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Statistical analysis of financial networks

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Abstract

Massive datasets arise in a broad spectrum of scientific, engineering and commercial applications. In many practically important cases, a massive dataset can be represented as a very large graph with certain attributes associated with its vertices and edges. Studying the structure of this graph is essential for understanding the structural properties of the application it represents. Well-known examples of applying this approach are the Internet graph, the Web graph, and the Call graph. It turns out that the degree distributions of all these graphs can be described by the *power-law model*. Here we consider another important application—a network representation of the stock market. Stock markets generate huge amounts of data, which can be used for constructing the *market graph* reflecting the market behavior. We conduct the statistical analysis of this graph and show that it also follows the power-law model. Moreover, we detect *cliques* and *independent sets* in this graph. These special formations have a clear practical interpretation, and their analysis allows one to apply a new data mining technique of classifying financial instruments based on stock prices data, which provides a deeper insight into the internal structure of the stock market.

Keywords: Market graph; Stock price fluctuations; Cross-correlation; Data analysis; Graph theory; Degree distribution; Power-law model; Clustering coefficient; Clique; Independent set; Classification; Diversified portfolio

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1. Introduction

A simple undirected graph $G = (V, E)$ is defined by its set of vertices V and the set of edges $E \subset V \times V$ connecting pairs of distinct vertices.

Various properties of graphs have been well studied, and a great number of practical applications of graph theory have been considered in the literature (Avondo-Bodino, 1962; Berge, 1976; Deo, 1974). Nowadays, very large objects of completely different nature and origin are thought of as graphs. One of the most remarkable example of the expansion of graph-theoretical approaches is representing the World Wide Web as a massive graph. One can also mention the Call graph arising in the telecommunications traffic data (Abello et al., 1999; Aiello et al., 2001; Hayes, 2000); metabolic networks arising in biology (Jeong et al., 2000), and social networks where real people are the vertices (Hayes, 2000; Watts, 1999; Watts and Strogatz, 1998). Therefore, graph theory has become a truly interdisciplinary branch of science.

All of the aforementioned graphs have been empirically studied, and one fundamental result was obtained. It turns out that all these graphs coming from diverse applications follow the *power-law model* (Aiello et al., 2001; Albert and Barabasi, 2002; Barabasi and Albert, 1999; Boginski et al., 2003a; Broder et al., 2000; Faloutsos et al., 1999; Hayes, 2000; Watts, 1999; Watts and Strogatz, 1998), which states that the probability that a vertex of a graph has a degree k (i.e. there are k edges emanating from it) is

$$P(k) \propto k^{-\gamma}.$$

Equivalently, one can represent it as

$$\log P \propto -\gamma \log k,$$

which demonstrates that this distribution would form a straight line in the logarithmic scale, and the slope of this line would be equal to the value of the parameter γ.

It should be noted that the degree distribution of a graph is an important characteristic of a real-life dataset corresponding to this graph. It reflects the large-scale pattern of connections in the graph, which in many cases reflects the global properties of the dataset this graph represents.

Another interesting observation is the fact that the aforementioned graphs tend to be *clustered* (i.e. two vertices in a graph are more likely to be connected if they have a common neighbor), so the *clustering coefficient*, which is defined as the probability that for a given vertex its two neighbors are connected by an edge, is rather high in these graphs.

In this paper, we study the characteristics of the graph representing the structure of the US stock market. It should be mentioned that network-based approaches are extensively used in various types of financial applications nowadays, and "financial networks" are widely discussed in the literature (Nagurney, 2003; Nagurney and Siokos, 1997).

A natural network representation of the stock market that we consider in this paper is based on the cross-correlations of stock price fluctuations. The *market graph* is constructed as follows: each financial instrument is represented by a vertex, and two vertices are connected by an edge if the correlation coefficient of the corresponding pair of instruments (calculated over a certain period of time) exceeds a certain threshold $\theta \in [-1, 1]$.

We show that, under certain conditions, the properties described above are also valid for the considered market graph.

Besides analyzing the degree distribution of the market graph, we also look for *cliques* and *independent sets* in this graph for different values of the correlation threshold. A *clique* in a graph is a set of completely interconnected vertices, and the *maximum clique problem* is to find the largest clique in the graph (Bomze et al., 1999). An *independent set* is a set of vertices without connections, and the *maximum independent set problem* is defined similarly to the maximum clique problem.

From the data mining perspective, locating cliques (quasi-cliques) and independent sets in a graph representing a dataset would provide valuable information about this dataset. Intuitively, edges in such a graph would connect vertices corresponding to "similar" elements of the dataset, therefore, cliques would naturally represent dense clusters of similar objects. On the contrary, independent sets can be treated as groups of objects that differ from every other object in the group, and this information can also be important in certain applications. Clearly, it is also useful to find a maximum clique or independent set in the graph, since it would give the maximum possible size of the groups of "similar" or "different" objects.

Cliques and independent sets in the market graph have a simple practical interpretation. A clique in the market graph with a positive value of the correlation threshold θ is a set of instruments whose price fluctuations exhibit a similar behavior (a change of the price of any instrument in a clique is likely to affect all other instruments in this clique), therefore, finding cliques in the market graph provides a natural technique of classifying financial instruments based on the dataset of daily price changes. An independent set in the market graph with a negative value of θ consists of instruments that are negatively correlated with respect to each other, therefore, it represents a "completely diversified" portfolio.

Other properties of the market graph, such as its connectivity and the size of connected components, are discussed in Boginski et al. (2003b).

2. Construction and statistical analysis of the market graph

The market graph considered in this paper represents the set 6546 of financial instruments traded in the US stock markets. We analyze daily fluctuations of their prices during 500 consecutive trading days in 2000–2002.

The formal procedure of constructing the market graph is rather simple. Let the set of financial instruments represent the set of vertices of the graph. For any pair of vertices i and j, an edge connecting them is added to the graph if the corresponding correlation coefficient C_{ij} based on the price fluctuations of instruments i and j is greater than or equal to a specified threshold θ ($\theta \in [-1, 1]$).

Let $P_i(t)$ denote the price of the instrument i on day t. Then $R_i(t) = \ln(P_i(t)/P_i(t-1))$ defines the logarithm of return of the instrument i over the one-day period from $(t-1)$

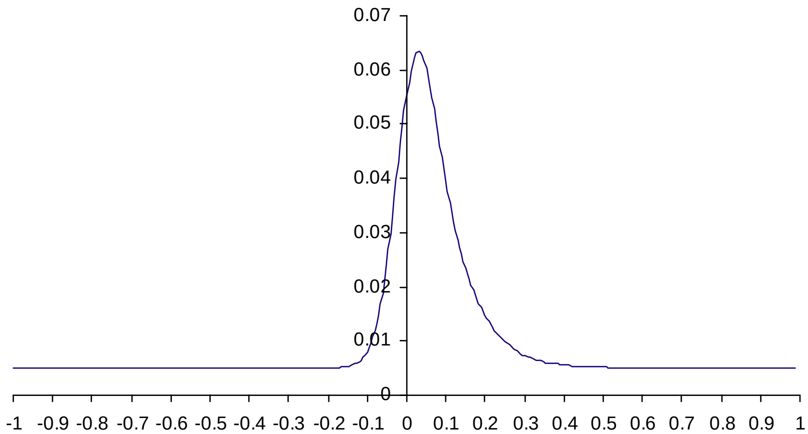

Fig. 1. Distribution of correlation coefficients in the stock market.

to t. The correlation coefficient between instruments i and j is calculated as

$$C_{ij} = \frac{E(R_i R_j) - E(R_i)E(R_j)}{\sqrt{Var(R_i)Var(R_j)}},$$

where $E(R_i)$ is defined simply as the average return of the instrument i over 500 days (i.e., $E(R_i) = \frac{1}{500}\sum_{t=1}^{500} R_i(t)$) (Mantegna and Stanley, 2000).

The correlation coefficients C_{ij} vary in the range from -1 to 1. The distribution of the correlation coefficients based on the considered data is shown in Fig. 1.

2.1. Degree distribution of the market graph

The first subject of our interest is the distribution of the degrees of the vertices in the market graph. We have conducted several computational experiments with different values of the correlation threshold θ, and these results are presented below.

It turns out that if a small (in absolute value) correlation threshold θ is specified, the distribution of the degrees of the vertices does not have any well-defined structure. Note that for these values of θ the market graph has a relatively high edge density (i.e. the ratio of the number of edges to the maximum possible number of edges). However, as the correlation threshold is increased, the degree distribution more and more resembles a power law. In fact, for $\theta \geqslant 0.2$ this distribution is approximately a straight line in the logarithmic scale, which represents the power-law distribution, as it was mentioned above. Fig. 2 demonstrates the degree distributions of the market graph for some positive values of the correlation threshold, along with the corresponding linear approximations. The slopes of the approximating lines were estimated using the least-squares method. Table 1 summarizes the estimates of the parameter γ of the power-law distribution (i.e., the slope of the line) for different values of θ.

From this table, it can be seen that the slope of the lines corresponding to positive values of θ is rather small. According to the power-law model, in this case a graph

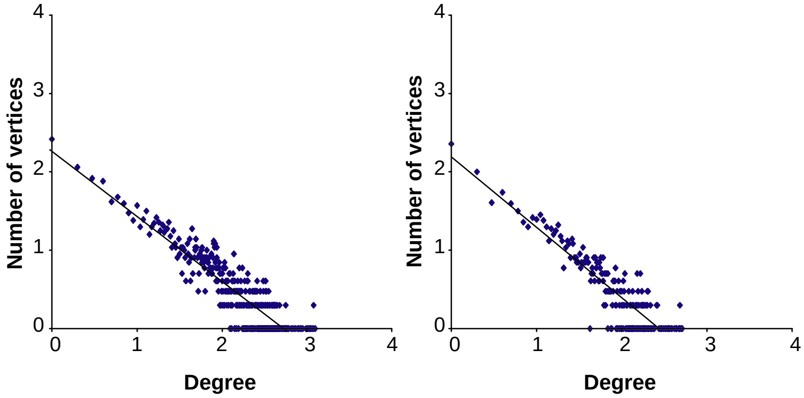

Fig. 2. Degree distribution of the market graph for $\theta = 0.4$ (left); $\theta = 0.5$ (right) (logarithmic scale).

Table 1
Least-squares estimates of the parameter γ in the market graph for different values of correlation threshold

θ	γ
−0.25[a]	1.2922
−0.2[a]	1.4088
−0.15[a]	1.4072
0.2	0.4931
0.25	0.5820
0.3	0.6793
0.35	0.7679
0.4	0.8269
0.45	0.8753
0.5	0.9054
0.55	0.9331
0.6	0.9743

[a]Complementary graph.

would have many vertices with high degrees, therefore, one can intuitively expect to find large cliques in a power-law graph with a small value of the parameter γ.

We also analyze the degree distribution of the *complement* of the market graph, which is defined as follows: an edge connects instruments i and j if the correlation coefficient between them $C_{ij} \leqslant \theta$. Studying this complementary graph is important for the next subject of our consideration—finding maximum independent sets in the market graph with negative values of the correlation threshold θ. Obviously, a maximum independent set in the initial graph is a maximum clique in the complement, so the maximum independent set problem can be reduced to the maximum clique problem in the complementary graph. Therefore, it is useful to investigate the degree distributions of the complementary graphs for different values of θ. As it can be seen from Fig. 1, the distribution of the correlation coefficients is nearly symmetric around $\theta = 0.05$,

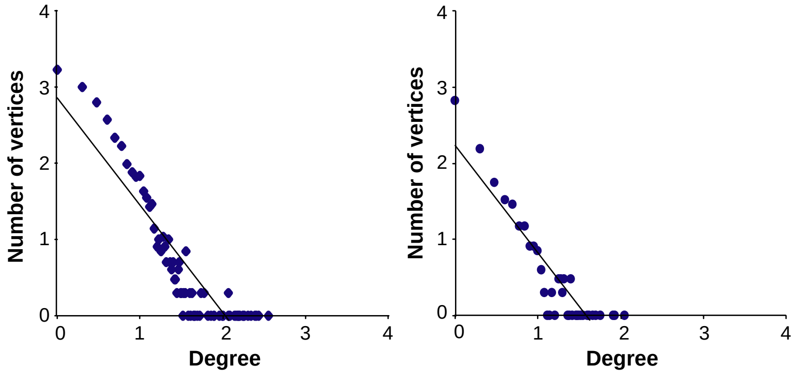

Fig. 3. Degree distribution of the complementary market graph for $\theta = -0.15$ (left); $\theta = -0.2$ (right) (logarithmic scale).

so for the values of θ close to 0 the edge density of both the initial and the complementary graph is high enough. For these values of θ the degree distribution of a complementary graph also does not seem to have any well-defined structure, as in the case of the corresponding initial graph. As θ decreases (i.e., increases in the absolute value), the degree distribution of a complementary graph starts to follow the power law. Fig. 3 shows the degree distributions of the complementary graph, along with the least-squares linear regression lines. However, as one can see from Table 1, the slopes of these lines are higher than in the case of the graphs with positive values of θ, which implies that there are fewer vertices with a high degree in these graphs, so intuitively, the size of a cliques in a complementary graph (i.e., the size of independent sets in the original graph) should be significantly smaller than in the case of the market graph with positive values of the correlation threshold.

2.2. Clustering coefficients in the market graph

Next, we calculate the *clustering coefficients* in the original and complementary market graphs for different values of θ. Interestingly, clustering coefficients in the original market graph are large even for high correlation thresholds, however, in the complementary graphs with a negative correlation threshold the values of the clustering coefficient turned out to be very close to 0. These results are summarized in Table 2. For instance, as one can see from this table, the market graph with $\theta = 0.6$ has almost the same edge density as the complementary market graph with $\theta = -0.15$, however, their clustering coefficients differ dramatically.

2.3. High-degree vertices in the market graph

One more issue that we address here is finding the vertices with high degrees in the market graph. This allows us to detect the instruments highly correlated with many others, and therefore reflecting the behavior of large segments of the stock market. For

Table 2
Clustering coefficients of the market graph

θ	Edge density	Clustering coef.
-0.15[a]	0.0005	2.64×10^{-5}
-0.1[a]	0.0050	0.0012
0.3	0.0178	0.4885
0.4	0.0047	0.4458
0.5	0.0013	0.4522
0.6	0.0004	0.4872
0.7	0.0001	0.4886

[a]Complementary graph.

this purpose, we consider the market graph with a high correlation threshold ($\theta = 0.6$) and calculate the degree of each vertex in this graph. Interestingly, even though this graph is very sparse (its edge density is only 0.04%), there are a lot of vertices with high degrees. This can be explained by the fact that the values of the clustering coefficient in the market graph are rather high, as it was pointed out above.

The vertex with the highest degree in this market graph corresponds to the NASDAQ 100 index tracking stock. The degree of this vertex is 216, which is more than twice higher than the number of stocks included into the NASDAQ index. Not surprisingly, almost all other vertices with high degrees also correspond to indices incorporating various groups of stocks.

3. Analysis of cliques and independent sets in the market graph

In this section, we discuss the methods of finding maximum cliques and maximum independent sets in the market graph and analyze the obtained results.

The maximum clique problem (as well as the maximum independent set problem) is known to be NP-hard (Garey and Johnson, 1979). Moreover, it turns out that the maximum clique is difficult to approximate (Arora and Safra, 1992; Håstad, 1999). This makes these problems especially challenging in large graphs. However, as we will see in the next subsection, even though the maximum clique problem is generally very hard to solve in large graphs, the special structure of the market graph allows us to find the exact solution relatively easily.

3.1. Cliques in the market graph

The maximum clique problem can be formulated as an integer programming problem (Bomze et al., 1999), however, before solving this problem, we applied a special pre-processing technique to reduce its size. In order to apply this procedure, we need to find a sufficiently large clique in the market graph, which will be the lower bound on the size of the maximum clique. For this purpose, we apply a greedy heuristic algorithm. The main idea of this algorithm is as follows: a clique is constructed "step by step" by recursively adding a vertex from the neighborhood of the clique adjacent to the most vertices in the neighborhood of the clique. More detailed description of this algorithm can be found in (Boginski et al., 2003b).

After finding a clique using this algorithm, the following preprocessing procedure was applied: all vertices in the graph with the degree less than the size of the found clique were recursively removed from the graph (Abello et al., 1999). Clearly, these vertices cannot be included into the maximum clique, therefore we can equivalently consider the maximum clique problem on the reduced graph. Let $G'(V',E')$ denote the reduced graph. Then the maximum clique problem can be formulated and solved for G'. We use the following integer programming formulation (Bomze et al., 1999):

$$\begin{array}{ll} \max & \sum x_i \\ \text{s.t.} & x_i + x_j \leqslant 1, \quad (i,j) \notin E', \end{array} \qquad x_i \in \{0,1\}.$$

In the case of market graphs with positive correlation thresholds, the aforementioned preprocessing procedure turned out to be very efficient and significantly reduced the number of vertices in a graph (Boginski et al., 2003b). Therefore, the resulting integer programming problem could be relatively easily solved using CPLEX (ILOG, 2000).

Table 3 summarizes the exact sizes of the maximum cliques found in the market graph for different values of θ. It turns out that these cliques are rather large, which agrees with the analysis of degree distributions and clustering coefficients in the market graphs with positive values of θ.

These results show that in the modern stock market there are large groups of instruments whose price fluctuations behave similarly over time, which is not surprising, since nowadays different branches of economy highly affect each other.

3.2. Independent sets in the market graph

Here we present the results of solving the maximum independent set problem in the market graphs with nonpositive values of the correlation threshold θ. As it was pointed out above, this problem is equivalent to the maximum clique problem in a complementary graph. However, the preprocessing procedure that was very helpful for finding maximum cliques in the original graph could not eliminate any vertices in

Table 3
Sizes of the maximum cliques in the market graph with positive values of the correlation threshold (exact solutions)

θ	Edge density	Clique size
0.35	0.0090	193
0.4	0.0047	144
0.45	0.0024	109
0.5	0.0013	85
0.55	0.0007	63
0.6	0.0004	45
0.65	0.0002	27
0.7	0.0001	22

Table 4
Sizes of independent sets in the complementary graph found using the greedy algorithm (lower bounds)

θ	Edge density	Indep. set size
0.05	0.4794	45
0.0	0.2001	12
−0.05	0.0431	5
−0.1	0.005	3
−0.15	0.0005	2

the case of the complement, and we were not able to find the exact solution of the maximum independent set problem in this case. Recall that the clustering coefficients in the complementary graph were very small, which intuitively explains the failure of the preprocessing procedure. Therefore, solving the maximum independent set in the market graph is more challenging than finding the maximum clique. Table 4 presents the sizes of the independent sets found using the greedy heuristic that was described in the previous section.

This table demonstrates that the sizes of computed independent sets are rather small, which is in agreement with the results of the previous section, where we mentioned that in the complementary graph the values of the parameter of the power-law distribution are rather high, and the clustering coefficients are very small.

The small size of the computed independent sets means that finding a large "completely diversified" portfolio (where all instruments are negatively correlated to each other) is not an easy task in the modern stock market.

A natural question now arises: how many completely diversified portfolios can be found in the market? In order to find an answer, we have calculated maximal independent sets starting from each vertex, by running 6546 iterations of the greedy algorithm mentioned above. That is, for each of the considered 6546 financial instruments, we have found a completely diversified portfolio that would contain this instrument. Interestingly enough, for every vertex in the market graph, we were able to detect an independent set that contains this vertex, and the sizes of these independent sets were rather close. Moreover, all these independent sets were distinct. Fig. 4 shows the frequency of the sizes of the independent sets found in the market graphs corresponding to different correlation thresholds.

These results demonstrate that it is always possible for an investor to find a group of stocks that would form a completely diversified portfolio with any given stock, and this can be efficiently done using the technique of finding independent sets in the market graph.

4. Data mining interpretation of the market graph model

As we have seen, the analysis of the market graph provides a practically useful methodology of extracting information from the stock market data. In this subsection,

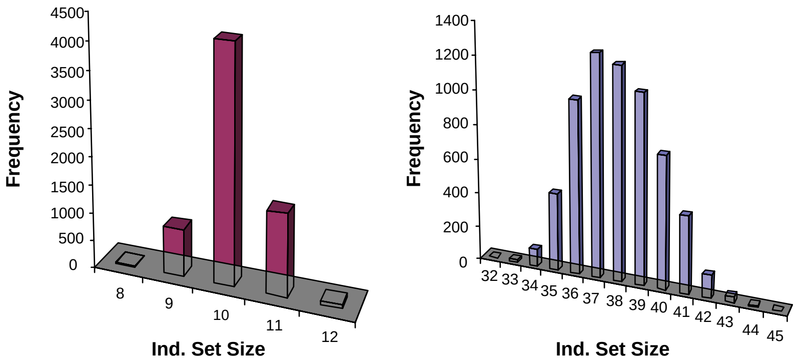

Fig. 4. Frequency of the sizes of independent sets found in the market graph with $\theta = 0.00$ (left), and $\theta = 0.05$ (right).

we discuss the conceptual interpretation of this approach from the data mining perspective. An important aspect of the proposed model is the fact that it allows one to reveal certain patterns underlying the financial data, therefore, it represents a structured data mining approach.

Non-trivial information about the global properties of the stock market is obtained from the analysis of the degree distribution of the market graph. Highly specific structure of this distribution suggests that the stock market can be analyzed using the power-law model, which can theoretically predict some characteristics of the graph representing the market. As we have mentioned, the power-law structure is typical for many real-life datasets coming from diverse areas. This fact gave a rise to the term "self-organized networks", and it turns out that this phenomenon also takes place in the case of financial data.

On the other hand, the analysis of cliques and independent sets in the market graph is also useful from the data mining point of view. As it was pointed out above, cliques and independent sets in the market graph represent groups of "similar" and "different" financial instruments, respectively. Therefore, information about the size of the maximum cliques and independent sets is also rather important, since it gives one the idea about the trends that take place in the stock market. Besides analyzing the maximum cliques and independent sets in the market graph, one can also divide the market graph into the smallest possible set of distinct cliques (or independent sets). Partitioning a dataset into sets (clusters) of elements grouped according to a certain criterion is referred to as *clustering*, which is one of the well-known data mining problems (Bradley et al., 1999).

The main difficulty one encounters in solving the clustering problem on a certain dataset is the fact that the number of desired clusters of similar objects is usually *not known* a priori, moreover, an appropriate *similarity criterion* should be chosen before partitioning a dataset into clusters.

Clearly, the methodology of finding cliques in the market graph provides an efficient tool of performing clustering based on the stock market data. The choice of the grouping criterion is clear and natural: "similar" financial instruments are determined according to the correlation between their price fluctuations. Moreover, the minimum number of clusters in the partition of the set of financial instruments is equal to the minimum number of distinct cliques that the market graph can be divided into (the *minimum clique partition* problem). Similar partition can be done using independent sets instead of cliques, which would represent the partition of the market into a set of distinct diversified portfolios. In this case the minimum possible number of clusters is equal to a partition of vertices into a minimum number of distinct independent sets. This problem is called the *graph coloring* problem, and the number of sets in the optimal partition is referred to as the *chromatic number* of the graph.

We should also mention another major type of data mining problems with many applications in finance. They are referred to as *classification problems*. Although the setup of this type of problems is similar to clustering, one should clearly understand the difference between these two types of problems.

In classification, one deals with a pre-defined number of classes that the data elements must be assigned to. Also, there is a so-called *training dataset*, i.e., the set of data elements for which it is *known* a priori which *class* they belong to. It means that in this setup one uses some initial information about the classification of existing data elements. A certain classification model is constructed based on this information, and the parameters of this model are "tuned" to classify new data elements. This procedure is known as "training the classifier". An example of the application of this approach to classifying financial instruments can be found in Bugera et al. (2003).

The main difference between classification and clustering is the fact that unlike classification, in the case of clustering, one does not use any initial information about the class attributes of the existing data elements, but tries to determine a classification using appropriate criteria. Therefore, the methodology of classifying financial instruments using the market graph model is essentially different from the approaches commonly considered in the literature in the sense that it does not require any a priori information about the classes that certain stocks belong to, but classifies them only based on the behavior of their prices over time.

5. Conclusion

The statistical analysis of the degree distribution of the market graph has shown that the power-law model is valid in financial networks. It confirms an amazing observation that many real-life massive graphs arising in diverse applications have a similar power-law structure, which indicates that the global organization and evolution of massive datasets arising in various spheres of life follow similar laws and patterns.

The analysis of cliques and independent sets in the market graph provides a novel alternative data mining approach to the classification of financial instruments. It would be also helpful for investors for making decisions of forming their portfolios. Therefore, this technique is useful from both theoretical and practical points of view.

As it was mentioned above, the classification methodology using cliques and independent sets in the market graph is conceptually different from classification methods commonly applied in the literature, therefore, it would be interesting to conduct a comparative analysis of various methodologies used in this field.

Among other issues that can be addressed, one should mention a possibility to study another type of the market graph based on the data of the *liquidity* of financial instruments. The comparison of the properties of this graph and the market graph considered in this paper would be useful for studying the relationship between return and liquidity, which is one of the fundamental problems of the modern finance theory.

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