

Statistical Finance (STAT W4290)

Assignment 4

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Rupert Problem 3

$$1200 = \frac{40}{r} \left(1 - \frac{1}{(1+r)^{30}} \right) + \frac{1000}{(1+r)^{30}}$$

Solving for r , we get $r = 0.03241618$

```
1 > zero = function(r) {(40/r)*(1-1/(1+r)^60) + 1000/(1+r)^60  
  - 1200}  
2 > uniroot(zero, c(0.03, 0.04), maxiter=100)  
3 $root  
4 [1] 0.03241618  
5  
6 $f.root  
7 [1] -0.5497406  
8  
9 $iter  
10 [1] 3  
11  
12 $init.it  
13 [1] NA  
14  
15 $estim.prec  
16 [1] 6.103516e-05
```

Rupert Problem 4

$$9800 = \frac{280}{\frac{r}{2}} \left(1 - \frac{1}{\left(1 + \frac{r}{2}\right)^{8*2}} \right) + \frac{10000}{\left(1 + \frac{r}{2}\right)^{8*2}}$$

Solving for r , we get

```
1 > zero = function(r) {(280/(r/2)) * (1-1/(1+(r/2))^(8*2)) +  
    10000/(1+(r/2))^(8*2) - 9800}  
2 > uniroot(zero, c(0.05, 0.06), maxiter=100)  
3 $root  
4 [1] 0.05920323  
5  
6 $f.root  
7 [1] -1.794659  
8  
9 $iter  
10 [1] 2  
11  
12 $init.it  
13 [1] NA  
14  
15 $estim.prec  
16 [1] 6.103516e-05
```

Rupert Exercise 1

$$y(t) = \frac{1}{t} \int_0^t r(s) ds$$

(a)

Setting $t = 20$,

$$\begin{aligned} y(20) &= \frac{1}{20} \int_0^{20} 0.028 + 0.00042s \, ds \\ &= \frac{1}{20} (0.028 * 20 + 0.00021 * (20^2)) \\ &= 0.0322 \end{aligned}$$

(b)

$$y(15) = 0.03115$$

```
1 > r = function(t) {0.028 + 0.00042*t}  
2 > y = integrate(r, 0, 15)$value / 15  
3 > y  
4 [1] 0.03115
```

Then the price of the bond is

$$P = \frac{1000}{(1 + 0.03115)^{15}} = 631.2077$$

Rupert Exercise 3

$$y(t) = \frac{1}{t} \int_0^t r(s) ds$$

(a)

Setting $t = 5$, $y(5) = 0.03616667$

```
1 > r = function(t) {0.032+0.001*t+0.0002*t^2}  
2 > y = integrate(r, 0, 5)$value / 5  
3 > y  
4 [1] 0.03616667
```

(b)

Assume bond has par \$100, then

$$P = \frac{100}{(1 + 0.03616667)^5} = 83.72438$$

Ruppert Exercise 5

Now the sum of a geometric series $C + Cm + Cm^2 + \dots = \frac{C}{1-m}$. If $m = \frac{1}{1+r}$, then $C + Cm + Cm^2 + \dots = \frac{C(1+r)}{r}$. If the cash flow C begins a year later, we discount the entire amount by $(1+r)$ thereby making the present value of the sum $\frac{C}{r}$.

Then,

$$\begin{aligned} & \sum_{t=1}^{2T} \frac{C}{(1+r)^t} + \frac{\text{PAR}}{(1+r)^{2T}} \\ &= \sum_{t=1}^{\infty} \frac{C}{(1+r)^t} - \sum_{t=2T+1}^{\infty} \frac{C}{(1+r)^t} + \frac{\text{PAR}}{(1+r)^{2T}} \\ &= \frac{C}{r} - \frac{C}{r(1+r)^{2T}} + \frac{\text{PAR}}{(1+r)^{2T}} \\ &= \frac{C}{r} + \left(\text{PAR} - \frac{C}{r} \right) (1+r)^{-2T} \end{aligned}$$

Ruppert Exercise 7

(a)

Given that

$$\begin{aligned} 818 &= \frac{1000}{\exp 5r} \\ r &= \frac{\log \frac{1000}{818}}{5} = 0.04017859 \end{aligned}$$

This means that $r = 0.04017859$

(b)

$$P = \frac{1000}{\exp 4 * 0.042} = 845.3538$$

(c)

$$R_2 = \frac{P_2}{P_1} - 1 = \frac{845.3538}{818} - 1 = 0.03343985$$

Ruppert Exercise 8

(a)

$$P = \frac{\frac{22}{0.04}}{2} \left(1 - \frac{1}{1.02^{20}} \right) + \frac{1000}{1.02^{20}} = 1032.703$$

(b)

Coupon rate is $\frac{22}{1000} = 0.022$. Coupon rate is higher than yield rate of 0.02, hence selling above par.

Ruppert Exercise 9

(a)

$$1050 = \frac{24}{\frac{r}{2}} \left(1 - \frac{1}{\left(1 + \frac{r}{2}\right)^{14}} \right) + \frac{1000}{\left(1 + \frac{r}{2}\right)^{14}}$$

Solving for r , we get $r = 0.03975274$

```
1 > zero = function(r) {(24/(r/2))*(1-1/(1+r/2)^14) + 1000/(1+
  r/2)^14 - 1050}
2 > uniroot(zero, c(0.001, 1))
3 $root
4 [1] 0.03975274
5
6 $f.root
7 [1] -0.03484846
8
9 $iter
10 [1] 5
11
12 $init.it
13 [1] NA
14
```

```
15 $estim.prec
16 [1] 6.103516e-05
```

(b)

Current price is \$1050. Coupon payment is \$24. Current yield is $2 * \frac{24}{1050} = 0.04571429$

(c)

Yield to maturity is less than current yield. This is because the bond is selling at above par (price > face value), hence the eventual face value principal payment would be discounted at a level steeper than the current yield, thereby resulting in a lower yield to maturity.

Ruppert Exercise 14

(a)

$$Y_T = 0.04 + 0.001T$$

Then,

$$P = \frac{1000}{(1 + Y_T(10))^{10}} = 613.9133$$

(b)

$$P = \frac{1000}{(1 + Y_T(9))^9} = 639.1099$$

Then, return R_2 is

$$R_2 = \frac{P_2}{P_1} - 1 = \frac{639.1099}{613.9133} - 1 = 0.0410426$$

Ruppert Exercise 16

(a)

To find the 5 year yield to maturity,

$$y(t) = \int_0^t r(s) ds = \int_0^t 0.03 + 0.001s + 0.0002s^2 ds$$

Then, $y(5) = 0.03416667$

```
1 > r = function(s) {0.03 + 0.001*s + 0.0002*s^2}
2 > integrate(r, 0, 5)$value/5
3 [1] 0.03416667
```

(b)

Assume face value of \$100,

$$P = \frac{100}{(1 + 0.03416667)^5} = 84.53709$$

Ruppert Exercise 20

(a)

To find the 4 year yield to maturity,

$$y(t) = \int_0^t r(s) ds = \int_0^t 0.022 + 0.005s - 0.004s^2 + 0.0003s^3 ds$$

Then,

$$P = \sum_{t=1}^8 \frac{21}{\left(1 + y\left(\frac{t}{2}\right)\right)^t} + \frac{1000}{\left(1 + y\left(\frac{8}{2}\right)\right)^8}$$

```

1 > r = function(s) {0.022 + 0.005*s - 0.004*s^2 + 0.0003*s^3}
2 > y = function(t) {integrate(r, 0, t)$value/t}
3 > yields = sapply(n/2, y)
4 > yields
5 [1] 0.02292604 0.02324167 0.02300312 0.02226667 0.02108854
   0.01952500 0.01763229 0.01546667
6 > pv_coupon = mapply(function(x, yield) {21/(1+yield)^x}, n,
   yields, SIMPLIFY=TRUE)
7 > pv_coupon
8 [1] 20.52934 20.05686 19.61500 19.22926 18.91918 18.69959
   18.58160 18.57351
9 > pv_face = 1000/(1+y(4))^8
10 > sum(pv_coupon) + pv_face
11 [1] 1038.657

```

The price of the bond is \$1038.657.

(b)

Assume that we are asking for **Macauley duration** (not modified duration)

$$D = \frac{\sum_{t=1}^8 \left[\frac{t}{2} \left(\frac{21}{(1+y(\frac{t}{2}))^t} \right) \right] + 4 * \frac{1000}{(1+y(4))^8}}{P}$$

```

1 > coupons = sapply(n, function(x) {(x/2) * (21/(1+y(x/2)))^x})
2 > coupons
3 [1] 10.26467 20.05686 29.42251 38.45851 47.29795 56.09876
   65.03560 74.29403
4 > sum(coupons) + 4*pv_face
5 [1] 3878.74
6 > (sum(coupons) + 4*pv_face)/(sum(pv_coupon) + pv_face)
7 [1] 3.73438

```

The duration is 3.73438