Chapter 10

Networked Markets

Classic theories of supply and demand, and the competitive models that underly them, are built on the trade of precisely defined and known commodities. This allows for markets that are largely anonymous and unmodeled. The idea that modern societies can produce regular goods and deliver them across large and effectively anonymous markets was championed by by Adam Smith [**?**] and has become a cornerstone of modern economic analysis. Indeed, there are many goods and services are such that the quality and reliability are predictable and delivery is easy and contractable, and where transactions can occur between parties who need not know each other outside of a single instance. This is true of many final goods, such as various consumer products. However, even in the most modern societies there are many goods and services that are not so uniform across large numbers of buyers and sellers, and might involve specific features tailored to a particular situation or involve smaller numbers of buyers and sellers. Labor markets are an example. Because of this, many markets operate through decentralized networks or at least partly throughnetworked interactions. This is true of many intermediate goods and services, where parts or inputs are supplied from one firm to another, or one individual to another. The importance of social networks in the market is evident to anyone who has ever searched for employment, and the extent and importance of the embeddedness of economic activity in social networks in developed markets is cogently argued by Granovetter [292].[1]

This chapter explores the role of social networks in various markets. A series of questions arise. To what extent are social and economic networks used in the exchange of goods and services? Which markets tend to be 'networked' and why? Does the use

[1] For an early discussion of the social embeddedness of economic activity see Polanyi [517].

of networked markets affect the terms of trade, prices and efficiency of a market? How do social networks impact wages and employment patterns? How does position in a network affect the trade and welfare of an individual or firm? Which networks are likely to emerge in the context of networked markets? This chapter addresses these questions and also provides groundwork for further research on this important subject.

10.1 The Social Embeddedness of Markets and Exchange

Social networks have been the primary fabric of many economic interactions for centuries, if not millenia (e.g., recall the discussion of fifteenth century Florence in Section **??**), and there has been detailed research on the embeddedness of various markets in social networks for over six decades. To begin this chapter, I provide an overview of some of the empirical work on this subject, and some discussion of which markets we should expect to be networked and why.

10.1.1 The Use of Job-Contacts in Labor Markets

The importance of social contacts in obtaining information about jobs and in the referral process is so prevalent, that it comes to embody part what we understand by the term "networking". In fact, a definition of "networking" in the Merriam-Webster dictionary (online, 2007) is "the cultivation of productive relationships for employment or business." Indeed, a substantial portion of jobs are filled through referrals from social contacts of current employees of a firm. This observation is not only important as an example of the role of social networks, but also because it has implications for employment, wages, and the efficiency of labor markets. It shows how understanding social networks can help us to gain new and deeper insights into the workings of a market.

To get an impression of the extent to which social networks play a role in labor markets, let us consider statistics from some studies.[2] An early study by Myers and Shultz [472] of textile workers found that 62 percent of interviewed workers found out about their job openings through a social contact, while only 23 percent found the job by a direct application, and 15 percent through an employment agency, advertisement, or some other means. This sort of pattern is not unique to the textile industry, it is

[2] See Montgomery [455] and Ioannides and Loury [**?**] for more references and discussion.

also found across other professions. For instance, Rees and Shultz [531] interviewed workers in a Chicago neighborhood and kept track of what percentage of them found out about their current jobs through friends or relatives. They considered twelve different occupations and found rates of jobs obtained through social contacts ranging from a low of 23.5 percent (for accountants) to a high of 73.8 percent (for material handlers). The array of occupations they considered was broad, including typists (37.3 percent from social contact), janitors (65.5 percent), electricians (57.4 percent), and truck drivers (56.8 percent), to take a few examples. While these are mostly manual labor jobs, similar patterns are exhibited across different types of work. Granovetter's [289], [290] interviews of residents of Amherst Massachusetts found similar patterns across various types of occupations. He found that 44 percent of technical workers found their jobs through a social contact, as did 56 percent of professional workers, and 65 percent of managerial workers. Corcoran, Datcher, and Duncan [**?**] examine the Panel Study in Income Dynamics data set (the "PSID") and compare across race and gender. They found the following percentages for finding jobs by social contact: Black Males 58.5 percent, Black Females 43 percent, White Males 52 percent, and White Females 47.1 percent.

These numbers provide an impression of the extent to which social networks play a role in labor markets. There are also a variety of studies that examine how the role of social networks in labor markets varies across different groups and professions (see Ioannides and Loury [**?**] for more of an overview). As an example, Pellizzari [510] examines data from European countries and finds a range of differences of how prevalent social contacts are in the labor market, as well as whether jobs obtained through social means lead to higher or lower wages. While going into the details of those differences is beyond the scope of this text, it is clear that it is very important to develop models that help us to understand why and when employers will use referrals as a means of filling job vacancies, and how social network structure plays into this. While there are some models that shed light on these issues, this area is still under-developed.

10.1.2 The Features of some Networked Markets

To provide further background, it is helpful to discuss a couple of examples of studies of networked markets and some of the features that those markets exhibit.[**?**, **?**,]

An influential study is Uzzi's [598] investigation of the importance of social relationships in the apparel industry in New York. First, Uzzi interviewed the executive officers of 43 "better dress" firms in New York City with annual sales between five hundred thousand dollars and one billion dollars. The firms are basically divided into two groups: manufacturers and contractors. For instance, a contractor might have a design for a particular garment and want a given number of them produced and delivered. A manufacturer would then take the garment design and produce and deliver the garments. The firms were selected via a stratified random sample. Uzzi's focus was on the type of relationships that firms had with each other. One type of relationship he categorized as "market" or "arm's length" relationships which included many one-time transactions, and the other he categorized as "special" or "close" relationships which included many relationships with repeated interaction and ones that involved idiosyncracies in products or special investments. Based on the interviews, Uzzi identified three main ingredients that are associated with close ties but not with arm's length ties: trust, fine-grained information transfer, and joint problem solving. This view is derived from interpretations of the interviews and associated anecdotes. Uzzi quotes an example where trust is associated with dealing with an unforeseen problem, such as a fabric not producing the sort of garments that a contractor desires, and then reaching some agreement on how to deal with this problem rather than leaving the manufacturer at a loss. The fine-grained information transfer refers to passing along useful information, for instance about a new design, fabric, or technique, to someone with whom one is in a repeated relationship. Firms involved in close relationships have more of an incentive to pass along such information, and can also do so in a credible manner. The joint problem solving refers to being able to quickly deal with problems that arise, which often hinges on knowing what the capabilities and situations of both firms in a relationship are.

Using these insights of how close relationships might help firms, Uzzi builds a hypothesis that firms that have more embedded relationships will have a higher probability of survival. To test this hypothesis, he examines 1991 data from the International Ladies Garment Workers' Union, which keeps detailed records of inter-firm transactions in the industry. Over 80 percent of the firms are unionized, and these data cover most of the active firms in New York that year. Based on these data, Uzzi examines the contractors who failed versus those who did not. Out of the 479 contractors with complete records in the data set, 125 failed during this year.[3] Uzzi measures how embedded a contractor i is by examining $\sum_j P_{ij}^2$ where P_{ij} is the percentage of all of contractor i's output contracted with manufacturer j. If a contractor deals with only

[3]Only 8 of 89 manufacturers in the data set failed, and so Uzzi does not report the relationship between survival and embeddedness for manufacturers.

one manufacturer then this will be 1, whereas if the contractor spreads its business out among many manufacturers then this will tend towards 0. 15 percent of the contractors send 100 percent of their business to one manufacturer, and 45 percent send at least 50 percent of their business to one manufacturer, while almost 20 percent send less than 25 percent to any one manufacturer. Thus, there is some variation in the data, but also substantial embeddedness according to this measure. Uzzi then regresses the survival variable on this embeddedness measure, as well as a series of other background variables (geographic location, age of the firm, size of the firm, some network centrality measures, and other neighborhood variables). He finds a positive and significant relationship (at the 95 percent confidence level) between survival and embeddedness. Based on the fitted regression line, a typical firm (setting other controls to average levels) that has an embeddedness score of 0 will fail to exist at the end of the year with a probability of .27, while a firm with an embeddedness score of 1 will fail to exist at the end of the year with a probability of .14.

As with any cross-sectional statistical analysis, it is difficult to draw causal conclusions from an observed correlation. It could be that embeddedness helps firms to survive, or it could be that firms that are near failure have a harder time establishing close relationships. Uzzi argues that it is the former. There could also be many other factors that are not observed and that contribute to the type of arrangements that firms have. For example, it might be that more complicated garments require specialized relationships and also are in a less competitive part of the business. Regardless of what the precise explanation for the observed correlations, or which direction the causation goes, Uzzi's [598] study provides evidence that many firms in this industry do a substantial portion of their business with just a few, and in many cases just one, partner; and also provides some insight into the potential differences between a network of close relationships versus a more distant market-style relationships.

Another example of a partly-networked market is that of the Marseille fish market as analyzed by Kirman [378] and Weisbuch, Kirman, and Herreiner [626]. The fish market has several critical features. First, the fish are perishable. This means that the ability for buyers and sellers to smooth inventory is quite limited, as the "fresh" fish need to be consumed soon after they are caught.[4] Second, the supply of fish is variable

[4]This is changing over time, as methods of freezing and transporting fish are advancing. "Fresh" is in quotes, as in fact some types of fish that are considered fresh are frozen on board the boats on which they are caught. Nevertheless, there is a demand for local fish in the Marseille market (e.g., as ingredients in Marseille's famous bouillabaisse).

as the catch is random and affected by various conditions including weather, water conditions, and fish populations. Third, the buyers of the fish differ in their demand elasticities for the fish; that is, they differ in the importance of fish for them and how their willingness to pay varies with quantity. A famous restaurant that has built a reputation on serving Marseille's finest bouillabaisse has to be able to consistently buy fish of a reliable quantity. Another local restaurant that does not specialize in bouillabaisse can adjust its menu and ingredients as availability changes.

Weisbuch, Kirman, and Herreiner [626] examine the market between 1988 and 1991 considering buyers and sellers in the market for more than 8 months. There are 45 sellers and over 1400 buyers, and a wide range of fish. The buyers are restaurants and retailers and there are no posted prices so that each price transaction is decided bilaterally between the buyer and seller. In terms of a network structure, they find that a sizeable fraction of buyers are loyal to a single seller, while other buyers buy from many sellers. For example, with regards to cod, almost half (48 percent) of the buyers purchase more than 95 percent of their cod from just one seller. With regards to whiting and sole, more than half of the buyers buy more than 80 percent of their fish from just one seller. They break this down by the patterns of buyers and note that buyers purchasing large quantities are significantly more likely to be loyal than other buyers. For example, dividing buyers into those who bought more than two tons per month and those who bought less, the ones who bought more transact on average 85 percent of the time with their most visited seller while this drops to 56 percent of the time for those who buy less. This suggests that one sees some established relationships, as well as some shopping around. Weisbuch, Kirman, and Herreiner [626] examine a simple model of this, where the driving force is predictability of available fish and of demand for fish. The model builds on what one might think of as a complicated version of musical chairs: buyers must choose to go to just one seller each day and the seller is either able to meet the buyer's demand. Buyers follow an adaptive updating rule, with a higher tendency to visit sellers who have met their demands in the past. This is a sort of metaphor that captures the idea that as a buyer shops around during the day, time is lost and fewer fish are available. On a day where there is a relatively large catch this might result in a better price, but on a day where there is a relatively small catch this might result in a relatively high price or being closed out of the market. Thus, there are some frictions in this type of market so that it does not simply clear as a classic supply and demand model all at once, but requires some search by buyers, and price-setting by individual and only partly-informed sellers. Buyers face some coordination issues and also learn about the available supply of different sellers over time. Having a larger demand, among other things, tilts a buyer towards settling down with just one (large) seller, partly due to the predictability of the supply.

10.1.3 Which Markets Should be Networked?

These examples point to some of the issues that underly why specific relationships might emerge in the trade of goods and services. The garment industry analysis highlights "trust" as a central part of the importance of repeated relationships. The fish market analysis highlights predictability as a central part of the importance of repeated relationships. There are a variety of situations that lead to some advantages of a repeated or close relationship, basically having to do with difficulties in contracting. This is related to some of the theory developed by Williamson [629] explaining the organization of firms. Williamson examines a variety of different frictions that can make it difficult for two parties to completely contract upon the exchange of goods or services. Williamson then argues that if a single organization or firm is on both sides of the transaction, then it can internalize the difficulty and overcome the obstacle to contracting. Granovetter [292] critiques Williamson's argument pointing out that it is not so clear that placing things within a firm provides a solution that could not be handled through some sort of other relationship. For example, reputations and repeated interactions can help discipline transactions and this could take place within the fabric of a network instead of being internal to a firm. Also, contracting parties being within the same firm does not imply that their incentives are aligned.

Drawing from the above examples and studies of incomplete contracting, let us examine some of the features of given transactions that might favor placing them within the context of a network of transactions where reputations and/or repeated relationships are relied upon to help circumvent difficulties inherent in a given transaction.

In many situations, there are unforeseen contingencies that arise between two parties involved in a contractual relationship. It may be that some specific input is not available, or a new regulation appears that requires the redesign of a product and some delay, and so forth. One can try to write a contract that completely covers all possible contingencies, but complex transactions (for instance, the construction of a building) this might not be possible. If the relationships between the parties are repeated, so that they deal with each other on an ongoing basis then these problems are not viewed as a one-time expense on one of the two parties' side, but instead can be evened out over time. This can make the bargaining over such unforseen issues smoother, as suggested by some of Uzzi's [598] interviews.

Contractual incompleteness can arise not only because of unforeseen contingencies, but also because of specific investments that might need to be made for particular transactions and require long-term use to realize their full value. Contractual incompleteness can also arise because of asymmetries in information. Asymmetries in information manifest themselves in the form of moral hazard problems, where one party to a contract cannot fully observe the actions of the other, and adverse selection problems where one party does not fully observe some attributes of the good being traded. Once again, long-term relationships can help resolve these issues. For example, in terms of moral hazard problems, having a repeated interaction allows one party to examine long run performance of the other. If a customer takes a car to a local mechanic on a regular basis, and the mechanic frequently claims that the car requires extensive and expensive repairs, the customer can look for a new mechanic. The fact that current performance can influence future business helps temper the moral hazard problem. In contrast, if a car needs a repair far from home, and it is a one time interaction, the incentives for the mechanic to suggest a more expensive repair than is necessary can be substantially larger. Similar reasoning favors long term relationships in the face of adverse selection. If a firm buys parts whose reliability or longevity cannot be observed except with the passage of time, then having repeated transactions can help provide incentives for the supplier to deliver a specified quality of parts.

There are tradeoffs to maintaining such closed relationships, as it limits one's ability to shop around for alternative prices. It might be that a given firm wants to work with two or three suppliers of the same parts over time. One can also rely on the transmission of information through a network to help temper asymmetric information problems. Maintaining a "reputation" for providing high-quality parts or service can help provide incentives that overcome some of the difficulties arising from asymmetries in information, if word of mouth can spread information about outcomes to other potential future business partners. Thus, we might see more complex network relationships for various reasons.

Beyond these asymmetric information considerations, we also saw the issues of predictability and more basic uncertainty as potential explanations for the relationships in the Marseille fish market.[5] As an incentive for a continued relationship, a regular customer might be given access to better produce or a higher chance at getting a desired

[5]See also the study by Podolny [514] of investment banking showing an increased concentration of transaction relationships when market uncertainty increases.

quantity of produce. In situations where the uncertainties of the crop or production totals are not fully insurable, risk-aversion can then favor the formation of repeated interactions.

There are other aspects of networks that can be valuable in the trade of goods and services beyond those which arise between the parties directly involved in the transaction. Networks also serve as an integral part of many markets in terms of putting different buyers and sellers in touch with each other. Labor markets serve as an excellent example. A firm wanting to hire a new employee might ask its existing employees for referrals. This could happen for a variety of reasons.[6] A very basic reason is simply that it wants to hire people similar to the employees that it already has. Given the homophily in many social networks, a firm can take advantage of its existing workforce in order to find other people with similar characteristics. For example, if a fast-food restaurant wants to hire someone willing to work part-time on weeknights and for minimum wage with low benefits, it might ask its employees if they know of anyone else who would be available in similar circumstances. This can save the time and cost of advertising and then sifting through applications. Beyond this, using current employees might reach potential hires that might not be reached via the advertising. It can also be that current employees are good at communicating with potential employees regarding whether a potential job is a good match. In addition to the benefits of locating potential hires who fit well with the firm, current employees might be credible sources of information about the quality of a potential hire. A recommendation coming from a current employee or a friend or other sort of acquaintance could carry more weight than that of a stranger.

This discussion has sketched potential benefits and reasoning behind networked relationships. Let us now examine some models of such interactions in order to get a fuller understanding of some of the potential implications of network structure for economic transactions and welfare, and also for incentives to form such networks.

[6] See Fernandez, Castilla, and Moore [228] for discussion of some factors favoring the use of referrals and evidence from a study of hiring practices in a phone bank that there can be economic benefits to firms who hire through referrals. In addition to some of the benefits from better matching, they also examine the extent to which hiring friends of current employees affects turnover in the firm, which might be related to the social environment of the firm.

10.2 Networks in Labor Markets

The pervasiveness of networks in labor markets makes them a leading example of networked markets and a source of a variety of insights. So, let us begin by analyzing different aspects of networked labor markets.

10.2.1 Strong and Weak Ties

As discussed in Section 3.2.7, the role of social networks in finding jobs was central to Granovetter's [289], [290] influential research that distinguished between "strong" and 'weak" social ties. To recall those data, Granovetter measured the strength of a tie by the number of times that individuals had interacted in a past year (strong = at least twice a week, medium = less than twice a week but more than once a year, and weak = once a year or less). Of the 54 people whom he interviewed who had found their most recent job through a social contact, he found that 16.7 percent had found their jobs through a strong tie, 55.7 percent through a medium tie, and 27.6 percent through a weak tie.[7]

Building on a distinction between strong and weak ties, Boorman [85] modeled individuals' decisions of how to allocate their time between maintaining these two different forms of ties in one of the first "economic" models of social networks. Boorman's model is based on the following structure. An individual has to divide his or her time between maintaining strong and weak ties. Strong ties take more time, and so the individual is faced with a tradeoff having more ties, but weak ones, or fewer ties, but strong ones. Boorman represents this by requiring that an individual have T units of time spent maintaining relationships. If W is the number of weak ties that an individual has, and S is the number of strong ties, then they must satisfy:

$$W + \lambda S = T \tag{10.1}$$

where $\lambda > 1$ is a factor indicating how much more time must be spent to maintain a strong tie.

Boorman's ties also lead to different benefits. Strong ties have priority in obtaining job information from social contacts. This operates as follows. Time ticks by in discrete periods. In any period, with probability μ an individual has need of a job. This is

[7] There is also a series of studies that have examined how strong and weak ties affect labor outcomes, and some debate about the relative effectiveness of weak ties. For example, see Lin, Ensel and Vaughn [412] and Bridges and Villemez [98] and the literature that follows.

the same across individuals and independent of history and the state of the system in the previous period. Effectively, the system restarts in each period. If an individual needs a job then he or she can find a job in two ways. First, the individual can hear about a job directly, which again happens at some exogenous rate δ, independent of the state of the system. In that case, the individual takes the job. Second, the individual might not hear directly, but instead might have a friend who is employed who happens to randomly hear about a job. In that case, the employed friend looks around at his or her strong ties and weak ties. If some of the strong ties are unemployed, then the employed friend passes the job to one of them uniformly at random. If all of the strong ties are employed, then the employed friend passes the job on to one of the weak ties uniformly at random. Thus, strong ties have a priority in hearing about a job. Boorman examines networks that are trees with infinite numbers of nodes so that one does not have to worry about the issue of two neighbors being neighbors of each other. He also considers situations where all individuals choose the same allocation of ties, so that the network is regular in a strong sense. Let $q_s \leq q_w$ be the probability that one does *not* hear about a job through a given strong tie and weak tie respectively, when in need. The chance of getting a job when in need can then be written as

$$\delta + (1-\delta)\left(1 - q_s^S q_w^W\right).$$

A given agent thus trades off the higher probability that strong ties lead to job information against being able to maintain fewer of them. One can derive the expressions for q_s and q_w as a function of the parameters of the model. For instance, q_s will depend on the probability that a given strong friend will be employed and how many other strong ties might be competing for job information at a given time. With expressions for q_s and q_w in hand, one can then look for an equilibrium of the system where individuals are optimally choosing S, W given the anticipated q_s and q_w, and the anticipated q_s and q_w correspond to the ones generated by the choices that individuals have made concerning S, W. This is hard to solve for directly, and can involve multiple equilibria, but one can at least work out simulations for some parameter values, as Boorman does.

There are several intuitive effects that Boorman reports from simulations. First, as λ increases, the relative cost of strong versus weak ties goes up and so the equilibrium involves fewer strong and more weak ties. Second, as μ decreases, so that one is less likely to need a job, the relative value of weak ties goes up. One only gets job information via a weak tie when all of the weak acquaintance's strong neighbors are employed, which is more likely when μ is low.

The Boorman model makes important strides in terms of considering strong and weak ties to be choice variables, and in terms of deriving tradeoffs between different forms of ties. However, the model lacks a number of things that we might be interested in. As a model of strong and weak ties, it is missing one of the critical ingredients that was the basis for Granovetter's theory: the idea that weak ties were more likely to bridge to parts of a network that are not accessed more directly whereas strong ties are more likely to link to nodes that are already at a short path distance. Such aspects of network architecture are missing from Boorman's model. Also, the model is missing the correlation in employment and time series implications that we might be interested in from the perspective of explaining how labor markets work. The fact that the state of the network is history independent simplifies the model (with the need for jobs being independent of history and the previous state of the network), but then we miss interesting dynamics and patterns of employment as a function of social structure.

10.2.2 A Networked Model of Employment

Calvó-Armengol and Jackson [119], [120] examine a model that is similar to Boorman's in having job information arrive directly and through neighbors, but that brings network structure to the forefront to see how network structure can impact employment and wage dynamics and distribution across a society.[8] Before getting into the model in detail, let me start with a brief overview of it. In the simplest version of the model, workers are connected by an undirected network. Randomly, an employed worker can lose his or her job. If that happens, then the worker looks for work. Information about new job openings arrives randomly to the workers in the network. A worker can hear about a job directly. If the worker is unemployed, then he or she takes the job. Similarly to the Boorman model, if the worker is employed, then the worker randomly picks an unemployed neighbor to receive the information; but in this case treating all unemployed neighbors with equal weight. The system operates over time, so that the starting state at the beginning of one period is state the system ended in the last period, and so dynamic patterns can be studied. The network becomes important in several ways. Having more neighbors gives a worker potential access to more job information

[8] Such information arrival processes are also in Diamond [188], without networks. Calvó-Armengol [115] developed a networked variation of Diamond's and Boorman's models of information passing to study incentives for network formation. Calvó-Armengol and Jackson [119], [120]developed a richer model in bringing in wages and variations on job information passing and used it to study employment and wage patterns and dynamics.

and so a higher average employment rate and higher average wages (in the version of the model with wages). This also leads to correlation in neighbors' employment status. An unemployed worker who has an employed neighbor is more likely to hear about a job opening than a similar worker with an unemployed neighbor, leading to a higher probability that a worker becomes employed in any given period as a function of how many of his or her neighbors are employed. This leads to a positive correlation in employment.

Beyond this basic structure, there are a number of things that can be analyzed. The model described above leads to a Markov chain, so that one can keep track of what tomorrow's employment pattern is likely to look like given today's employment pattern. From this, one can deduce long run steady-state distributions of employment and how these vary with network structure. In addition to things like the correlation structure, one can also examine the time series of employment. For instance, a prevalent observation in the labor economics literature is what is known as duration dependence. That refers to the fact that workers who have been unemployed for more periods are less likely to find work in a given period than workers who are just recently unemployed. There are various partial explanations for this, but the network model exhibits this quite naturally. Conditional on a worker being unemployed for a longer time, it is likely that many of the worker's neighbors are also unemployed, and hence have not been passing information along. This means that it is less likely that such a worker will be hearing about a job in the next period, compared to a worker whose neighborhood has a higher employment level. One can also enrich the model to allow for different types of jobs, different wage levels, and decisions such as whether or not to pursue education.

The Model Description

I present the simplest version of the model. For variations with heterogeneous jobs and multiple wage levels, see Calvó-Armengol and Jackson [120]. In the version presented here, all jobs are identical and there is just one wage level, which simplifies the analysis.

There are n workers or agents who are connected by an undirected network, represented by the $n \times n$ symmetric matrix g, which has entries in $\{0, 1\}$. Time evolves in discrete periods indexed by $t \in \{1, 2, \ldots\}$.

The n-dimensional vector s_t describes the employment status of the agents at time t. If agent i is employed at the end of period t, then $s_{it} = 1$ and if i is unemployed then $s_{it} = 0$.

Period t begins with some agents employed and others unemployed, as described by the state s_{t-1}. The first thing that happens in a period is that information about new job openings arrives. Each agent directly hears about a job opening with a probability $a \in [0,1]$. This job arrival process is independent across agents. If an agent i is unemployed ($s_{i,t-1} = 0$) and hears about a job then he or she takes that job and becomes employed. If an agent i is employed ($s_{i,t-1} = 1$) and hears about a job then he or she picks an unemployed neighbor ($j \in N_i(g)$ such that $s_{j,t-1} = 0$) and passes the job information to that neighbor. If agent i has several unemployed neighbors, then the agent picks one uniformly at random. If agent i's neighbors are all employed, then the job information is lost.

The probability of the joint event that agent i learns about a job and this job ends up in agent j's hands is described by $p_{ij}(s_{t-1})$, where

$$p_{ij}(s_{t-1}) = \begin{cases} a & \text{if } s_{i,t-1} = 0 \text{ and } i = j; \\ \frac{a}{\sum_{k: s_{k,t-1}=0} g_{ik}} & \text{if } s_{i,t-1} = 1, s_{j,t-1} = 0, \text{ and } g_{ij} = 1; \text{ and} \\ 0 & \text{otherwise.} \end{cases} \tag{10.2}$$

At the end of a period some employed agents lose their jobs. This happens randomly according to an exogenous breakup probability, $b \in [0,1]$, independently across agents.

Some Simple Examples

It is useful to start with some simple examples to see how things evolve.

Example **10.2.1** *An Isolated Agent*

First, let us consider an isolated agent as a benchmark. Let μ be the long-run steady-state probability that the agent is employed.[9] This must satisfy

$$\mu = (1-b)\left(\mu + a(1-\mu)\right). \tag{10.3}$$

This keeps track of the two different ways the agent could be employed. First, it could be that at the end of last period the agent was employed, which has a probability μ, and then the agent did not lose his or her job at the end of this period, which happens

[9] This is a probability that may be thought of in two ways. First, regardless of what the agent's initial state is, this is the limit of the probability that the agent will be employed in a distant period in the future. Second, if one starts by randomly setting the agent's initial state with this probability, then it is the probability that the agent will be employed tomorrow, and at any date in the future.

with probability $1-b$. Second, it could be that the agent was unemployed at then end of last period and then heard about a job in the beginning of this period, which has a probability $a(1-\mu)$, and then the agent did not lose his or her job at the end of this period, which happens with probability $1-b$. Solving (10.3) for μ leads to

$$\mu = \frac{(1-b)a}{b+(1-b)a} = \frac{1}{1+\frac{b}{(1-b)a}}. \tag{10.4}$$

As one would expect, the steady-state employment probability is increasing in the probability of hearing about a job, a; and decreasing in the probability of losing a job, b; is no more than the probability of retaining a job $(1-b)$; but approaches $(1-b)$ as a approaches 1. Moreover, it is not the absolute values of a and b that matter, but their relative values. In particular, it is how $\frac{b}{1-b}$ compares to a that is critical.

Example **10.2.2** *A Dyad*

Next, let us consider a dyad. Here $n=2$ and $g_{12}=g_{21}=1$. Given the symmetry of this setting, the steady-state distribution can simply be kept track of through the probability that no agents are employed, μ_0, one agent is employed, μ_1, and both agents are employed, μ_2.

We can then keep track of the transitions as follows. The situation with two employed workers can happen in three ways. It could be that they were both employed at the end of last period and neither lost a job, which happens with probability $\mu_2(1-b)^2$, or it could be that only one was employed at the end of the last period and at least one heard about a job and neither lost a job, which happens with probability $\mu_1(1-(1-a)^2)(1-b)^2$, or it it could be that neither started out employed and both heard about jobs and then both kept those jobs, which happens with probability $\mu_0 a^2(1-b)^2$. Similar reasoning applied to the other states, and we can characterize the steady states as the solutions to:

$$\begin{pmatrix} \mu_0 \\ \mu_1 \\ \mu_2 \end{pmatrix} = \begin{pmatrix} (1-a+ab)^2 & (1-a)^2 b(1-b)+b^2 & b^2 \\ 2a(1-b)(1-a+ab) & (1-b)\left((1-a)^2(1-2b)+2b\right) & 2b(1-b) \\ a^2(1-b)^2 & \left(1-(1-a)^2\right)(1-b)^2 & (1-b)^2 \end{pmatrix} \begin{pmatrix} \mu_0 \\ \mu_1 \\ \mu_2 \end{pmatrix}. \tag{10.5}$$

We solve this (noting that the vector μ is a unit eigenvector of the transition matrix) to find

$$\begin{pmatrix} \mu_0 \\ \mu_1 \\ \mu_2 \end{pmatrix} = \begin{pmatrix} b^2\left(1+(1-b)(1-a)^2\right)/X \\ 2ab(1-b)\left(1+(1-b)(1-a)\right)/X \\ a^2(1-b)^2\left((1-a)(3-a)(1-b)+1\right)/X \end{pmatrix} \tag{10.6}$$

where

$$X = b^2\left(1+(1-b)(1-a)^2\right)+2ab(1-b)\left(1+(1-b)(1-a)\right)+a^2(1-b)^2\left((1-a)(3-a)(1-b)+1\right).$$

If we let $a = 1$, so that workers are sure to hear about jobs in any period, then the probability of having nobody employed goes to b^2, the probability of having one employed goes to $2b(1-b)$, and the probability of having both employed goes to $(1-b)^2$, as we should expect. As b goes to 0, μ_2 goes to 1, and as b goes to 1, μ_0 goes to 1.

As these expressions are cumbersome, we can also examine the model when the time between periods becomes small. Then, a and b both go to zero, and it is only the relative rates that matter. In particular, the chance that more than one change happens in a period, in terms of having more than one piece of information and/or loss of a job, goes to 0 relative to the probability that one change happens in a period. For instance, let us replace a and b with $\frac{a}{T}$ and $\frac{b}{T}$. As T becomes large, second order and larger terms become negligible relative to single changes and then (10.5) is approximated by

$$\begin{pmatrix} \mu_0 \\ \mu_1 \\ \mu_2 \end{pmatrix} = \begin{pmatrix} 1-\frac{2a}{T} & \frac{b}{T} & 0 \\ \frac{2a}{T} & 1-\frac{2a}{T}-\frac{b}{T} & \frac{2b}{T} \\ 0 & \frac{2a}{T} & 1-\frac{2b}{T} \end{pmatrix} \begin{pmatrix} \mu_0 \\ \mu_1 \\ \mu_2 \end{pmatrix}. \tag{10.7}$$

The solution to (10.7) is[10]

$$\begin{pmatrix} \mu_0 \\ \mu_1 \\ \mu_2 \end{pmatrix} = \begin{pmatrix} \frac{b^2}{b^2+2ab+2a^2} \\ \frac{2ab}{b^2+2ab+2a^2} \\ \frac{2a^2}{b^2+2ab+2a^2} \end{pmatrix}. \tag{10.8}$$

From this we can deduce a few things. First, the probability that a given agent is employed in any period is

$$\mu_{dyad} = \mu_2 + \frac{\mu_1}{2} = \frac{2a^2+ab}{b^2+2ab+2a^2}.$$

If we compare this to the same limit for (10.4) which is $\mu_{isolate} = \frac{a}{a+b}$, we see that it is larger. Indeed, simplifying the above expression leads to

$$\mu_{dyad} = \frac{a}{a+b-\frac{ba}{2a+b}} > \mu_{isolate} = \frac{a}{a+b}. \tag{10.9}$$

It is clear that this should hold, since having a neighbor increases the opportunities for hearing about employment.

[10] We can also obtain this by examining the limits in (10.6) directly.

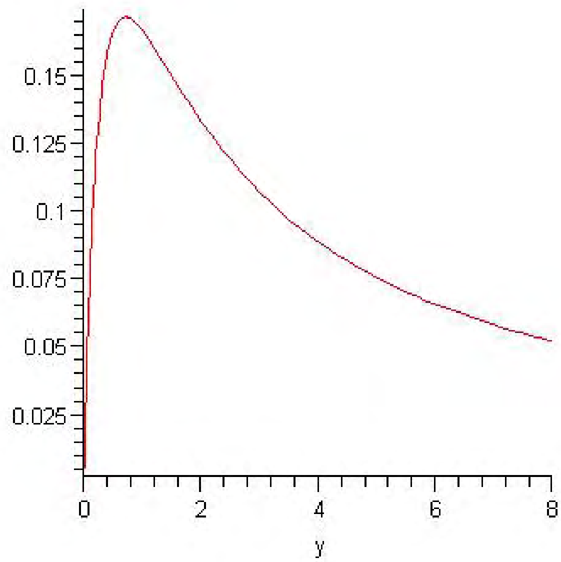


Figure 10.2.2. Correlation in Employment as a Function of $y = \frac{a}{b}$

We can also examine the correlation of employment across the two agents.

$$Corr_{dyad} = \frac{E[s_{1t}s_{2t}] - E[s_{1t}]E[s_{2t}]}{E[s_{it}^2] - E[s_{it}]^2} = \frac{\mu_2 - \mu_{dyad}^2}{\mu_{dyad} - \mu_{dyad}^2}.$$

Substituting from (10.8) and (10.9), the correlation is

$$Corr_{dyad} = \frac{ab}{3ab + 2a^2 + b^2} = \frac{1}{3 + 2\frac{a}{b} + \frac{b}{a}}. \tag{10.10}$$

This reaches a maximum when $\frac{a}{b}$ is $\frac{1}{\sqrt{2}}$, and is always positive. It is graphed in Figure 10.2.2 below as a function of $y = \frac{a}{b}$.

The correlation of two neighbors should clearly be positive, as one's probability of finding a job goes up when his or her neighbor is employed. It is less clear whether larger groups of agents should have positively correlated employment outcomes.

Complete Networks

Let us next examine settings where individuals live in cliques of n individuals who are all connected to each other. Here there is a full symmetry, so that we can keep track of the state simply in terms of how many agents are employed. Let μ_k be the steady-state probability that exactly k agents are employed in a given period.

Again, let us examine situations where the time periods become small, so the arrival rates are a/T and b/T for some large T. This means that the probability of having two or more events occur in a period (for instance two people hearing about a job) becomes infinitely less likely than having just one event occur, as the former is on the order of $1/T^2$ and the latter is on the order of $1/T$. To calculate what happens at the limit as T grows, we only need to keep track of transitions between neighboring states. This permits us to solve in closed form for the steady-state probability of any state.

PROPOSITION **10.2.1** *Consider a complete network of n agents, with arrival rate a/T and breakup rate b/T. As T grows, the steady state probability of having k agents employed converges to*

$$\mu_k = \frac{\frac{n!}{k!}\left(\frac{b}{na}\right)^{n-k}}{\sum_{j=0}^{n}\frac{n!}{j!}\left(\frac{b}{na}\right)^{n-j}} = \frac{1}{\sum_{j=0}^{n}\frac{k!}{j!}\left(\frac{b}{na}\right)^{k-j}}. \tag{10.11}$$

And, for $k' > k$,

$$\frac{\mu_{k'}}{\mu_k} = \left(\frac{na}{b}\right)^{k'-k}\frac{k!}{k'!} \tag{10.12}$$

The proof is straightforward, but helps in illustrating how to derive properties of such a Markov chain.

Proof of Proposition 10.2.1: In steady state, for $1 < k < n$, we can end up in a state with exactly k employed agents from three different states in the previous period. It could be that the previous period had $k-1$ agents employed, and then the probability of transitioning to having k employed agents is the probability that some agent heard about a job (as then it certainly reaches an unemployed agent in a completely connected clique), which is approximately na/T for large T. It could be that the previous period had $k+1$ agents employed and then an agent lost a job, which happens with probability approximately $(k+1)b/T$. It could also be that the previous period had k agents employed and nobody heard about a job or lost a job, which happens with probability $1 - na/T - kb/T$

Thus, for large T we approximate the steady-state probability of being in state k by

$$\mu_k = \mu_{k-1}n\frac{a}{T} + \mu_k(1 - n\frac{a}{T} - k\frac{b}{T}) + \mu_{k+1}(k+1)\frac{b}{T}. \tag{10.13}$$

The zero employment state satisfies

$$\mu_0 = \mu_0(1 - n\frac{a}{T}) + \mu_1\frac{b}{T},$$

or

$$\mu_0 = \frac{b}{na}\mu_1. \tag{10.14}$$

If we substitute (10.14) into (10.13) with $k = 1$ we can solve to find

$$\mu_1 = \frac{2b}{na}\mu_2. \tag{10.15}$$

Iterating, we find that for any $k < n$

$$\mu_k = \frac{(k+1)b}{na}\mu_{k+1}. \tag{10.16}$$

This implies that

$$\mu_k = \frac{n!}{k!}\left(\frac{b}{na}\right)^{n-k}\mu_n. \tag{10.17}$$

(10.12) then follows.

Noting that $\sum_{j=0}^{n}\mu_j = 1$, (10.17) implies that

$$\mu_n = \frac{1}{\sum_{j=0}^{n}\frac{n!}{j!}\left(\frac{b}{na}\right)^{n-j}}. \tag{10.18}$$

and more generally that

$$\mu_k = \frac{\frac{n!}{k!}\left(\frac{b}{na}\right)^{n-k}}{\sum_{j=0}^{n}\frac{n!}{j!}\left(\frac{b}{na}\right)^{n-j}} = \frac{1}{\sum_{j=0}^{n}\frac{k!}{j!}\left(\frac{b}{na}\right)^{k-j}}. \tag{10.19}$$

Thus, we have the claimed expression for the probabilities of the states. ∎

Having these expressions for the steady-state probabilities allows us to compute average employment rates as well as correlations, as in Table 10.1.

Although the correlation is decreasing between any two agents as we increase the network size, that does not mean that the network effect is decreasing. In fact, quite the contrary is true. The society swings more to situations where many agents are employed or many agents are unemployed at the same time. Although any two of them in a large society will have low correlations, there is still a large overall effect. One way of measuring the impact of the network effect is to examine the variance of the total employment under the steady-state distribution of the network and then normalize it

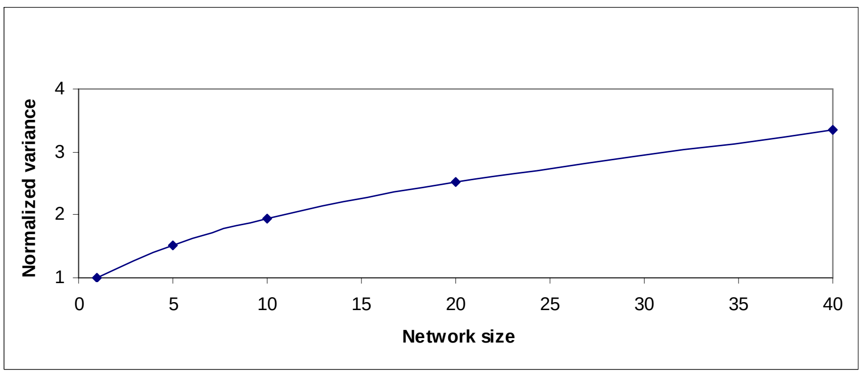


Figure 10.2.2. The Variance of Steady-State Total Employment Divided by the Variance of Total Employment if Each Agent were Independently Employed with the Same Average Probability, as a function of Network Size with $a = .5 = b$

Table 10.1: Average Employment and Correlation in Employment in Complete Networks

	Ratio of job arrival to breakup: a/b		
size n:	1/2	1	2
1	avg=.333, corr=—	avg=.500, corr=—	avg=.667, corr=—
2	avg=.400, corr=.167	avg=.600, corr=.167	avg=.769, corr=.133
4	avg=.452, corr=.135	avg=.689, corr=.139	avg=.851, corr=.099
8	avg=.485, corr=.099	avg=.764, corr=.111	avg=.910, corr=.067
16	avg=.498, corr=.061	avg=.825, corr=.087	avg=.948, corr=.043
32	avg=.500, corr=.032	avg=.871, corr=.066	avg=.972, corr=.025
64	avg=.500, corr=.016	avg=.907, corr=.049	avg=.985, corr=.014
128	avg=.500, corr=.008	avg=.933, corr=.036	avg=.992, corr=.007
256	avg=.500, corr=.004	avg=.952, corr=.026	avg=.996, corr=.004
512	avg=.500, corr=.002	avg=.966, corr=.019	avg=.998, corr=.002

to see how it compares to the variance of a society with the same average employment but where each agent's employment follows a binomial distribution, independent of the employment other agents. This is pictured in Figure 10.2.2.[11]

Figure 10.2.2 shows how the normalized variation in steady-state total employment increases with network size.

Other Networks and Correlation in Employment

When moving beyond simple network structures, solving for the steady-state employment rates and correlations in employment becomes difficult analytically, but is still possible numerically through simulations. To get a feeling for some comparative statics, Calvó-Armengol and Jackson [119] present a few examples. All of the following examples use an arrival rate of $a = 0.100$ and a breakup rate of $b = 0.015$. Based on a time period being a week, then an agent loses a job about once every 67 weeks, and hears about a job directly on average once every ten weeks. Figure 10.2.2 presents unemployment rates and the correlation in employment for four different four-person networks.

[11] I thank Toni Calvó-Armengol for suggesting this illustration.

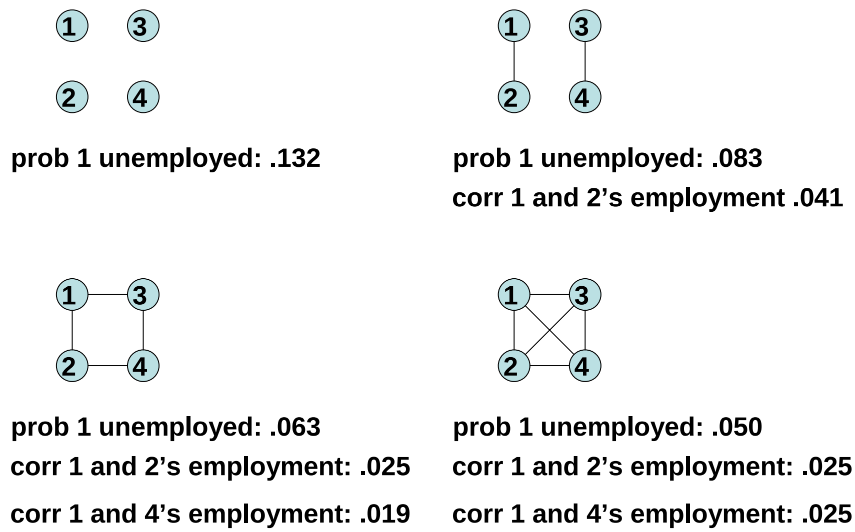


Figure 10.2.2. Calvó-Armengol and Jackson [119]: Unemployment Rates and Correlation in Employment with $a = .100$ and $b = .015$

As the network becomes more connected, the unemployment rate falls as the information about jobs has a lower probability of being lost. The correlation is higher for agents who are directly connected than for those who are indirectly connected. The correlation between the employment of any two agents falls as the number of ties that they have increases, as there are more sources of information affecting their employment.

Figure 10.2.2 shows that asymmetries in network position can lead to differences in steady-state employment rates even when the degree of agents is identical.

In Figure 10.2.2, the agents whose link forms a bridge (agents 1 and 6) have higher employment rates. The effect here is due to the fact that those two agents are more diversified in their social connections than the other agents: none of their neighbors are linked to each other. In contrast, each of the other agents has some clustering in their neighborhoods, and has higher correlation in their neighbors employment. The correlation in the employment of neighbors makes it more likely that an agent will end up hearing about either no jobs or multiple jobs at once, while an agent would rather instead have a higher probability of hearing about (at least) one job.

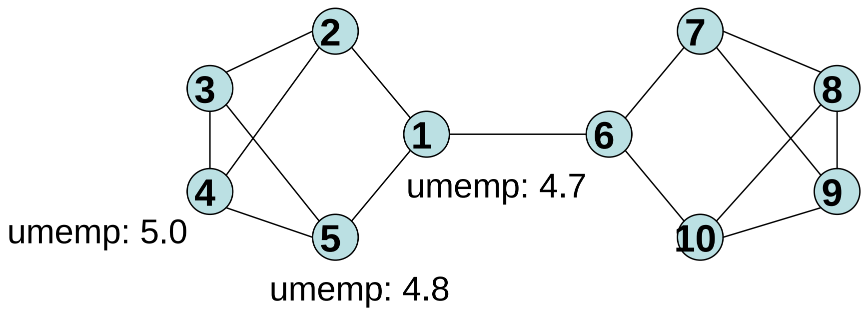


Figure 10.2.2. Calvó-Armengol and Jackson [119]: Unemployment Rates as a Function of Position in a Network with $a = .100$ and $b = .015$

The correlation observed in the above networks is not unique to these examples, but holds more generally. In fact, Calvó-Armengol and Jackson [119] show that as a and b each converge to 0, but a/b converges to some positive limit, the correlation in the employment of any two path-connected agents is positive. The interpretation of this limit is that the time between periods is shrinking.[12]

10.2.3 Duration Dependence

An important aspect of networked interaction models of this type is that they generate specific correlation patterns over time and can help us to understand time-series patterns of behavior that are observed in various data that have not been very well understood in the absence of the social setting.

A good example of this is what is called "duration dependence" in the labor economics literature. This refers to the fact that if we examine an unemployed worker

[12] In the short run, two indirectly connected agents might be competitors for a mutual neighbor's job information and so might have negatively correlated employment conditional on some states (see Exercise 10.7). Looking at this limit allows one to look more directly at longer-term effects.

and ask what the probability is that the worker will be employed in the next month, that probability goes down conditional on the worker having a longer history of unemployment, holding all else equal. This has been found in a variety of studies including Schweitzer and Smith [553], Heckman and Borjas [306], Flinn and Heckman [233], and Lynch [421]. As an illustration, Lynch [421] finds average probabilities of a typical worker finding employment on the order of 0.30 after one week of unemployment, 0.08 after eight weeks of unemployment, and 0.02 after a year of unemployment, after correcting for other observable characteristics like skill level, local employment rates, and so forth.

A standard explanation for duration dependence is that there must be some features of workers that we cannot observe in the data but that are observable to firms, and the workers who are unemployed for long time periods have unattractive features from firms' perspectives or other characteristics that make them less likely to become employed. However, the magnitude of the residual effects on employment (e.g., a 15-fold difference in the probability of becoming employed after one week of unemployment compared to one year above) even after including a wide variety of characteristics, has been a puzzle. As pointed out by Calvó-Armengol and Jackson [119], such a networked model of employment generates duration dependence as a general proposition. Let us illustrate this effect with some examples.

While the networks in Figure 10.2.3 are small, they show that understanding a worker's social context can account for some of the observed duration dependence. The idea behind a networked labor market exhibiting duration dependence is as follows. The longer that a worker has been unemployed, the greater the probability of the worker's neighbors being unemployed. This reflects both that the worker has not been able to pass the neighbors any job information, and also that the worker has not heard about a job from the neighbors and so it is less likely that the neighbors are employed. As more of a worker's neighbors are unemployed, it becomes less likely that the worker will hear about a job in the coming period. For instance, if a worker has been unemployed for only a week, it is still quite possible that many of the worker's neighbors are employed and so the worker will hear about a job shortly. However, if a worker has been unemployed for a year, then that suggests that many of the worker's neighbors are also unemployed, and makes it unlikely that the worker will will hear about a job from a neighbor. Thus, if an unobserved feature of a worker is the status of his or her neighbors, then could affect the employment probability of a worker. This presents a complementary explanation of duration dependence to the usual unobserved

Probability Employed Next Period
If unemployed for at least:

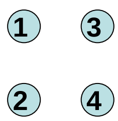
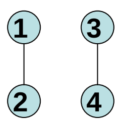
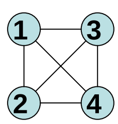
	1 period	2 periods	10 periods
	.099	.099	.099
	.176	.175	.170
	.305	.300	.278

Figure 10.2.3. Calvó-Armengol and Jackson [119]: Probability of Becoming Employed as a Function of Network and Previous Periods Unemployed, with $a = .100$ and $b = .015$

characteristics explanation.

This feature is not unique to networks in a labor market context, but also appears in other settings where behaviors across individuals are complementary. Observing the behavior of one individual tells us something about the likely behavior of the neighbors and vice versa. When we look at the time series of individual behavior in isolation, it will take on greater history dependence than one would otherwise expect.

10.2.4 Education and Drop-Out Decisions

Networked labor markets also provide interesting incentives for investment in education and human capital. This ties back to our discussion of strategic complementarities and graphical games from Chapter **??**.

Suppose that agents start out in a network g. Each agent has an idiosyncratic cost c_i of investing in education. If the agent invests in education then he or she becomes eligible for jobs, and otherwise simply gets a payoff of 0.[13] Let $x_i \in \{0, 1\}$ be 1 if agent i invests in education and 0 if not. If an agent is educated then the labor market described in the previous sections applies, and the agent can hear about jobs directly and also from an employed neighbor j who has also chosen to be educated.

Thus, we start with a network g and end up with a network $g(x)$, which is the subnetwork of g restricted to the nodes i such that $x_i = 1$. Based on $g(x)$, each agent will then have a long-run employment rate. We can then examine Nash equilibria of the game where the payoff to investing in education is the long run expected employment rate minus the cost, c_i. If this is greater than 0, then the agent invests.

Note that this is a graphical game of strategic complements. To get a feeling for the structure of the game, consider a dyad. Let us consider a situation where $a = b$ and look at the limiting process as in Proposition 10.2.1.

Figure 10.2.4 illustrates the complementarities well. The employment probability, and hence the payoff, of an agent goes up as he or she has more neighbors, since they pass job information. This leads to multiple equilibria and also to contagion effects in actions. To see the possibility of multiple equilibria explicitly, consider 4 agents and the payoffs pictured in Figure 10.2.4. If each agent has a cost c_i of .6, then it is an equilibrium for no agent to get an education, and it is also an equilibrium for all

[13]This is clearly a simplification. It could be that if one does not invest, then one gets information about unskilled jobs from neighbors who did not invest, while if one invests then one gets information about skilled jobs from neighbors who also invested. The important thing is that neighbors' decisions to invest affect one's own decision.

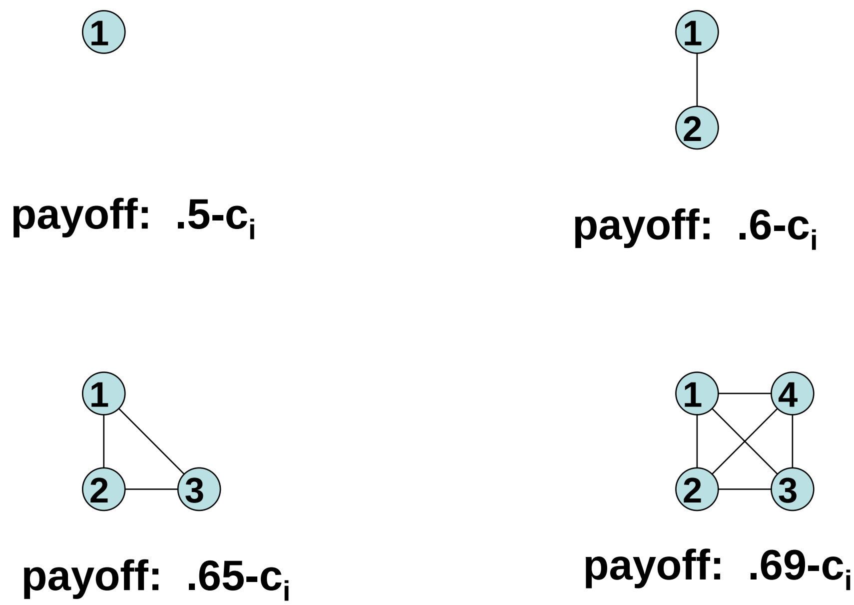


Figure 10.2.4. Expected Payoffs for Agents who Become Educated as a Function of the Job Network with $a = b$.

agents to get an education. There is a complementarity in their incentives. The better equilibrium for all involved is for all of them to get an education, but it is possible to end up at the other equilibrium.

We can also see contagion effects here. Suppose that the 4 agents have costs of education that are $c_1 = .51$, $c_2 = .61$, $c_3 = .66$ and $c_4 = .70$. Then we end up with a unique equilibrium such that no agents become educated. Agent 4 would not find it worthwhile no matter what the other agents do. This leaves at most three agents to become educated. Under that circumstance, agent 3 does not find it worthwhile to become educated, and so forth. So, Agent 4's decision has wide consequences. If we change costs so that $c_4 = .68$, so that we lower agent 4's cost just slightly, then suddenly it is an equilibrium for all four agents to become educated.[14]

This also provides some intuition for "poverty traps". The idea is that initial conditions can be very important, especially if there is even the slightest sequentiality in agents' decisions. For instance, if historically no agents have become educated, and then we ask whether some agent wants to become educated, he or she has to be willing to do

[14] It still remains an equilibrium for none of them to invest. One can also create examples where slight changes lead that equilibrium to disappear.

so without any neighbors having invested. This can lead whole groups to stay under-invested relative to what the best equilibrium would be. This complementarity can lead to dramatic differences in behavior between different groups of agents embedded in highly segregated networks; similar to what we saw in Section [**?**], so that some subgroup invests highly while another does not. When we bring homophily into the picture, then we can see how these ideas can help explain dramatic differences in pursuit of higher education across different ethnicities. If most of one's friends are of the same ethnicity and almost none go on to higher education then that will tend to be a best response, while if almost all go on to higher education then that will tend to be a best response. This has feedback effects and can exacerbate effects that arise in part from other socio-economic factors.

10.2.5 A Labor Market in a Homophilous Network

The model of networked labor markets of Calvó-Armengol and Jackson [119] discussed above only analyzes half of the market. That is, firms play no role in the analysis, as jobs simply appear at an exogenous rate.

A model of labor markets where firms play a richer role (and the structure of workers' networks plays a lesser role) was developed by Montgomery [455]. This model provides a reason for firms to use referrals as a means of hiring, as workers' social ties are homophilous in a sense associated with their productivity; it also provides different insights into wage dispersion than those we found in the Calvó-Armengol and Jackson [119], [120] model discussed above. Montgomery's [455] model is described as follows.

- The economy lasts two periods.
- There are N workers in each period and they live for only one period.
- Half of the workers produce no output and half produce one unit of output.
- Firms cannot observe the workers' types until output is delivered.
- Firms employ at most one worker in any period.
- A firm's profit is the output minus any wage paid.
- Wages are paid upon the hiring of the worker and cannot be contingent on the output.

- There is free entry into the market so that firms can enter the market in either period.

The potential workers are connected in a network which is formed as follows.

- Each worker from the first period knows one second-period worker with probability τ and does not know any second-period worker with probability $1 - \tau$.
- If a first-period worker knows a second-period worker, then it is a worker of the same "type" (productivity) with a probability $a > 1/2$.
- Each first-period worker who gets a link has that tie assigned to a second period worker by first choosing whether it will be a same or different type worker (with probability a and $1 - a$, respectively) and then choosing uniformly at random from workers of the selected type. This means that second period workers can have multiple ties.

The timing of decisions is as follows.

- Firms hire first-period workers at an (equilibrium) wage w_1.
- A firm that hired a first-period worker observes that worker's (and only that worker's) first-period output. If that firm desires, it may then make a "referral" wage offer to its worker's social tie (if the worker has a social tie).
- Second-period workers who receive offers from the firms of their first-period friends may accept one of those offers or decide to go on the second-period market.
- The second-period market is such that any second-period workers who have not accepted a job through the referral process are hired at an (equilibrium) wage of w_2.

The equilibrium notion that Montgomery employs is a variation on a competitive equilibrium, requiring that no firm want to enter or exit, that firms optimize given their information, and workers take the best offer they get. It also involves aspects of Nash equilibrium, since in offering a referral wage a firm is entering an auction against other potential employers who might also be making a referral offer to the same worker. The result that Montgomery proves is as follows.

Proposition **10.2.2** *[Montgomery [455]] A firm makes a referral offer if and only if it has a productive worker in the first period, and it then randomly picks a wage to offer from an interval with lower bound of w_2 and an upper bound below 1.*[15]

The idea behind the proposition is as follows. Let us work backwards from the second period. In the second period market for the workers who have not accepted an offer, the wage will be equal to the expected value of those workers, given their distribution of types in equilibrium. Firms all have the same information about those workers' values, and will not over pay, and cannot under pay given that new firms can enter the market. Given the claim in Proposition 10.2.2, we will eventually be able to conclude that the expected value of such a worker is below $1/2$, as those will be workers who have not received and accepted an offer, and so are conditionally more likely to either be connected to a low output first-period worker or to lack connections. Given that a second-period worker can get a wage of w_2 by waiting for the open market, any non-degenerate referral offer has to be at least w_2. The fact that the equilibrium involves a mixed strategy can be understood from the fact that offering a wage through a referral is like bidding in an auction with an unknown number of other bidders. The second-period worker could have other ties to first-period workers and thus could be receiving other wage offers. The worker's expected value will generally be above w_2, given that the worker is tied to a high-output first-period worker, and given that the wage w_2 will be below $1/2$. If referrals were all hired at a given fixed wage below the expected value of the worker, then by slightly raising the wage one would hire the second-period worker for certain. If the wage were at or above the expected value of the worker, then one could lower the offer and still win in situations where there turn out not to be any other bidders.

The important aspects of the equilibrium can be summarized as follows.

- Having more social ties leads to higher expected wages for second-period workers.

  This is true since each additional tie has some chance of being a high-output worker and thus resulting in an extra wage offer.

- A low-output second-period worker with social ties has a higher expected wage than a high-output second-period worker who has no social ties.

[15] In fact, one can reason that the upper bound on bids will be no more than a, which is the expected value of a second-period worker conditional on having a tie to a first-period worker. Conditional on the worker accepting the wage offer, it is less likely that the worker has ties to other high-output first period workers, and so the conditional expectation of the worker's value will generally be less than a.

This follows from the observation that the high-output worker without social ties has to go on the second-period market, while any worker (regardless of actual productivity) with social ties has some probability of being connected to a high-output first-period worker and getting a wage offer above the second-period open-market wage.

- There is a dispersion of wages in the second period.

  This follows from the randomization in referral wages and the fact that workers differ in the number of social connections that they have.

- Firms earn positive profits in the second period from using referrals.

  This happens since firms have a higher chance of finding high-output workers through referrals, and as a lower bound, there is at least some chance that they can hire the worker with a wage just above w_2 in cases where the worker ends up having no other social ties.[16]

While highly stylized, this model provides insights into several aspects of networks in labor markets: why referrals can be attractive for firms, why they can lead to dispersion in wages,[17] and how workers who are more connected can fare better.

10.2.6 Evidence and Effects of Networked Labor Markets

Topa [595], Conley and Topa [156], and Bayer, Ross, and Topa [**?**] fit models of social interactions and employment which have some features similar to that described in Section [**?**], where networked workers should exhibit correlated employment and wages. Topa [595] and Conley and Topa [156] examine census-tract data in Chicago, focusing on data from the 1980 and 1990 censuses. As a proxy for network neighborhood relationships, Topa uses geographic neighborhood relationships. He examines the correlation of unemployment across census tracts and finds statistically significant

[16] The full argument here is a bit tricky, as the conditional expectation of a worker's value depends on what wage is offered and accepted. Hiring the worker with a lower wage provides some indication that the worker received fewer other offers. But in equilibrium, the expected profit is the same at all wages that are offered, and the workers who do receive offers are biased towards being of higher output.

[17] See Arrow and Borzekowski [18] for another model of wage dispersion based on the number of ties and a calibration to wage data.

correlation patterns between adjacent census tracts. He also finds significantly positive correlation between tracts that are not immediately adjacent, but are still both adjacent to a common tract.

As it is possible that social connections are not simply related to geographic proximity, Conley and Topa also examine a series of other distance measures. In addition to census tract distance measures, they examine travel time distance, ethnic distance, occupational distance, and education measures, as well as some other socio-economic co-variates. However, it could be that these measures are related to other characteristics that relate to employment patterns, and so the similar outcomes in employment which seem to indicate the role of some social relationships along these dimensions might simply be related to the fact that being close on these measures is related to other characteristics which account for common employment outcomes. To deal with this issue, they perform different exercises. First, they examine how the raw unemployment rate correlations depend on some combinations of these proximity measures. When combining various measures, being close on the ethnic dimension seems to explain the majority of covariation in unemployment rates. In view of this, they examine the residual employment rates when adjusting for a number of observable tract level characteristics. So, these residuals are obtained by subtracting out the variation in employment that can be explained directly by tract level characteristics. They then examine how these residuals correlate with the distance measured in tracts or some other characteristic (ethnicity, occupation, etc.). When they do this, the correlation patterns across these measures largely disappear. This seems to indicate that it is not that people have similar outcomes because they are nearby and thus socially related; but instead it could be that nearby tracts are quite similar in their characteristics and hence their workers tend to be employed or unemployed at the same time for other reasons.

The Conley and Topa [156] study could cast significant doubt on social proximity being important in employment outcomes. However, not finding a relationship when looking at census tract data does not imply that social relationships do not affect employment. Social relationships are not so obviously related to census tracts, especially when aggregated. Here a study by Bayer, Ross and Topa [**?**] cuts one level deeper. They examine census data from Boston where they can pinpoint residence down to the block level. This allows them to examine whether people living on the same block have more correlated employment outcomes than people living on nearby blocks that have similar characteristics. First, they find that living on the same block compared to a nearby block with similar demographics significantly increases the probability that two individuals work together, and this effect is magnified when the individuals are of similar ages and backgrounds. Then, examining pairs of individuals who have a strong predicted referral effect, they find a substantial effect on employment and wages. They also examine other questions. For instance, they find that there is assortative matching along education, income and age, so that similarity along these dimensions with those in one's city block improves labor market outcomes significantly. The study is also careful to examine "reverse causation" explanations, where people end up on the same block because they are similarly employed.

There are other ways of studying social network effects on labor outcomes, such as examining immigrant populations and the social networks that they move into, or other exogenous factors that affect social networks. The difficulty is in finding extensive social network data together with rich measures of employment outcomes. Some examples of clever proxies for social networks appear in Munshi [470], Laschever [401], and Beaman [46]. Laschever [401] examines the formation of military units via the U.S. draft in World War I. He examines units that were formed at random, and then examines subsequent employment outcomes via the 1930 census. If the friendships formed within a given military unit did not matter, then there would not be any correlation among employment outcomes in the later period (after correcting for other factors). He finds statistically significant effects, so that a ten percent increase in the employment rate of a veteran's unit's unemployment rate decreases that veteran's employment rate by over three percentage points. Munshi examines Mexican immigrants to the U.S., using rainfall in Mexico to estimate the number of immigrants during various time periods[18] and then shows that having a larger number of immigrants arrive more than three years prior to one's own arrival leads to a significant increase in the probability of one's employment. Beaman [46] finds a more direct measure of the size of waves of specific immigrant groups, with a rich variation in sizes, countries of origin, and locations, as she examines the assigned relocation of political refugees into the U.S. The network size proxy is the number of refugees from the same country who did not have prior family members in the US and who are relocated to the same city. She finds several effects. First, the larger the number of political refugees relocated to a particular location at

[18] This might, at first, seem to be a strange method; but one wants something which influences immigration but is not correlated with employment possibilities within the U.S. This is something which can lead to emigration from Mexico, and yet is unlikely to be correlated with job opportunities in the U.S.

the same time or within a year of each other, the lower their average employment rate; which is consistent with the new arrivals competing for jobs and job information. More pointedly in terms of evidence of the effect of social networks, the larger the number of refugees who were relocated to a given area at least two years prior, the higher the employment rate and wages. For instance, a standard deviation increase in the number of refugees arriving two years prior leads to an increase in the probability of becoming employed by 4.6 percent along with a fifty cent increase in the average hourly wage, while a similar increase in the number of refugees arriving in the prior year leads to a decrease of 4.9 percent in the probability of becoming employed and a seventy cent decrease in the average hourly wages.

10.3 Models of Networked Markets

The above studies were tailored to labor markets. Beyond labor settings, it is important to have a more general understanding how the structure of the network of interactions affects the terms of trade.

10.3.1 Exchange Theory

An area of research known as "exchange theory" (see Cook and Whitmeyer [162] for an overview) concerns how the structure of relationships among agents affects "exchanges" between them. Such exchanges could be economic transactions of goods and services, the trading of favors, communication of information, or a variety of social interactions which convey direct benefits and costs to the involved agents. The term "exchange theory" has its origins in work by Homans [318], [**?**] on "social behavior as exchange," which initiated a theory of socially embedded behavior based on psychological reinforcement ideas applied to dyadic exchanges. This view was complemented by work by Thibaut and Kelley [594], and more direct connections to economic ideas of exchange were brought into the picture by Blau's [68] influential work. The ideas have been extensively developed and applied to a range of economic interactions that involve explicit relationships, such as decentralized markets, the formation of corporate boards, and international relations. Networks have played an increasing role in exchange theory, especially since the work of Emerson [209]. Emerson considered explicitly networked interactions to understand the power and dependencies that underly exchange. Critical to Emerson's theory is the idea that the exchange that occurs between two agents depends on their outside options and influences, and thus on their other relationships. Thus, one cannot examine an exchange between two individuals without understanding the influences of the network on their behavior.

To get a feeling for this theory, it is useful to discuss the work of Cook and Emerson [161] who laid out hypotheses about how power derives from social network structure. They also examined the role of equity considerations in exchanges and conducted some of the first experiments on this subject. Some of these ideas provide a nice background for the predictions and observations that we will see below when considering models of economic transactions in social networks.

Cook and Emerson [161] operationalize the idea of equity through a condition that two agents involved in an exchange should equilibrate their respective profits or net gains from a given transaction. They examine this in a context where agent 1 holds good X and agent 2 holds good Y, agent 1 has a higher per unit value for Y than X, and agent 2 has a higher value for X than Y. The idea is that if x units of good X are given from agent 1 to 2 in exchange for y units of good Y, and v_{iz} represents the marginal value to agent i for good z, then equity requires that

$$v_{1y}y - v_{1x}x = v_{2x}x - v_{2y}y.$$

Cook and Emerson define the power of 1 over 2 as "the potential of agent 1 to obtain favorable Y minus X outcomes at agent 2's expense."[19] They do not provide a formal recipe for how to evaluate this as a function of the network, but they do distinguish between how relationships affect each other. Consider agent 1 who has links to both agent 2 and agent 3 in a social network. They say that these relationships have a positive connection if a transaction across one link is contingent upon a transaction across the other, and have negative connection if a transaction on one link precludes a transaction on the other. To analyze how power depends on the network of relationships and potential exchanges, Cook and Emerson examine the networks in Figure 10.3.1.

Here, Cook and Emerson say that A has power over each B, but the B's have equal power relative to each other and the C's have equal power relative to each other. In terms of a specific measure of the power that A has over a B, Cook and Emerson reason based on the "comparison levels" that the agents will have. In particular, they argue that A's comparison level, in terms of the expected value of the transaction, should be 20 units. The idea is that A can trade with any of the B's. The B's that do not trade with A are balanced and so should end up splitting their 8 units equally for a value of

[19] This quote substitutes "agent 1" for "A" and "agent 2" for "B" in the original quote.

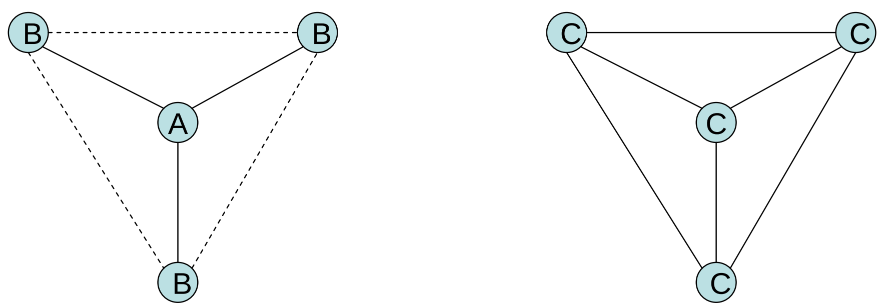


Figure 10.3.1. Exchange Networks from Cook and Emerson [161]. The solid lines indicate a potential transaction worth 24 units and a dashed line indicates a potential transaction worth 8 units.

4 units, which is their comparison level. Thus, if the B who transacts with A gets more than 4 units, then another B would have an incentive to offer to split 24 with A in a more favorable way.[20] Cook and Emerson then think of the comparison levels as some indication of power used, and then the excess of the power of an A with comparison level of 20 over a B with comparison level of 4 is 16 units. So here power is a measure of how much extra resources A can extract from the transaction with B compared to what B gets.

Cook and Emerson [161] ran a series of experiments on these networks, working with human subjects playing the roles of the above agents for cash earnings in proportion to the units of transaction. The subjects interacted through computers and were not aware of the identities of the other agents. The bargaining protocol was such that players could make direct offers of units (up to 8 between two B's and up to 24 between any other pair) to agents in their network, and if that agent agreed then they would complete that transaction, with the offering player keeping the total less the offer. Subjects played the game (in a fixed position) forty times. For the first twenty, players were only aware of the values of their potential transactions, and not of the transactions of the other players other than those that they were involved with. For the next twenty times, players observed each other's cumulative earnings. Cook and Emerson hypothesize that having knowledge of others' payoffs will lead to more equitable behavior and less exercise of power. The Cook and Emerson [161] data provides insight into several different things: first, whether or not there was an exercise of power so that A's earned more than B's; next, how this compared with the behavior of the evenly balanced C's; and, also how the knowledge of others' payoffs affected behavior. Furthermore, Cook and Emerson had an even pool of 56 male and 56 female subjects, and so compared behavior across genders. A brief summary of the results is that the A's did exercise power, beginning by earning between 2 and 4 units more than B's and tending up to 10 to 12 units more than B's just before the cumulative earnings were shown, but exercising less than the full 16 units of power that they had.[21] Among the C's, there were imbalances between the even partners, with the average imbalances beginning at 7 to 8 units (out of 24) and tending downwards to end up at 4 to 5 units by the end of the 40 periods. When looking across genders, Cook and

[20] This (20 to A and 4 to each B) turns out to be the unique core allocation in this problem, as defined via a standard cooperative game theory concept. See Section **??** and Exercise 12.3. This differs from the Shapley Value or Myerson Value allocations for this problem.

[21] These numbers actually look more consistent with the Myerson Value predictions than the core predictions for this network. See Exercise 12.3.

Emerson found significant differences between how males and females act in position A in the periods after the cumulative histories of payoffs are revealed. Males exhibit a short term lowering in their exercise of power, but eventually return back to about 12 units, while the females (significantly) lower their power usage to around 4 units by the end of the experiment. These experiments provide evidence that network position matters in bargaining and in helping to operationalize notions of "power." The data show that network-based bargaining power ends up being exercised even in situations where agents are not fully aware of the values of the possible transactions that can occur, but also that some equity concerns can mitigate the exercise of power. The differences between female and male behavior provide some interesting puzzles.

10.3.2 Bilateral Trading Models

The Cook and Emerson [161] exchange studies provide some insight into how bargaining power might be exercised, and suggest that agents are sensitive to the network of potential transactions that they are embedded in. To examine this in more detail, let us consider models networks of buyers and sellers. The following analyses use game theoretic models of the bargaining on networks to make predictions about which networks will form as agents try to maximize the value of their transactions.

A Networked Trading Model Based on Alternating Offers Bargaining

A natural starting point is a simple model of networks with bilateral bargaining that is due to Corominas-Bosch [167].

Each seller has a single object to sell which has no value to the seller. Buyers have a valuation of 1 for an object and do not care from whom they purchase it. If a buyer and seller exchange at a price p, then the buyer receives a payoff of $1-p$ and the seller a payoff of p. A link in the network represents the opportunity for a buyer and seller to bargain and potentially exchange a good.

Corominas-Bosch models the bargaining process explicitly via an alternating move game between the various buyers and sellers. That game leads to a particular solution. A link is necessary between a buyer and seller for a transaction to occur, but if an individual has several links then there are several possible trading patterns. Thus, the network structure essentially determines bargaining power of various buyers and sellers.

The game that Corominas-Bosch examines to predict the prices and transactions is described as follows. In the first period sellers simultaneously call out prices. A buyer can only select from the prices that she has heard called out by the sellers to whom she is linked. Buyers simultaneously respond by either choosing to accept some single price offer they received, or to reject all price offers they received. If there are several sellers who have called out the same price and/or several buyers who have accepted the same price, and there is any discretion under the given network connections as to which trades should occur, then there is a careful protocol for determining which trades occur (which is designed to maximize the number of eventual transactions). At the end of the period, trades are made and buyers and sellers who have traded are cleared from the market. In the next period the situation reverses and buyers call out prices. These are then either accepted or rejected by the sellers connected to them in the same way as described above. Each period the roles of proposer and responder switch, and this process repeats itself indefinitely until all remaining buyers and sellers are not linked to each other. Buyers and sellers are impatient and discount according to a common discount factor $0 < \delta < 1$. So a transaction at price p in period t is worth $\delta^t p$ to a seller and $\delta^t(1-p)$ to a buyer.

In an equilibrium with very patient agents (so that δ is close to 1), there are effectively three possible outcomes for any given agent: either he or she gets most of of the available gains from trade, or roughly half of the gains from trade, or a small portion of the available gains from trade. Which of these three cases ensues depends on that agent's position in the network. Some easy special cases are as follows. First, consider a seller linked to two buyers, who are only linked to that seller. Competition between the buyers to accept the price will lead to an equilibrium price of close to 1 if agents are sufficiently patient. So the payoff to the seller in such a network will be close to 1, while the payoff to the buyers will be close 0. This is reversed for a single buyer linked to two sellers.

More generally, which side of the market outnumbers the other is a bit tricky to determine as it depends on the overall link structure, which can be much more complicated than that described above. Quite cleverly, Corominas-Bosch describes an algorithm which has some roots in Hall's Theorem (recall Theorem **??**) and subdivides any network into three types of sub-networks: those where a set of sellers are collectively linked to a larger set of buyers, sellers get payoffs of close to 1, and buyers get payoffs of close to 0; those where the collective set of sellers is linked to a same-sized collective set of buyers and each get payoff of around 1/2; and those where sellers outnumber buyers, sellers get payoffs close to 0, and buyers get payoffs close to 1. The limiting payoffs, as the discount factor approaches 1, are found via the following algorithm.

- [1a.] Identify groups of two or more sellers who are all linked only to the same buyer. Regardless of that buyer's other connections, take that set of sellers and buyer out, and that buyer gets a payoff of 1 and the sellers all get payoffs of 0.
- [1b.] On the remaining network, repeat this process but with the role of buyers and sellers reversed.
- [k] Proceed, inductively in k, each time to identify subsets of at least k sellers who are collectively linked to some set of fewer than k buyers, or some collection of at least k buyers are collectively linked to some set of fewer than k sellers.
- [Stop] When all such subgraphs are removed, the buyers and sellers in the remaining network are such that every subset of sellers is linked to at least as many buyers and vice versa, and the buyers and sellers in that subnetwork earn payoffs of 1/2.

The limiting payoffs found by this algorithm are illustrated in Figure 10.3.2 .

To see how the algorithm works on the last network, consider Figure 10.3.2.

First, step 1a is concluded with no sets identified. Next, at step 1b, the set of two buyers linked to just one seller are eliminated. The remaining two buyers and two sellers are each linked to the same number on the other side and so the algorithm concludes by stopping with the remaining subset of 4 agents each getting a payoff of 1/2.

The intuition behind why the algorithm identifies the unique equilibrium outcome is as follows. If there are two or more sellers who are linked to just one buyer, they will compete in bargaining and the buyer can obtain a price of 0 (in the limit with high patience). This then implies that any other sellers linked to that buyer cannot expect that buyer to bid for their goods. At a later step, when we find three sellers linked to just two buyers, then it must be that each of the buyers is linked to at least two of the sellers (as otherwise two of the sellers would have been linked to just one buyer and removed at an earlier step). In this case, it is sort of a game of "musical chairs" among the sellers. At most two can sell an object, and so sellers who are quoting the highest price in some round have an incentive to cut their price slightly to avoid being left with an object; if it is the buyers who are quoting prices then no seller wants to be the only one not accepting as he or she would be left without selling an object. Again,

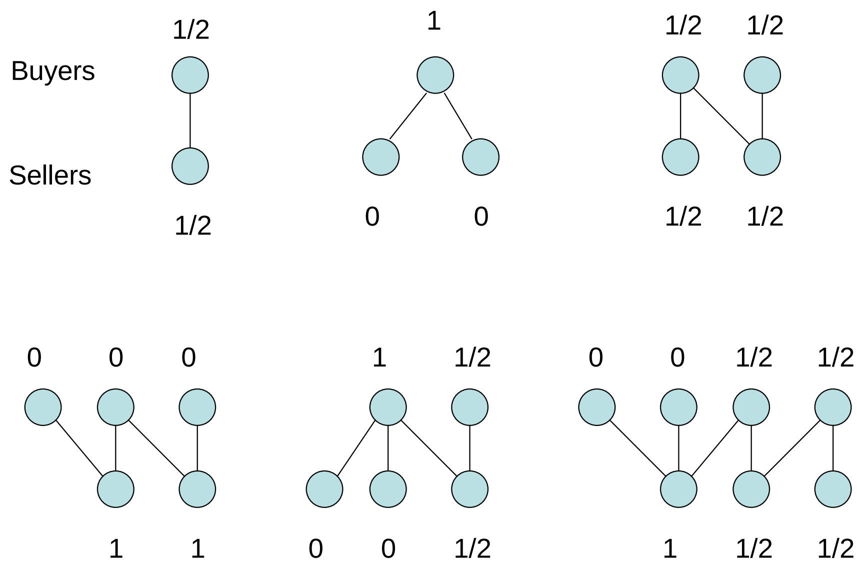


Figure 10.3.2. Limit Payoffs in the Corominas-Bosch [167] Model for Selected Networks

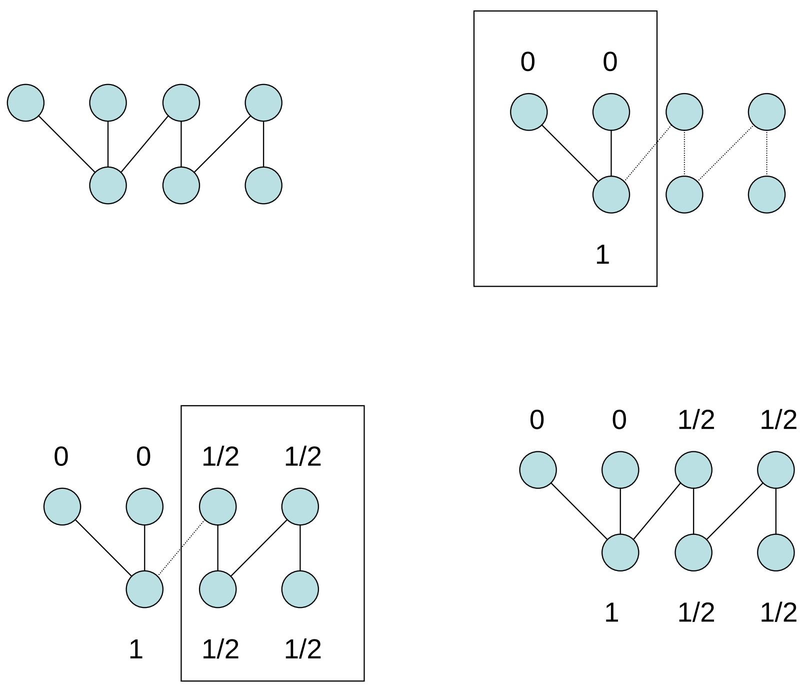


Figure 10.3.2. An Illustration of the Algorithm in the Corominas-Bosch [167] Model.

this leads to an unraveling of the price and so the buyers obtain all of the gains from trade.

For quite involved networks, the logic of this derivation pushes quite heavily on induction. However, the process itself naturally involves induction just as the algorithm does. That is, with some patience, several sellers linked to just one buyer will tend to compete away their surplus. As such transactions occur, the more complicated games of "musical chairs" begin to play out. A series of experiments by Charness, Corominas-Bosch, and Frechette [135] examine the extent to which human subjects play as predicted in such games. They examine a game of bargaining on a fixed network that proceeds just as the Corominas-Bosch model except that there are only a finite (but uncertain) number of rounds. While the payoffs in those experiments rarely reach the extremes that are predicted under the limit of full patience and potentially infinitely many periods of bargaining, the payoffs do share patterns that are similar to those predictions. Figure 10.3.2 pictures the predicted payoffs for one of the networks that was tested. This figure reports the percentage of potential pies to be split that each role received (so the total sums to 3 as there were three possible transactions in total), and also provides the benchmark prediction for the same network in the case with infinite patience and repetition.

Critical aspects of limiting payoffs under the bargaining can be summarized as follows.

(i) if a buyer gets a payoff of 1, then some seller linked to that buyer must get a payoff of 0, and similarly if the roles are reversed;

(ii) a buyer and seller who are only linked to each other get payoffs of 1/2; and

(iii) the subnetwork when restricting to the set of agents who get payoffs of 1/2 is such that any subgroup of k buyers in the subnetwork is linked with at least k distinct sellers in the subnetwork and vice versa for any k.

These observations about the payoffs, coupled with a cost per link, lead to the following sharp predictions concerning network formation using pairwise stability (see Section **??**).

PROPOSITION **10.3.1** *Consider a version of the Corominas-Bosch model where agents get the limiting payoffs as described just above. If the cost of a link for each agent*

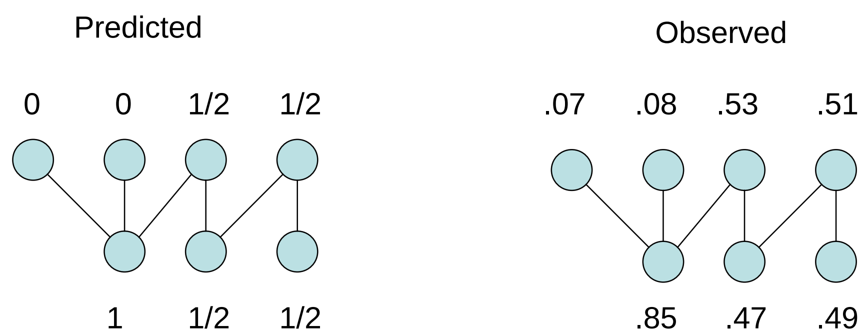


Figure 10.3.2. Predicted and Observed Payoffs from Charness, Corominas-Bosch, and Frechette [135].

involved lies strictly between 0 and 1/2, then the pairwise stable networks coincide with the set of efficient networks.[22]

The proof of Proposition is straightforward so I sketch it here and the details are left for Exercise 10.1. An individual getting a payoff of 0 cannot have any links, as by severing a link he or she could save the link cost and not lose any benefit. Thus, all individuals who have links must get payoffs of 1/2. Then, one can show that if there are extra links in such a network (relative to the efficient network, which consists of a maximal number of disjoint linked pairs) that some could be severed without changing the bargaining payoffs, thus saving link costs. This builds on the fact that for payoffs to be 1/2, then it must be that buyers and sellers are evenly balanced. It cannot be that there is a buyer and seller who each have no links, as by linking they could both be made better off. So it must be that the network consists of pairs and that the maximum number of potential pairs forms.

[22] This contrasts with Corominas-Bosch's [167] analysis, which considers a formation process where no cost is saved by severing a link. That can lead to players having links even when they know that they receive a 0 payoff from the bargaining and trade.

The conclusion of an equitable split of the value in each transaction contrasts with the conclusion from a supply and demand model. If there are more buyers than sellers, then in a competitive model would have the sellers collect all of the surplus. What is it that accounts for the difference? Here there is a cost to connecting, and connecting occurs *before* the bargaining. Buyers and sellers only enter if they expect that they will have a positive payoff. The model is extreme, so that slight imbalances in the network of matchings of buyers to sellers leads to extreme outcomes and zero payoffs to some agents. Thus, the equilibrium entry into the market results in a balanced set of buyers and sellers. The limited set of outcomes, so that transactions are either even splits or completely favor one side of the exchange, is critical to the result. As we shall now see, if we enrich the set of possible transaction outcomes, we will see richer networks emerge.

A Networked Trading Model Based on Auctions

While the model above provides a benchmark, the conclusion that agents will form an efficient network relies on the complete information setting and the fact that buyers are identical as are sellers. In situations where there can be some heterogeneity and uncertainty in valuations, there can be benefits from having a more connected network so that the set of potential transactions is larger. This leads to a more complicated analysis and can lead to inefficient networks being stable. To see this, let us consider a simple example, based on a model of Kranton and Minehart [390]. It is similar to the Corominas-Bosch model described above except that the valuations of the buyers for an object are random and the determination of prices is made through an auction.[23]

The potential sources of inefficiencies can be seen from a situation with one seller and two buyers; and so I focus on that setting, and refer the reader to Kranton and Minehart [390] for a fuller analysis. The buyers each have a valuation for the good which is uniformly and independently distributed on $[0, 1]$. The good is sold via a second-price auction.[24] This is an auction where the highest bidder obtains the object and pays the highest bid among the bidders who are not getting the object (with ties in

[23] In many applications one would also see random and heterogeneous valuations for the sellers, but the main ideas can be seen without introducing such complications.

[24] Any of a variety of auctions will have the same property in this example, as there is an equivalence between any two different auctions and corresponding equilibria, provided that the equilibrium lead to the same allocation of the good as a function of valuations are are such that buyers with 0 values pay 0 if they "win."; The equivalence is that each buyer has the same expected payment in the two auctions, and the seller expects the same revenue.

the highest bid broken uniformly at random). If only one buyer links to the seller, then he or she gets the object for a price of 0. If both buyers are linked to the seller, then it is a dominant strategy for each buyer to bid his or her value, and the corresponding revenue to the seller is the minimum value of the two buyers.

In this model the potential value of a transaction is random, depending on the realized valuations of the buyers. The auction on the network with just one link has an expected value of a transaction of 1/2, and an equilibrium price of 0 so that the full expected gains from trade go to the buyer. In the auction with two links, the ex ante expected payoff to each buyer (before he or she sees his or her value for the object) is $\frac{1}{6}$. Each buyer has a $\frac{1}{2}$ chance of having the high value, an the expected valuation of the highest bidder out of 2 draws from a uniform distribution on [0,1] is $\frac{2}{3}$, and the expected price is the expected second highest valuation, which is $\frac{1}{3}$. So, the ex ante total gains from trade in the two-link network is 2/3, with 1/3 going to the seller and 1/6 to each buyer.

Now consider a situation where there is a cost of links of c to each individual. Efficient network structures are an empty network if $c \geq 1/4$, a one-link network if $1/4 \geq c \geq 1/12$, and a two-link network if $1/12 \geq c$.

Let us consider whether or not the pairwise stable networks are efficient. Given the seller's payoffs as a function of the network, the seller will only link to both buyers or else to neither. Thus, if link costs lie between 1/12 and 1/4, then the efficient network will not be pairwise stable. If the cost is above 1/6 and below 1/4, then the only pairwise stable network is empty as buyers do not expect a high enough payoff to maintain a link in a two-link network and the seller will not maintain a link in a one-link network. If the cost is between 1/12 and 1/6, then the two-link network will be pairwise stable, but it is over-connected relative to the efficient network. Thus, an efficient network is only pairwise stable in the cases where costs are below 1/12 or above 1/4.[25]

To see the intuition for the inefficiency in this setting note that the increase in expected price to the seller from adding a link comes from two sources. One is the expected increase in willingness to pay of the winning bidder, since the sale is to the highest valuation out of a set of independent draws from the same distribution, and

[25] This conclusion contrasts with that in Kranton and Minehart [390]. However, they analyze a case where link costs are 0 for sellers and positive for buyers. If sellers bear no costs of links, then the efficient networks are pairwise stable. Kranton and Minehart do discuss the fact that costly investment by the seller can lead to inefficiency.

we get one more draw when a link is added. This increase is of social value, as it means that the good is going to someone who values it more. The other source of price increase to the seller from adding a link comes from the increased competition among the bidders in the auction. This source of price increase is not of social value since it only increases the proportion of value which is transferred to the seller.

While the pairwise stable networks in this example are not efficient (or even constrained efficient), they are Pareto efficient. This is not true with more sellers as shown in Exercise 10.3, which shows pairwise stable networks that are Pareto inefficient.

10.3.3 Price Dispersion on Networks

Beyond the strategic formation of networks in exchange settings, there are also studies which examine how the terms of trade depend on network structure, when networks are (exogenously) generated via random graph models. For instance, Kakade et al [357] (as well as Kakade, Kearns and Ortiz [356]) examine a model of exchange on random graph-generated networks.

Buyers have cash endowments and a constant marginal value for a consumption good. Sellers have unit endowments of the consumption good (which they do not value) and desire cash. Buyers buy from the least expensive seller(s) with whom they are connected until they have exhausted their cash budgets. Prices are seller-specific and determined to clear markets. An equilibrium is a set of prices and transactions such that the market clears. In this setting, market clearing implies that each buyer who is connected to at least one seller exhausts his or her budget and each seller that is connected to at least one buyer sells all of his or her endowment.

The simplest version of this model is such that all buyers have the same marginal valuation and endowments, say each normalized to 1. In that case, there is an equilibrium of the following form.

- A buyer buying from multiple sellers sees the same price from each seller.
- The price of a given seller can be found by computing, for each buyer, the fraction of the buyer's total purchases that come from that seller, and then summing across buyers.
- The price of a given seller is no higher than the seller's degree and no lower than the minimum degree of the buyers connected to the seller.

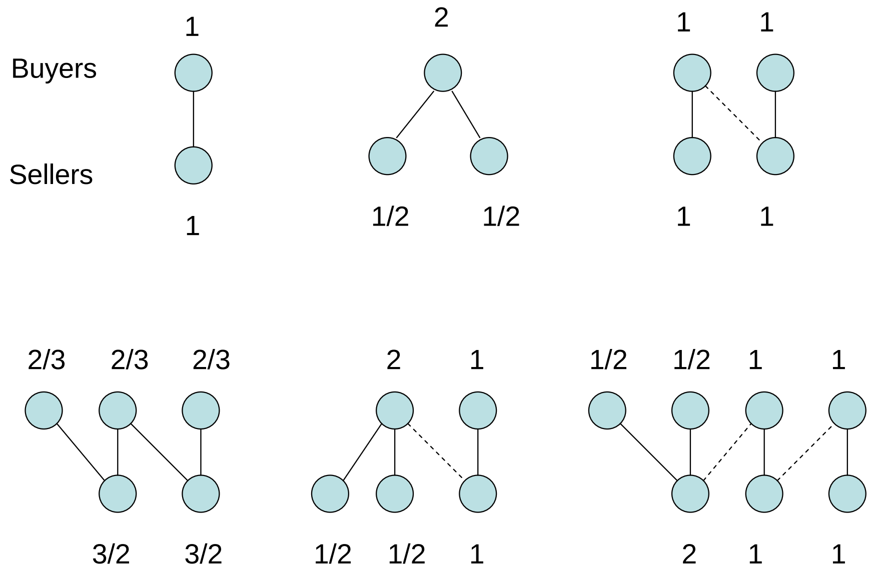


Figure 10.3.3. Equilibrium Prices and trades for Some Network Configurations in the Competitive Model of Kakade et al [357]. The number under each seller is the price that the seller charges and also the total cash that the seller ends up with. The number above each buyer is the amount of the consumption good that the buyer purchases. The solid links involve trades, whereas there are no trades along the dashed links.

The outcomes for a few networks are pictured in Figure 10.3.3.

The pattern of trades in Figure 10.3.3 turns out to be very similar to that in the model of Corominas-Bosch and Figure 10.3.2. There is a richer variation in prices in the competitive model, but the basics of which buyers and sellers do well and which do poorly are qualitatively similar. The richer variation in prices reflects that buyers and sellers are assumed not exercise any monopoly power they might have; instead they adjust prices to clear markets. This also results in differences if one examines the networks that are pairwise stable, as shown in Exercise 10.5. In this competitive model, there are still gains from trade to be earned by both sides of the market, even when there are large imbalances among buyers and sellers. This leads too many agents to connect to each other relative to the efficient network, which involves pairing agents.[26]

Figure 10.3.3 already demonstrates that the configuration of prices that emerges as a function of the network is very network specific, and so deriving general conclusions for complex networks is difficult. There are simple observations, such as the fact that adding (costless) connections will weakly benefit the agents involved with that connection, but there is no general relationship between degree and welfare, as terms of trade depend on the full network configuration. Even though the model is difficult to solve analytically, Kakade et al [357] show that if links are formed uniformly at random, and the probability of forming a link is high enough and the number of agents grows, then there is no limiting price dispersion. In the case of a network formed via preferential attachment, there is a greater asymmetry in the degrees of nodes and such networks can maintain some price dispersion.[27]

10.3.4 Collaboration Networks Among Firms

Most of the discussion in this chapter has been either about labor markets and information transmission or about exchange networks, where trades occur between linked agents. There are also a number of other ways in which networks play a role in markets. For example, firms collaborate in research and development, firms merge, firms produce joint products and ventures, firms contract on specific supply relationships, and so forth.[28]

[26] This reflects the linear utility functions. With concavities, more interesting architectures emerge as being efficient.

[27] For more on how network shape affects trading behavior, see Judd and Kearns [351] for a set of experiments of networked trade.

[28] See Bloch [70] for an overview of some related literature.

To get some feeling for how various relationships among firms might affect how they act in the market, let us examine an example due to Goyal and Joshi [**?**]. They examine a setting where a link between two firms lowers their respective costs of production. Since firms eventually compete in the market, the costs of production affect the overall market outcome and profits.

In particular, each firm produces identical goods. If firm i produces q_i units of the good and the network structure is g, then firm i's cost is

$$q_i \left(a - bd_i(g)\right). \tag{10.20}$$

where $a > (n-1)b > 0$ and n is the number of firms, so that costs are always positive. So, a firm's marginal cost of production is decreasing in the number of collaborative links it forms with other firms, which in this model is its degree.

The profits to a firm then depend on how much each firm produces and what the resulting price in the market is. To model this, let us consider the textbook-case of "Cournot competition," which works as follows. The market price is described by an (inverse) demand function, such as

$$p = \alpha - \sum_j q_j,$$

where $\alpha > 0$ is scalar. Thus, the price decreases as firms produce more.

So firm i's profits are

$$q_i p - q_i \left(a - bd_i(g)\right) = q_i \left(\alpha - \sum_j q_j - a + bd_i(g)\right).$$

If q_i maximizes this, then it must be that the derivative with respect to q_i is 0, or

$$-q_i + \alpha - \sum_j q_j - a + bd_i(g) = 0. \tag{10.21}$$

Solving this simultaneously across i leads to[29]

$$q_i = \frac{\alpha - a + nbd_i(g) - b\sum_{j \neq i} d_j(g)}{n+1}.$$

A firm's profits are $(p - c_i)q_i$ (where $c_i = a - bd_i(g)$ is the marginal cost of firm i), and so noting from 10.21 that $q_i = p - c_i$ it follows that each firm's Cournot equilibrium profits are q_i^2, where q_i is given above.

[29] A sufficient condition for all quantities to be positive is that α is large, or that $\alpha - a - (n-1)(n-2)b > 0$.

This makes it easy to deduce pairwise stable networks. The profits of a firm are increasing in the equilibrium q_i, and the network enters q_i in proportion to $nd_i(g) - \sum_{j\neq i} d_j(g)$. Thus, firm i gains $n-1$ (noting that d_j increases for some j) with each link that it adds. If the link costs are the same across links, then the set of pairwise stable networks falls in one of two extremes: it is either the complete network or the empty network, depending on the cost of a link. In the case where the link costs are heterogeneous across firms, then the pairwise stable network would be a complete network among the subset of firms whose costs are lower than $n-1$ (presuming no firm has costs exactly at $n-1$). More interesting configurations would require some nonlinearities in link costs.

In terms of efficiency, there are a variety of different benchmarks to consider. In particular, it matters whether one just considers firm profits or also considers the welfare of the consumers buying the firms' products; and it also matters whether competition is restricted to Cournot equilibrium or can take some other form (see Exercise 10.6). From the industry profit standpoint, in most cases the highest industry profits would actually involve a star network where the center firm enjoys a low cost and also sees higher costs, and thus prices, from its competitors. This would be the efficient network structure if link costs are small and only firms' profits are considered. If we also include the consumers' welfare, and link costs are small enough, then it is best to see a low price and high production, which emerges when the complete network forms.[30]

10.4 Some Concluding Remarks

The above-described models, empirical analyses, and experiments on networked markets offer some insights that are fairly general. In the context of labor markets, we saw how links lead to correlated outcomes across neighbors, at a given time and across time. Such linked outcomes lead to complementarities in incentives to make investments in things like education. These patterns are not unique to labor markets, but also hold for some other economic transactions and behaviors where there are complementarities between the states of neighbors. In the context of exchange, we saw that the relation

[30] This conclusion, however, depends on how firms compete. If they compete via prices, then it can be enough to have two low-cost firms to push prices down (again, see Exercise 10.6). In such a case, the efficient network is what Goyal and Joshi [?] call "interlocking stars," such that two firms, i and j, are each linked to every other firm, and firms other than i and j are only linked to i and j.

between network structure and the terms of trade can be complex, but that an important determinant of favorable terms of trade is having connections to other agents whose other trading options are somewhat limited. So it is not simply direct connections that are important for terms of trade, but having connections to others who have more limited connections. This contrasts with other sorts of applications, such as information networks, where having well connected neighbors is desired. Moreover, high connection to low-degree neighbors leads to good outcomes for an agent not only in the exchange setting, but also in settings of collaboration among firms, and other settings where there is some sort of competition among agents.

While we have seen that networked labor markets are pervasive, and seen that there are some general insights to be gained from observing and modeling networked markets, there is much still left to be learned in this extensive area of application where networks play such a central and critical role.[31]

10.5 Exercises

Exercise **10.1** *Proof of Proposition 10.3.1.*

Provide a full proof of Proposition 10.3.1.

Exercise **10.2** *Payoffs in the Corominas-Bosch [167] Model.*

Find the predicted payoffs in the networks in Figure 10.5 under the Corominas-Bosch [167] model.

Exercise **10.3** *Pareto Inefficient Pairwise Stable Networks with Trade by Auction*

The following example from Jackson [329] shows that it is possible for (non-empty) pairwise stable networks in the Kranton-Minehart model to be Pareto inefficient. For this we need more than one seller. In that case, the auction works as follows. Prices rise simultaneously across all sellers. Buyers drop out when the price exceeds their valuations. As buyers drop out, there emerge sets of sellers such that the set of buyers

[31] This is not to say that this chapter provides an exhaustive survey of the literature, as there are a number of areas that I did not cover, including international trade (e.g., Furusawa and Konishi [246]), financial contagion and the role of networks in financial markets (see Allen and Babus [12]), collusion and market-sharing agreements among firms (e.g., Belleflamme and Bloch [48]), and models of competition among buyers and sellers with added heterogeneity (e.g., Wang and Watts [612]).

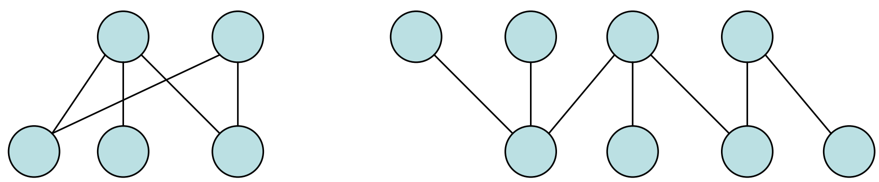


Figure 10.5. Find the payoffs under the Corominas-Bosch [167] Model

still linked to those sellers is no larger than the set of sellers. Those sellers transact with the buyers still linked to them. (The exact matching of who trades with whom given the link pattern is done carefully to maximize the number of transactions.) Those sellers and buyers are cleared from the market, and the prices continue to rise among remaining sellers, and the process repeats itself.

Consider a population with 2 sellers and 4 buyers. Let individuals 1 and 2 be the sellers and 3,4,5, and 6 be the buyers. Let the cost of a link to a seller be $c_s = \frac{5}{60}$ and the cost of a link to a buyer be $c_b = \frac{1}{60}$.

Expected payoffs to buyers in sellers in some of the relevant network configurations are:

$g^a = \{13\}$: $u_1(g^a) = -\frac{5}{60}$ and $u_1(g^a) = \frac{29}{60}$.

$g^b = \{13, 14\}$: $u_1(g^b) = \frac{10}{60}$ and $u_3 = u_4(g^b) = \frac{9}{60}$.

$g^c = \{13, 14, 15\}$: $u_1(g^c) = \frac{15}{60}$ and $u_3 = u_4 = u_5(g^c) = \frac{4}{60}$.

$g^d = \{13, 14, 15, 16\}$: $u_1(g^d) = \frac{16}{60}$ and $u_3 = u_4 = u_5(g^d) = \frac{2}{60}$.

$g^e = \{13, 14, 25, 26\}$: $u_1 = u_2(g^e) = \frac{10}{60}$ and $u_3 = u_4 = u_5 = u_6(g^e) = \frac{9}{60}$.

$g^f = \{13, 14, 15, 25, 26\}$: $u_1(g^f) = \frac{13}{60}$, $u_2(g^f) = \frac{8}{60}$, and $u_3 = u_4(g^f) = \frac{6}{60}$, while $u_5(g^f) = \frac{10}{60}$ and $u_6(g^f) = \frac{11}{60}$.

$g^g = \{13, 14, 15, 24, 25, 26\}$: $u_1 = u_2(g^g) = \frac{9}{60}$ and $u_3 = u_4 = u_5 = u_6(g^g) = \frac{8}{60}$.

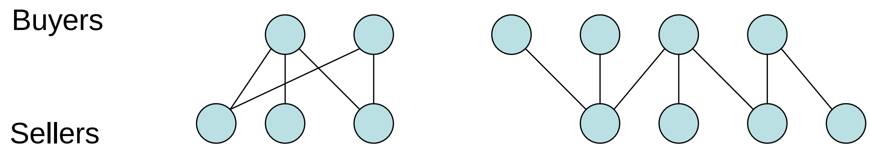


Figure 10.5. Find the Equilibrium Prices and Trades for These Network Configurations in the Competitive Model of Kakade et al [357].

Show that out of these networks and the empty network, the pairwise stable networks are the empty network, g^d, and g^g. Show that that none of the pairwise stable networks is efficient. Show that g^g is not Pareto efficient.

Exercise **10.4** *Competitive Trades on a Network.*

Find the equilibrium prices and allocations for the networks in Figure 10.5 under the competitive model of Kakade et al [357] in Section 10.3.3.

Exercise **10.5** *Pairwise Stable Networks with Competitive Trades.*

Consider the competitive trading model of Kakade et al [357] as discussed in Section 10.3.3, when there are N_B buyers and N_S sellers. Each seller has a single unit for sale, and buyers have valuations of 1 and endowments of 1. Show that if N_B is an integer multiple of N_S then there is a pairwise stable network where each buyer links to one seller and each seller links to N_B/N_S sellers, and the reverse is true if N_S is an integer multiple of N_B. Assume that there is a cost $c > 0$ per link, which is less than the inverse of the integer multiple and is paid out of the final goods (so in cash for sellers and in the consumption good for buyers).

Is such a network structure Pareto efficient when $N_B > N_S$? Show that no efficient network is pairwise stable when $N_B > N_S$.

[

Exercise **10.6** *Collaboration Networks Among Firms with Bertrand Competition.*

Consider a collaboration network among firms as in Section 10.3.4 where production costs are given by (10.20). Let firms compete for the sale of their products via pure "Bertrand competition," where each firm simultaneously quotes a price and the firms charging the lowest price evenly split the market. Consider a situation where the total amount purchased is Q independently of the price.[32] If there are at least two firms who have the lowest cost level (highest degree), then there is a Nash equilibrium of the Bertrand game where all firms quote prices equal to their per unit costs, and do not make any profits. If there is a single firm with the lowest cost, then there is an equilibrium where that firm charges the second lowest per unit cost and sells Q.[33]

If there is a positive cost to a link, show that the empty network is the only pairwise stable network.

Exercise **10.7** *Negative Correlation in Short-Run Labor Networks.*

Consider a triad (three-agent completely connected network) in model of Section [**?**]. Suppose that at the end of one period agents 1 and 2 are unemployed and agent 3 is employed. Show that the next-period employment states of agents 1 and 2 are negatively correlated.

Exercise **10.8** *Association**

Consider a network where each agent has an employment state $s_i \in \{0, 1\}$. Suppose that state of i in period t is a function of the previous vector of all agents' states and an undirected network g. In particular, the probability that $s_{it} = 1$ depends on the states $s_{j,t-1}$ for $j \in N_i(g) \cup \{i\}$, it lies strictly between 0 and 1 and is increasing in $s_{j,t-1}$ for each $j \in N_i(g) \cup \{i\}$ holding the other agents' states constant, and it is independent

[32] The shape of the demand curve is not important for this exercise.

[33] The precise equilibrium with asymmetric costs in Bertrand competition involves mixed strategies where the firm with the lowest cost charges the second lowest cost, and then the firm(s) with the second lowest cost employ a mixed strategy that has some atomless weight on prices just above the second lowest cost. See Blume [**?**] for details.

of the states $s_{j,t-1}$ for each $j \notin N_i(g) \cup \{i\}$.[34] Show that the steady-state distribution of the vector $(s_1, \ldots, s_n)$ exhibits strong association (see Section 4.5.7) relative to the components of the network.

[34]The same conclusion holds under weaker conclusions, but involves substantial complications in the proof. See Calvó-Armengol and Jackson [119] for details.