Matrix-Circle Assignment
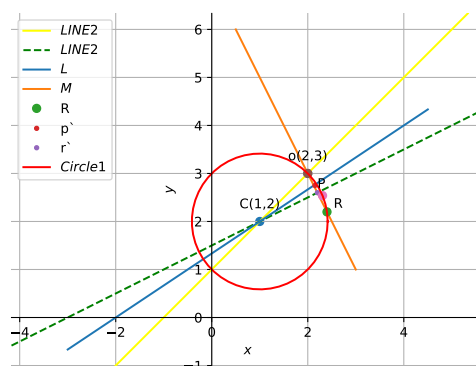
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Figure 1: locus of the given point

2 Construction

1 Problem Statement

locus of the image of the point (2,3) in the line (2x-3y+4)+K (x-2y+3)=0,K ϵ R,is a ?

3 Solution

Given that resultant locus of the image of the point(2,3) in the line 2x-3y+4+K(x-2y+3)=0 here the K is a rational number assuming the locus of image of point (2,3) is

Symbol	Value	Description
P	$\begin{pmatrix} x \\ y \end{pmatrix}$	image of given point
Q	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$	Point of Intersection
R	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$	Given Point

Table 1: Parameters

P)

Given family of the line:

(2x-3y+4)+K(x-2y+3)=0

From the above given family of line we, can write as follows

$$\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{X} = -4 \tag{1}$$

$$\begin{pmatrix} 1 & -2 \end{pmatrix} \mathbf{X} = -3 \tag{2}$$

The above equation can be written in matrix form as follows

$$\begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix} \mathbf{X} = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \tag{3}$$

The Augmented can be expressed as

$$\begin{pmatrix} 2 & -3 & -4 \\ 1 & -2 & -3 \end{pmatrix} \tag{4}$$

Through,pivoting the Augmented matrix will become as

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{5}$$

On solving the above equation the cross point of the given family of line equation and we get point **Q**

$$\mathbf{Q} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{6}$$

Given point **R**=(2,3) and asumming its locus of image as point **P**=(x,y) and we have obtain the point**Q**=(1,2)

we, know that locus of the image of the point is equidistance from point Q thus,

Distance between point PQ and QR is same

$$\mathbf{PQ} = \mathbf{QR} \tag{7}$$

$$Distance between \mathbf{QR} = ||\mathbf{Q} - \mathbf{R}|| \tag{8}$$

$$\mathbf{QR} = || \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} || \tag{9}$$

by, solving we get

$$distance between \mathbf{QR} = \sqrt{2} \tag{10}$$

From eq7 $\mathbf{PQ} = \sqrt{2}$

$$Distance between \mathbf{PQ} = ||\mathbf{P} - \mathbf{Q}|| \tag{11}$$

$$||\mathbf{P} - \mathbf{Q}|| = \sqrt{2} \tag{12}$$

$$||\mathbf{P} - \mathbf{Q}||^2 = 2 \tag{13}$$

by solving we get,

$$||\mathbf{P}||^2 - 2\mathbf{Q}^\top\mathbf{P} + ||\mathbf{Q}||^2 = 2 \tag{14}$$

by solving we get equation of cirlce as,

$$\mathbf{P}^\top\mathbf{P} - 2\mathbf{Q}^\top\mathbf{P} + 3 = 0 \qquad (15)$$

$$\mathbf{X}^\top\mathbf{X} - 2\begin{pmatrix}-1 & -2\end{pmatrix}^\top\mathbf{X} + 3 = 0 \qquad (16)$$

hence The Resultant is a **circle**

4 finding the center of circle and radius

$$center = -u = \begin{pmatrix}1\\2\end{pmatrix} \qquad (17)$$

radius of the circle$=\sqrt{2}$

5 verfication of image of point with respect to line

taken a line

$$\mathbf{n}^\top\mathbf{X} = C \qquad (18)$$

$$\begin{pmatrix}1 & -2\end{pmatrix}\mathbf{X} = -3 \qquad (19)$$

we have a point P $=\begin{pmatrix}2\\3\end{pmatrix}$

by using the formula image of point with respect to line

$$\mathbf{R} = \mathbf{P} + 2\frac{C - \mathbf{n}^\top\mathbf{P}}{||\mathbf{n}||^2}\mathbf{n} \qquad (20)$$

by solving above equation we the image of point $\mathbf{R} = \begin{pmatrix}\frac{12}{5}\\\frac{11}{5}\end{pmatrix}$

6 Software

Download the following code using,

https://github.com/imran111888/fwc2/blob//matrix/line

and execute the code by using command

Python3 line.py

7 Conclusion

We found the locus of the image of point(2,3) in a line is a circle.