→ Time Complexity & Space Complexity
→ Asymptotic Analysis
→ Big O
→ TLE - Time Limit Exceeded

TC - 2

Today's :- How to calculate No. of iterations ?

Quiz1 Sum of N Natural Numbers $= \frac{N * (N+1)}{2}$

Quiz2 [3, 10] ⇒ 3, 4, 5, 6, 7, 8, 9, 10 :→ 8

→ 10 − 3 + 1

[a, b] :→ b − a + 1

Quiz3 $N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \cdots \rightarrow 1$: $\log_2 N$

Arithmetic Progression :-

$\boxed{\text{Common difference} = d}$

4 7 10 13 16 19

3 3 3 3 3 $\Rightarrow d = 3$

In general

1 2 3 4 N^{th}

a , $a+d$, $a+2d$, $a+3d$, ... $a+(N-1)d$

Sum of N terms in AP $\Rightarrow$ $\boxed{\frac{N}{2}\left[2a+(N-1)d\right]}$

First term = a
Common Diff = d

Geometric Progression (GP)

$\boxed{\text{Common Ratio} = r}$

3 6 12 24 48

2 2 2 2

In general

1 2 3 N^{th} term

a , ar, ar^2, ... ar^{N-1}

Sum of N terms in GP :- $\boxed{a * \left[\frac{r^N - 1}{r - 1}\right]}$ $r != 1$

r is common ratio

Q1)

```
void fun(int N){
    s = 0
    for(i = 1; i <= N ; i++){
        s = s + i ;
    }
    return s ;
}
```

a , b

$i \in [1, N]$

$\Rightarrow N - 1 + 1$

$\Rightarrow N$

$\boxed{\Rightarrow O(N)}$

Q2)

```
Void fun(int N, int M){
    for(i = 1 ; i <= N ; i++){
        if(i % 2 == 0){
            print(i);
        }
    }
    for(i = 1; i <= M; i++){
        if (i % 2 == 0){
            print (i);
        }
    }
}
```

N

M

$(N + M)$

$\boxed{\Rightarrow O(N + M)}$

Q3)
```
int fun(int N){
    s=0
    for(i=1; i<=N ; i+=2){
        s = s+i
    }
}
```

$\Rightarrow O(N)$

$i = 1 \xrightarrow{+2} 3 \xrightarrow{+2} 5 \xrightarrow{+2} 7 \xrightarrow{+2} 9$

$\Rightarrow$ covering odd Nos

Iterations = Number of odd Numbers from $[1, N] = \boxed{\frac{(N+1)}{2}}$

$N=10 \Rightarrow \frac{10+1}{2}$??

$i = 1, 3, 5, 7, 9$

$N=13$

$i = 1, 3, 5, 7, 9, 11, 13$

iterations

5 $\frac{10}{2}$

7 $\frac{13+1}{2}$

$\frac{(N+1)}{2}$

$\left(\frac{N+1}{2}\right)$

Q4)
```
int fun(int N){
    s=0
    for(i=0; i<=100 ; i++){
        s = s+ i+i^2 ;
    }
    return s;
}
```

$[0, 100]$

$\Rightarrow 100-0+1$

$\Rightarrow 101$

$\Rightarrow O(1)$

Q5)

```
void fun(int N){
    for(i=1; (i*i<=N); i++){
        s = s + i^2
    }
    return s;
}
```

$i <= \sqrt{N}$

$[1, \sqrt{N}]$

$\Rightarrow \sqrt{N} - 1 + 1$

$\Rightarrow \sqrt{N}$

$\Rightarrow O(\sqrt{N})$

Q6)

```
void fun(int N){
    i = N
    while(i>1){
        i = i/2
    }
}
```

$\Rightarrow \log_2 N$

$O(\log N)$

iterations	i value	
1	N/2	$N/2^1$
2	N/4	$N/2^2$
3	N/8	$N/2^3$
4	N/16	$N/2^4$
⋮	⋮	
K	$N/2^K$	

$N \to \frac{N}{2} \to \frac{N}{4} \to \frac{N}{8} \dots \to 1$

$\to \boxed{\log a^b = b \log a}$

$\to \boxed{\log_a a = 1}$

$\Rightarrow$ After K iterations, loop breaks

$i = \frac{N}{2^K} = 1 \Rightarrow \boxed{N = 2^K}$

$\to$ Take $\log_2$ both sides $\Rightarrow \log_2 N = \log_2 2^K$

$\Rightarrow \log_2 N = K \cancel{\log_2 2}$ 1

$\Rightarrow \boxed{K = \log_2 N} \Leftarrow$

Q7) void fun(N) { $N \geq 0$

$s = 0$

for($i=0$; $i \leq N$; $i = i*2$){

$s = s + i$

}

}

0, 0, 0, 0, 0, 0, 0

⇒ ∞

Infinite

⇓ O(   )

Q8) void fun(N) {

$s = 0$

→ for ($i = 1$; $i < N$; $i = i*2$) ←

$i = i*3$

}

}

$i >= N$

$i = N$

⇓ $\log_2 N$

⇓ $O(\log N)$

iterations	value of i	
1	$2 = 2^1$	3^1
2	$4 = 2^2$	3^2
3	$8 = 2^3$	3^3
4	$16 = 2^4$	3^4
⋮	⋮	⋮
K	2^K	3^K

After K iterations, the loop breaks,

$$2^K = N$$

$$\log_2 2^K = \log_2 N$$

$$\Rightarrow K \log_2 2 = \log_2 N$$

$$\Rightarrow \boxed{K = \log_2 N}$$

$$3^K = N$$

$$\log_3 3^K = \log_3 N$$

$$\boxed{K = \log_3 N}$$

8 Min Break

Nested Loops

9) void fun(N){

```
for(i=1; i<=10; i++){
    for(j=1; j<=N; j++){
        print("Sky");
    }
}
```

Tables

i	j	
1	[1, N]	→ N +
2	[1, N]	→ N +
3	[1, N]	→ N +
¦	¦	
10	[1, N]	→ N

$\}$ 10

Total # of iterations $\Rightarrow$ 10N

10 * N times

O(N)

10) Void fun(N) {

```
for(i=1; i<=N; i++){
    for(j=1; j<=N; j++){
        print(i*j)
    }
}
}
```

i	j	iterati
1	[1, N]	N +
2	[1, N]	N +
¦	¦	¦ +
N	[1, N]	N

$\Rightarrow N + N + N + \ldots + N$

$\Rightarrow N * N$

$\Rightarrow N^2$ $\Rightarrow O(N^2)$

11) void fun(N){ $\boxed{i<N}$

```
for(i=0; i<N; i++){
    for(j=0; j<=i; j++){
        print("monkey");
    }
}
}
```

i	j	iterations
0	[0,0]	1 +
1	[0,1]	2 +
2	[0,2]	3 +
3	[0,3]	4 +
⋮	⋮	⋮ +
N-1	[0,N-1]	N

$1 + 2 + 3 + 4 + \; --- + N$

$\Rightarrow$ Sum of first N Natural Nos.

$$\Rightarrow \boxed{\frac{N*(N+1)}{2}} \Rightarrow \frac{N^2+N}{2} \Rightarrow \frac{N^2}{2} + \frac{N}{2}$$

$$\Downarrow$$

$$\frac{N^2}{2} + \frac{N}{2}$$

$$\Rightarrow N^2 + \cancel{N} \qquad N^2 + \cancel{N}$$

$$\Rightarrow O(N^2)$$

$N = 10^6$

$10^{12} + 10^6$

$\approx 10^{12}$

Q12) void fun(int N) {

for(i=1; i<=N; i++) {

[1, N] ← for(j=1; j<=N; j*=2) {

$\downarrow$

$\log_2 N$

print(i*j);

}

}

}

i	j	iteration
→ 1	[1, N]	→ $\log_2 N$ +
→ 2		$\log_2 N$ +
		+
→ N	[1, N]	$\log_2 N$

$\underbrace{\log N + \log N + \log N + \dots \log N}_{N}$

$N * \log N$

$\boxed{N \log_2 N}$

$O(N \log N)$

2 + 2 + 2 + 2

2 * 4

Q13) void fun(N) { $i <= 2^N$

for(i=1 ; $i <= 2^N$; i++) {

print(i)

}

}

$2 * N = N$

$2^N = N$

$\Downarrow$

$O(2^N)$

$[1, 2^N]$ $2^N - 1 + 1$

2^N iterations

$\boxed{1 << i = 2^i}$ ←

Q14)

```
void fun(N){
    for(i=1; i<=N; i++){
        for(j=1; j<=2^i; j++){
            print(i*j)
        }
    }
}
```

$\Downarrow$

O(

i	j	iterations
1	$[1, 2^1]$	$\Rightarrow 2^1$ +
2	$[1, 2^2]$	$\Rightarrow 2^2$ +
3	$[1, 2^3]$	$\Rightarrow 2^3$ +
|	|	| +
N	$[1, 2^N]$	$\Rightarrow 2^N$

Total iterations

$$\Rightarrow 2^1 + 2^2 + 2^3 + 2^4 + - - + 2^N$$

$\Rightarrow$ Sum of N terms in GP $= \dfrac{a * (r^N - 1)}{r - 1}$

$$\left\{ \begin{array}{l} a = 2 \\ r = 2 \\ N = N \end{array} \right\}$$

$$\Rightarrow \frac{2 * (2^N - 1)}{2 - 1}$$

$$\Rightarrow 2 * (2^N - 1)$$

$$\Rightarrow \cancel{2} * 2^N - \cancel{2}$$

$$\Downarrow$$

$$O(2^N)$$

Amazon's MCQ :-

Q15)

```
void fun(N){
    for(i = N ; i > 0 ; i = i/2) {
        for( j = 1 ; j <= i ; j++ ){
            print(i+j);
        }
    }
}
```

i	j	iterations
N	$[1, N]$	N +
$N/2$	$[1, N/2]$	$N/2$ +
$N/4$	$[1, N/4]$	$N/4$ +
$N/8$	$[1, N/8]$	$N/8$ +
⋮	⋮	⋮ +
1	$[1, 1]$	1

Total iterations :

$$\Rightarrow N + \left\{\frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \cdots + 1\right\} \Rightarrow N + N - 1 \Rightarrow 2N-1$$

$$N + \{N-1\} \Rightarrow 2N-1$$

$a = N$

$r = 1/2$

$t =$

$$\text{Sum of GP} = \frac{a \times (r^t - 1)}{r - 1}$$

$$2 \to 2^2 \to 2^3 \to \ldots 2^k$$

$$N \to \left[\frac{N}{2} \to \frac{N}{4} \to \frac{N}{8} \to \cdots\cdots 1\right]$$

$$\left[\frac{N}{2^1} \quad \frac{N}{2^2} \quad \frac{N}{2^3} \to \cdots \frac{N}{2^k}\right]$$

$$\Rightarrow \frac{N}{2^k} = 1$$

$$\Rightarrow N = 2^k$$

$$\Rightarrow \boxed{k = \log_2 N}$$

$$\frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \cdots + \frac{N}{2^k} \Rightarrow$$

$a = N/2$

$r = 1/2$

$t = \log_2 N$

Sum of GP = $\frac{a*(r^N-1)}{r-1}$

$$\Rightarrow \frac{\frac{N}{2}*\left(\left(\frac{1}{2}\right)^{\log_2 N}-1\right)}{\frac{1}{2}-1}$$

$$\frac{1^{\log_2 N}}{2^{\log_2 N}}$$

$$\left(\frac{a}{b}\right)^k = \frac{a^k}{b^k}$$

$$\left(\frac{1}{2}\right)^{\log_2 N} = \frac{1^{\log_2 N}}{2^{\log_2 N}}$$

$$\Rightarrow \frac{\frac{N}{2}*\left[\frac{1}{2^{\log_2 N}}-1\right]*(-1)}{-\frac{1}{2}*(-1)}$$

$$\boxed{2^{\log_2 N} = N}$$

$$\Rightarrow \frac{\cancel{2}*\frac{N}{\cancel{2}}*\left[1-\frac{1}{2^{\log_2 N}}\right]}{1/\cancel{2}}$$

$$\Rightarrow N*\left[1-\frac{1}{N}\right]$$

$$\Rightarrow N-\frac{N}{N}$$

$$\Rightarrow N-1$$

$$\boxed{\log a^b = b\log a}$$

$$\boxed{\#\text{ of iterations:} \Rightarrow 2N-1} \Rightarrow O(N)$$

$$2^{\log_2 N} = N$$

⇒ Take log both sides

$$\Rightarrow \log_2 2^{\log_2 N} = \log_2 N$$

1

$$\Rightarrow \log_2 N \cancel{\log_2 2}^{\,1} = \log_2 N$$

$$\Rightarrow \log_2 N = \log_2 N$$

$$2^{\log_2 N} = N$$

How to write Big O?

1) Calculate No. of iterations
2) Neglect constant & lower order terms.

For very large value of N

$$\log_2 N < \sqrt{N} < N < N\log N < N\sqrt{N} < N^2 < 2^N$$

$$a + a+d + a+2d + \cdots + a+(n-1)d.$$

$$l = a+(n-1)d$$

$a+d+l-d$

$a+l$

$$S = a + (a+d) + (a+2d) + \cdots + l$$

$$S = l + (l-d) + (l-2d) + \cdots + a$$

$$2S = (a+l) + (a+l) + (a+l) + \cdots + (a+l)$$

$$2S = N * (a+l)$$

$$S = \frac{N*(a+l)}{2}$$

l

$$S = \frac{N}{2}\left[2a+(n-1)d\right]$$

Thank You