Welcome everyone ☺

Today's Agenda :-

→ Modulas Basics

→ Modulas Properties

→ Interesting Problem (asked in Google).

Range of an int :→ $[-2 * 10^9, 2 * 10^9]$

Range of a long :- $[-8 \times 10^{18}, 8 \times 10^{18}]$

N % a = Remainder when N is divided by a

10 % 4 = 2 , 13 % 5 = 3

Dividend = Divisor x Quotient + Remainder

Remainder = Dividend − Divisor x Quotient

Divisor x Quotient → Greatest Multiple of Divisor <= Dividend.

$\boxed{11 \times 13}$

Divisor ← 11) 150 (13 → Quotient

150 → Dividend

11

40

33

7 → Remainder

Q1) $150 \% 11 = 150 - \{\underbrace{\text{Greatest Multiple of } 11 <= 150}\}$

$\Rightarrow 150 - \{143\}$

$\Rightarrow \boxed{7}$

Q2) $100 \% 7 = 100 - \{\underbrace{\text{Greatest Multiple of } 7 <= 100}\}$

$= 100 - 98$

$\Rightarrow \boxed{2}$

Q3) $-40 \% 7 = -40 - \{\underbrace{\text{Greatest Multiple of } 7 <= -40}\}$

$\Rightarrow -40 - (-42)$

$\Rightarrow -40 + 42 \Rightarrow \boxed{2}$

Q4) $-60 \% 9 = -60 - \{\underbrace{\text{Greatest Multiple of } 9 <= -60}\}$

$\Rightarrow -60 - (-63)$

$\Rightarrow -60 + 63 \Rightarrow \boxed{3}$

Python :-	C/C++/Java/JS
$-40 \% 7 = 2$	$-40 \% 7 = -5$
$-60 \% 9 = 3$	$-60 \% 9 = -6$
$-10 \% 3 = 2$	$-10 \% 3 = -1$

if $a < 0$, $a \% m = (a \% m) + m$

$40 \% -7 = 40 - \{\text{Greatest Multiple of } -7 <= 40\}$

$\Rightarrow 40 - 35$

$\Rightarrow \boxed{5}$

Why %? → limit our data to some required range.

Ex :→ Hashing – Hashmap
Consistent Hashing
Cryptography.

$\left.\begin{matrix}\infty \\ \vdots \\ -\infty\end{matrix}\right\} \% M = [0, M-1]$

Modular Arithmetic :-

$[0, M-1] + [0, M-1] > (M) \% M \rightarrow [0, M-1]$

1) $\boxed{\underbrace{(a+b) \% M}_{[0,\,M-1]} = (a \% m + b \% m) \% m}$

Eg $a = 6, b = 8, M = 10$

$(14) \% 10 \qquad (6 \% 10 + 8 \% 10)$

$\qquad\qquad\qquad 6 + 8$

$4 \neq 14 \% 10$

$4 = 4$

2) $\boxed{(a * b) \% M = (a \% m * b \% m) \% m}$

Eg $a = 6, b = 8, M = 10$

$(6 * 8) \% 10 \qquad (6 * 8) \% 10$

$\Rightarrow 48 \% 10 \qquad \Rightarrow 48 \% 10$

$\underline{\underline{8}} \qquad\qquad \underline{8}$

// No inbuilt functions.

// power(a, n, p) => Calculate $a^n \% p$.

$a = 2,\ n = 5,\ p = 7 \Rightarrow 2^5 \% 7 = 32 \% 7 = 4$

$a = 3,\ n = 4,\ p = 6 \Rightarrow 3^4 \% 6 = 81 \% 6 = 3$

4 Minutes

Constraints:

$1 <= a <= 10^9$
$1 <= N <= 10^5$
$1 <= p <= 10^9$

$10^{N-1} \% p$

we implemented.

$a^N \% p$

power($\underset{10}{a}$, $\underset{N-1}{n}$, $\underset{p}{p}$) {

Input:

$a = 2,\ N = 40,\ ans = 2^{40}$

$a = 2,\ N = 100,\ ans = 2^{100}$

long ans = 1

for(i = 1 ; i <= N; i++) {

In the for loop, ans can overflow.

ans = ans * a *Overflow

// ans = $\underbrace{(ans * a) \% p}_{[0,\ p-1]}$ ✓

$[0, p-1] \to 10^9$

$* \simeq 10^{18}$

10^9

ans * a

↳ overflow

$\boxed{ans = (ans \% p * a \% p) \% p}$ ✓

}

return ans % p

}

$a \to 10^{18}$

$p \to 10^9$

ans = (ans * a) % p

$[0, 10^9]$

$(10^9 * 10^{18})$

TC :→ O(N)

SC :→ O(1)

ans = (ans % p * a % p) % p

$(10^9 * 10^9)$

$(10^{18})\ \%\ p \to 10^9$.

Dry Run $\underline{a, N=5, p}$ $\Rightarrow$ $\underline{a^5 \% p}$, ans = 1

i	ans = (ans * a) % p	ans
1	ans = (1 * a) % p	a % p
2	ans = (a % p * a) % p	a^2 % p
3	ans = (a^2 % p * a) % p	a^3 % p
4	ans = (a^3 % p * a) % p	a^4 % p
5	ans = (a^4 % p * a) % p	a^5 % p

// Divisibility Rule of 3 ?

↳ (Sum of all digits) % 3 = 0 } ⇒ child hood.

$(3458) = (3 * 10^3 + 4 * 10^2 + 5 * 10^1 + 8 * 10^0)$

$1 \% 3 = 1$

$10 \% 3 = 1$

$10^2 \% 3 = 1$

$10^3 \% 3 = 1$

$10^x \% 3 = 1$

$(3458) \% 3 = (3 * 10^3 + 4 * 10^2 + 5 * 10^1 + 8 * 10^0) \% 3$

$\Rightarrow ((3 \times 10^3) \% 3 + (4 \times 10^2) \% 3 + (5 \times 10^1) \% 3 + (8 \times 10^0) \% 3) \% 3$

$\Rightarrow (3 + 4 + 5 + 8) \% 3$

// Divisibility Rule of 9 ⇒ (Sum of digits % 9 == 0)

$1 \% 9 = 1$

$10 \% 9 = 1$

$10^2 \% 9 = 1$

|

$10^x \% 9 = 1$

// Divisibility Rule of 4

$\hookrightarrow$ (last 2 digits) % 4 == 0

$$(4328) = (4 \times 10^3 + 3 \times 10^2 + 2 \times 10^1 + 8 \times 10^0)$$

$$(4328) \% 4 = (4 \times 10^3 + 3 \times 10^2 + 28) \% 4$$

$$= ((4 \times \underbrace{10^3) \% 4}_{0} + \cancel{3} \times \underbrace{10^2) \% 4}_{0} + 28 \% 4) \% 4$$

$$\Rightarrow (28) \% 4$$

$1 \% 4 \neq 0$

$10 \% 4 \neq 0$

$10^2 \% 4 = 0$

$10^3 \% 4 = 0$

$10^4 \% 4 = 0$

$10^x \% 4 = 0$

5 Min

Que :- Given a number in arr[ ] . Calculate number % p

each ar[i] contains single digit of number.

Constraints

$\{ 1 <= N <= 10^5 \ ; \ 1 <= p <= 10^9 \}$

10^{10}

Ex N = 5

	0	1	2	3	4
arr[5]:	3	2	4	6	4

p = 4

$\hookrightarrow (32464) \% 4 \Rightarrow 0 \rightarrow$ o/p.

Idea1: Convert entire arr[ ] into a number % p

If N = 9, 9 digits = 10^9.

N = 18, 18 digits $\Rightarrow 10^{18}$

N = 100 : 100 digits = 10^{100}

N = 10^5 10^5 digits – 10^{10^5}

Idea 3:

0	1	2	3	4	5
3	2	8	4	2	9

arr[6] :-

$((328429)) \% 4 \Rightarrow$

$((3 * 10^5)) \% 4$
$+$
$\Rightarrow (2 \times 10^4) \% p$
$+$
$(8 \times 10^3) \% p$
$+$
$(4 \times 10^2) \% p$
$+$
$(2 \times 10^1) \% p$
$+$
$((9 \times 10^0) \% p) \% 4$

In general arr[N],

0

$\underbrace{(arr[0] * 10^{N-1})}_{a \quad b} \% p$

$a^N \% p$

$\Rightarrow [arr[0] \% p * ((10^{N-1} \% p))] \% p$

$N = 10^5$

Math.pow(10, N-1) % P

Math.pow(10, 10^5) ⇒ 10^{10^5}

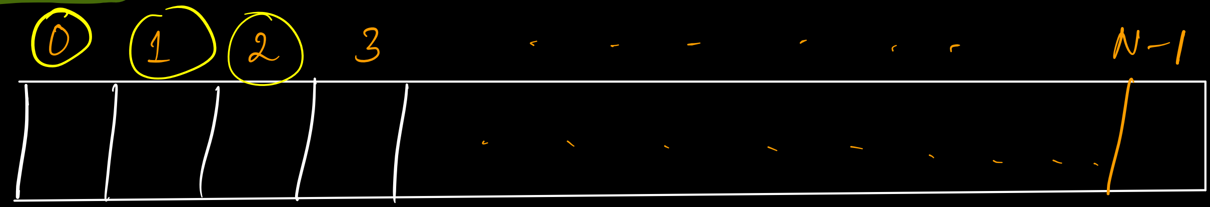
$(10^{N-1} \% p)$

$\Rightarrow [arr[0] \% p * [power(10, N-1, p)]] \% p$

// Generalization :-

$i \rightarrow N-i-1$

A[N] :→



$$\left[(arr[0] * 10^{N-1}) + (arr[1] * 10^{N-2}) + (arr[2] \times 10^{N-3}) + \dots (arr[N-1] * 10^{0}) \right] \% p$$

$$\Rightarrow \left[(arr[0] * 10^{N-1}) \% p + (arr[1] * 10^{N-2}) \% p + (arr[2] \times 10^{N-3}) \% p \dots \right] \% p$$

$$\Rightarrow \big(arr[0]^{\%p} * power(10, N-1, p) \big) + \big(arr[1]^{\%p} * power(10, N-2, p) \big) + \big(arr[2]^{\%p} * power(10, N-3, p) \big) \dots \big) \% p$$

```
ans = 0
→ for(i=0; i<N; i++){
    ans = ans + [(arr[i] %p) * (power(10, N-i-1, p))] %p
  → ans = ans %p;
}
return ans;
```

power(10, N-i-1, p) → O(N)

exp

↳ [arr[i] * 10^{N-i-1}] %p.

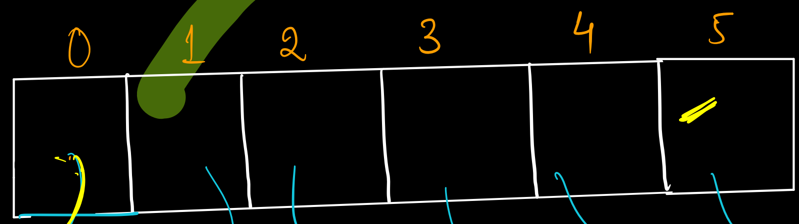
↳ TC : O(N^2)

SC : O(1)

// Optimization :-

arr[i] * 10^{N-i-1} ↳ exponent

arr[6] ⇒



exp = 1

(arr[0] * 10^5) % p (arr[1] * 10^4) % p

10^5 % p	10^4 % p	10^3 % p	10^2 % p	10^1 % p	10^0 % p
exp = ((exp)*10) % p	exp = (exp*10) % p	exp = (exp*10) % p	exp = (exp*10) % p	exp = (exp*10) % p	
exp = 10^5 % p	exp = 10^4 % p	exp = 10^3 % p	exp = 10^2 % p	exp = 10^1 % p	

Calculate exp from Right to left.

TC : O(N)
SC : O(1)

```
long ans = 0
long exp = 1
for(i = N-1; i >= 0; i--){
    ans = ans + ((arr[i] % p) * exp) % p
    ans = ans % p
    exp = (exp * 10) % p;
}
return ans;
```

// Division doesn't hold true :-

$$\left(\frac{a}{b}\right) \% m = \left(\frac{a \% m}{b \% m}\right) \% m$$