# Linear Filtering

- smoothing is implemented with linear filters
- given an image $x(n_1,n_2)$, filtering is the process of convolving it with a kernel $h(n_1,n_2)$

$$y(n_1,n_2) = \sum_{k_1 k_2} x(k_1,k_2)h(n_1-k_1,n_2-k_2)$$

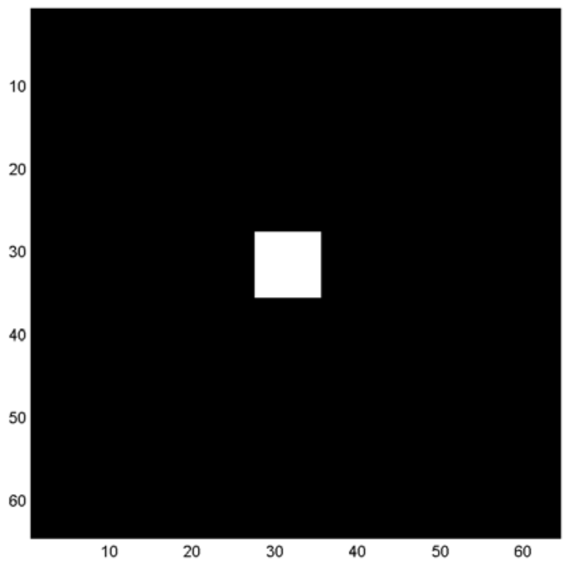
- some very common operations in image processing are nothing but filtering, e.g.
  - smoothing an image by low-pass filtering
  - contrast enhancement by high pass filtering
  - finding image derivatives
  - noise reduction

# Popular filters

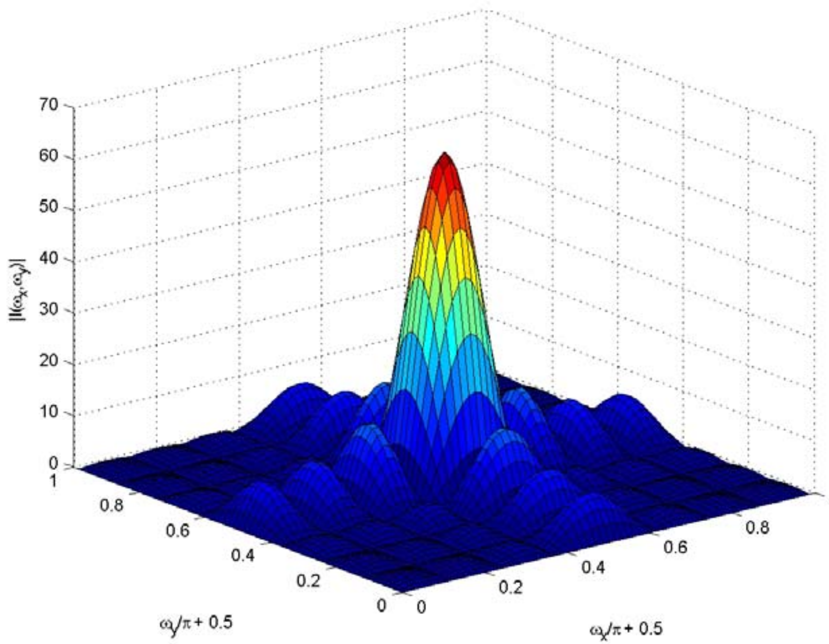
- box function

$$R_{N_1 x N_2}(n_1, n_2) = \begin{cases} 1, & 0 \le n_1 \le N_1 - 1, 0 \le n_2 \le N_2 - 1 \\ 0 & otherwise \end{cases}$$

- Fourier transform of a box is the sinc, low-pass filter




$\xleftrightarrow{\mathcal{F}}$

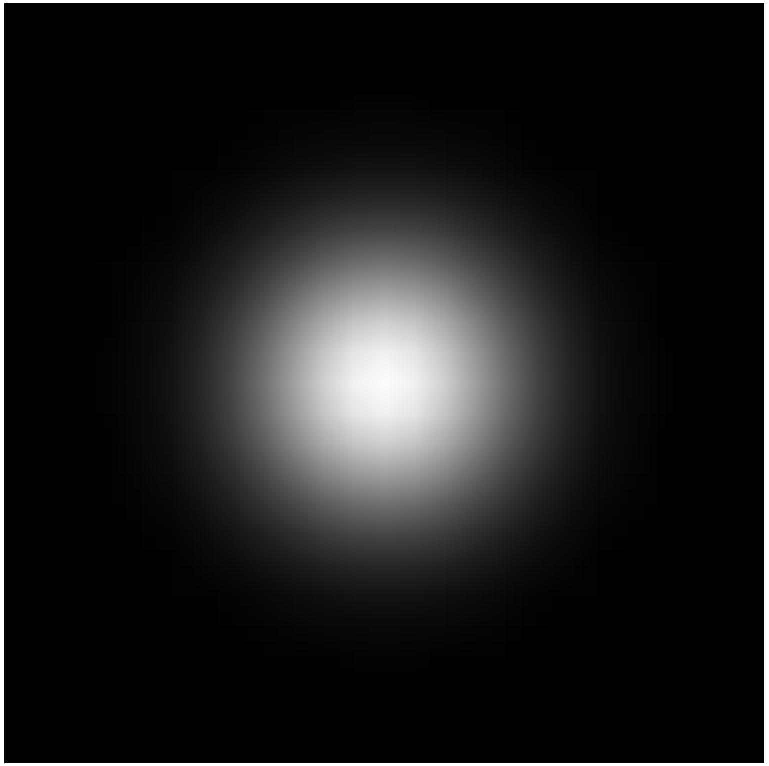



- side-lobes produce artifacts, smoothed image does not look like the result of defocusing

# Example: Smoothing by Averaging

# Camera defocusing

- if you point an out-of-focus camera at a very small white light (e.g. a light-bulb) at  night, you get something like this
- the light can be thought of as an impulse
- this must be the impulse response
- well approximated by a Gaussian
- more natural filter for image blur than the box



$$h(x,y) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

# The Gaussian

- the discrete space version is

$$h(n_1, n_2) = \frac{1}{2\pi\sigma^2} \exp\left( -\frac{n_1^{\,2} + n_2^{\,2}}{2\sigma^2} \right)$$

- obviously separable

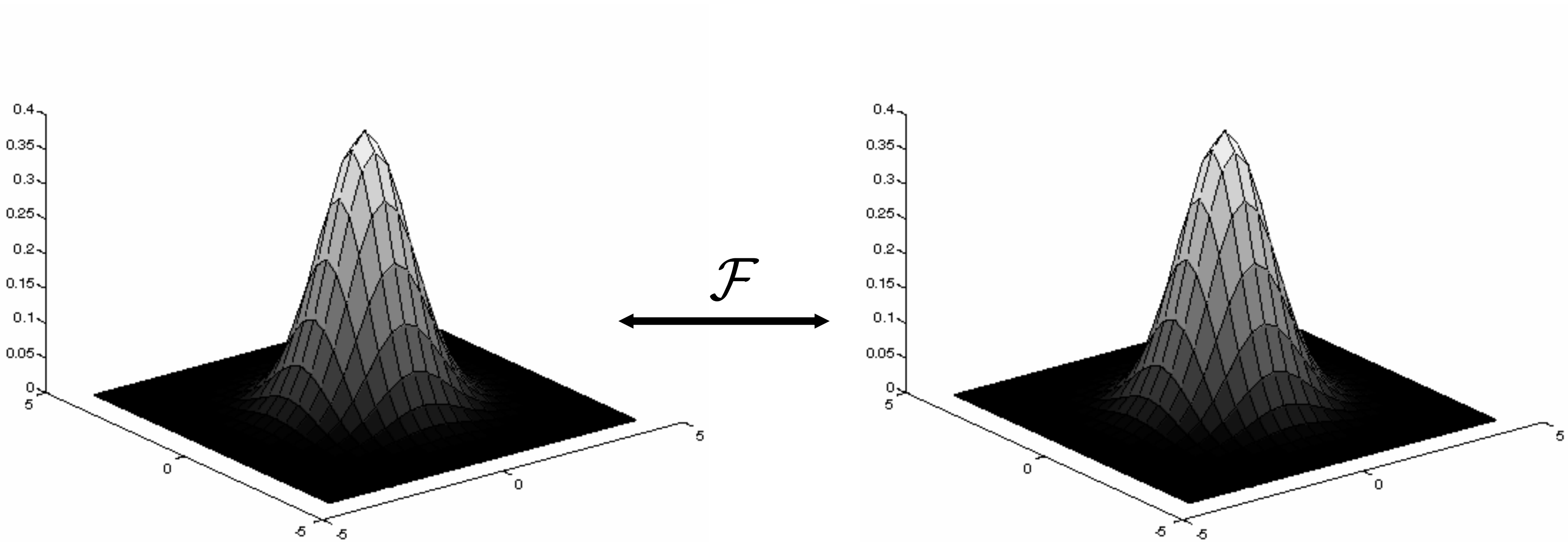
$$h(n_1, n_2) = \underbrace{\frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{n_1^{\,2}}{2\sigma^2}}}_{h(n_1)} \times \underbrace{\frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{n_2^{\,2}}{2\sigma^2}}}_{h(n_2)}$$

- $h(n_1,n_2)$ has Fourier transform

$$H(\varpi_1, \varpi_2) = \exp\left( -\frac{\sigma^2(\varpi_1^{\,2} + \varpi_2^{\,2})}{2} \right)$$

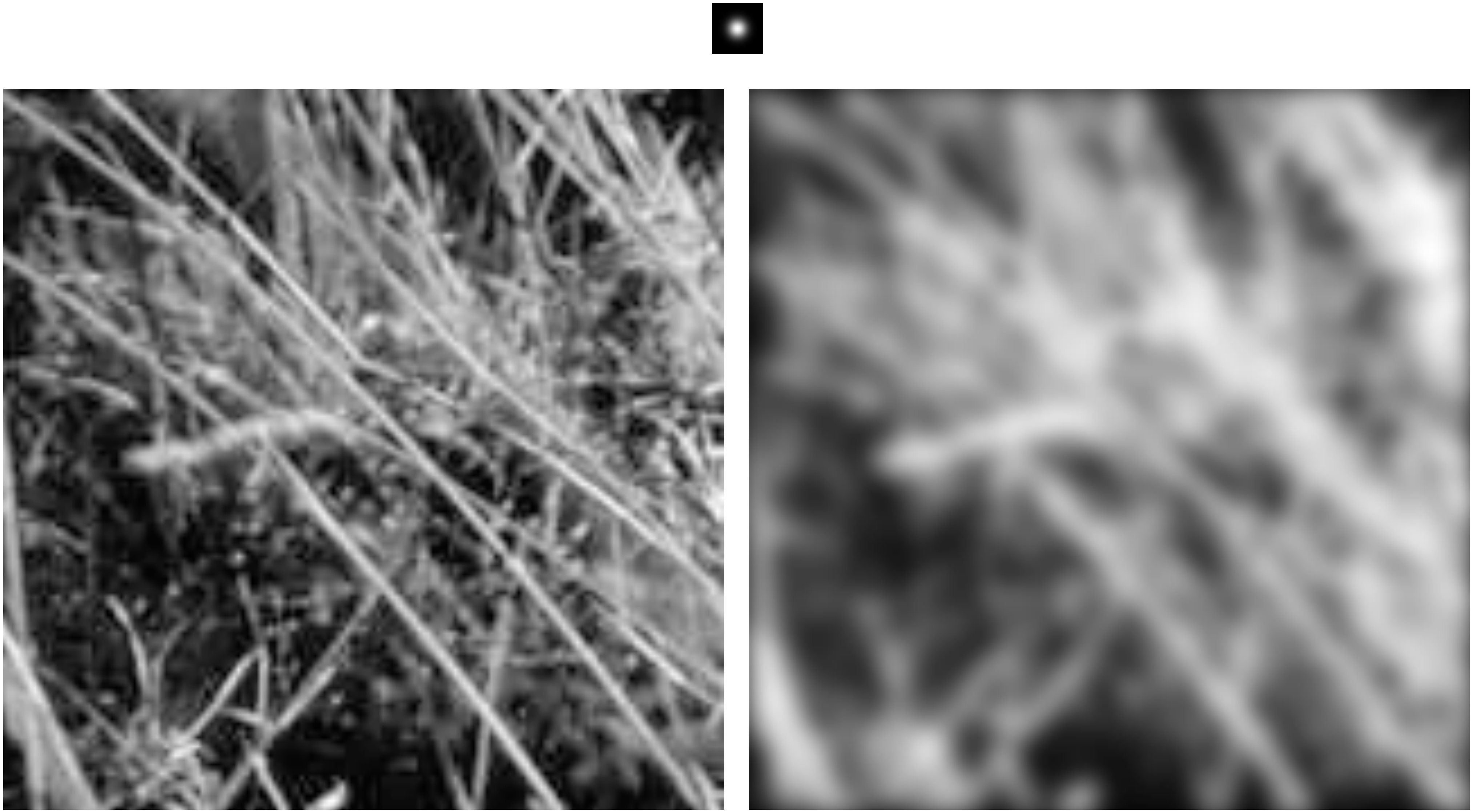
# The Gaussian filter

- the Fourier transform of a Gaussian is a Gaussian
  $(\sigma_x, \sigma_y) \propto (1/\sigma_{w1}, 1/\sigma_{w2})$

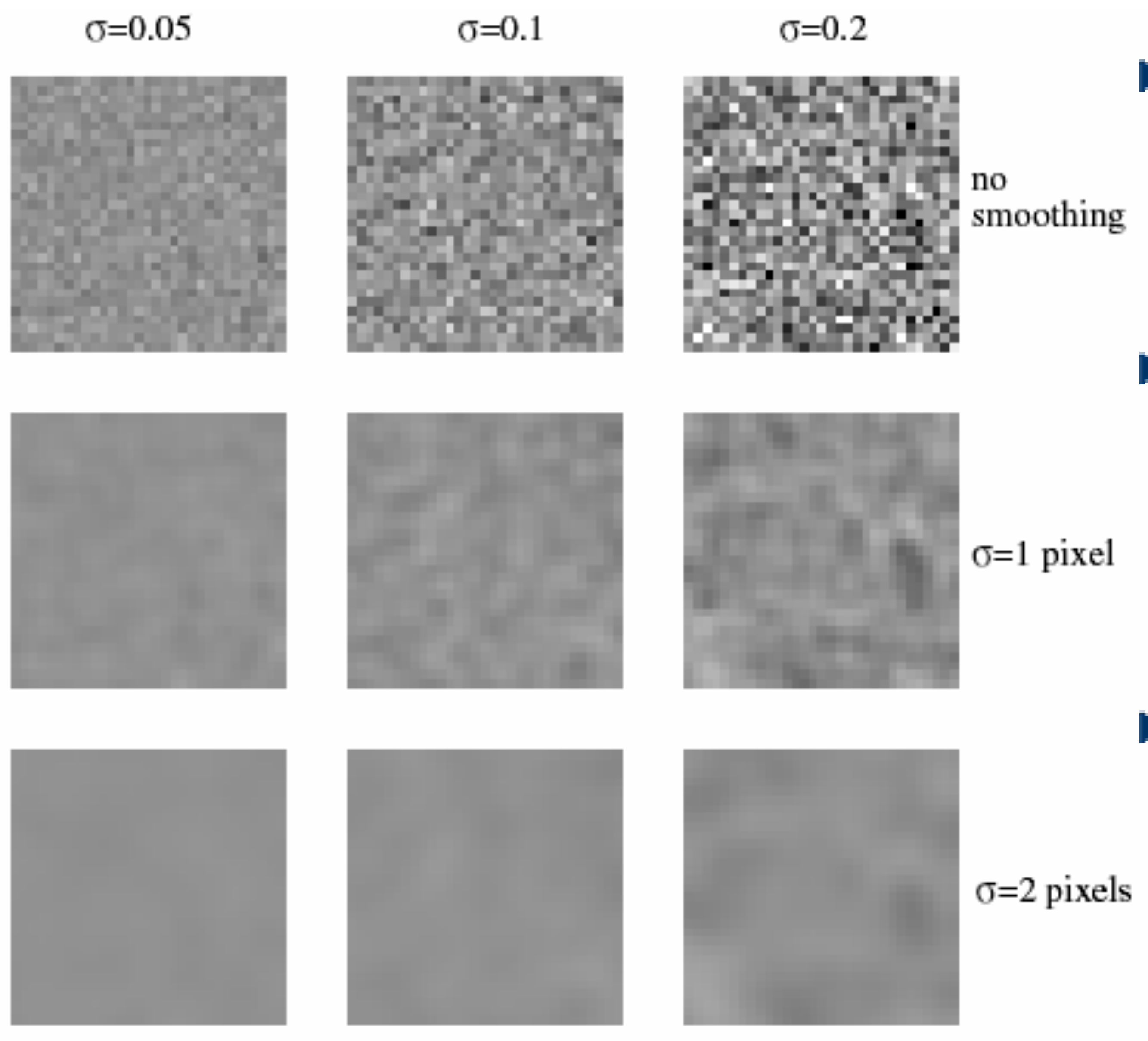



- note that there are no annoying side-lobes

# Smoothing with a Gaussian



# Role of the variance




- the variance controls the amount of smoothing
- each column shows different realizations of an image of gaussian noise
- each row shows smoothing with gaussians of different $\sigma$