Announcements

Homework 1 will be posted today after (depending on how much I'm able to cover.

STAT 430: Lecture 3

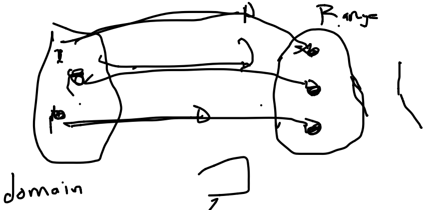
Using Probability

Reviewing and Reframing

Course page: imouzon.github.io/stat430

A Quick Rundown

Measurement, Probability, Kolmogorov



Quick Rundown

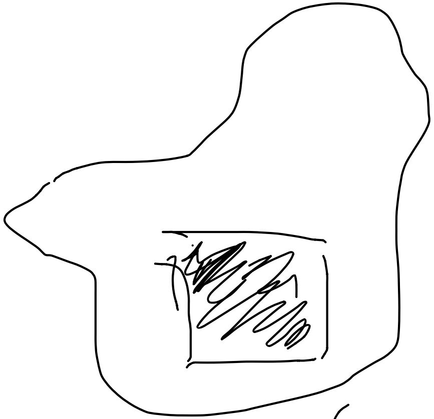
Sets and Functions

- We talked about sets, the common notation for them, and how to go about proving that sets were eqaul
- We revisited the definition of functions: that they are mappings from one set to another set that obey special ~~rules~~

Measurements

We did an illustration of what it means to measure something in the mathematical sense:

- We described a tool that can measure simple things exactly (the SVMT)
- We then discussed how measuring more complicated things with the tool is possible *if we look at the limits of over-measurements an under-measurements*
- If the largest under-measurement and the smallest over-measurement we can find agree, we call the more complicated object *measurable* by our tool.
- The measure of the complicated object is the value at which the over- and under-measurements met



gnarly field

V_L = volume from under measuring

V_U = volume from over measuring

If our $V_L \to V$ (some value) and $V_U \to V$ (some value) then the non-regular object no measure V

Recap

Sets and Functions

Measurements

Quick Rundown (cont)

Measurements

Notationally, we say that

M_*

- ~~μ~~ μ is our tool for directly measuring simple objects
- $\mathbf{A}$ is a complicated object that is not clearly measurable by μ
- M_* is the smallest over-measurement of a complicated object
- ~~m^{*}~~ m^* is the largest under-measurement of a complicated object
- if $\cancel{m^{*}} = M_{*}$ then we say that (i) $\mathbf{A}$ is μ-measurable and (ii) $\mu(\mathbf{A}) = m^{*} = M_{*}$

Quick Rundown (cont)

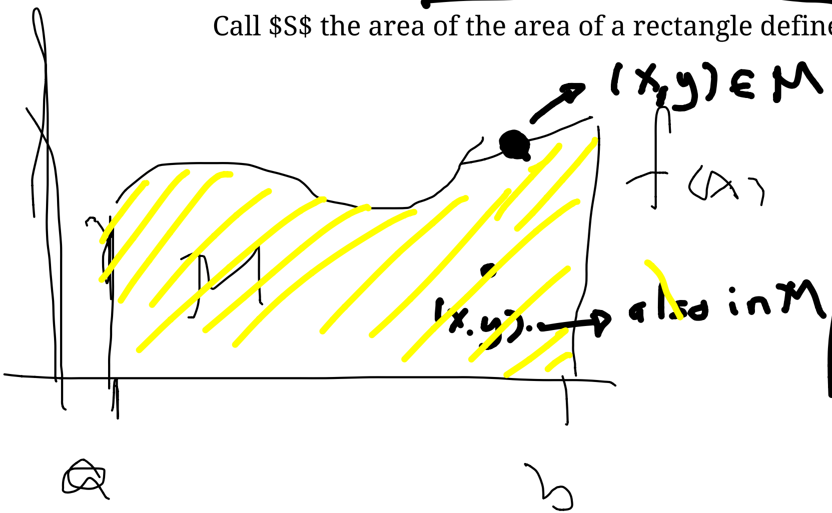
Example: Integration with measurements

Let's consider the Jordan Measure

$f(x)$ be a bounded, nonnegative function on the interval $[a,b]$

Consider the set $M = \{(x,y) : x \in [a,b], y \in [0, f(x)]\}$.

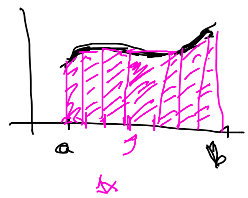
Call S the area of the area of a rectangle defined by $[0,S] \times [0,max(f(x))]$



So if the set M is measurable by the jordan measure (i.e. rectangles)

$f: [a, b] \to \mathbb{R}$

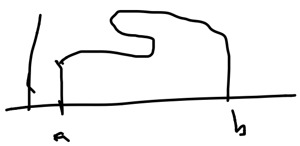
When f is bounded on its Range

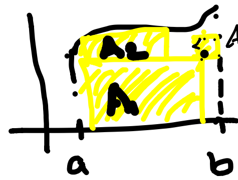


Riemann ("normal")

as $\Delta x \to 0$ the area of the rectangles may converge, and if does, we say that $f(x)$ is integrable

Ex





A_3 under covering $\to$ has area $A_1 + A_2 + A_3$

So if the set M is measurable then the function is integrable

more importantly All sets that are reimann integrable are Jordan measurable.

Ω

Quick Rundown (cont)

$\mathcal{F}$

Probability

Probabilites are just special types of measurements. For a specific sample space Ω we define ~~$\mathcal{F}$~~ to be all the events (or subsets of Ω) that are *measurable* by P. In order to ensure that this system is well-defined, the measure P must follow these rules:

Properties of $\mathcal{F}$

$\mathcal{F}$ is a set whose elements are subsets of Ω

1. $\emptyset \in \mathcal{F}$
2. for any subset A of Ω if $A \in \mathcal{F}$ then $A^c \in \mathcal{F}$
3. if $A_1, A_2, \ldots \in \mathcal{F}$ then $A_1 \cup A_2 \cup \ldots \in \mathcal{F}$.

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$\mathcal{F}$ is closed under countable unions of pairwise disjoint sets

1. $\mathcal{F}$ contains the empty set
2. $\mathcal{F}$ is closed under complement

Properties on P

~~$P: \mathcal{F} \rightarrow [0,1]$~~ is a function such that:

$P: \mathcal{F} \to [0,1]$

1. $P(\emptyset) = 0$ and $P(\Omega) = 1$
2. if $A_1, A_2, ...$ are pairwise disjoint sets in $\mathcal{F}$ then $P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots$

1. $P(\Omega) = 1, P(\emptyset) = 0$
2. If $A_1, A_2, \ldots$ are pairwise disjoint then $P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$

If we let $\mathcal{F}$ = the set containing all possible subsets of Ω and choose P so that for any $A \in \mathcal{F}$, $P(A) = \dfrac{|A|}{|\Omega|}$ then we have a valid probability system.

In this case, we call P a simple probability

Ex

$$|\Omega| = \underset{\substack{\#\text{ of}\\ \text{possible}\\ \text{coin flips}}}{\underline{2}} \cdot \underline{6} = 12$$

P({(H, ⚁), (H, ⚂)}) = $\dfrac{2}{12}$

Sample spaces

Probability

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Continuing Probability

Assigning Measures

Samples spaces and Probability Measurements

Samples spaces and assigning probabilities

Scenario

A person flips a fair coin and at the same time rolls a fair die

"fair" = all outcomes have the same chance of occuring.

Let (H, ⚂) be the outcome where the coin flip was a "heads" and the die roll was a 3

The sample space:

Ω = {(H, ⚀), (H, ⚁), (H, ⚂), ..., (T, ⚃), (T, ⚄), (T, ⚅)}

An Event

ω ⊆ Ω so {(H, ⚀), (H, ⚃)} is an event.

Assigning Measures

Coin toss/die roll

Random variables

Samples spaces and assigning probabilities

Scenario

A research team roll a fair red die and a fair blue die. They record the number of dots facing up on the red die as X and the number of die facing up on the blue die as Y. Further, they define the total number of dots facing up as Z

Describe the probability system created for X, Y, and Z

Continuing Probability

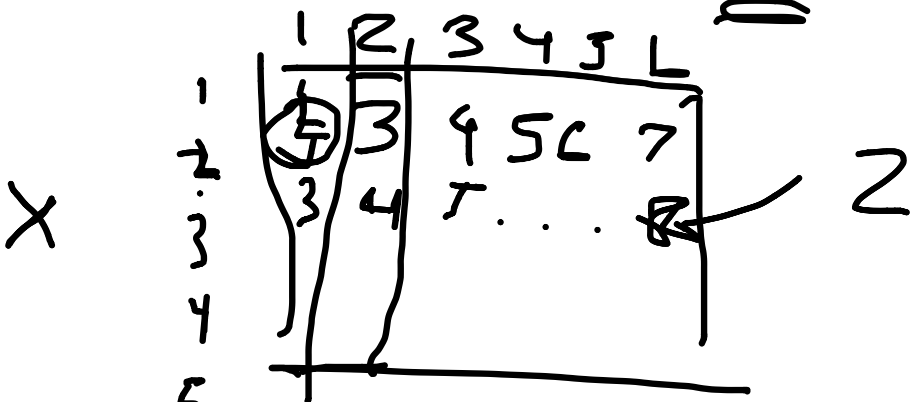
Additional Properties

Conditional probability

Suppose we have a complicated experiment with many possible steps, and the the ultimate outcome depends on each of these steps (for example, when rolling two die, the sum of the die depends on both the roll of the first and the roll of the second die).

Now consider that we know the result of one of the steps - what happens to the probability?

For instance, suppose that in the last example we know that $Y = 2$. What happens to the probability that $Z = 7$?



we changed the question - the sample has changed and that means that the way we we determine the probability has to change to.

Assigning Measures

Properties of Probability

Conditional Probability

Conditional probability

Conditional probability is the probability of seeing an outcome given additional information while staying consistent with the original probability values.

Suppose that there are two events, A and B. Then, if $P(A) \ne 0$, the conditional probability of B given that A is known to have occurred is \[P(B|A) = \dfrac{P(A \cap B)}{P(A)} \]

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Independence

One of the most important concepts related to conditional probability is **independence**.

If events are independent, then the probability of an outcome B occurring does not change based on knowing that A has occurred.

To write it mathematically,

$$ P(B | A) = \dfrac{P(A \cup B)}{P(A)} = P(B) $$

which we can rearrange to get: $$ P(A \cup B) = P(A) \cdot P(B) $$

Independence

System Reliability

Consider two systems, each composed of four independent operating parts.

In system A, the components are arranged in serial

In system B, the components are arranged in parallel.

Suppose the parts fail with probability 0.1.

What is the probability that System A fails?

What about System B?

Independence

Detecting and Disabling Rockets

Independence

Balls in an Urn

Bayes Rule