red). How many degrees is the blue angle $\angle BDC$?



Question 19 (INT):

Viera made less than 200 cookies and now wishes to distribute them into paper bags, such that each bag has exactly the same number of cookies. Sadly, this never seems to work out: If she wants to distribute them into 5 bags, there are four cookies left. The same thing happens if she wants to distribute them into 6 bags. If she wants to have 7 bags, there are three cookies left. How many cookies did Viera make in total?

Question 20 (INT):

If a right angled triangle has a side of length 15 and a side of length 12, what is its smallest possible area?

Question 21 (INT):

What integer has the property that if you square it and subtract 49 you get the same number as if you first subtract 49 and then square it?

Question 22 (INT):

Nicole's favourite ice cream store offers eight different flavours. Nicole wants to buy three scoops of ice cream which are not all of the same flavour. How many possibilities does she have to do this?
Remark: The order of the flavours doesn't matter.

Question 23 (MTF):

If some positive integers a, b and c satisfy $a^2 + b^2 = c^2$, it follows that...

A: $a + b > c$.

B: c is odd.

C: a and b are not equal.

D: c is not divisible by 7.

Question 24 (MTF):

There are 51 distinct positive integers on the blackboard, none exceeding 100. Which statements about this board must be true?

A: There are two consecutive numbers.

B: There are two numbers differing by 50.

C: There are two numbers summing to 100.

D: There are six numbers with the same last digit.

Question 25 (MTF):

The expression $n^2 + n + 41$, for any positive integer n, is always...

A: prime.

B: bigger than $(n + 1)^2$.

C: odd.

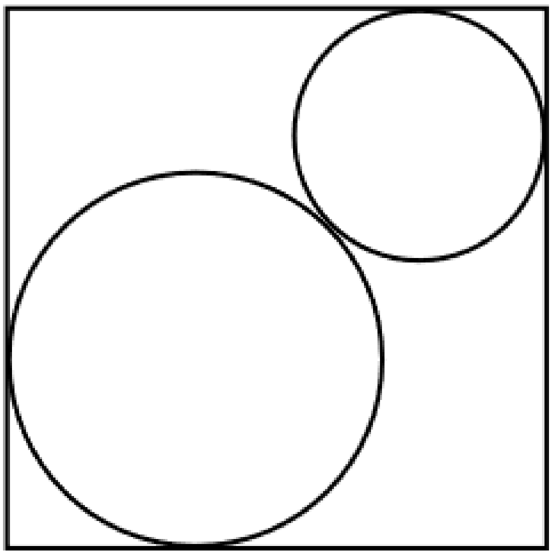
D: not a square.

Level 3

MC:	+16 for the correct answer,	–4 for a wrong answer,	0 for unanswered
T/F:	+4 for each correct answer,	–4 for each wrong answer,	0 for unanswered
NUM:	+16 for the correct answer,	0 for wrong or unanswered	

Question 26 (MC):

Two circles are inscribed in a square of side length 1, as shown in the picture below. What is the sum of the two radii?



A: $\frac{1}{2}$

B: $\frac{1}{\sqrt{2}}$

C: $\sqrt{2}-1$

D: $2-\sqrt{2}$

E: Different choices of circles will lead to different answers.

Question 27 (MC):

There are 100 coins in a line, all showing heads. Louis now turns every coin, then every second coin, then every third coin and so on up to every 100th coin. Assuming that the first coin was flipped every time, how many coins show heads at the end?

A: 50

B: 89

C: 90

D: 91

E: 98

Question 28 (MC):

All the numbers from 1 up to 10 are written on the blackboard. Viviane now repeatedly replaces two numbers by their (non-negative) difference until only one number is left. Which of the following could this number possibly be?

A: 0

B: 1

C: 4

D: 6

E: 11

Question 29 (MC):

For a given positive integer $n > 1$, we write down all its positive divisors in ascending order: $1 < d_1 < \ldots < d_k < n$. How many different n satisfy $d_k = 11 \cdot d_1$?

A: 0

B: 2

C: 3

D: 4

E: 5

Question 30 (INT):

Quirin has four sticks of length 12. He breaks exactly one of them into two sticks and arranges all five pieces to a right-angled triangle. What is the area of that triangle?

Question 31 (INT):

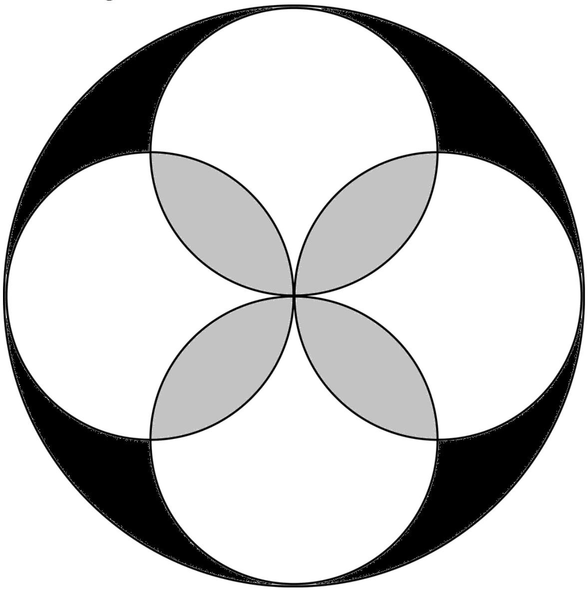
Let n be the smallest positive integer such that $10n$ is a square number and $6n$ is a cube number. What is n?

Question 32 (INT):

In how many different ways can a 2×11 rectangle be filled with 11 indistinguishable dominoes of size 1×2?

Question 33 (INT):

In the figure below, each of the grey regions has an area of 72. What is the total area of the black regions?



Question 34 (MTF):

There exists a three-digit number $\overline{abc}$, such that...

A: $\overline{abc}$ is divisible by c and the two-digit number $\overline{ab}$.

B: $\overline{abc}$ is divisible by b and the two-digit number $\overline{ac}$.

C: $a > c > 0$ and $\overline{abc} - \overline{cba}$ is a prime number.

D: $\overline{abc} + \overline{bca} + \overline{cab} = 2021$.

Question 35 (MTF):

The five friends Anaëlle, Bibin, Cyril, David and Ema are playing a social deduction game. Good guys always have to tell the truth and bad guys always have to lie. This is their conversation:
Anaëlle: "*Cyril and David are either both good or both bad.*"
Bibin: "*If Ema is good, Anaëlle is telling the truth!*"
Cyril: "*There is an even number of bad guys in the game.*"
David: "*At least one among Anaëlle, Bibin and Cyril has to be bad.*"
Ema: "*Anaëlle and Cyril are not both good.*"
Which of the following statements are true?

A: It is possible that Anaëlle and Cyril are both good.

B: David is certainly good.

C: Bibin is certainly bad.

D: It's possible that there is only one good player.