Ralph Arvin De Castro
0923223

Assignment 3
CIS*2910
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(1) a) E Probability that at least one of the results was 6

$= 1 -$ probability that there is no 6

$= 1 - \left(\frac{5}{6} \times \frac{5}{6}\right)$

$= 1 - \frac{25}{36}$

$= \frac{11}{36}$

F Probability that the sum of the result is seven and at least one is 6 $=$ $(6,1)$ $(1,6)$ $= \frac{2}{36}$

$p(E|F) = \frac{p(E \cap F)}{p(F)}$

$= \frac{\frac{2}{36}}{\frac{11}{36}}$

$= \frac{2}{11}$

b) The answer would also be $\frac{2}{11}$ since probability that at least one of the result of 5 is $\frac{11}{36}$ and probability that the sum of the result is seven and at least one is 5 $(5,2)$ $(2,5)$ is $\frac{2}{36}$

$= \frac{\frac{2}{36}}{\frac{11}{36}}$

$= \frac{2}{11}$

(2) • Probability by not changing is still $\frac{1}{4}$.

• Probability by changing the door is $\frac{3}{8}$ since probability of getting the goat initially is $\frac{3}{4}$ and since Monty eliminates a door with a goat, there is $\frac{1}{2}$ (one of two remaining doors) chance to win. Therefore

$$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

(3) • Let the bipartite graph be divided into two Graph p and q and total number of edges is $n = p+q$.

The maximum number of edges is therefore pq: therfore, product of pq is highest when $p = q$ which is equal to $\frac{n^2}{4}$. This is because when we set $q = n-p$ and we try ~~are trying~~ to maximize $f(p) = p(n,p)$ on $[0, n]$, ~~[illegible]~~ we have $f'(p) = n-2p$ and this is 0 when $p = \frac{n}{2}$. Therefore, $f(p) = p(n-p)$

$$f(p) = \frac{n}{2}\left(n - \frac{n}{2}\right)$$

$$= \frac{n^2}{4}$$

Therefore, ~~[illegible]~~ a bipartite simple graph with V nodes and e edges would have $\leq \frac{V^2}{4}$ number of edges $\boxed{e \leq \frac{V^2}{4}}$