Indian Institute of Technology Hyderabad

Digital Communication Assignment

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1 Problem

I . Given

$$V = -2\ln(1-U)$$

Find a Theoratical expression for $F_V(x)$.

II . Generate samples of

$$V = -2\ln(1-U)$$

and plot its CDF.

2 Solution

Let U be a uniform random variable between 0 and 1.
Now, CDF of U is

$$F_U(x) = \Pr(U \le x) \tag{1}$$

$$F_U(x) = \begin{cases} 0 & 0 < x \\ x & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases} \tag{2}$$

Similarly, CDF of V is given as

$$\begin{aligned} F_V(x) &= \Pr(V \le x) && (3)\\ &= \Pr(-2\ln(1-U) \le x) && (4)\\ &= \Pr\left(\ln(1-U) \ge \frac{-x}{2}\right) && (5)\\ &= \Pr\left(1-U \ge e^{\frac{-x}{2}}\right) && (6)\\ &= \Pr\left(U \le 1-e^{\frac{-x}{2}}\right) && (7) \end{aligned}$$

From (1) and (7) $F_V(x)$ can be simplied as,

$$F_V(x) = F_U(1-e^{\frac{-x}{2}}) \tag{8}$$

From (2) and (8)

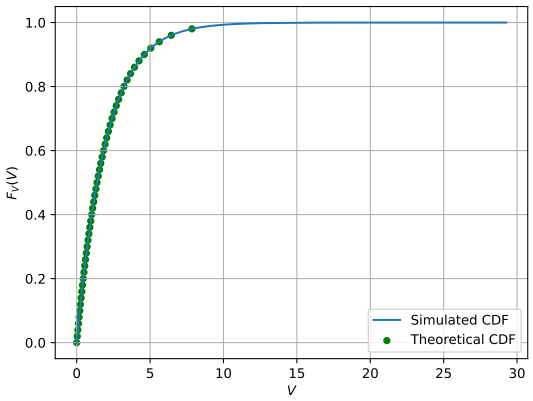
$$F_V(x) = \begin{cases} 0 & 0 < x \\ 1-e^{\frac{-x}{2}} & 0 \le x < \infty \\ 1 & x = \infty \end{cases} \tag{9}$$

3 Code Link

The following code generates samples and plots the CDF of V.

https://github.com/imran111888/fwc2/blob/main/Module2/Digital_comm/code/digi.py

4 Figure


1.0
0.8
0.6
0.4
0.2
0.0
$F_V(V)$
0
5
10
15
20
25
30
V
Simulated CDF
Theoretical CDF