

1 Theoretical Part

1.1 Rotation Quaternions

The rotation quaternion with the rotation angle as θ and rotation axis $\mathbf{v} = (v_x, v_y, v_z)^T$ is given as:

$$q(\theta, v) = \cos(\theta/2) + iv_x \sin(\theta/2) + jv_y \sin(\theta/2) + kv_z \sin(\theta/2). \quad (1)$$

The length of q is

$$|q| = \sqrt{\cos^2(\theta/2) + (v_x^2 + v_y^2 + v_z^2) \sin^2(\theta/2)} = \cos^2(\theta/2) + \sin^2(\theta/2) = 1.$$

1.2 3-D Gyro Integration

(i)

The refresh rate is 1 Hz so the time step is given as 1 s. The angle of rotation is given as $\|\omega\|\Delta t$ and the rotation axis is $\frac{\omega}{\|\omega\|}$. Therefore, the rotation axis and amount of rotation for each 1, 2, 3, 4 is:

$$\begin{aligned} \|\theta^{(1)}\| &= 90^\circ, \text{ axis}^{(1)} = (1, 0, 0)^T \\ \|\theta^{(2)}\| &= -90^\circ, \text{ axis}^{(2)} = (0, 0, 1)^T \\ \|\theta^{(3)}\| &= -90^\circ, \text{ axis}^{(3)} = (0, 1, 0)^T \\ \|\theta^{(4)}\| &= 90^\circ, \text{ axis}^{(4)} = (0, 0, 1)^T \end{aligned}$$

Or,

1. Rotate around x-axis by 90° ;
2. Rotate around z-axis by -90° ;
3. Rotate around y-axis by -90° ;
4. Rotate around z-axis by 90° ;

(ii)

Following the procedure summarized in (i), we get the full rotation as:

$$R_{tot} = R_z(\pi/2)R_y(-\pi/2)R_z(-\pi/2)R_x(\pi/2) \quad (2)$$

$$= \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad (3)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (4)$$

So the sequence of operation is equivalent of getting the inverse of the y- and z- coordinates.

(iii)

Following the first-order Taylor series, the angle at each time point is:

$$\theta^{(0)} = (0, 0, 0)^T \rightarrow \theta^{(1)} = (\frac{\pi}{2}, 0, 0)^T \rightarrow \theta^{(2)} = (\frac{\pi}{2}, 0, -\frac{\pi}{2})^T \rightarrow \theta^{(3)} = (\frac{\pi}{2}, -\frac{\pi}{2}, -\frac{\pi}{2})^T \rightarrow \theta^{(4)} = (\frac{\pi}{2}, -\frac{\pi}{2}, 0)^T$$

To verify, the total rotation for Euler angle is given as:

$$R = R_z(\theta_z)R_x(-\theta_x)R_y(-\theta_y). \quad (5)$$

Therefore the rotation matrix for the Euler angle for each step is:

$$R^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \quad (6)$$

$$R^{(2)} = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \quad (7)$$

$$R^{(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad (8)$$

$$R^{(4)} = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad (9)$$

The resulting R4 is not the same as the rotation matrix in (ii).

(iv)

Due to the non-commute nature of rotation operation, the actual procedure matters for rotation. The effect of a sequence of rotation cannot be equivalently represented as a full rotation given as the final outcome angle (Euler angle). From the operational perspective, the Euler angle is ill-defined when the vector is along certain axis around which the rotation happens, introducing ambiguity.

2 Programming Part

2.1 Noise and Bias Estimation

2.1.1 Bias Estimation

The reported bias values:

	$x-$	$y-$	$z-$
gyr_bias	-3.57494	2.23188	-0.10047
acc_bias	1.059	0.719	8.899

2.1.2 Noise Variance Estimation

The reported variance values:

	$x-$	$y-$	$z-$
gyr_var	0.01021	0.01014	0.01017
acc_var	0.000	0.000	0.001