y_{11} values $\qquad\qquad$ y_{12} values

1a)

$$\bar{y}_{1\cdot} = \frac{1}{6}(4.23 + 4.08 + 3.81 + 9.87 + 9.23 + 8.86)$$
$$= 6.68$$

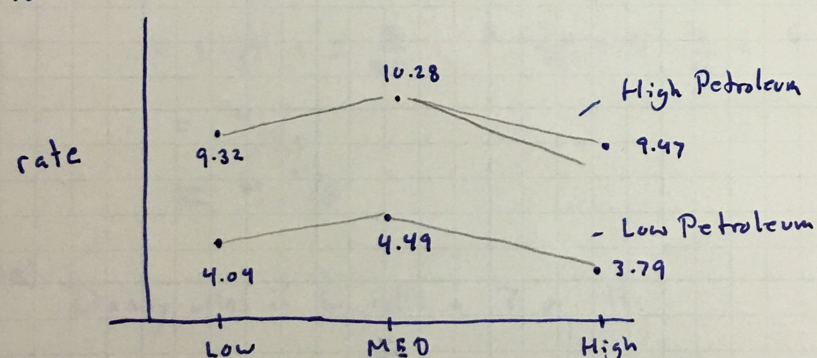
1b)

$$\bar{y}_{3\cdot} = \frac{1}{2}(3.79 + 9.47)$$
$$= 6.63$$

1c)

$$\bar{y}_{\cdot 2} = \frac{1}{3}(\bar{y}_{12} + \bar{y}_{22} + \bar{y}_{32})$$
$$= \frac{1}{3}(9.32 + 10.28 + 9.47)$$
$$= 9.69$$

1d)



1e) No - both lines are basically made of parallel segments. If there was a significant interaction, we would expect to see these profiles crossing, or at least having very different overall shapes.

1f) $a_1 = \bar{y}_{1\cdot} - \bar{y}_{\cdot\cdot} = 6.68 - 6.9 = -.22$

1g) $\hat{y}_{12} = a_1 + b_1 + \bar{y}_{\cdot\cdot}$ $\qquad$ $b_1 = \bar{y}_{\cdot 1} - \bar{y}_{\cdot\cdot} = 4.11 - 6.9$

$$= -2.79$$

$$= -.22 - 2.79 + 6.9$$
$$= 3.89$$

2a) $P(Y=1) = f_Y(1) = 6/21$

2b) Since $\sum_{\text{all } x} f_X(x) = 1$, we need $f_X(1)$ so that this holds.

$2/21 + 3/21 + 4/21 + 5/21 + 6/21 = 20/21$

So $f_X(1)$ must be $1/21$

2c) $P[Y=6] = f_Y(1) = 1/21$

2d) $EX = \sum_{\text{all } x} x \cdot f_X(x)$

$= 1 \cdot f_X(1) + 2 \cdot f_X(2) + 3 \cdot f_X(3) + 4 \cdot f_X(4) + 5 \cdot f_X(5) + 6 \cdot f_X(6)$

$= 1 \cdot \frac{1}{21} + 2 \cdot \frac{2}{21} + 3 \cdot \frac{3}{21} + 4 \cdot \frac{4}{21} + 5 \cdot \frac{5}{21} + 6 \cdot \frac{6}{21}$

$= 91/21$

$\simeq 4\ 1/3$

2e) Danny wins if he rolls a 7 or 11.

$P(X+Y=7) = f(1,6) + f(2,5) + f(3,4) + f(4,3) + f(5,2) + f(6,1)$

$= f_X(1) \cdot f_Y(6) + f_X(2) \cdot f_Y(5) + \ldots + f_X(6) \cdot f_Y(1)$

$= \left(\frac{1}{21}\right)\left(\frac{1}{21}\right) + \left(\frac{2}{21}\right)\left(\frac{2}{21}\right) + \left(\frac{3}{21}\right)\left(\frac{3}{21}\right) + \left(\frac{4}{21}\right)\left(\frac{4}{21}\right) + \left(\frac{5}{21}\right)\left(\frac{5}{21}\right) + \left(\frac{6}{21}\right)\left(\frac{6}{21}\right)$

$= \frac{91}{441}$

≈ 0.206

2 f) - We have 3 attempts

- The probability that we win ~~every~~ on any single attempt is $p = 0.206$
- We are asked for P(Denny wins twice)

<u>NOTE</u> If Danny wins all 3 times, he has <u>still</u> won twice.

Let D = # times danny wins in 3 tries.

D is a binomial random variable with $n=3$, $p = 0.206$

$$P(D=2) = \frac{3!}{(3-2)!\,2!}(0.206)^2(1-0.206)^1$$

$$= 3\,(0.042436)(.794)$$

$$\approx 0.101$$

$$P(D=3) = \frac{3!}{(3-3)!\,3!}(0.206)^3(1-0.206)^0$$

$$= 1\cdot(.206)^3(.794)^0$$

$$= (0.206)^3$$

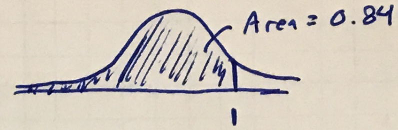
$$= 0.00874$$

So $P(\text{Danny wins twice}) = P(D=2) + P(D=3)$

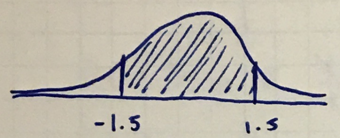
$$\approx 0.110$$

3 (a)

i) $P(Z \le 1) = 0.841$



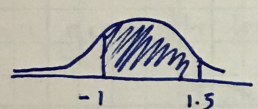
ii) $P(|Z| \le 1.5)$



$$\Rightarrow P(|Z| \le 1.5) = P(Z \le 1.5) - P(Z \le -1.5)$$
$$= 0.933 - 0.067$$
$$= 0.866$$

iii) $P(-1 \le Z \le 1.5) = P(Z \le 1.5) - P(Z \le -1)$

$$= 0.933 - 0.159$$
$$= 0.774$$

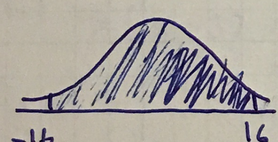


iv) $P(X > 13) = P\left(\frac{X-10}{\sqrt{3}} > \frac{13-10}{\sqrt{3}}\right)$

$$= P(Z > 3/\sqrt{3})$$
$$= 1 - P(Z \le \sqrt{3})$$
$$\approx 0.0426$$

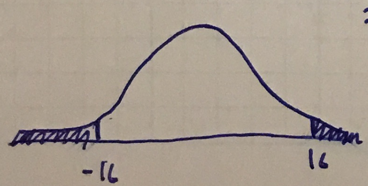
v) $P[|X| \le 16] = P(-16 \le X \le 16)$

$$= P\left(\frac{-16-10}{\sqrt{3}} \le \frac{X-10}{\sqrt{3}} \le \frac{16-10}{\sqrt{3}}\right)$$
$$= P\left(-2\sqrt{3} \le \frac{X-10}{\sqrt{3}} \le 2\sqrt{3}\right)$$
$$\approx P(-2\sqrt{3} \le Z \le 2\sqrt{3})$$
$$\approx 0.999$$



vi) $P(|X| > 16) = 1 - P(|X| \le 16)$

$$= 1 - 0.999$$
$$= 0.001$$



3 (b)

$$P(-a+10 < X < a+10) = .60$$

$$\Downarrow$$

$$P(-a < X-10 < a) = .6$$

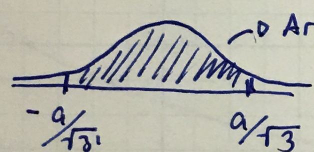
$$\Downarrow$$

$$P\left(-\frac{a}{\sqrt{3}} < \frac{X-10}{\sqrt{3}} < \frac{a}{\sqrt{3}}\right)$$

$$\Downarrow$$

$$P\left(-\frac{a}{\sqrt{3}} < Z < \frac{a}{\sqrt{3}}\right)$$

This gives the following picture



The tails must have an area of 0.4 combined. (total Area = 1)
So each tail must have an area of .2

Thus $P\left(Z \leq \frac{a}{\sqrt{3}}\right) = .8$

From the table, we get $\frac{a}{\sqrt{3}} \approx 0.842$

$$\Rightarrow \quad a = 0.842\sqrt{3}$$

4a)
$$\begin{aligned} P(X>3) &= 1 - P(X \leq 3) \\ &= 1 - F(3) \\ &= 1 - (1 - e^{-3\cdot 3}) \\ &= e^{-9} \end{aligned}$$

$$\begin{aligned} P(1 \leq X \leq 3) &= P(X \leq 3) - P(X \leq 1) \\ &= F(3) - F(1) \\ &= (1 - e^{-3(3)}) - (1 - e^{-3(1)}) \\ &= e^{-3} - e^{-9} \end{aligned}$$

c) $f(x) = F'(x)$

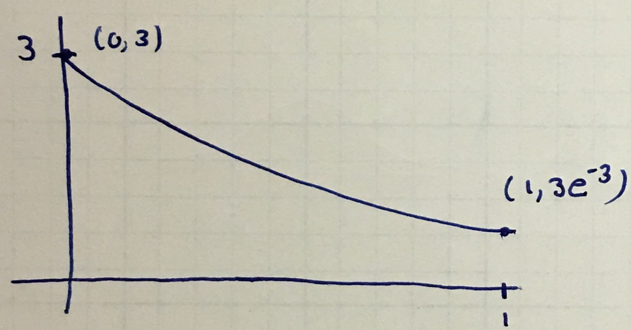
$= (1-e^{-3x})'$ for $x \geq 0$

$= 3e^{-3x}, \; x \geq 0$

Since $F'(x) = 0$ for $x < 0$, we can say

$$f(x) = \begin{cases} 0 & x < 0 \\ 3e^{-3x} & x \geq 0 \end{cases}$$

d) $f(0) = 3, \quad f(1) = 3e^{-3}$



5) a) $f_Y(3) = \sum_{\text{all } x} f(x, 3)$

$= f(1,3) + f(2,3)$

$= f_{Y|X}(3 \mid X=1) \cdot f_X(1) + f_{Y|X}(3 \mid X=2) \cdot f_X(2)$

$= (\frac{1}{3})(\frac{1}{2}) + (\frac{1}{3})(\frac{1}{2})$

$= \frac{1}{3}$

b) $f(1,3) = (\frac{1}{3})(\frac{1}{2}) = \frac{1}{6}$ (done above)

c) $f(2,5) = f_{Y|X}(5 \mid X=2) \cdot f_X(2)$

$= (\frac{1}{3})(\frac{1}{2})$

$= \frac{1}{6}$

d) $f_X(2 \mid Y=3) = \frac{f(2,3)}{f_Y(3)} = \frac{1/6}{1/3} = 3/6 = 1/2$

e) $P(X=2 \mid Y=1) = \frac{f(2,1)}{f_Y(1)} = \frac{0}{f_Y(1)} = 0$ (there is no way for $X=2$ and $Y=1$ simultaneously)