MWG 6.B.1

Bloody Micro! by Impatient Researcher

December 24, 2020

Question

Refer to the textbook for the actual wordings of the question. The main idea is to show that if Independent Axiom (IA) is satisfied, then:

$$L \succ L^{'} \iff \alpha L + (1-\alpha)L^{''} \succ \alpha L^{'} + (1-\alpha)L^{''}, \forall \alpha \in (0,1)$$

Where $L, L^{'}, L^{''} \in \mathcal{L}$.

Answer

We will try to find a contradiction.

1. Recall from the definition of IA:

$$L \succcurlyeq L^{'} \iff \alpha L + (1-\alpha)L^{''} \succcurlyeq \alpha L^{'} + (1-\alpha)L^{''}, \forall \alpha \in (0,1)$$

2. Suppose $L \succ L^{'}$. Notice that $L \succ L^{'} \implies L \succcurlyeq L^{'}$, IA applies! Thus

$$\alpha L + (1-\alpha)L^{''} \succcurlyeq \alpha L^{'} + (1-\alpha)L^{''}, \forall \alpha \in (0,1)$$

3. Then there are two distinct possibilities:
   (a) $\alpha L + (1-\alpha)L^{''} \sim \alpha L^{'} + (1-\alpha)L^{''}, \forall \alpha \in (0,1)$
   (b) $\alpha L + (1-\alpha)L^{''} \succ \alpha L^{'} + (1-\alpha)L^{''}, \forall \alpha \in (0,1)$

4. Suppose it is the indifference case, then it must be the case that[1]:

$$\alpha L^{'} + (1-\alpha)L^{''} \succcurlyeq \alpha L + (1-\alpha)L^{''}, \forall \alpha \in (0,1)$$

5. However, using IA again, this implies that $L^{'} \succcurlyeq L$. But this contradicts the claim that $L \succ L^{'}$. Thus it must be the case that $\alpha L + (1-\alpha)L^{''} \succ \alpha L^{'} + (1-\alpha)L^{''}, \forall \alpha \in (0,1)$. $\square$

[1] By the defintion of $\sim$,

$$A \sim B \implies A \succcurlyeq B \text{ and } B \succcurlyeq A$$