Indian Institute of Technology Hyderabad

Matrix Assignment - Conic

mohammad imran

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1 Problem

Tangent to the curve y=x^2 + 6 at a point (1,7) touches the the circle $x^2 + y^2 + 16x + 12y + c = 0$ at a point Q then the coordinates of Q are ?.

2 Solution

The equation of a curve is given as,
$y = x^2 + 6$

the above equation can be expressed in the form

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$$

where,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } \mathbf{u} = \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix} \text{and f=6}$$

for the given conic

the point where the tangent touches the conic is $Q(1, 7)$

given a point of contact Q, then Equation of tangent is given by,

$$(\mathbf{V}\mathbf{Q} + \mathbf{u})^\top \mathbf{x} + \mathbf{u}^\top \mathbf{Q} + f = 0$$

$$(\mathbf{V}\mathbf{Q})^\top \mathbf{x} = -f$$

which can be written as

$$\mathbf{n}^\top \mathbf{x} = C$$

$$\begin{pmatrix} 2 & -1 \end{pmatrix} \mathbf{X} = -5 \tag{1}$$

$$n = \mathbf{VQ} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \text{ and } C = f = -5$$

from equation of line we can write directional vector

$$\mathbf{m} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

given circle equation

$$x^2 + y^2 + 16x + 12y + c = 0$$

the above equation can be expressed as
$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$
where

$$\mathbf{u} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

the point of intersection of the line with the conic section is given as

$$\mathbf{x} = \mathbf{q} + \mu \mathbf{m} \tag{2}$$

where, $\mu = \frac{1}{\mathbf{m}^T \mathbf{v}\mathbf{m}}(-\mathbf{m}^T(\mathbf{v}\mathbf{q} + \mathbf{u}))$

by substituting all the values we get

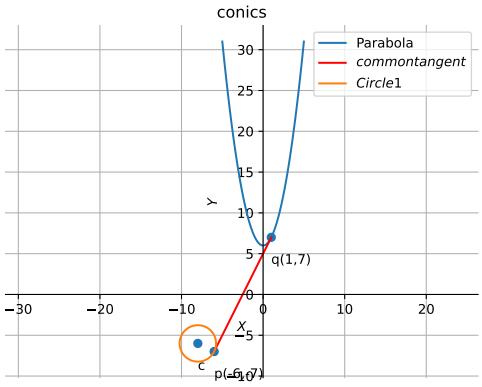
μ=-7

as we have point of contact q and directional vector m and μ

by substituting all the values in eq 2 we get point of intersection on the line

$$\mathbf{X} = \begin{pmatrix} -6 \\ -7 \end{pmatrix}$$

3 Figure



4 Code Link

https://github.com/imran111888/fwc2/tree/main/matrix/conic/codes

Execute the code by using the command **python3 conic.py**