CHAPTER 9

LINKS AND ACTIONS IN INTERPLAY

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9.1 Introduction

In social contexts, agents not only choose their actions but often also have substantial control over whom they interact with when choosing those actions.[1] Therefore, in the end, if every agent behaves in this fashion, there must be a joint determination of both choice dimensions (i.e., what, in common jargon, is described as a *co-evolution of links and actions*). In the study of many phenomena of interest, such an integrated consideration of the link-action interplay is key to understand the problem at hand. In particular, if we abstract from it, either the model may fail to produce sharp predictions or, if it does, those predictions could be quite misleading.

In this chapter, I cannot review exhaustively the wide range of theoretical research that bears on the issue. Thus what I will do is illustrate some of the main ideas involved through a collection of paradigmatic applications. Specifically, I will start by focusing on three key but abstract problems: *coordination, cooperation, and intermediation*. Then, somewhat more concretely, I will turn to briefly summarizing how network co-evolution shapes the outcome for other important phenomena such as bargaining, local public goods, learning, and conflict. For all of these cases, I shall argue that a modeling approach that jointly accounts for the adjustment of links and actions provides important insights on the problem at hand that would be largely missed if they were separately considered.

[1] See Chapters 5 and 8 in this volume, the first coauthored by Yann Bramoullé and Rachel Kranton, and the second by Ana Mauleón and Vincent Vannetelbosch. Whereas the former focuses on how agents play on given networks, the latter considers the issue of how the network itself is shaped by agents' decisions.

9.2 Coordination

Coordination is certainly one of the key aspects of social interaction. The easiest way to model it is through a simple bilateral game with two actions, A and B, and a payoff table of the following form:

$i \backslash j$	A	B
A	a, a	c, d
B	d, c	b, b

(9.1)

where i and j are generic players connected by the network and a, b, c, d are payoffs satisfying $a > c$ and $b > d$. Thus the bilateral game has two pure strategy equilibria, and let us suppose that $a > b$, so that the one given by (A,A) defines the unique Pareto efficient configuration. Suppose first that the social network is given by an undirected graph $\Gamma = \{N,L\}$, where $N = \{1,2,\ldots,n\}$ is the set of players and L the set of links of the form ij (equivalently, ji) for any pair $\{i,j\}$ of connected players.

To make our point in a particularly transparent manner, let us suppose that the network Γ is a ring and every player i is connected to $i-1$ and $i+1$ (with $n+1$ identified with 1 and 0 with n). An important assumption is that every player must choose the same action, A or B, when playing the game with each of her neighbors in Γ. This is the only case in which the network could have any effect on the pattern of play. Then, it is clear that at least two action profiles define Nash equilibria of the population-wide (network-based) game where every player chooses her action independently. These are the two homogeneous configurations: $s_A = \{A,A,\ldots,A\}$ and $s_B = \{B,B,\ldots,B\}$. In fact, if we rule out the knife-edge case where $a+c = d+b$, they are the only two equilibria.

This begs the question of whether these two action profiles are equally robust in the face of some dynamic process of adjustment, say the classical best-response dynamics. In the absence of any noise or perturbation, the answer must be positive: essentially, both appear equally "solid" (i.e., locally stable for those dynamics). This is why the early evolutionary literature concerned with tackling the resulting equilibrium-selection issue introduced the possibility that, over time, the agents may be subject to what is often labeled as "mutations" (i.e., rare and random changes in their current action).[2] In this context, the focus then turned to identify those states that are called stochastically stable (i.e., those that have positive weight in the invariant distribution of the process

[2] The seminal papers are Kandori, Mailath, and Rob (1993) and Young (1993), which studied the problem under the assumption that interaction is global (i.e., every player confronts everyone else). In a subsequent paper, Ellison (1993) analyzed the case considered here, with interaction being local and players placed along a ring.

when the probability of mutation becomes arbitrarily small). It was then found that the so-called risk-dominant action is selected in the long run—or, more precisely, the only *stochastically stable state* is the one where every agent chooses that action. How does one define such an action? It is the one that yields the higher expected payoff when facing an equal probability that the other player chooses any of the two actions, A or B. If, for example, $b + d > a + c$, action B qualifies as risk-dominant, while action A may continue being the Pareto-efficient action. The intuition for the selection of the risk-dominant action is widely understood by now, so we do not elaborate on it here. Suffice to say that risk dominance makes the transition from s_A to s_B so much easier in terms of the number of mutations required that, when the probability of mutation is small,[3] the process spends "most of the time" at, or in the vicinity, of s_B.

Suppose now that the network is not fixed but can change over time, either by agents creating new links or/and destroying existing ones, in addition to their ability to adjust actions. Jackson and Watts (2002) and Goyal and Vega-Redondo (2005) have studied this problem in different frameworks—the former in a two-sided network-formation context, while the latter considers a one-sided one. Here I summarize the approach pursued by Jackson and Watts. Their framework is essentially that described for a fixed network but with the *added* possibility that agents can also form or/and destroy existing links when randomly given the opportunity. Links are taken to be costly, with a constant creation/maintenance cost for every prevailing link, which is split equally between the two agents involved. Being a two-sided mechanism, the assumption is that any given link under consideration remains in place (when existing) or it is formed (when not present) if, and only if, its net benefits are positive for both players connected by it. Then, the possibility of mutation (now affecting both actions and links) is again added to the dynamics, and the question is posed as to what is the long-run outcome where the system spends most of the time when the mutation probability is very small (i.e., infinitesimal). The main conclusion is that, in this context, an efficient action carries a positive long-run weight, even if it is risk-dominated, provided that the linking cost lies at an intermediate level (higher than the off-diagonal payoffs, c and d, but still lower than the highest payoff a).

What is the intuition for this conclusion? It is based on the following simple insight. If the costs are as described, any configuration with two completely connected and homogeneous components where every player chooses the same action (A in one and B in the other) is stationary in the absence of mutations. But then, through drift, the relative sizes of the two components can change in either direction and at the same rate (in terms of single mutations). The only case in which a single mutation will not work to produce a persistent transition is when the whole population is in a single component, all choosing the same action, A or B. In this case, *two* mutations are required to escape the situation, since this is what is needed to create a non-trivial

[3] This conclusion presumes that all mutations are infinitesimals of the same order. However, in some cases there may be reasons why this is not a good assumption, and deviating from it alters significantly the selection results. For an early discussion of this issue, see Bergin and Lipman (1996).

component consisting of at least two agents. But then, given that the same number of mutations are needed to transit away from an homogeneous population in A or B, these two configurations should obtain in the long run with positive frequency, even as the mutation rate becomes infinitesimally small.

The former analysis has the following limitations. On the one hand, by relying crucially on low-probability mutation, its analysis, in effect, must be viewed as having a very (ultra-)long-run character and therefore questionable practical relevance. On the other hand, it does not shed light on what may be the topology, or the relative sizes of the components. But when we think of social coordination, those relative sizes are clearly an important aspect of the problem. However, for low mutation, it is easy to see that the prediction of the model by Jackson and Watts (2002)—also of that by Goyal and Vega-Redondo (2005)—is that most of the time the whole population will be in a *single and completely connected homogeneous component*.

To arrive at a less drastic conclusion, it seems necessary to relax the assumption that the perturbation component of the model operates at an infinitesimal level throughout. This was the route pursued by Ehrhardt, Marsili, and Vega-Redondo (2008) in a model where, to focus the discussion, the coordination game is assumed to have $a = b > 0$ and $c = d = 0$—hence it is a pure and action-symmetric coordination game. Over time (modeled continuously), agents form links when they meet (at random) some other agent with whom they can obtain positive payoffs. Their actions, on the other hand, are chosen as a (weak) best response to the current state. Finally, the "perturbation" required to make the analysis interesting is conceived in a minimalist manner as *link volatility*. That is, it is assumed that all existing links vanish at a constant rate. Combining all these features, an ergodic process is obtained whose unique invariant distribution can be characterized. In particular, the long-run states (i.e., those that have positive weight in the invariant distribution) are found to display the following features. If the population is large, there is a threshold value $\hat{\eta}$ for the volatility rate η such that

1. if $\eta > \hat{\eta}$, in every long-run state the population is fragmented into "maximum miscoordination" (i.e., half of the population chooses each action) and the underlying network is sparsely connected (no giant component exists);
2. if $\eta < \hat{\eta}$, in every long-run state one action attracts a majority of the population and the underlying network displays a giant component, whose fractional size increases as volatility decreases.

Thus the magnitude of volatility has a major effect not only on the extent of coordination but also on the corresponding density of connections prevailing in the society. This conclusion contrasts with that obtained in the aforementioned models of Jackson and Watts (2002) and Goyal and Vega-Redondo (2005), where coordination and connectivity are at a maximum in every long-run state. This is not the case in the model of Ehrhardt, Marsili, and Vega-Redondo (2008) because, since noise (i.e., volatility) operates at a non-infinitesimal level, full connectivity is no longer maintained at long-run states, and hence it interplays with the extent of action coordination in richer ways.

9.2.1 Summary of Additional Literature

The co-evolution of links and actions in coordination games has been studied from a number of other different perspectives. Hojman and Szdeil (2006) consider a one-sided model of network formation similar to that of Goyal and Vega-Redondo (2005) but assume that the partners of any given player are not just her direct neighbors but indirect ones as well (i.e., everyone in her component). They find that equilibria are selected by a criterion that embodies some trade-off between efficiency and risk dominance. Staudigl (2011) posits a model where the underlying coordination game is a partnership (in our notation, $d = c$) and both links and actions are adjusted according to a logit formulation. The dynamics also includes a process of volatility as described before, by which links are destroyed at a constant rate. The paper derives the invariant distribution of the process and shows that the efficient action is selected in the long run when the noise displayed by the logit formulation becomes arbitrarily small.

Another specific scenario where actions and links co-evolve under strategic complementarities is the one studied by König, Tessone, and Zenou (2014), which is based on the linear-quadratic model introduced by Ballester, Calvó-Armengol, and Zenou (2006). They show that the model leads to networks called "nested split graphs," which are networks that exhibit a strict hierarchical structure. More general games displaying strategic complementarities have been studied by Hiller (2012) and Baetz (2014). They model the network-formation problem as a simultaneous game where agents decide jointly on actions and links, the key difference between them being that the former assumes that the value function is convex while the latter postulates that it is concave. Baetz also restricts to one-sided network formation. They both show that some interesting structures arise at equilibria, such as core-periphery networks in the first paper and stratified networks in the second.

Finally, we refer to those papers that have addressed essentially the same issue (the co-evolution of conventions and connections in coordination games), but in a model where agents, rather than choosing their neighbors directly, do it indirectly by selecting a "location." More specifically, the key assumption is that each agent interacts with all those placed at the same location. In general, the main insight gained from these models (see, e.g., Ely 2002 or Bhaskar and Vega-Redondo 2004)—is that allowing for such a possibility of migration tends to favor the rise of efficient conventions. This approach is reminiscent of the celebrated work of Schelling (1972, 1978), who also explored the dynamic implications of individual self-motivated moves on some measure of social welfare (e.g., ethnic integration). There, however, individual mobility can lead to undesired consequences (segregation), even if individuals do not display a marked preference for it. Recently, a quite versatile model studying such interplay between individual incentives, local peer effects, and social welfare has been proposed by Badev (2013), who has applied it to the study of teenage smoking.

9.3 Cooperation

Cooperation is another key phenomenon where the interplay of network-based interaction and link adjustment may play a key role. In principle, however, it is possible to conceive "behavioral" setups where such an interplay is not needed for cooperation to arise. That is, local interaction alone can still support cooperation in some cases, provided agents behave in a somewhat less than fully rational manner. A good example is provided by the well-known model of Eshel, Samuelson, and Shaked (1998), which we outline next.

In their model agents are arranged on a ring and interact with their two neighbors. If they behave altruistically (say, by producing a local public good), they benefit their neighbors by one unit but have to pay a cost of $C > 0$ (net of the gross benefits they might derive from the public good). This obviously makes behaving egoistically (i.e., not providing the public good) a dominant strategy. Hence no truly rational agent will ever provide the public good, and the prediction in that case is that full egoism must prevail throughout the population.

Assume instead that agents do not guide their actions by direct payoff maximization but simply adjust their behavior by mimicking the action of whoever achieves the highest average payoff in their neighborhood. Then, Eshel et al. show that some altruism can be supported as a stable configuration. To see this, first note that, of course, an all-altruism situation will remain stable if the alternative egoist behavior is not present at all. (This simply follows from the fact that imitation by itself cannot bring up any new behavior.) The problem, however, is that such a homogeneous cooperative state is extremely fragile and any deviation from it will lead to a large change away from altruism. Thus suppose instead that the initial state is chosen at random. What is then the likely outcome? It is not difficult to see that, if altruism is not too costly (specifically, if $C \leq 1/2$), the final outcome of an imitation-based adjustment process as described is very likely to lead to a configuration where at least 60% of the agents are altruists. In fact, the same happens as well if, as we contemplated in our discussion of coordination games, the imitation dynamics is perturbed by some small "mutation"—it turns out that at least 3/5 of the population must be altruist in any stochastically stable state.

The problem with this apparently promising conclusion is that it is quite fragile. For, as shown by Mengel (2009), even if agents enjoy a "radius of information" that is just slightly longer than the range at which they interact with others (for example, they interact with their immediate neighbors but observe instead the payoffs obtained by their first- and second-order neighbors), full defection is the only stochastically stable state. This happens as well if, instead of the ring, we allow for some irregular networks. Such an ambiguous state of affairs is indeed underscored by recent experimental evidence, which shows that cooperation on *fixed networks* generally remains low and is hardly affected by the type of local interaction considered. For example, Grujić et al. (2010) and Gracia-Lázaro et al. (2012) have conducted controlled Prisoner's Dilemma experiments on large networks, varying their size and degree of heterogeneity.

Comparing these treatment scenarios with a control setup in which the network is "reshuffled" at every round (and hence the network is largely irrelevant), they find no significant differences in the amount of cooperation displayed by the treatment and control groups for the different cases.

Given the theoretical and empirical considerations outlined above, the question arises as to what mechanism can be effective in supporting cooperation in a network context. And again, the natural option that is now considered is that an endogenous co-evolving network could prove an effective way to overcome the aforementioned limitations. In fact, experimental evidence (see, e.g., the interesting paper by Wang, Suri, and Watts 2012)[4] does provide strong support for this suggestion. Next, I discuss two papers (Fosco and Mengel 2011 and Vega-Redondo 2006) that study, in two very different theoretical frameworks, the significant contribution that co-evolving networks may lend to the rise of cooperation.

The paper by Fosco and Mengel (2011) considers what is probably the simplest *co-evolutionary counterpart* of the model proposed by Eshel, Samuelson, and Shaked (1998). It can be seen, therefore, as a natural way of assessing whether network endogeneity can indeed remedy some of the aforementioned problems. The model involves a stochastic dynamic process through which agents play a Prisoner's Dilemma with their current neighbors and can also adjust their links over time. Its main novelty revolves around the contrast between two different ways in which local effects can be conceived and formalized. A first one is in terms of what is called the *radius of interaction*, which specifies the network range at which any given agent interacts (i.e., plays the game) with others and obtains corresponding payoffs. The second one is in terms of the *radius of information*, which indicates how far into the network an agent can access payoff-relevant information. (This information includes not only the payoffs others obtain and the actions they choose, but also who are the available new partners.) Then, given how payoffs and information are jointly determined by these two radii, the postulated adjustment involves a natural imitation dynamics, both for actions and links.

The methodology used by Fosco and Mengel to analyze their model is a standard one: the outlined dynamics is perturbed with some small noise/mutation and the focus is on the stochastically stable states—those states that are visited with significant frequency even for infinitesimal noise. The main conclusion is that all stochastically stable states are polymorphic—hence they display *some* amount of cooperation—but the degree of cooperation achieved and the details of its implementation crucially depends on both the radius of information and that of interaction. For example, if the former is longer than the latter, the population is split into two components (the cooperative and the

[4] Their experimental setup involved groups of 24 individuals who were involved in an iterated Prisoner's Dilemma lasting 12 rounds. At every such round, they had to choose whether to be cooperators and defectors and also could update their links (only a certain fraction, which was a design parameter). The extent of cooperation was substantially increased by link flexibility, and the effect became stronger the higher the fraction of links available for revision, even if the cost of a cooperator meeting a defector grew higher.

uncooperative one), which means that, in this case, cooperation is maintained by having the cooperators being shielded from the exploitation of opportunistic defectors.

If, instead of allowing for some bounded rationality (e.g., imitation, as above) one insists on sticking to the traditional full-rationality paradigm, repeated play has been the classical equilibrium approach in which the game-theoretic literature has understood the rise of altruistic behavior in social contexts. That is, if the same set of agents are facing each other over time, the (credible) threat of future punishment can sometimes be enough of a deterrence against opportunistic behavior. But if interaction is purely bilateral, the possibilities of supporting altruism in this manner may be quite limited due to a number of factors (e.g., highly impatient agents or too-tempting rewards for opportunistic behavior). And when this happens, it is conceivable that embedding the bilateral interaction in a larger social network can remedy the problem by enlarging significantly the scope of punishment and reward. But this then begs the question of whether the given ("initial") network will display the features required for such a state of affairs—or, if not, whether there are endogenous forces at play that may lead the network on that direction.

The model studied by Vega-Redondo (2006) aims at shedding light on this issue. In it agents are involved in a *separate* Repeated Prisoner's Dilemma with each of their neighbors on an evolving social network. The focus is on grim-trigger (subgame-perfect) equilibria, which embody the threat of reversion to indefinite defection with any neighbor who is found to have violated the cooperative "social norm." Two different social norms (or, equivalently, types of equilibria) are considered: the network-based and the network-free ones.

(a) Under the *network-based norm*, agents are supposed to punish (in equilibrium) not only those partners who have defected in their own bilateral interaction but also those who have done it with third parties.[5] Thus, in this case, even though the actions played in different bilateral interactions are independently chosen by players, agents' behavior is *not strategically* independent across those interactions.
(b) Under the standard *network-free norm*, each bilateral repeated game is played in a strategically independent manner across connected pairs. Thus, the behavior in any given bilateral interaction only depends on what has happened in that same interaction.

The main objective of the chapter is to contrast the implications—both on the resulting network as well as on the corresponding ability to support cooperation—of the two previous social norms. This is done in terms of a co-evolutionary process of network and strategy change that includes the following two components, in operation at each point in time.

[5] This punishment, however, is only implemented with some delay since information about the defection on other agents is assumed to be channeled gradually though the network itself.

1. *Link creation*: Individuals explore the network and search for new linking opportunities; when some are found new links are created iff, given the prevailing network, cooperation can be strategically supported on them.
2. *Volatility*: The payoffs associated to existing links are redrawn with some given probability; once this is done, all links are reconsidered and only those that can still support cooperation are maintained.

The analysis combines mean-field techniques (i.e., the study of the deterministic dynamics given by the expected law of motion) as well as numerical simulations. Some of the conclusions are as expected. For example, it is found that volatility has a negative impact on the density of the networks that are sustainable in the long run (recall that each prevailing link must be a cooperative one). The social norm in place, however, has a very substantial effect on how the population confronts the detrimental consequences on increased volatility. While under the network-based norm, the impact is gradual and moderate, in the alternative network-free norm it is abrupt (i.e., of the threshold type) and much larger in magnitude. Such contrasting behavior can be understood as follows. Under the network-based norm, the links *endogenously reconfigure* themselves so that the social structure becomes more cohesive, and hence the population can support cooperation more effectively. Instead, under the network-free norm, no such reconfiguration occurs (nor is relevant, for that matter), which means that the population ends up being much less successful in coping with volatility.

9.3.1 Summary of Additional Literature

The literature that studies how alternative interaction structures, endogenous or not, bear on the rise of cooperation is so large and diverse that my account of it here will be unavoidably sketchy and very partial. I started this section by illustrating the fact that, in order for cooperation to be robustly sustained in a network setup, agents displaying simple (say, imitative) behavior must enjoy not only the flexibility of adjusting their actions but also exhibit some plasticity in their linking behavior. In a network setup, a further illustration of this point can be found in the work of Eguíluz et al. (2005), Pacheco, Traulsen, and Nowak (2006), Ule (2008), and Bilancini and Boncinelli (2009). A similar idea arises as well in network-free environments if agents can endogenously determine when to maintain a given partnership or return to a common matching pool. Fujiwara-Greve and Okuno-Fujiwara (2012), and Izquierdo, Izquierdo, and Vega-Redondo (2014) study evolutionary models of this sort, in which the evolutionary stable equilibria is found to support cooperation under some conditions. Still a different evolutionary mechanism of partner selection is given by group selection. In this case, evolution proceeding at different levels (within and across groups) can again support cooperative behavior if the discipline imposed by group selection is strong/fast enough.

Simple illustration of this mechanism can be found in Vega-Redondo (1996) and Traulsen and Nowak (2006).

Concerning repeated interaction, the role of embeddedness (or what Nowak 2006 has called indirect reciprocity) in supporting cooperation has been highlighted by the literature studying this phenomenon at the interface of the theories of games and networks.[6] An early instance is the paper by Raub and Weesie (1990), whereas recent contributions include those of Ali and Miller (2009), Lippert and Spagnolo (2011), Haag and Lagunoff (2011), Jackson, RodrÃ-guez-Barraquer, and Tan (2012), Fainmesser (2014), and Immorlica, Lucier, and Rogers (2014). The first three papers stress the role that network structure (exogenously postulated) has in the possibility of supporting cooperation at equilibrium. The latter two instead introduce network endogeneity into the analysis, although only concerning link destruction within some exogenously given network of ex-ante linking possibilities.

9.4 Intermediation

Often, the social network not only determines the pattern of *direct* interaction among agents but also specifies how these agents connect indirectly—say to communicate, collaborate, or compete. This is the phenomenon we may call intermediation. It reflects the natural idea that, even if agents are far from each other in the social network, they can rely on others lying along a path, joining them in order to establish a valuable connection. Then, as a natural follow-up question, the issue arises as to whether those agents facilitating such indirect connection will obtain (and possibly compete for) some share in the value/surplus thus generated. In line with the theme of this chapter, our emphasis here will be on how these considerations shape the incentives to form links (i.e., direct connections) and how these links feed back on the actual distribution of the surplus among all the agents involved.

The notion of intermediation can play an important role in many different contexts (e.g., trade brokerage, technological or scientific research, trust and "social collateral," job search and referral, etc). (see Chapter 32). Here I shall approach it from a general and abstract viewpoint, focusing my discussion on a model studied by Goyal and Vega-Redondo (2007) that proposes a stylized formulation of the problem that is not specifically tailored to any given scenario.

Given any fixed population of n ex-ante identical agents and some given social network, the starting assumption of the model is that any pair of agents i and j who can connect, *directly or indirectly*, can earn a unit of surplus. If their connection is direct (i.e., there is a link between them in the social network), they need no one else to earn this surplus, so any efficient and symmetric bargaining procedure should have them divide the "pie" equally. Instead, if they need "intermediaries" to connect indirectly, the

[6] For a discussion of how networks bear on strategic play in repeated games see Chapter 6 by Francesco Nava in this volume.

key issue is whether some of these agents are essential (i.e., they are in *every* path joining them). If so, they should be expected to also demand a share of the surplus. And since, in fact, they are as crucial as either i or j in generating the surplus, all of them should demand the same share, and only they (the truly essential individuals) will obtain a positive cut. Formally, this reasoning can be provided with microfoundations as the Kernel of a coalitional bargaining game involving the whole population.

Clearly, the preceding analysis allows for a wide range of payoff possibilities, depending on the underlying network. Just to illustrate this richness, consider the following two polar cases: a ring network and a star network. In the case of the ring, every two agents are of course connected, and the important point is that there are two possible and disjoint paths in every case. This means that no player is an essential intermediary and, therefore, every pair of agents in the population can not only generate a unit of surplus but divide it between the two of them alone. In the end, every player obtains the same total payoff, equal to $\frac{1}{2}(n-1)$.

Now contrast the former situation with the one where the social network is a star and, say, agent 1 is at the center of it. Clearly, on the one hand, agent 1 can generate a unit of surplus with each of the $n-1$ spoke agents without any intermediation required. But, being at the center, the central agent can also intermediate, in an essential manner, the connections between every pair $\{i,j\}$ of other agents in the population ($i \neq 1 \neq j$). Thus, for each of these $\frac{1}{2}(n-1)(n-2)$ cases, agent 1 will get $1/3$ of the unit surplus generated. Overall, therefore, the payoff of agent 1 will be much higher than that of the other agents. In the terminology put forward by Burt (1992), agent 1 fills a "structural hole" in the social network and is able to extract hefty rents from that.

But then the important issue is whether such asymmetry in agents' network positions can be understood/rationalized as the outcome of a network-formation mechanism that reflects the strategic incentives of the agents involved. In other words, one would like to know which structures and corresponding outcomes should obtain when agents' linking decisions are taken in anticipation of the intermediation rents induced. For, in general, it is clear that in parallel to the incentives of agents to create (and fill in) structural holes, there are the opposing incentives to destroy essential intermediation through the creation of additional links.

To explore this tension, Goyal and Vega-Redondo (2007) propose a network-formation model where, as usual, any (two-sided) link can be created by bilateral consensus of the two agents to be connected while it is destroyed if any of the agents involved objects to its remaining in place.[7] On the other hand, links are assumed costly, so that only those links that provide a benefit above their cost are formed. The key two assumptions of the model are as follows. First, any two agents directly or indirectly connected can earn a unit surplus. Second, that surplus is equally divided among the two agents in question and any other agent who might be a crucial intermediary in establishing the indirect connection between the former two. Under these conditions, the main conclusion is

[7] Specifically, the concept used to model network formation is *Strict Bilateral Equilibrium*, which is a refinement of (strict) Nash equilibrium that requires the robustness of both individual and bilateral deviations.

very clear-cut: there is a certain threshold such that if the linking cost is below it, then the only equilibrium network is the complete one; instead, if the cost lies above the threshold, the only equilibrium architecture is a star.

The result for a low linking cost below the threshold is clear. For, if links are very cheap, it must be worthwhile establishing direct connections to every other agent and thus save on intermediation payments. But why is the star architecture the only strategically stable architecture for linking costs above the threshold? The reason is that, for any structure other than the star, at least one of the two following statements is true:

(a) some *individual agents* have at least weak incentives to *unilaterally* severe their links (i.e., they have no strict incentives to maintain all of them);
(b) there is some pair of agents who can divide the network into separate components (each of them lying in one of them) by destroying some of the links under their control.

To understand the essential point, suppose that the two alternative configurations to be considered are just a ring and a star. In both of them, if the linking costs are sizable and the population is large, agents have *strict individual incentives* to keep all their links.[8] The ring, however, allows for the possibility that any two agents at opposite sides of it can act in coordination and achieve the following outcome: first, they may break the ring into two separate (line) components, opening a "structural hole"; second (but simultaneously), they can establish a new link between themselves, close the aforementioned hole, and by so doing become central and crucial to many pairs of other agents. This is precisely the profitable bilateral deviation that renders the ring an unstable configuration. Obviously, the same argument does *not* apply to the star, which is stable in the face of any possible deviation, unilateral or bilateral. So, in sum, again we find that by endogenizing the prevailing network one obtains sharper predictions and important insights that hinge upon the action-link interplay embodied in the network-formation game.

9.4.1 Summary of Additional Literature

The general issue of intermediation has received significant attention in the recent literature, with a particular focus on bargaining and trade (see Chapter 27 by Condorelli and Galeotti and Chapter 28 by Thomas Chaney). In an abstract context, stylized models have been proposed by Gale and Kariv (2007), Blume et al. (2009), Manea (2013), Nava (2014), and Siedlarek (2014). These papers differ in their specific details

[8] Note that, in a ring, the elimination of one of the links would put the two agents who were involved in that link at the end of a line. Thus, in order to still connect with many others, a high number of crucial intermediaries would then be needed, and hence high intermediation rents should be paid. The ring, instead, provides the two-path redundancy that allows agents to save paying those rents.

of the trading protocol (e.g., whether agreements are bilateral or multilateral) but have in common two key features: the network is taken to be exogenously given and both the network as well as all the characteristics of the market participants (costs and valuations) are common knowledge. In contrast, a recent paper by Condorelli and Galeotti (2012) studies trade and intermediation in a context where agents have incomplete information on agents' values.

On the specific context of financial networks, the issue of intermediation has drawn much recent attention, in light of the widely held that it has an important bearing on the robustness of the financial system. Interesting examples of this growing literature are Gofman (2011), Babus (2012), Farboodi (2014), Fainmesser (2012),[9] and Glode and Opp (2014). Gofman focuses on the issue of how the (given) network of trading relationships impinges on the efficiency of the system, whereas Babus and Farboodi study a context where the network is endogenous and responds to the incentives of the agents involved (say, banks or other financial institutions). Both Babus and Farboodi show that agents' incentives lead to polarized hub-spoke networks, a prediction consistent with empirical evidence on financial markets. Finally, the papers by Fainmesser and Glode-Opp explore the role of intermediation in financial markets under two different important extensions: repeated interaction (Fainmesser) and asymmetric information (Glode and Opp). Both show that intermediation can mitigate (and even eliminate) inefficiencies—those due to adverse selection in the first case, and to opportunism in the second.

Finally, in line with the issue of structural holes discussed in abstract terms by Goyal and Vega-Redondo (2007), there are two other papers that have studied the problem in a similar vein: Buskens and van der Rijt (2008) and Kleinberg et al. (2008). Both focus as well on the implications on network formation but, in contrast with the model of Goyal and Vega-Redondo, assume that the payoff for an agent is defined locally (i.e., it depends only their own set of neighbors). In the first case, an agent's payoff is identified with the negative of a magnitude introduced by Burt (1992), which he called *network constraint*.[10] Instead, in the second paper, the gross payoff of an agent i is identified with both the number of neighbors he has as well as the number of neighbor pairs he can intermediate, each of the latter being associated a weight that decreases with the number of alternative two-step paths that compete with the one provided by i.[11] Both papers single out complete multipartite (or multilevel) networks as a prominent prediction of their models.

[9] This is an earlier version of the aforementioned Fainmesser (2014). In that earlier paper the focus is on financial markets.

[10] Intuitively, the network constraint experienced by an individual is high is for the pair of neighbors he can intermediate there are many alternative two-step paths that could perform a similar function.

[11] In the setup proposed by Kleinberg et al. (2008), net payoffs are obtained by subtracting from gross payoffs the cost of links, which are assumed one-sided (and thus paid by one party) but permit two-way flows of payoffs.

9.5 Additional Contexts

As advanced, now I briefly review other socio economic phenomena where an explicit account of the interplay between actions and links also delivers important insights.

9.5.1 Bargaining

The traditional paradigm used in economics to model a market economy has been that of Walrasian equilibrium (see Chapter 2 by Alan Kirman in this volume). The standard version of this approach presumes that prices of homogeneous goods are uniform throughout the economy and trade is *anonymously* conducted at those prices. But at least since the seminal work of Rubinstein and Wolinsky (1985)—see also Gale (1987) and Rubinstein and Wolinsky (1990)—economic theory has made a substantial effort to enrich such a description/model of a market system with the explicit introduction of so-called micro-structure. Much of this recent literature (see Chapter 26 in this volume by Mihai Manea) has adopted a network approach, with buyers and sellers assumed to bargain bilaterally over time with those partners from the other side of the market to whom they are connected in some underlying trading network.

When such a trading network is bipartite-complete (every buyer and every seller are connected to each other) and the discount factor converges to one (i.e., agents become arbitrarily patient), it is straightforward to see that there must be a unique limit price at which all transactions are conducted—that is, the outcome is "arbitrage-free." But, naturally, the most interesting setup is one where the network is *not* complete and, possibly, the agents may occupy asymmetric positions in the network and thus enjoy significantly different bargaining power. In this case, the key question arises as to whether the network in place displays enough connectivity (and a suitable topology) to ensure the "law of one price." This question was addressed by Manea (2011) in an homogeneous context, where all buyers and sellers respectively have the same valuations and costs. Note that, in this case, a single prevailing price is equivalent to a *uniform payoff* for each side of the market. Manea shows that such a price and payoff uniformity situation obtains under buyer and seller homogeneity if, and only if, the underlying trading network displays what is called a perfect matching (i.e., there is a collection of buyer-seller pairs (links) where every buyer and every seller is involved in exactly one such pair).[12]

Then, in line with our approach here, from the previous characterization we are led to the following additional question: Should one expect arbitrage-free networks to obtain if, endogenously, the links are shaped by agents' own (optimizing) decisions? The answer to this question, provided as well by Manea (see the online appendix

[12] In general, a matching—possibly non-perfect—is defined as a collection of links satisfying the restriction that every node is in involved it at most one of them (possibly none).

to the aforementioned paper), is again a sharp one: a network is *pairwise stable* (cf. Jackson and Wolinsky 1996 and Chapter 5) if, and only if, it is arbitrage-free (or nondiscriminatory[13]). Thus, in this sense, price/payoff uniformity is intimately associated to the possibility that the price and the underlying trading networks be endogenously co-determined. A conceptual limitation of this result is that freedom of arbitrage imposes too little structure on the trading network. For, in general, a wide range of arbitrage-free networks—including in *every case* the complete one—are pairwise-stable.

Polanski and Vega-Redondo (2014a) extend the previous analysis to a general context, with an arbitrary distribution of buyers' valuations and sellers' costs. In this context, price uniformity no longer entails payoff uniformity within each side of the market and the outcome (in particular, the payoffs) depends on a rich interplay between network topology and types (valuations and costs). They characterize those networks that induce an arbitrage-free outcome in terms of a condition that can be viewed as a market-based version of the well-known result of Graph Theory—the so-called Marriage Theorem by Hall (1935)—that characterizes the existence of perfect matchings in binary networks. But then, we can ask once more: How are matters affected if (under type heterogeneity) the trading network is endogenous?

Polanski and Vega-Redondo address the previous question but, in contrast with Manea (2011), they do it under the assumption that links are costly (infinitesimally so). This, in essence, implies that the unilateral and bilateral incentive conditions embodied by the notion of pairwise stability require a *strictly positive* gain of linking. Under these conditions, every pairwise stable network is still found to be arbitrage-free. However, the converse conclusion is no longer true because costly linking rules out those networks that include "strategically irrelevant" links—in particular, the complete network is always discarded. An important consequence is that pairwise stability introduces a specific network structure, which is generally nontrivial. It turns out, in particular, that pairwise-stable networks lead to inherent waste (allocation inefficiency) if the matching procedure is decentralized. (Roughly, decentralized matching does not allow the reliance on some global mechanism and prescribes that pairs of agents bargain and trade when they can profitably do so.) This illustrates once more the interesting insights that may arise when a network-based model of economic interaction—trade, in the present case—is subject to the discipline of incentive compatibility in the process of link creation and destruction.

Two additional recent papers in a similar vein are those by Elliot (2014) and Elliott and Nava (2014). The former considers a setup where, in a first stage, the agents have to decide on whether to devote resources to partner-specific investments that will allow them to bargain with those partners in a second stage. The first stage, therefore, can be essentially conceived as determining the trading network on which subsequent bargaining (modeled as the outcome of a cooperative matching procedure)

[13] Manea (2011) uses this terminology since, as explained, under buyer and seller homogeneity they imply a uniform payoff across buyers and sellers.

takes place in the second stage. Elliott finds that, depending of the protocol through which linking costs are divided between partners, inefficiencies of two polar sorts (under- or over-investment) can arise. On the other hand, the paper by Elliott and Nava (2014) models both partner selection and bargaining as part of a single non-cooperative dynamic game. Their analysis relies on an intuitive comparison of the "Rubinstein-type payoffs" that would arise in bilateral bargaining with those belonging to the Core of the economy. They show, specifically, that an efficient equilibrium exists if, and only if, the aforementioned Rubinstein payoffs lie in the Core. These two papers therefore provide an additional illustration of how a modeling approach where links and actions are endogenously determined can enhance the sharpness of the conclusions. In their case, it sheds light on the important issue of how the primitives of the model (e.g., valuation and costs) impinge on the efficiency of the outcome.

9.5.2 Local Public Goods

In Section 9.2, I discussed coordination games, which is a particularly stark example where the actions of players are strategic complements—that is, the incentives for choosing a particular action grow with the number of partners who choose that same action. A polar context is one where players' actions are strategic substitutes (see Chapter 8). In this case, as the number of partners who choose a certain action grows, it is instead the incentive to use the *alternative* strategies that becomes stronger. One example of this type of network games are anti-coordination games, which are studied, for example, by Bramoullé et al. (2004). They focus on how actions and links are jointly co-determined as part of an equilibrium in an overall simultaneous-move game, and characterize how the networks thus induced depend on the cost of forming links.

Another interesting case is provided by the study of local public goods, as studied for example by Bramoullé and Kranton (2007) in a context where agents are located on a *given* network. These authors suppose that agents have to choose a level of costly effort that generates positive spillovers on their immediate neighbors. A natural interpretation is that individual effort is devoted to gathering information, which is then freely shared with neighbors. Depending on the network structure, a wide multiplicity of *very different* equilibria exist. Thus, if agents are arranged in a star, one equilibrium has the center be the sole contributor of effort, while the opposite configuration where the peripheral agents contribute (and the center not) defines an equilibrium as well. Clearly, the former is much more cost-effective (i.e., efficient) than the latter. This begs the question of how matters would be affected if the network were allowed to be determined endogenously. The issue has been addressed by Galeotti and Goyal (2010), who pose a network-formation model where agents can choose (unilaterally and at an individually born cost) with whom to connect and hence enjoy spillovers from.[14] Their

[14] Another way to tackle the problem is to postulate specific mechanisms/criteria of equilibrium selection. This is the approach pursued by López-Pintado (2008), who studies classical (myopic)

main result is that any arbitrarily small asymmetry in the cost of effort induces a single robust equilibrium where the lowest-cost individual is the center of a star and all others establish connections to him. So, in this sense, allowing for a joint determination of action and links supports an efficient effort profile even if, as explained, the associated network configuration (a star) is one where a large inefficiency could in principle obtain at equilibrium if that same star network were to be considered exogenous.

9.5.3 Conflict

Conflict is present in many economic and social interactions, and indeed most games of theoretical and practical interest do have a "conflict" component (see Chapter 10 in this volume by Sanjeev Goyal and coauthors). As a paradigmatic representation of "pure" conflict situations, there are two leading formulations in the literature: the contest-function approach pioneered by Buchanam and Tullock (1962) and that going under the general term of Colonel Blotto games (see Roberson 2006, or Kovenock and Roberson 2012 for a recent survey). Somewhat surprisingly, however, the literature that approaches the problem from a network viewpoint is scarce.

There is some recent literature, discussed in Chapter 13, where a network defines the nature of the conflict by specifying the routes or nodes through which value is generated—and, correspondingly, where attack and defense should gravitate. Another approach is the one pursued by the recent paper of König et al. (2014), which relies on a *binary signed network* to represent the patterns of alliances (friends and enemies) in an all-out conflict set out to determine the shares in some fixed surplus. Just very few papers, however, do model a network of interrelated bilateral conflicts, with the important spillovers that should typically flow across them. A notable exception is the paper by Franke and Öztürk (2009), which explores a context where agents are involved in a set of bilateral conflicts, each one of them modeled as a contest game. The interconnection among different conflicts derives from the fact that, for every agent, the resources devoted to all conflicts in which he is involved induce a total cost that is given by a *convex* function of the aggregate amount. They consider different stylized contexts (star, regular, complete networks) and study how the intensity of conflict depends on the parameters of the model.

All of the papers mentioned above posit a fixed network, signed or not, which is exogenously given. In contrast, the recent papers by Hiller (2012) and Huremovic (2014) extend the analysis to contexts where the network itself is chosen by the agents as part of a network-formation game. In Hiller's model, agents have to choose with what other agents they want to have positive and negative relationships (i.e., their friends and foes). These one-sided decisions, in the end, determine a signed, binary, and directed

best-response dynamics through which a large population adjusts their behavior over time. Relying on a mean-field approach, she shows that a unique stable equilibrium exists, and the fraction of agents devoting effort decreases as the connectivity of the network rises or becomes more spread out.

network. The key feature of his model is that, for any individual agent, the strength he musters in each of the conflicts he has with his enemies increases with the number of his friends. Instead, Huremovic's approach involves two-sided links, with two agents becoming enemies if at least one of them decides so. Then, between all pairs of enemies, a corresponding collection of interrelated contest games are simultaneously played, in a setup quite similar to that considered by Franke and Öztürk (2009).

The notions of network stability are formally different in the aforementioned papers by Hiller and Huremovic, but conceptually they are quite similar. On the one hand, positive links are created and maintained as usually postulated in the literature (i.e., by bilateral consensus). Instead, the formation of negative (or conflict) links is carried out in a polar fashion: whereas such links can be created unilaterally (i.e., if just one of the agents involved decides to "fight"), their removal requires bilateral agreement (i.e., "peace treaties" must be signed by both parties). Interestingly, despite their very significant differences, the two models deliver quite parallel predictions and insights. Specifically, they show that at a strategically stable outcome, the population must be arranged as a complete k-partite network. Each of the k parts can be conceived as coalitions, with all agents in each of them being friends among themselves but in an all-out conflict with those from all other coalitions. This is an interesting conclusion in two respects. First, empirically, it is in line with evidence observed in real-world contexts so diverse as large online social media, or the pattern of country alliances in international conflict (see, e.g., Leskovec et al. 2010 and Antal et al. 2006). Second, theoretically, it is consistent with the predictions of Structural Balance Theory (see Heider 1946 and Cartwright and Harary 1956), which has been long used in sociology as a canonical framework to study signed social networks. In a sense, one can view the models of Hiller and Huremovic as providing alternative game-theoretic foundations of Structural Balance Theory.

9.5.4 Learning

The study of how the structure of social networks impinges on social learning has spanned a large literature, which is the subject of Chapter 12. Here, in order to illustrate the phenomenon at hand (i.e., the effect on learning of an endogenous action-link interplay), I choose a particularly simple model that goes back to the work of De Groot (1974) and has received fresh attention from recent work by Golub and Jackson (2010, 2012). In this model, the pattern of "influence" in a society is represented by a directed weighted network, with an agent i being influenced by another one j as captured by the corresponding weight a_{ij} in the adjacency matrix A defining the social network. The basic modeling assumption is that the action (or belief) displayed by any given agent at some point in time t is simply a convex combination of the actions displayed at $t-1$ by his neighbors (i.e., those who have an influence on him, generally including himself). The main implication is then that, if the network is connected (everyone is at least indirectly connected to everyone else), as the number of rounds grows unboundedly the system converges to a state of consensus where everyone in the population ends up sharing the same action/belief. Furthermore, the effect that the initial action of a player has on such a final outcome depends on some suitable measure of his network centrality.

The approach just summarized presumes that the social network is exogenously given, independently of the learning outcome. That is, the assumption is that there is no converse effect, from the learning outcome to the influence matrix. But, in fact, empirical evidence suggests that such a reciprocal relationship is strong in many real-world contexts. For example, it has been amply documented in the political arena, where an agent's ideology (or *a priori* position in the political spectrum) affects significantly the range of agents with whom he interacts (and hence the set of individuals who can influence him) (see, e.g., Adamic and Glance 2005; Boutyline and Willer 2013; or Colleoni et al. 2014). In quite different context, a similar phenomenon has also been shown to arise among Wikipedia editors, as they edit different articles and correspondingly change their editing concerns—see Crandall et al. (2008). The main feature that transpires from all this evidence is that agents tend to display significant homophily (i.e., a tendency to connect to those who share their actions or beliefs). This then must have important implications on the learning process. Intuitively, one expects that homophily tends to segment the interaction of the population into distinct communities and hence must also affect the extent to which the actions/beliefs of the different agents tend to converge—or, at the very least, the rate at which this happens.[15] And then, by having the forces at work feed on each other, the overall interplay of action and link dynamics may well lead to an exacerbation of social segmentation.

To study the problem, a model displaying an endogenous co-determination of influence and learning in the De Groot framework has been recently proposed by Polanski and Vega-Redondo (2014b). In contrast to the received approach, it is assumed that, for any given network of interagent influence, a large number of learning rounds are conducted so that:

(a) at the the start of each of them, agents receive individual signals (possibly correlated among them but stochastically independent in time) that determine their initial action/belief;
(b) social learning then proceeds for only a finite (possibly large) number of steps.

Because of (b), full convergence of actions is generally not achieved at the end of each round. Hence, in the absence of complete consensus, (a) implies that one can *non-trivially* define the degree of correlation among the actions displayed by the different agents across all the different rounds, each of these starting with a fresh pattern of signals. Then, given the induced pattern of correlations, an influence network is said to be in *equilibrium* if it satisfies the following homophily condition: for each

[15] Golub and Jackson (2012) study how homophily affects the rate at which the population converges to consensus in a connected network.

pair of individuals connected by the social network, their bilateral influence weight is proportional to their corresponding bilateral correlation.

The equilibrium analysis just outlined sheds light on the following important questions:

(i) Will the population be fragmented in separate influence components?
(ii) What is relationship between link weights, informational/influence redundancy, and segmentation?

The relevance of (i) is illustrated by the empirical evidence on action/belief segmentation discussed above. The interest of (ii), on the other hand, relates to the celebrated hypothesis of Granovetter's that strong ties are (informationally) weak. As explained in Easley and Kleinberg (2010, Chapter 3), the usual motivation for this hypothesis is based on the notion of "strong triadic closure," the postulate that when two agents are *strongly* connected to a common third agent, they themselves will tend to become connected as well. Instead, the model of Polanski and Vega-Redondo (2014b) relies on the converse idea that whenever two agents have common neighbors, these act as shared "anchors" that render their actions correlated and hence make their link strong. The two approaches highlight different mechanisms. Whereas the first has link strength impinge on link formation, in the second it is the topology of existing connections that determines the pattern of link strengths. One expects that, in the real world, both of these mechanisms should act in a complementary manner whenever link strengths and action choices evolve in full interplay.

9.6 Summing Up

In this chapter I have stressed the importance of modeling socioeconomic network phenomena in a theoretical setup where actions and links are in reciprocal interplay. By endogenously determining the prevailing network, this approach often succeeds as well in producing sharp predictions on agents' behavior. In a sense, formulated in such a general fashion, the previous point is a trivial one: extending the range of endogenous features can never harm the modeling effort, the only possible drawback being that it makes the analysis more involved. This is why I have chosen to illustrate its practical significance by discussing a set of concrete and paradigmatic contexts. Many of these contexts concern important problems that are discussed in more detail in other chapters of this volume. Specifically, I have focused on coordination, cooperation, and intermediation in Sections 9.2 through 9.4 while, more briefly, I have discussed bargaining, public goods, conflict, and learning in Section 9.5.

In all of those cases, we have seen that, allowing for a genuine co-determination of actions and links sheds new and sharper light on the phenomena at hand, by (a) singling out definite network structure where otherwise there would be little basis to select it, and (b) identifying specific behavior associated to that structure, in cases where a wide (equilibrium) multiplicity would be *a priori* possible. Obviously, the empirical relevance of the analysis has to be qualified by the fact that, in the real world, network formation is also subject to many factors other than just action choice (anticipated or effective). For example, individuals tend to obtain important economic information from friends, but these are often not chosen with such "instrumental" considerations in mind. However, the fact that so many important phenomena (e.g., learning, cooperation, or intermediation) do hinge upon the reciprocal feedback of link and action choice suggests that, even if carried out in a stylized manner, a proper study of such interplay can hardly be avoided.

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