

Theorem.

If x isn't divisible by 10, but x is divisible by either 5 or 2, then x is not divisible by both 5 and 2.

Proof.

Suppose x is not divisible by 10. Suppose x is divisible by either 5 or 2. Now we have 2 cases.

Case 1. $x|2$. Then we have an integer k such that $2k = x$. Suppose $x|5$. Then we have an integer m such that $5m = x$. Then $m = \frac{2k}{5} = 2 * \frac{x}{10}$. But 10 doesn't divide x , so we have a contradiction. Thus, 5 doesn't divide x , so either 5 doesn't divide x or 2 doesn't divide x .

Case 2. $x|5$. Then we have an integer m such that $5m = x$. Suppose $x|2$. Then we have an integer k such that $2k = x$. Then $k = \frac{5m}{2} = 5 * \frac{x}{10}$. But 10 doesn't divide x , so we have a contradiction. Thus, 2 doesn't divide x , so either 5 doesn't divide x or 2 doesn't divide x .

Therefore, we know that 5 doesn't divide x or 2 doesn't divide x . Thus, if x isn't divisible by 10, and x is divisible by either 5 or 2, then x is not divisible by both 5 and 2. \square