CSE 152 Discussion
Week 3

April 15, 2019
Yu-Ying Yeh

Homework 1: Due on Apr 24 (Wed) 23:59

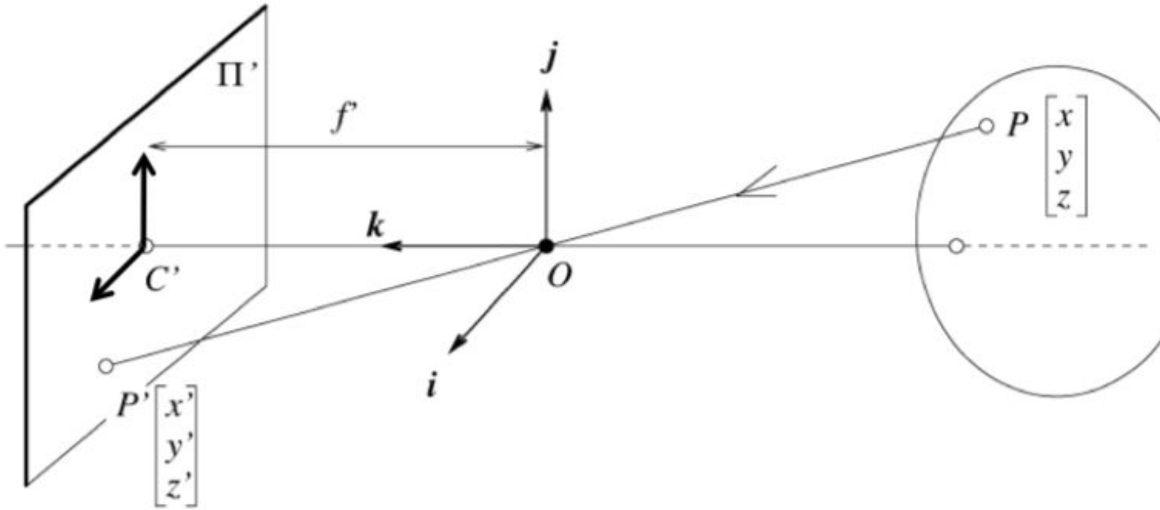
- Geometric Image Formation
- Filtering
- Template Matching
- Corner Detection
- Feature Matching using SIFT

Outline

- Geometric Image Formation
- Filtering
- Template Matching
- Corner Detection
- Feature Matching using SIFT

Geometric Image Formation

- Pinhole perspective projection

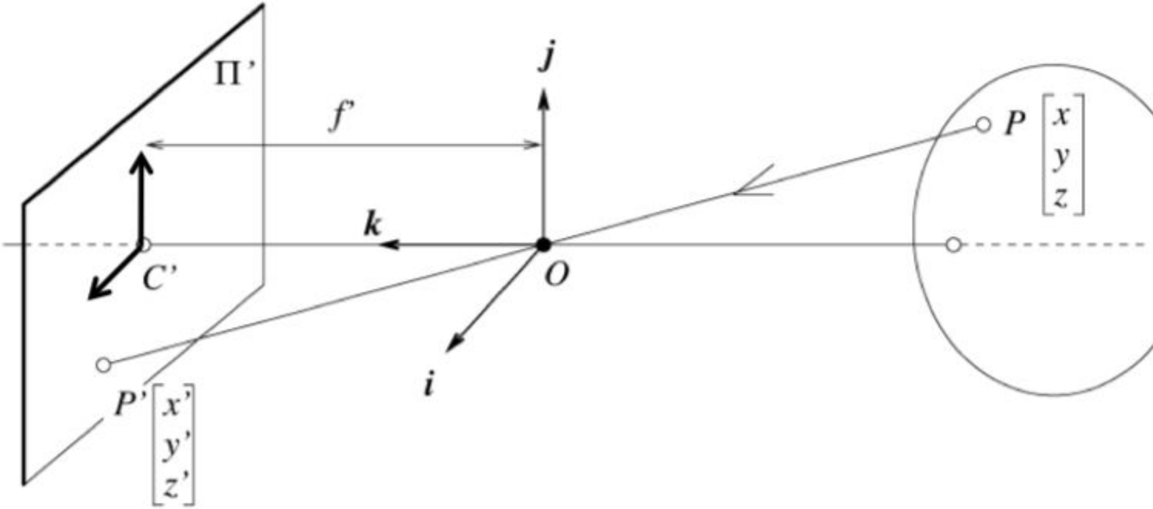


$$(x, y, z) \rightarrow (f'\frac{x}{z}, f'\frac{y}{z})$$

Not linear transformation

Geometric Image Formation

- Pinhole perspective projection



$$(x, y, z) \rightarrow (f'\frac{x}{z}, f'\frac{y}{z})$$

Not linear transformation

Perspective projection become "linear"
In Homogeneous Coordinates

Geometric Image Formation

- Project 4 points in 3D space onto camera 2D image under different cases

Geometric Image Formation

- Project 4 points in 3D space onto camera 2D image under different cases
    - Transform 3D point from Euclidean coordinates to Homogeneous coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Geometric Image Formation

- Project 4 points in 3D space onto camera 2D image under different cases
  - Transform 3D point from Euclidean coordinates to Homogeneous coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

  - Camera Projection

$$M = K\Pi_w^c T = \underbrace{\begin{bmatrix} f & s & c_x \\ 0 & \alpha f & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\substack{\text{Intrinsic} \\ \text{Parameters} \\ 3 \times 3}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\substack{\text{Projection} \\ 3 \times 4}} \underbrace{\begin{bmatrix} {}_w^c R & {}^c O_w \\ \mathbf{0}^T & 1 \end{bmatrix}}_{\substack{\text{Extrinsic} \\ \text{Parameters} \\ 4 \times 4}}$$

Rigid Transformation

$${}^B P = {}_A^B R \, {}^A P + {}^B O_A$$

Remember:

Inverse $R^{-1} = R^T$

Geometric Image Formation

- Project 4 points in 3D space onto camera 2D image under different cases
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Rigid Transformation

$${}^BP = {}_A^BR\,{}^AP + {}^BO_A$$

Remember:

Inverse $R^{-1} = R^T$

  - Transform 2D point from Homogeneous coordinates to Euclidean coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

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- Geometric Image Formation
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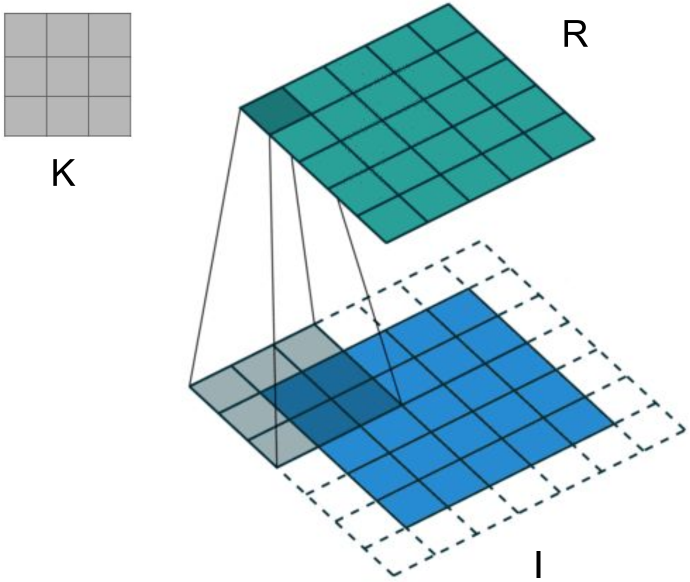
Recap: Filtering

- 2D Convolution: Apply linear filters on image
  - Kernel is **flipped** over both axes

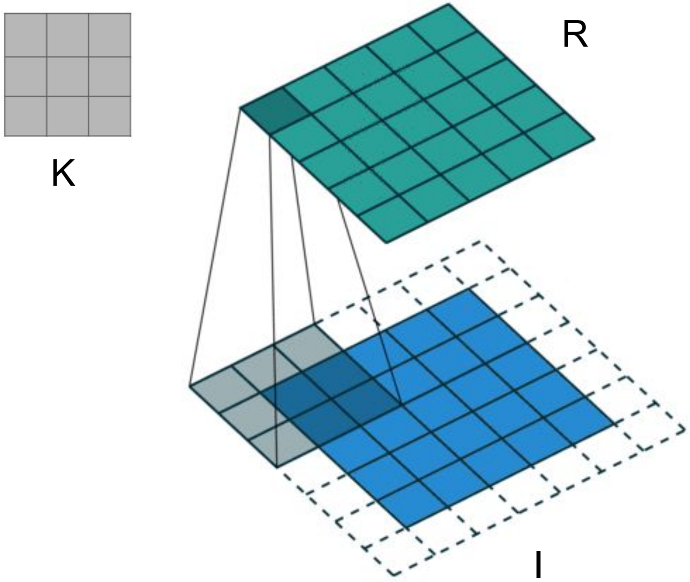
$$R(i,j) = \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} K(h,k) I(i-h, j-k)$$

- 2D Correlation:
  - Kernel is **not flipped**

$$R(i,j) = \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} K(h,k) I(i+h, j+k)$$



Recap: Filtering



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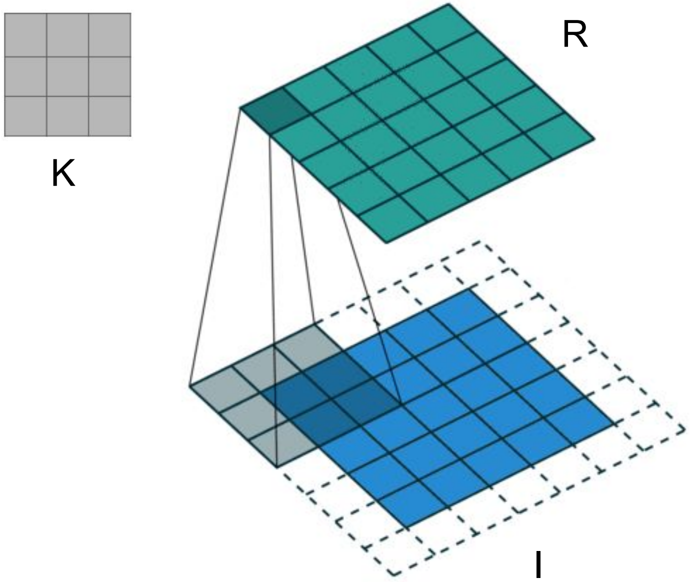
- Zero-padding:
  - Pad zeros outside the original image
  - Question: how many pixels should we pad to maintain the same size after convolution?

Recap: Filtering



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- Zero-padding:
  - Pad zeros outside the original image
  - Question: how many pixels should we pad to maintain the same size after convolution?
  - Ans: $\left\lfloor \frac{m}{2} \right\rfloor$ on each side, m is kernel size

Recap: Filtering

original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Pixel offset

Filtered
(no change)

original

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

BOX filter

Blurred (filter applied in both dimensions).

original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Pixel offset

Shifted one
Pixel to the left

Original

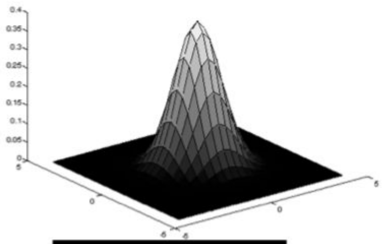
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

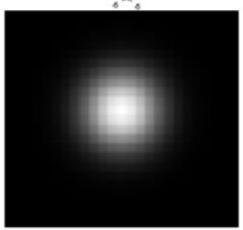
Sharpening filter
- Accentuates differences with local average

Smoothing

- Gaussian Smoothing

An Isotropic Gaussian





- Circularly symmetric Gaussian with variance σ^2

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

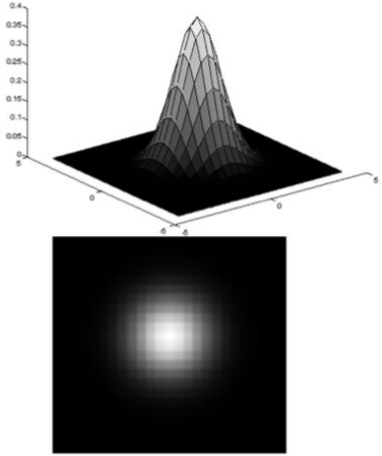
$\frac{1}{273}$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

Smoothing

- Gaussian Smoothing

An Isotropic Gaussian



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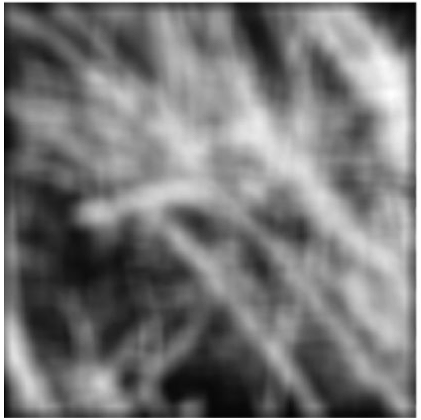
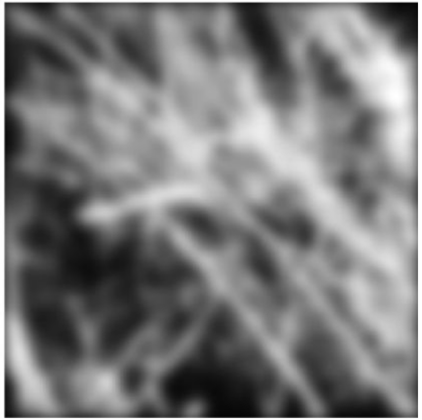
1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

- Smoothing: BOX v.s. Gaussian

Box filter

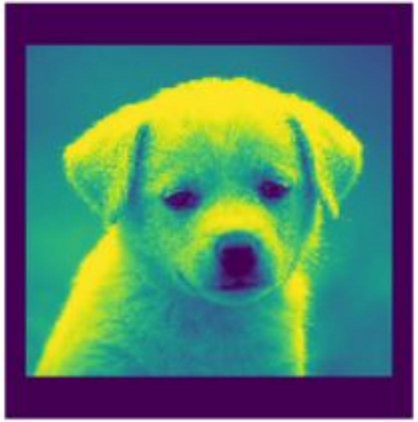


Gaussian filter



Filtering

Filtering

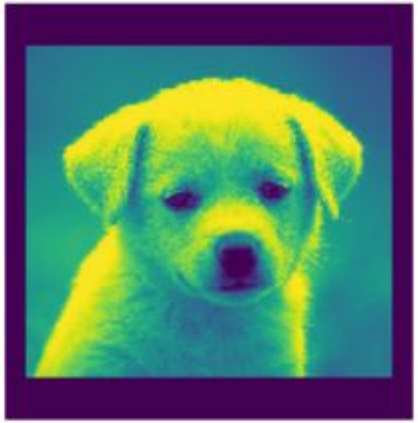


Filtering

1	0	-1
2	0	-2
1	0	-1

Sobel Filter

Zero Padding



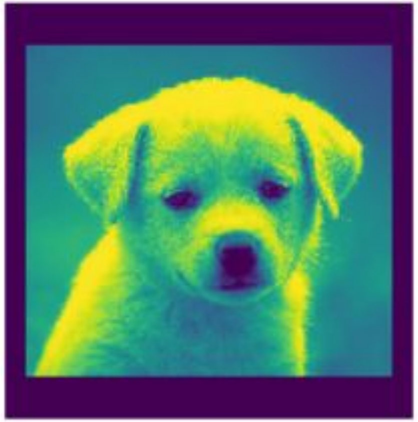
Convolution

Gaussian Filter

Sharpening Filter

Filtering

Zero
Padding



Convolution

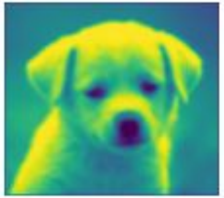
1	0	-1
2	0	-2
1	0	-1

Sobel
Filter

Image
Gradient

Gaussian
Filter

Blurred



Sharpening
Filter

Sharpened

Try other filters yourself …
E.g., median filter, BOX filter ...

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- Geometric Image Formation
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- Feature Matching using SIFT

Template Matching

- Template Image 
- Find the location with maximum response from the larger image

Template Matching

- Template Image 
- Find the location with maximum response from the larger image

- Find the correlation instead of convolution
    - flip back template then apply convolution
- Subtract off the mean value of the image or template to make result not biased toward higher-intensity (white) regions

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Corner Detection

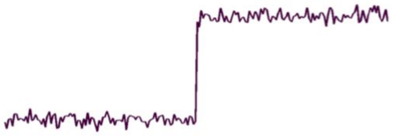
1. Filter image with a Gaussian
    a. Convolution with Gaussian filter
2. Compute the gradient everywhere
    a. Convolution with [1 0 -1] and [1 0 -1].T to get $I_x = \frac{\partial I}{\partial x}$ and $I_y = \frac{\partial I}{\partial y}$
3. Compute C(x,y) over the window at every point
    a. Sum the I_x^2 , $I_x I_y$ and I_y^2 over the window by convolution with a window of ones
4. Find λ_1 and λ_2 at each location by calculating eigenvalues of C
5. Detect corner if both λ_1 and λ_2 are large
    a. r(x,y) = min ($\lambda_1(x,y)$, $\lambda_2(x,y)$)
    b. Non-Maximum Suppression
        i. Detect corner if r(x,y) is local maximum within a window
        ii. Set response map of other non-maximum pixels to zeros

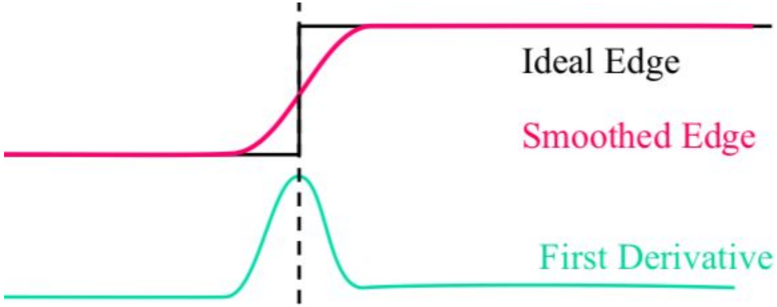
$$C(x,y) = \begin{bmatrix} \sum_W \left(\frac{\partial I}{\partial x}\right)^2 & \sum_W \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \sum_W \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \sum_W \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}$$

Corner Detection

1. Filter image with a Gaussian
   a. Convolution with Gaussian filter (from previous question)
2. Compute the gradient everywhere
   a. Convolution with [1 0 -1] and [1 0 -1].T to get $I_x = \frac{\partial I}{\partial x}$ and $I_y = \frac{\partial I}{\partial y}$

Noisy step edge





$$f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}$$

First Derivative: [1 0 -1]

Corner Detection

3. Compute C(x,y) over the window at every point
   a. Sum the I_x^2 , $I_x I_y$ and I_y^2 over the window by convolution with a window of ones

I_x

I_y

$$C(x,y) = \begin{bmatrix} \sum_W \left(\frac{\partial I}{\partial x}\right)^2 & \sum_W \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \sum_W \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \sum_W \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}$$

Corner Detection

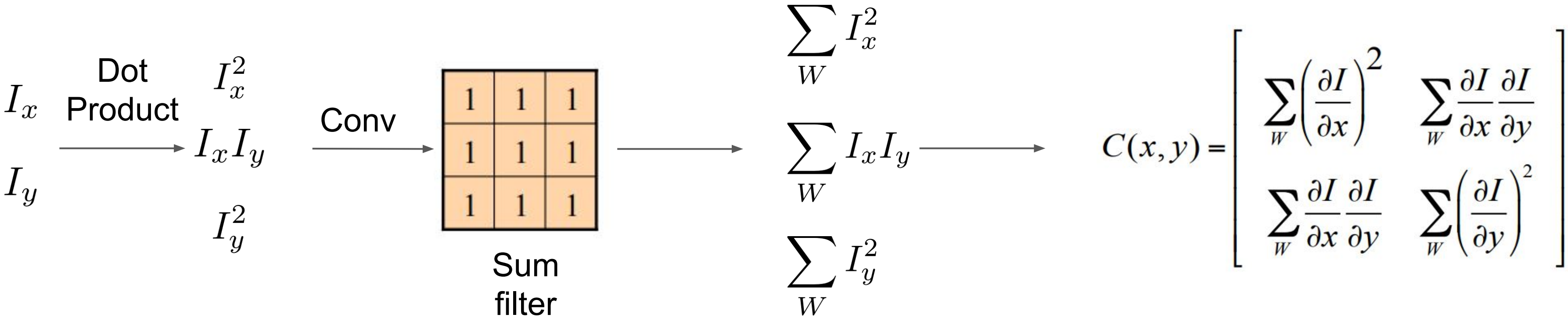
3. Compute C(x,y) over the window at every point
    a. Sum the I_x^2, I_xI_y and I_y^2 over the window by convolution with a window of ones

$$\begin{array}{ccc} & & I_x^2 \\ I_x & \xrightarrow{\text{Dot Product}} & I_xI_y \\ I_y & & I_y^2 \end{array}$$

$$C(x,y) = \begin{bmatrix} \sum_W \left(\frac{\partial I}{\partial x}\right)^2 & \sum_W \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \sum_W \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \sum_W \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}$$

Corner Detection

3. Compute C(x,y) over the window at every point
   a. Sum the I_x^2 , $I_x I_y$ and I_y^2 over the window by convolution with a window of ones



Corner Detection

4. Find λ_1 and λ_2 at each location by calculating eigenvalues of C

Because C is a symmetric positive definite matrix, it can be factored as:

$$C = R^T \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

where R is a 2x2 rotation matrix. λ_1, λ_2 are non-negative and are Eigenvalues of C.

$$C(x,y) = \begin{bmatrix} \sum_W \left(\frac{\partial I}{\partial x}\right)^2 & \sum_W \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \sum_W \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \sum_W \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}$$

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Solve $\boldsymbol{A}\boldsymbol{v} = \lambda \boldsymbol{v}$

Is equivalent to solve $(\boldsymbol{A} - \lambda \boldsymbol{I})\,\boldsymbol{v} = \boldsymbol{0}$

When the linear system has non-trivial solutions,

$$\det(\boldsymbol{A} - \lambda \boldsymbol{I}) = \begin{vmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda \end{vmatrix} = 0.$$

Corner Detection

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Because C is a symmetric positive definite matrix, it can be factored as:

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Not necessarily need for loop until now

Solve $\boldsymbol{A}\boldsymbol{v} = \lambda \boldsymbol{v}$

Is equivalent to solve $(\boldsymbol{A} - \lambda \boldsymbol{I})\,\boldsymbol{v} = \boldsymbol{0}$

When the linear system has non-trivial solutions,

$$\det(\boldsymbol{A} - \lambda \boldsymbol{I}) = \begin{vmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda \end{vmatrix} = 0.$$

Corner Detection

5. Detect corner if both λ_1 and λ_2 are large
   a. r(x,y) = min ($\lambda_1(x,y)$, $\lambda_2(x,y)$)
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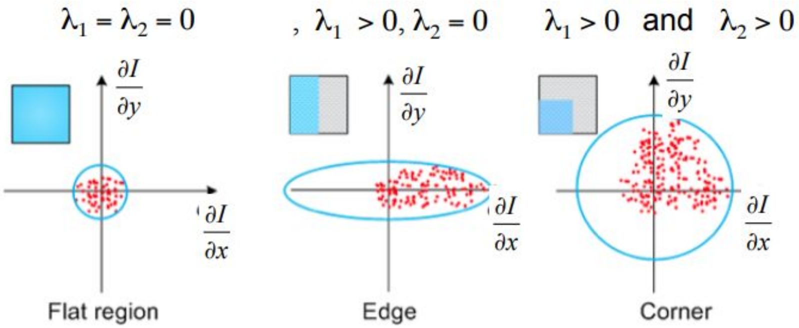
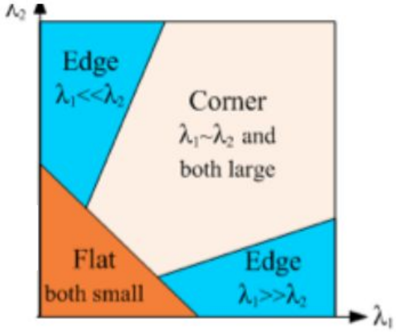
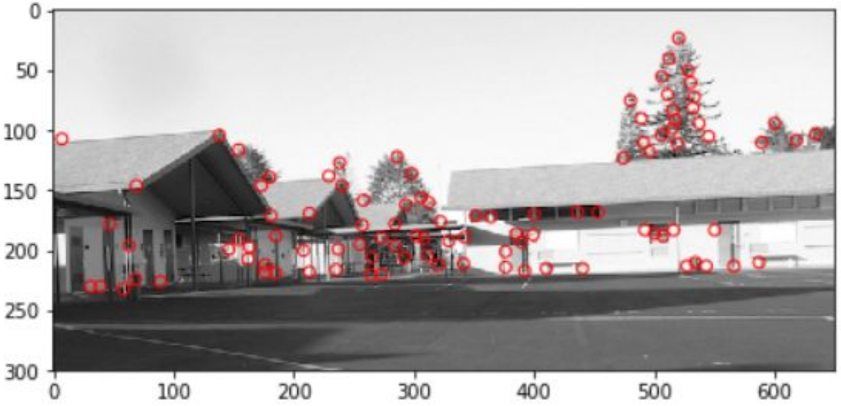
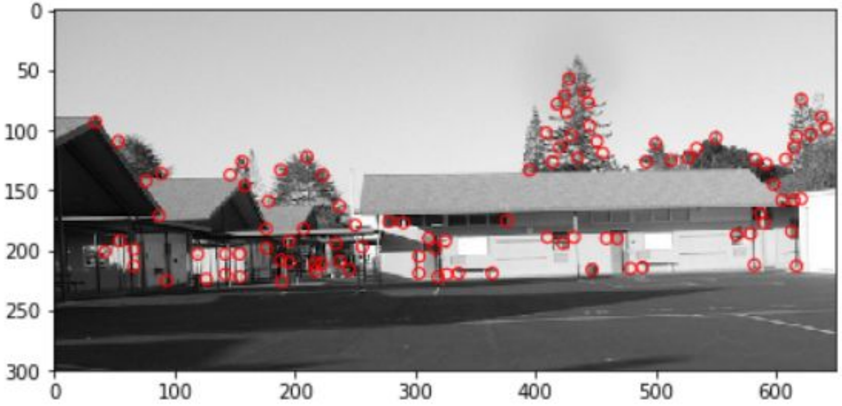




Corner Detection

- Sample results





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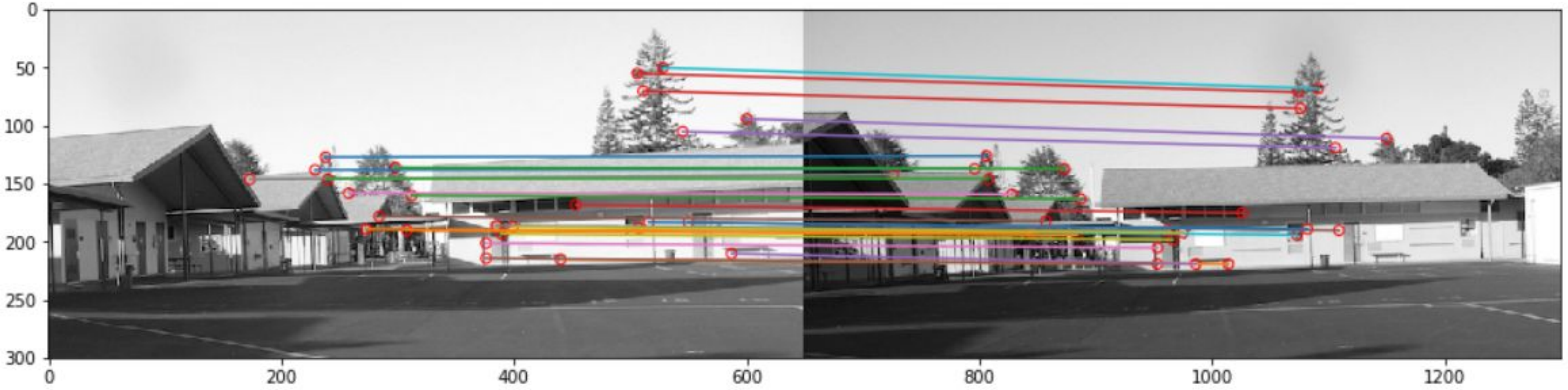
Feature Matching using SIFT

- SIFT feature extractor -> 128-d descriptor
- Distance: SSD (Sum of Squares Differences)
- Accept matching if best match’s distance is small enough (SSD threshold) and $\frac{best\ match's\ distance}{second\ best\ match's\ distance} \leq 0.3$ (Nearest Neighbor threshold)

$$\mathrm{SSD}(u, v) = \sum_i (u_i - v_i)^2$$



Q & A