STAT 105 Exam II
Reference Sheet

Factorial Analysis (Two Factors)

Assuming

- Factor A with levels $1, 2, ..., I$,
- Factor B with levels $1, 2, ..., J$,
- n is the total number of observations,
- n_{ij} is the total number of observations with Factor A at level i and Factor B at level j,
- $n_{i\cdot}$ is the total number of observations with Factor A at level i,
- $n_{\cdot j}$ is the total number of observations with Factor B at level j.
- y_{ijk} is the kth observation where Factor A is at level i and Factor B is at level j.

$$y_{ij\cdot} = \sum_{k=1}^{n_{ij}} y_{ijk} \qquad \bar{y}_{ij} = \frac{1}{n_{ij}} \sum_{k=1}^{n_{ij}} y_{ijk}$$

$$\bar{y}_{i\cdot} = \frac{1}{J} \sum_{j=1}^{J} \bar{y}_{ij} \qquad \bar{y}_{\cdot j} = \frac{1}{I} \sum_{i=1}^{I} \bar{y}_{ij}$$

$$\bar{y}_{\cdot\cdot} = \frac{1}{I} \sum_{i=1}^{I} \bar{y}_{i\cdot} = \frac{1}{J} \sum_{j=1}^{J} \bar{y}_{\cdot j}$$

Main effect of Factor A at level i	$a_i = \bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot}$
Main effect of Factor B at level j	$b_j = \bar{y}_{\cdot j} - \bar{y}_{\cdot\cdot}$
Interaction of Factor B at level j and Factor A at level i	$ab_{ij} = \bar{y}_{ij} - a_i - b_j + \bar{y}_{\cdot\cdot}$
Fitted Value (no interactions)	$\hat{y}_{ij} = a_i + b_j + \bar{y}_{\cdot\cdot}$
Fitted Value (including interactions)	$\hat{y}_{ij} = a_i + b_j + ab_{ij} + \bar{y}_{\cdot\cdot}$

Basic Probability Rules

Probability A given B	$P[A\|B] = \frac{P[A,B]}{P[B]}$
Probability A and B	$P[A, B] = P[A\|B]P[B] = P[B\|A]P[A]$
Probability A or B	$P[A \text{ or } B] = P[A] + P[B] - P[A, B]$

Discrete Random Variables

Probability function	$P[X = x] = f_X(x)$
Cumulative probability function	$P[X \leq x] = F_X(x)$
Expected Value	$\mu = E(X) = \sum_x x f_X(x)$
Variance	$\sigma^2 = Var(X) = \sum_x (x - \mu)^2 f_X(x)$
Standard Deviation	$\sigma = \sqrt{Var(X)}$

Geometric Random Variables

X is the trial count upon which the first successful outcome is observed performing independent trials with probability of success p.

Possible Values	$x = 1, 2, 3, \ldots$
Probability function	$P[X = x] = f_X(x) = p^x(1 - p)^{x-1}$
Expected Value	$\mu = E(X) = \frac{1}{p}$
Variance	$\sigma^2 = Var(X) = \frac{1-p}{p^2}$

Binomial Random Variables

X is the number of successful outcomes observed in n independent trials with probability of success p.

Possible Values	$x = 0, 1, 2, \ldots, n$
Probability function	$P[X = x] = f_X(x) = \frac{n!}{(n-x)!x!} p^x(1 - p)^{n-x}$
Expected Value	$\mu = E(X) = np$
Variance	$\sigma^2 = Var(X) = np(1 - p)$

Poisson Random Variables

X is the number of times a rare event occurs over a predetermined interval (an area, an amount of time, etc.) where the number of events we expect is λ.

Possible Values	$x = 0, 1, 2, 3, \ldots$
Probability function	$P[X = x] = f_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}$
Expected Value	$E(X) = \lambda$
Variance	$Var(X) = \lambda$

Continuous Random Variables

Probability density function	$P[a \leq X \leq b] = \int_a^b f_X(x)dx$
Cumulative probability function	$P[X \leq x] = F_X(x) = \int_{-\infty}^x f_X(t)dt$
Expected Value	$\mu = E(X) = \int_{-\infty}^{\infty} x f_X(x)dx$
Variance	$\sigma^2 = Var(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f_X(x)dx$
Standard Deviation	$\sigma = \sqrt{Var(X)}$