Today's Agenda :-

→ Basic Number System

→ Decimal → Binary & Vice Versa

→ Add 2 Decimal Numbers

→ Negative Numbers in Binary

→ Most Significant Bit (MSB)

→ Data type Ranges

→ Basic Bitwise Operators

→ Properties of Operators

→ Simple Problems on them.

$\Rightarrow \; 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$

<u>GP</u> : Sum of first N terms in GP = $\dfrac{a * (r^N - 1)}{(r-1)}$

$a = 1$

$N = 7$

$\underline{r} = 2$

$\Rightarrow \dfrac{\cancel{1} * (2^7 - 1)}{\cancel{2-1}} \Rightarrow \boxed{\underline{\underline{2^7 - 1}} < 2^7}$

base = (No of unique digits allowed)

Decimal Number System

- Base :- 10
- Digits :- [0 - 9]

$734 \rightarrow 700 + 30 + 4 \Rightarrow 7 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$

$6594 \rightarrow 6000 + 500 + 90 + 4 \Rightarrow 6 \times 10^3 + 5 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$

$2456 \rightarrow 2 \times 10^3 + 4 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$

$10^3 \quad 10^2 \quad 10^1 \quad 10^0$

B=10

1) Octal :-

- Base = 8
- Digits = (0-7)

Q1 $(0\ 1\ 3\ 2)_8 = 0 \times 8^3 + 1 \times 8^2 + 3 \times 8^1 + 2 \times 8^0 = 90$

$8^3 \quad 8^2 \quad 8^1 \quad 8^0$

Q2 $(0125)_8 =$

$8^3 \quad 8^2 \quad 8^1 \quad 8^0$

$\Rightarrow 0 \times 8^3 + 1 \times 8^2 + 2 \times 8^1 + 5 \times 8^0$

$\Rightarrow (85)_{10}$

2) Terinary :-

- Base = 3
- Digits = [0, 2]

Q1 $(0\ 2\ 1\ 0\ 1)_3 \Rightarrow 0 \times 3^4 + 2 \times 3^3 + 1 \times 3^2 + 0 \times 3^1 + 1 \times 3^0$

$3^4 \quad 3^3 \quad 3^2 \quad 3^1 \quad 3^0$

$\Rightarrow 54 + 9 + 1$

$\Rightarrow (64)_{10}$

3) Binary Number System :-

- Base = 2
- Digits = [0, 1]

Binary to Decimal :-

Q. $(1\ 0\ 1\ 1\ 0)_2$

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$= 1\times2^4 + 0\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0$

$\Rightarrow 16 + 4 + 2$

$\Rightarrow \underline{22}$

$(0\ 1\ 1\ 0\ 1\ 0)_2$

$2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$= 2 + 8 + 16$

$\Rightarrow \underline{26}$

Decimal to Binary :-

1.

2	28
2	14 — 0
2	7 — 0
2	3 — 1
2	1 — 1
	0 — 1

$2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$(28)_{10} \Rightarrow (0\ 1\ 1\ 1\ 0\ 0)_2$

$\Rightarrow 4 + 8 + 16$

$\Rightarrow \underline{28}$

2.

2	19
2	9 — 1
2	4 — 1
2	2 — 0
2	1 — 0
	0 — 1

$(19)_{10} = (0\ 1\ 0\ 0\ 1\ 1)_2$

3.

2	25
2	12 — 1
2	6 — 0
2	3 — 0
2	1 — 1
	0 — 1

$\Rightarrow (25)_{10} = (0\ 1\ 1\ 0\ 0\ 1)_2$

Add 2 Decimal Numbers

$d = sum \% 10$ ✓

$c = sum / 10$ ✓

0 1 1 1

3 4 5 9

2 8 4 7

0 6 3 0 6

0 1 1 1

2 4 5 8

3 7 6 4

0 6 2 2 2

Add 2 Binary Numbers

$c = \frac{sum}{2}$ ✓, $d = sum \% 2$ ✓

①

0 0 1 1 0

1 0 1 1 0

\+ 0 0 1 1 1

0 1%2 1%2 3%2 2%2 1%2

0 1 1 1 0 1

②

0 0 0 1 1

1 0 0 1 1

\+ 0 1 0 0 1

0 1%2 1%2 1%2 2%2 2%2

0 1 1 1 0 0

③

0 1 2 1 1 1 0

1 0 1 1 0

1 1 0 1 0

1 0 1 0 1

0 1%2 2%2 4%2 2%2 3%2 2%2 1%2

0 1 0 0 0 1 0 1

Some Naming Conventions :-

$(\overset{4}{1}\ \overset{3}{0}\ \overset{2}{\underline{1}}\ \overset{1}{1}\ \overset{0}{0})_2$

2nd bit ⇒ 1 // set

3rd bit ⇒ 0 // unset

Bit → 1 : set / ON | true
Bit → 0 : unset / OFF | false.

⇒ Bit Positions starts from 0 (Right to left)

Negative Numbers in Binary :-

8 bit Binary Representation :-

	7	6	5	4	3	2	1	0
10 :	0	0	0	0	1	0	1	0
−10 :	1	0	0	0	1	0	1	0

* You are assuming 7th bit as sign bit.

↓

Wrong Way

	0	0	0	0	0	0	0	0
4 :	1	0	0	0	0	1	0	0
10 :	0	0	0	0	1	0	1	0

1 0 0 0 1 1 1 0

$2^3\ 2^2\ 2^1\ 2^0$

Indicating the No' is ⊝ive

8 + 4 + 2 ⇒ 14

(−14)

Ex:-

0 0 0 0 0 0 0 0 → 0

1 0 0 0 0 0 0 0 → −0

for 0, we have 2 different Binary Representations.

$$-a = 2'a = 1'a + 1$$

$$\sim a + 1$$

10 :	0	0	0	0	1	0	1	0
1's 10 :	1	1	1	1	0	1	0	1
+1 :	0	0	0	0	0	0	0	1

Left Most Bit

Most Significant Bit

$\Downarrow$

Base value of MSB = $\ominus$ive

$-128 + 64 + 32 + 16 + 4 + 2 \Rightarrow -10$

	-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
-10 :	1	1	1	1	0	1	1	0

$\rightarrow 128 + 64 + 32 + 16 + 4 + 2$

$\Rightarrow 246$

$-4 = 2's\,4 = 1's\,4 + 1$

4 :	0	0	0	0	0	1	0	0
1's 4 :	1	1	1	1	1	0	1	1
+1 :	0	0	0	0	0	0	0	1
-4 :	1	1	1	1	1	1	0	0

0 0 0

10 :	0	0	0	0	1	0	1	0
-4 :	1	1	1	1	1	1	0	0

1 % 2   1 % 2   0 % 2

$\cancel{1}$ 0 0 0 0 0 1 1 0

$\Downarrow$

6

Some Examples :-

Ex1 8 bit :

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	0	0	0	1	1	1	0

$\rightarrow 8+4+2 -$

$\Rightarrow \underline{\underline{14}}$

Ex2 8 bit :

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	1	0	0	0	1	1	0

$\rightarrow$

$\Rightarrow -128 + 64 + 4 + 2$

$\Rightarrow -128 + 70 \Rightarrow -58$

Ex3 8 bit :

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	1	0	0	1	0

$\rightarrow$

$\Rightarrow -128 + 16 + 2$

$\Rightarrow -128 + 18$

$\Rightarrow -110$

// 4 bit Number :-

MSB 3	2	1	0
-2^3	2^2	2^1	2^0

$\boxed{-2^{N-1}}$

// In a N bit Number :-

-2^{N-1}				2^4	2^3	2^2	2^1	2^0
N-1	—	—	—	4	3	2	1	0

N-1 $\downarrow$ MSB

Data Type Ranges :-

4 bit → [-8, 7]

	0	1	1	1
	1	0	0	0
	-2^3	2^2	2^1	2^0

Max :- 7 [0111]

Min :- -8 [1000]

No of bits	Min Value	Max Value
2	10 (-2) ; -2^1 2^0 ; -2^1	01 (1) ⇒ $2^1 - 1$; -2^1 2^0
4.	$-8 \Rightarrow -2^3$	$7 \Rightarrow 2^3 - 1$
5.	-2^4 2^3 2^2 2^1 2^0 ; 1 0000 ⇒ $-16 = -2^4$	01111 ⇒ 15 ⇒ $2^4 - 1$
6.	100000 ⇒ -32 ⇒ -2^5	011111 ⇒ 31 ⇒ $2^5 - 1$
7	1000000 ⇒ -2^6	⇒ $2^6 - 1$
N	-2^{N-1}	$2^{N-1} - 1$

→ N bits : $\boxed{\left[-2^{N-1},\ 2^{N-1} - 1\right]}$ → Range

Min   Max

1 byte = 8 bits

① byte → 1 byte → 8 bits $\{-2^7, 2^7-1\}$ $\{-128, 127\}$

② Short int → 2 bytes → 16 bits

$\Rightarrow \{-2^{15}, 2^{15}-1\}$ $\{-32768, 32767\}$

③ int → 4 bytes → 32 bits

$\Rightarrow \{-2^{31}, 2^{31}-1\}$

$\Rightarrow \{-2 * 2^{30}, 2 * 2^{30}\}$

$\Rightarrow \{-2 * 10^9, 2 * 10^9\}$

INT_MIN $\{-2,147,483,648\}$ Min Int value

INT_MAX $\{2,147,483,647\}$ Max Int value

④ Long → 8 bytes → 64 bits $\Rightarrow \{-2^{63}, 2^{63}-1\}$

$\Rightarrow \{-8 * 2^{60}, 8 * 2^{60}\}$

$\Rightarrow \{-8 * 10^{18}, 8 * 10^{18}\}$

$2^{10} = 1024 \simeq 10^3$

$\{2^{10} \simeq 10^3\}$

$\boxed{\{2^{30} \simeq 10^9\}} \Rightarrow \{2^{60} \simeq 10^{18}\}$

Bitwise Operators :- { &, |, ^, ~ }

Operators operates on bits

// Truth table

if both bits are 1, ⇒ 1

xor → if both bits same → 0, diff ⇒ 1

1st complement ! Negation

a	b	&	|	^	~(a)
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

if any of the bits are set ⇒ 1

$\boxed{1's\ a = -a - 1}$ ⇒ $-a = 1's\ a + 1$

$-a = \sim a + 1$ → 2's a

byte a = 28 : 0 0 0 1 1 1 0 0 ⇒ 28

byte b = 18 : 0 0 0 1 0 0 1 0

print (a & b) : 0 0 0 1 0 0 0 0

print (a | b) : 0 0 0 1 1 1 1 0

print (a ^ b) : 0 0 0 0 1 1 1 0

print (~a) : 1 1 1 0 0 0 1 1 ⇒ $-128 + 64 + 32 + 2 + 1$

-2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0 ⇒ -29

$a = 10$

	3	2	1	0
$a =$	1	0	1	0
$\& 1$	0	0	0	1
	0	0	0	0

$a = 11$

$a =$	1	0	1	1
$\& 1 =$	0	0	0	1
	0	0	0	1

for even Nos :- $a \& 1 == 0$ ✓

for all odd Nos :- $a \& 1 == 1$

```
if (a & 1 == 0) {
    //even
}
```

```
if (a & 1 == 1) {
    //odd
}
```

$\wedge \rightarrow$ xor