

H.W.



M.E possible \Rightarrow 2:

$> N/3$:



3 distinct elements.

Moore's Voting algo

2 distinct



<u>N</u>	<u>M.E.1</u>	<u>M.E.2</u>
$\sum_{i=1}^{20}$	(c)	(d)

Agenda: Why sorting?

Problems

Comparators.

moribis

data spec^r

Sorting : Arranging integers in asc/desc order
based on a parameter.

Buying on Amazon → sort by relevance/
popularity/
low to high price }

sort a deck of cards

- ↳ By color
- ↳ By value
- ↳ Combination of value/type

Sorted in Asc order by value

Final array [8 | 3]

Sorted in Desc order by value.

1	13	a	6	12
↓	↓	↓	↓	↓
1	2	3	4	6

Sorted in Asc order by no.
of factors.

factors

Library sorting methods (T.C) $\boxed{O(N \log N)}$
S.C. : Depends ($O(1) \rightarrow O(N)$)

Sorting \rightarrow Helps in searching

Q.1 Given an array of size N. You can remove one array element at a time.

Cost of removal = sum of elements in the array just before removal.

Find min. cost to remove all elements.

$$A : [2, 1, 4]$$

$$\begin{cases} \text{remove } 2 \rightarrow 2 + 1 + 4 \Rightarrow 7 \\ \text{remove } 1 \rightarrow 1 + 4 \Rightarrow 5 \\ \text{remove } 4 \rightarrow 4 \Rightarrow 4 \end{cases}$$

Total cost $\Rightarrow \underline{\underline{16}}$.

Try diff order

$$\begin{cases} \text{remove } 4 \rightarrow 2 + 1 + 1 \Rightarrow 7 \\ \text{remove } 2 \rightarrow 2 + 1 \Rightarrow 3 \\ \text{remove } 1 \rightarrow 1 \Rightarrow 1 \end{cases}$$

Total cost $\Rightarrow \underline{\underline{11}}$.

Quiz: 4, 6, 1

[4, 6, 1]

[4, 1]

[1]

$$\begin{array}{l}
 \text{remove } 6 \rightarrow 4 + 6 + 1 \Rightarrow 11 \\
 \text{remove } 4 \rightarrow 4 + 1 \Rightarrow 5 \\
 \text{remove } 1 \rightarrow 1 \Rightarrow 1 \\
 \hline
 \text{Total} \quad . \quad \Rightarrow \underline{\underline{17}}
 \end{array}$$

times.

Quiz 2:

[3, 5, 1, -3]

[3, 1, -3]

[1, -3]

[-3]

$$3, 5, 1, -3 \xrightarrow[\text{sort in desc}]{\text{sort in desc}} (5, 3, 1, -3)$$

$$\begin{array}{l}
 5 \rightarrow 1 \\
 3 \rightarrow 2
 \end{array}$$

$$\begin{array}{l}
 1 \rightarrow 3 \\
 -3 \rightarrow 4
 \end{array}$$

$$\text{remove } 5 \rightarrow 3 + 5 + 1 + (-3) \Rightarrow \underline{\underline{6}}$$

$$\text{remove } 3 \rightarrow 3 + 1 + (-3) \Rightarrow 1$$

$$\text{remove } 1 \rightarrow 1 + (-3) \Rightarrow -2$$

$$\text{remove } (-3) \rightarrow -3 \Rightarrow -3$$

$$\begin{array}{l}
 \text{Total} \quad . \quad \Rightarrow \underline{\underline{12}}
 \end{array}$$

[a, b, c, d]

$$\text{Remove } a \rightarrow \frac{a}{1} + \frac{b}{2} + \frac{c}{3} + \frac{d}{4}$$

Remove b \rightarrow
 Remove c \rightarrow
 Remove d \rightarrow
Total cost $\Rightarrow \underline{1a} + \underline{2b} + \underline{3c} + \underline{4d}$
 $a > b > c > d \Rightarrow \min \text{ sum}$

Pseudo:

cost = 0
 sort(A) (in desc order)

for($i=0$; $i < N$; $i++$)

 cost += A[i] * ($i+1$)



}

$T.C \Rightarrow \underbrace{O(N \log N)}_{\text{sort}} + \underbrace{O(N)}_{\text{finding cost}} \Rightarrow \boxed{O(N \log N)}$

- analogous on sorting algorithm.

S.C \Rightarrow Dependence = 0 0

Amayor
zero

Given N elements. Count no. of 'Nobel integers' present in the array. (Distinct elements)

Nobel integer: $A[i]$ is a Nobel int

No. of elements less than $A[i] = A[i]$

$A : [1 \quad -5 \quad 3 \quad 5 \quad -10 \quad 4]$
 $\# \text{less} \Rightarrow [2 \quad 1 \quad 3 \quad 5 \quad 0 \quad 4]$

Ans = 3

Ques $[-3, 0, 2, 5]$
 $[0, 1, 2]$
Ans = 1

Bottom:

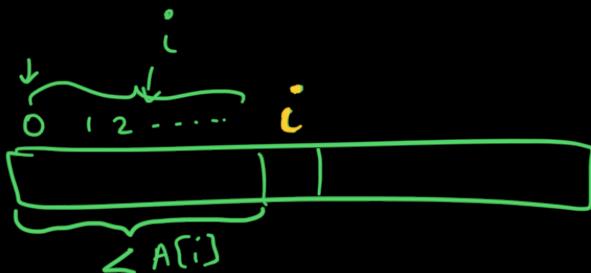
$\text{ans} = 0$
 $\text{for } (i = 0 \rightarrow N - 1)$

T.C $\Rightarrow O(N^2)$

S.C $\Rightarrow O(1)$

```
{  
    count = 0  
    for (j = 0 → N-1)  
    {  
        if (A[j] < A[i])  
            count++  
        if (count == A[i]) ans++;  
    }  
    set ans;  
}
```

Use sorting



e.g 1.

0	1	2	3	4	5
-10	-5	1	13	4	5

A red bracket underlines the values from index 0 to index 3. The value at index 3, which is 13, is circled in red.

T.C $\Rightarrow O(N \log N)$

```
{  
    sort(A)  
    for (i = 0 → N-1)  
    {  
        if (A[i] == i)  
            ans++;  
    }  
}
```

{
 return ans;

Q. 2.b with duplicates.

Quiz : $\left[\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 \\ -10, 1, 1, 3, 100 \end{array} \right]$
 #less
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 0 1 1 3 4

Quiz : $\left[\begin{array}{cccccccccc} 0 & 1 & 1 & 2 & 4 & 4 & 4 & 8 & 10 \\ -10, 1, 1, 2, 4, 4, 4, 8, 10 \end{array} \right]$
 #less
 $\downarrow \quad \downarrow \quad \downarrow$
 0 1 1 3 4 4 4 7 8

ANS = 5

Quiz : $\left[\begin{array}{cccccccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ -3, 0, 2, 2, 5, 5, 5, 5, 8, 8, 10, 10, 10, 14 \end{array} \right]$
 $\downarrow \quad \downarrow \quad \downarrow$
 2 2 4 4 5 5 5 5 8 8 10 10 10 13

#les

0

Ans = 7

steps

① Sort the array } $O(N \log N)$

② } $O(N)$

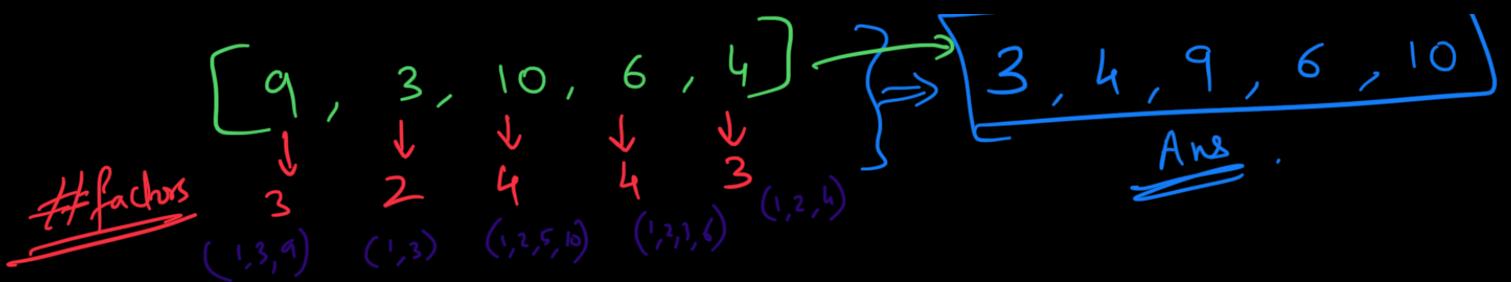
Sorted data

index → 0 1 2 3 4 5 6 7 8 9 10 11 12 13

element → a a b b c c c c d d d d e f
#les → 0 2 4 8 12 13

| Break \Rightarrow 10:25 |

Q. Sort an array in ASC order by the no. of factors.
If no. of factors is same, then sort based on val.



Comparator

Check if array is sorted

$[1, \underline{3}, \underline{5}, 7, 9, 10]$

$a[i] < a[i+1]$

$[1, 4, \underline{6}, 9, 10]$

{
 for ($i = 0 \rightarrow N-2$)
 if ($A[i] \leq A[i+1]$)
 else
 break
 }
 }

! h $\underline{16}$ 3 10

At a time we only need

In any sorting algo → To compare 2 data points.

↓
(comparator
based)

Compare f^n

↳ boolean
↳ 1/-1/0

↳ 2 argument (data-type that has to be compared)

↳ Based on parameters, returns which one is smaller & which one is larger (or come after the first element).

C++ : sort (v.begin(), v.end(), comp())

↳ int
↳ string
↳ obj.

Java : Array.sort (A, new Comparator<Integer>() {
... })

$[2, 3, 7, 1, 4]$ $\xrightarrow[\text{asc.}]{\text{sort}} [1, 2, 3, 4, 7]$

boolean Comp(int a, int b)

{

if($a \leq b$)
return true

else ()
return false .

}

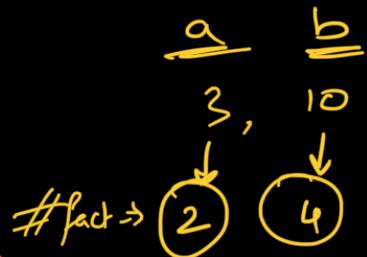
$[2, 3, 7, 1, 4]$ $\xrightarrow[\text{Desc}]{\text{sort in}}$ $[7, 4, 3, 2, 1]$

boolean ~~Comp(int a, int b)~~
{
if($a > b$)
return true

else return false .

}

a : [9, 3, 10, 6, 4]



bool comp (int a, int b)

{
 int fa = countfactor(a)
 int fb = countfactor(b)}

if (fa < fb)

return true .

else if (fa == fb)
 if (a < b)
 return true

else return false



(3), 3
↓ ↓
3, 3

r })

Custom comparator
for sorting based on any parameter

python : A.sort (key = comp-to-key(cmp))

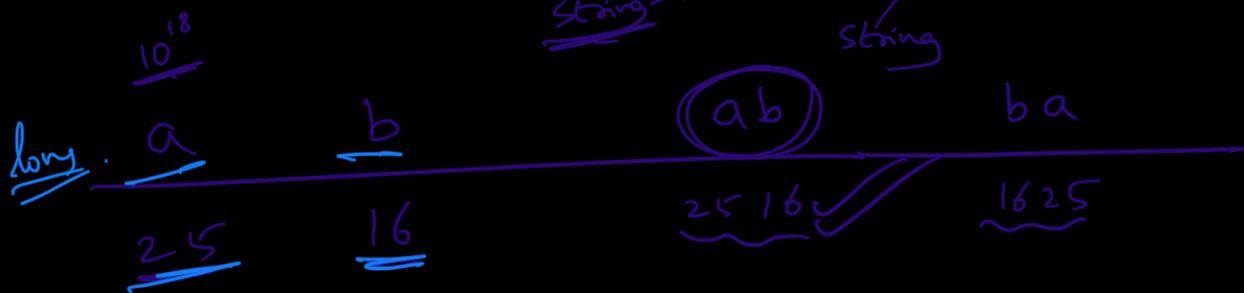
Given array of +ve integers . Return largest no.
Q. than can be formed . using these integers .

A : [3, 30, 34, 5, 9]

3 30 34 5 9
3 4 5 30 9 3
largest → 9 5 34 3 30

long 10^{18}
18 18

↓ Comp (int a, int b)



10	9	109	910
49	9	499	949

ab → 2516 → string
ba → 1625 → string

if($ab < ba$)
return true
else

Doubts :

$\frac{N}{2}$ | $\frac{N}{2}$

~~N₂~~ →

M.E → 1 element possible.

N₃



M.E → 2 elements possible



$$N = 20$$

$$\frac{N}{3} = \frac{20}{3} = 6\overline{2}$$

$$M.E = 3$$

$$\text{Contr} = 7$$



(3)

~~3~~

distinct
elements

~~6(-1)~~
→ 0

(, ~~3, 2, 5~~)

~~3, 6, 7~~

~~(3, 6, 7)~~

$\overrightarrow{[1, 1, 1, 2, 3, 5, 7]}$ } . Validate.
M.E = ~~7~~

M.E.1

M.E.2 .

