

Theorem: A truth table covering n variables (where n is a positive integer) has 2^n rows.

Proof by induction: First, we see that a single-variable truth table has two rows: one for truth and one for falsehood. Suppose a truth table with k variables has 2^k rows. Then there are 2^k unique combinations of truth and falsehood for those k variables. Suppose we add another variable to the table, giving it $(k+1)$ variables. Now for each unique combination in the table before modification, our new variable can be either true or false, replacing each combination with two potential unique combinations, so that we now have $2 * 2^k$ total combinations, or 2^{k+1} . Thus we may conclude that if this property holds for k , then it holds for $(k+1)$. Since the property holds for $k=1$, we know it holds for all of the positive integers. Thus, for any positive integer n , a truth table covering n variables has 2^n rows.