Today's Content

- Submatrix Sum Queries
- Sum of all Submatrices
- Max Submatrix Sum

1) 2D arrays.
2) Sum of Subarray 1D
3) Kadane

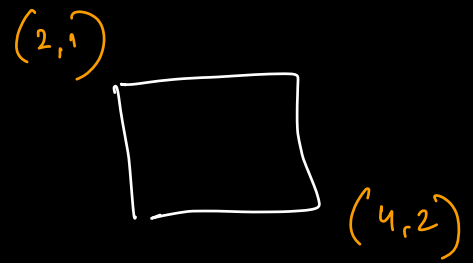
Q1) Given a matrix of size N×M. Find sum of given submatrix

↳ part of matrix
↳ Single element

Eg

	0	1	2	3
0	2	-1	3	2
1	3	2	6	2
2	10	9	8	2
3	4	-1	2	3
4	3	2	6	9

Q : TL : (2,1)
BR : (4,2)



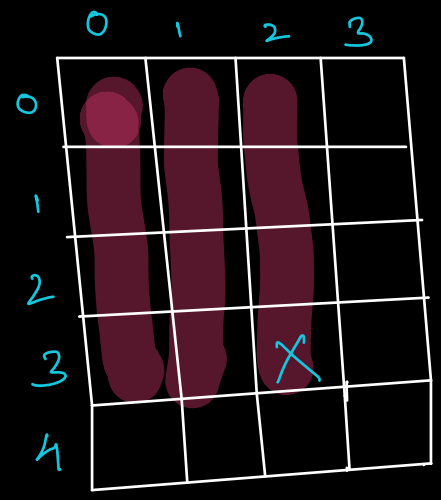
Brute force: iterate through the submatrix & find sum.

TC : O(QMN)
SC : O(1)

Idea 2 :

p[][] = prefix matrix

p[i][j] = Sum of all elements [(0,0) - (i,j)]



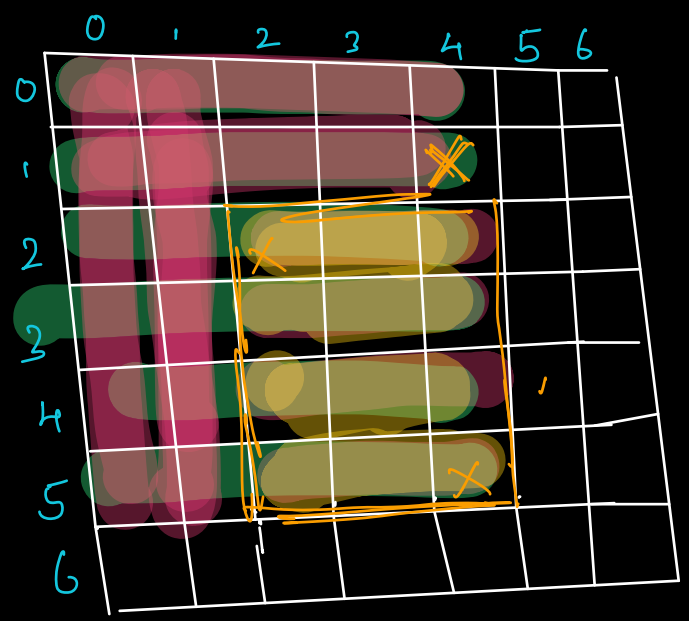
$p[i][j] = Sum\{(0,0) - (i,j)\}$

D $p[3][2]$

(0,0)

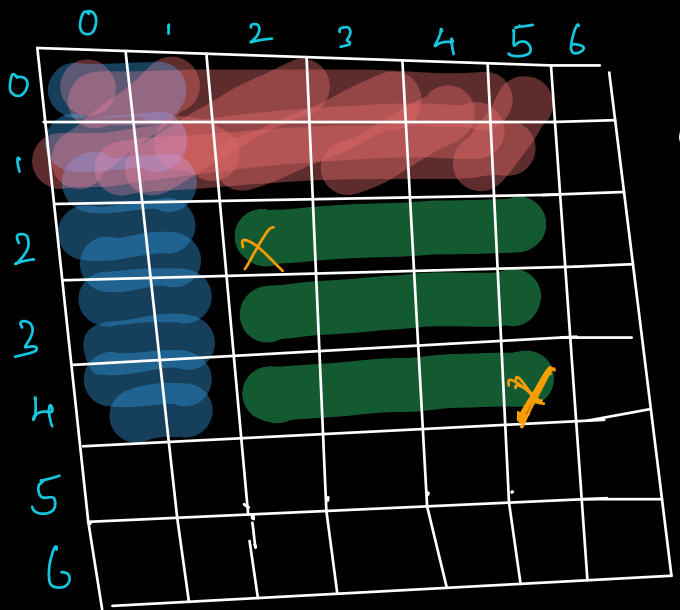
(i,j)

Assuming we have $p[r][c]$



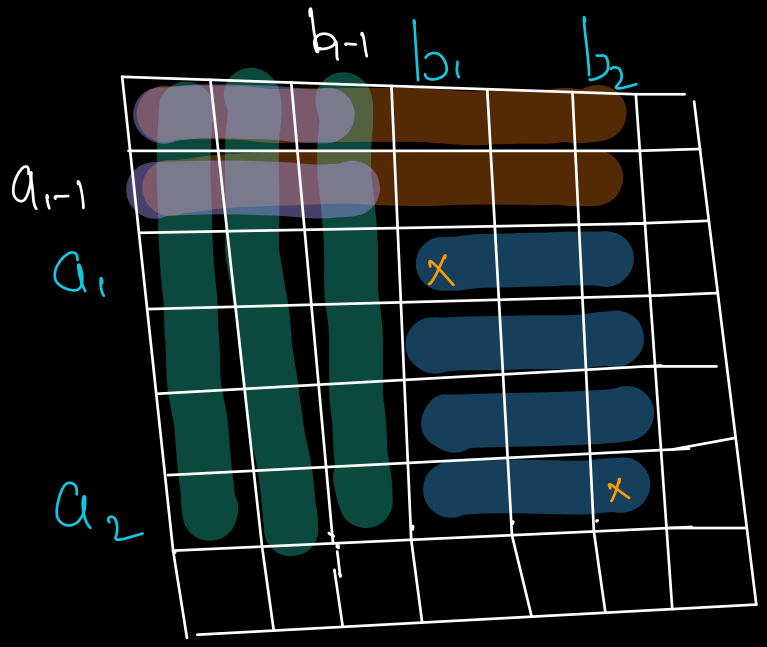
Sum { TL (2,2) · BR (5,4) }

Sum = Pf[5][4]
− Pf[1][4]
− Pf[5][1]
+ Pf[1][1]



Sum { TL (2,2) · BR (4,5) }

Pf[4][5]
− Pf[1][5]
− Pf[4][1]
+ Pf[1][1]



TL BR

$Q : (a_1, b_1) \quad (a_2, b_2)$

$pf[a_2][b_2]$

$-$

$pf[a_{1-1}][b_2]$

$-$

$pf[a_2][b_{1-1}]$

$+$

$pf[a_{1-1}][b_{1-1}]$

$a_1 > 0$

$ans = pf[a_2][b_2] - pf[a_{1-1}][b_2] - pf[a_2][b_{1-1}]$

$+ pf[a_{1-1}][b_{1-1}]$

$b_1 > 0$

$a_1 > 0 \;\&\; b_1 > 0$

If you update original

For Query $= O(1)$

SC $\rightarrow O(1)$ / $O(mn)$

Calculating Prefix[][]

a_0	b_0	c_0
a_1	b_1	c_1
a_2	b_2	c_2

↓ row wise prefix sum

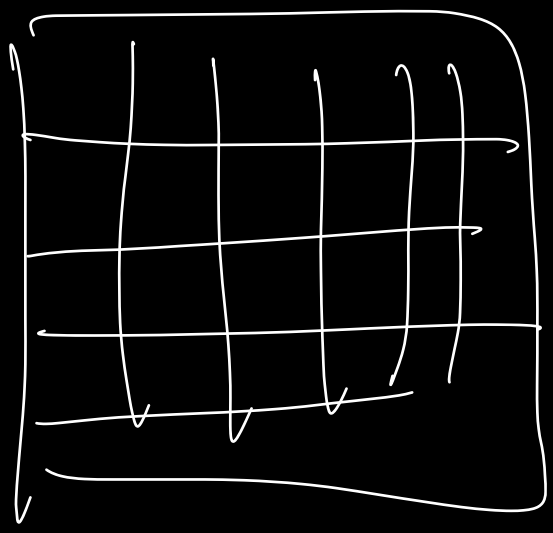
a_0	$a_0 + b_0$	$a_0 + b_0 + c_0$
a_1	$a_1 + b_1$	$a_1 + b_1 + c_1$
a_2	$a_2 + b_2$	$a_2 + b_2 + c_2$

→ Prefix sum column wise

a_0	$a_0 + b_0$	$a_0 + b_0 + c_0$
$a_0 + a_1$	$a_0 + b_0 + a_1 + b_1$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1$
$a_0 + a_1 + a_2$	$a_0 + b_0 + a_1 + b_1 + a_2 + b_2$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1 + a_2 + b_2 + c_2$

Tc: n×m + m×n

⇒ O(mn)



Entire Question with

Q Queries

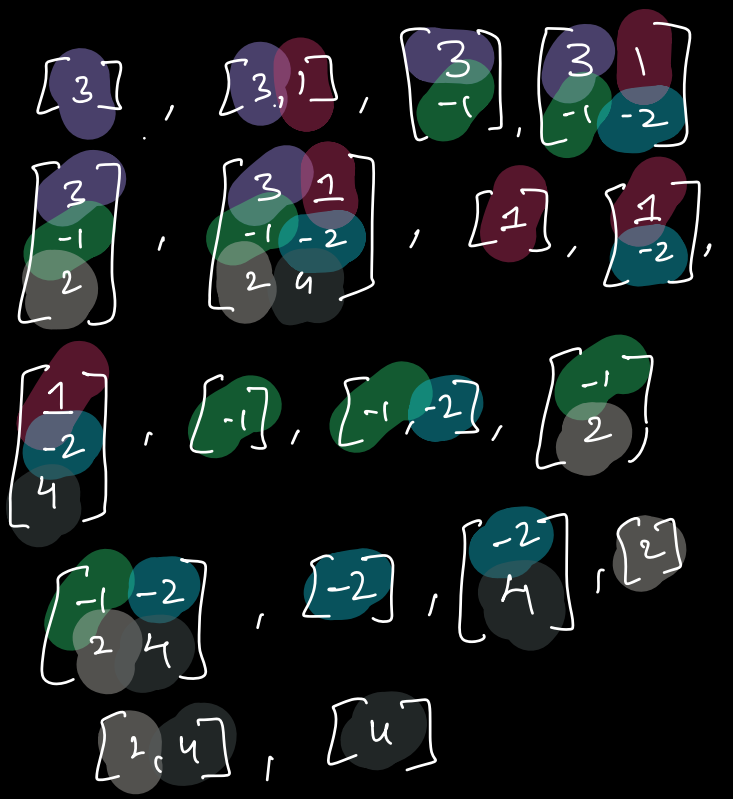
Tc: O(mn + q)

Q2 Given a matrix of size N x M, find sum of all Submatrix Sum.

Ex:

	0	1
0	3	1
1	-1	-2
2	2	4

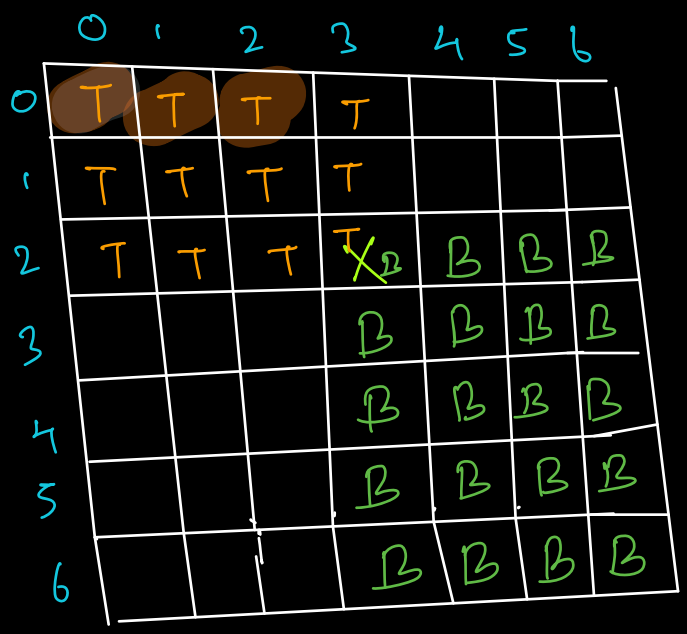
=



$3 \times (6) + 1 \times (6) + -1 \times (8) + -2 \times (8)$

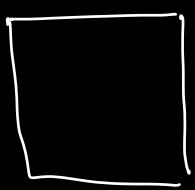
$2 \times (6) + 4 \times (6) \Rightarrow (36)$

Given element (i, j), how many submatrices would it be in ??



How many submatrix cell [2,3] is in = ?

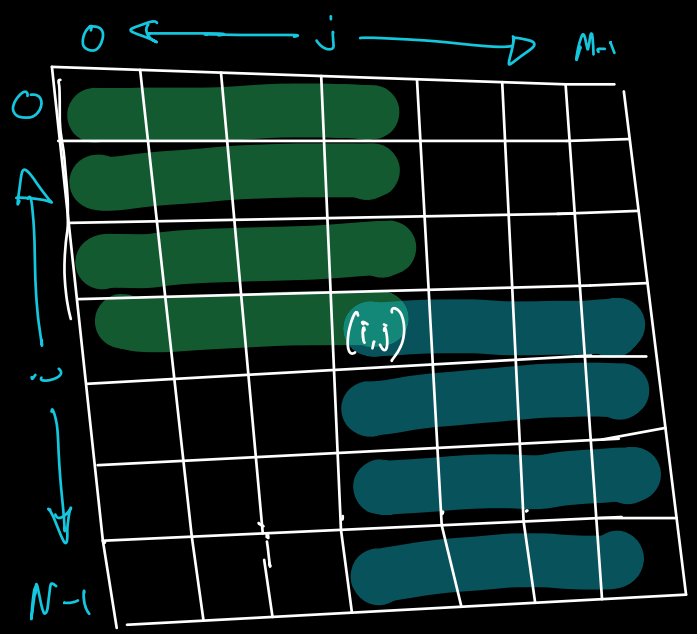
(TL) ⇒ 12 possibilities



(BR)

(20 possibilities)

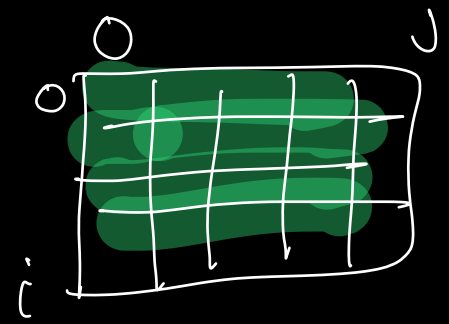
Generalize



How many submatrix cell [i,j] is in = ?

T ⇒ (i+1) x (j+1)

B ⇒ (n-i) x (m-j)



(i+1) X (j+1)

```
mat[][]
ans = 0
for (int i=0 ; i<n ; i++) {
    for (int j=0 ; j<m ; j++) {
        int contri ⇒ (i+1)x(j+1)x(n-i)x(m-j)
        ans = ans + contri * mat[i][j];
    }
}
return ans;
```

10:38

Tc: O(M x N)

Sc: O(1)

Q3 Given a matrix [N x M], find max submatrix sum, where submatrix starts row = 0 and ends row = n - 1

	0	1	2	3	4	5
0	-3	2	3	4	-6	4
1	5	5	-5	2	2	-7
2	-4	-3	1	-1	1	4

0	1	2	3	4	5
-2	4	-1	5	-3	1

⇒ 8

Q4 Given a matrix [N x M], find max submatrix sum, where submatrix starts row = 0 and ends anywhere

	0	1	2	3	4	5
0	2	-4	1	3	-1	2
1	1	3	2	-7	3	3
2	0	-1	1	3	4	-7
3	1	-1	-6	4	-4	6

Apple Radare

Case 1 start 0 end 0 → 2 | -4 | 1 | 3 | -1 | 2 ⇒ 5

Case 2 start 0 end 1 → 3 | -1 | 3 | -4 | 2 | 5 ⇒ 8

Case 3 start 0 end 2 → 3 | -2 | 4 | -1 | 6 | -2 ⇒ 10

Case 4 start 0 end 3 → 4 | -3 | -2 | 3 | 2 | 4 ⇒ 9

Pseudo code

```
ans = INT.MIN;
st = 0
sum[m] = {0};
for (int end = 0 ; end < n ; end++) {
    for (int i = 0 ; i < m ; i++) {
        sum[i] = sum[i] + mat[end][i];
    }
```

O(m) = current_max = kadane(sum)

```
    ans = max(ans, current_max);
}
return ans;
```

TC : O(nm)

SC : O(m)

Q4 Given a matrix [N x M], find max submatrix sum, where submatrix starts anywhere and ends anywhere.

```
for (st = 0; st < n; st++) {
    sum[m] = {0};
    for (int end = st; end < n; end++) {
        for (int i = 0; i < m; i++) {
            sum[i] = sum[i] + mat[end][i];
        }
        current_max = kadane(sum);
        ans = max(ans, current_max);
    }
}
kadane
```

O(m)

O(m)

TC : $(n \times mn) = (n^2 m)$

SC : O(m)

Core 1	start	end
	0	0
Core 2	0	1
Core 3	0	2
Core 4	0	3

start	end
1	1
1	2
1