

Today's Content

- prefix Sum
- problems on prefix Sum
- LeftMax, RightMax
- Water accumulated
- Max Subarray Problem

$$\# \quad \text{P}f[i] = x$$
$$\quad \quad \quad [0 \text{ --- } i]$$

Prefix Sum ?

Q1) Given N array elements = 0

$1 \leq n \leq 10^5$
 $1 \leq Q \leq 10^5$

arr[7] = $\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$

Q = 4
 a val

$\begin{matrix} & & & 4 & 4 & 4 & 4 & 4 \\ & & & -1 & -1 & -1 & -1 \\ 2 & 4 & & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & -1 & & & & & 1 & 1 & 1 \\ 0 & 2 & & & & & & & \\ 4 & 1 & & & & & & & \end{matrix}$

$\begin{matrix} 2 & 2 & 6 & 5 & 6 & 6 & 6 \end{matrix}$

1) for each query iterate array & update

T.C: $O(nq)$

Prefix Sum

$\boxed{a_1 \mid a_2 \mid a_3 \mid a_4 \mid a_5}$

Pr[7]:

$\begin{matrix} a_1 & a_1 & a_1 & a_1 & a_1 \\ + & + & + & + \\ a_2 & a_2 & a_2 & a_2 \\ + & + & + \\ a_3 & a_3 & a_3 \\ + & + \\ a_4 & a_4 & + a_5 \end{matrix}$

Q=4

a	val
2	4
3	-1
0	2
4	<u>1</u>

0	1	2	3	4	5	6
2	0	4	-1	1	0	0

0	1	2	3	4	5	6
2	2	6	5	6	6	6

$$Tc: O(Q + N)$$

$$Sc: O(1)$$

Pseudo Code

while (Q--)

// a, val

arr[a] += val

}

for (int i=1; i<=n; i++)

arr[i] = arr[i] + arr[i-1];

}

Input + Extra

Sc

Q Given N array elements = 0

arr[10] =

0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0

1 1 1 1

3 3 3 3 3 3

-3 -3 -3 -3

2 2 2 2 2 2 2 2 2 2

0 2 5 6 6 3 3 2 -12

Q = 4

s	e	val
---	---	-----

3 6 1

2 7 3

5 8 -3

1 9 2

Observation

Q

s	e	val
---	---	-----

3 6 1

[3, 6] ~~[7, 9]~~

2 7 3

[2, 7] ~~[8, 9]~~

↗ [e+1, n-1]

Step 1: arr[s] += val

arr[e+1] -= val

e = n-1

Pseudo Code

while ($Q > 0$) {

// s, e, val

arr[s] += val;

if ($e \neq n-1$) {

arr[e+1] -= val;

}

Q--;

}

Q = 4

s e val

3 6 1

⇒ [3] [7]

2 7 3

⇒ [2] [8]

5 8 -3

⇒ [5] [9]

1 9 2

⇒ [1]

[0, 1]

0	1	2	3	4	5	6	7	8	9
	2	3	1		-3		-1	-3	+3

0	1	2	3	4	5	6	7	8	9
0	2	5	6	6	3	3	2	-1	2

Q3 Find $Pfm[i] = \max$ of all Elements from $[0, i]$

arr = $[-5, 1, 2, 3, 2]$ $Tc: O(n)$

Left
max

Pfm = $[-5, 1, 2, 3, 3]$

TODO

Q4 Find $Sfm[i] = \max$ of all Elements from $[i, n-1]$

arr = $[-5, 1, 2, 3, 2]$ $[3, 4]$

Sfm = $[3, 3, 3, 3, 2]$ Right Max

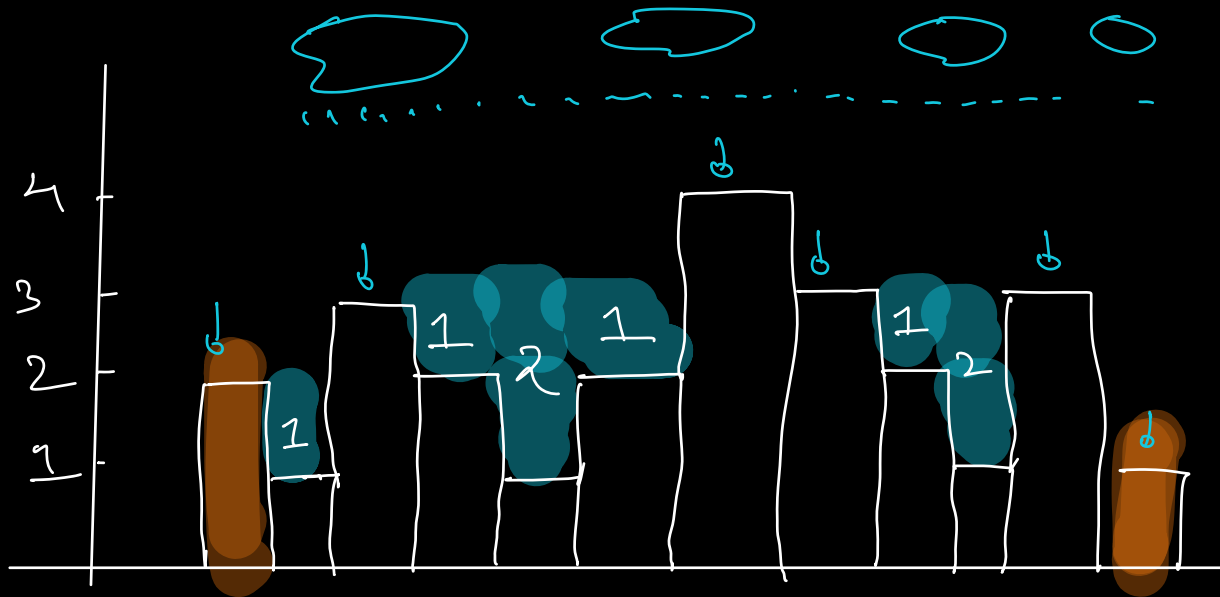
$Tc: O(n)$

Q5) Rain water trapped

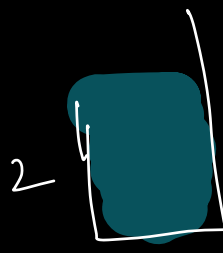
Given N array elements, where $arr[i]$ represents height of the building.

Return the amount of water trapped between building.

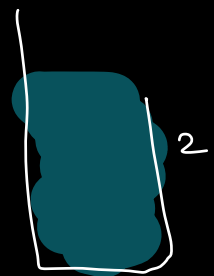
Ex $arr[] = \{2, 1, 3, 2, 1, 2, 4, 3, 2, 1, 3, 1\}$



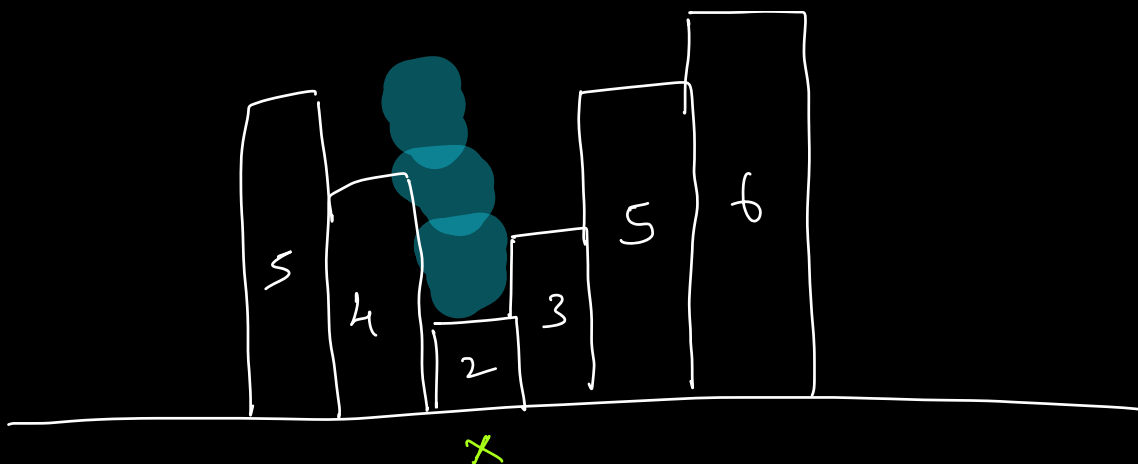
Both ends
need support



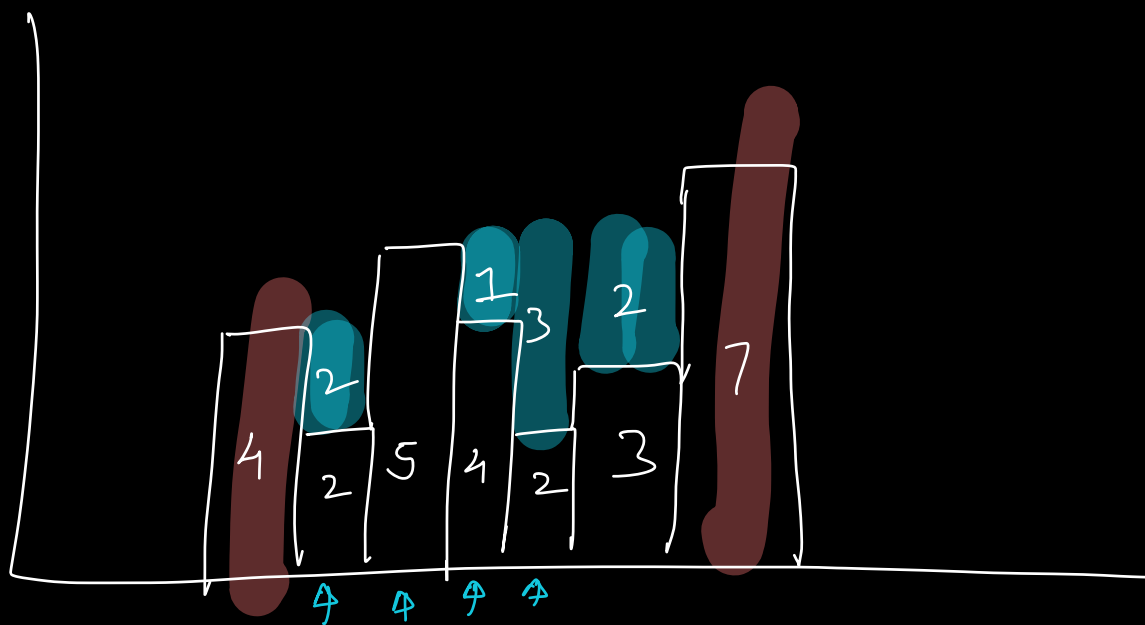
Ans = 2



Ans = 2



$$\min \{ \text{left max}, \text{right max} \} - \text{height}$$



⇒ 8

Pseudo Code

// left_max $O(n)$

// right_max $O(n)$

int ans = 0

Tc: $O(n)$

Sc: $O(n)$

for (int i = 1; i < (n-1); i++) {

int support $\Rightarrow \min(\text{left_max}[i-1],$
 $\text{right_max}[i+1])$

int water $\Rightarrow \text{support} - \text{arr}[i];$

if (water > 0)

ans += water;

}

return ans;

10:47

Q6) Given N array Elements, Calculate Maximum Subarray Sum.

Ex array = $\{-1, 4, 2, 8, -2, 3\}$
ans \Rightarrow 15

Approach 1 : $T.C : O(n^3)$

Approach 2 : $T.C : O(n^2)$
using prefix sum

Scenario 1 : All element > 0

3	4	5	6
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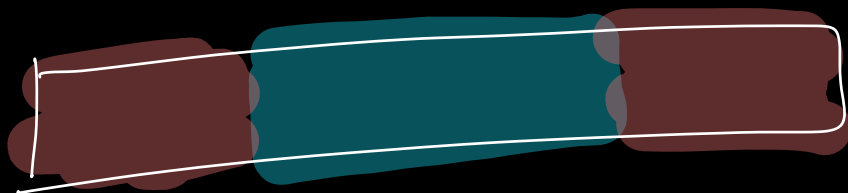
 \Rightarrow 18

Scenario 2 : All element < 0

-1	-2	-3	-4
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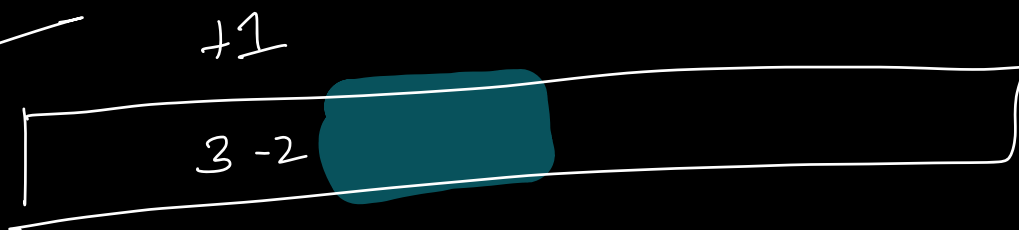
 \Rightarrow -1

Scenario 3



\hookrightarrow max subarray sum

Scenario 4



\downarrow
If $\text{sum} > 0$, then carry sum



arr[] :	5	1	7	-3	2	-10	-12	8	12	21	-4	7
Sum = 0	5	11	18	15	17	7	0	8	20	41	37	44
ans = INT.MIN	5	11	18	18	18	18	18	18	20	41	41	44

$$S = 0$$

$$ans = INT.MIN$$

// find maximum

$O(n)$

if (max < 0)
return max;

for (int i = 0; i < n; i++) {

$S = S + arr[i];$

$ans = \max(ans, S);$

if (S < 0)

{
S = 0;

$Tc: O(n)$

$Sc: O(1)$

return are;

~~TODO~~

- 1) Can we find the indices of max Subarray .
- 2) Can you handle edge case in a different way.