At the end of class I wrote the following

$$\begin{aligned} P(B \cap A^c \cap M^c) &= P(B \mid A^c \cap M^c) \cdot P(A^c \cap M^c) \\ &= P(B \mid M^c) \cdot P(M^c) \\ &= (0.1)(0.98) \\ &= 0.098 \end{aligned}$$

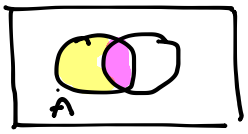
The highlighted part is incorrect.

This would be the correct way of working that step out

So M^c = the set of all outcomes in which the missile hits

We know that for any set, A, we can write

$$A = (A \cap B) \cup (A \cap B^c)$$



$A \cap B^c$ ∪ $A \cap B$

So that means that $M^c = (M^c \cap A) \cup (M^c \cap A^c)$

in the context of our problem, we would say that "$M^c \cap A$" is the set of all outcomes where the antimissile system worked and the missile hit the base.

But! If the antimissile system worked ("$x \in A$") then the missile would have been destroyed ("$x \notin M^c$").

So $A \cap M^c \neq \emptyset$ -

That means that

$$\begin{aligned} M^c &= (M^c \cap A) \cup (M^c \cap A^c) \\ &= \emptyset \cup (M^c \cap A^c) \\ &= M^c \cap A^c \end{aligned}$$

and that

$$\begin{aligned} P(M^c) &= P(M^c \cap A^c) && \text{(since } M^c = M^c \cap A^c\text{)} \\ &= P(M^c \mid A^c) \cdot P(A^c) && \text{(by the definition of conditional probability)} \\ &= P(M^c \mid A^c)[1 - P(A^c)] \end{aligned}$$

Now imagine: if we know that the anti missile system fails (meaning we know A^c) the the missile will hit the target 98% of the time.

So $P(M^c \mid A^c) = .98$ and $P(M^c) = (.98)[1 - P(A)]$

Thus: $P(B \cap M^c \cap A^c) = (0.1)(.98)(1 - P(A))$ and we are done.

Here is my error in context (graded out of 10)

$$P(B \cap A^c \cap M^c) = P(B \mid \underbrace{A^c \cap M^c}_{M^c}) \cdot P(\underbrace{A^c \cap M^c}_{M^c})$$

$$= P(B \mid M^c) \cdot P(M^c)$$

$$= (.10)(.98) \quad -2$$

$$= .098$$

$$P(B) = P(B \cap A) + P(B \cap A^c \cap M) + P(B \cap A^c \cap M^c)$$

$$= \# \quad + \quad \# \quad + \quad \#$$

So what went wrong?

I think that it comes down to this

M^c = the set of all possible outcomes where the missile is able to hit the base

While 0.98 is the probability the missile hits a target in a general sense.

If I had been more careful that what 0.98 meant in our problem wasn't "the probability of the missile hitting the target, but was instead "the probability of the missile hitting the target if we know the missile has made it through all the defenses" then I would have likely caught that $P(M^c \mid A^c) = 0.98$ instead

Thanks,

Ian