

# STAT 105 Exam I

## Reference Sheet

### Numeric Summaries

mean	$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$
population variance	$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$
population standard deviation	$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$
sample variance	$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$
sample standard deviation	$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$

### Linear Relationships

Form	$y \approx \beta_0 + \beta_1 x$
Fitted linear relationship	$\hat{y} = b_0 + b_1 x$
Least squares estimates	$b_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$ $b_1 = \frac{\sum_{i=1}^n x_i y_i - n \bar{x} \bar{y}}{\sum_{i=1}^n x_i^2 - n \bar{x}^2}$ $b_0 = \bar{y} - b_1 \bar{x}$
Residuals	$e_i = y_i - \hat{y}_i$
sample correlation coefficient	$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}}$ $r = \frac{\sum_{i=1}^n x_i y_i - n \bar{x} \bar{y}}{\sqrt{(\sum_{i=1}^n x_i^2 - n \bar{x}^2)(\sum_{i=1}^n y_i^2 - n \bar{y}^2)}}$
coefficient of determination	$R^2 = (r)^2$ $\frac{\sum_{i=1}^n (y_i - \bar{y})^2 - \sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$

### Multivariate Relationships

Form	$y \approx \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k$
Fitted relationship	$\hat{y} \approx b_0 + b_1 x_1 + b_2 x_2 + \dots + b_k x_k$
Residuals	$e_i = y_i - \hat{y}_i$
Sums of Squares	$SSTO = \sum_{i=1}^n (y_i - \bar{y})^2$ $SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2$ $SSR = SSTO - SSE = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2$
coefficient of determination	$R^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2 - \sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$ $R^2 = \frac{SSTO - SSE}{SSTO}$ $R^2 = \frac{SSR}{SSTO}$ $\frac{\sum_{i=1}^n (y_i - \bar{y})^2 - \sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$

### Functions

**Quantile Function**  $Q(p)$  For a dataset consisting of  $n$  values that are ordered so that  $x_1 \leq x_2 \leq \dots \leq x_n$  and value  $p$  where  $0 \leq p \leq 1$ , let  $i = \lfloor n \cdot p + 0.5 \rfloor$ . Then the quantile function at  $p$  is:

$$Q(p) = \begin{cases} x_i & \lfloor n \cdot p + 0.5 \rfloor = n \cdot p + 0.5 \\ x_i + (n \cdot p - i + 0.5)(x_{i+1} - x_i) & \lfloor n \cdot p + 0.5 \rfloor \neq n \cdot p + 0.5 \end{cases}$$