

**Duration:** 3 hours

Zürich, Lausanne, Lugano

**Difficulty:** The problems of each topic are ordered by difficulty.

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**Points:** Each problem is worth 7 points.

## Geometry

**G1)** Let  $\Gamma_1$  and  $\Gamma_2$  be two circles intersecting at the points  $X$  and  $Y$ . Let  $\ell$  be a line intersecting  $\Gamma_1$  at points  $A$  and  $C$ , and intersecting  $\Gamma_2$  at points  $B$  and  $D$ , such that  $A, B, C, D$  lie on  $\ell$  in this order. Let  $P$  be the intersection of  $XA$  and  $YB$ , and  $Q$  be the intersection of  $XD$  and  $YC$ .

Prove that  $PQ$  is parallel to  $\ell$ .

**G2)** Let  $A, B, C, D, E$  be five points lying on a circle in this order, such that  $AE$  is a diameter and  $BC > CD$ . Let  $F$  be the intersection of  $AD$  and  $BE$ . Let  $O$  be the circumcenter of the triangle  $BFD$ . Let the reflection of the line  $BD$  across  $BC$  and the reflection of the line  $BD$  across  $DC$  intersect at  $Y$ .

Prove that  $\angle OYC = 90^\circ$ .

## Combinatorics

**C1)** Let  $k \leq a \leq b$  be positive integers. Ruhi has a sequence of  $a + b$  letters that contains  $a$  copies of the letter  $M$ , and  $b$  copies of the letter  $O$ . She wants the sequence to have exactly  $k$  pairs of consecutive letters that form the word  $MO$ . In how many ways can she arrange the letters?

**C2)** Mark has 20 bookshelves of heights  $1, 1, 2, 2, \dots, 10, 10$ , where two bookshelves of the same height are indistinguishable. He wants to place them in a line in such a way that his cat can jump from one bookshelf to the next, starting on the leftmost bookshelf and ending on the rightmost bookshelf. His cat is lazy and has two conditions:

- Exactly one of the cat's nineteen jumps is from a bookshelf to a strictly taller bookshelf.
- Between two bookshelves of the same height, no strictly taller bookshelf can be placed.

In how many ways can Mark place his bookshelves satisfying both conditions?

## Number Theory

**N1)** Determine all triples  $(p, q, n)$  of positive integers where  $p$  and  $q$  are prime numbers, and

$$20p + 26q = n^2 + 48.$$

**N2)** Determine all positive integers  $n$  such that

$$n^3 \mid (n+1)(n+2)(n+3)(n+4)(n+5)(n+6).$$

Good luck!