

$$\langle \rho \rangle = \frac{2\Gamma(2-\frac{\beta}{2})}{2^{\frac{\beta-1}{2}} \sqrt{\pi}} \rho_0 \left(\frac{M}{T_0} \right)^{\frac{\beta-1}{2}}$$

$$\frac{dn}{ds} = -n^2 \langle \rho \rangle$$

$$\frac{dr}{dt} = \langle \rho \rangle \frac{r^2 s}{H^2}$$

$$= \frac{r^2 (2\pi^2 g_s)^3}{H^2} \cdot A \frac{2\Gamma(2-\frac{\beta}{2})}{2^{\frac{\beta-1}{2}} \sqrt{\pi}} \left(\frac{M}{T_0} \right)^{\frac{\beta-1}{2}}$$

$$= \frac{r^2 T^2}{H} \frac{(2\pi^2 g_s)}{45} \frac{(M T_0)^{\frac{\beta-1}{2}}}{T^{\beta-1}} \frac{2\Gamma(2-\frac{\beta}{2})}{2^{\frac{\beta-1}{2}} \sqrt{\pi}} =$$

$$= \frac{r^2 T^{3-\beta}}{H} \left(\frac{2\pi^2 g_s}{45} \right) (M T_0)^{\frac{\beta-1}{2}} \frac{2\Gamma(2-\frac{\beta}{2})}{2^{\frac{\beta-1}{2}} \sqrt{\pi}} \rho_0$$

$$dr = D_s \frac{r^2}{T^{\beta+1}} = \left\{ \begin{array}{l} D_{RD} \frac{r^2}{T^{\beta-1}} = \frac{r^2}{T^{\beta-1}} \frac{2\pi^2 g_s (M T_0)^{\frac{\beta-1}{2}}}{\left(\frac{4\pi^2 g_e}{45} \right)^{1/2} \frac{1}{m_{Pl}}} \frac{2\Gamma(2-\frac{\beta}{2})}{2^{\frac{\beta-1}{2}} \sqrt{\pi}} \\ D_{MD} \frac{r^2}{T^{\beta-3/2}} = \frac{r^2}{T^{\beta-3/2}} \frac{2\pi^2 g_s (M T_0)^{\frac{\beta-1}{2}}}{\frac{2}{3} \frac{1}{t_{ec}} \cdot \frac{1}{T_{ec}^{3/2}}} \frac{2\Gamma(2-\frac{\beta}{2})}{2^{\frac{\beta-1}{2}} \sqrt{\pi}} \end{array} \right.$$

$$-\frac{1}{r} = \left\{ \begin{array}{l} D_{RD} \frac{1}{r^{\beta}} \\ D_{MD} \frac{1}{r^{\beta-1/2}} \end{array} \right. \Rightarrow \frac{1}{r} + C = \left\{ \begin{array}{l} \frac{D_{RD}}{\beta} \frac{1}{T^{\beta}} \\ \frac{D_{MD}}{\beta-1/2} \frac{1}{T^{\beta-1/2}} \end{array} \right.$$

$$\frac{1}{r} - \frac{1}{r_0} = \left\{ \begin{array}{l} \frac{D_{RD}}{\beta} \left(\frac{1}{T^{\beta}} - \frac{1}{T_0^{\beta}} \right) \\ \frac{D_{MD}}{\beta-1/2} \left(\frac{1}{T^{\beta-1/2}} - \frac{1}{T_0^{\beta-1/2}} \right) \end{array} \right.$$

$$r = \left(\frac{1}{r_0} + \frac{D_s}{\beta} \left(\frac{1}{T^{\beta}} - \frac{1}{T_0^{\beta}} \right) \right)^{-1}$$

$$r = \frac{r_0}{1 + \frac{D_s r_0}{\beta} \left(\frac{1}{T^{\beta}} - \frac{1}{T_0^{\beta}} \right)}$$

$$H = \left\{ \begin{array}{l} \sqrt{\frac{4\pi^2 g_e}{45}} \frac{T^2}{m_{Pl}} \\ \frac{2}{3 t_{ec}} \left(\frac{T}{2.7K} \right)^{3/2} \end{array} \right.$$

$$= \sqrt{\frac{g_s^2}{45 g_e}} \frac{\Gamma(2-\frac{\beta}{2})}{2^{\frac{\beta-1}{2}-1}} m_{Pl} \rho_0$$

$$r(\tau) = r_0 \left\{ \begin{array}{l} \frac{1}{1 + \frac{D_{RD}}{\beta} (\dots)} \\ \frac{1}{1 + \frac{D_{MD}}{\beta-1/2} (\dots)} \end{array} \right.$$

$$D_{RD} = D_{MD} \sqrt{T_{RH}} = \frac{T(2-\frac{\beta}{2}) g_s}{2^{\frac{\beta-3}{2}} \sqrt{45 g_e}} \rho_0 m_{Pl} (M T_0)^{\frac{\beta-1}{2}}$$