Show **all** of your work on this assignment and answer each question fully in the given context.

Please staple your assignment!

1. **Chapter 5, Exercise 35 (page 330)**:

   Hints:

   (a) if a and b are two constants, $x^a \cdot x^b = x^{(a+b)}$.
   (b) if a and b are two constants, $a^x \cdot b^x = (a \cdot b)^x$.
   (c) if a and b are two constants, $a^x \cdot b^{-x} = (a/b)^x$.
   (d) if you are taking a sum that depends on x then you can factor out terms that don't depend on x. For example,

   $$\begin{aligned}\sum_{x=0}^{\infty} \frac{x!}{(x-y)!y!}(.8)^y(.2)^{x-y}\frac{e^{-3}3^x}{x!} &= \sum_{x=0}^{\infty} \frac{x!}{1}\frac{1}{(x-y)!}\frac{1}{y!}(.8)^y(.2)^x(.2)^{-y}\frac{e^{-3}}{1}\frac{3^x}{1}\frac{1}{x!} \\ &= \frac{1}{y!}(.8)^y(.2)^{-y}\frac{e^{-3}}{1}\sum_{x=0}^{\infty}\frac{x!}{1}\frac{1}{(x-y)!}(.2)^x\frac{3^x}{1}\frac{1}{x!}\end{aligned}$$

   since each term that was factored out in the second line had nothing to do with x.
   (e) For any value c, $\sum_{x=0}^{\infty} \frac{e^{-c}c^x}{x!} = 1$ and $\sum_{x=0}^{\infty} \frac{c^x}{x!} = e^c$ (notice that the function $f_X(x)$ used in this problem is a probability function and thus $\sum_{x=0}^{\infty} f_X(x) = 1$).

2. **Chapter 5, Exercise 37 (page 331)**

   Hint: the limits over which you integrate in this problem matter - notice that if $y < x$ then $f(x, y) = 0$.

3. (*This problem is now a bonus problem worth 15 points*)

   Suppose that X and Y are two independent random variables with probability density functions given by:

   $$f_X(x) = \begin{cases} 5e^{-5x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

   and

   $$f_Y(y) = \begin{cases} 2e^{-2y} & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

   respectively.

   Further, define random variable U as

   $$U = \begin{cases} 1 & Y > X \\ 0 & \text{otherwise} \end{cases}$$

   Meaning that if the observed value of the random variable Y is larger than the observed value of the random variable X then $U = 1$ and if the observed value of the random variable X is larger than the observed value of the random variable Y then $U = 0$.

(a) Sketch the pdf of X and Y on the same plot. Include the points when the input is 0, 5, and 10 for each function.

(b) Find the probability that X is greater than 3.

(c) Find the probability that Y is greater than 3.

(d) Provide the joint probability of (X, Y).

(e) Find the probability that $U = 1$.

4. Suppose that $Z_1, Z_2, \ldots, Z_n$ are n independent standard normal random variables. It may be helpful to recall that $\mathbb{E}(aZ_i + b) = a\mathbb{E}(Z_i) + b$ and that $\text{Var}(aZ_i + b) = a^2\text{Var}(Z_i)$ for any constants a, b in addition to knowing that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

(a) Find the expected value and variance of X where $X = 3Z_1 + 5$

(b) Find the expected value and variance of Y where $Y = Z_1 - Z_2$

(c) Find the expected value and variance of U where $U = Z_1 - Z_1$

(d) Find the expected value and variance of W where $W = \sum_{i=1}^{n} \frac{i}{n}\left(Z_i + \frac{i}{n}\right)$.