Дн по дт

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$$-\frac{dn}{dt} = \beta(T_a)\, n_б n_a n_б + 3HT$$

$$n = rs = r\cdot\frac{2\pi^2 g_s}{45}T^3 \qquad dn = dr\cdot s + \frac{3rs\,dT}{T}$$

$$-dt = \frac{dT}{HT} \quad \{ g_s = const$$

$$\frac{s\,dr + 3rs\frac{dT}{T}}{\frac{1}{HT}dT} = n_a\cdot n_б\cdot n_б\,\beta(T_б) + 3HT$$

Считаю, что В не участвует в столкновениях

$$sHT\frac{dr}{dT} + 3\cancel{H}rs = n_a\cdot n_б^2\,\beta(T_б) + \cancel{3HT}$$

$$\frac{dr}{dT} = \frac{r_a r_б^2 s^2}{HT}\beta(T_б) \sim \frac{1}{eV^5}$$

$$T_б = T_a \approx \frac{T^2}{T_{ay}} \qquad T_{ay} = \frac{T_{ay}}{\sqrt{\pi}\,\varkappa^{1/3}} \qquad T_\gamma = \varkappa^{1/3}T$$

T_{ay} – т отсоединения T_{rec}

$$\varkappa = \frac{g_{s0}(T)}{g_{s0}(T_{rec})}$$

$$n_a = n_б \Rightarrow r_a = r_б$$

$$\frac{dr}{dT} = \frac{r^3 s^2}{HT}\cdot\frac{2\sqrt{m}}{M}e^{10}\cdot\frac{1}{T_a^{9/2}}(1-\cos\theta)$$

$$\frac{dr}{r^3} = \frac{s^2}{H}\cdot e^{10}\frac{2\sqrt{m}}{M}(1-\cos\theta)\cdot\frac{1}{T}\cdot\frac{T_{ay}^{9/2}}{T^9}$$

RD: $$\frac{dr}{r^3} = \frac{\left(\frac{2\pi^2 g_s}{45}\right)^2 T^6}{h_{RD}\frac{T^2}{m_{Pl}}}\, e^{10}\frac{2\sqrt{m}}{M}(1-\cos\theta)\frac{T_{ay}^{9/2}}{T^{4+6}}$$

$$d\left(\frac{1}{-2r^2}\right) = \underbrace{\frac{\left(\frac{2\pi^2 g_s}{45}\right)^2}{h_{RD}/m_{Pl}}\, e^{10}\frac{2\sqrt{m}}{M}(1-\cos\theta)\,T_{ay}^{9/2}}_{D_{RD}}\; d\frac{1}{T^5}$$

$$n = rs = r\frac{2\pi^2 g_s}{45}T^3,$$

$$g_s = \Sigma_{bos}(T_{bos}/T)^3 + 7/8\Sigma_{ferm}(T_{ferm}/T)^3, \qquad (14)$$

where sums are over sorts of ultrarelativistic bosons and fermions, T is the temperature of photons. Hubble constant on radiation and matter dominated (RD and MD) stages can be represented as following

$$H = \begin{cases} \sqrt{\frac{4\pi^3 g_\varepsilon}{45}}\frac{T^2}{m_{Pl}} = h_{RD}\frac{T^2}{m_{Pl}} \text{ at RD stage,} \\ \frac{2}{3t_{mod}}\left(\frac{T}{2.7\,\mathrm{K}}\right)^{3/2} = h_{MD}\frac{T^{3/2}}{m_{Pl}^{1/2}} \text{ at MD stage.} \end{cases} \qquad (15)$$

Here m_{Pl} is the Plank mass, $t_{mod} \approx 4.4\cdot10^{17}$ s is the modern age of Universe, $h_{RD} \approx 5.5\sqrt{g_\varepsilon/11}$, $h_{MD} \approx 3.08\cdot10^{-14}$ and analogously to g_s of Eq.(14)

$$g_\varepsilon = \Sigma_{bos}(T_{bos}/T)^4 + 7/8\Sigma_{ferm}(T_{ferm}/T)^4. \qquad (16)$$

Value $T = T_{RM} \approx 1.2$ eV provides good transition between given dependences of H on RD- and MD-stages, connected them with factor $\sqrt{\frac{T}{T_{RM}}}$. For time intervals when $g_{\varepsilon,s} \approx const$ it is just the following transition between differentials

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$$\frac{1}{2r^2} = \frac{D_{RD}}{5T^5} + C \qquad r(T_{rec}) = r_{rec}$$

$$C = \frac{1}{2r_{rec}^2} - \frac{D_{RD}}{5T_{rec}^5} \qquad \frac{5}{2}\left(\frac{1}{r^2} - \frac{1}{r_{rec}^2}\right) = D_{RD}\left(\frac{1}{T^5} - \frac{1}{T_{rec}^5}\right)$$

$$\frac{1}{r^2(T)} = \frac{1}{r_{rec}^2} + \frac{2D_{RD}}{5}\left(\frac{1}{T^5} - \frac{1}{T_{rec}^5}\right)$$

$$r^2(T) = \frac{r_{rec}^2}{1 + \frac{2}{5}D_{RD}\left(\frac{1}{T^5} - \frac{1}{T_{rec}^5}\right)r_{rec}^2}$$

$$\boxed{r_{RD}(T) = r_{rec}\sqrt{\frac{1}{1 + \frac{2}{5}D_{RD}r_{rec}^2\left(\frac{1}{T^5} - \frac{1}{T_{rec}^5}\right)}}}$$