

Show **all** of your work on this assignment and answer each question fully in the given context.

*Please staple your assignment!*

1. **Chapter 5, Exercise 35 (page 330)**

2. **Chapter 5, Exercise 37 (page 331)**

3. Suppose that  $X$  and  $Y$  are two independent random variables with probability density functions given by:

$$f_X(x) = \begin{cases} 5e^{5x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_Y(y) = \begin{cases} 2e^{2y} & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

respectively.

Further, define random variable  $U$  as

$$U = \begin{cases} 1 & Y > X \\ 0 & \text{otherwise} \end{cases}$$

Meaning that if the observed value of the random variable  $Y$  is larger than the observed value of the random variable  $X$  then  $U = 1$  and if the observed value of the random variable  $X$  is larger than the observed value of the random variable  $Y$  then  $U = 0$ .

- (a) Sketch the pdf of  $X$  and  $Y$  on the same plot. Include the points when the input is 0, 5, and 10 for each function.
  - (b) Find the probability that  $X$  is greater than 3.
  - (c) Find the probability that  $Y$  is greater than 3.
  - (d) Provide the joint probability of  $(X, Y)$ .
  - (e) Find the probability that  $U = 1$ .
4. Suppose that  $Z_1, Z_2, \dots, Z_n$  are  $n$  independent standard normal random variables. It may be helpful to recall that  $\mathbb{E}(aZ_i + b) = a\mathbb{E}(Z_i) + b$  and that  $\text{Var}(aZ_i + b) = a^2\text{Var}(Z_i)$  for any constants  $a, b$  in addition to knowing that  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$  and  $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$ .
- (a) Find the expected value and variance of  $X$  where  $X = 3Z_1 + 5$
  - (b) Find the expected value and variance of  $Y$  where  $Y = Z_1 - Z_2$
  - (c) Find the expected value and variance of  $U$  where  $U = Z_1 - Z_1$
  - (d) Find the expected value and variance of  $W$  where  $W = \sum_{i=1}^n \frac{i}{n} (Z_i + \frac{i}{n})$ .