PRML Assignment 1 (Team 33)

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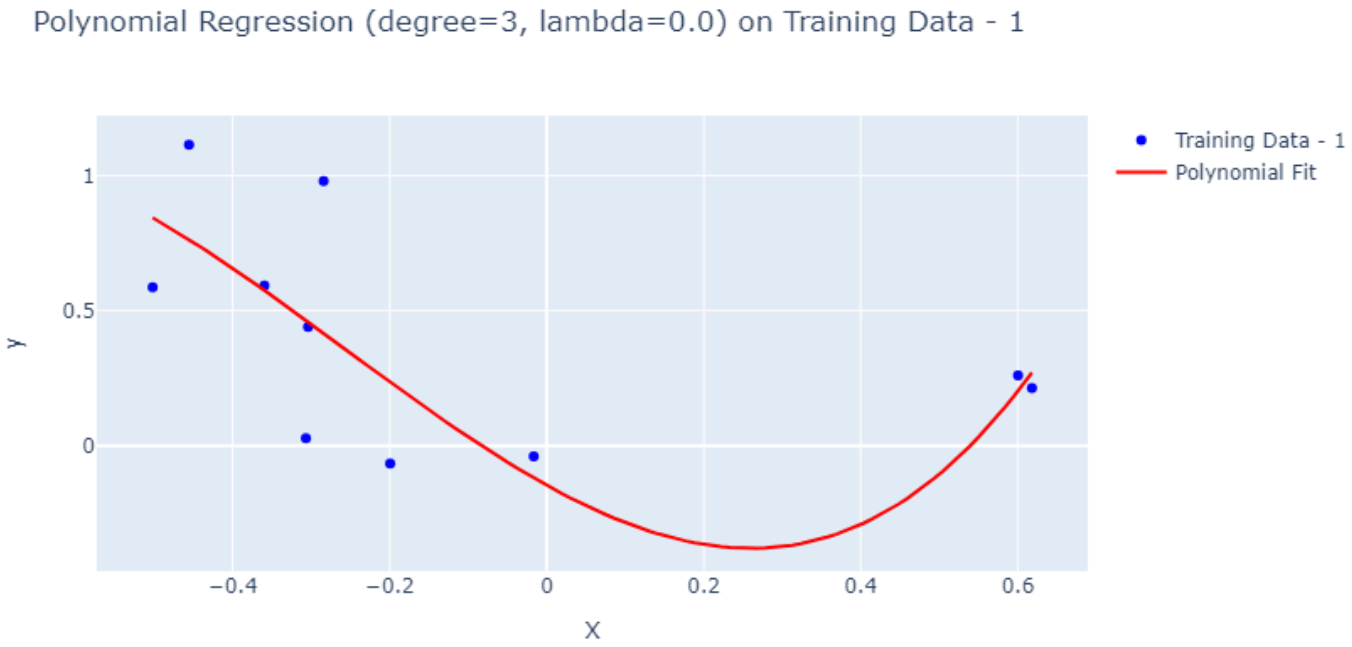
Results for Assignment 1

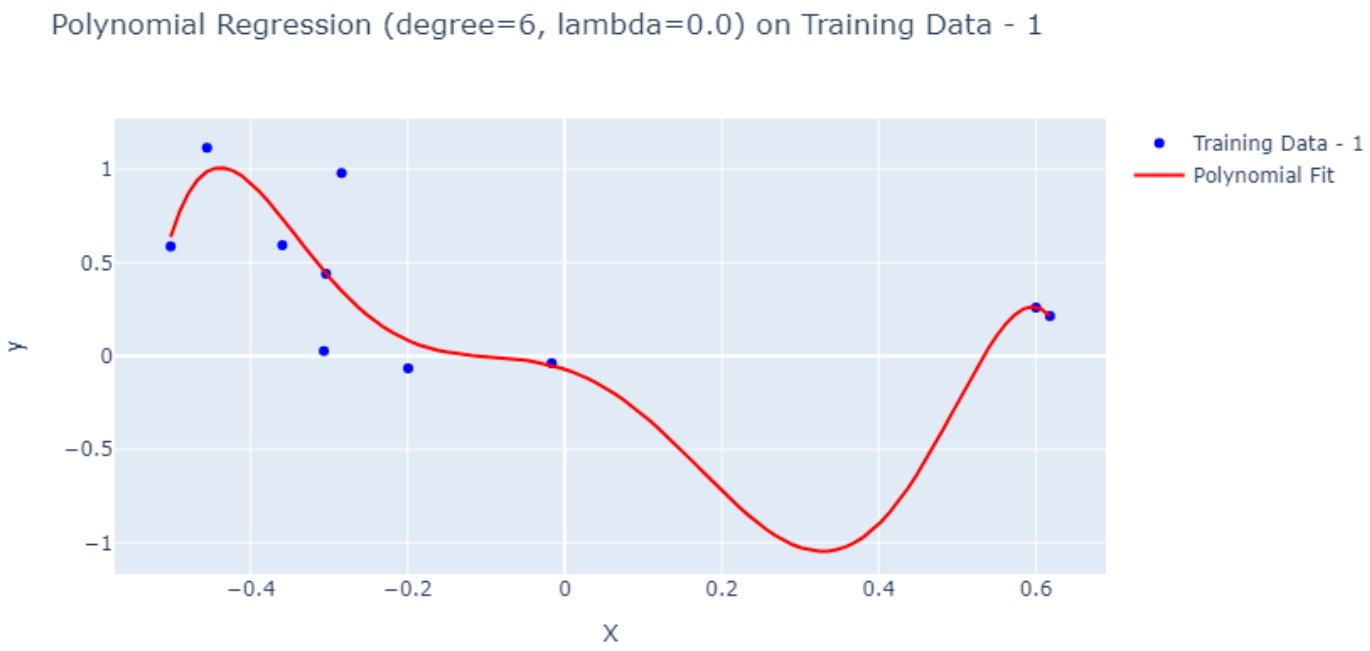
- **Regression Model:** Linear model for regression using polynomial basis functions
- **Regularization method:** Quadratic regularization

Training Dataset 1(a): 10 examples

> 1-dimensional (Univariate) input data

Without Regularization

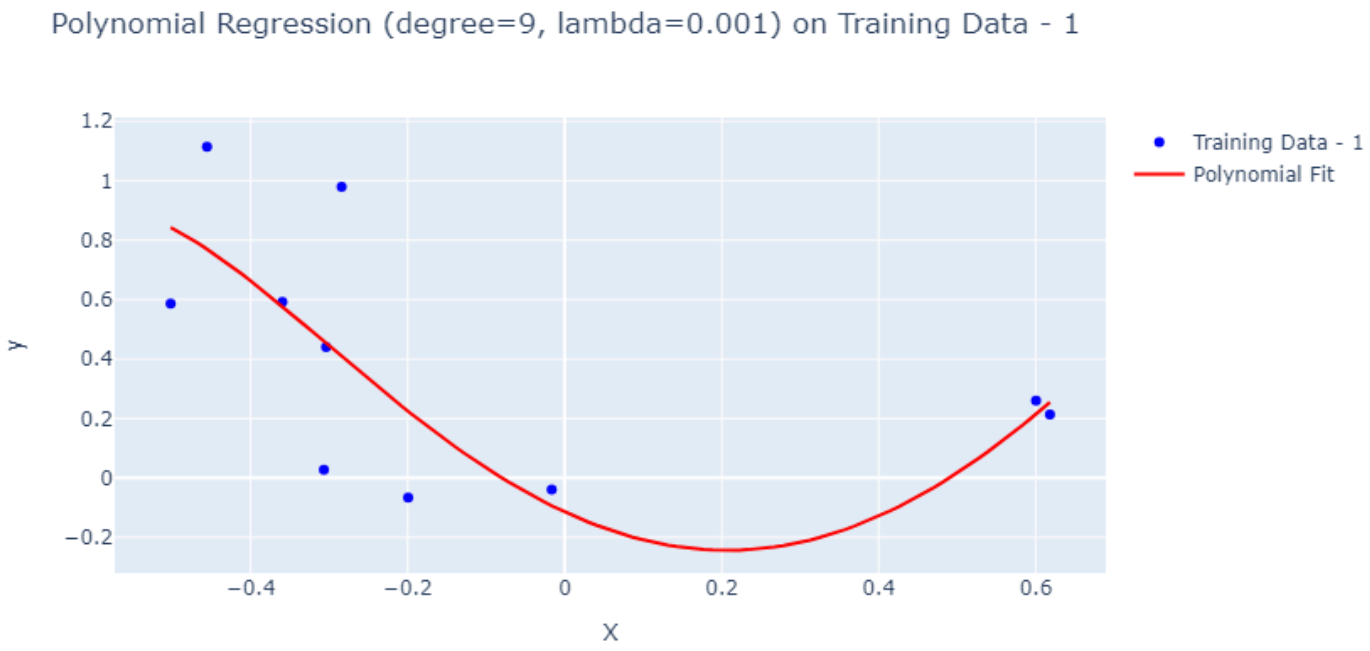


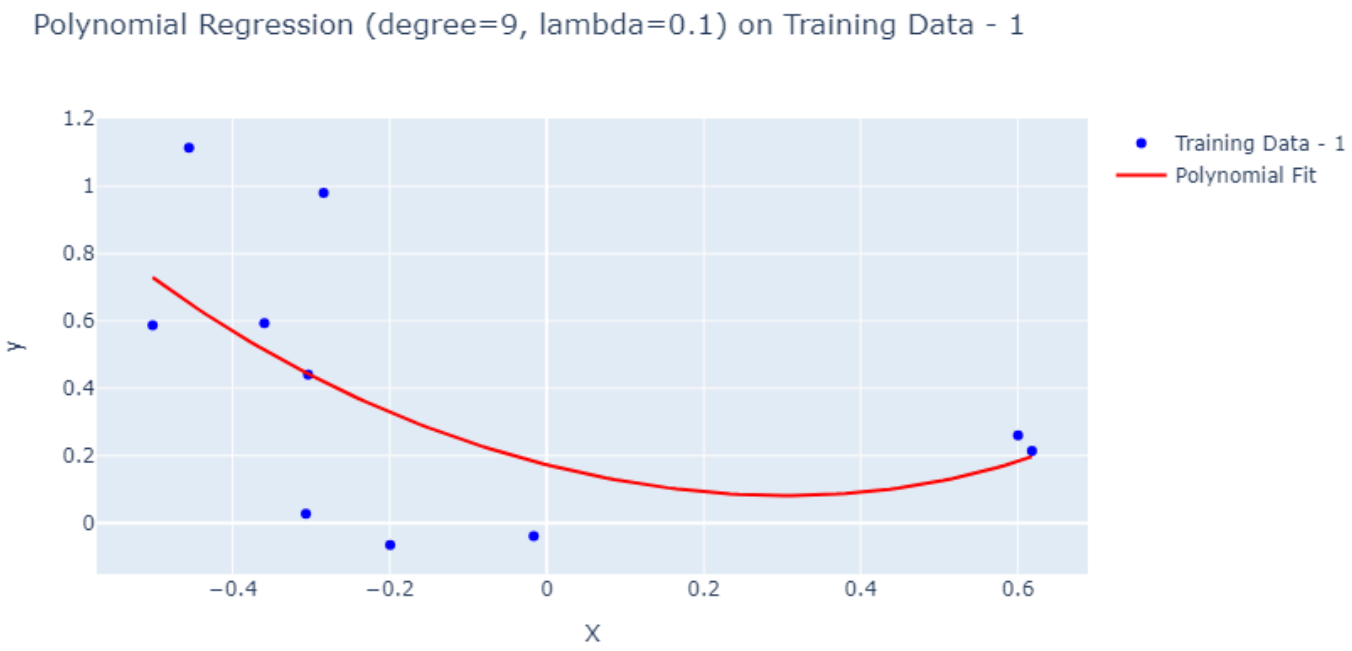


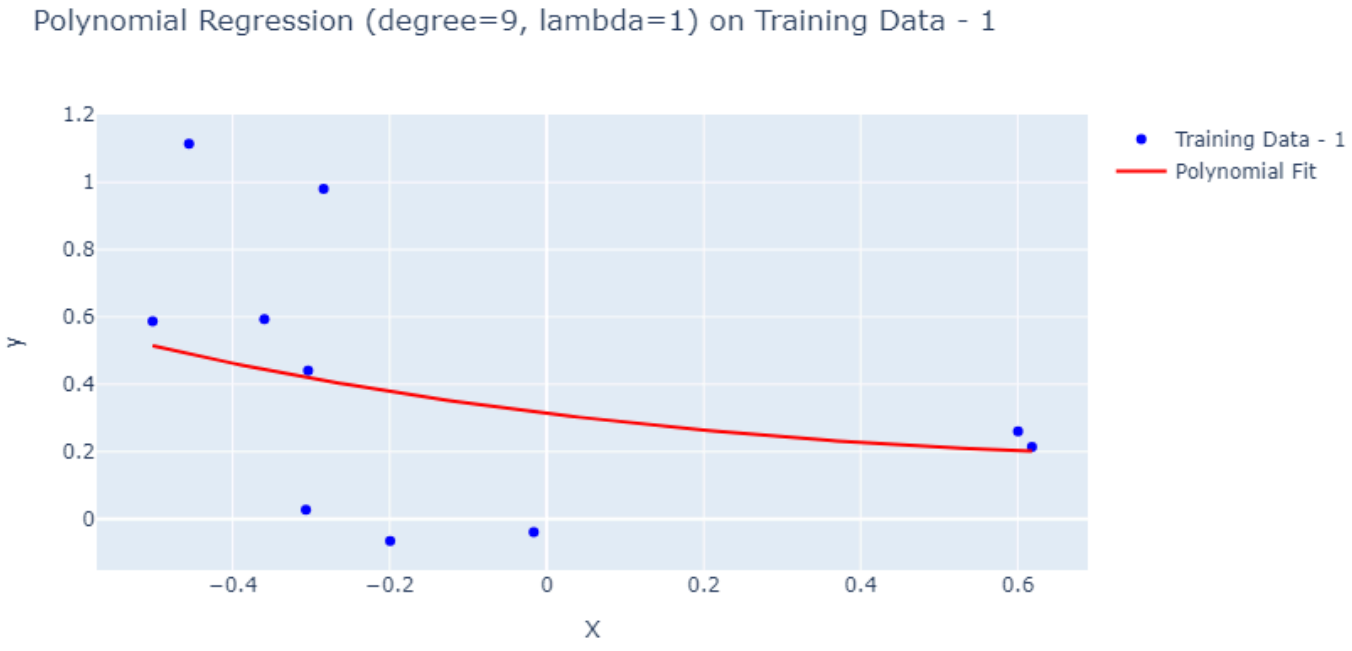
Observations

1. **Degree 3 without Regularization**: Comparable performance with regularization, slightly better in test MSE.
2. **Degree 6 without Regularization**: Severe overfitting with extremely high MSE values in validation and test.

With Regularization







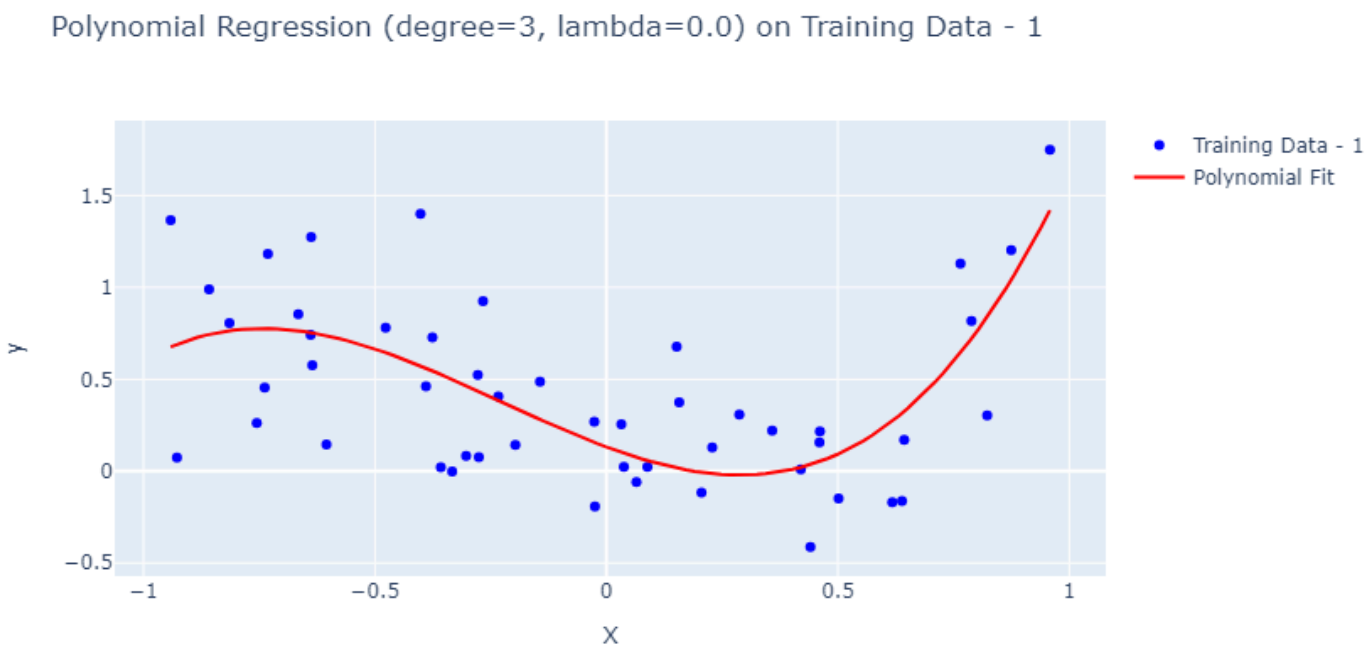
Observations

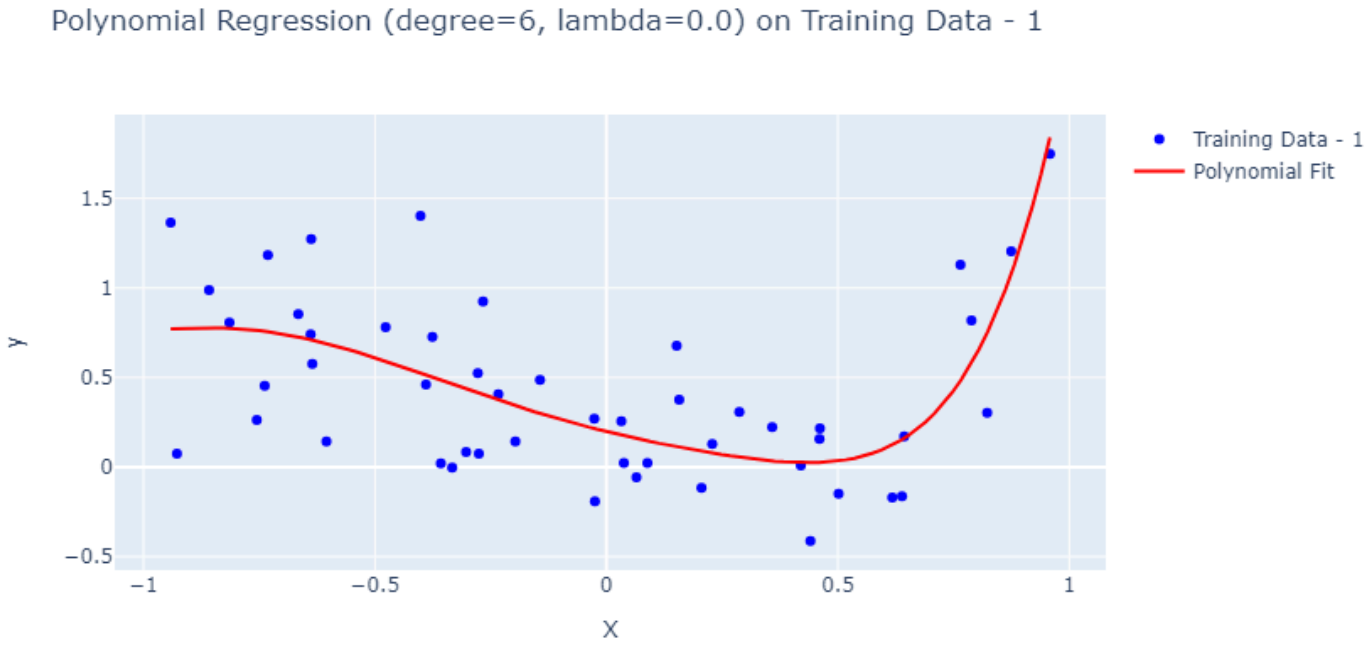
1. **Degree 9 with Regularization (λ = 0.001)**: Lower MSE in training, but higher in test, suggesting some overfitting.
2. **Degree 9 with Regularization (λ = 0.1 and 1)**: Increasing lambda decreases the test MSE, improving generalization.

Training Dataset 1(b): 50 examples

> 1-dimensional (Univariate) input data

Without Regularization

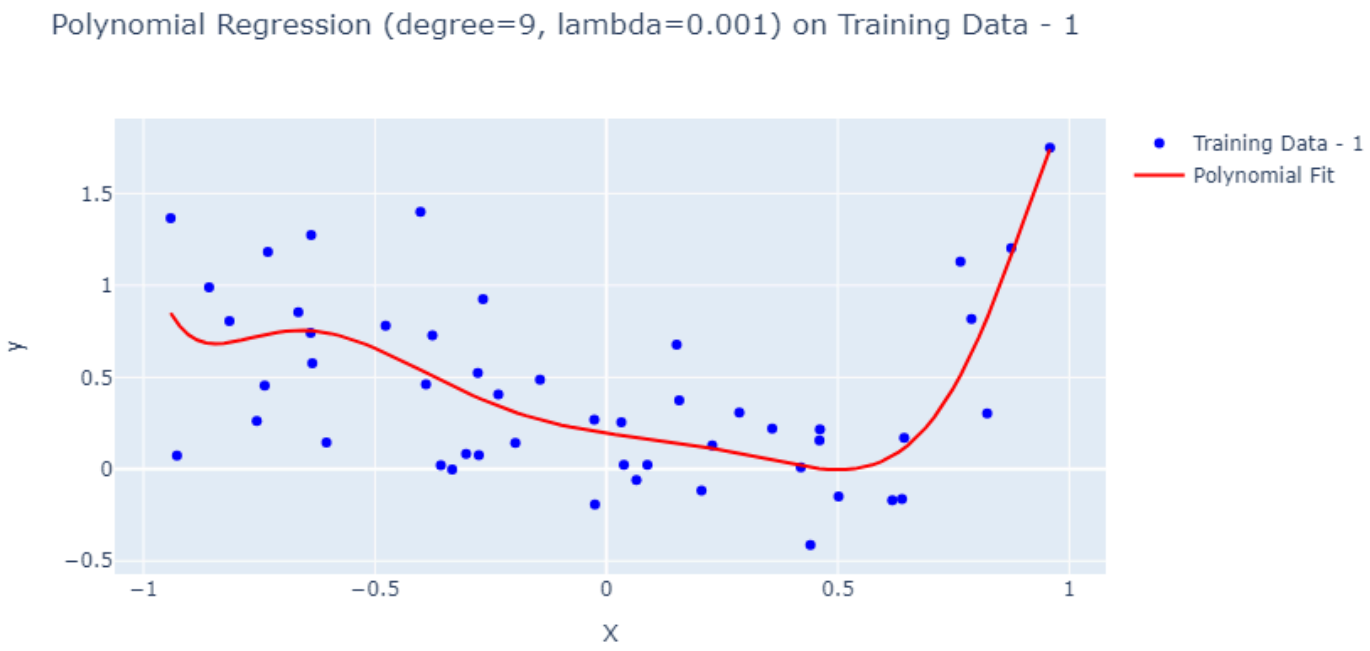


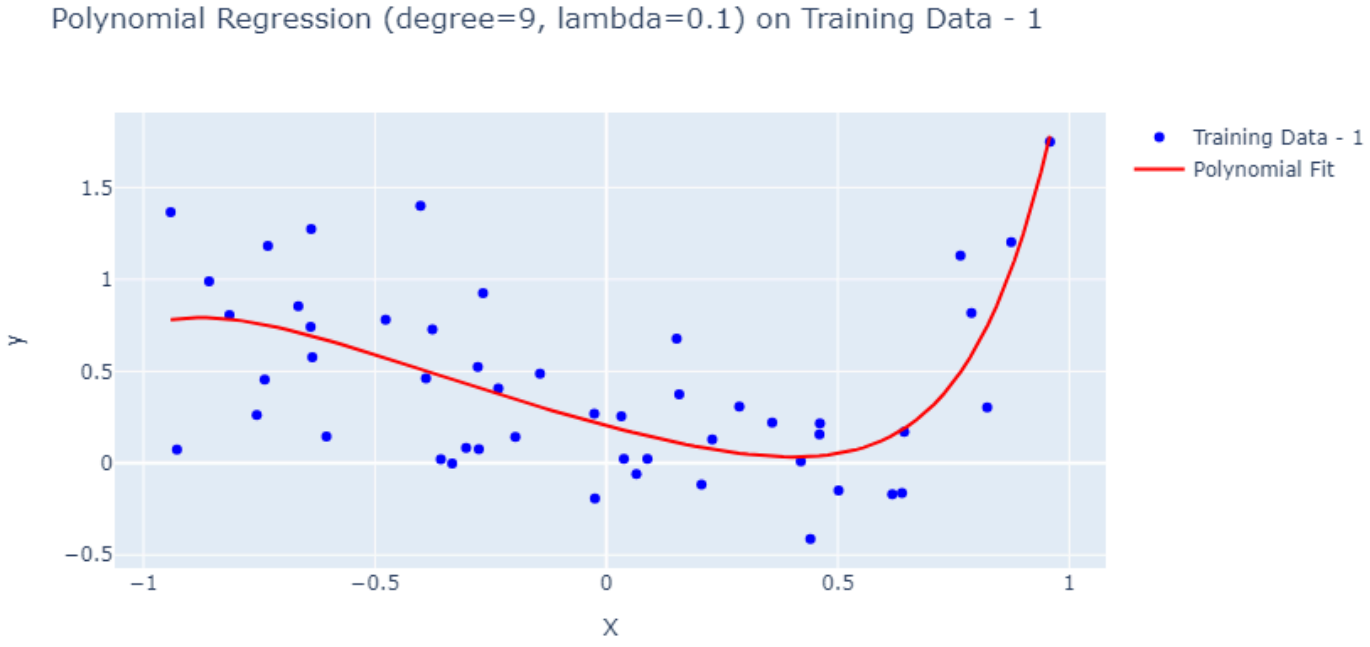


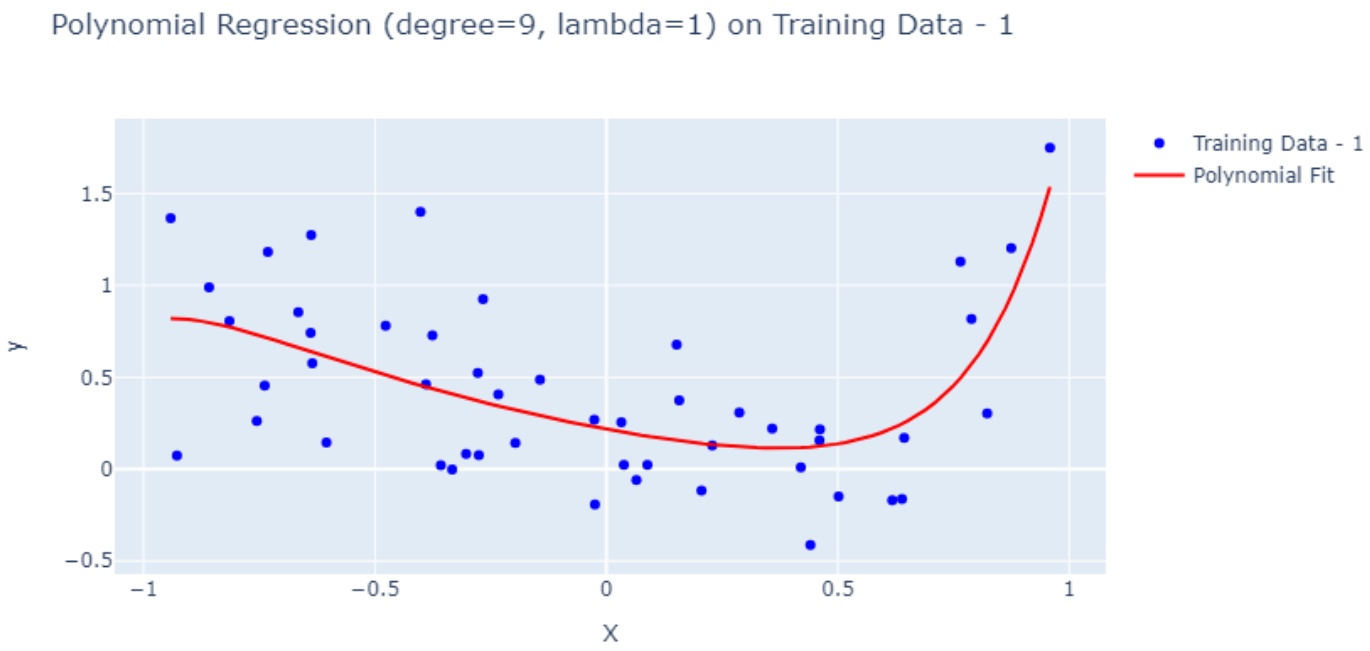
Observations

1. **Degree 3 without Regularization**: Slightly worse test performance, but better than some regularized cases.
2. **Degree 6 without Regularization**: Similar performance to Degree 9 with regularization, but slightly less effective.

With Regularization







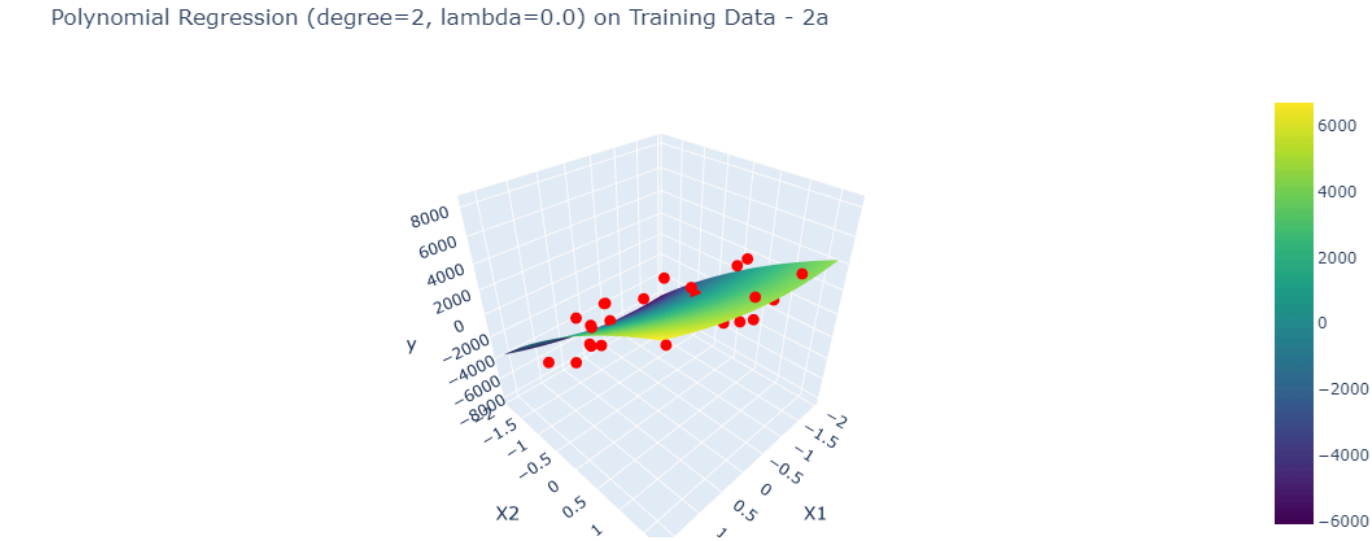
Observations

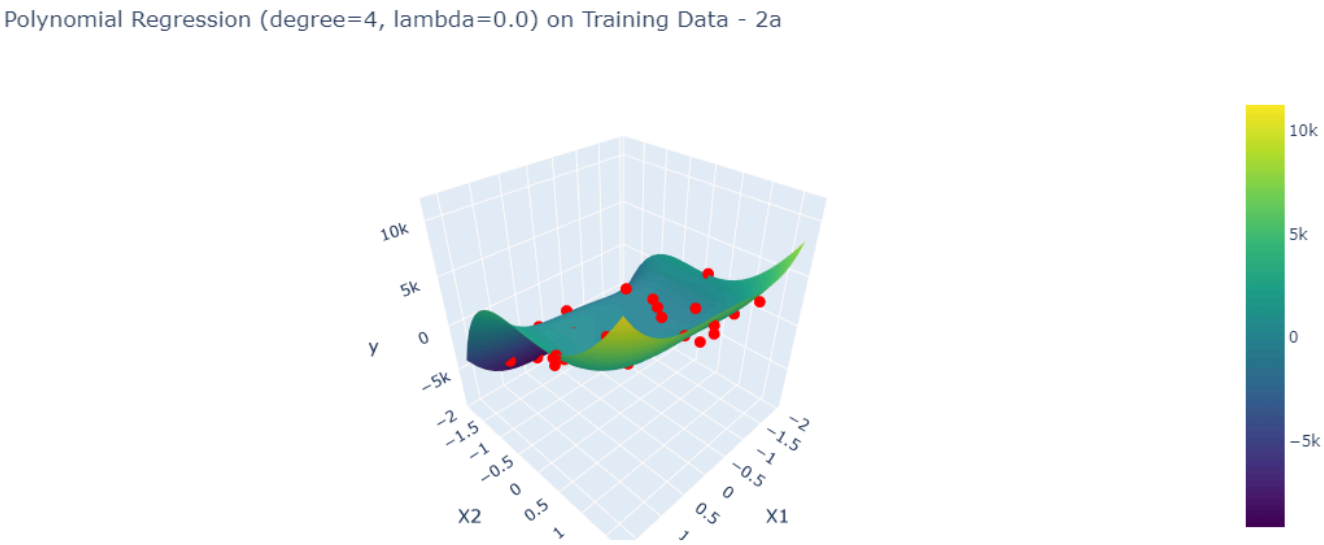
1. **Degree 9 with Regularization (λ = 0.001)**: Good generalization, with slightly better MSE in validation and test than 1a.
2. **Degree 9 with Regularization (λ = 0.1 and 1)**: Increasing lambda results in marginal increases in test MSE.

Training Dataset 2(a): 25 examples

> 2-dimensional (Bivariate) input data

Without Regularization

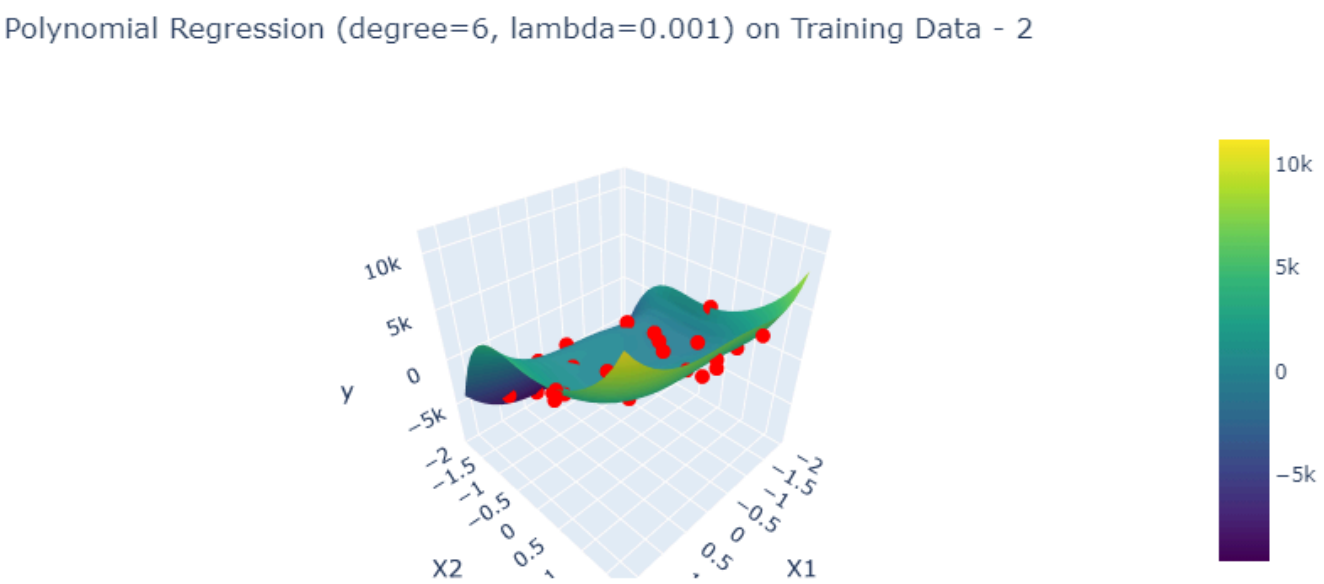


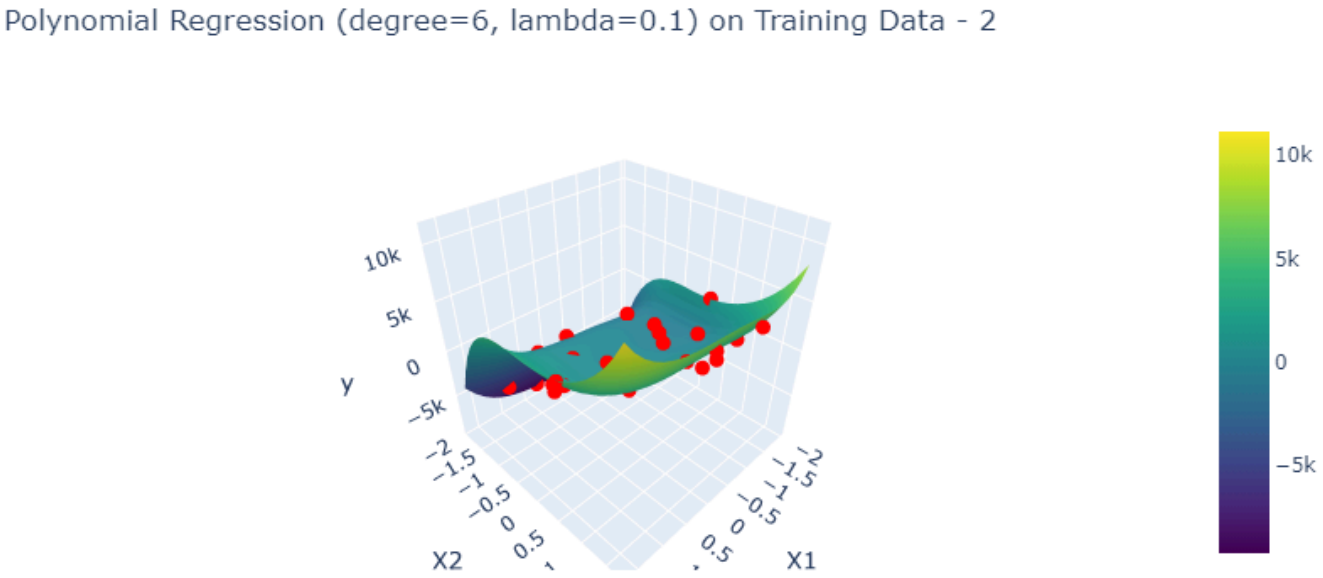


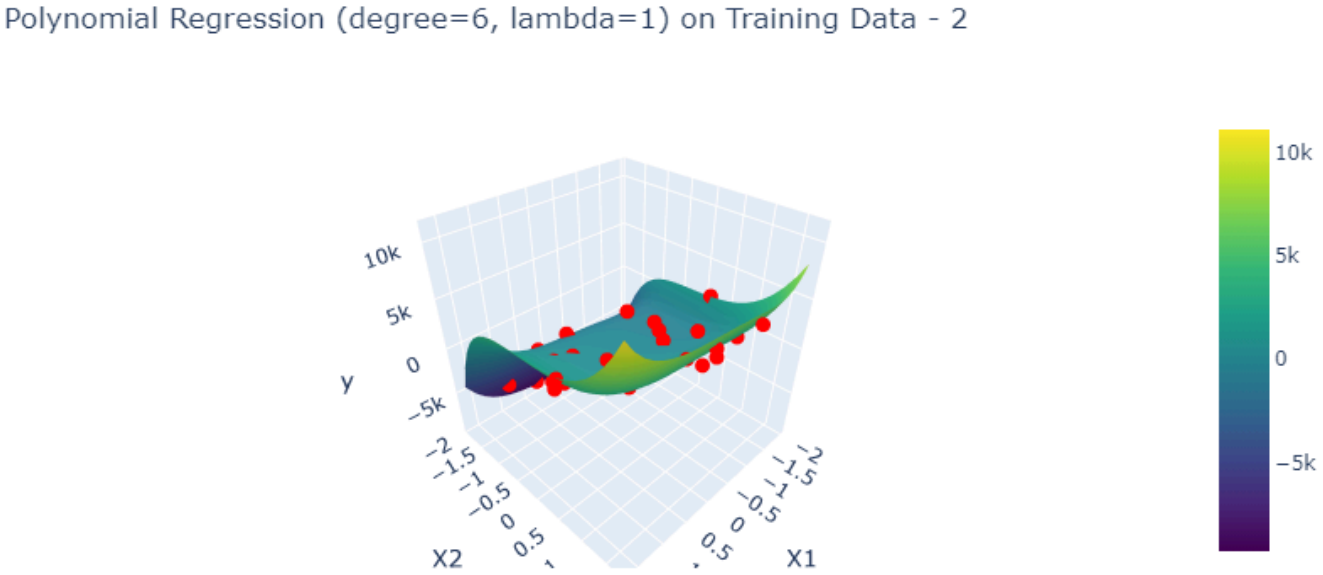
Observations

1. **Degree 2 and 4 without Regularization**: High MSE, showing inability to generalize.

With Regularization







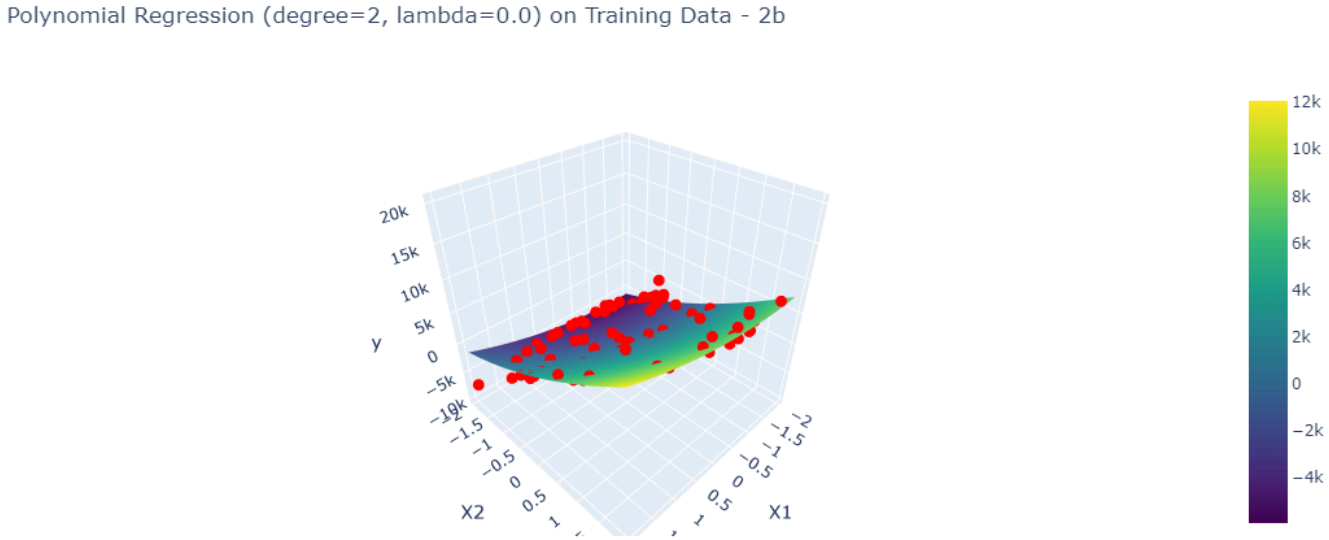
Observations

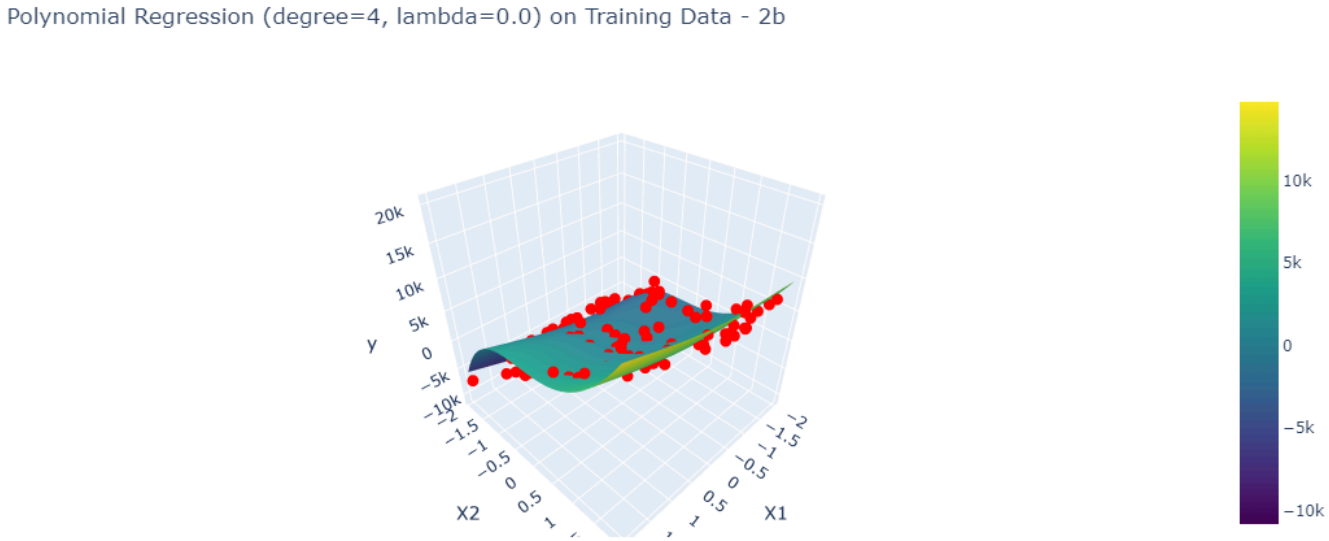
1. **Degree 6 with Regularization**: Extremely high MSE values across the board, suggesting poor fit.

Training Dataset 2(b): 100 examples

> 2-dimensional (Bivariate) input data

Without Regularization



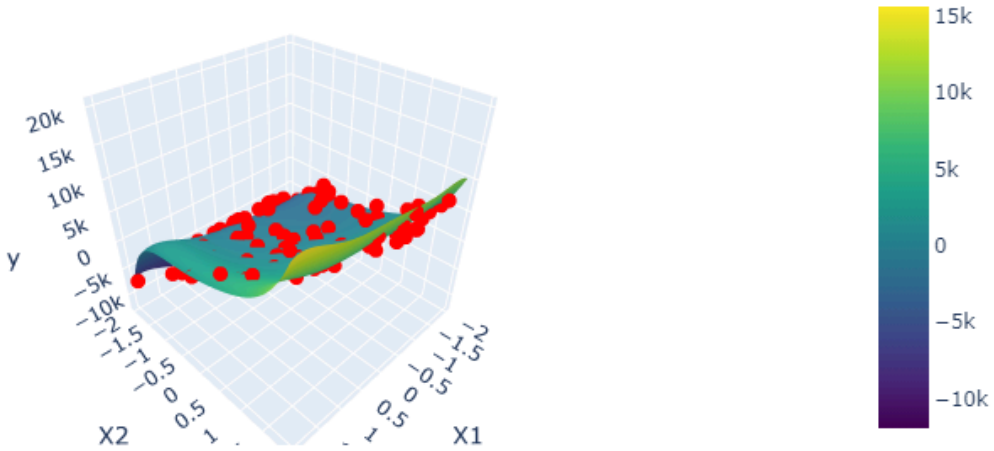


Observations

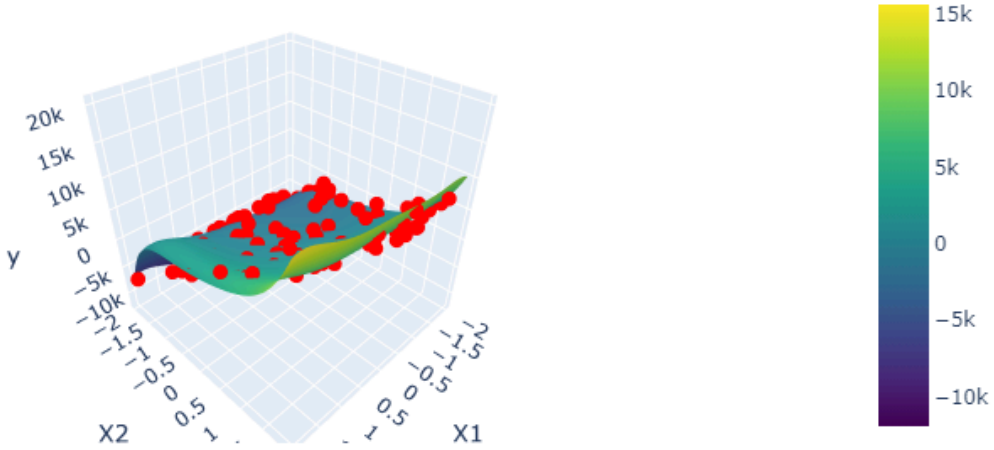
1. **Degree 2 and 4 without Regularization**: High MSE, similar poor generalization as in 2a.

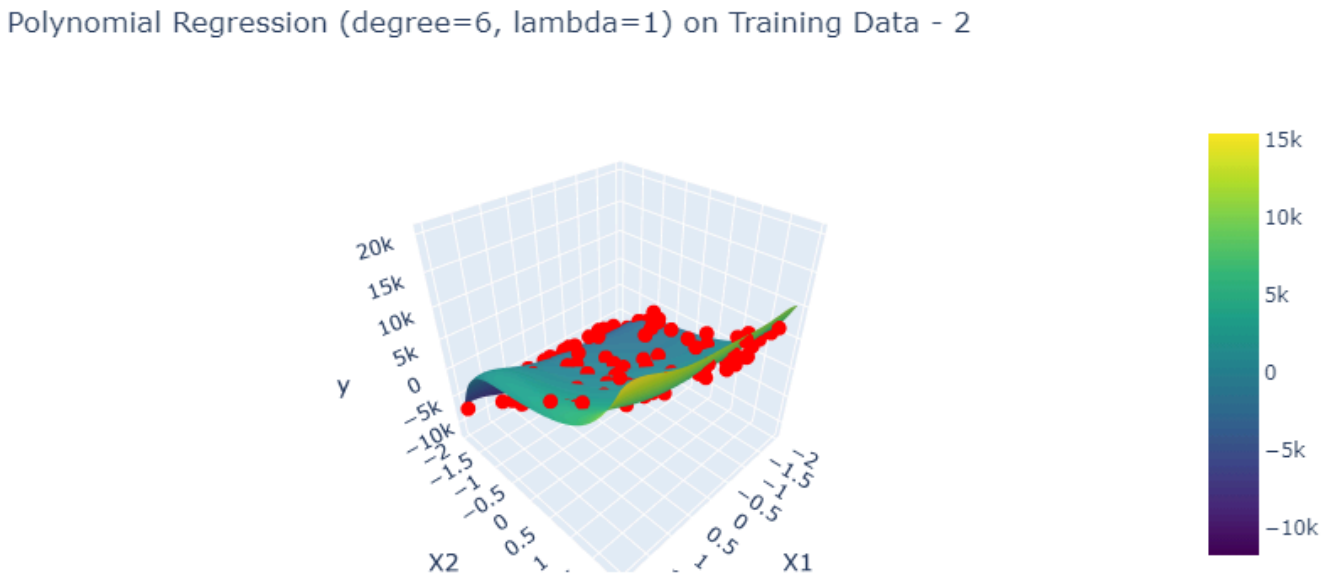
With Regularization

Polynomial Regression (degree=6, lambda=0.001) on Training Data - 2



Polynomial Regression (degree=6, lambda=0.1) on Training Data - 2





Observations

1. **Degree 6 with Regularization**: Similar to 2a, very high MSE values.

Dataset 3

> Multivariate data

Observations

1. **Degree 3 with Regularization**: Stable MSE values across all datasets, suggesting good generalization.
2. **Degree 2 without Regularization**: Significantly lower MSE, showing effective performance without regularization.

E_RMS Table

Dataset	Degree	λ	Regularization	Training E_{RMS}	Validation E_{RMS}	Test E_{RMS}
1a	9	0.001	Yes	0.280719	0.5396	0.90606
1a	9	0.1	Yes	0.306639	0.560843	0.662793
1a	9	1	Yes	0.350065	0.63514	0.567244
1a	3		No	0.283378	0.669069	0.578373
1a	6		No	0.25371	27.7856	42.2011
1b	9	0.001	Yes	0.338586	0.411546	0.511182
1b	9	0.1	Yes	0.344825	0.403183	0.513352
1b	9	1	Yes	0.352835	0.369344	0.527465
1b	3		No	0.355957	0.40061	0.508245
1b	6		No	0.343583	0.41459	0.51277
2a	6	0.001	Yes	843.572	4496.38	2427.39
2a	6	0.1	Yes	845.591	4640.6	2476.11
2a	6	1	Yes	857.566	4729.2	2508.36
2a	2		No	1593.56	6032.11	2520.07
2a	4		No	849.883	4849.5	2582.12
2b	6	0.001	Yes	1776.35	4310.96	1861.26
2b	6	0.1	Yes	1776.39	4316.3	1861.48
2b	6	1	Yes	1778.32	4348.22	1862.82
2b	2		No	2318.17	5335.96	2390.16
2b	4		No	1819.4	4597.95	1849.14
3	3	1e-06	Yes	5.21479	5.17761	5.20719
3	3	0.0001	Yes	5.21479	5.17761	5.20719
3	3	0.1	Yes	5.21479	5.17761	5.20719
3	2		No	2.24179	2.24129	2.30676

Best Combinations

Here are the best combinations for each dataset as observed in the report:

1. **Dataset 1(a) - 10 examples, Univariate Input Data:**

- **Best Combination:** Degree 9 with Regularization (λ = 1)
  - **Reason:** This combination provides the lowest test Mean Squared Error (MSE) while maintaining generalization, making it the best choice.

2. **Dataset 1(b) - 50 examples, Univariate Input Data:**

   - **Best Combination:** Degree 9 with Regularization (λ = 0.001)
     - **Reason:** This configuration shows good generalization and better MSE in validation and test compared to other regularized cases.

3. **Dataset 2(a) - 25 examples, Bivariate Input Data:**

   - **Best Combination:** None of the tested combinations are particularly good, but Degree 2 or 4 without regularization might be the least poor options.
     - **Reason:** High MSE values across the board indicate poor generalization ability, even with regularization.

4. **Dataset 2(b) - 100 examples, Bivariate Input Data:**

   - **Best Combination:** Similar to Dataset 2(a), none of the combinations performed exceptionally well, but lower-degree polynomials without regularization might be marginally better.

5. **Dataset 3 - Multivariate Data:**

   - **Best Combinations:**
     - **Degree 3 with Regularization:** Provides stable MSE values across all datasets, indicating good generalization.
     - **Degree 2 without Regularization:** Shows significantly lower MSE, which suggests effective performance without the need for regularization.