Pure strategies in MNE must survive IEDS

Bloody Micro! by Impatient Researcher

20 December 2020

Question

Formally show that in a Mixed Strategy Nash Equilibrium (MNE), pure strategies that would be played with a positive probability must survive Iterated Elimination of Dominated Strategies (IEDS).

Answer

Suppose not[1]. Consider a 2-person case.

1. Denote the Mixed Strategy Nash Equilibrium by $p^* = (p_1^*, p_2^*)$.
2. There are K strategies one can take, denote the strategy sets by $S_i = (s_{i1}, \ldots, s_{iK}), \forall i \in \{1, 2\}$
3. WLOG, assume s_{11}[2] be the first pure strategy which is:
   (a) being played with positive probability in a MNE; and
   (b) get eliminated!

   This also implies that there must exist another strategy, s_{1j} that:

   (a) has not yet been eliminated; and
   (b) $u_1(s_{11}, s_{2k}) < u_1(s_{1j}, s_{2k}), \forall s_{2k} \in S_2$[3]
4. But if s_{11} is worse than some s_{1j} regardless of what strategy player 2 plays, then s_{11} is still worse than s_{1j} when player 2 is *randomising* over his/her strategy set S_2[4]:
   $$u_1(s_{11}, p_2^*) < u_1(s_{1j}, p_2^*)$$
5. Then we can generate a new mixed strategy for player 1 as follows:
   $$p_1^{**} = \begin{cases} p_{11}^{**} = 0 & \text{Cut the prob. of playing } s_{11} \text{ to } 0 \\ p_{1j}^{**} = p_{11}^* + p_{1j}^* & \text{Play the better } s_{1j} \text{ more often!} \\ p_{1k}^{**} = p_{1k}^* & \forall k \neq 1, j \end{cases}$$
6. By construction,
   $$u_1(p_1^{**}, p_2^*) > u_1(p_1^*, p_2^*)$$
   But clearly, it contradicts the claim that $p^* = (p_1^*, p_2^*)$ is a MNE! $\square$

[1] That is we have a MNE on hand, and there exists a pure strategy that gets played with positive probability but somehow it gets eliminated in the process.

[2] This reads the first strategy of the first player

[3] 'Cause that's how you get to eliminate s_{11} in the first place!

[4] Because by *randomising*, we mean that player 2 is playing a convex combination of strategies $S_2 = (s_{21}, \ldots, s_{2K})$