Introduction to Control Theory

Renan G. Maidana

Porto Alegre, 2018

About me

- **Autonomous Systems Laboratory (LSA)**
  - 2014 - 2016
- **BSc Control and Automation Engineering**
  - PUCRS, 2009 - 2016
  - Monocular Visual Odometry for Mobile Robot Localization
- **MSc Computer Science**
  - PUCRS, 2016 - 2018
  - Outdoor Localization System for Mobile Robots based on RSSI
- **Ph.D. Computer Science**
  - PUCRS, 2018 - present
  - Control Theory and Autonomic Computing for Self-Management in Embedded Mobile Robots

Course Syllabus

- **Introduction to Control Theory**
  - What is feedback? What is control?
  - Advantages and Disadvantages
  - Classical Applications
  - Applications in Robotics
  - Practical Examples and Exercises

Course Syllabus

- **Dynamic Systems Modelling**
  - Introduction
  - Differential Equations and Transforms
  - Transfer Functions
  - State-spaces
  - Examples and Exercises

- **Frequency Domain Analysis**
  - Transient Response
  - Root Locus Analysis
  - Stability
  - Examples and Exercises

Course Syllabus

- **Digital Control Systems Design**
  - Discretization
  - PID Controller Design
  - PID Controller Tuning
  - Examples and Exercises

Bibliography

- K. Aström and R. Murray, "***Feedback Systems: An Introduction for Scientists and Engineers***", Princeton University Press, 2008, 408 p.
- J. DiStefano, A. Stubberud and I. Williams, "***Schaum's Outline of Feedback and Control Systems***", McGraw-Hill, 2nd ed., 2013, 528 p.
- P. Janert, "***Feedback Control for Computer Systems***", O'Reilly, 2013, 330 p.
- S. Frank, "***Control Theory Tutorial: Basic Concepts Illustrated by Software Examples***", Springer, 1st ed., 2018, 111 p.
- P. Albertos and I. Mareels, "***Feedback and Control for Everyone***", Springer, 2010, 318 p.
- N. Nise, "***Control Systems Engineering***", Wiley, 6th ed., 2010, 944 p.

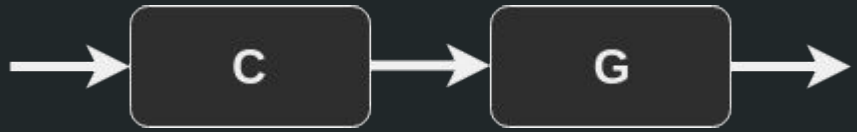
What is feedback?

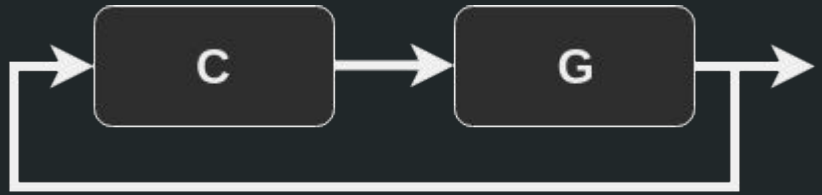
- A **system** is an arrangement, set or collection of things connected or related in a manner as to form a whole

- A **dynamical system** is a system whose behavior changes with time, often in reaction to an external stimulus

What is feedback?

- **Feedback** is when two (or more) **dynamical systems** are connected such that each one influences the other





"*Feedback is a central feature of life. The process of feedback governs how we grow, respond to stress and challenge, and regulate factors such as body temperature, blood pressure and cholesterol level…*"

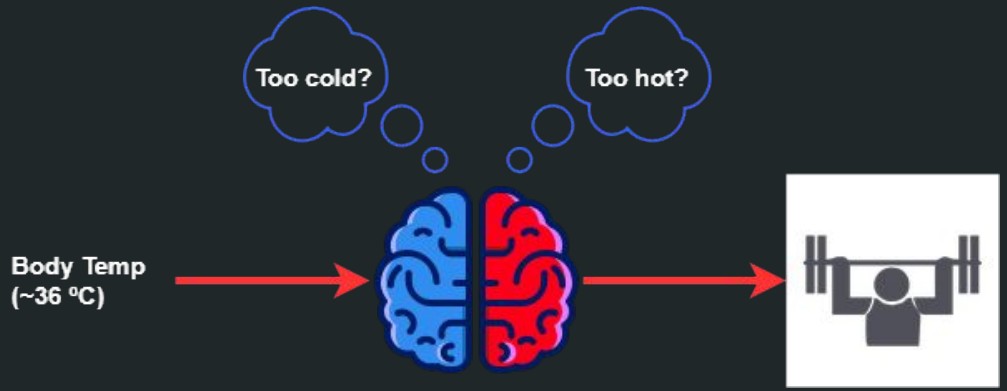
M. Hoagland and B. Dodson, "*The way life works*", 1995.

What is feedback?

- Example: Body Temperature

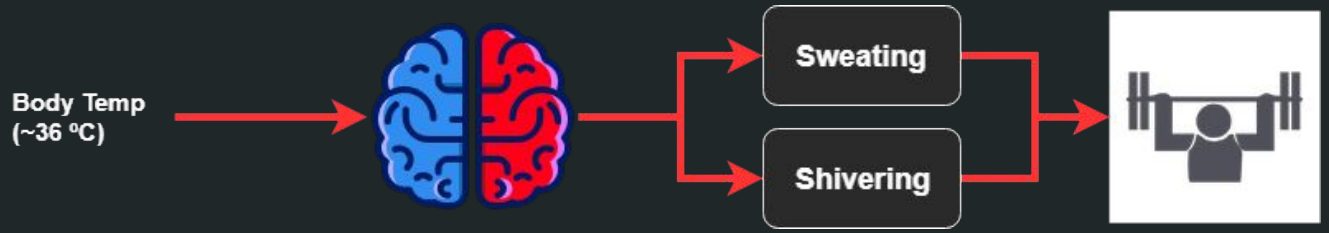
What is feedback?

- Example: Body Temperature



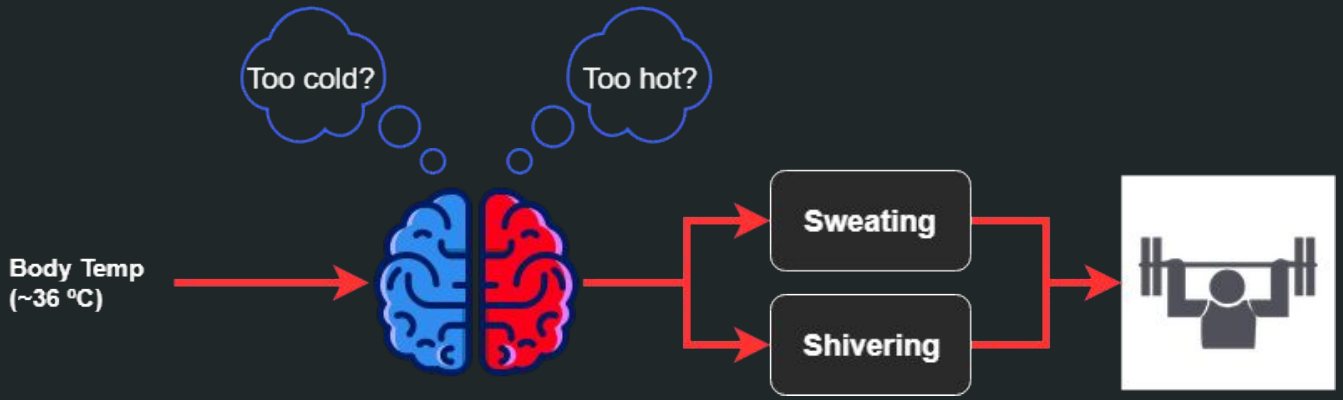
What is feedback?

- Example: Body Temperature



What is feedback?

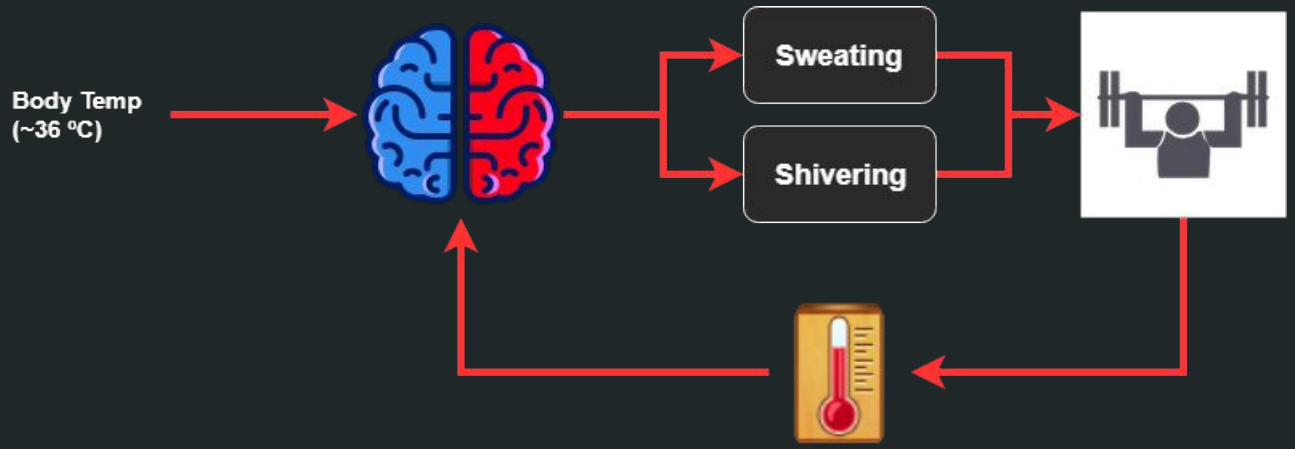
- Example: Body Temperature



- ***Open-Loop System***

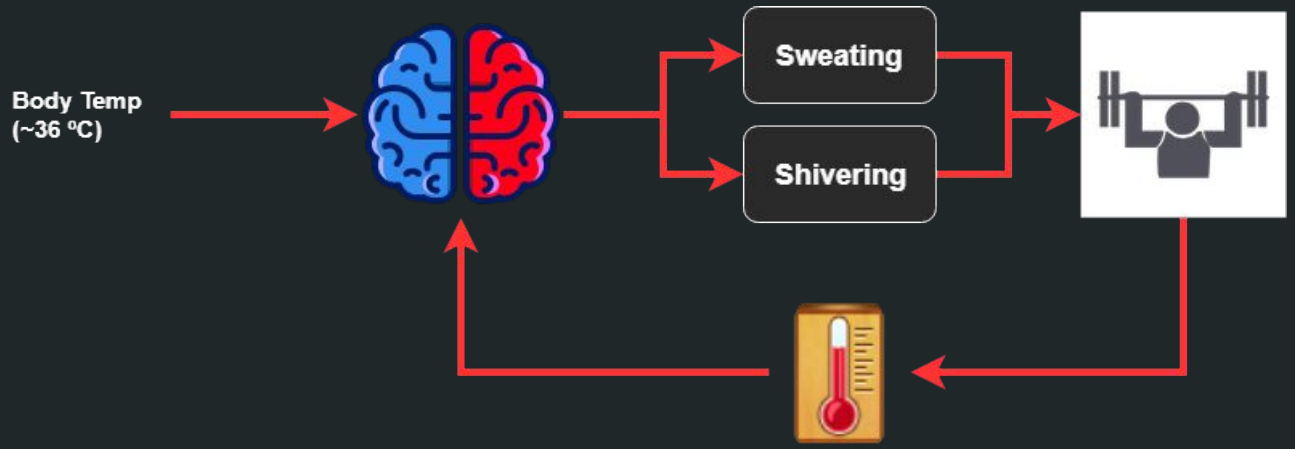
What is feedback?

- Example: Body Temperature



What is feedback?

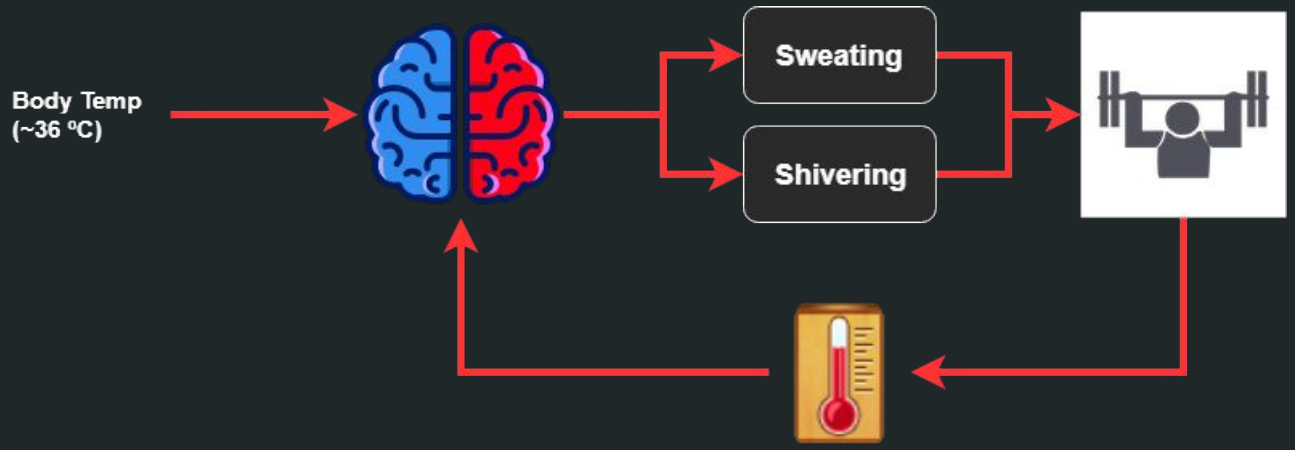
- Example: Body Temperature



- ***Closed-Loop System***

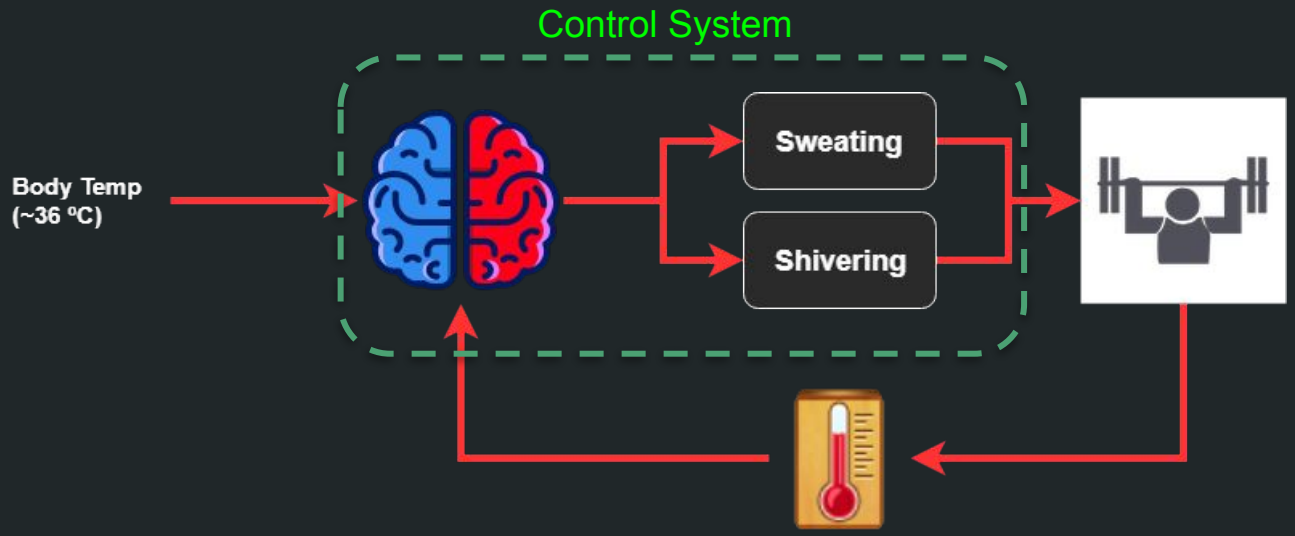
What is control?

- Example: Body Temperature Control



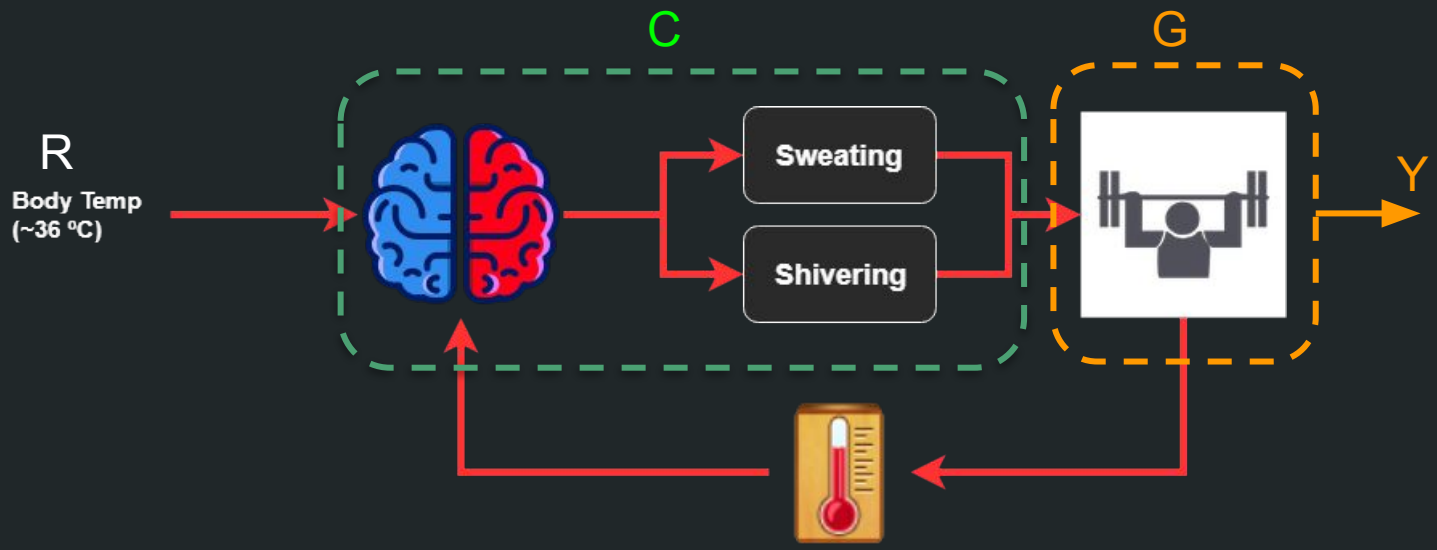
What is control?

- A **control system** is one or a set of systems connected/related as to command, direct, or regulate itself or another system



What is control?

- A **control system** is one or a set of systems connected/related as to command, direct, or regulate itself or another system

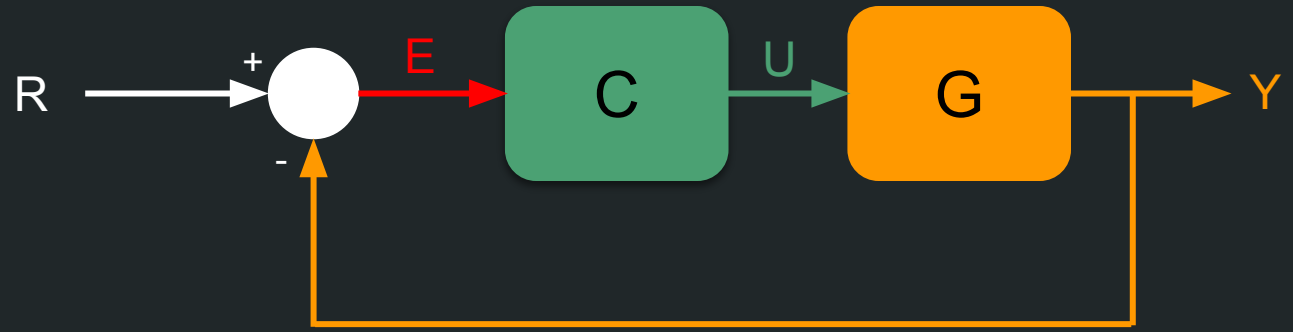


What is control?

- A **control system** is one or a set of systems connected/related as to command, direct, or regulate itself or another system

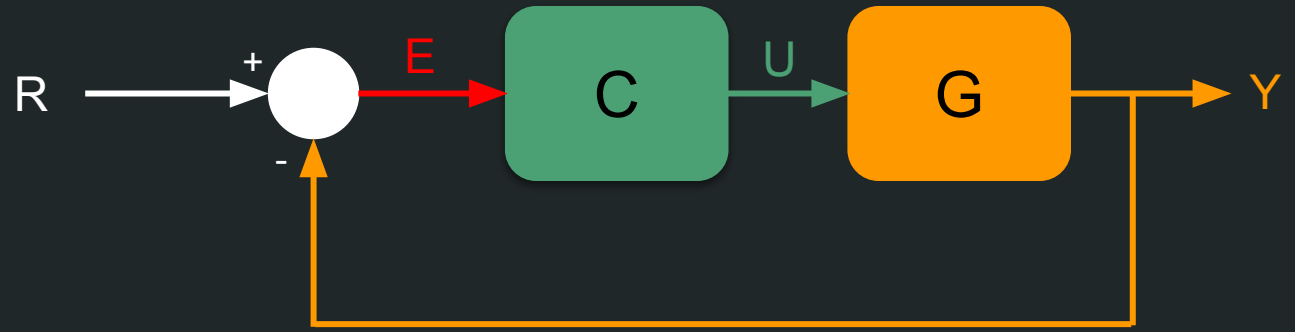
What is control?

- A **control system** is one or a set of systems connected/related as to command, direct, or regulate itself or another system



What is control?

- A **control system** is one or a set of systems connected/related as to command, direct, or regulate itself or another system

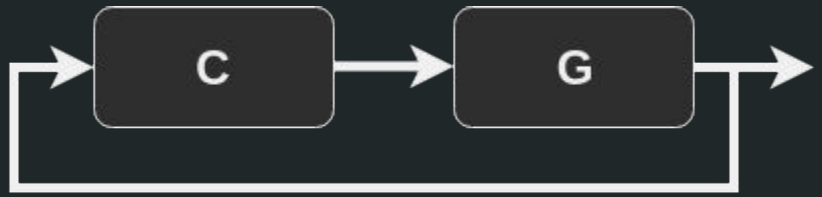


- ***Feedback Control System***

Terminology

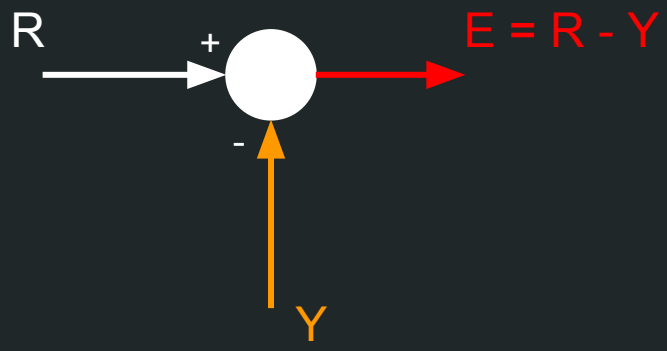
- **Blocks** are useful visual representations of the dynamical systems

- **Block diagrams** are useful representations of the relationships between dynamical systems

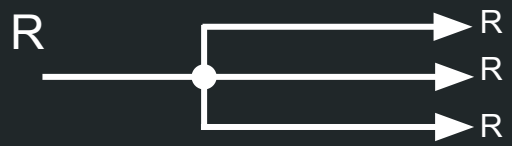


Terminology

- Addition and subtraction

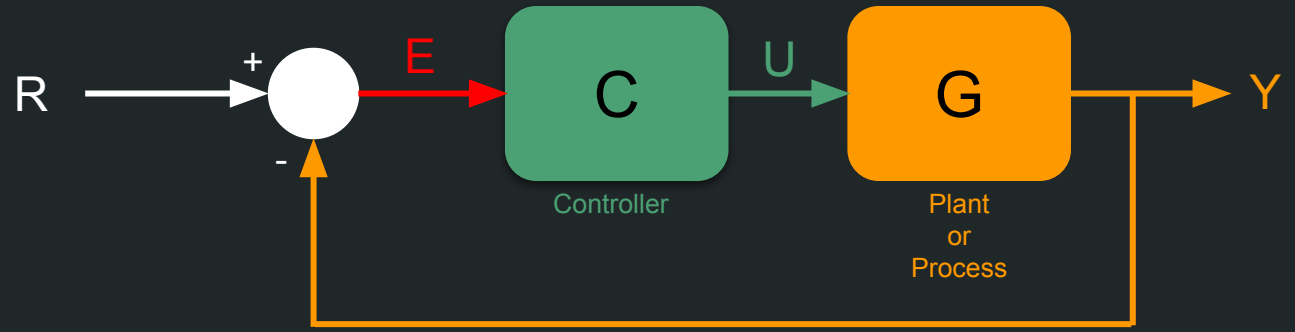


- Takeoff points



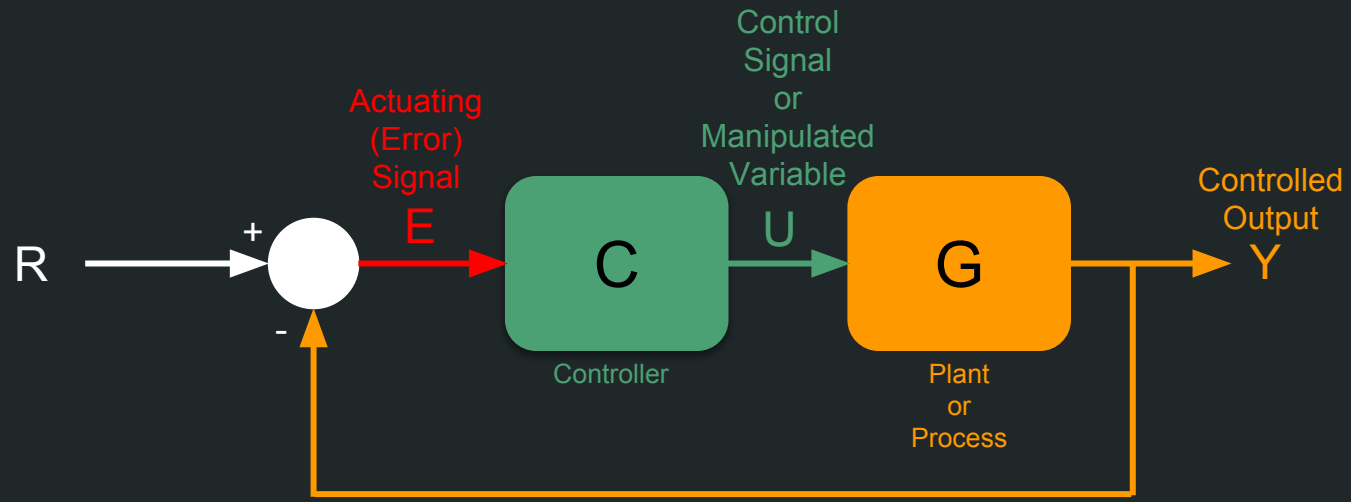
Terminology

- Block diagram of feedback control systems



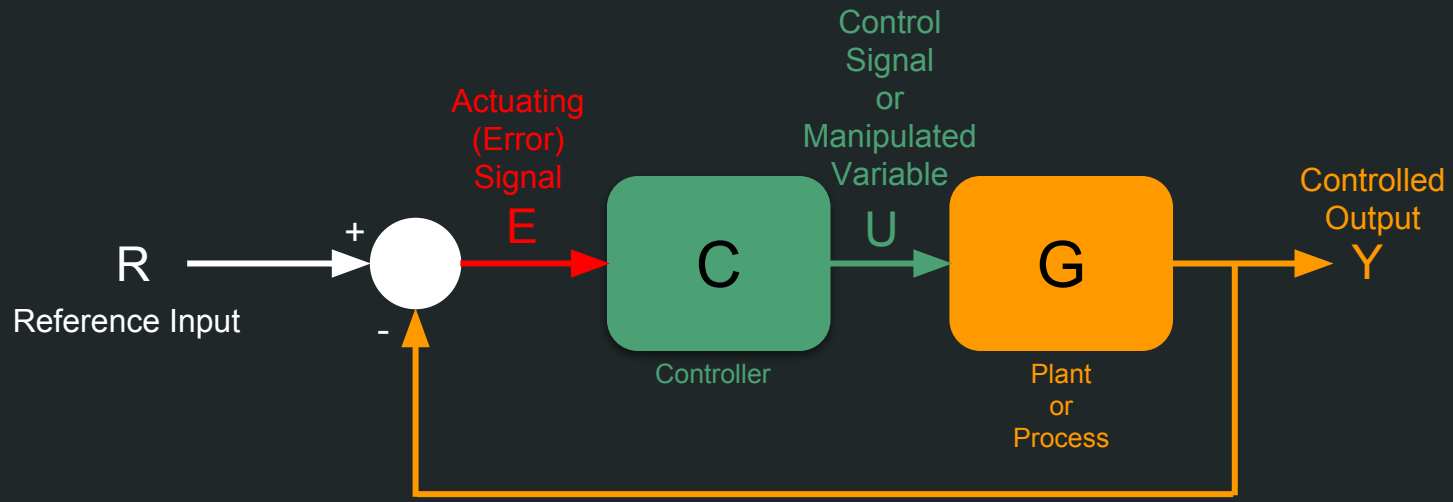
Terminology

- Block diagram of feedback control systems



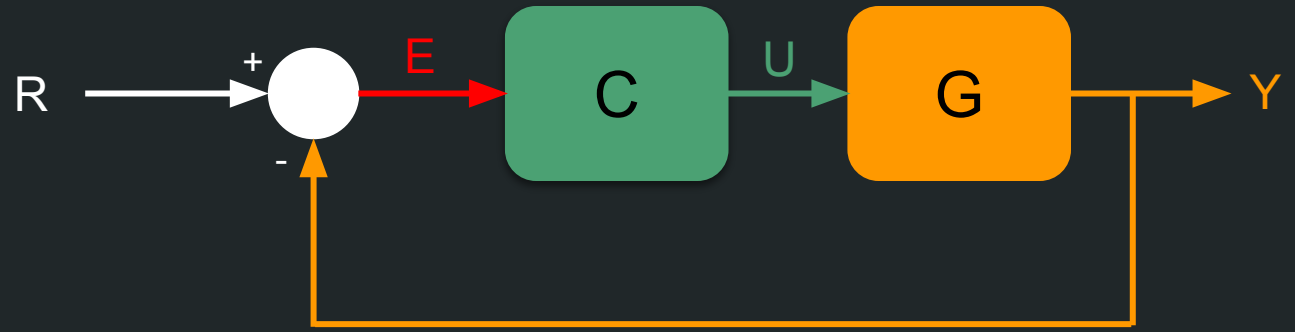
Terminology

- Block diagram of feedback control systems



Terminology

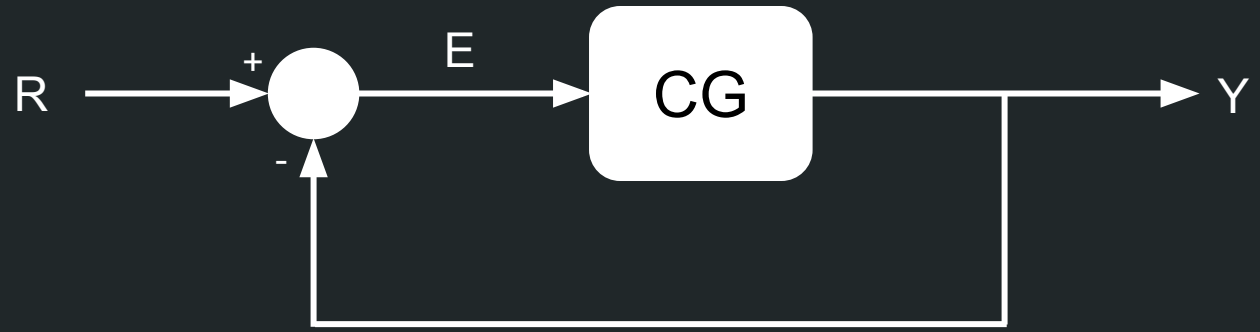
- Block diagram of feedback control systems



- Can we represent this in a single block?

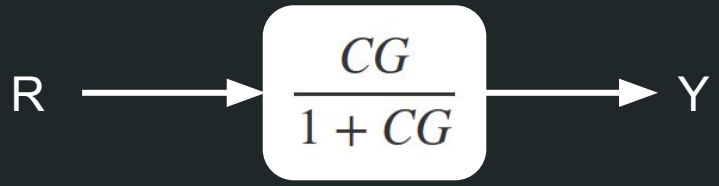
Block Algebra

- Block diagram of feedback control systems



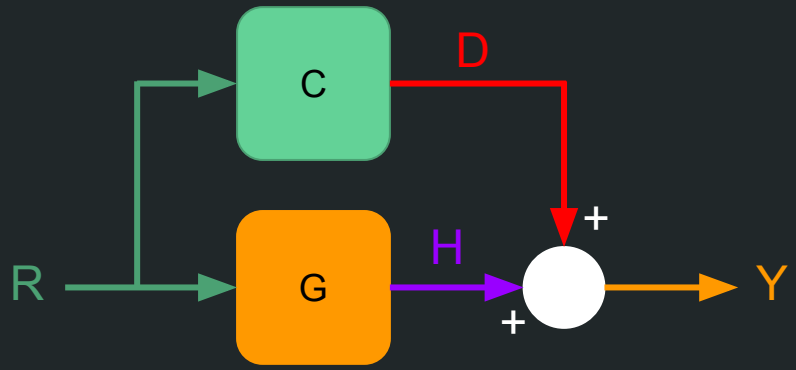
Block Algebra

- Block diagram of feedback control systems



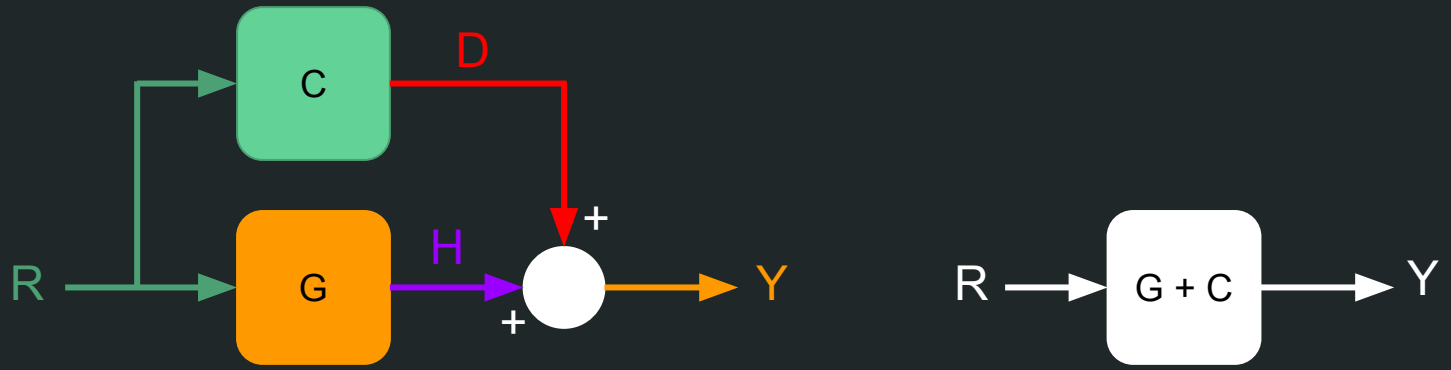
Block Algebra

- Another example: Feedforward Control



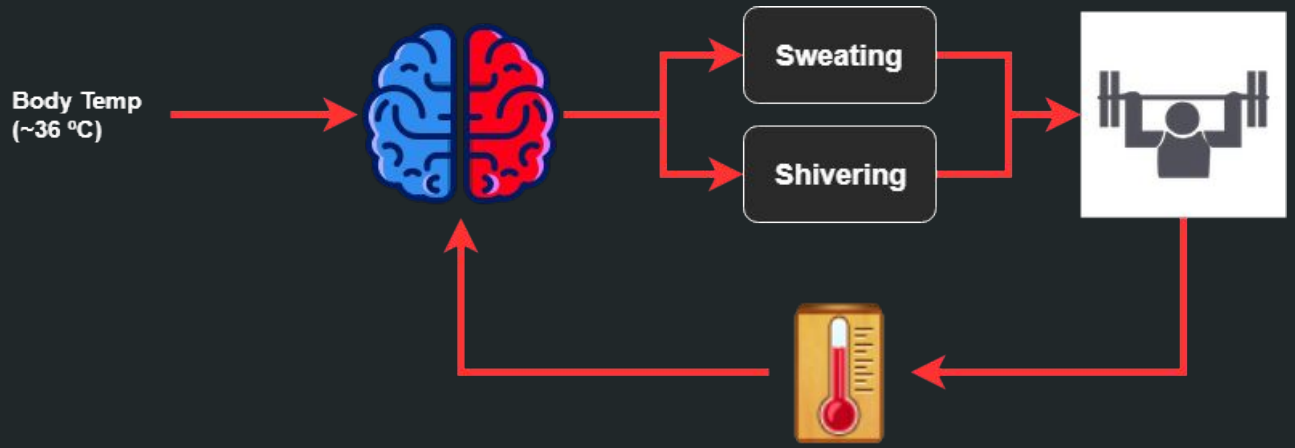
Block Algebra

- Another example: Feedforward Control



Practical Example

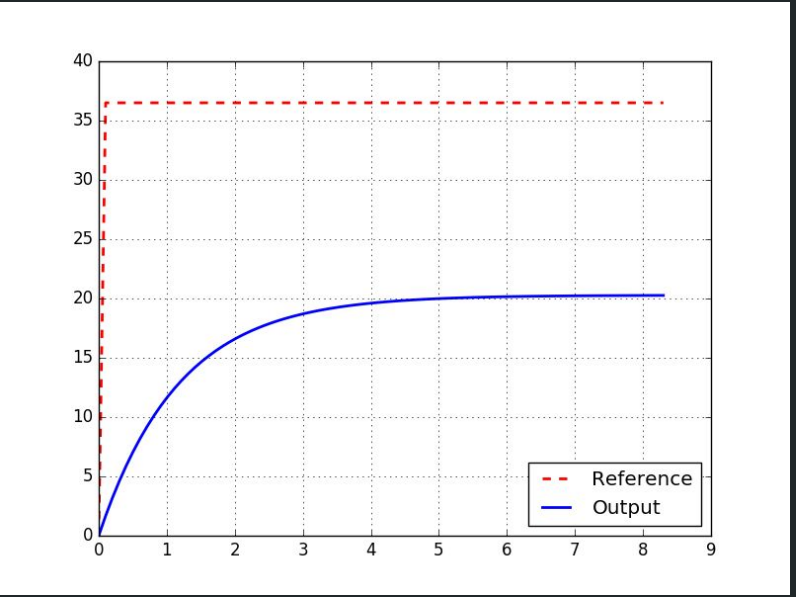
- Body Temperature Control



Practical Example

```
import matplotlib.pyplot as plt
import numpy as np
from time import sleep

T = 0.1      # Sampling time
r = [0]      # Reference vector
y = [0]      # Output vector

def bodyTemp(u):
    # Model
    a, b = 0.5, 0.9
    return (T*a*u + y[-1])/(1+T*b)

if __name__ == '__main__':
    try:
        while True:
            r.append(36.5)              # Reference is always 36.5 C
            y.append(bodyTemp(r[-1]))
            sleep(T)
            print 'Elapsed: %.2f seconds' % float(len(y)*T)

    except KeyboardInterrupt:
        t = np.linspace(0,len(y)*T,len(y))
        plt.plot(t,r,'--r',linewidth=2.0)
        plt.plot(t,y)
        plt.grid()
        plt.legend(['Reference', 'Output'], loc='lower right')
        plt.show()
```

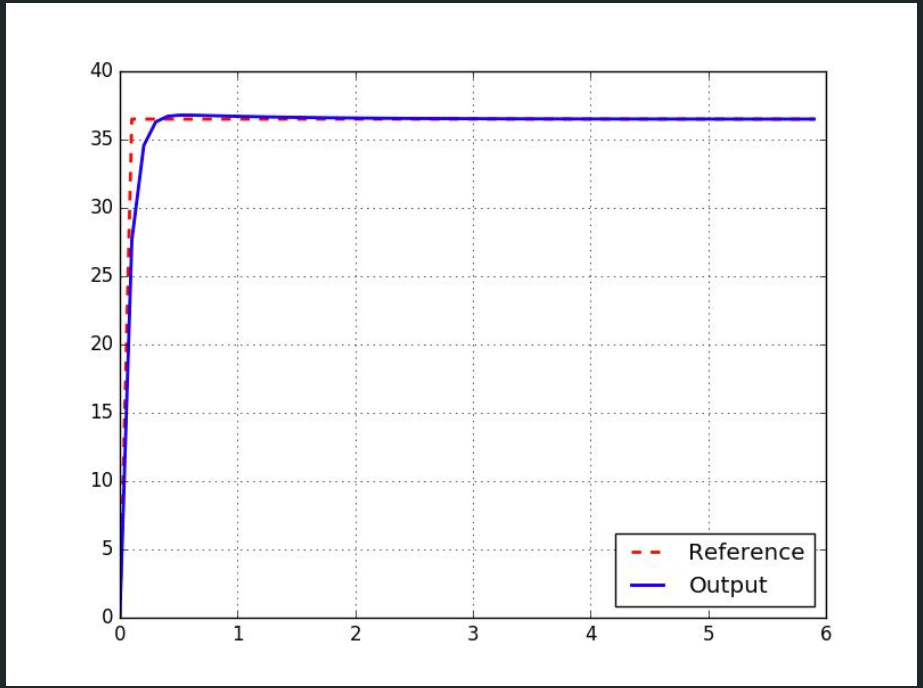
Open Loop



Practical Example

```
import matplotlib.pyplot as plt
import numpy as np
from time import sleep

T = 0.1      # Sampling time
r = [0]      # Reference vector
u = [0]      # Process input vector
y = [0]      # Process output vector
e = [0]      # Error vector

def control(r):
    # Controller
    c, d = 15, 15
    e.append(r-y[-1])      # Error
    return (e[-1]*(c+T*d) - c*e[-2] + u[-1])

def bodyTemp(u):
    # Model
    a, b = 0.5, 0.9
    return (T*a*u + y[-1])/(1+T*b)

if __name__ == '__main__':
    try:
        while True:
            r.append(36.5)            # Reference is always 36.5 C
            u.append(control(r[-1]))
            y.append(bodyTemp(u[-1]))
            sleep(T)
            print 'Elapsed: %.2f seconds' % float(len(y)*T)

    except KeyboardInterrupt:
        t = np.linspace(0,len(y)*T,len(y))
        plt.plot(t,r,'--r',linewidth=2.0)
        plt.plot(t,y)
        plt.grid()
        plt.legend(['Reference', 'Output'], loc='lower right')
        plt.show()
```

Feedback Control



Advantages

- What can we do with feedback control?
  - Accurately track a reference signal

Advantages

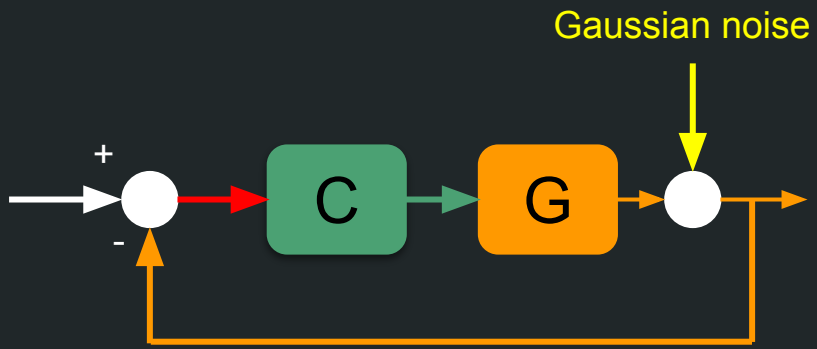
- What can we do with feedback control?
  - Accurately track a reference signal
  - Modify the dynamics of a known system

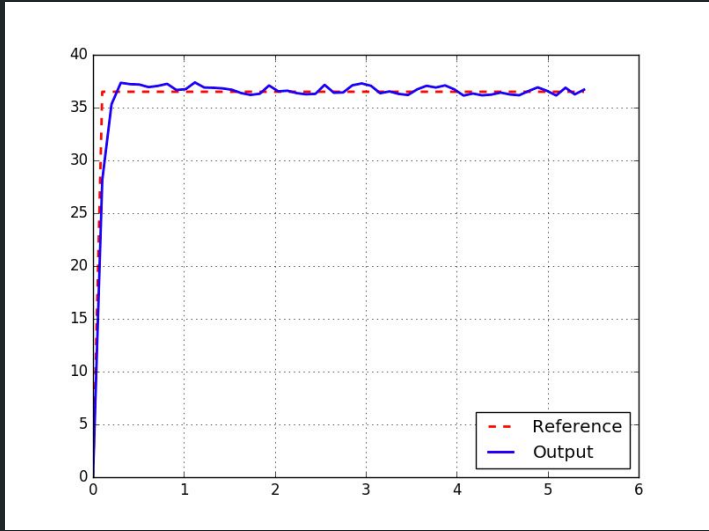
Advantages

- What can we do with feedback control?
  - Accurately track a reference signal
  - Modify the dynamics of a known system
  - Achieve robustness to noise and external disturbances (to an extent)

Advantages

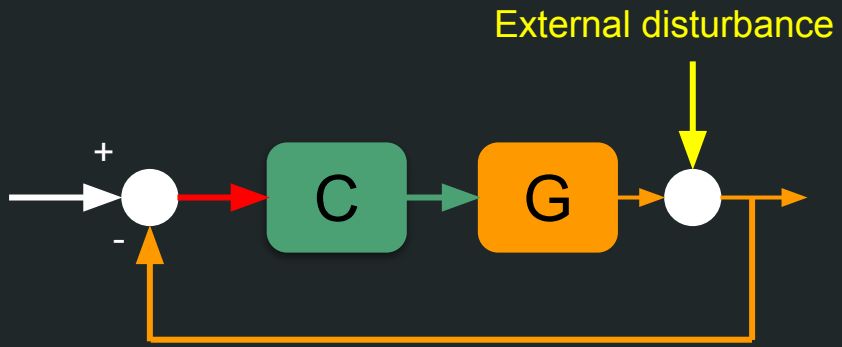
- What can we do with feedback control?
  - Accurately track a reference signal
  - Modify the dynamics of a known system
  - Achieve robustness to noise and external disturbances (to an extent)

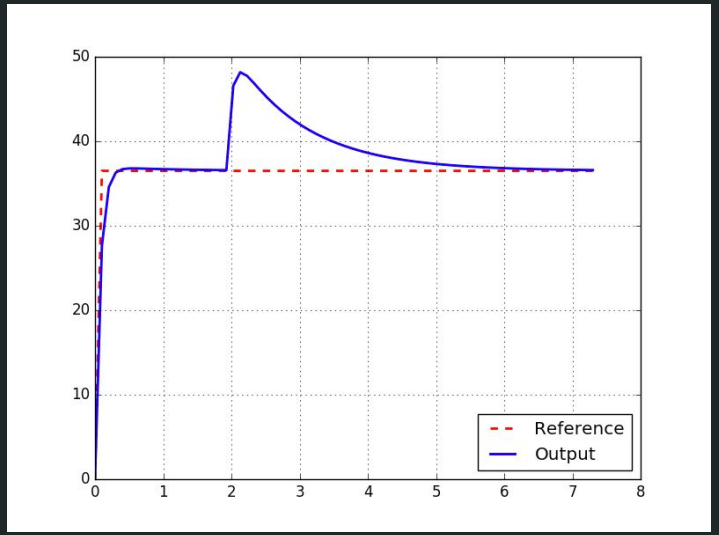




Advantages

- What can we do with feedback control?
  - Accurately track a reference signal
  - Modify the dynamics of a known system
  - Achieve robustness to noise and external disturbances (to an extent)



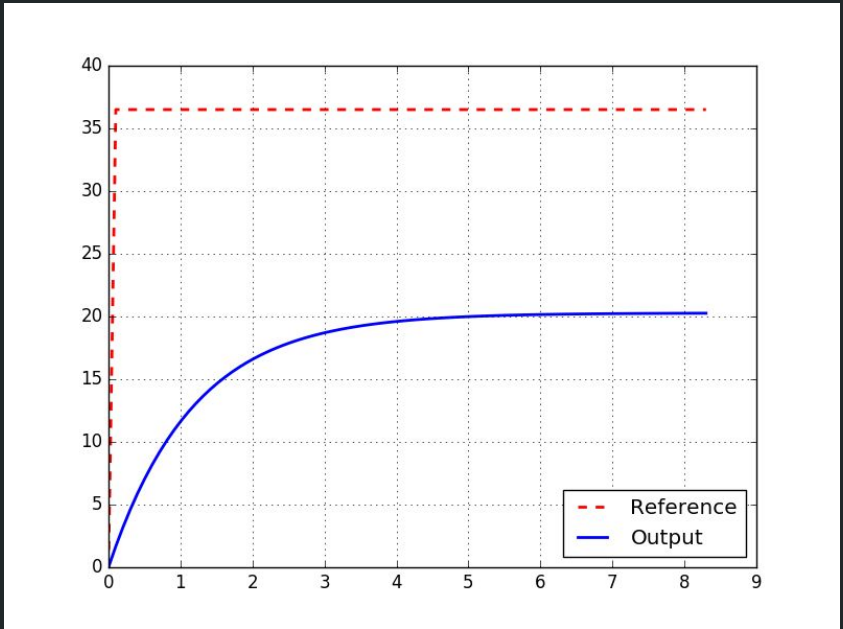


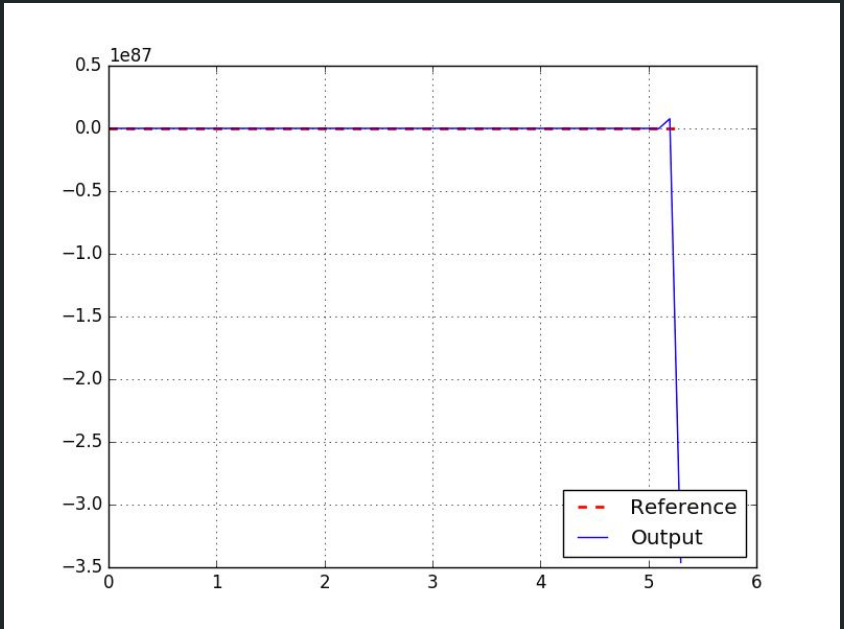
Advantages

- What can we do with feedback control?
  - Accurately track a reference signal
  - Modify the dynamics of a known system
  - Achieve robustness to noise and external disturbances (to an extent)
  - Stabilize an unstable system

Disadvantages

- What problems may arise from feedback control?
  - Dynamic instabilities, oscillations and runaway behavior



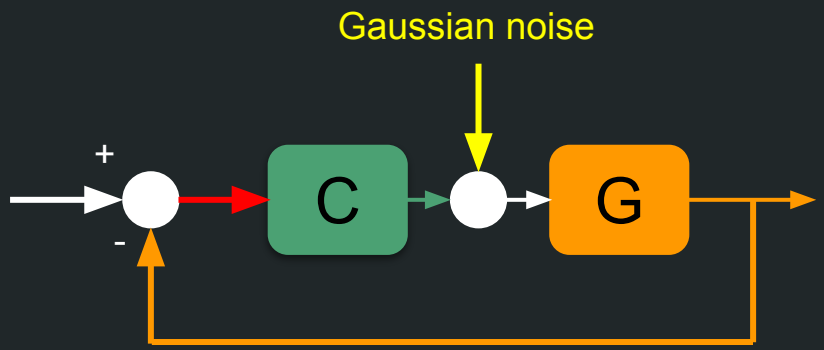


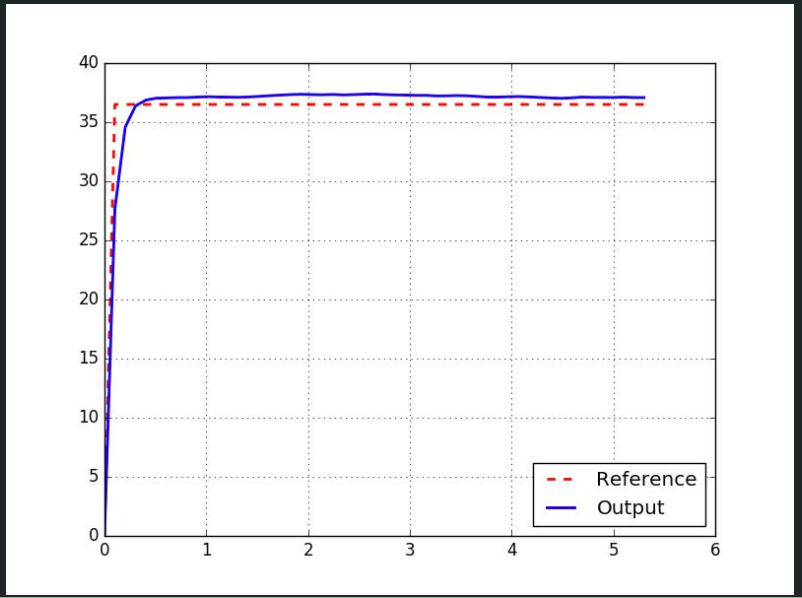
Disadvantages

- What problems may arise from feedback control?
  - Dynamic instabilities, oscillations and runaway behavior
  - Sensitivity to noise and external disturbances (???)

Disadvantages

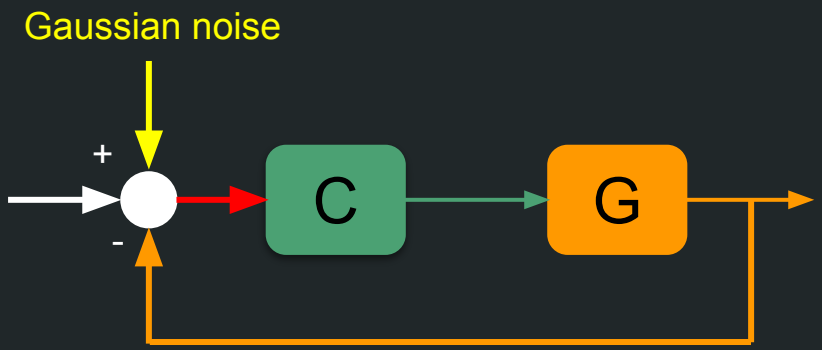
- What problems may arise from feedback control?
  - Dynamic instabilities, oscillations and runaway behavior
  - Sensitivity to noise and external disturbances

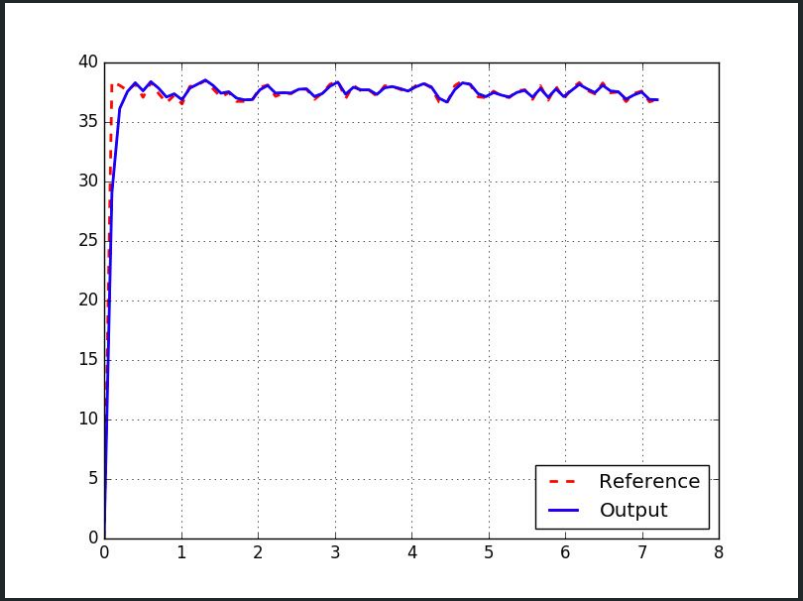




Disadvantages

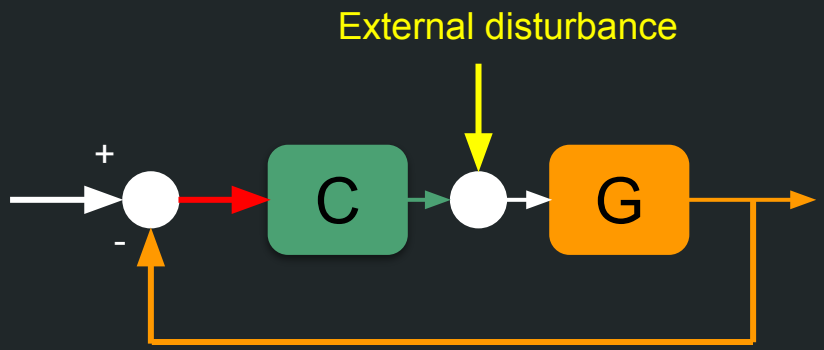
- What problems may arise from feedback control?
  - Dynamic instabilities, oscillations and runaway behavior
  - Sensitivity to noise and external disturbances

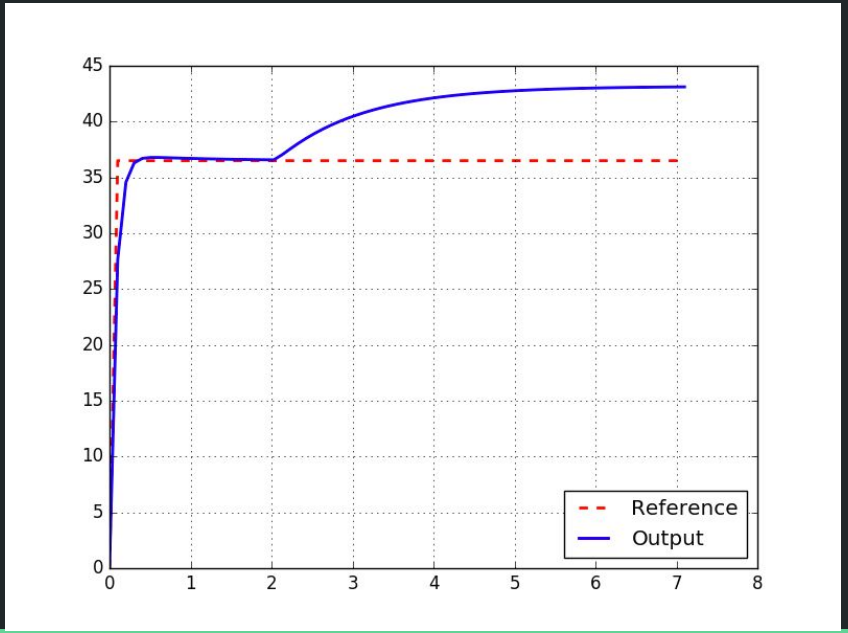




Disadvantages

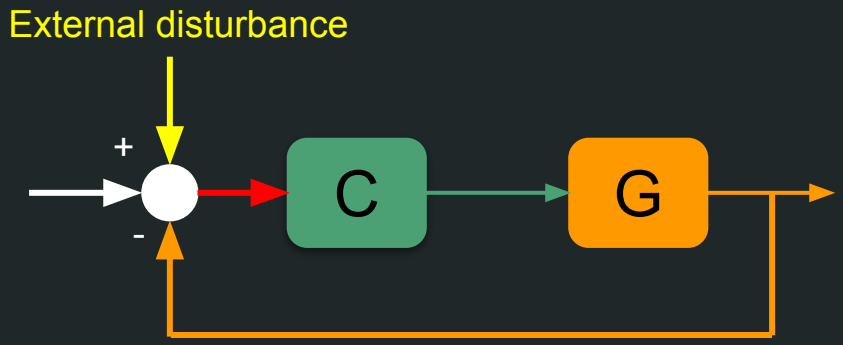
- What problems may arise from feedback control?
  - Dynamic instabilities, oscillations and runaway behavior
  - Sensitivity to noise and external disturbances

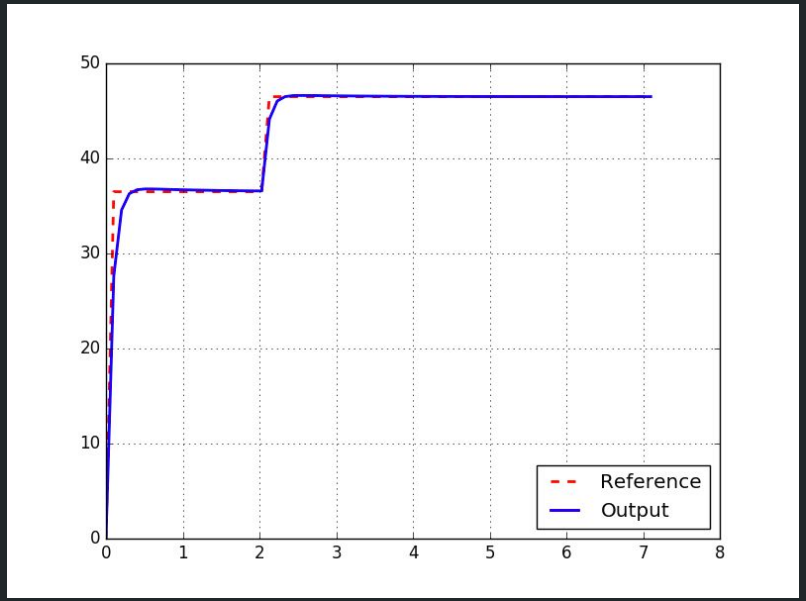




Disadvantages

- What problems may arise from feedback control?
  - Dynamic instabilities, oscillations and runaway behavior
  - Sensitivity to noise and external disturbances





Disadvantages

- What problems may arise from feedback control?
  - Dynamic instabilities, oscillations and runaway behavior
  - Sensitivity to noise and external disturbances
  - Huge search state-space for tuning

Disadvantages

- What problems may arise from feedback control?
  - Dynamic instabilities, oscillations and runaway behavior
  - Sensitivity to noise and external disturbances
  - Huge search state-space for tuning
  - Inability to actuate past saturation (limitation)
  - Purely reactionary behaviour (limitation)

Classical Applications

- Early examples

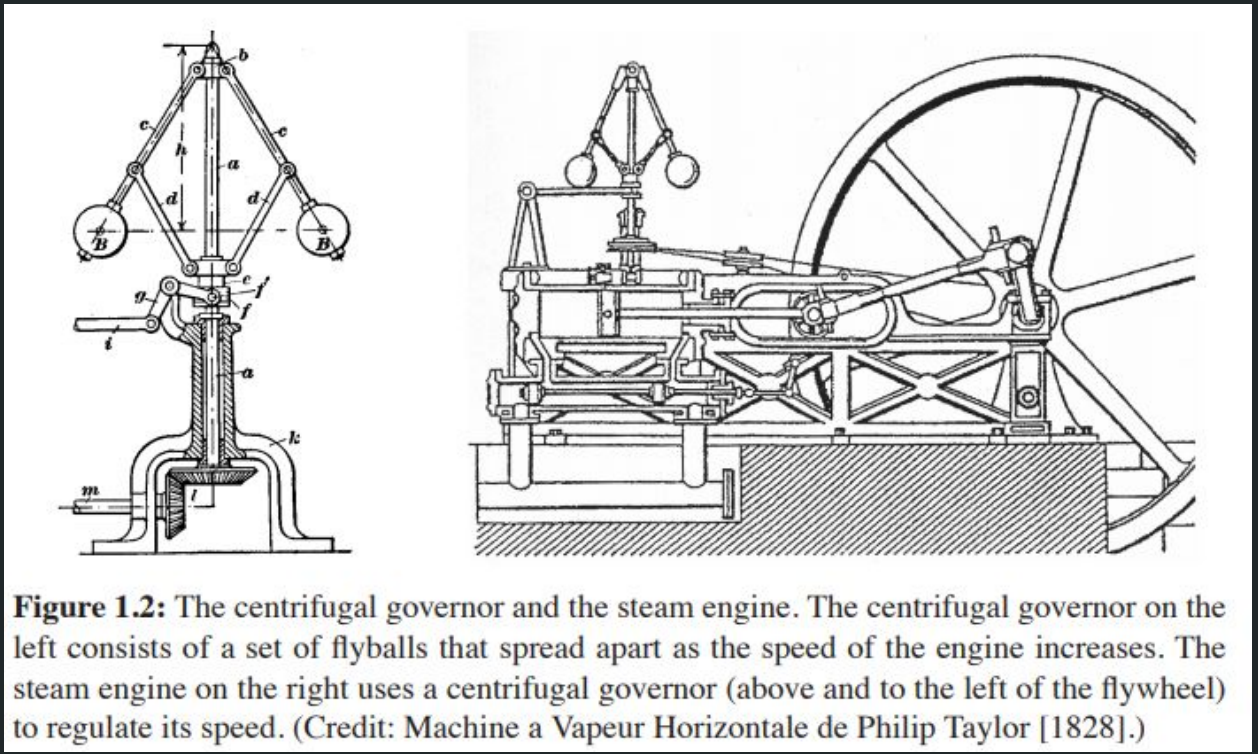
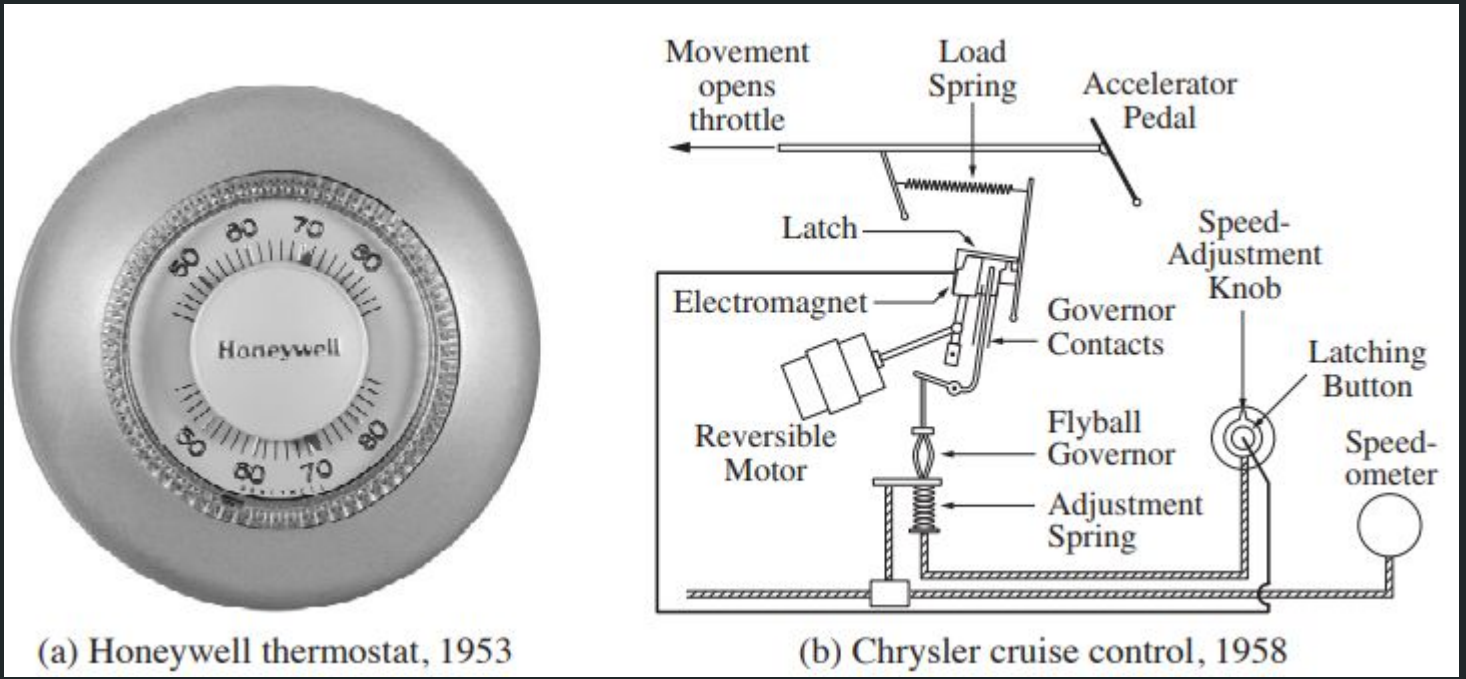


Figure 1.2: The centrifugal governor and the steam engine. The centrifugal governor on the left consists of a set of flyballs that spread apart as the speed of the engine increases. The steam engine on the right uses a centrifugal governor (above and to the left of the flywheel) to regulate its speed. (Credit: Machine a Vapeur Horizontale de Philip Taylor [1828].)

Classical Applications

- Early examples

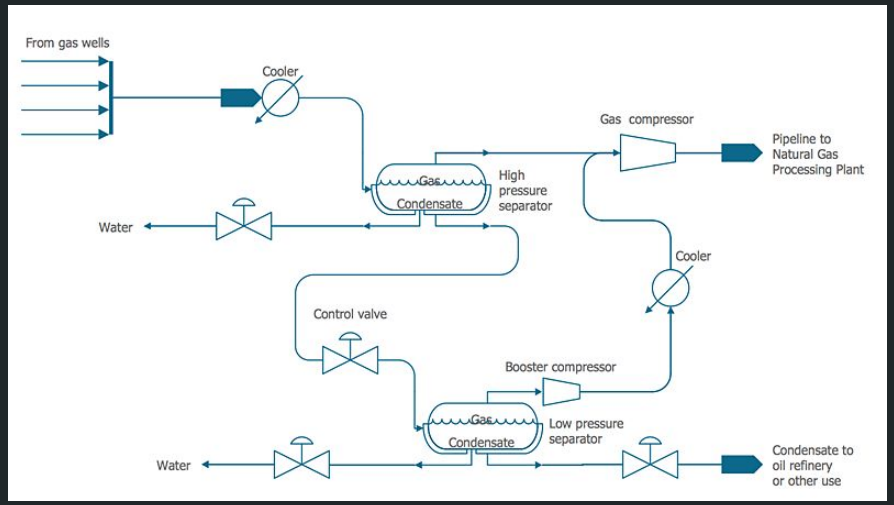


(a) Honeywell thermostat, 1953

(b) Chrysler cruise control, 1958

Classical Applications

- Industrial



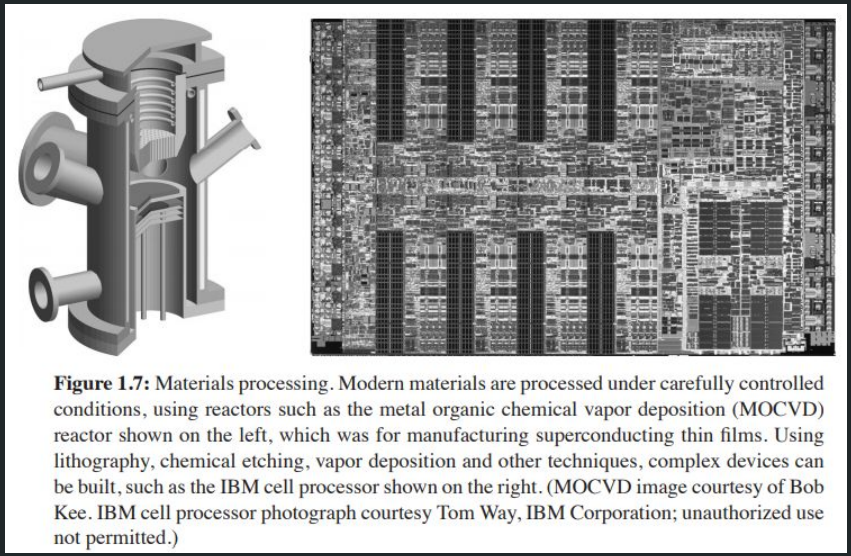


Figure 1.7: Materials processing. Modern materials are processed under carefully controlled conditions, using reactors such as the metal organic chemical vapor deposition (MOCVD) reactor shown on the left, which was for manufacturing superconducting thin films. Using lithography, chemical etching, vapor deposition and other techniques, complex devices can be built, such as the IBM cell processor shown on the right. (MOCVD image courtesy of Bob Kee. IBM cell processor photograph courtesy Tom Way, IBM Corporation; unauthorized use not permitted.)

Classical Applications

- Industrial

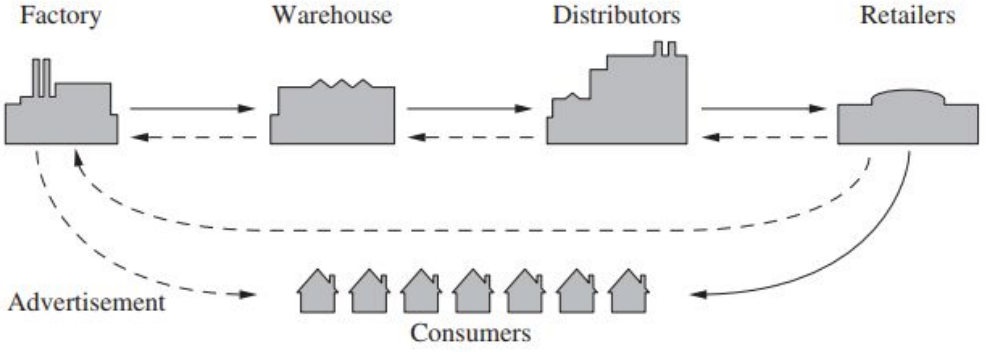
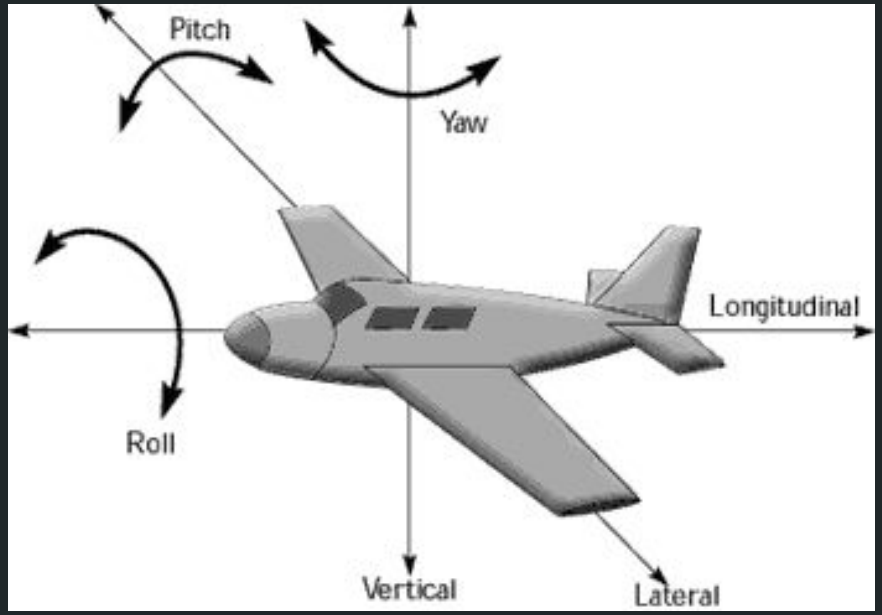
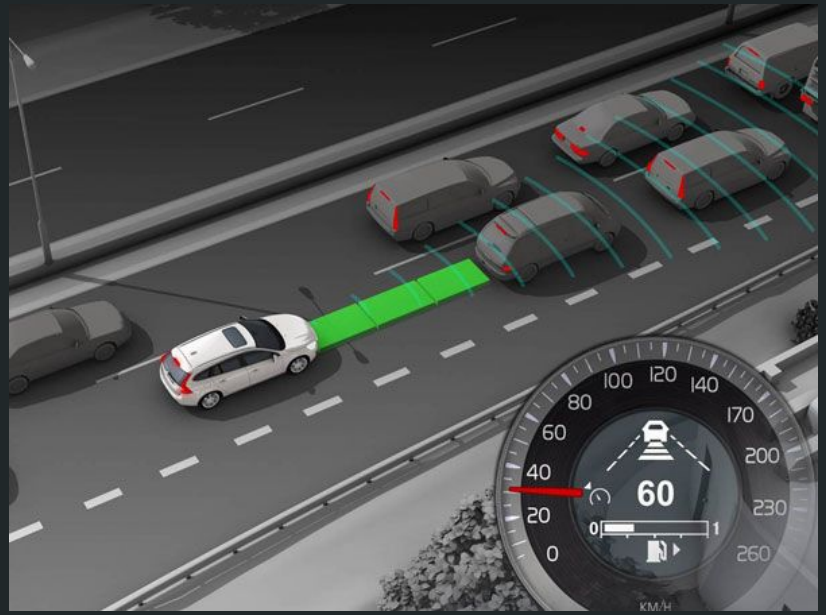


Figure 1.11: Supply chain dynamics (after Forrester [75]). Products flow from the producer to the customer through distributors and retailers as indicated by the solid lines. There are typically many factories and warehouses and even more distributors and retailers. The dashed lines show the upward flow of orders. The numbers in the circles represent the delays in the flow of information or materials. Multiple feedback loops are present as each agent tries to maintain the proper inventory level.

Classical Applications

- Commercial



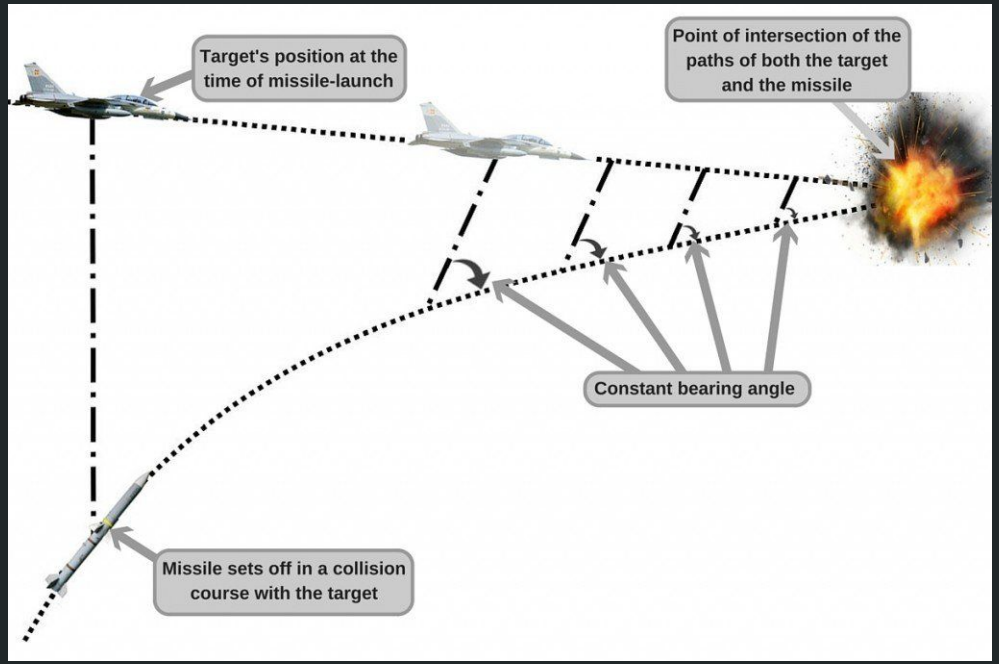


Classical Applications

- Commercial

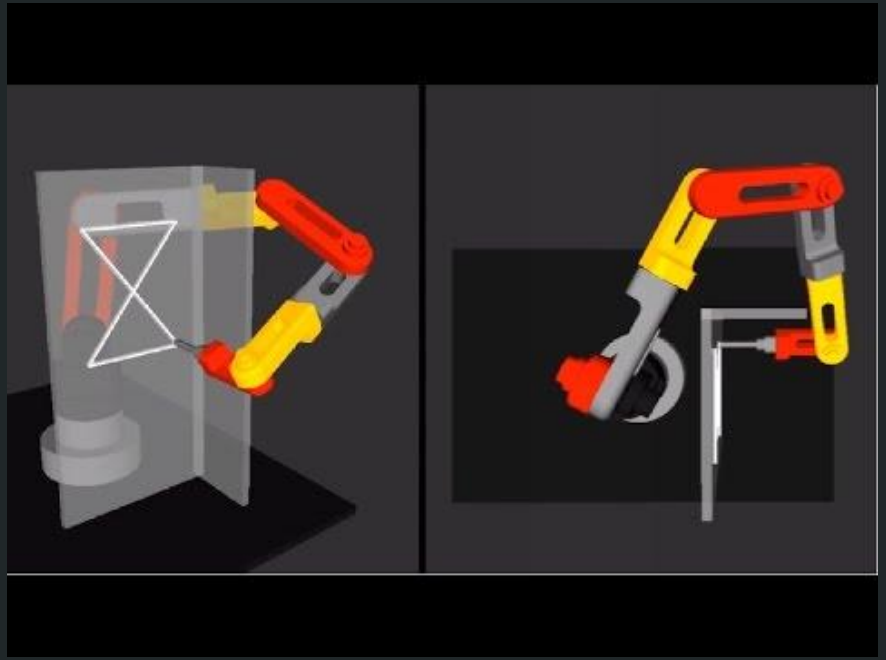
Classical Applications

- Military



Applications in Robotics

- Robot manipulators



https://www.youtube.com/watch?v=-FYs5INtVM4

Applications in Robotics

- Mobile Robots

https://www.youtube.com/watch?v=cNZPRsrwumQ

Applications in Robotics

- Mobile Robots

https://www.youtube.com/watch?v=cNZPRsrwumQ

Applications in Robotics

- Mobile Robots

https://www.youtube.com/watch?v=cNZPRsrwumQ

https://www.youtube.com/watch?v=-7xvqQeoA8c

Applications in Robotics

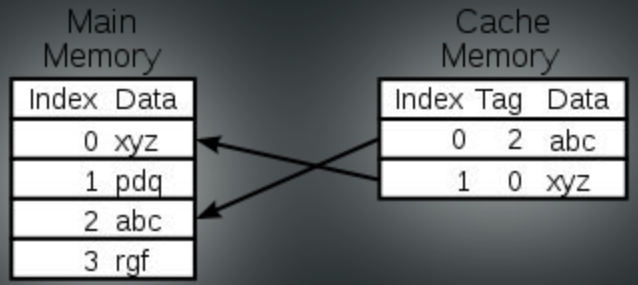
- Mobile Robots

https://www.youtube.com/watch?v=eznMokFQmpc

https://www.youtube.com/watch?v=YQIMGV5vtd4

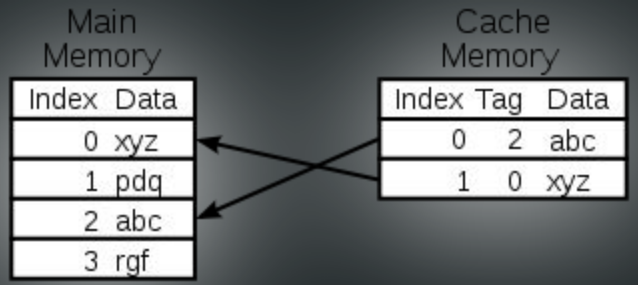
Practical Exercise

- Feedback Control in Computer Systems



Practical Exercise

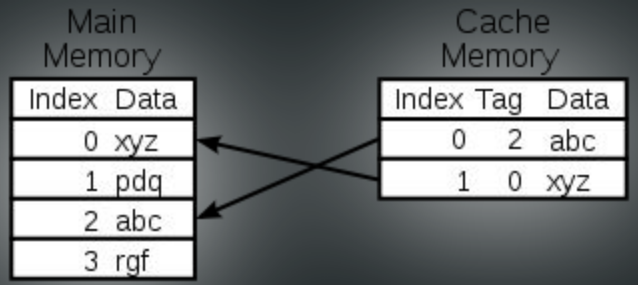
- Feedback Control in Computer Systems



- **Cache hit rate:** Percent of times a memory access results in a non-null response (i.e., the desired data is in the cache)

Practical Exercise

- Feedback Control in Computer Systems



- **Challenge:** Design a feedback control system which maintains the cache hit rate at 60%