1 Theoretical Part

(i) Vergence

(a)

The angle $\theta_{1,2}$ can be calculated using cosine rule:

$$\cos(\theta_1) = \frac{|e_1p_1|^2 + |e_2p_1|^2 - |e_1e_2|^2}{2|e_1p_1||e_2p_1|} \tag{1}$$

$$\cos(\theta_2) = \frac{|e_1p_2|^2 + |e_2p_2|^2 - |e_1e_2|^2}{2|e_1p_2||e_2p_2|} \tag{2}$$

The coordinates of the e points are given as:

$$e_1 = (-32,\ 0),\ e_2 = (32,\ 0),$$

therefore the distance are given as:

$$|e_1p_1| = 551,\ |e_2p_1| = 527,\ |e_1p_2| = 301,\ |e_2p_2| = 341,\ |e_1e_2| = 64,$$

Thus the angles:

$$\theta_1 = 6.32^\circ,\ \theta_2 = 8.96^\circ.$$

(b)

For this part, we would need to rederive the expressions for the angles. Here we use the difference and inverse tan() functions. The angles are given:

$$\theta_1 = \tan^{-1}(\frac{x_1 + \text{ipd}/2}{z_1}) - \tan^{-1}(\frac{x_1 - \text{ipd}/2}{z_1}) \tag{3}$$

$$\theta_i = \tan^{-1}(\frac{x_i + \text{ipd}/2}{z_i}) - \tan^{-1}(\frac{x_i - \text{ipd}/2}{z_i}). \tag{4}$$

Since inverse tangent function is odd and using the given formulae for simplification, we can get:

$$\theta_1 = \tan^{-1}(\frac{\text{ipd}\ z_1}{(-\text{ipd}/2)^2 + x_1^2 + z_1^2}) \tag{5}$$

$$\theta_i = \tan^{-1}(\frac{\text{ipd}\ z_i}{(-\text{ipd}/2)^2 + x_i^2 + z_i^2}). \tag{6}$$

Since inverse tangent function is monotonically increasing, we need the argument to be equal in order to have the angle equal as well. Therefore,

$$\frac{\text{ipd}\ z_i}{(-\text{ipd}/2)^2 + x_i^2 + z_i^2} = \tan\theta_1. \tag{7}$$

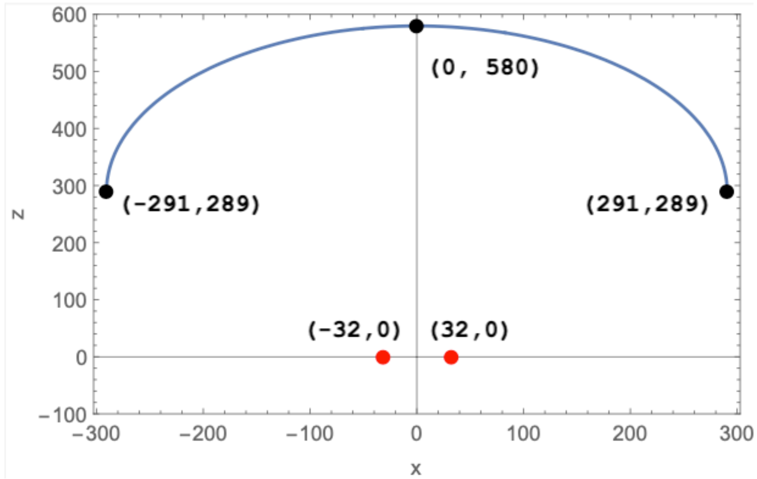
After simplification, we get:

$$x_i^2 + \left(z_i - \frac{ipd}{2\tan(\theta_1)}\right)^2 \ = \ \left(\frac{ipd}{2\sin(\theta_1)}\right). \tag{8}$$

Therefore the function:

$$z_i \ = \ \frac{ipd}{2\tan(\theta_1)} + \sqrt{\left(\frac{ipd}{2\sin(\theta_1)}\right)^2 - x_i^2}. \tag{9}$$

Using the above formulae we can get the plot:



(ii) Retinal Blur

(a)

The focal length can be calculated via:

$$1/f = 1/D_e + 1/d_3, \tag{10}$$

thus $f =$ 23.4 mm.

(b)

For point p_4, the circle of confusion in the image plane can be calculated via the equation:

$$c = S_e \frac{|d_4 - d_3|}{|d_4|} \frac{f}{d_3 - f} = 5f/(1000 - f) = 0.12 \text{ mm}. \tag{11}$$

Visual Acuity

For 13.3" Macbook Pro, the diagonal distance is 13.3". With a screen resolution of 2560×1600, the diagonal has the number of pixel given as: $\sqrt{2560^2 + 1600^2} = 3020$. Therefore the number of pixel in 1 cm is: $3020/33.78$ cm $= 89.4$ cm^{-1}. Therefore the visual angle of one pixel is $1/89.4/50/\pi \times 180° = 0.0128°$. For 1 degree, at a viewing distance of 50 cm, the arc length will be $50\ cm \times \pi/180 = 0.873$ cm. The number of pixel that one can placed into such distance is: $0.873 \times 89.4 = 78$. Therefore the number of dark-bright alternating pair of lines is $78/2 = 39$ cpd. It is higher than 30 cpd that humans can perceive.

Eccentricity and Visual Acuity

(a,b)

For p5, the angle $\theta_e = 36.9°$. Using the equation of eccentricity, one can get: $\omega = m\theta_e + \omega_0 = 0.0275 \times 36.8 + 1/48 = 1.03°$. The acuity is the reciprocal of the MAR, therefore the highest frequency one can resolve would be: $1/1.03° = 0.97$ cpd.