Announcements

- HW2 is up on course page – due this coming Tuesday
- HW1 is in TA's hands

STAT 430: Lecture 6

Problem-Pa-Looza

You've Got Questons

I've Got Answers

Course page: imouzon.github.io/stat430

Wrapping Up Chapter 1

Pivoting to Chapter 2

Recap

Phase I

Lecture 1 - 2

- Sets
  - Terminology and notation (element, contains, subset, ...)
  - Basic properties and operations (defining with rules, compliments, intersections,...)
  - Proofs to show equality
- Functions
  - Terminology and notation (domains, ranges, ...)
  - Functions where the elements of the domain are sets
- Measurements
  - Special set functions
  - Sets are measureable or not measurable

$$(A \cup B)^c = A^c \cap B^c$$

and

$$(A \cap B)^c = A^c \cup B^c$$

Recap

Phase I

Phase II

Lecture 3 - 5

- Probability
  - A special measure where the domain, $\mathcal{F}$ has events of a sample space ($\Omega)
  - $\mathcal{F}$: a set of sets
    1. contains empty set (\empty $\in \mathcal{F}$)
    2. if $A \in \mathcal{F}$ then $A^c \in \mathcal{F}$
    3. if $A_1, A_2, \ldots \in \mathcal{F}$ then $A_1 \cup A_2 \cup \ldots \in \mathcal{F}$
  - P: a special measurement
    1. $P(\emptyset) = 0, P(\Omega) = 1$
    2. if $A_1, A_2, \ldots \in \mathcal{F}$ are pairwise disjoint then $P(A_1 \cup A_2 \cup) = P(A_1) + P(A_2) + \ldots$
- Using Probability
  - Defining probabilities given a scenario
  - Conditional probabilities
  - Independence
  - Bayes Rule

domain

function

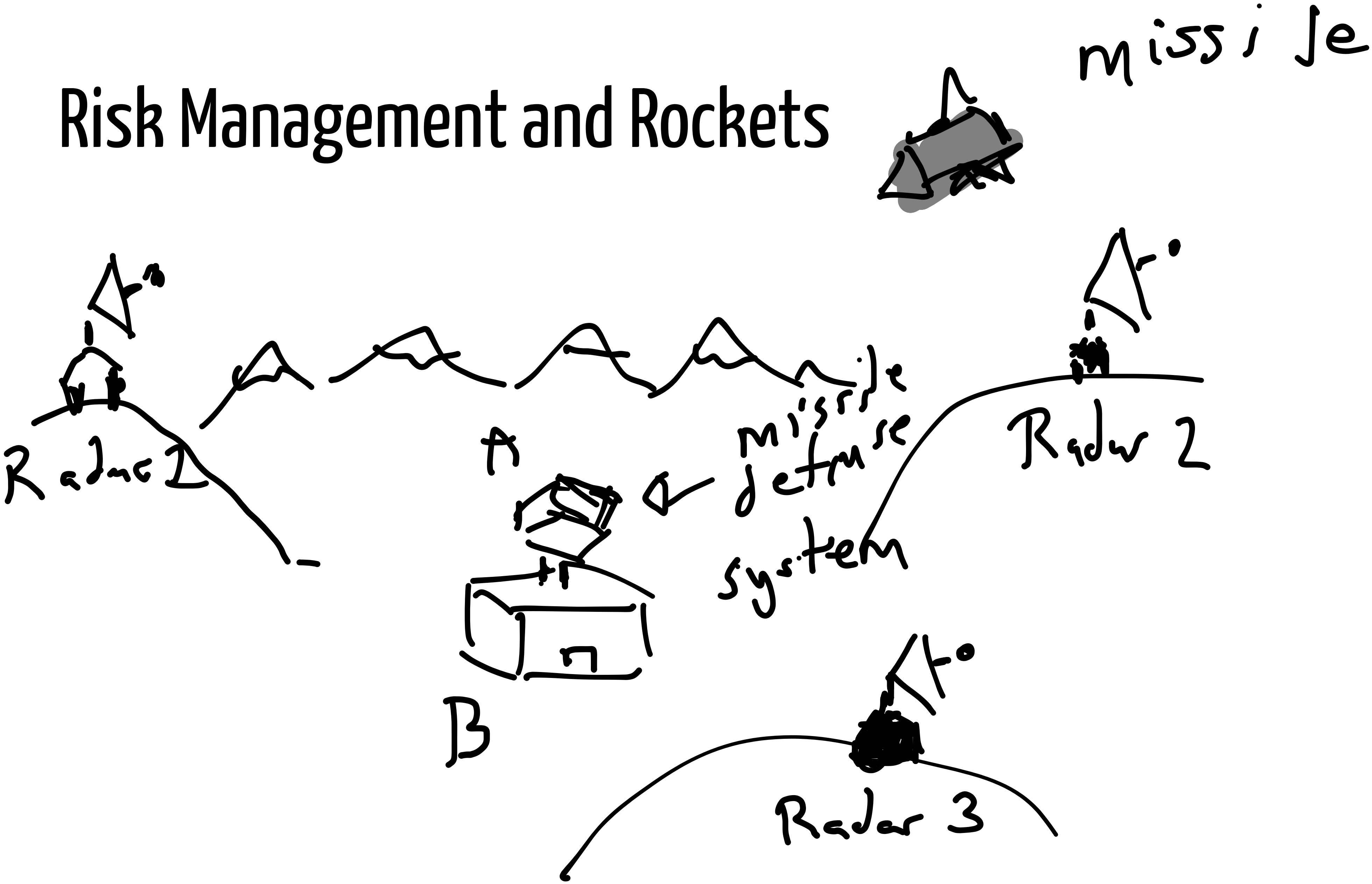
$P: \mathcal{F} \to [0,1]$

$\hookrightarrow$ sets made up of possible outcomes (i.e, events)

Problem-Pa-Looza

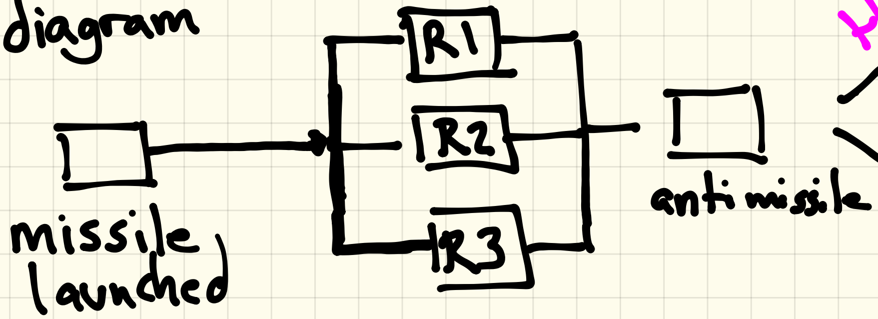
Lots and lots of examples

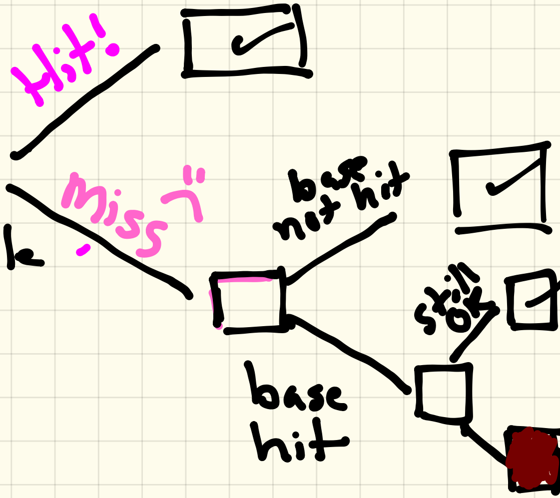
Risk Management and Rockets



- Radars operate independently of each other
- Historical experience tells us that a individual radar has a 80% chance of detecting a rocket
- The anti missile system has a battle tested accuracy of 50% unaided by radar location information
- with each radar that detects an incoming missile, the chance of missing the missile goes down by 50%
- The base has a 10% chance of staying battlefield operational even if directly hit by a missile
- The missile, undestroyed, will hit its target 98% of the time.

diagram





Radar Detection

3 radars, each either detecting or not

R1	R2	R3	# detecting
✓	✓	✓	3
✓	✓	✗	2
✓	✗	✓	2
✗	✓	✓	2
✓	✗	✗	1
✗	✓	✗	1
✗	✗	✓	1
✗	✗	✗	0

Notice

this is just $2 \cdot 2 \cdot 2$ possible outcomes

<u>consider:</u> R_i = the outcomes i^{th} radar detects

R_i^c = the outcomes where i^{th} radar does not

$$P(3 \text{ radars detect}) = P(R_1 \cap R_2 \cap R_3) = P(R_1) \cdot P(R_2) \cdot P(R_3)$$

by indep

$$= (0.8)(0.8)(0.8)$$

$$= (0.8)^3$$

$$P(2 \text{ radars}) = \ldots = P(R_1) \cdot P(R_2) \cdot P(R_3^c) +$$
$$P(R_1) \cdot P(R_2^c) \cdot P(R_3) +$$
$$P(R_1^c) \cdot P(R_2) \cdot P(R_3)$$

$$= (0.8)(0.8)(0.2) +$$
$$(0.8)(0.2)(0.8) +$$
$$(0.2)(0.8)(0.8)$$
$$= 3 \cdot (0.8)^2 (0.2)^1$$

$$P(1 \text{ radar}) = 3 \cdot (0.8)^1 (0.2)^2$$

$$P(0 \text{ radar}) = (0.2)^3$$

A = all events where anti-missile success

$$P(A) = ?$$

$$P(A \mid \#R=3) = 0.9375$$

$$P(A \mid \#R=2) = 0.875$$

$$P(A \mid \#R=1) = 0.75$$

$$P(A \mid \#R=0) = 0.5$$

$$\Longrightarrow$$

$$R(A \cap \#R=3) = P(A \mid \#R=3) \cdot P(\#R=3)$$

$$= (0.9375)(0.8)^3$$

$$P(A \cap \#R=2) = P(A \mid \#R=2) \cdot P(\#R=2)$$

$$= (0.875)(3)(.8)^2(.2)$$

$$P(A \cap \#R=1) = (.75) \cdot 3 \cdot (.8)^1 (.2)^2$$

$$P(A \cap \#R=0) = (0.5)(0.2)^3$$

$$P(A) = P\left[(A \cap \#R=3) \cup (A \cap \#R=2) \cup (A \cap \#R=1) \cup (A \cap \#R=0)\right]$$

$$= P(A \cap \#R=3) + \dots + P(A \cap \#R=0)$$

$$P(A) = (0.9375)(.8)^3 + 3(.875)(.8)^2(.2) + 3(0.75)(.8)^1(.2)^2 + (.5)(.2)^3$$

B = base survives

$$B = (B \cap A) \cup (B \cap A^c)$$

$$P(B \cap A) = P(A)$$

$$P(B \cap A^c) = P(B \mid A^c) P(A^c)$$

Now the only mystery left is the prob.s associated with missing the missile. (M = missile misses)

$$B \cap A^c = (B \cap A^c \cap M) \cup (B \cap A^c \cap M^c)$$

i) $$P(B \cap A^c \cap M) = P(\underbrace{B \cap M} \cap A^c)$$

$$= P(B \cap M \mid A^c) \cdot P(A^c)$$

$$= P(M \mid A^c) \cdot P(A^c)$$

$$= P(M) \cdot P(A^c)$$

$$= (0.02) \cdot (1 - P(A))$$

$$P(B \cap \underbrace{A^c \cap M^c}) = P(B \mid \underbrace{A^c \cap M^c}_{M^c}) \cdot P(\underbrace{A^c \cap M^c}_{M^c})$$

$$= P(B \mid M^c) \cdot P(M^c)$$

$$= (.10)(\cancel{.98}) \qquad -2$$

$$= \cancel{.098}$$

$$P(B) = P(B \cap A) + P(B \cap A^c \cap M) + P(B \cap A^c \cap M^c)$$

$$= \# + \# + \#$$