

PROBABILITY ASSIGNMENT

MOHAMMAD IMRAN

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1 Problem

Two dice are thrown simultaneously, if x denotes number of sixes, find the expectation of x

2 Solution

Consider each trial results in success (i.e getting sixes on dice) or failure (i.e not getting sixes on dice) represented by 1 and 0 respectively.

Let $X_i \in \{1, 2, 3, 4, 5, 6\}$, $i = 1, 2$ be the random variables representing the outcome for each die. Assuming the dice to be fair, the probability mass function (pmf) is expressed as

$$p_{X_i}(k) = \Pr(X_i = k) = \begin{cases} \frac{1}{6} & 1 \leq k \leq 6 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

p and $q = (1 - p)$ are the probability of success and failure respectively.

$$p = P_{X_i}(1) = \frac{1}{6} \quad (2)$$

$$q = 1 - p = P_{X_i}(0) = \frac{5}{6} \quad (3)$$

The generating function (or z -transform) of $\Pr(X_i = k)$ is defined as

$$M_{X_i}(z) = E[z^{X_i}] = \sum_{k=-\infty}^{\infty} P_{X_i}(k)z^{-k} \quad (4)$$

$$\begin{aligned} E[z^{X_1+X_2+\dots+X_n}] &= E[z^{X_1} z^{X_2} \dots z^{X_n}] \\ &= E[z^{X_1}] E[z^{X_2}] \dots E[z^{X_n}] \end{aligned} \quad (5)$$

($\because X_1, X_2, \dots, X_n$ are independent and identically distributed)

\therefore For a simultaneous throw of two dice,

$$E[z^{X_i}] = \sum_{k=0}^2 P_{X_i}(k)z^{-k} \quad (6)$$

$$= (q + pz^{-1})^2 \quad (7)$$

$\Rightarrow M_{X_i}(z)$ for n number of dice is

$$M_{X_i}(z) = (q + pz^{-1})^n \quad (8)$$

Expectation of x or mean is defined as the 1st moment of $M_{X_i}(z^{-1})$ at $z=1$ i.e,

$$\mu = \frac{dM_{X_i}(z^{-1})}{dz} \Big|_{z=1} \quad (9)$$

$$= \frac{d(q + pz^{-1})^n}{dz} \Big|_{z=1} \quad (10)$$

$$= np(q + pz^{-1})^{n-1} \Big|_{z=1} \quad (11)$$

$$= np(q + p)^{n-1} \quad (12)$$

$$\therefore \text{Mean} = np \quad \because (p + q = 1) \quad (13)$$

$$= 2 \times \frac{1}{6} = 0.333 \quad (14)$$

The Expectation of x is 0.33