STAT 105 Exam I
Reference Sheet

Numeric Summaries

mean	$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
population variance	$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$
population standard deviation	$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$
sample variance	$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$
sample standard deviation	$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$

Quantile Function $Q(p)$ For a dataset consisting of n values that are ordered so that $x_1 \leq x_2 \leq \ldots \leq x_n$ and value p where $0 \leq p \leq 1$, let $i = \lfloor n \cdot p + 0.5 \rfloor$. Then the quantile function at p is:

$$Q(p) = \begin{cases} x_i & \lfloor n \cdot p + 0.5 \rfloor = n \cdot p + 0.5 \\ x_i + (n \cdot p - i + 0.5)(x_{i+1} - x_i) & \lfloor n \cdot p + 0.5 \rfloor \neq n \cdot p + 0.5 \end{cases}$$

Linear Relationships

Form	$y \approx \beta_0 + \beta_1 x$
Fitted linear relationship	$\hat{y} = b_0 + b_1 x$
Least squares estimates	$b_1 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$
	$b_1 = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}$
	$b_0 = \bar{y} - b_1\bar{x}$
Residuals	$e_i = y_i - \hat{y}_i$
sample correlation coeffecient	$r = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2 \sum_{i=1}^{n}(x_i-\bar{x})^2}}$
	$r = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sqrt{(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2)(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2)}}$
coeffecient of determination	$R^2 = (r)^2$
	$\frac{\sum_{i=1}^{n}(y_i-\bar{y})^2 - \sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{\sum_{i=1}^{n}(y_i-\bar{y})^2}$

Factorial Analysis (Two Factors)

Assuming

- Factor A with levels $1, 2, ..., I$,
- Factor B with levels $1, 2, ..., J$,
- n is the total number of observations,
- n_{ij} is the total number of observations with Factor A at level i and Factor B at level j,
- $n_{i\cdot}$ is the total number of observations with Factor A at level i,
- $n_{\cdot j}$ is the total number of observations with Factor B at level j.
- y_{ijk} is the kth observation where Factor A is at level i and Factor B is at level j.

$y_{\cdot\cdot} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} y_{ijk}$	$\bar{y}_{\cdot\cdot} = \frac{1}{n}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} y_{ijk}$
$\bar{y}_{i\cdot} = \frac{1}{n_{i\cdot}}\sum_{j=1}^{J}\sum_{k=1}^{K} y_{ijk}$	$\bar{y}_{\cdot j} = \frac{1}{n_{\cdot j}}\sum_{i=1}^{I}\sum_{k=1}^{K} y_{ijk}$
Main effect of Factor A at level i	$a_i = \bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot}$
Main effect of Factor B at level j	$b_j = \bar{y}_{\cdot j} - \bar{y}_{\cdot\cdot}$
Fitted Value	$\hat{y}_{ij} = a_i + b_j + \bar{y}_{\cdot\cdot}$

Discrete Random Variables

Probability function	$P[X = x] = f_X(x)$
Cumulative probability function	$P[X \leq x] = F_X(x)$
Expected Value	$\mu = E(X) = \sum_x x f_X(x)$
Variance	$\sigma^2 = Var(X) = \sum_x (x-\mu)^2 f_X(x)$
Standard Deviation	$\sigma = \sqrt{Var(X)}$

Joint Distributions and Related Distributions

Joint Probability Function	$P[X = x, Y = y] = f(x, y)$
Marginal Probability Function	$P[X = x] = f_X(x) = \sum_{\text{all } y} f(x, y)$ $P[Y = y] = f_Y(y) = \sum_{\text{all } x} f(x, y)$
Conditional Probability Function	$P[X = x \mid Y = y] = \frac{f(x,y)}{f_Y(y)}$ $P[Y = y \mid X = x] = \frac{f(x,y)}{f_X(x)}$

Geometric Random Variables

X is the trial count upon which the first successful outcome is observed performing independent trials with probability of success p.

Possible Values	$x = 1, 2, 3, \ldots$
Probability function	$P[X = x] = f_X(x) = p(1-p)^{x-1}$
Expected Value	$\mu = E(X) = \frac{1}{p}$
Variance	$\sigma^2 = Var(X) = \frac{1-p}{p^2}$

Binomial Random Variables

X is the number of successful outcomes observed in n independent trials with probability of success p.

Possible Values	$x = 0, 1, 2, \ldots, n$
Probability function	$P[X = x] = f_X(x) = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x}$
Expected Value	$\mu = E(X) = np$
Variance	$\sigma^2 = Var(X) = np(1-p)$

Continuous Random Variables

Probability density function	$P[a \le X \le b] = \int_a^b f_X(x)dx$
Cumulative probability function	$P[X \le x] = F_X(x) = \int_{-\infty}^x f_X(t)dt$
Expected Value	$\mu = E(X) = \int_{-\infty}^{\infty} x f_X(x)dx$
Variance	$\sigma^2 = Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f_X(x)dx$
Standard Deviation	$\sigma = \sqrt{Var(X)}$

Normal Random Variables

Let X be a normal random variable with mean μ and variance σ^2.

Probability density function	$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
Expected Value	$E(X) = \mu$
Variance	$Var(X) = \sigma^2$

Standard Normal Random Variables (Z)

A normal random variable with mean 0 and variance σ^2.
If X is normal(μ,σ^2) then $P[a \le X \le b] = P\left[\frac{a-\mu}{\sigma} \le Z \le \frac{b-\mu}{\sigma}\right]$

Probability density function	$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

Functions of random variables

For $X_1, X_2, \ldots, X_n$ independent random variables and $a_0, a_1, a_2, \ldots, a_n$ constants if $W = a_0 + a_1 X_1 + \ldots + a_n X_n$:

- $E(W) = a_0 + a_1 E(X_1) + a_2 E(X_2) + \ldots + a_n E(X_n)$
- $Var(W) = a_1^2 Var(X_1) + a_2^2 Var(X_2) + \ldots + a_n^2 Var(X_n)$

Confidence Intervals and Hypothesis Tests

Confidence Intervals $n \ge 25$

$(1-\alpha) \cdot 100\%$ Confidence interval for population mean	$\bar{x} \pm z_{1-\alpha/2}\sqrt{\frac{\sigma^2}{n}}$
$(1-\alpha) \cdot 100\%$ Confidence lower bound	$\bar{x} - z_{1-\alpha}\sqrt{\frac{\sigma^2}{n}}$
$(1-\alpha) \cdot 100\%$ Confidence upper bound	$\bar{x} + z_{1-\alpha}\sqrt{\frac{\sigma^2}{n}}$

Confidence Intervals $n < 25$

$(1-\alpha) \cdot 100\%$ Confidence interval for population mean	$\bar{x} \pm t_{1-\alpha/2,n-1}\sqrt{\frac{\sigma^2}{n}}$
$(1-\alpha) \cdot 100\%$ Confidence lower bound	$\bar{x} - t_{1-\alpha,n-1}\sqrt{\frac{\sigma^2}{n}}$
$(1-\alpha) \cdot 100\%$ Confidence upper bound	$\bar{x} + t_{1-\alpha,n-1}\sqrt{\frac{\sigma^2}{n}}$

Test statistics in hypothesis tests for population mean

$n \ge 25$	$\frac{\bar{x}-\mu}{\sqrt{\sigma^2/n}}$	$N(0,1)$
$n < 25$	$\frac{\bar{x}-\mu}{\sqrt{\sigma^2/n}}$	t with $\nu = n-1$ degrees of freedom

Standard Normal Probabilities

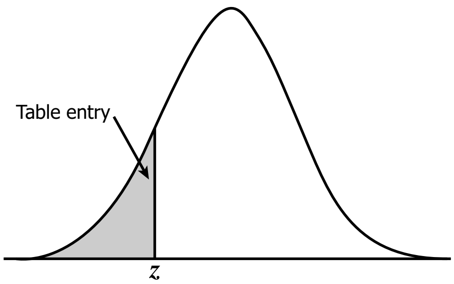

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
–3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
–3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
–3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
–3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
–3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
–2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
–2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
–2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
–2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
–2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
–2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
–2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
–2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
–2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
–2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
–1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
–1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
–1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
–1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
–1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
–1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
–1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
–1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
–1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
–1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
–0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
–0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
–0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
–0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
–0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
–0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
–0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
–0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
–0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
–0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

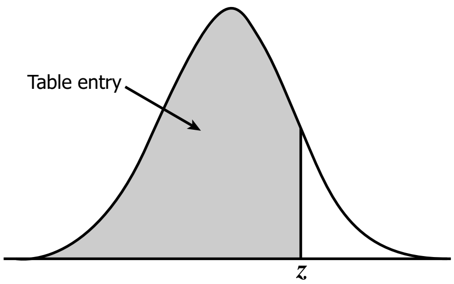

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998