MWG 6.B.3

Bloody Micro! by Impatient Researcher

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Question

Refer to the textbook for the actual wordings of the question. The main idea is to show that if:

1. the set of outcome is finite
2. the individual's preference over the set of outcomes is *rational*[1]
3. the independence axiom is satisfied

Then you can always find the best and the worst lottery.

[1] That is preference is complete and transitive

Answer

To prove this, we construct the best and the worst lottery directly.

1. Denote the set of outcomes by C and the rational relation by $\succcurlyeq$.
2. As the set C is finite, $\succcurlyeq$ is complete and transitive, there must exist at least one outome that is deemed to the best and and at least one deemed to be the worst.[2]
3. WLOG, let the number of outcomes be N such that:
$$c_1 \succ c_2 \succ c_3 \succ \cdots \succ c_N$$
4. Let L be a generic lottery which takes the form:
$$L = (p_1 \circ c_1, p_2 \circ c_2, p_3 \circ c_3, \ldots, p_N \circ c_N)$$
5. We can make the lottery L more attractive[3] by replacing c_2 by c_1:
$$L' = \left(p_1 \circ c_1, p_2 \circ c_1, p_3 \circ c_3, \ldots, p_N \circ c_N\right)$$
6. Following this logic, we can create an even better lottery by successively replacing $c_3, \ldots, c_N$ by c_1. Clearly, there's no room for improvement when we have replaced all outcomes in L with c_1 that is:
$$\overline{L} = \left(p_1 \circ c_1, p_2 \circ c_1, p_3 \circ c_1, \ldots, p_N \circ c_1\right)$$
Implying that $\overline{L}$ constructed this way is indeed the best lottery.
7. Similarly one can construct the worst lottery $\underline{L}$ by replacing all outcomes by c_N. $\square$

[2] C being finite means there are just that many pairwise comparisons to be done, and rational preference implies that the individual can compare any pair of outcomes (completeness) and form a uncontradictory ordering of outcomes (transitivity).

[3] This is because except c_2 all other items are the same. And by IA, $L' \succ L \iff c_1 \succ c_2$