SEQUENCE / SERIES

SEQUENCE: 4, 5, 14, 19, 24

SERIES $4 + 9 + 14 + 19 + 24 + 29$ A.P

A.P - Arithmetic Progression

a - 1st term $= 4$

d - common difference $= 5$

n - number of terms $= 6$

To get nth term $a + (n-1)d$

e.g. $4 + (5-1)5 = 24$

is 5th term.

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$S_6 = \frac{6}{2}\{2 \times 4 + (6-1)5\}$$

$$= 99$$

$Q^{\underline{n}}$1. The 6th and 9th terms of an arithmetic progression are 200 and 245. Calculate the:

a/ 10th term

b) Sum of the first 20 terms.

$Q^{\underline{n}}$2. H/W

Given the series 20+26+32+38+...,
Find the number of terms that will give you a sum of 328. Use the formula

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Solution $Q^{\underline{n}}$ 1

a)

$$a + 5d = 200$$
$$a + 8d = 245$$

$$3d = 45$$
$$d = 15$$
$$a = 200 - 5 \times 15$$
$$a = 125$$

b)

$$S_{20} = \frac{20}{2}\{125 \times 2 + (20-1)15\}$$
$$= 5350$$

GEOMETRIC PROGRESSION

Sequence: 3, 6, 12, 24, 48, 96

Series: 3 + 6 + 12 + 24 + 48 + 96

First term: a

Common ratio: r

nth term: ar^{n-1}

Sum of series formula: $S_n = \dfrac{a(r^n - 1)}{r - 1}$

Sum of series formula: $S_n = \dfrac{a(1 - r^n)}{1 - r} \qquad -1 < r < +1$

Sum of the series above: $S_6 = \dfrac{3(2^6 - 1)}{2 - 1} = 189$

Example:

The 4th and 7th terms of a geometric progression are 54 and 1456 respectively. Determine the:

a) 5th term
b) Sum of the first 10 terms.

Solution:

$$ar^3 = 54$$

$$ar^6 = 1456$$

$$\frac{ar^6}{ar^3} = r^3 = \frac{1456}{54} = 27$$

$ar^3 = 54$ but r = 3 ; 27a =54; a=2.

$$r = 3$$

$$S_{10} = \frac{2(3^{10} - 1)}{3 - 1} = 59048$$

Simple Interest

1. A man deposited Sh. 10,000 in a bank offering a simple interest of 12% per year. Calculate the:
   a) Total amount after 5 years.
   b) Time the accumulated amount will be Sh. 35,000.

Interest = PRT; where P is principal, R is rate and T is time

a) $A = 10{,}000 + \dfrac{10000 \times 12 \times 5}{100} = 16000$

b) Interest =I = 35000-10000 = 25000

$$25000 = \frac{10000 \times 12 \times T}{100}$$

$$T = 20.83$$

$20+26+32+ (n^2-n)6 = 328$

$S_n = \frac{n}{2}[2 \cdot 20 + (n-1)6] = 656$

$40n + 6n^2 - 6n$

2. A lady deposited some money in a bank offering a simple interest of 10% per year. After 4 years, the accumulated amount was Sh. 20300. Calculate the amount deposited.

$6n^2 + 34n - 656 = 0$

Compound interest

$A = P\left(\dfrac{100+R}{100}\right)^n$ A is Amount, R is rate of interest, P is principal, n is time.

1. Mary deposited Sh. 5,500 in a bank offering a compound interest of 10% per annum. Calculate the:
   a) Total amount after 8 years
   b) Time the accumulated amount will be Sh. 22,000.

Solution:

5500×1.1^8

$\log a^n = n \log a$

a) $A = 5500\left(\dfrac{100+10}{100}\right)^8 = 11789.74$

$5500 \times 1.1^n = 22000$

b) $5500 \times 1.1^n = 22000$

$$1.1^n = \frac{22000}{5500} = 4$$

b) $\log_{10} 1{,}1^n = \log_{10} 4$

$n \log_{10} 1.1 = \log_{10} 4$

$n \log_{10} 1.1 = \log_{10} 4$

$$n = \frac{\log_{10} 4}{\log_{10} 1.1} = 14.55$$

$n = \log 4 \div \log 1.1 =$

2. A lady deposits Sh. 5,000 at the beginning of each year in a bank offering a compound interest of 10% per year. Determine the total amount after 10 years.
Solution:

$$1st \quad year \qquad amount = 5000\left(\frac{100+10}{100}\right)^1 = 5000\times 1.1$$

$$A = P\left[\frac{100+R}{100}\right]^n$$

2nd year amount $\quad (5000 + 5000\times 1.1)\times 1.1 = 5000\times 1.1^1 + 5000 \times 1.1^2$

10th year amount $\quad \overset{a}{5000\times 1.1} + 5000\times 1.1^2 + 5000\times 1.1^3 + \ldots + 5000\times 1.1^{10}$

$$S_{10} = 5000\times 1.1\left(\frac{1.1^{10}-1}{1.1-1}\right) = 87655.83$$

$$S_n = a\left[\frac{r^n-1}{r-1}\right] \qquad \frac{R}{100} = r$$

$$1.1^{10} - 1 = A \quad \frac{A}{0.1} \times 5000 \times 1.1$$

DISCOUNTING

$$\left[P = \frac{S}{(1+r)^n}\right]$$ P is present amount, future value is S, r is discounting rate.

$$A = P\left[\frac{100+R}{100}\right]^n$$

$$A = P\left[1+r\right]^n$$

$$S = P(1+r)^n$$

$$\frac{S}{(1+r)^n} = P$$

Example

Determine the present value of $ 20,000 received in 5 years time at a discount rate of 10%.

Solution:

$$\frac{10}{100} = 0.1$$

$$20000 \div 1.1^5$$

$$P = \frac{20000}{(1+0.1)^5} = 12,420$$

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DISCOUNTING A SERIES

$$i = 1, 2, 3, 4, \ldots, n$$

$$P = \sum_{i=1}^{n} \frac{A_i}{(1+r)^i} = \frac{A_1}{(1+r)^1} + \frac{A_2}{(1+r)^2} + \ldots + \frac{A_n}{(1+r)^n}$$

Example

Determine the present value of receiving Sh. 1000 in 1 year's time, Sh. 2000 in 2 years time and Sh. 3000 in 3 years time. at discount rate of 10%.

$$r = \frac{10}{100} = 0.1$$

Solution: $$P = \frac{1000}{1.1} + \frac{2000}{1.1^2} + \frac{3000}{1.1^3} = 4814$$

ANNUITIES

OA ordinary annuity

1 $$FV_{OA} = C\left[\frac{(1+i)^n - 1}{i}\right]$$

2 $$PV_{OA} = C\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$i = r$

0.1

AD Annuity due

$i = r = \frac{R}{100}$

3 $$FV_{AD} = C\left[\frac{(1+i)^n - 1}{i}\right](1+i)$$

PV

4 $$PV_{AD} = C\left[\frac{1-(1+i)^{-n}}{i}\right](1+i)$$

Example

OA

Determine the present value of an ordinary anuuity of $500 per year received for 10 years when the discount rate is 10%.

Solution: $$P = 500\left[\frac{1-1.1^{-10}}{0.1}\right] = 3072$$