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EFFECTS OF NEW LONG-RANGE INTERACTION OF RELIC HEAVY NEUTRINOS

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Non-dominating DM component in form of Heavy neutrinos possessing its own Coulomb-like interaction is considered. It is shown that a recombination of pairs of Heavy neutrinos and antineutrinos has a crucial significance in cosmological evolution of this component. Recombination with successive annihilaiton of bound pairs can account for γ-flux observed by EGRET.

Title in Russian
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Text of abstract in Russian

1. Introduction

This work continues investigations of model of subdominant component of dark matter in form of Heavy neutrinos [1] and is devoted to its special case when given component of dark matter possesses its own long range interaction. Let us briefly render Heavy neutrino (N) properties, inheritted from [1]: it belongs to new 4th generation of fermions with the charges attributed analogiously to other three Standard Model generations, mass of 4th neutrino lies in range about $m = 45 - 80$ GeV. It is supposed that new interacton is Coulomb-like, being described with unbroken $U(1)$-gauge group; we will call it in this paper as y-interaction and its particle-mediater and charge as y-photon and y-charge respectively. Supposition of existence of such an interaction as well as of Heavy neutrinos of 4th generation (4th neutrinos) theirselves follows from superstring inspired models. .....

Effects of new interaction of primordial 4th neutrinos can be shared on those in early Universe and in modern Universe - in Galaxy. In early Universe the meaningfull processes are the freezing out of y-charged 4th neutrinos, which gives their amount preserved to present time, freezing of y-photons, an influence of whose background not only on 4th neutrinos evolution but also on predictions of primordial helium abundance should be assessed, and other. After decoupling of 4th neutrinos from y-background in course of their cosmological evolution the formers can form bound pairs of 4th neutrinos and antineutrinos due to long-range y-interaction. These bound systems must annihilate soon. So, such an effect of "recombination" of 4th neutrinos and antineutrinos changes their relic density and, dependently on corresponding period, can lead to observable effects.

An existence of longliving quarks of 4th generation [2] would change evolution of 4th neutrinos, in case of y-interaction exists, drastically, requiring a separate consideration.

In second section the evolution of 4th neutrinos and y-background in early Universe is analysed and their relic densities are estimated in the result. In the third section the case of simultaneous presence in early Universe of y-charged 4th generation both neutrinos and quarks is discussed. Fourth section is devoted to the possible observable effects of primordial 4th neutrinos recombination. In fifth section observable effects of y-charged 4th neutrinos in Galaxy are considered. In final sixth section discussion is put.

2. y-charged Heavy neutrinos and y-background in early Universe

As was repeatedly asserted in previous works, 4th neutrinos are frosen out in early Universe at the temperature [4] $T_f \approx m/30$. Their amount in comoving volume does not change essentially in course of subsequent evolution of cooling Universe down to modern epoch. New y-interaction of 4th neutrinos would change their annihilitation cross-section not on principle and respectively their frosen out density. However, if y-interaction esists, 4th neutrinos are frosen out at $T \sim m/30$ just for a while. At lower temperatures, after decoupling

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[4]Throughout in the text the system of units $h = c = k = 1$, where k is the Boltzman constant, is used.

4th neutrinos from y-background, formation of bound systems neutrino-antineutrino (recombination) becomes possible. Annihilation time of systems with the size a_b bound due to interaction with the fine-structure consntant α_y

$$\tau_{ann} \sim \frac{m^2 a_b^3}{\alpha_y^2} \tag{1}$$

is quite short so process of such recombination is in fact equivalent to pair annihilation. Number of primordial 4th neutrinos reduces more while recombination occurs.

Let us first consider early freezing out (at $T \sim m/30$) of 4th neutrinos. New interaction existence leads to appearence of new channel of 4th neutrinos annihialtion analogous to 2γ-annihilation of e^+e^-

$$N\bar{N} \to yy. \tag{2}$$

Furthemore for slow 4th neutinos and antineutrinos (with relative velocity $v < 2\pi\alpha_y$) it leads to the incsrease of cross section of all their annihilation channels, including one via intermediate Z-boson ($N\bar{N} \to Z \to$ ...), due to Coulomb factor

$$C = \frac{C_0}{1 - e^{-C_0}},\ C_0 = \frac{2\pi\alpha_y}{v}. \tag{3}$$

However, at neutrino mass of interest ($m \sim 50$ GeV) channel $N\bar{N} \to Z \to ...$ undergoes nearly resonant amplification ($m_{res} = m_Z/2 \approx 46$ GeV) so extra channel (2) has a suppressed effect being displayed as strong as far m from m_{res}. The ratio of cross sections of annihililation channels of (2) and through Z-boson in nonrelativistic limit is, independently on Coulomb factor,

$$\frac{\sigma_{yy}}{\sigma_Z} \approx 0.144 \left(\frac{\alpha_y}{1/30}\right)^2 \left(\frac{50\,\mathrm{GeV}}{m}\right)^4 \frac{P_Z(m)}{P_Z(50\,\mathrm{GeV})}, \tag{4}$$

$$P_Z(m) = (4m^2/m_Z^2 - 1)^2 + \Gamma_Z^2/m_Z^2,$$

where m_Z and Γ_Z are the mass and decay width of Z-boson, $P_Z(50\,\mathrm{GeV}) \approx 0.0418$. Effect of Coulomb factor does not also lead to the changes of principle, since at $T \sim m/30$ velocity of 4th neutrinos is still great enough. On the figure 1 the frosen out number density of 4th neutrinos in units of entropy density, $r = \frac{n}{s}$ (definitions are given below), is presented. Both mentioned effects induce correction of density of 4th neutrinos within factor 2-10.

After this early freezing out of 4th neutrinos their gas cools down being for a some period in thermal equilibrium with ambient ordinary matter and y-photon background. For qualitive and quantitive description we will use mainly energetic consideration of thermodynamics. Different components of matter will swap by energy with other components, also will experience energy losses due to cosmological expansion. The latter makes up, being interpretted here as a positive work of gas,

$$\delta A = pdV = pV3Hdt, \tag{5}$$

Figure 1: Number densities of 4th neutrinos frosen out at $T \sim m/30$ in units of entropy density for the cases without and with y-interaction with different magnitudes of its constant.

where p is the pressure of gas, V is the volume (being proportianl to cube of scale factor a), $H = \dot{a}/a$ is the Hubble constant and t is the cosmological time. If particles of ith gas interact ellastically with particles of jth gas with cross section σ_{ij} and in collision an energy exchange $\overline{\Delta E_{ij}}(\vec{p_i}, \vec{p_j})$ (the transferred from ith to jth particle energy averaged over all its possible magnitudes at fixed initial particles momentums) is happened then energy exchange between i- and j- gases in volume V for time dt will be

$$\delta Q_{ij} = \langle \overline{\Delta E_{ij}} \sigma_{ij} v_{ij} \rangle n_i n_j V dt. \tag{6}$$

Here brackets $\langle\ \rangle$ means averaging over distributions in velocities (momentums) of particles of ith and jth gases, v_{ij} is the relative velocity of colliding particles[5], $n_{i,j}$ are the number densitites of respective gases. We will use below notation $\langle \overline{\Delta E_{ij}} \sigma_{ij} v_{ij} \rangle = \langle \overline{\Delta E} \sigma v \rangle_{ij}$.

4th neutrinos interact with each other due to long-range y-interaction. This interaction with a great, Rutherford-like cross secton provides perfectly a thermally equilibrium (Maxwell) distribution for them whatever energy losses or/and exchanges with other matter components they experience. Internal energy of such gas is defined, as wellknown from thermodynamics, by its temperature (T_N) and by number of particles ($N = nV$) as $U = 3/2NT_N$, and the change of it can be induced by the changes of these two values

$$dU = \frac{3}{2} N dT_N + \frac{3}{2} T_N dN. \tag{7}$$

Interacting with other matter after freezing out (when $dN = 0$) the change of 4th neutrinos temperature with the time will be described by the equation (1st law of

[5] In relativistic case, in form of Eq.(6) independent on reference frame choice a Möller velocity should be meant instead of relative velocity [3],[4].

thermodynamics)[6]

$$\begin{gathered}\delta Q = dU + \delta A\\ \langle\,\overline{\Delta E}\sigma v\,\rangle_{Ny} n n_y V dt + \langle\,\overline{\Delta E}\sigma v\,\rangle_{No} n n_o V dt = \\ = \frac{3}{2} n V dT_N + n T_N V 3 H dt. \end{gathered}\tag{8}$$

Relationship $p = nT_N$ was used for 4th neutrinos in Eq.(8). Making simple cancellations one obtains

$$\frac{3}{2}\frac{dT_N}{dt} = \langle\,\overline{\Delta E}\sigma v\,\rangle_{Ny} n_y + \langle\,\overline{\Delta E}\sigma v\,\rangle_{No} n_o - 3T_N H. \tag{9}$$

In Eqs.(8,9) the terms of inelastic 4th neutrinos interaction are skipped since they have a suppressed influence on 4th neutrinos evolution with respect to the given terms of elastic interaction while the latters hold. Eq.(9) allows to determine the moment when 4th neutrinos gas goes out from equilibrium with (decouples from) other components of matter comparing corresponding term with $3T_N H$. The term of energy exchange with ordinary matter "No" should be shared on a few ones in accordance with available types of ordinary particles. At $T < 100$ MeV, when only (quasi)stable particles compose mainly the matter of Universe, "No"-terms will consist of "$N\nu$"-, "$Ne^{\pm}$"- and "$N\,nucleon$"-ones. However, nucleons have in this period small number density, on about 9 orders of magnitude less than that of ν and $e^{\pm}$, so, inspite on more effective energy exchange in each collision of 4th neutrino with nucleon and greater cross section of this process as compared to light or massless ν, $e^{\pm}$ and y, "$N\,nucleon$"-term is negligible. There can be a small admixture in surrounding matter of other 4th generation particles (quarks), if they are longliving enough, to be taken into account; they will be considered separately.

Gas of y-photons interacts in period of question with only 4th neutrinos and, as mentioned, with possibly admixture of 4th generation quarks. Because of absence of 'internal' mechanism of establishment of equilibrium distribution for y-photon gas (due to yy-interaction), distribution of y-photon in energy should be in principle assumed to be unknown. It, being followed from kinetic equation, depends on effectivety of energy exchange with 4th neutrino gas and inelastic interaction with y-photon production. Y-photon production proceeds copiously in NN-interaction as bremstralung emission which, being inappreciable in energy, has a probability as great as Rutherford-like scattering of 4th neutrinos [5]. Thus, effectivity of energy exchange between N- and y- gases (including the heating of soft y) is determinant in evolution of y-background. Let us for a trial evaluation of this effectivity assume that y-gas has a thermal equilibrium distribution in Planck form (without chemical potential). Then for y-gas, which state is determined only by its temperature T_y, the change of internal energy ($\propto T_y^4$) is represented as

$$dU_y = d(\varepsilon_y V) = \varepsilon_y dV + V4\varepsilon_y \frac{dT_y}{T_y}. \tag{10}$$

Analogously to Eq.(8) one writes

$$\begin{gathered}\langle\,\overline{\Delta\omega}\sigma v\,\rangle_{yN} n n_y V dt = \\ = \varepsilon_y V 3H dt + 4\varepsilon_y V\frac{dT_y}{T_y} + \frac{1}{3}\varepsilon_y V 3H dt.\end{gathered}\tag{11}$$

For the mean energy transferred between y and N with given their momentums one has $\overline{\Delta\omega_{yN}} = -\overline{\Delta E_{Ny}}$. Relationship $p_y = \varepsilon_y/3$ was used in Eq.(11). Also reminding other relations for "Planck" gas

$$n_y = \frac{2\zeta(3)}{\pi^2}T_y^3, \qquad \varepsilon_y = \frac{\pi^2}{15}T_y^4 = bT_y n_y, \tag{12}$$

where $b = \frac{\pi^4}{30\zeta(3)}$, from Eq.(11) one obtains

$$4b\frac{dT_y}{dt} = -\langle\,\overline{\Delta E}\sigma v\,\rangle_{Ny} n - 4bT_y H. \tag{13}$$

In Appendix A the values $\langle\,\overline{\Delta E}\sigma v\,\rangle_{Ny}$ and $\langle\,\overline{\Delta E}\sigma v\,\rangle_{N\nu,e}$ are calculated. Scatterings Ny, $N\nu$, $Ne^{\pm}$ are considered as a scattering of free particles and Maxwell distribution is applied for all particles in averaging $\langle\ \rangle$. This does not lead to the error greater than 10% [6].

Number density of 4th neutrinos is expressed from r and entropy density s of matter

$$\begin{gathered}n = rs = r\frac{2\pi^2 g_s}{45}T^3,\\ g_s = \Sigma_{bos}(T_{bos}/T)^3 + 7/8\Sigma_{ferm}(T_{ferm}/T)^3,\end{gathered}\tag{14}$$

where sums are over sorts of ultrarelativistic bosons and fermions, T is the temperature of photons. Hubble constant on radiation and matter dominated (RD and MD) stages can be represented as following

$$H = \begin{cases} \sqrt{\frac{4\pi^3 g_\varepsilon}{45}}\frac{T^2}{m_{Pl}} = h_{RD}\frac{T^2}{m_{Pl}} \text{ at RD stage,}\\ \frac{2}{3t_{mod}}\left(\frac{T}{2.7\,\mathrm{K}}\right)^{3/2} = h_{MD}\frac{T^{3/2}}{m_{Pl}^{1/2}} \text{ at MD stage.}\end{cases}\tag{15}$$

Here m_{Pl} is the Plank mass, $t_{mod} \approx 4.4\cdot 10^{17}$ s is the modern age of Universe, $h_{RD} \approx 5.5\sqrt{g_\varepsilon/11}$, $h_{MD} \approx 3.08\cdot 10^{-14}$ and analogously to g_s of Eq.(14)

$$g_\varepsilon = \Sigma_{bos}(T_{bos}/T)^4 + 7/8\Sigma_{ferm}(T_{ferm}/T)^4. \tag{16}$$

Value $T = T_{RM} \approx 1.2$ eV provides good transition between given dependences of H on RD- and MD-stages, connected them with factor $\sqrt{\frac{T}{T_{RM}}}$. For time intervals when $g_{\varepsilon,s} \approx$ const it is just the following transition between differentials

$$-dt = \frac{1}{H}\frac{dT}{T}. \tag{17}$$

Let us for T_y introduce parameter κ as

$$T_y^3 = \kappa T^3. \tag{18}$$

[6] We avoid to use the index "N" in symbols of 4th neutrino values except of its temperature. Symbol T without any index is reserved for photon temperature.

Inserting $\langle\,\overline{\Delta E}\sigma v\,\rangle_{Ny}$ from Eq.(88) of Appendix A, Eq.(14) and Eq.(15) for RD-stage into Eq.(13) one qets

$$\frac{dT_y}{dt} = -A\cdot r\frac{T^5}{m^3}\left(1-\frac{T_N}{\kappa^{1/3}T}\right) - B\frac{T^3}{m_{Pl}}, \tag{19}$$

where $A \sim 10^{-2}$, $B \sim 10$. Comparing the terms $A\cdot rT^5/m^3$ and $B\cdot T^3/m_{Pl}$ we obtain that energy transfer from N-gas to y-gas would be effective at

$$T \gtrsim \frac{1\,\mathrm{keV}}{\sqrt{r}}, \tag{20}$$

what is not realizable taking into consideration Fig.1. One can conclude that y-background does not experience any significant changes except cosmological expansion (is frozen out) after 4th neutrinos were frozen out at $T \sim m/30$, exacter a little earlier when r is suppressed enough. Their temperature at $T < T_f \approx m/30$ is inferred from entropy conservation, so

$$\kappa(T) = \frac{g_{s\,o}(T)}{g_{s\,o}(T_f)}. \tag{21}$$

Here the factors $g_{s\,o}$ take into account only ordinary matter (without contribution of y). Factor κ gives y-photon number density and entropy density in respect to that of photons. For instance, at the period of cosmological nucleosynthesis $\kappa(T \lesssim 1\,\mathrm{MeV}) = \kappa_{nucl} \approx 1/6.5$, at modern period $\kappa(2.7\,\mathrm{K}) = \kappa_{mod} \approx 1/18$. Contribution into energy density of Universe at those times relatively to that of photons are respectively $\kappa_{nucl}^{4/3} \approx 1/12$ and $\kappa_{mod}^{4/3} \approx 1/47$. Predicted magnitude of primordial abundance of ^{4}He alters with this addition of y-background relatively on $\delta Y_{He}/Y_{He} \approx 0.45\cdot 10^{-2}$, what is less than the error of magnitude deduced from observation.

For N-gas from Eq.(9) taking Eqs.(88,92) from Apprendix A, transformation Eq.(17), H for RD-stage and also using that $n_{\nu\bar\nu} = n_{e^-,e^+} = 3/4 n_\gamma$, where photon number density n_γ is defined in the same way as Eq.(12), after elementary transformations we get

$$\frac{1}{2}\frac{dT_N}{dT} = -\frac{T(\kappa^{1/3}T - T_N)}{T_{Ny}^2} - \frac{T^3(T-T_N)}{T_{No}^4} + \frac{T_N}{T}. \tag{22}$$

$T_\nu = T_e = T$ was put. Coefficients in Eq.(22) are

$$T_{Ny} = \left(\frac{\pi g_\varepsilon}{5}\right)^{\frac14}\left(\frac{m}{2\zeta(3)\kappa^{4/3}m_{Pl}}\right)^{\frac12}\frac{\pi m}{4\alpha_y} \approx \approx 15\,\mathrm{keV}\left(\frac{m}{50\,\mathrm{GeV}}\right)^{\frac32}\frac{1/30}{\alpha_y}, \tag{23}$$

$$T_{No} = \left(\frac{\pi g_\varepsilon}{5}\right)^{\frac18}\left(\frac{m}{135\zeta(3)(3+2\xi_e)m_{Pl}}\right)^{\frac14}\frac{\pi}{\sqrt{G_F}} \approx \approx 10\,\mathrm{MeV}\left(\frac{m}{50\,\mathrm{GeV}}\right)^{\frac14}. \tag{24}$$

They have the sense of the temperature of decoupling of 4th neutrinos from respective matter component (at $T = T_{Na}$, $T_N = T_a$ the term describing expansion (T_N/T) is equal to the term describing the heat acquirement rate (terms with brackets without T_N in them); illustration will be for the case of Ny). Quantities κ and g_ε in T_{Ny} were taken to be respectively κ_{mod} and $g_\varepsilon(13\,\mathrm{keV}) = 3.363 + 2\kappa_{mod}^{4/3} \approx 3.4$, and g_ε in T_{No} to be $g_\varepsilon(10\,\mathrm{MeV}) = 10.75 + 2\kappa_{nucl}^{4/3} \approx 10.9$.

Eq.(22) gives that even before decoupling of 4th neutrinos from ordinary matter their temperature follows to that of y-photons, i.e. $T_N = \kappa^{1/3}T$. After decoupling of 4th neutrinos from y-background (at $T < T_{Ny}$) their temperature is regulated by term T_N/T in Eq.(22), so in this period $T_N \propto T^2$. For the following we will need exact dependence of T_N, for that we will solve Eq.(22) at $T \ll T_{No}$. Removing "No"-term and requiring $T_N = T_y = \kappa^{1/3}T$ at $T \gg T_{Ny}$ one gets

$$T_N(T) = \frac{\sqrt{\pi}\kappa^{1/3}T^2}{T_{Ny}}\,\mathrm{e}^{(T^2/T_{Ny}^2)}(1-\mathrm{erf}(T/T_{Ny})). \tag{25}$$

At $T \ll T_{Ny}$

$$T_N \approx T^2/\bar T_{Ny}, \quad \bar T_{Ny} = T_{Ny}/(\sqrt{\pi}\kappa^{1/3}). \tag{26}$$

A probability for 4th neutrinos and antineutrinos to form their bound states grows when particles slows down, so at $T \ll T_{Ny}$ the rate of this process increases. For estimation of this effect we will use classical approxiamtion following [7]. [[**Motivaton of it seems to be necessary.**]] Pair $N\bar N$ moving to meet each other must expirience dipole emission due to y-interaction. If the lost energy on emissoin makes up initial energy of their relative motion then they are bound. Note, that the typical length passed off by N and $\bar N$ to emit sufficient energy is much less than the mean distance between 4th neutrinos (a fortiori than Debye radius of N-plasma) in cases of interest, and the typical time of this process is much less than the mean time between N-y interactions. It provides that such a process of binding can proceed freely, independently on other interacton processes. [[**I am not sure that this argument is correct. That is if that typical length $\ll$ inter-N distance then Coulomb interaction of surrounding N will not inhibit to the binding of given pair.**]] Cross section of bound system formation in this approach [3] is obtained to be

$$\sigma_b = \pi\rho_b^2(v) = \frac{(4\pi)^{7/5}r_{cl}^2}{v^{14/5}}, \tag{27}$$

where ρ_b is the maximal impact parameter of moving $N\bar N$ to be bound, v is the initial $N\bar N$ relative velocity.

The rate of binding with cross section of form $\sigma_b = \sigma_0/v^\beta$ for particles, distributed by Maxwell, is

$$\Gamma_{rec} = \langle\,\sigma_b v\,\rangle n = \frac{4\Gamma(2-\frac{\beta}{2})}{2^\beta\sqrt{\pi}}\sigma_0\left(\frac{m}{T_N}\right)^{\frac{\beta-1}{2}} rs. \tag{28}$$

This turns out with Eq.(27) at $T \gg T_{Ny}$ to be $\propto T^{2.1}$ and very close to the expansion rate $H \propto T^2$. At such a similarity of the dependences a difference of T_N from T ($T_N = \kappa^{1/3} T < T$) and temperature dependence of factors g_s, g_ε can play role. At some parameters Γ_{rec} exceeds a little H in given period, so recombination [[**being independent on other processes**]] can proceed flabbily (see comments below). After T_N changes its slope (Eqs.(25,26)), condition $\Gamma_{rec} > H$ becomes fulfilled virtually at once, at T below

$$T_{H\,rec} \approx 15\,\mathrm{keV} \left(\frac{m}{50\,\mathrm{GeV}}\right)^{\frac{5}{16}} \left(\frac{\alpha_y}{1/30}\right)^{\frac{11}{8}} r_{50,1/30}^{\frac{5}{4}}. \quad (29)$$

Here $r_{50,1/30} = \frac{r(m,\alpha_y)}{r(50\,\mathrm{GeV},1/30)}$, $r(50\,\mathrm{GeV}, 1/30) = 1.22 \cdot 10^{-15}$. The fact that $T_{H\,rec}$ of Eq.(29) can at some m, α_y be formally greater than T_{Ny} means that in this case the condition $\Gamma_{rec} > H$ happens before $T = T_{Ny}$ [[**the fact that recombination can be before T_{Ny} can induce question that 'is in this approach the recombination possible (in principle) whatever temperature is'? Nonetheless it was so in case of 4th quarks. - the question of criterion of the recombination beginning (CRB)**]]. Since as was noted Γ_{rec} decreases in this period just a little faster than H with the temperature falling we will assume that recombination reduces r insomuch as provides condition $\Gamma_{rec} = H$. [[**In fact, it would be wrong if to consider recombination before T_{Ny} as usually. Because of Γ_{rec} slows down faster than H the result would depend stronger on initial moment of recombination rather than final. - the problem of CRB**]] The latter at $T = T_{Ny}$ gives

$$r = r_{H\,Ny} \approx 1.22 \cdot 10^{-15} \left(\frac{m}{50\,\mathrm{GeV}}\right)^{\frac{19}{20}} \left(\frac{1/30}{\alpha_y}\right)^{\frac{19}{10}}. \quad (30)$$

Note, that recombination in period $T > T_{Ny}$ does not virtually influence on Eq.(22) and hence on magnitude of T_{Ny} itself.

On the base of said above, we will assume that (main) recombiantion, which final result softly depends on initial conditions, starts at $T = T_{rec} = \min\{T_{H\,rec}, T_{Ny}\}$ with $r = r_{rec}$ being equal to respectively $\min\{r_f, r_{H\,Ny}\}$, where r_f is defined by early freezing out and given by Figure 1. Thus, at the beginning of recombination we have

$$\Gamma_{rec}(T_{rec}) = H(T_{rec}), \;\; T_{rec} \le T_{Ny}. \quad (31)$$

A big part of the period to be considered corresponds to MD-stage. Inhomogenities of matter distribution are developing. In course of these processes the form of distribution of 4th neutrinos in space and in velocities deviates from that assumed for them on RD-stage. However, this becomes significant when relative density perturbations of matter grow up to 1, what takes place at $z \sim 10$ (galaxies formation). In galaxies 4th neutrinos obey another velocity distribution - with much greater typical velocities, and as a consequence effect of recombination reduces. We will assume primordial recombination stops at $z + 1 = 10$ what corresponds to $T = T_{fin} = (z+1)T_{mod} = 27$ K, where $T_{mod} = 2.7$ K is the modern temperature of photons. Evolution of 4th neutrinos in Galaxy will be considered in separate section.

Typical size of the most of created bound systems is less than $\rho_b \sim r_{cl}/v^{7/5} \sim r_{cl}(m/T_N)^{7/10}$; annihilation time Eq.(1) turns out to be much less than the mean time of bound systems distruction and than cosmological time.

Since mostly slow 4th neutrinos and antineutrinos create (owing to dipole emission) bound states (see Eqs.(27,28)), their subsequent annihilation leads effectively to heating of N-gas. Besides, all $N\bar{N}$ scatterings, not ending by bounding, are accompanied by dipole emission too, what contrarily leads to cooling of N-gas. Let us take into account these effects (being insignificant before Ny-decoupling).

Let $\Delta E(v,\rho)$ be the kinetic energy lost due to dipole emission in process of $N\bar{N}$-scattering [7] with impact parameter ρ and relative velocity $v = |\vec{v}_N - \vec{v}_{\bar{N}}|$. If $\rho < \rho_b$ then $\Delta E(v,\rho) = E_N + E_{\bar{N}} = \frac{mv_N^2}{2} + \frac{mv_{\bar{N}}^2}{2}$. Energy losses rate by $N + \bar{N}$-gas per unit volume will be given by

$$\dot{\varepsilon} = n_N n_{\bar{N}} \int \left\{ \int_0^{\rho_b} (E_N + E_{\bar{N}}) v 2\pi\rho d\rho + \right.$$
$$\left. + \int_{\rho_b}^{\rho_{max}} \Delta E v 2\pi \rho d\rho \right\} f_N(\vec{v}_N) d^3 v_N f_{\bar{N}}(\vec{v}_{\bar{N}}) d^3 v_{\bar{N}}. \quad (32)$$

Upper limit ρ_{max} should be given by Debye radius of $N\bar{N}$-plasma. The first term in Eq.(32) is $2n^2 \langle E\sigma_b v \rangle$, where was taking into account equivalence of N- and $\bar{N}$-gases (velocity distribution functions $f_N = f_{\bar{N}}$, $n_N = n_{\bar{N}} = n$). Averaging over distribution gives

$$\langle E\sigma_b v \rangle = \frac{(7-\beta)\Gamma(2-\frac{\beta}{2})}{2^\beta \sqrt{\pi}} \sigma_0 \left(\frac{m}{T_N}\right)^{\frac{\beta-1}{2}} T_N. \quad (33)$$

The value $\Delta E(v,\rho)$ is accordingly [3] (in large scattering angle limit)

$$\Delta E(v,\rho) = \frac{2^5 \pi \alpha_y^5}{m^4 v^5 \rho^5} = \frac{mv^2}{4} \left(\frac{\rho_b}{\rho}\right)^5. \quad (34)$$

Note, $mv^2/4$ is the relative motion energy, E_{rel}. Large scattering angle approxiamtion holds when $\rho \ll 2r_{cl}/v^2$. However, thanks to fast convergence of dependence of $\Delta E(v,\rho)$ with $\rho \to \infty$ any upper restriction of ρ can be avoided, so in Eq.(32) one can apply $\rho_{max} \to \infty$. For second term in Eq.(32) we get, denoting it as $\dot{\varepsilon}_{dip}$,

$$\dot{\varepsilon}_{dip} = n^2 \int_0^\infty \frac{mv^2}{4} \frac{2}{3} \sigma_b v f(v) dv = \frac{2}{3} n^2 \langle E_{rel}\sigma_b v \rangle,$$
$$\langle E_{rel}\sigma_b v \rangle = \frac{2(4-\beta)\Gamma(2-\frac{\beta}{2})}{2^\beta \sqrt{\pi}} \sigma_0 \left(\frac{m}{T_N}\right)^{\frac{\beta-1}{2}} T_N. \quad (35)$$

[7] NN-, $\bar{N}\bar{N}$-scatterings do not lead to dipole emission

Here it was passed to integration over relative velocity distribution $(f_N(\vec{v}_N)d^3v_N f_{\bar{N}}(\vec{v}_{\bar{N}})d^3v_{\bar{N}} \to f(v)dv)$.

For description of temperature evolution of 4th neutrinos in period when recombination occurs we are oblidged to take into account in Eq.(7) the term $\frac{3}{2}T_N dN = -\frac{3}{2}T_N\langle\sigma_b v\rangle n^2 V dt$. Then 1st law of thermodynamics gives

$$\frac{3}{2}\frac{dT_N}{dt} = \langle(\frac{3}{2}T_N - E - \frac{1}{3}E_{rel})\sigma_b v\rangle n - 3T_N H. \quad (36)$$

As in previous evolution equations it relates to N or $\bar{N}$ (therefore $\dot{\varepsilon}_{dip}/2$ was taken). First couple terms in the right side of Eq.(36), $\propto \frac{3}{2}T_N - E \propto (\beta-1)$, originates from the fact of disappearence of 4th neutrinos (because of recombination) and lead to heating of N-gas (at $\beta > 1$). Term $\propto \frac{1}{3}E_{rel}$ does not dominate for σ_b equal to Eq.(27) and diminishes this effect. It is seen after accounting of Eqs.(28,33,35)

$$\langle(\frac{3}{2}T_N - E - \frac{1}{3}E_{rel})\sigma_b v\rangle = \frac{5\beta-11}{3}\times$$
$$\times\frac{\Gamma(2-\frac{\beta}{2})}{2^\beta\sqrt{\pi}}\sigma_0\left(\frac{m}{T_N}\right)^{\frac{\beta-1}{2}} T_N. \quad (37)$$

It is convinient to pass from T_N to new variable θ:

$$T_N = \theta T^2/\bar{T}_{Ny}. \quad (38)$$

Eq.(36) using Eqs.(17,38) takes the form, abbreviating Eq.(37) with $\langle ...\rangle$,

$$\frac{d\theta}{dT} = -\frac{2}{3}\frac{\theta}{T_N}\langle...\rangle\frac{rs}{HT} = -\gamma D_s\frac{r\,\theta^{\frac{3-\beta}{2}}}{T^{\beta_s+1}}. \quad (39)$$

Here the following notations are introduced: index $s='R'$ or $'M'$ means RD- or MD- stage,

$$\gamma = \frac{5\beta-11}{18},\;\; \beta_R = \beta-2,\;\; \beta_M = \beta - \frac{5}{2}.$$

Note, that in absence of $\dot{\varepsilon}_{dip}$ γ would be equal to $(\beta-1)/6$. The quantities $D_{R,M}$ are

$$D_R = D_M\sqrt{T_{RM}} = \frac{4\Gamma(2-\frac{\beta}{2})g_s}{2^\beta\sqrt{45 g_\varepsilon}}\sigma_0 m_{Pl}(m\bar{T}_{Ny})^{\frac{\beta-1}{2}} \approx$$
$$\approx 1.6\cdot 10^{18}\,\mathrm{eV}^{\frac{4}{5}}\left(\frac{m}{50\,\mathrm{GeV}}\right)^{\frac{1}{4}}\left(\frac{\alpha_y}{1/30}\right)^{\frac{11}{10}}. \quad (40)$$

Taking into account Eq.(31) they can be represented as

$$D_R = \frac{T_{rec}^{\beta_R}}{r_{rec}},\;\; D_M = \frac{T_{rec}^{\beta_M+\frac{1}{2}}}{r_{rec}T_{RM}^{1/2}}. \quad (41)$$

Number density parameter r itself is derived from equation

$$\frac{dn}{dt} = -n^2\langle\sigma_b v\rangle - 3Hn. \quad (42)$$

With Eqs.(14,17) and introduced notations it gets

$$\frac{dr}{dT} = \langle\sigma_b v\rangle\frac{r^2 s}{HT} = D_s\frac{r^2}{\theta^{\frac{\beta-1}{2}}T^{\beta_s+1}}. \quad (43)$$

Equations (43,39) form the system with initial conditions $\theta(T_{rec}) = 1$, $r(T_{rec}) = r_{rec}$. To solve given system one devides Eq.(43) onto Eq.(39) from where one gets independently on the stage

$$r(\theta) = r_{rec}\,\theta^{-1/\gamma}. \quad (44)$$

Being interested in $r(T)$ on MD-stage, we will put the solution of system for RD-stage to be initial conditions for that for MD-stage. Substituting $r(\theta)$ of Eq.(44) in this manner into Eq.(39) yields

$$\theta(T) = \left\{1 + \frac{\bar{\gamma}}{\beta_R}D_R r_{rec}\left(\frac{1}{T_{RM}^{\beta_R}} - \frac{1}{T_{rec}^{\beta_R}}\right) + \right.$$
$$\left. + \frac{\bar{\gamma}}{\beta_M}D_M r_{rec}\left(\frac{1}{T^{\beta_M}} - \frac{1}{T_{RM}^{\beta_M}}\right)\right\}^{\gamma/\bar{\gamma}}, \quad (45)$$

where $\bar{\gamma} = 1 + \gamma\frac{\beta-1}{2}$. Using relations Eq.(41) one can express Eq.(45) in terms of T_{rec}

$$\theta(T) = \left\{1 - \frac{\bar{\gamma}}{\beta_R} - \frac{\bar{\gamma}}{2\beta_R\beta_M}\left(\frac{T_{rec}}{T_{RM}}\right)^{\beta_R} + \right.$$
$$\left. + \frac{\bar{\gamma}}{\beta_M}\left(\frac{T_{rec}}{T_{RM}}\right)^{\frac{1}{2}}\left(\frac{T_{rec}}{T}\right)^{\beta_M}\right\}^{\gamma/\bar{\gamma}}. \quad (46)$$

Note, that Eq.(45) unlike Eq.(46) depends on r_{rec} explicitly.

Function $\theta(T)$, with $\beta = 14/5$, is very slowly growing with decrease of T ($\beta_R = 4/5$, $\beta_M = 3/10$, $\gamma = 1/6$, $\bar{\gamma} = 23/20$). At $T \ll T_{RM} \ll T_{rec}$, $\theta(T) \propto T^{-1/23}$. Nonetheless, at $T_{fin} \approx 27$ K for $m = 50$ GeV and $\alpha_y = 1/30$ $\theta \approx 4.7$.

For finding $r(T)$ one can use Eq.(44).

$$r = r_{rec}\left\{1 + \frac{\bar{\gamma}}{\beta_R}D_R r_{rec}\left(\frac{1}{T_{RM}^{\beta_R}} - \frac{1}{T_{rec}^{\beta_R}}\right) + \right.$$
$$\left. + \frac{\bar{\gamma}}{\beta_M}D_M r_{rec}\left(\frac{1}{T^{\beta_M}} - \frac{1}{T_{RM}^{\beta_M}}\right)\right\}^{-1/\bar{\gamma}}. \quad (47)$$

Or in terms of T_{rec}

$$r = r_{rec}\left\{1 - \frac{\bar{\gamma}}{\beta_R} - \frac{\bar{\gamma}}{2\beta_R\beta_M}\left(\frac{T_{rec}}{T_{RM}}\right)^{\beta_R} + \right.$$
$$\left. + \frac{\bar{\gamma}}{\beta_M}\left(\frac{T_{rec}}{T_{RM}}\right)^{\frac{1}{2}}\left(\frac{T_{rec}}{T}\right)^{\beta_M}\right\}^{-1/\bar{\gamma}}. \quad (48)$$

At $\gamma \to 0$ ($\bar{\gamma} \to 1$) it tends to solution without account of energy losses due to dipole emission, i.e. to solution of Eq.(43) without factor θ. The first two terms inside $\{..\}$ of Eqs.(46,48) and respectively the terms 1 and $1/T_{rec}^{\beta_R}$ in Eqs.(45,47) can be neglected. At $T \ll T_{RM} \ll T_{rec}$ (with $\beta_{R,M} > 0$) $r(T)$ approaches to

$$r \approx \left\{r_{rec}^{\bar{\gamma}-1}\frac{\beta_M}{\bar{\gamma}}\frac{T^{\beta_M}}{D_M}\right\}^{1/\bar{\gamma}} = r_{rec}\left\{\frac{\beta_M}{\bar{\gamma}}\frac{T_{RM}^{1/2}}{T_{rec}^{1/2}}\frac{T^{\beta_M}}{T_{rec}^{\beta_M}}\right\}^{1/\bar{\gamma}}. \quad (49)$$

Figure 2: Relic densities of 4th neutrinos in units of critical density for the cases with recombination, without it and without y-interaction at all. $\alpha_y = 1/30,\ 1/60$ were taken.

The part of Eq.(49) after '$\approx$' illustrates a soft sensitivity of final density from initial conditions, which vanishes fully at $\bar{\gamma} \to 1$.

On Figure 2 the relic densities of 4th neutrinos in units of critical density of Universe obtained with the help of Eq.(48) with $T = T_{fin} = 27\,\mathrm{K}$ for $\alpha_y = 1/30$ and $1/60$ are shown. For comparison the relic densities which would be predicted without effects of recombination and without y-interaction at all are put on Fig.2. At given parameters approximate equation Eq.(49) gives result on 10% less than that of Eqs.(47,48). The carried out account of the temperature change due to dipole emission, including pairs disappearence, (Eq.(39)) gave rise to 3-4 times greater density at given parameters than without it (in 3.5 for $m = 50$ GeV, in 3.7 for $m = 75$ GeV with $\alpha_y = 1/30$). Disregard of the effect of cooling due to dipole emisson (the term $\propto 1/3E_{rel}$ in Eq.(36)), accounting only pairs disappearence, would increase the obtained result for $m = 50,\ 75$ GeV in 2.2 and 2.3 times respectively.

3. The case of 4th generation quarks presence

The quarks of 4th generation (Q) could be present in early and modern Universe too provided they are longliving enough [2]. The y-charge of Q in 3 times less than that of N (for y-charge of N we set $|e_{yN}| = 1$) would provide cancellation of triangle axial anomalies for particles of 4th generation possessing new interaction. After hadronization 4th generation quarks can come into hadron states $\{Qqq\}^{1/3}$, $\{QQQ\}^1$, $\{\bar{Q}q\}^{1/3}$, $\{\bar{Q}\bar{Q}\bar{Q}\}^1$, where the modules of y-charges are marked, q is the quark of first three generations.

Gas of Q-hadrons in early Universe is in equilibrium with ordinary matter, due to mainly electromagnetic interaction, and also with y-background while $T > T_{Qy}$ being given analogously to Eq.(23). Their temperature is supposed to be T. [[**We will not investigate which temerature they have. If it is apparent that Coulomb Q-Nucl or/and Q-e(-relat. and non-relat.) interactions is most effective, then one needs to say affirmetively that $T_Q = T$. Otherwise, simple comparison of only Q-y and Q-γ interactions does not allow to say it.**]] Equilibrium between N- and Q-gases is established if respective energy exchange is fast in respect to expansion rate. For N- and Q- components of matter interacting Coulomb-likewise one has accordingly to [8]

$$\frac{\delta Q_{NQ}}{Vdt} = \sum_Q 2\pi\alpha_y^2 e_{yQ}^2 Lnn_Q \int \left(\frac{1}{T_N} - \frac{1}{T_Q}\right) \times \\ \times \frac{v^2(\vec{v}_N\vec{v}_Q) - (\vec{v}\vec{v}_N)(\vec{v}\vec{v}_Q)}{v^3} f_N f_Q d^3v_N d^3v_Q. \quad (50)$$

Here the sum is over sorts of Q-hadrons, e_{yQ} is their y-charge, L is the Coulomb logarithm which is defined with the help of Debye radius R_D as [[**I did not clear up how it must look like in case of different temperatures of components**]]

$$L \approx \ln\frac{R_D T}{\alpha_y |e_{yQ}|} \approx \ln\frac{1}{\alpha_y^{3/2} r^{1/2}} \approx 22, \quad (51)$$

the rest of notation resembles already introduced ones. Integration over (Maxwell) distribution gives us

$$\frac{\delta Q_{NQ}}{nVdt} = \sum_Q 4\sqrt{2\pi}\alpha_y^2 e_{yQ}^2 L n_Q \times \\ \times \sqrt{\frac{m m_Q}{(m_Q T_N + m T_Q)^3}}(T_Q - T_N). \quad (52)$$

This term must enter right side of equation (9). Comparison of it with $3HT_N$ will give us the limiting moments when Q-N equilibrium is established.

For Q-hadrons, analogously to N, we have $n_Q = s \cdot r_Q$. So, equlibrium between 4th neutrinos and Q-hadrons, having $T_Q = T$ (it can fail for electro-neutral $\{\bar{Q}q\}$ mesons), would take place on RD-stage at the temperature below

$$T_{NQ} = \frac{32\pi^2}{405}\frac{g_s^2}{g_\varepsilon}\alpha_y^4 L^2 m_{Pl}^2 m \left(\sum_Q \frac{m_Q^{1/2} e_{yQ}^2 r_Q}{(m_Q + m)^{3/2}}\right)^2. \quad (53)$$

On MD-stage energy exchange due to Coulomb interaction turns out to slow down with the same rate as external conditons change (expansion got faster). Maybe, a pecularity of nonhomogenities growth can change equality of these slopes. So, if equilibrium had not been established on RD-stage (i.e. at $T > 1$ eV), then it can hardly happen on MD-stage.

One can show that whatever composion of Q-plasma is (provided that all types of Q-hadrons, including $\{\bar{Q}q\}$, have $T_Q = T$) the sum in Eq.(53) is the same, if m is neglected in it. Indeed, let m_4 and r_4 be the mass of Q-quark and their relative concentration like r (the same for Q-antiquarks), Qi ($\bar{Q}i$) and w_i ($\bar{w}_i$)

denote respectively hadron containing i Q-(anti)quarks and the share of all Q-(anti)quarks contained in them ($\sum_i w_i = \sum_i \bar{w}_i = 1$). Then $m_{Qi} = i \cdot m_4$, $r_{Qi} = w_i \cdot r_4/i$, $e^2_{yQi} = i^2/9$. So, for sum in Eq.(53) we have

$$\sum_Q = \sum_{Qi} + \sum_{\bar{Q}i} = 2\sum_i \frac{i^2/9 \cdot w_i r_4/i}{i m_4} = \frac{2r_4}{9m_4}. \quad (54)$$

The value r_4 is given by Eq.(13) of [2] which relates to number density of Q-quarks after their freezing out due to hadronic recombination at $T = T_{QCD} \approx 150$ MeV. For T_{NQ} in three cases A, B, C (see [2]) one formally gets (provided $T_{QCD} > T_{NQ} > T_{RM}$) [[**in case C denominator in r_4 was rejected**]]

$$T_{NQ} = \begin{cases} 0.12\,\mathrm{eV}\frac{m}{50\,\mathrm{GeV}}\left(\frac{250\,\mathrm{GeV}}{m_4}\right)^3\left(\frac{\alpha_y}{1/30}\right)^2\frac{\bar{g}}{4.7} - \mathrm{A}, \\ 102\,\mathrm{eV}\frac{m}{50\,\mathrm{GeV}}\left(\frac{250\,\mathrm{GeV}}{m_4}\right)^3\left(\frac{\alpha_y}{1/30}\right)^2\frac{\bar{g}}{4.7} - \mathrm{B}, \\ 69\,\mathrm{GeV}\frac{m}{50\,\mathrm{GeV}}\frac{m_4}{250\,\mathrm{GeV}}\left(\frac{\alpha_y}{1/30}\right)^4\frac{\bar{g}}{70} - \mathrm{C}, \end{cases} \quad (55)$$

where $\bar{g} = g_s^2/g_\varepsilon$. It is clear, that inspite on strong dependence on parameters the equilibrium between N and Q in case C takes place during all period of interest. The equilibrium in case A is hardly possible.

T_{NQ} obtained for case B implies intermediate situation. If $T_{NQ} \sim 100$ eV then it corresponds to period when recombination of 4th neutrinos proceeds (compare with Eq.(23)). Number density of 4th neutrinos by $T = T_{NQ}$ is reduced down to the value (being given by Eq.(47) with $T_{RM} \leftrightarrow T_{NQ}$ and without last term relating to recombination on RD-stage)

$$r = r_{rec}\left\{1 + \frac{\bar{\gamma}}{\beta_R}D_R r_{rec}\left(\frac{1}{T_{NQ}^{\beta_R}} - \frac{1}{T_{rec}^{\beta_R}}\right)\right\}^{-1/\bar{\gamma}} = $$
$$= r_{rec}\left\{1 + \frac{\bar{\gamma}}{\beta_R}\left(\left(\frac{T_{rec}}{T_{NQ}}\right)^{\beta_R} - 1\right)\right\}^{-1/\bar{\gamma}} \quad (56)$$

and does not decrease further since N-Q equilibrium provides $T_N = T$ bringing to nought recombination rate. Minimal estimation for r in case of Q-N equilibrium establishment is provided by $T_{NQ} = T_{RM}$. It is

$$r_{min} \approx r_{rec}^{1-\frac{1}{\bar{\gamma}}}\left(\frac{\beta_R T_{RM}^{\beta_R}}{\bar{\gamma}D_R}\right)^{\frac{1}{\bar{\gamma}}} \approx 1.6\cdot 10^{-18} \times$$
$$\times\left(\frac{r_{rec}}{1.22\cdot 10^{-15}}\right)^{\frac{3}{23}}\left(\frac{50\,\mathrm{GeV}}{m}\right)^{\frac{5}{23}}\left(\frac{1/30}{\alpha_y}\right)^{\frac{22}{23}}, \quad (57)$$

where, remind, $r_{rec} = \min\{r_f, r_{H\,Ny}\}$.

If recombination of 4th neutrinos proceeds, what takes place in cases A and, at least for a while, in case B, then cross recombination between N and Q can seem to be possible. Let us study this opportunity.

Cross section of N-Q recombination will be given by Eq.(27) with $r_{cl} \leftrightarrow \frac{|e_{yQ}|\alpha_y}{2\mu}$, where μ is the reduced mass of Q and N, and v being their relative velocity. In period of question N and Q have different temperatures. Recombination rate given by Eq.(28) is generalized on this case by replacement $\frac{m}{T_N} \leftrightarrow \frac{2mm_Q}{mT_Q + m_Q T_N}$, so

$$\Gamma_{NQrec} = \frac{(2\pi)^{\frac{9}{10}} e^2_{yQ}\alpha_y^2\Gamma(\frac{3}{5})}{\mu^2}\left(\frac{mm_Q}{mT_Q + m_Q T_N}\right)^{\frac{9}{10}} n_Q. \quad (58)$$

The same value for Q-hadrons (Γ_{QNrec}) is obtained from Eq.(58) by replacment $n_Q \leftrightarrow n$. Moreover one can assume $T_N \ll T_Q$ so that $m_Q T_N \ll mT_Q$. It is seen that at formal transition from the case of N-Q equilibrium to the case of its absence recombination rate increases in $(m_Q/m)^{9/10}$ times and does not change its slope of temperature dependence. It makes difference between theses cases be not of principle from point of view of cross recombination. Comparison of Γ_{NQrec} with H gives that recombination does not happen in any significant for 4th neutrinos degree (moreover if it took place then the talk would be about just a small fraction of 4th neutrinos to be bound with Q since the latters in cases A, B are less numerous than N during all periods up to galactic stages). [[**It is strange but something seems possible in case C. - the question of CRB. And it is not clear in case B. What is about before T_{QCD} when r_4 is big? - the problem of CRB.**]]

For Q-hadrons the behavior of recombination rate is qualitively the same but quantitively apparently not because of different amounts of Q and N. Cross recombination can proceed in substantional degree for Q-hadrons, though exact prediction is strongly dependent on parameters because of similarity of the temperature dependences of Γ_{QNrec} and H on RD-stage. [[**Calculation is not possible because it is not clear when recombination must start** ($r_Q(t) = r_Q(t_0)\exp\{-\int_{t_0}^t \Gamma_{QNrec}dt\} \propto (r = \mathrm{const}, RD) \propto \exp\{-C(T_0^{0.1} - T^{0.1})\}$)**.**]] [[**The situation seems entangled more in case C and in case B if Q reach equilibrium with N. Will Q capture N during nonhomogeneities growth? If yes then it can seem to exclude this case at all. It would be required to consider this case further.**]]

So, consideration of N and Q being present together in primordial plasma does not result in essential changes of 4th neutrinos evolution in case A, does change it in case C, so 4th neutrinos do not recombinate preserving their density to be $r = r_{rec}$. [[**or less if they might recombinate if $\Gamma_{rec} > H$ even at $T_N = T$ - the problem of CRB. Remind, that in r_{rec} recombination at $T_N = T$ is partially taken into account already!**]] In case B 4th neutrinos can recombinate but partially so their density does not reduce below Eq.(57). Q-quarks can enter partially bound states with 4th neutrinos, e.g. $N\{QQQ\}$-like.

4. γ-emission from primordial recombination of 4th neutrinos

In case A and in case B if T_{NQ} of Eq.(53) < 1 eV NN-recombination proceeds on MD stage. γ-radiation induced by annihilaton in this period will contribute into extragalactic γ-emission in energy range measured by EGRET.

Annihilation effects produced by NN-recombination on RD-stage is rather inconspicuous. Spectrum of the relic photons (CMB) would be distorted in perceptible degree if energy realization were greater in unit volume than $10^{-4}\varepsilon_\gamma$ and happened later than $T \approx 5$ keV [9]. Total energy realized in the result of recombination at $T < 5$ keV is given by (see Eqs.(56,57) with $T_{RM} \leftrightarrow 5$ keV)

$$\frac{\delta\varepsilon}{\varepsilon_\gamma} \lesssim \frac{2ms(5\,\mathrm{keV})r(5\,\mathrm{keV})}{\varepsilon_\gamma(5\,\mathrm{keV})} \approx 2.5\cdot 10^{-8}\left(\frac{5\,\mathrm{keV}}{T}\right)^{\frac{7}{23}} \times$$
$$\times\left(\frac{r_{rec}}{1.22\cdot 10^{-15}}\right)^{\frac{3}{23}}\left(\frac{50\,\mathrm{GeV}}{m}\right)^{\frac{5}{23}}\left(\frac{1/30}{\alpha_y}\right)^{\frac{22}{23}}. \qquad (59)$$

This estiamtion makes annihilation effects be hardly constrained by data on CMB. Other types of data (EGRET,...) are to be analyzed.

During recombination, within interval, when Universe temperature falls down on dT, $s\cdot dr$ pairs of $N\bar{N}$ in unit volume annihilate, where dr is given by Eq.(43). Let $B_Z = \frac{1}{1+\sigma_{yy}/\sigma_Z}$ be fraction of annihilation acts going through the channel $N\bar{N} \to Z \to ...$, being determined with the help of Eq.(4), and $dN_\gamma(E_0) = f_{\gamma\,0}(E_0)dE_0$ be averaged multiplicity of created γ in this channel within energy interval $E_0 - E_0 + dE_0$. Due to redshift modern energy $E - E + dE$ of photons emitted at the temperature T (redshift z) differs from initial accordingly

$$E_0 = (z+1)E = \frac{T}{T_{mod}}E, \qquad (60)$$

where $T_{mod} = 2.7$ K. Taking into account that number density evolves as $\propto s$, for modern number density of annihilation γ we have

$$dn_\gamma(E) = B_Z\cdot dN_\gamma(E_0)\cdot s(T_{mod})\cdot dr(T) =$$
$$= B_Z s_{mod}\cdot f(E_0(T,E))\frac{T}{T_{mod}}dE\cdot r'_T dT, \qquad (61)$$

where $r'_T = \frac{dr}{dT}$ of Eq.(43), $s_{mod} = s(T_{mod}) \approx 3\cdot 10^3$ cm^{-3}. One can pass from integration dT to dE_0. For intensity from Eq.(61) we have

$$I_\gamma(E) = \frac{c}{4\pi}\frac{dn_\gamma}{dE} = \qquad (62)$$

$$= \frac{B_Z s_{mod} c}{4\pi T_{mod}}\int_{T_{fin}}^{\frac{E_{0max}}{E}T_{mod}} f(E_0(T,E))\cdot r'_T(T)\cdot T dT =$$

Figure 3: Gamma fluxes from reombination of 4th neutrinos in early Universe in comparison with EGRET data. $\alpha_y = 1/30,\ 1/60,\ 1/137$ and $m = 50,\ 75$ GeV were taken.

$$= \frac{B_Z s_{mod} c T_{mod}}{4\pi E^2}\int_{\frac{T_{fin}}{T_{mod}}E}^{E_{0max}} f(E_0)\cdot r'_T(T(E,E_0))\cdot E_0 dE_0.$$

Here E_{0max} is the upper limit of annihilation γ spectrum, c is the light speed. All transformations between E_0, E, T are in accordnace with Eq.(60).

Using results of previous section one obtains intensity of γ from priomordial 4th neutrino recombination as shown on Figure 3. Two magnitudes of m and α_y were taken. EGRET data on extragalactic γ-radiation are put for comparison. Spectra $f(E_0)$ had been obtained with the help of code PYTHIA 6.2.

Effect of T_N variation due to dipole emission (including pairs disappearence), taken into account here, raises predicted γ flux in 2-3 times.

Note, that because of restricted maximal energy of the created γ (by mass of 4th neutrino) annihilation photons born in period $T \lesssim 3000\,\mathrm{K} < T_{rec}$ only contribute to the energy range of EGRET. By this period the most of 4th neutrinos had been annihilated, so full effect in interested range is not proportional integrally to the total initial r_{rec} but rather to the value given by Eq.(49). It explains the dispalyed dependence of predicted γ-flux from α_y, so $I_\gamma(\alpha_y) \propto r'_T(\alpha_y)B_Z(\alpha_y) \propto D_M(\alpha_y)^{-\frac{20}{23}} r_{rec}^{\frac{3}{23}}(\alpha_y)B_Z(\alpha_y) \to D_M(\alpha_y)^{-1}B_Z(\alpha_y)$, where the last limiting transition for $\bar{\gamma} \to 1$.

Comparison with EGRET data allows to restrict admissible magnitudes of α_y as $\alpha_y \gtrsim 1/137$ for $m = 50$ GeV and a little more severe constraint for greater m.

5. Effects of new interaction of 4th neutrinos in Galaxy

If 4th neutrinos had been recombinating on pre-galactic stage, so their density had been reduced as shown on Fig.2, entering Galaxy they recombinate much slower. The mean time of recombination of 4th neutrinos in Galaxy exceeds strongly the age of Universe t_{mod} [[**ev-**

erywhere here the talk is about ordinary case whithout effects of Q]]. However, the typical lifetime of created bound systems in Galaxy is much less than t_{mod}. Slow recombination can produce γ-flux being measurable.

We will assume that 4th neutrinos are distributed in Galaxy accordingly to isothermal halo model. So

$$\rho(R) = \rho_{N\,loc}\frac{(1\,\mathrm{kpc})^2 + (8.5\,\mathrm{kpc})^2}{(1\,\mathrm{kpc})^2 + R^2}, \tag{63}$$

where R is the distance to the galactic center, $\rho_{N\,loc} = \xi \cdot 0.3\ \mathrm{GeV/cm^3}$. Velocity disribution is Maxwellian with the maximal likelihood value $u = 220$ km/s. Dependence of γ-flux prediction from halo model choice is not of princpile here, formulated just by quantitative factor (as rule, within 1-2).

However, said above relates to homogenously distributed density, without small scale inhomogeneities (clumps), being predicted for Cold Dark Matter (CDM). Possibility of such a small scale structure induces its uncertainty in predictions of annihilation effects. This uncertainty is put into an addition (quasi)free paramer - degree of possible enhancement of recombination (annihilation) rate due to inhomogenous structure (η). As it was estimated in [1] this parameter can reach the magnitudes 20-30, while able to be vanishing because of its crucial dependence on primordial perturbation spectrum, exactly unknown [10]. However this estiamtion relates to the case of N without y-interaction. Consideration of the effect of clumpiness of 4th neutrinos with y-interaction should take into account both the change of clump mass scale and velocity dependence of recombination rate what changes drastically this effect. Let us consider it in more detail following the work [10] as previously.

Clumps are predicted to be distributed in mass so starting from minimal value $M = M_{min}$ their number falls down with the increase of M. The value M_{min} is determined by free-streaming scale of DM particles at the moment when inhomogeneities start to grow. It in its term is determined by the moment of decoupling of DM particles from surrounding matter which for y-charged 4th neutrinos should be taken to be $\bar{T}_{Ny}$. For minimal mass of clump containing N we get from [10] approximately

$$M_{N\,min} \approx 10^{-2} M_{\odot}\left(\frac{50\,\mathrm{GeV}}{m}\right)^{15/4}\left(\frac{\alpha_y}{1/30}\right)^{3/2}, \tag{64}$$

where $M_{\odot}$ is the solar mass. One supposes [1], as a basic approximation, that 4th neutrino density **is distributed** at scales larger than $M_{N\,min}$ **proportionally** to that of CDM, so their relative contribution is $\xi = \Omega_N/\Omega_{CDM}$, where Ω_N is given by Fig.2, $\Omega_{CDM} = 0.3$. Here we will neglect the possibility for 4th neutrinos to enter the clumps of mass less than $M_{N\,min}$ considered in [1].

Clumps of mass around minimal one give the main contribution into annihilation enhancement effect. Non-interacting DM with $M_{min} = M_{N\,min}$ could have enhancement factor in range (see Figures 3-5 of [10])

$$\eta_0 = 1...5. \tag{65}$$

Note, that this factor $\eta_0 \propto \bar{n}_{cl}^2/\bar{n}_{Gal}^2$, where inserted values are respectively the averaged squared number densities of DM particles inside the clumps and homogeneously distributed in Galaxy.

In case of 4th neutrinos, annihilaiton rate strongly increases due to its velocity dependence so their number in clump can experience essential changes. For proper estimations other parameters of clump are needed to know. Radius of clump of $M = M_{N\,min}$, being in fact dependent on other model patrameters, is [10]

$$R \approx 0.7 \cdot 10^{18}\,\mathrm{cm}\left(\frac{50\,\mathrm{GeV}}{m}\right)^{5/4}\left(\frac{\alpha_y}{1/30}\right)^{1/2}. \tag{66}$$

Distribution of number density n inside the clump is needed to know as detail as

$$S = \frac{V}{N^2}\int n^2 dV, \tag{67}$$

where $V = \frac{4}{3}\pi R^3$, $N = \int n dV$ is the total amount of given particles inside the clump; $\bar{n} = \frac{N}{V}$ gives the mean number density. The value S is given by Eq.(4) of [10] and with admissible paramerters is obtained to be $S = 5.2$. Typical velocity of particles inside the clump with $M = M_{N\,min}$ is determined by

$$u_{cl} = \sqrt{\frac{GM_{N\,min}}{R}} \approx 14\,\mathrm{m/s}. \tag{68}$$

We will assume that this velocity parameter is constant over all clump volume.

For study of evolution of 4th neutrinos in the clump we will take into account effects of heating due to slow pairs disappearence and of dissipation due to dipole emisson (the latter, as was shown above, stands down the former). Parameter Eq.(68) defines initial temperature of N: $T_{N0} = \frac{mu_{cl}^2}{2}$. Any suppression of 4th neutrinos number in the clump will not virtually affect its gravitational potential and mass being determined by dominant DM component. We will assume that the shape of distribution of 4th neutinos inside the clump does not alter during time matching with that of non-interacting DM (the comments will be below), so $S = 5.2 = \mathrm{const}$. Then, analogously to Eqs.(36,42) for total amount of 4th neutrinos inside the clump one has

$$\left\{\begin{array}{r} \frac{dN}{dt} = -\langle\,\sigma_b v\,\rangle N^2\frac{S}{V} \\ \frac{3}{2}\frac{dT_N}{dt} = \langle\,(\frac{3}{2}T_N - E - \frac{1}{3}E_{rel})\sigma_b v\,\rangle N\frac{S}{V}. \end{array}\right. \tag{69}$$

Solution of this system is analogous to that of Eqs.(39,43) and yields

$$N = N_0\left(\frac{T_{N0}}{T_N}\right)^{1/\gamma},$$

$$T_N = T_{N0}\left\{1+\frac{t-t_0}{\tau}\right\}^{\gamma/\bar{\gamma}},$$
$$\dot{N}_{ann} = \dot{N}_{ann0}\left\{1+\frac{t-t_0}{\tau}\right\}^{-(1+\frac{1}{\bar{\gamma}})},$$
$$\tau = \frac{1}{\bar{\gamma}\langle\sigma_b v\rangle_0 \bar{n}_0 S}. \qquad (70)$$

Here $\dot{N}_{ann}$ means the total annihilaton rate given by first equation of system (69). Index "0" corresponds to the initial moment $T_N = T_{N0}$. The mean number density of 4th neutrinos in the clump and characteristic time of recombination in it τ are estimated as

$$\bar{n}_0 \approx 1.5\cdot 10^{-8}\mathrm{cm}^{-3}\xi_{50,1/30}\frac{50\,\mathrm{GeV}}{m},$$
$$\tau \approx \frac{0.077\,\mathrm{Gyr}}{\xi_{50,1/30}}\left(\frac{m}{50\,\mathrm{GeV}}\right)^{3/4}\left(\frac{1/30}{\alpha_y}\right)^{11/10}, \qquad (71)$$

where $\xi_{50,1/30}$ is ξ in units $\xi(m = 50\,\mathrm{GeV}, \alpha_{\mathrm{y}} = 1/30) = 2.32\cdot 10^{-8}$; note $\xi(50, 1/137) = 1.08\cdot 10^{-7}$. Roughly, nedlecting by dependence of r on r_{rec}, $\xi \propto m^{18/23}\alpha_y^{-22/23}$, so τ is almost independnet on m and α_y. [[**Sergey, could you check relations between typical times (of binding, destruction) for clumps what you did for early Universe?**]]

Obtained τ exceeds typical dynamical time of clump $\sim R/u_{cl} \approx 0.016$ Gyr what tells in favour of that the shape of 4th neutrinos distribution inside the clump has time to take the "equilibrium" form during recombination. However, this time is rather smaller than duration of clump formation so effects of recombination can be significant on pre-clump stages already. The temperature T_N during $t - t_0 \sim 10$ Gyr increases in 2 times. It implies that "equilibrium" form of N distribution must experience washing out and an effect of N evaporation increases its role. Mentioned arguments, from them the effect of evaporation can be crucial, must lead to the additional slowing down of annihilation rate in the clump in respect to estimated in given approximation.

Enhancment factor η for 4th neutrinos at the modern moment will be given by the product of (formally) initial factor, determining by annihilation rate $\dot{N}_{ann0}$ in Eq.(70), and of factor in brackets {...} of that equation. It is the value Eq.(65) multiplied by $(u/u_{cl})^{9/5}$ what gives initial factor. Unit in {...} can be neglected. So

$$\eta = \eta_0\left(\frac{u}{u_{cl}}\right)^{9/5}\left(\frac{\tau}{t-t_0}\right)^{1+\frac{1}{\bar{\gamma}}} < 1.1\cdot 10^4. \qquad (72)$$

The quoted magnitude relates to $m = 50$ GeV, $\alpha_y = 1/30$, its dependence from this parametrers, neglecting by mentioned soft such depndence of τ, is $\propto m^{9/4}\alpha_y^{-9/10}$. Obtained estiamtion does not take into account spectrum of clumps in mass. Account of large mass tail of clump distribution will just a little decrease η. Indeed, in Eq.(72), except the value η_0, the value u_{cl} only depends on M giving additional suppression $\propto M^{-3/5}$.

Figure 4: γ-fluxes from recombination of 4th neutrinos in early Universe and in Galaxy with enhancement factor 200 in comparison with EGRET data for $\alpha_y = 1/137$ and $m = 50$ GeV.

Active annihilation of 4th neurions inside the clumps, mainly at $z \sim 20...10$, will give its contribution into γ-flux. However, estimated contribution from annihilaiton of homogenously distributed 4th neutrinos even at the same period by intensity is comparable with that of clumps and can not be integrally less than that because in the clumps just a small remains of 4th neutrinos, after primordial recombination, annihilated. Provided a big fraction of the rest primordial 4th neutrinos enters clumps and clump destruction time is greater τ, homogenous 4th neutrino population of Galaxy will be reduced.

Intensity of γ is determined by

$$I(E) = \eta\cdot\frac{B_Z\langle\sigma_b v\rangle}{4\pi}f_{\gamma 0}(E)\int_0^\infty\left(\frac{\rho}{m}\right)^2 dl. \qquad (73)$$

For $m = 50$ GeV, $\alpha_y = 1/137$ with factor $\eta \approx 200$ one obtains γ-flux from annihilation of 4th neutrinos in halo as shown on Figure 4.

For greater α_y, to provide explanation of respective part of EGRET data, a similar factor η is required.

6. Discussion

Considered case of y-interacting 4th neutrinos shows that this component of DM, being strongly suppressed in density due to primordial recombination, can account in principle for observation data on extragalactic γ-emission EGRET at $m \approx 50$ GeV and $\alpha_y \approx 1/100...1/137$.

However, such a small fine structure constant, in considered case, implies rather specific requirements for Grand Unification (GUT) models involved y-interaction. Indeed, y-interaction, being restricted with particles just of 4th generation, to be unified with other interactions at large energy scale, should have constant at low energy scale greater than electromagnetic one, which grows with energy due to screening by virtual particles of all four generations.

In case of another WIMP possessing its Coulomb-like interaction, the consideration above can be mostly applied. Early freezing out, determined by annihilaiton cross section of WIMPs, influences on the final results very weakly. The main differences will concern to B_Z and $f_{\gamma 0}$.

7. On references

References are numbered in the text in the form [1], [2], etc.; the format of the list of references is indicated in the sample that follows, so PLEASE CAREFULLY FOLLOW IT, paying attention to the order in which the information appears, spaces, quotation marks, etc. The titles of articles are (unlike books) not necessary, but desirable for preprints and e-prints.

The order of references in the list may either correspond to first citations in the text, or be alphabetic.

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8. Appendix A

Here the values $\langle\overline{\Delta E}\sigma v\rangle_{Na}$, where $a = y$, $\nu_{e,\mu,\tau}$, $\bar{\nu}_{e,\mu,\tau}$, e^-, e^+, are calculated. In all scattering processes of interest N is non-relativistic, a is ultra-relativistic. Many calculations we will do in laboratory system (l.s.), where initial N is at rest, and then transform the result in comoving reference frame, where N- and a- gases are at rest as whole. Let us assign 4th-momentums for initial and final particles N and a as $p_N = \{m+E, \vec{p}\}$, $p'_N = \{m+E', \vec{p'}\}$ and $k_a = \{\omega, \vec{k}\}$, $k'_a = \{\omega', \vec{k'}\}$; $p = |\vec{p}| = \sqrt{2mE}$, $k = |\vec{k}| = \omega$. The values in l.s. will be endued with index "l.s.", in comoving system - without any indexes.

In given approximation for Ny-scattering (Compton-effect) Thomson cross section can be taken

$$\sigma_{Ny} = \sigma_T = \frac{8\pi}{3} r_{cl}^2, \tag{74}$$

where $r_{cl} = \alpha_y/m$. For estimation of $\overline{\Delta E}_{Ny}$ we can use distribution in $\cos\theta_{\vec{k}_{l.s.}\vec{k'}_{l.s.}}$ [5]

$$\frac{dw}{d\cos\theta_{\vec{k}_{l.s.}\vec{k'}_{l.s.}}} \sim 1 + \cos^2\theta_{\vec{k}_{l.s.}\vec{k'}_{l.s.}},$$
$$-1 < \cos\theta_{\vec{k}_{l.s.}\vec{k'}_{l.s.}} < 1, \tag{75}$$

where $\theta_{\vec{k}_{l.s.}\vec{k'}_{l.s.}}$ means angle between respective vectors. From Eq.(75) for the mean value it follows

$$\overline{\cos\theta_{\vec{k}_{l.s.}\vec{k'}_{l.s.}}} = 0, \tag{76}$$

what will be usefull later on. In current case ($\omega_{(l.s.)} \ll m$) $\omega_{l.s.} \approx \omega'_{l.s.}$ and with accuracy $\sim (\omega/m)^2$

$$\Delta E_{l.s.} = E'_{l.s.} - E_{l.s.} = \omega_{l.s.} - \omega'_{l.s.} \approx$$
$$\approx \frac{\omega_{l.s.}^2}{m}(1 - \cos\theta_{\vec{k}_{l.s.}\vec{k'}_{l.s.}}). \tag{77}$$

Here $E_{l.s.} = 0$. For the mean transferred energy knowing Eq.(76) one gets from Eq.(77) at once (or through the proper transformation $dw/d\cos\theta_{\vec{k}_{l.s.}\vec{k'}_{l.s.}} \to dw/d(\Delta E_{l.s.})$)

$$\overline{\Delta E_{l.s.}} = \frac{\omega_{l.s.}^2}{m}. \tag{78}$$

For mean transferred energy in comoving system (at the fixed $\vec{p}$ and $\vec{k}$) we have accordingly to Galilei transformation for kinetic energy

$$\overline{\Delta E} = \overline{\Delta E_{l.s.} + \vec{v}\vec{p'}_{l.s.}} = \overline{\Delta E_{l.s.}} + \overline{\vec{v}\vec{p'}_{l.s.}}, \tag{79}$$

where $\vec{v}$ is the velocity of initial N in comoving system (velocity of laboratory reference frame in respect to comoving one). Vector $\vec{p'}_{l.s.}$ is distributed homogenously in azimuth angle around vector $\vec{k}_{l.s.}$ so ($\overline{\vec{p'}_{l.s.}} \sim \vec{k}_{l.s.}$)

$$\overline{\vec{v}\vec{p'}_{l.s.}} = v \cdot \overline{p'_{l.s.}\cos\theta_{\vec{k}_{l.s.}\vec{p'}_{l.s.}}} \cdot \cos\theta_{\vec{k}_{l.s.}\vec{v}}. \tag{80}$$

Let us denote $z = p'_{l.s.}\cos\theta_{\vec{k}_{l.s.}\vec{p'}_{l.s.}}$. In given approxiamtion (when $\omega_{l.s.} \approx \omega'_{l.s.}$)

$$z \approx \omega_{l.s.}(1 - \cos\theta_{\vec{k}_{l.s.}\vec{k'}_{l.s.}}) \quad \Rightarrow \quad \bar{z} = \omega_{l.s.}. \tag{81}$$

The value $\cos\theta_{\vec{k}_{l.s.}\vec{v}}$ in Eq.(80) should be represented through the comoving system values too. From Lorentz transformations it follows

$$\cos\theta_{\vec{k}_{l.s.}\vec{v}} = \frac{\cos\theta_{\vec{k}\vec{v}} - v}{1 - v\cos\theta_{\vec{k}\vec{v}}}. \tag{82}$$

Note, that it is a small variation of this cosine with the given reference frame transformation what contributes into final answer (see Eq.(85) below). Also one transforms

$$\omega_{l.s.} = \omega\gamma(1 - v\cos\theta_{\vec{k}\vec{v}}), \quad \gamma = (1 - v^2)^{-1/2}. \tag{83}$$

In expression $\langle\overline{\Delta E}\sigma v\rangle_{Na}$ a Möller velocity should be taken (see footnote 5) which in case $v_a = 1$ is [4]

$$v_{Na} = 1 - v\cos\theta_{\vec{k}\vec{v}}. \tag{84}$$

Its difference from unit is important here. Denoting appropriate distributions in kinematic varable q (velocity, momentum, energy) as $f_i(\vec{q_i}) = dw_i/d^3q_i$, $f_i(q_i) = dw_i/dq_i$ one writes using Eqs.(78-84)

$$\langle\overline{\Delta E}\sigma v\rangle_{Ny} = \sigma_T\langle\overline{\Delta E}(1 - v\cos\theta_{\vec{k}\vec{v}})\rangle = \sigma_T \times$$
$$\times \int\left(\frac{\omega^2}{m}\gamma^2(1 - v\cos\theta_{\vec{k}\vec{v}})^3 + \omega v\gamma(\cos\theta_{\vec{k}\vec{v}} - v)\times\right.$$
$$\left.\times\ (1 - v\cos\theta_{\vec{k}\vec{v}})\right) f_y(\vec{k})d^3k f_N(\vec{v})d^3v. \tag{85}$$

After averaging over angles $\cos^{odd\,degree} \to 0$, $\cos^2 \to 1/3$. Keeping terms only $\sim 1/m$ one gets

$$\langle\overline{\Delta E}\sigma v\rangle_{Ny} \approx \sigma_T\int\left(\frac{\omega^2}{m} - \frac{8\omega E}{3m}\right) f_y(\omega)d\omega f_N(E)dE =$$

$$= \sigma_T \left(\frac{\langle \omega^2 \rangle}{m} - \frac{8\langle \omega \rangle \langle E \rangle}{3m} \right). \quad (86)$$

For Maxwell distribution in ultra-relativistic and non-relativistic cases one has

$$\langle \omega \rangle = 3T_y, \quad \langle \omega^2 \rangle = 12T_y^2, \quad \langle E \rangle = \frac{3}{2}T_N. \quad (87)$$

So

$$\langle \overline{\Delta E} \sigma v \rangle_{Ny} = \frac{32\pi\alpha_y^2}{m^3} T_y(T_y - T_N). \quad (88)$$

For estimation of $\langle \overline{\Delta E} \sigma v \rangle_{N\nu,e}$ one needs to get appropriate cross sections. Squared invariant matrix elements of the processes of elastic scattering $N\nu$, $N\bar{\nu}$, Ne^-, Ne^+, summed over all spin states, are

$$|M_{N\nu}^2| = 32G_F^2(p_N k_\nu)^2,$$
$$|M_{Ne^-}^2| = 32G_F^2\left((1-2\xi)^2(p_N k_e)^2 + 4\xi^2(p_N k'_e)^2\right) \quad (89)$$

with the change $k \leftrightarrow k'$ for $a \leftrightarrow \bar{a}$, where G_F is the Fermi constant, $\xi = \sin^2\theta_W \approx 0.23$ is the weak mixing parameter. Since in l.s. of considered case $\omega_{l.s.} \approx \omega'_{l.s.}$ (see above) then $(p_N k_a) \approx (p_N k'_a) \approx m\omega_{l.s.}$. So in case of question the squared matrix elements are independent on $\cos\theta_{\vec{k}_{l.s.}\vec{k'}_{l.s.}}$. It implies as in case of Ny-scattering $\overline{\cos\theta_{\vec{k}_{l.s.}\vec{k'}_{l.s.}}} = 0$ and all estimates for $\overline{\Delta E}$ are the same (Eqs.(78-84)). There is a difference in total cross sections, which are energy dependent. From Eq.(89) one gets

$$\sigma_{N\nu} = \sigma_{N\bar{\nu}} = \frac{G_F^2\omega_{l.s.}^2}{2\pi},$$
$$\sigma_{N\,e^-} = \sigma_{N\,e^+} = \frac{G_F^2\xi_e\omega_{l.s.}^2}{2\pi}, \quad (90)$$

where $\xi_e = 1 - 4\xi + 8\xi^2 \approx 0.50$.

Estimation of $\langle \overline{\Delta E} \sigma v \rangle_{N\nu,e}$ is analogous to Eqs.(85,86), where additional factor ω^2 should be taken into account. Necessary values are

$$\langle \omega^3 \rangle = 60T_a^3, \quad \langle \omega^4 \rangle = 360T_a^4. \quad (91)$$

Finally

$$\langle \overline{\Delta E} \sigma v \rangle_{N\nu(\bar{\nu})} = \frac{180G_F^2}{\pi m} T_\nu^3(T_\nu - T_N),$$
$$\langle \overline{\Delta E} \sigma v \rangle_{Ne^\pm} = \frac{180G_F^2\xi_e}{\pi m} T_e^3(T_e - T_N). \quad (92)$$

References

[1] K.M. Belotsky, M.Yu. Khlopov, D. Fargion, R. Konoplich, K.I. Shibaev, *Grav. & Cosmol.* **this volume**, (2005).

[2] K.M. Belotsky, M.Yu. Khlopov, D. Fargion, R. Konoplich, M.G. Ryskin, K.I. Shibaev, *Grav. & Cosmol.* **this volume**, (2005).

[3] L.D. Landau, E.M. Lifshts, *Field Theory.*

[4] P. Gondolo, G. Gelmini, *Nucl. Phys.* **B 360**, 145-179 (1991).

[5] V.B. Berestetsky, E.M. Lifshts, L.P. Pitaevsky, *Quantum Electrodynamics.*

[6] A.D. Dolgov, Phys.Rep. **370** (2002), 333; hep-ph/0202122.

[7] Ya.B. Zeldovich, M.Yu. Khlopov, Phys. Lett. **B 79** (1978), 239.

[8] Yu.B. Rumer, M.Sh. Ryvkin, *Termodynamics, Statistic Physics and Kinetics*, Novosibirsk University Press, 2001 (in rus).

[9] M.Yu. Khlopov, *Cosmoparticle Physics.*

[10] V. Berezinsky, V. Dokuchaev, Yu. Eroshenko, astro-ph/0301551.

[11] G. Green, B. Blue and W. White, "Theory of Colour", North Pole Univ. Press, North Pole, 1900.

[12] A.A. Ivanov and B.B. Petrov, "The Universe and Its Neighbourhood", gr-qc/9909999; to appear in *Phys. Rev. Z.*

[13] B.B. Petrov, *in:* "Selected Discoveries", ed. A.A. Ivanov et al., North Pole Univ. Press, North Pole, 2999.

[14] F.F. Sidorov, "Tops of Gravitational Science", PhD thesis, Everest University, 2999.

[15] L.L. Large and S.S. Small, *Proton* **931**, 1 (1901); M.M. Medium, *Neutron* **934**, 101 (2008), hep-th/0800001.