Show **all** of your work on this assignment and answer each question fully in the given context.

Please staple your assignment!

1. **Chapter 5, Exercise 35 (page 330)**
2. **Chapter 5, Exercise 37 (page 331)**
3. Suppose that X and Y are two independent random variables with probability density functions given by:

$$f_X(x) = \begin{cases} 5e^{5x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_Y(y) = \begin{cases} 2e^{2y} & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

respectively.

Further, define random variable U as

$$U = \begin{cases} 1 & Y > X \\ 0 & \text{otherwise} \end{cases}$$

Meaning that if the observed value of the random variable Y is larger than the observed value of the random variable X then $U = 1$ and if the observed value of the random variable X is larger than the observed value of the random variable Y then $U = 0$.

(a) Sketch the pdf of X and Y on the same plot. Include the points when the input is 0, 5, and 10 for each function.

(b) Find the probability that X is greater than 3.

(c) Find the probability that Y is greater than 3.

(d) Provide the joint probability of (X, Y).

(e) Find the probability that $U = 1$.

4. Suppose that $Z_1, Z_2, \ldots, Z_n$ are n independent standard normal random variables. It may be helpful to recall that $\mathbb{E}(aZ_i + b) = a\mathbb{E}(Z_i) + b$ and that $\text{Var}(aZ_i + b) = a^2 + \text{Var}(Z_i)$ for any constants a, b in addition to knowing that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

(a) Find the expected value of variance of X where $X = 3Z_1 + 5$

(b) Find the expected value of variance of Y where $Y = Z_1 - Z_2$

(c) Find the expected value of variance of U where $U = Z_1 - Z_1$

(d) Find the expected value of variance of W where $W = \sum_{i=1}^{n} \frac{i}{n}\left(Z_i + \frac{i}{n}\right)$.