CHAPTER 8

NETWORK FORMATION GAMES

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8.1 Introduction

The organization of agents into networks plays an important role in the determination of the outcome of many social and economic interactions. Networks of relationships help determine the careers that people choose, the jobs they obtain, the products they buy, and how they vote. Such networks also matter for the trade of goods and services, the provision of insurance in developing countries, R&D collaborations among firms, and trade agreements between countries. In many economic networks, the network is not exogenous, but agents decide about the links they want to build. A central question is predicting the networks that agents will form.

Jackson and Wolinsky (1996) propose the notion of pairwise stability to predict the networks that one might expect to emerge in the long run. A network is pairwise stable if no agent benefits from deleting a link and no two agents benefit from adding a link between them. It suffices to check whether two agents have incentives or not to add a link between them, and whether a single agent has incentives or not to delete one of her links. Mutual consent is only required for adding a link. Pairwise stability is a very important tool in network analysis. It is simple and tractable. In some applications it is powerful enough for ruling out most network architectures. But, in other applications, pairwise stability does not yield precise predictions. There are situations where an agent has no incentive to delete one of her links but would benefit from deleting simultaneously more links. Myerson's (1991) linking game allows for such a possibility and models the network formation as a noncooperative game where agents simultaneously choose the links they want to form. A link between two agents is formed if and only if both agents wish to build the link. A network is pairwise Nash stable if it corresponds to a Nash equilibrium of Myerson's linking game and any pair of agents have no incentive to form a link that does not exist in the network. Pairwise Nash stability is a refinement of pairwise stability. However, one shortcoming of pairwise

(Nash) stability is the lack of farsightedness. Agents do not anticipate that other agents may react to their changes. For instance, farsighted agents might not add a link that appears valuable to them as this might induce the formation of other links, ultimately leading to lower payoffs for them.

Another central question about network formation is whether the networks formed by the agents are efficient from an overall societal perspective. Moreover, as the relevance of social and economic networks has been recognized, there are more and more policies that provide incentives to build links. The effectiveness of such policies is highly dependent on our understanding of how such networks form.

Some of the literature on network formation has been motivated by concrete empirical evidence on the properties that networks have, and has explored the circumstances under which networks will or will not exhibit those properties (see Goyal and Joshi 2006a, among others). In this chapter we rather focus on solution concepts (Section 8.2), and we illustrate the bites that these solution concepts have on economic applications (Section 8.3). So, we first propose some myopic and farsighted definitions for modeling network formation. We then investigate in three models of economic networks whether the networks formed by farsighted agents are efficient and different from those formed by myopic agents. There are alternative methodologies to analyze network formation, like stochastic network formation models (see Chapter 7 by Pin and Rogers in this handbook). More recently, the study of network formation has been combined with games on networks (see Chapter 9 by Vega-Redondo in this handbook).

We focus on situations where the formation of a link requires the consent of both agents. An alternative approach was developed by Bala and Goyal (2000), who propose a connections model of network formation where each agent unilaterally decides the links she wants to form and the costs of link formation are incurred only by the agent who initiates the link, while the benefits accrue to both agents linked. Bala and Goyal (2000) find that the only strict Nash networks are the center-sponsored star (where one agent forms all the links) and the empty network. Galeotti, Goyal, and Kamphorst (2006) introduce heterogeneous agents in the connections model, and they show that strict Nash equilibrium networks exhibit high centrality and short average distances between agents even in presence of considerable heterogeneity. Hojman and Szeidl (2008) assume that benefits from connections exhibit decreasing returns and decay with network distance. They find that the unique equilibrium network is a periphery-sponsored star (where a single agent—the center—maintains no links and all other agents maintain one link to the center). We refer to Goyal (2007) for an extensive analysis of one-sided link formation models where each agent can unilaterally form links with any subset of the other players.

8.2 Network Formation: Solution Concepts

Let $N = \{1, \ldots, n\}$ be the finite set of players who are connected in some network relationship. The network relationships are reciprocal and the network is thus modeled as a nondirected graph. A network g is a list of players who are linked to each other. We write $ij \in g$ to indicate that i and j are linked in the network g. Let g^N be the set of all subsets of N with cardinality 2, so g^N is the complete network. The set of all possible networks on N is denoted by $\mathcal{G}$ and consists of all subsets of g^N. The network obtained by adding link ij to an existing network g is denoted $g + ij$, and the network obtained by cutting link ij from an existing network g is denoted $g - ij$. For any network g, we denote by $N(g) = \{i \mid \exists\, j \text{ such that } ij \in g\}$ the set of players who have at least one link in the network g. A path in a network g between i and j is a sequence of players $i_1, \ldots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$ with $i_1 = 1$ and $i_K = j$. A non-empty network $h \subseteq g$ is a component of g, if for all $i \in N(h)$ and $j \in N(h) \setminus \{i\}$, there exists a path in h connecting i and j, and for any $i \in N(h)$ and $j \in N(g)$, $ij \in g$ implies $ij \in h$.[1] We denote by $C(g)$ the set of components of g. A component h of g is minimally connected if h has $\#N(h) - 1$ links (i.e., every pair of players in the component are connected by exactly one path). The partition of N induced by g is denoted by $\Pi(g)$, where $S \in \Pi(g)$ if and only if either there exists $h \in C(g)$ such that $S = N(h)$ or there exists $i \notin N(g)$ such that $S = \{i\}$.

A network utility function (or payoff function) is a mapping $u : \mathcal{G} \to \mathbb{R}^N$ that assigns to each network g a utility $u_i(g)$ for each player $i \in N$. A network $g \in \mathcal{G}$ is strongly efficient relative to u if it maximizes $\sum_{i \in N} u_i(g)$. A network $g \in \mathcal{G}$ Pareto dominates a network $g' \in \mathcal{G}$ relative to u if $u_i(g) \geq u_i(g')$ for all $i \in N$, with strict inequality for at least one $i \in N$. A network $g \in \mathcal{G}$ is Pareto efficient relative to u if it is not Pareto dominated, and a network $g \in \mathcal{G}$ is Pareto dominant if it Pareto dominates any other network. In Figure 8.1 we provide an example of the networks that could be formed by three players and their utilities. For instance, g_4 is a star network where player 1 gets 5 as utility and players 2 and 3 obtain 1 as utility. We will use this example to illustrate the solution concepts.

8.2.1 Pairwise Stability and Closed Cycles

A simple way to analyze the networks that one might expect to emerge in the long run is to examine a sort of equilibrium requirement that players not benefit from altering the structure of the network. A weak version of such condition is the pairwise stability notion defined by Jackson and Wolinsky (1996). A network is pairwise stable if no player benefits from severing one of their links and no other two players benefit from adding a link between them, with one benefiting strictly and the other at least weakly.

Definition 1 (Jackson and Wolinsky 1996). A network g is pairwise stable with respect to u if and only if (i) for all $ij \in g$, $u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$, and (ii) for all $ij \notin g$, if $u_i(g) < u_i(g + ij)$ then $u_j(g) > u_j(g + ij)$.

[1] We use the notation $\subseteq$ for weak inclusion and $\subsetneq$ for strict inclusion, and # refers to the notion of cardinality.

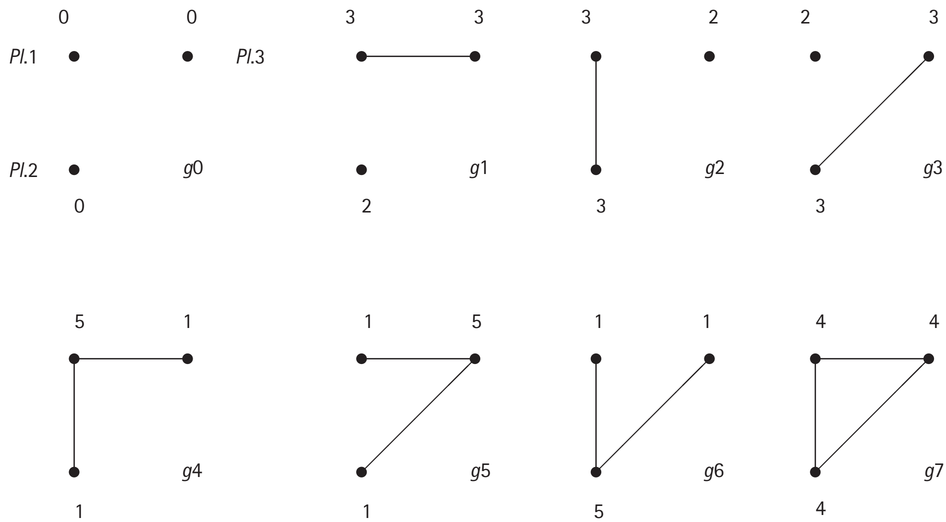

FIGURE 8.1 The networks that can be formed among three players with their utilities.

Two networks g and g' are adjacent if they differ by one link. That is, g' is adjacent to g if $g' = g + ij$ or $g' = g - ij$ for some ij. A network g' defeats g if either $g' = g - ij$ with $u_i(g') > u_i(g)$ or $u_j(g') > u_j(g)$, or if $g' = g + ij$ with $u_i(g') \geq u_i(g)$ and $u_j(g') \geq u_j(g)$ with at least one inequality holding strictly. Hence, a network is pairwise stable if and only if it is not defeated by another (necessarily adjacent) network. We say that the network utility function u exhibits no indifference if for any g and g' that are adjacent either g defeats g' or g' defeats g. In the 3-player example of Figure 8.1, both the partial networks g_1, g_2, and g_3 and the complete network g_7 are pairwise stable. Nobody has an incentive to delete one of her links in the complete network. She would end up worse off in the star network after having cut one of her two links. In any partial network, the player who has no link does not want to link with another player to form a star network. Moreover, both players who are linked have no incentives to cut this link to move to the empty network. The empty network g_0 is not pairwise stable because two players have incentives to link to each other and the star networks g_4, g_5, and g_6 are not pairwise stable since the peripheral players have incentives to add the missing link to form the complete network.

Pairwise stable networks do not always exist. Jackson and Watts (2002) introduce the notion of improving paths. An improving path is a sequence of networks that can emerge when players form or sever links based on the improvement the resulting network offers relative to the current network. If a link is added, then the two players involved must both prefer the resulting network to the current network, with at least one of the two strictly preferring the resulting network. If a link is deleted, then it must be that at least one of the two players involved in the link strictly prefers the resulting network. Formally, an improving path from a network g to a network $g' \neq g$

is a finite sequence of networks $g_1,\ldots,g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1,\ldots,K-1\}$ either (i) $g_{k+1} = g_k - ij$ for some ij such that $u_i(g_{k+1}) > u_i(g_k)$ or $u_j(g_{k+1}) > u_j(g_k)$, or (ii) $g_{k+1} = g_k + ij$ for some ij such that $u_i(g_{k+1}) > u_i(g_k)$ and $u_j(g_{k+1}) \geq u_j(g_k)$. If there exists an improving path from g to g', then we write $g \mapsto g'$. For a given network g, let $M(g) = \{g' \in \mathcal{G} \mid g \mapsto g'\}$ be the set of networks that can be reached by an improving path from g. Notice that g is pairwise stable if and only if $M(g) = \emptyset$.

Improving paths emanating from any network lead either to some pairwise stable network or to some closed cycle. Jackson and Watts (2002) define the notion of a closed cycle. A set of networks C is a cycle if for any $g \in C$ and $g' \in C$, we have $g \in M(g')$. A cycle C is a closed cycle if no network in C lies on an improving path leading to a network that is not in C.

Proposition 1 (Jackson and Watts 2002). *For every* g $\in \mathcal{G}$, *either* g *is pairwise stable or there is a closed cycle* C *such that* C $\subseteq$ M(g).

Jackson and Watts (2001) provide a condition on the network utility function that rules out cycles.[2] The network utility function u is exact pairwise monotonic if g' defeats g if and only if $\sum_{i\in N} u_i(g') > \sum_{i\in N} u_i(g)$ and g' is adjacent to g. Exact pairwise monotonicity implies that strongly efficient networks are pairwise stable. Jackson and Watts (2001) show that, if u is exactly pairwise monotonic, then there are no cycles.[3]

8.2.2 Pairwise Nash Stability

An alternative way to model network formation is Myerson's (1991) linking game where players choose simultaneously the links they wish to form and where the formation of a link requires the consent of both players. A strategy of player $i \in N$ is a vector $\sigma_i = (\sigma_{i1},\ldots,\sigma_{ii-1},\sigma_{ii+1},\ldots,\sigma_{in})$ where $\sigma_{ij} \in \{0,1\}$ for each $j \in N \setminus \{i\}$. If $\sigma_{ij} = 1$, player i wishes to form a link with player j. Given the strategy profile $\sigma = (\sigma_1,\ldots,\sigma_n)$, the network $g(\sigma)$ is formed where $ij \in g(\sigma)$ if and only if $\sigma_{ij} = 1$ and $\sigma_{ji} = 1$.

Definition 2 (Belleflamme and Bloch 2004; Goyal and Joshi 2006a). A strategy profile σ is a pairwise Nash equilibrium of Myerson's linking game if and only if, for each player i, each strategy $\sigma_i' \neq \sigma_i$, $u_i(g(\sigma)) \geq u_i(g(\sigma_i',\sigma_{-i}))$ and there does not exist a pair of players i and j such that $u_i(g(\sigma)+ij) \geq u_i(g(\sigma))$, $u_j(g(\sigma)+ij) \geq u_j(g(\sigma))$ with strict inequality for one of the two players.

[2] Jackson and Watts (2002) and Tercieux and Vannetelbosch (2006) propose random dynamic models of network formation to select from the set of pairwise stable networks.

[3] The connections model of Jackson and Wolinsky (1996) satisfies exact pairwise monotonicity for certain values of the parameters.

A network g is pairwise Nash stable with respect to a network utility function u if there exists a pairwise Nash equilibrium σ such that $g = g(\sigma)$.[4] Pairwise Nash stability is a refinement of pairwise stability. Pairwise Nash stability requires that a network is immune both to the formation of a new link by any two players and to the deletion of any number of links by any player.[5] In the three-player example of Figure 8.1, both the partial networks g_1, g_2, and g_3 and the complete network g_7 are pairwise Nash stable since these networks are pairwise stable and no player has incentives to cut more than one link at a time. If the players who have no link in the partial networks g_1, g_2, and g_3 would get a utility of 5 instead of 2, then the complete network g_7 would be pairwise stable but not pairwise Nash stable. However, g_1, g_2, and g_3 would remain pairwise stable and pairwise Nash stable. Calvo-Armengol and Ilkilic (2009) provide conditions on the network link marginal utilities such that the sets of pairwise and pairwise Nash stable networks coincide. Let $\alpha \geq 0$. The network utility function u is α-submodular in own current links on $G \subseteq \mathcal{G}$ if and only if $u_i(g) - u_i(g - l) \geq \alpha \sum_{ij \in l}(u_i(g) - u_i(g - ij))$ for all $g \in G$, $i \in N$ and $l \subseteq \{jk \in g \mid j = i \text{ or } k = i\}$. This condition applies only to marginal utilities from existing links, and it imposes that the marginal benefits from a group l of links already in the network are higher than the sum of marginal benefits of each single link in l scaled by α. Under the condition that utilities are α-submodular on the set of pairwise stable networks for some $\alpha \geq 0$, if a player does not gain from cutting any single link, then she does not gain from deleting more links simultaneously.

Proposition 2 (Calvo-Armengol and Ilkilic 2009). *The set of pairwise stable networks coincides with the set of pairwise Nash stable networks if and only if u is α-submodular on the set of pairwise stable networks, for some $\alpha \geq 0$.*

Calvo-Armengol and Ilkilic (2009) show that the condition of α-submodularity holds in the connections and co-author models of Jackson and Wolinsky (1996) and in the information transmission model of Calvo-Armengol (2004).[6]

Goyal and Vega-Redondo (2007) strengthen the concept of pairwise Nash stability by proposing the notion of bilateral equilibrium that allows for simultaneous deletion and addition of a link. A network can be supported in a bilateral equilibrium of Myerson's linking game if it is pairwise Nash stable and no pair of players benefit, at least one

[4] A strategy profile σ is a Nash equilibrium of Myerson's linking game if and only if, for each player i, each strategy $\sigma_i' \neq \sigma_i$, $u_i(g(\sigma)) \geq u_i(g(\sigma_i', \sigma_{-i}))$. A network g is Nash stable with respect to a network utility function u if there exists a Nash equilibrium σ such that $g = g(\sigma)$. But the concept of Nash stability is too weak for analyzing network formation when the formation of a link needs the approval of both players. For instance, the empty network is always Nash stable regardless u. De Sinopoli and Pimienta (2010) show that all Nash equilibria are regular when players incur a positive cost to propose links.

[5] Gilles and Sarangi (2010) extend Myerson's linking game to include additive link formation costs: if player i attempts to form a link with player j (i.e., $\sigma_{ij} = 1$), then player i incurs a cost $c_{ij} \geq 0$ regardless of σ_{ji}. See also Slikker and van den Nouweland (2001); Gilles, Chakrabarti, and Sarangi (2012).

[6] Goyal and Joshi (2006a) explore conditions on the network utility function under which pairwise Nash stable networks are or are not asymmetric. Hellman (2013) studies how externalities between links affect the existence and uniqueness of pairwise stable networks.

of them strictly, from simultaneously deleting some of their links and adding the link between them. Denote $\sigma_{-i,j} = (\sigma_1, ..., \sigma_{i-1}, \sigma_{i+1}, ..., \sigma_{j-1}, \sigma_{j+1}, ..., \sigma_n)$ the strategy profile σ less the strategies of players i and j.

Definition 3 (Goyal and Vega-Redondo 2007). A strategy profile σ is a bilateral equilibrium of Myerson's linking game if and only if, for each player i, each strategy $\sigma_i' \neq \sigma_i$, $u_i(g(\sigma)) \geq u_i(g(\sigma_i', \sigma_{-i}))$ and there does not exist a pair of players i and j and a pair (σ_i', σ_j') such that $u_i(g(\sigma_i', \sigma_j', \sigma_{-i,j})) \geq u_i(g(\sigma))$, $u_j(g(\sigma_i', \sigma_j', \sigma_{-i,j})) \geq u_j(g(\sigma))$ with strict inequality for one of the two players.

In the 3-player example of Figure 8.1, the partial networks g_1, g_2, and g_3 cannot be supported in a bilateral equilibrium of Myerson's linking game. For instance, g_1 is not a bilateral equilibrium outcome because from $\sigma_1 = (\sigma_{12}, \sigma_{13}) = (0,1)$, $\sigma_2 = (\sigma_{21}, \sigma_{23}) = (0,0)$, $\sigma_3 = (\sigma_{31}, \sigma_{32}) = (1,0)$, players 1 and 2 can now deviate to $\sigma_1' = (\sigma_{12}', \sigma_{13}') = (1,0)$, $\sigma_2' = (\sigma_{21}', \sigma_{23}') = (1,0)$, $\sigma_3 = (\sigma_{31}, \sigma_{32}) = (1,0)$, and player 2 is strictly better off at σ' while player 1 is indifferent between σ and σ'. However, the complete network g_7 is a bilateral equilibrium outcome since it is pairwise Nash stable and no pair of players have incentives to cut some of their links.

8.2.3 Farsighted Stability

Pairwise stability and pairwise Nash stability are myopic definitions. Players are not farsighted in the sense that they do not anticipate how others might react to their actions. For instance, adding or severing one link might lead to subsequent addition or deletion of another link. If players have very good information about how others might react to changes in the network, then these are things we want to allow for in the definition of the stability concept. For instance, a network could be stable because players might not add a link that appears valuable to them given the current network, as that might in turn lead to the formation of other links and ultimately lower the payoffs of the original players. The notion of farsighted improving paths captures the farsightedness of the players.

A farsighted improving path is a sequence of networks that can emerge when players add or delete links based on the improvement the end network offers relative to the current network. Each network in the sequence differs by one link from the previous one. If a link is added, then the two players involved must both prefer the end network to the current network, with at least one of the two strictly preferring the end network. If a link is deleted, then it must be that at least one of the two players involved in the link strictly prefers the end network.

Definition 4 (Jackson 2008; Herings, Mauleon, and Vannetelbosch 2009). A farsighted improving path from a network g to a network $g' \neq g$ is a finite sequence of networks $g_1, \ldots, g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, \ldots, K-1\}$ either: (i) $g_{k+1} =$

$g_k - ij$ for some ij such that $u_i(g_K) > u_i(g_k)$ or $u_j(g_K) > u_j(g_k)$, or (ii) $g_{k+1} = g_k + ij$ for some ij such that $u_i(g_K) > u_i(g_k)$ and $u_j(g_K) \geq u_j(g_k)$.

If there exists a farsighted improving path from g to g', then we write $g \to g'$. For a given network g, let $F(g) = \{g' \in \mathcal{G} \mid g \to g'\}$ be the set of networks that can be reached by a farsighted improving path from g. Notice that $g \to g'$ means that g' is the endpoint of at least one farsighted improving path from g. In the 3-player example of Figure 8.1, we have $F(g_0) = \{g_1, g_2, g_3, g_7\}$, $F(g_1) = \{g_2, g_3, g_7\}$, $F(g_2) = \{g_1, g_3, g_7\}$, $F(g_3) = \{g_1, g_2, g_7\}$, $F(g_4) = \{g_1, g_2, g_3, g_7\}$, $F(g_5) = \{g_1, g_2, g_3, g_7\}$, $F(g_6) = \{g_1, g_2, g_3, g_7\}$, and $F(g_7) = \emptyset$. The computations of farsighted improving path are not always obvious. For instance, the only way to go from g_1 to g_2 is via g_4 (and $g_2 \in F(g_1)$). Indeed, players 1 and 2 make a link to go from g_1 to the intermediate network g_4 in the anticipation that player 3 subsequently cuts her link with player 1. At the same time it holds that $g_4 \notin F(g_1)$. To go from g_1 to the terminal network g_4 is worse for player 2 and player 3. The only thing player 1 can do at g_1 is to delete her link with player 3, which leads to g_0. This is not helpful for player 1, since once at g_0 she can only form a link with player 2 (or 3) to go to g_2 (or g_1), but player 2 (or 3) will never form the missing link to go to g_4.

Jackson (2008) defines a network to be farsightedly pairwise stable if there is no farsighted improving path emanating from it. That is, g is farsightedly pairwise stable if $F(g) = \emptyset$. This concept refines the set of pairwise stable networks, and so may fail to exist in economic networks. Another drawback of the definition is that it does not require that a farsighted improving path ends at a network that is stable itself. Hence, Herings, Mauleon, and Vannetelbosch (2009) propose the concept of pairwise farsightedly stable set.[7]

A set of networks G is pairwise farsightedly stable if three conditions are satisfied. First, all possible pairwise deviations from any network $g \in G$ to a network outside G are deterred by a credible threat of ending worse off. Second, there exists a farsighted improving path from any network outside the set leading to some network in the set (external stability condition). Third, there is no proper subset of G satisfying the first two conditions.

Definition 5 (Herings, Mauleon, and Vannetelbosch 2009). A set of networks $G \subseteq \mathcal{G}$ is pairwise farsightedly stable if

(i) $\forall\, g \in G$,

 (ia) $\forall\, ij \notin g$ such that $g + ij \notin G$, $\exists g' \in F(g + ij) \cap G$ such that $(u_i(g'), u_j(g')) = (u_i(g), u_j(g))$ or $u_i(g') < u_i(g)$ or $u_j(g') < u_j(g)$,

 (ib) $\forall\, ij \in g$ such that $g - ij \notin G$, $\exists g', g'' \in F(g - ij) \cap G$ such that $u_i(g') \leq u_i(g)$ and $u_j(g'') \leq u_j(g)$.

[7] An alternative approach to model network formation is simply to model it explicitly as a non-cooperative extensive form game. But, the equilibrium outcome is usually quite sensitive to the exact network formation process. See Aumann and Myerson (1988), among others.

(ii) $\forall g' \in \mathcal{G} \setminus G, F(g') \cap G \neq \emptyset$.
(iii) $\nexists\, G' \subsetneq G$ such that G' satisfies conditions (ia), (ib), and (ii).

Condition (ia) captures that adding a link ij to a network $g \in G$ that leads to a network outside of G, is deterred by the threat of ending in g'. Here g' is such that there is a farsighted improving path from $g + ij$ to g'. Moreover, g' belongs to G, which makes g' a credible threat. Condition (ib) is a similar requirement, but then for the case where a link is deleted. Condition (ii) requires that from any network outside G there is a farsighted improving path leading to some network in G. Condition (iii) is a minimality condition motivated by the fact that the set $\mathcal{G}$ (trivially) satisfies the first two conditions. In the 3-player example, $\{g_7\}$ is pairwise farsightedly stable. Since $g_7 \in \bigcap_{g \in \mathcal{G} \setminus \{g_7\}} F(g)$, condition (ii) of the definition is satisfied. In addition, condition (i) is also satisfied, since any deviation from g_7 may lead back to g_7. The set $\{g_7\}$ is clearly minimal, so condition (iii) is satisfied too. Since $F(g_7) = \emptyset$, condition (ii) implies that g_7 belongs to any pairwise farsightedly stable set. Using condition (iii) it follows that $\{g_7\}$ is the only pairwise farsightedly stable set.

Proposition 3 (Herings, Mauleon, and Vannetelbosch 2009)**.** *A pairwise farsightedly stable set of networks exists.*

Herings, Mauleon, and Vannetelbosch (2009) provide easy to verify conditions for a set G to be pairwise farsightedly stable.

Proposition 4 (Herings, Mauleon, and Vannetelbosch 2009)**.**

(i) *If for every $g' \in \mathcal{G} \setminus G$ we have $F(g') \cap G \neq \emptyset$ and for every $g \in G$, $F(g) \cap G = \emptyset$, then G is a pairwise farsightedly stable set.*
(ii) *The set $\{g\}$ is a pairwise farsightedly stable set if and only if for every $g' \in \mathcal{G} \setminus \{g\}$ we have $g \in F(g')$.*

Notice that the minimality condition implies that if $\{g\}$ is a pairwise farsightedly stable set, then g does not belong to any other pairwise farsightedly stable set. But there may be pairwise farsightedly stable sets not containing g.

The next proposition provides a full characterization for unique pairwise farsightedly stable sets.

Proposition 5 (Herings, Mauleon, and Vannetelbosch 2009)**.**

(i) *The set G is the unique pairwise farsightedly stable set if and only if $G = \{g \in \mathcal{G} \mid F(g) = \emptyset\}$ and for every $g' \in \mathcal{G} \setminus G$, $F(g') \cap G \neq \emptyset$.*
(ii) *The set $\{g\}$ is the unique pairwise farsightedly stable set if and only if for every $g' \in \mathcal{G} \setminus \{g\}$ we have $g \in F(g')$ and $F(g) = \emptyset$.*

Thus, if G is the unique pairwise farsightedly stable set and the network g belongs to G, then $F(g) = \emptyset$, which implies that g is pairwise stable. So, pairwise farsighted stability is a refinement of pairwise stability when there is a unique pairwise farsightedly stable set.[8]

In Section 8.3 we show that propositions 4 and 5 turn out to be helpful and powerful for characterizing pairwise farsightedly stable sets in economic networks. If for every $g' \in \mathcal{G} \setminus \{g\}$ we have $g \in F(g')$, then $\{g\}$ is a pairwise farsightedly stable set. If, moreover, $F(g) = \emptyset$, then $\{g\}$ is the unique pairwise farsightedly stable set. If, on the other hand, $F(g) \neq \emptyset$, then there exists another pairwise farsightedly stable set.[9]

Dutta, Ghosal, and Ray (2005) propose an alternative approach to model network formation among farsighted players. They develop a model of dynamic network formation where players are farsighted and evaluate the formation of links in terms of its consequences on the entire discounted stream of payoffs. Dutta, Ghosal, and Ray (2005) provide an example where there is a network g that strictly Pareto dominates all other networks, but which is not reached in equilibrium.[10] However, $\{g\}$ is the unique pairwise farsightedly stable set.

8.2.4 Coalitions, Side Payments, and Bargaining

Pairwise (farsighted) stability only considers deviations by at most a pair of players at a time. It might be that some coalition of players could all be made better off by some complicated reorganization of their links, which is not accounted for under pairwise stability. Dutta and Mutuswami (1997) and Jackson and van den Nouweland (2005) define the notion of strong stability where a strongly stable network is a network which is stable against changes in links by any coalition of players. A network g' is said to be obtainable from g via deviations by coalition $S \subseteq N$ if (i) any new links that are added can only be between players belonging to S and (ii) at least one player of any deleted link is member of S.

Definition 6 (Jackson and van den Nouweland 2005). A network g is strongly stable if for any $S \subseteq N$, g' that is obtainable from g via deviations by S, and $i \in S$ such that $u_i(g') > u_i(g)$, there exists $j \in S$ such that $u_j(g') < u_j(g)$.

[8] If G is the unique pairwise farsightedly stable set, then G is the set of farsightedly pairwise stable networks. In addition, if g is a farsightedly pairwise stable network then it belongs to all pairwise farsightedly stable set of networks.

[9] Incorporating the notion of farsighted improving paths into the original definition of the vNM stable set (von Neumann and Morgenstern 1944) leads to the vNM pairwise farsightedly stable set. From Proposition 4, it follows that a vNM pairwise farsightedly stable set is a pairwise farsightedly stable set. However, vNM pairwise farsightedly stable sets do not always exist. Another farsighted concept is the largest pairwise consistent set (Chwe 1994; Page, Wooders, and Kamat 2005). But, it often fails to eliminate implausible pairwise stable networks.

[10] Page and Wooders (2009) propose a model of network formation whose primitives consist of a feasible set of networks, player preferences, rules of network formation, and a dominance relation on feasible networks. Rules may range from noncooperative, where players may only act unilaterally, to cooperative, where coalitions of players may act in concert.

Strong stability is a refinement of pairwise stability. Jackson and van den Nouweland (2005) provide some conditions on the network utility function so that the set of strongly efficient networks coincides with the set of strongly stable networks. The network utility function u is top convex if some strongly efficient network maximizes the per-capita sum of utilities among players. Let $\rho(u,S) = \max_{g \subseteq g^S} \sum_{i \in S} u_i(g)/\#S$. The network utility function u is top convex if $\rho(u,N) \geq \rho(u,S)$ for all $S \subseteq N$. Suppose that u is such that (i) players belonging to the same component get the same utility, and (ii) there are no externalities across components (i.e., payoffs of players belonging to a component in a given network do not depend on the structure of other components). It turns out that, under the conditions they impose, the notion of strong stability eliminates the inefficient pairwise stable networks. Moreover, Grandjean, Mauleon, and Vannetelbosch (2011) show that the set of strongly efficient networks is the unique pairwise farsightedly stable set if and only if u is top convex. So, pairwise farsighted stability selects the pairwise stable networks that are immune to deviations by coalitions if and only if u is top convex.

Proposition 6 (Jackson and van den Nouweland 2005; Grandjean, Mauleon, and Vannetelbosch 2011)**.** *Take any u such that* (**i**) $u_i(g) = u_j(g)$ *for all* $i, j \in S \in \Pi(g)$ *and* (**ii**) $u_i(g) = u_i(h)$ *with* $h \in C(g)$ *and* $i \in N(h)$.

(**a**) *The set of strongly efficient networks is the set of strongly stable networks if and only if u is top convex.*
(**b**) *The set of strongly efficient networks is the unique pairwise farsightedly stable set if and only if u is top convex.*

Strong stability makes sense in situations where players have substantial information about the overall structure and potential payoffs and can coordinate their actions.[11]

There are a number of papers that look at the endogenous determination of payoffs together with network formation. Currarini and Morelli (2000) develop a sequential network formation game, where players propose links and demand payoffs. Following an exogenously given order, each player proposes in turn the links she wants to form and she demands some payoff. Once all proposals are made, links are formed if both players involved in the link proposed it and the demands of the players are compatible. The payoffs in Currarini and Morelli (2000) are endogenously generated but are highly asymmetric and sensitive to the order in which players make proposals.[12] Bloch and Jackson (2006, 2007) investigate the role played by transfers payments in

[11] The definition of strong stability of Dutta and Mutuswami (1997) considers a deviation to be valid only if all members of a deviating coalition are strictly better off, while the definition of Jackson and van den Nouweland (2005) is slightly stronger by allowing for a deviation to be valid if some members are strictly better off and others are weakly better off. Under the weaker definition of Dutta and Mutuswami, a network is strongly stable if it corresponds to a strong Nash equilibrium of Myerson's linking game.

[12] Goyal and Vega-Redondo (2007) develop a model where players form links with others to create surplus and where rents are split among buyers, sellers and intermediaries.

the formation of networks. They study whether different forms of transfers (direct transfers, indirect transfers, or contingent transfers) can solve the conflict between stability and strong efficiency when there are network externalities that usually lead to the emergence of inefficient networks when transfers are not feasible. They find that indirect transfers together with contingent transfers are needed to guarantee that strongly efficient networks form. Indirect transfers enable players to take care of positive externalities by subsidizing the formation of links by other players; while contingent transfers enable players to overcome negative externalities by preventing the formation of links.[13]

8.3 Some Models of Economic Networks

We now investigate in some models of economic networks whether the pairwise farsightedly stable sets of networks coincide or not with the set of pairwise (Nash) stable networks and the set of strongly efficient networks. We mostly focus on three models of network formation: (1) networks of R&D collaborations, (2) networks of free trade agreements, and (3) criminal networks. In (1), myopia predicts that firms partition themselves into two nearly symmetric coalitions while farsightedness leads to two asymmetric coalitions, with the largest one comprising roughly three-quarters of the total number of firms. Both myopia and farsightedness do sustain networks that do not maximize the social welfare nor the sum of the profits. In (2), the global free trade network is the unique strongly efficient network, and both myopia and farsightedness can sustain it. However, myopia and farsightedness do not impede the emergence of some inefficient networks. In (3), farsightedness reduces the conflict between stability and efficiency by destabilizing some strongly inefficient networks and sustaining one of the strongly efficient networks, namely the complete network.

Thus, depending on the application, myopia and farsightedness may lead to divergent predictions, and farsightedness can help to support the emergence of efficient networks. An interesting question is to find properties on the network utility function so that either myopia and farsightedness lead to the same conclusion, or farsightedness solves the conflict with efficiency while myopia does not.[14] Whenever some of the properties are not satisfied, it is important to understand whether the agents are myopic or farsighted in order to use the appropriate stability concept. Knowing which

[13] Navarro (2014) studies a model of dynamic network formation with farsighted players, similar to the Dutta, Ghosal, and Ray (2005) model, but when side payments can be made between connected players.

[14] In many-to-one matching problems with substitutable preferences, Mauleon, Vannetelbosch, and Vergote (2011) show that, contrary to the vNM (myopically) stable sets (Ehlers 2007), vNM farsightedly stable sets cannot include matchings that are not in the core. For one-to-one matching problems, Mauleon, Molis, Vannetelbosch, and Vergote (2014) provide conditions on preference profiles such that farsightedness coincides with myopia.

networks are likely to be formed and to evolve can help to shape more effective policy recommendations.

8.3.1 R&D Networks

We consider Goyal and Joshi's (2003) two-stage game in a setting with n competing firms that produce some homogenous good. In the first stage, firms decide the bilateral R&D collaborations they are going to establish in order to maximize their respective profits. R&D collaborations reduce marginal costs of production. Given a network g, the marginal cost for firm i is given by $c_i(g) = c_0 - (1 + \sum_{j \neq i} \delta^{t(ij)-1})$ where c_0 is the initial marginal cost, $\delta \in (0, 1]$ and $t(ij)$ is the number of links in the shortest path between i and j (setting $t(ij) = \infty$ if there is no path between i and j). Each firm benefits both from her own R&D (reducing her marginal cost by 1) and from the R&D done by the firms she is connected to (reducing her marginal cost by $\sum_{j \neq i} \delta^{t(ij)-1}$). Let $N_i^k(g) = \{j \mid t(ij) = k\}$ be the set of firms that are connected to firm i by a path of at least k links. Then, $c_i(g) = c_0 - 1 - \sum_{k=1}^{n-1} \# N_i^k(g) \delta^{k-1}$. We focus as in Mauleon, Sempere-Monerris, and Vannetelbosch (2014) on the case $\delta = 1$ where each firm fully benefits from the research done by the firms she is connected to.[15] There are small but positive costs to forming links, $\gamma > 0$.

In the second stage, firms compete in quantities in the oligopolistic market, taking as given the costs of production. Let $p = a - \sum_{i \in N} q_i$ with $a > 0$ be the linear inverse demand function. Thus, firm i's profits in a R&D network g is given by

$$u_i(g) = (q_i(g))^2 - d_i(g)\gamma$$

where the equilibrium output is $q_i(g) = (a - c_0 + (n+1)\#S(i) - \sum_{S \in \Pi(g)} (\#S)^2)/(n+1)$, $d_i(g)$ is the number of links of firm i and $S(i)$ is the group of firms to which firm i is connected.

Lemma 1 (Mauleon, Sempere-Monerris, and Vannetelbosch 2014). *For R&D networks,*

- **(i)** *any network g where all components are not minimally connected is not pairwise stable;*
- **(ii)** *any minimally connected network g is not pairwise stable;*
- **(iii)** *any network g where $\#\Pi(g) > 2$ and all components are minimally connected is not pairwise stable.*

Since each firm benefits perfectly from the research done by the firms she is connected to ($\delta = 1$) and there is an infinitesimal small cost for forming links ($\gamma > 0$), any R&D network where a component is not minimally connected cannot be pairwise

[15] In Goyal and Joshi's (2003) original model, each firm benefits only from her own R&D and from the R&D done by the firms she is directly linked to. In Goyal and Moraga-Gonzalez (2001), firms even benefit, although imperfectly, from the R&D done by firms to whom they are not connected, while in Mauleon, Sempere-Monerris, and Vannetelbosch (2008), benefits decrease with the distance.

stable. Mauleon, Sempere-Monerris, and Vannetelbosch (2014) also show that any minimally connected network is not pairwise stable because any firm who has more than one link in the R&D network increases her profits from cutting a link with a firm who has only one link. In addition, any R&D network consisting of at least three components and where all components are minimally connected is not pairwise stable because two minimally connected components of cardinality smaller than $(n+1)/2$ have incentives to add a link between them to form one component.

Thus, the only candidates for being pairwise stable are R&D networks consisting of two minimally connected components. In fact, Mauleon, Sempere-Monerris, and Vannetelbosch (2014) find that a R&D network g is pairwise stable if and only if g consists of two minimally connected components with the cardinality of the largest component equal to $\mathrm{int}((n+3)/2)$ for n even and to $(n+1)/2$ for n odd (where $\mathrm{int}(\cdot)$ denotes the integer part).[16]

Proposition 7 (Mauleon, Sempere-Monerris, and Vannetelbosch 2014)**.** *For R&D networks, a network g is pairwise stable if and only if $C(g) = (h_1, h_2)$, h_1 and h_2 are minimally connected, $N(h_1) \cup N(h_2) = N$, and $\#N(h_1) = \mathrm{int}((n+3)/2)$ if n even and $\#N(h_1) = (n+1)/2$ if n odd.*

Pairwise Nash stability coincides with pairwise stability since no firm has an incentive to delete more than one link at the pairwise stable R&D networks.[17] The R&D networks that maximize social welfare include any minimally connected network, and the strongly efficient R&D networks consist of one minimally connected component of cardinality greater than $3n/4$ with the remaining firms having no links.[18] Hence, we observe a conflict between pairwise stability and efficiency since pairwise stability leads to the emergence of R&D networks that split the firms into two coalitions of nearly equal size.[19]

[16] Firms in the largest component have incentives to delete one link isolating one firm until the cardinality of the component is equal to $\mathrm{int}((n+3)/2)+1$ if n even or to $(n+3)/2$ if n odd. So, only any network g such that $C(g) = (h_1, h_2)$, h_1 and h_2 are minimally connected, $N(h_1) \cup N(h_2) = N$, and $\#N(h_1) = \mathrm{int}((n+3)/2)$ if n even and $\#N(h_1) = (n+1)/2$ if n odd, is a candidate for being pairwise stable. Since firms in the smallest component h_2 have no incentives to delete links, firms in the largest component h_1 have no incentives to delete one link isolating more than one firm, and two firms $i \in N(h_1)$ and $j \in N(h_2)$ do not have incentives to form the link ij, this R&D network is pairwise stable.

[17] The pairwise (Nash) stable R&D networks cannot be supported in a bilateral equilibrium of Myerson's linking game because there is a pair of firms who benefit from simultaneously deleting some of their links and adding the link between them. For instance, one firm belonging to the smallest component cuts all her links and adds a new link to some firm belonging to the largest component.

[18] Yi (1998) shows that a network consisting of one minimally connected component h^* with the remaining firms organized as singletons maximizes industry profits, where $\#N(h^*)$ ($\leq n$) is the solution to $4(a-c_0)+3(n+1)^2(\#N(h^*)+1)-4(n+2)(\#N(h^*)^2+(n-\#N(h^*)))=0$.

[19] Goyal and Joshi (2003) find that, in case each firm only benefits from R&D done by firms she is directly linked to, the complete network maximizes the social welfare and is the unique pairwise stable network when linking costs are small.

Once firms are farsighted, Mauleon, Sempere-Monerris, and Vannetelbosch (2014) show that the set of all networks g consisting of two minimally connected components h_1 and h_2 such that $\#N(h_1) = \text{int}((3n+1)/4)$ and $N(h_1) \cup N(h_2) = N$ is a pairwise farsightedly stable set of networks.[20]

Proposition 8 (Mauleon, Sempere-Monerris, and Vannetelbosch 2014). *The set* $G^* = \{g \mid C(g) = (h_1, h_2)$, h_1 *and* h_2 *are minimally connected,* $\#N(h_1) = \text{int}((3n+1)/4)$ *and* $N(h_1) \cup N(h_2) = N\}$ *is a pairwise farsightedly stable set.*

First, profitable deviations from any network $g^* \in G^*$ are deterred. For instance, for $n > 5$, the only profitable deviations from g^* to $g^* - ij$ are such that j is isolated in $g^* - ij$. But such deviations are deterred because there is a farsighted improving path from $g^* - ij$ to some $g' \in G^*$ and the initial deviator, firm i, is worse off at g'. At each step of a farsighted improving path, one firm belonging to the largest component in g^* becomes isolated, and this firm then links to the smallest component in g^* looking forward to the end network g' where the smallest component in g^* becomes now in g' the largest component with size $\text{int}((3n+1)/4)$. Firm i, who initially deleted the link ij, ends in the smallest component and so is worse off in g'. Second, there is a farsighted improving path from any network g consisting of minimally connected components to some $g^* \in G^*$.[21] For instance, in case g has only components of cardinality smaller than $\text{int}((3n+1)/4)$, first at each step of a farsighted improving path from g, two firms belonging to the two largest components form a link until we reach a network g' with one component of cardinality greater or equal than $\text{int}((3n+1)/4)$ and some smaller components. From g', at each step, two firms belonging to the two smallest components form a link until we reach a network g'' consisting of only two components (with one of them having cardinality larger than $\text{int}((3n+1)/4)$). Next, at each step, a firm of the largest component is isolated, and then this firm is linked to one firm of the smaller component, and so forth until we reach g^*.[22] Finally, since $g \notin F(g')$ for all $g, g' \in G^*$, any proper subset violates condition **(ii)** in Definition 5. Hence, this set G^* is also a vNM farsightedly stable set.

However, the set of all pairwise stable networks is neither a pairwise farsightedly stable set of networks nor a vNM farsightedly stable set. Indeed, the set of all pairwise stable networks violates the external stability condition (i.e., condition **(ii)** in Definition 5).

[20] Roketskiy (2012) studies collaboration between farsighted firms competing in a tournament and finds that von Neumann-Morgenstern farsightedly stable sets of networks consist of two asymmetric mutually disconnected complete components. See also Grandjean and Vergote (2014).

[21] If $g^* \in F(g)$ then $g^* \in F(g')$ for any $g' \supset g$ with $\Pi(g') = \Pi(g)$ because profits only depend on the cardinality of the components and forming links is costly.

[22] Firms who have linked the largest components along the sequence as well as firms who have isolated some other firms from the largest component once there were only two components prefer the end network g^* to the network of the sequence from which they moved since they belong to the component of cardinality $\text{int}((3n+1)/4)$ in g^*.

Remember that strongly efficient R&D networks consist of one minimally connected component of cardinality greater than $3n/4$ with the remaining firms having no links. Hence, we obtain that efficient R&D networks may not emerge in the long run even if firms are farsighted. Thus, in case of quantity competition and homogenous goods, farsightedness does not solve the conflict between stability and efficiency. In fact, farsightedly stable R&D networks lead to a collaboration architecture close to the equilibrium structure of Bloch's (1995) sequential game for forming research associations where firms partition themselves into two asymmetric associations, with the largest one comprising roughly three-quarters of industry members.[23] But, the network approach differs from the group formation approach in the decision making for establishing R&D collaborations. Mutual consent is needed for forming a new link between two firms, whereas the consent of all members of the association is required when a firm joins the association.

Finally, in case of price competition and homogenous goods, all networks would give zero profits for all firms. Since forming links is costly, farsighted stability would then only support the empty network, which is also the unique pairwise stable network.

8.3.2 Free Trade Networks

We consider the Goyal and Joshi (2006b) and Zhang, Xue, and Zu (2013) three-stage game in a setting with n countries. Each country has one firm producing some homogeneous good that can be sold in the domestic market and in each foreign market. A firm's ability to sell in foreign markets, however, depends on the level of import tariffs set by the foreign countries. In the first stage, countries decide the free-trade agreements (or links) they are going to establish in order to maximize their respective social welfare. The collection of bilateral links between the countries defines a network of free trade agreements. In the second stage, if two countries have negotiated a free trade agreement, then each country offers the other a tariff-free access to her domestic market. Otherwise, each country imposes an optimal non-zero tariff on the imports from the other country in order to maximize her social welfare. In the third stage, each firm chooses how much to produce for her domestic market and how much to export to each foreign country, taking as given the output decisions of the other firms, the settled tariffs, and the network of free trade agreements. Firms compete in quantities in each country's market and markets are of equal size. In country i's market, firms face an inverse linear demand, $p_i = a - q_i$, where $a > 0$, q_i^j is the export level of firm j to country i, and $q_i = \sum_j q_i^j$ is the aggregate quantity in country i. Firms have a constant and identical marginal cost $c > 0$, where $a > c$.

The social welfare of country i is defined as the sum of consumer surplus, firm's profits and tariff revenues. Given a free trade network g, Zhang, Xue, and Zu (2013) shows that

[23] Bloch (2005) provide a survey on group and network formation in industrial organization.

the social welfare of country i at equilibrium is given by

$$u_i(g) = \frac{\left[d_i(g)(2n+1)-(n-4)\right]}{2\left[d_i(g)(2n+5)-(n-2)\right]}(a-c)^2 + \sum_{j\in N_i(g)} \left[\frac{2(d_j(g)+1)}{d_j(g)(2n+5)-(n-2)}\right]^2 (a-c)^2 + \sum_{k\in N\setminus N_i(g),\, k\neq i} \left[\frac{\left(2d_k(g)-1\right)}{d_k(g)(2n+5)-(n-2)}\right]^2 (a-c)^2,$$

where $N_i(g)$ is the set of countries with whom country i has negotiated a free trade agreement and $d_i(g)$ is the number of links or free trade agreements of country i. Thus, country i's welfare is a function of the number of links of country i, $d_i(g)$, the number of links of countries linked to country i, $d_j(g)$ with $j \in N_i(g)$, and the number of links of countries without link to country i, $d_k(g)$ with $k \in N \setminus N_i(g)$. Given g, global welfare is defined as $\sum_{i\in N} u_i(g)$. A free trade network g is strongly efficient if $\sum_{i\in N} u_i(g) \geq \sum_{i\in N} u_i(g')$ for all $g' \neq g$. Goyal and Joshi (2006b) find that the complete network g^N is strongly efficient if tariffs are exogenously given. In case tariffs are endogenously determined, Zhang, Xue, and Zu (2013) show that the complete global free trade network is again the unique strongly efficient network.[24]

Proposition 9 (Goyal and Joshi 2006b). *The complete global free trade network g^N is pairwise stable.*

Goyal and Joshi (2006b) show that the complete global free trade network g^N is pairwise stable, implying that global free trade, if reached, will prevail. However, the global free trade network is not the unique pairwise stable network. Moreover, the global free trade network may not be pairwise Nash stable. For instance, Zhang, Xue, and Zu (2013) show that, in the case of 10 countries, the global free trade network is not pairwise Nash stable because a single country has incentives to simultaneously delete all her links. Furusawa and Konishi (2007) study the trading network generated by countries that trade a numeraire good and a continuum of differentiated goods. They find that, when all countries are symmetric, the global free trade network in which every pair of countries sign a free trade agreement is pairwise stable, and it is the unique pairwise stable network if goods are not highly substitutable. However, if countries are asymmetric in the market size, the global free trade network may not be attained.[25]

[24] Given g, let $\widehat{u}_i(g)$ be the welfare generated by country i. It consists of three parts: consumer surplus of country i, producer surplus provided by firm i to her country and her linked countries, and tariff revenues provided by firm i to her unlinked countries. Zhang, Xue, and Zu (2013) show that, in any g, $\sum_{i\in N} u_i(g) = \sum_{i\in N} \widehat{u}_i(g)$ and $\widehat{u}_i(g)$ is an increasing function of $d_i(g)$. Hence, $\widehat{u}_i(g)$ is maximized when g is the complete network g^N, and so it is the unique strongly efficient network.

[25] Mauleon, Song, and Vannetelbosch (2010) find that the asymmetry consisting of having unionized and non-unionized countries, could also impede the formation of the global free trade network.

Goyal and Joshi (2006b) leave open the question whether a sequence of bilateral free trade negotiations can lead to global free trade from the empty network or any preexisting network.[26] Zhang, Xue, and Zu (2013) answer this question in case of farsighted countries.

Proposition 10 (Zhang, Xue, and Zu 2013). *The complete global free trade network $\{g^N\}$ is a pairwise farsightedly stable set.*

Thus, the complete network constitutes a pairwise farsightedly stable set and, therefore, starting from any other free trade network, there is a farsightedly improving path leading to the complete network. In particular, there is a farsighted improving path from the empty network to the complete network involving only link addition. One could interpret this result in the sense that there is a suitable sequence of bilateral free trade agreements that constitutes building blocks towards global free trade. However, starting from an arbitrary free trade network, a farsighted improving path to the complete network may involve link deletion. When this is the case, some bilateral free trade agreements are stumbling blocs that should be eliminated to facilitate the convergence to global free trade. If, for some reasons, such bilateral free trade agreements cannot be eliminated, countries can get trapped in a partially connected free trade network.

Proposition 11 (Zhang, Xue, and Zu 2013). *The complete global free trade network $\{g^N\}$ may not be the unique farsightedly stable set.*

Since $F(g^N) \neq \emptyset$, from Proposition 5, there can be other pairwise farsightedly stable sets of networks that do not contain the complete network g^N. Therefore, common expectation of the eventual emergence of the complete network and suitable sequence of bilateral free trade agreements are essential in achieving global free trade.

8.3.3 Criminal Networks

We consider a simplified version of Calvo-Armengol and Zenou's (2004) model where criminals compete with each other in criminal activities but benefit from being friends with other criminals by learning and acquiring know-how on the crime business.[27] Players are referred to as criminals. If two players are connected, then they are part of

[26] Mauleon, Song, and Vannetelbosch (2010) show that, in presence of unionized and non-unionized countries, starting from the network in which no country has signed a free trade agreement, all sequences of networks due to continuously profitable deviations do not lead (in most cases) to the global free trade network, even when global free trade is pairwise stable.

[27] Calvo-Armengol and Zenou (2004) mostly focus on the case where the criminal network is exogenously given. In their original model, players choose first either to participate to the labor market or to be involved in criminal activities. In case they become criminals, they decide how much effort they devote to delinquent behavior.

the same criminal group. Each group S of criminals has a positive probability $p_S(g)$ of winning the loot $B > 0$. We assume that the bigger the criminal group, the higher its probability of getting the loot. Criminals learn from others belonging to the same group how to be more efficient in criminal activities,[28] and the probability of winning the loot is given by $p_S(g) = \#S/n$. The network also determines how the loot is shared among the criminals in the group. Let $S(i) \in \Pi(g)$ be the criminal group criminal i belongs to and $c_i(g) = \max_{j \in S(i)} d_j(g)$ be the maximum degree in the criminal group $S(i)$, where $d_j(g)$ is the number of links (or degree) of criminal j. A criminal i who belongs to $S(i) \in \Pi(g)$ expects a share $\alpha_i(g)$ of the loot given by

$$\alpha_i(g) = \begin{cases} 1/(\#\{j \in S(i) \mid d_j(g) = c_j(g)\}) & \text{if } d_i(g) = c_i(g) \\ 0 & \text{otherwise.} \end{cases}$$

That is, within each criminal group, the criminal who has the highest number of links obtains the loot. If two or more criminals have the highest number of links, then they share the loot equally among them. Criminals can be caught with some positive probability. In case criminal i is caught, i's rewards are punished at a rate $\phi > 0$ with $\phi < n/(n-1)$. We assume that the higher the number of links criminal i has, the lower i's probability of being caught. The probability of being caught is simply given by $q_i(g) = (n - 1 - d_i(g))/n$. Then, criminal i's payoff is given by

$$\begin{aligned} u_i(g)) &= p_{S(i)}(g)\alpha_i(g)(1 - q_i(g)\phi)B, \\ &= \begin{cases} \frac{\#S}{n}\frac{1}{\#\{j \in S(i) | d_j(g) = c_j(g)\}}(1 - \frac{n-1-d_i(g)}{n}\phi)B & \text{if } d_i(g) = c_i(g) \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

There are many networks that are pairwise stable. Calvo-Armengol and Zenou (2004) find that the complete network is pairwise stable and strongly efficient. In addition, it can be easily verified that any network consisting of complete components, where no two components have the same degree, is pairwise stable. However, such pairwise stable networks are not strongly efficient.

Proposition 12 (Herings, Mauleon, and Vannetelbosch 2014). *For criminal networks, each network g such that $g = \bigcup_{S \in \Pi(g)} g^S$ and $\#S \neq \#S'$ $\forall S, S' \in \Pi(g)$ is pairwise stable.*

In addition, any network consisting of complete components, where no two components have the same degree, is pairwise Nash stable since no criminal has an incentive to delete more than one link.[29] The set of strongly efficient networks consists

[28] Patacchini and Zenou (2008) provide evidence that peer effects and the structure of social interactions matter strongly in explaining criminal or delinquent behavior.

[29] However, such a network may not be supported in a bilateral equilibrium of Myerson's linking game. For instance, take g such that $g = g^{S_1} \cup g^{S_2}$ where $S_1, S_2 \in \Pi(g)$, $S_1 \cup S_2 = N$ and $\#S_1 = \#S_2 + 1$. Then, both criminals $i \in S_1$ and $j \in S_2$ benefit from simultaneously adding the link ij between them and deleting one link of i with some other criminal in S_1 to form g'. Indeed, $u_i(g) = (1-(n-\#S_1)\phi/n)B/n < u_i(g') = (1-(n-\#S_1)\phi/n)B/\#S_1$ and $u_j(g) = (1-(n-\#S_2)\phi/n)B/n < u_j(g') = u_i(g')$.

of all networks with a single component where at least one player has $n-1$ links. Some of them are not pairwise stable. Indeed, pairwise stable and strongly efficient networks consist of a single component where $d_i(g) = n-1$ for some i and either $d_j(g) \leq n-3$ or $d_j(g) = n-1$ for any $j \neq i$. In any such network, adding one link for criminals with $d_j(g) \leq n-3$ amounts to receiving more know-how on the crime business, but this additional know-how is not enough to have a chance to win the competition for the loot. So, when myopic criminals decide with whom to establish links, it is not excluded that their decisions lead to some strongly inefficient network.

Herings, Mauleon, and Vannetelbosch (2009) show that from any criminal network there is a farsighted improving path ending in the complete network. Hence, using Proposition 4 we obtain the following proposition.

Proposition 13 (Herings, Mauleon, and Vannetelbosch 2009). *For criminal networks, the set* $\{g^N\}$ *is a pairwise farsightedly stable set.*

From any network g' consisting of complete components, where no two components have the same size, farsighted criminals may add links anticipating that their moves ultimately lead to the complete network. Moreover, from g^N there is no farsighted improving path leading to any g'. Hence, the set $\{g'\}$ cannot be a pairwise farsightedly stable set and farsightedness helps in reducing the conflict between stability and efficiency.

8.3.4 Other Models

There are other economic models that have been analyzed under both the myopic and the farsighted perspective.[30] Grandjean, Mauleon, and Vannetelbosch (2011) find that, in the Jackson and Wolinsky (1996) connections model, farsightedness does not eliminate the conflict between stability and strong efficiency that may occur when costs are intermediate. However, farsightedness helps to reduce the conflict when costs are large enough. In the Kranton and Minehart (2001) model of buyer-seller networks, Grandjean, Mauleon, and Vannetelbosch (2011) show that pairwise farsighted stability may sustain the strongly efficient network while pairwise stability only sustains networks that are strongly inefficient or even Pareto dominated. Finally, Grandjean (2014) investigates the Bramoullé and Kranton (2007) model of risk-sharing networks in developing countries. When the cost for forming links is small, farsighted players form strongly efficient networks while myopic players do not.

[30] We do not intend to cover all economic applications of network formation. We only mention those that, up to now, have been studied adopting both perspectives.

8.4 Conclusion

The outcomes of real-life network formation are affected by the degree of farsightedness of the players. Consider the case where the worth of link creation turns non-negative after some threshold in the connectedness of the network is reached, both for the players and on aggregate, but the players' benefits are negative below this threshold. If network externalities take this form, myopic players can be stuck in insufficiently dense networks. Farsighted players may take care of this problem and achieve the efficient network. If players have a limited degree of farsightedness, their ability to pass the threshold will depend on the number of reactions to their moves they can foresee from the starting network.

Strategic models of network formation have been evaluated in the experimental laboratory.[31] Kirchsteiger, Mantovani, Mauleon, and Vannetelbosch (2013) design a simple network formation experiment to test between pairwise stability and farsighted stability, but find evidence against both of them. Their experimental evidence suggests that subjects are consistent with an intermediate rule of behavior, which can be interpreted as a form of limited farsightedness. Herings, Mauleon, and Vannetelbosch (2014) propose an intermediate concept, namely level-K farsighted stability, that can be used to study the influence of the degree of farsightedness on network stability. Level-K farsighted stability is a tractable concept with myopic and full farsighted behavior as extreme cases.

References

Aumann, R. and R. Myerson (1988). "Endogenous formation of links between players and coalitions: An application of the Shapley value." In *The Shapley value*, A. Roth, ed., Cambridge, UK: Cambridge University Press.

Bala, V. and S. Goyal (2000). "A non-cooperative model of network formation." *Econometrica* 68, 1181–1229.

Belleflamme, P. and F. Bloch (2004). "Market sharing agreements and collusive networks." *International Economic Review* 45, 387–411.

Berninghaus, S. K., K.-M. Ehrhart, and M. Ott (2012). "Forward-looking behavior in Hawk-Dove games in endogenous networks: Experimental evidence." *Games and Economic Behavior* 75, 35–52.

Bloch, F. (1995). "Endogenous structures of association in oligopolies." *Rand Journal of Economics* 26, 537–556.

[31] There is a growing literature on experiments with network formation. See Corbae and Duffy (2008), Goeree, Riedl, and Ule (2009), Berninghaus, Ehrhart, and Ott (2012), and Falk and Kosfeld (2012), among others.

Bloch, F. (2005). "Group and network formation in industrial organization: A survey." In *Group Formation in Economics: Networks, Clubs and Coalitions*. G. Demange and M. Wooders, eds., Cambridge, UK: Cambridge University Press.

Bloch, F. and M. O. Jackson (2006). "Definitions of equilibrium in network formation games." *International Journal of Game Theory* 34, 305–318.

Bloch, F. and M. O. Jackson (2007). "The formation of networks with transfers among players." *Journal of Economic Theory* 133, 83–110.

Bramoullé, Y. and R. Kranton (2007). "Risk-sharing networks." *Journal of Economic Behavior & Organization* 64, 275–294.

Calvó-Armengol, A. (2004). "Job contact networks." *Journal of Economic Theory* 115, 191–206.

Calvó-Armengol, A. and R. Ilkilic (2009). "Pairwise stability and Nash equilibria in network formation." *International Journal of Game Theory* 38, 51–79.

Calvó-Armengol, A. and Y. Zenou (2004). "Social networks and crime decisions: The role of social structure in facilitating delinquent behavior." *International Economic Review* 45, 939–958.

Chwe, M. S. (1994). "Farsighted coalitional stability." *Journal of Economic Theory* 63, 299–325.

Corbae, D. and J. Duffy (2008). "Experiments with network formation." *Games and Economic Behavior* 64, 81–120.

Currarini, S. and M. Morelli (2000). "Network formation with sequential demands." *Review of Economic Design* 5, 229–249.

De Sinopoli, F. and C. Pimienta (2010). "Costly network formation and regular equilibria." *Games and Economic Behavior* 69, 492–497.

Dutta, B., S. Ghosal, and D. Ray (2005). "Farsighted network formation." *Journal of Economic Theory* 122, 143–164.

Dutta, B. and S. Mutuswami (1997). "Stable networks." *Journal of Economic Theory* 76, 322–344.

Ehlers, L. (2007). "von Neumann-Morgenstern stable sets in matching problems." *Journal of Economic Theory* 134, 537–547.

Falk, A. and M. Kosfeld (2012). "It's all about connections: Evidence on network formation." *Review of Network Economics* 11(3), Article 2.

Furusawa T. and H. Konishi (2007). "Free trade networks." *Journal of International Economics* 72, 310–335.

Galeotti, A., S. Goyal, and J. Kamphorst (2006). "Network formation with heterogeneous players." *Games and Economic Behavior* 54, 353–372.

Gilles, R. P., S. Chakrabarti, and S. Sarangi (2012). "Nash equilibria in network formation games under consent." *Mathematical Social Sciences* 64, 152–158.

Gilles, R. P., and S. Sarangi (2010). "Network formation under mutual consent and costly communication." *Mathematical Social Sciences* 60, 181–185.

Goeree, J. K., A. Riedl, and A. Ule (2009). "In search of stars: Network formation among heterogeneous agents." *Games and Economic Behavior* 67, 445–466.

Goyal, S. (2007). *Connections: An Introduction to the Economics of Networks*. Princeton, NJ: Princeton University Press.

Goyal, S. and S. Joshi (2003). "Networks of collaboration in oligopoly." *Games and Economic Behavior* 43, 57–85.

Goyal, S. and S. Joshi (2006a). "Unequal connections." *International Journal of Game Theory* 34, 319–349.

Goyal, S. and S. Joshi (2006b). "Bilateralism and free trade." *International Economic Review* 47, 749–778.

Goyal, S. and J. L. Moraga-Gonzalez (2001). "R&D networks." *RAND Journal of Economics* 32, 686–707.

Goyal, S. and F. Vega-Redondo (2007). "Structural holes in social networks." *Journal of Economic Theory* 137, 460–492.

Grandjean, G. (2014). "Risk-sharing networks and farsighted stability." *Review of Economic Design* 18, 191–218.

Grandjean, G., A. Mauleon, and V. Vannetelbosch (2011). "Connections among farsighted agents." *Journal of Public Economic Theory* 13, 935–955.

Grandjean, G. and W. Vergote (2014). "Network formation among rivals." CEREC Discussion Paper 2014–9. Université Saint-Louis, Brussels, Belgium.

Hellmann, T. (2013). "On the existence and uniqueness of pairwise stable networks." *International Journal of Game Theory* 42, 211–237.

Herings, P. J. J., A. Mauleon, and V. Vannetelbosch (2009). "Farsightedly stable networks." *Games and Economic Behavior* 67, 526–541.

Herings, P. J. J., A. Mauleon, and V. Vannetelbosch (2014). "Stability of networks under level-K farsightedness." CORE Discussion Paper 2014–32. Université catholique de Louvain, Belgium.

Hojman, D. A. and A. Szeidl (2008). "Core and periphery in networks." *Journal of Economic Theory* 139, 295–309.

Jackson, M. O. (2008). *Social and Economic Networks.* Princeton, NJ: Princeton University Press.

Jackson, M. O. and A. Watts (2001). "The existence of pairwise stable networks." *Seoul Journal of Economics* 14, 299–321.

Jackson, M. O. and A. Watts (2002). "The evolution of social and economic networks." *Journal of Economic Theory* 106, 265–295.

Jackson, M. O. and A. Wolinsky (1996). "A strategic model of social and economic networks." *Journal of Economic Theory* 71, 44–74.

Jackson, M. O. and A. van den Nouweland (2005). "Strongly stable networks." *Games and Economic Behavior* 51, 420–444.

Kirchsteiger, G., M. Mantovani, A. Mauleon, and V. Vannetelbosch (2013). "Limited farsightedness in network formation." CORE Discussion Paper 2013–33. Université catholique de Louvain, Belgium.

Kranton, R. E. and D. F. Minehart (2001). "A theory of buyer-seller networks." *American Economic Review* 91, 485–508.

Mauleon, A., E. Molis, V. Vannetelbosch, and W. Vergote (2014). "Dominance invariant one-to-one matching problems." *International Journal of Game Theory* 43, 925–943.

Mauleon, A., J. J. Sempere-Monerris, and V. Vannetelbosch (2008). "Networks of knowledge among unionized firms." *Canadian Journal of Economics* 41, 971–997.

Mauleon, A., J. J. Sempere-Monerris, and V. Vannetelbosch (2014). "Farsighted R&D networks." *Economics Letters* 125, 340–342.

Mauleon, A., H. Song, and V. Vannetelbosch (2010). "Networks of free trade agreements among heterogeneous countries." *Journal of Public Economic Theory* 12, 471–500.

Mauleon, A., V. Vannetelbosch, and W. Vergote (2011). "von Neumann-Morgenstern farsightedly stable sets in two-sided matching." *Theoretical Economics* 6, 499–521.

Myerson, R. (1991). *Game Theory: Analysis of Conflict.* Cambridge, MA: Harvard University Press.

Navarro, N. (2014). "Expected fair allocation in farsighted network formation." *Social Choice and Welfare* 43, 287–308.

Page, F. H., Jr. and M. Wooders (2009). "Strategic basins of attraction, the path dominance core, and network formation games." *Games and Economic Behavior* 66, 462–487.

Page, F. H., Jr., M. Wooders, and S. Kamat (2005). "Networks and farsighted stability." *Journal of Economic Theory* 120, 257–269.

Patacchini, E. and Y. Zenou (2008). "The strength of weak ties in crime." *European Economic Review* 52, 209–236.

Roketskiy, N. (2012). "Competition and networks of collaboration." Mimeo, New York University.

Slikker, M. and A. van den Nouweland (2001). "A one-stage model of link formation." *Games and Economic Behavior* 34, 153–175.

Tercieux, O. and V. Vannetelbosch (2006). "A characterization of stochastically stable networks." *International Journal of Game Theory* 34, 351–369.

von Neumann, J. and O. Morgenstern (1944). *Theory of Games and Economic Behavior*. Princeton, NJ: Princeton University Press.

Yi, S-S. (1998). "Industry profit-maximizing joint-venture structure in a linear Cournot oligopoly." *Economics Letters* 58, 361–366.

Zhang, J., L. Xue, and L. Zu (2013). "Farsighted free trade networks." *International Journal of Game Theory* 42, 375–398.