PROBABILITY ASSIGNMENT

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1 Problem

Two dice are thrown simultaneously, if x denotes number os sixes, find the expectation of x

2 Solution

Consider each trial results in success (i.e getting sixes on dice) or failure (i.e not getting sixes on dice) represented by 1 and 0 respectively.

Let $X_i \in \{1,2,3,4,5,6\}, i = 1,2$ be the random variables representing the outcome for each die.Assuming the dice to be fair,the probability mass function(pmf) is expressed as

$$p_{X_i}(k) = \Pr(X_i = k) = \begin{cases} \frac{1}{6} & 1 \le k \le 6 \\ 0 & otherwise \end{cases} \tag{1}$$

p and $q = (1-p)$ are the probability of success and failure respectively.

$$p = P_{X_i}(1) = \frac{1}{6} \tag{2}$$

$$q = 1 - p = P_{X_i}(0) = \frac{5}{6} \tag{3}$$

The generating function (or z-transform) of $\Pr(X_i = k)$ is defined as

$$M_{X_i}(z) = E\left[z^{X_i}\right] = \sum_{k=-\infty}^{\infty} P_{X_i}(k) z^{-k} \tag{4}$$

For a throw of dice,

$$E\left[z^{X_i}\right] = \sum_{k=0}^{1} P_{X_i}(k) z^{-k} \tag{5}$$

$$= P_{X_i}(0) z^0 + P_{X_i}(1) z^{-1} \tag{6}$$

$$= q + pz^{-1} \tag{7}$$

$\therefore$n number of throws of dices can be represented as,

$$\begin{aligned} E\left[z^{X_1+X_2+\ldots+X_n}\right] &= E\left[z^{X_1} z^{X_2} \ldots . z^{X_n}\right] \\ &= E\left[z^{X_1}\right] E\left[z^{X_2}\right] \ldots . E\left[z^{X_n}\right] \end{aligned} \tag{8}$$

($\because$ $X_1, X_2, ..., X_n$are independent and identically distributed)

$\Rightarrow M_{X_i}(z)$ for n number of dices is

$$M_{X_i}(z) = (q + pz^{-1})^n \tag{9}$$

Expectation of x or mean is defined as the 1st moment of $M_{X_i}(z^{-1})$ at z=1 i.e,

$$\mu = \frac{dM_{X_i}(z^{-1})}{dz}|_{z=1} \tag{10}$$

$$= \frac{d(q+pz)^n}{dz}|_{z=1} \tag{11}$$

$$= np(q+pz)^{n-1}|_{z=1} \tag{12}$$

$$= np(q+p)^{n-1} \tag{13}$$

$$\therefore \text{Mean} = np \qquad \because (p+q=1) \tag{14}$$

$$= 2 \times \frac{1}{6} = 0.333 \tag{15}$$

The Expectation of x is 0.33