Statistical Finance (STAT W4290)

Assignment 3

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Ruppert 2

(a)

According to CAPM,

$$r = r_f + \alpha(r_m - r_f)$$

If $r_f = 0.03$ and $r_m = 0.14$, for r to be 0.11, then $\alpha = \frac{8}{11}$. The efficient way to invest is then $\frac{8}{11}$ market portfolio and the rest in risk free assets.

(b)

Since standard deviation of risk free asset is 0, the standard deviation of the portfolio is $\alpha\sigma_m = \frac{8}{11}0.12 = 0.08727273$

Ruppert 6

(a)

$$\beta = \frac{\sigma_{A,m}}{\sigma_m^2} = \frac{0.0165}{0.11^2} = 1.363636$$

(b)

$$E(r) = r_f + \beta(r_m - r_f) = 0.04 + 1.363636(0.12 - 0.04) = 0.1490909$$

(c)

Proportion of risk due to market risk is $\frac{\sigma_{A,m}}{\sigma_A^2} = \frac{165}{220} = 0.75$

Ruppert 8

(a)

$$E(r_m) = w_C * r_C + w_D * r_D = 0.6 * 0.04 + 0.4 * 0.06 = 0.048$$

(b)

$$\sigma_m^2 = w_C^2 \sigma_C + w_D^2 \sigma_D + 2 w_C w_D \sigma_C \sigma_D \rho_{CD}$$

Then, $\sigma_m = 0.1144727$

(c)

$$\sigma = \alpha \sigma_m$$

$\alpha = \frac{\sigma}{\sigma_m} = \frac{0.03}{0.1144727} = 0.2620712.$
Then $1 - 0.2620712 = 0.7379288$ of equity should be in risk free assets.

(d)

$$r = r_f + \alpha(r_m - r_f)$$

For $r = 0.07$,

$$\alpha = \frac{r - r_f}{r_m - r_f} = \frac{0.07 - 0.012}{0.048 - 0.012} = 1.611111$$

This means that 1.611111 of the equity should be in the risky portfolio, and -0.611111 should be in risk free. In the risky portfolio, the proportion of C and D stays the same. Hence, overall:

- 0.9666666 in C
- 0.6444444 in D
- -0.611111 in risk free assets (short position)

Ruppert 10

(a)

$$r = r_f + \alpha(r_m - r_f)$$

For $r = 0.11$,

$$\alpha = \frac{r - r_f}{r_m - r_f} = \frac{0.11 - 0.07}{0.14 - 0.07} = 0.5714286$$

0.5714286 in risky assets, $1 - 0.5714286 = 0.4285714$ in risk free assets.

(b)

$$\sigma = \alpha\sigma_m = 0.5714286 * 0.12 = 0.06857143$$

Additional Problem A1

(a)

The general equation for CML is

$$\mu = \mu_f + \frac{\mu_m - \mu_f}{\sigma_m}\sigma$$

The slope is then $\frac{\mu_m - \mu_f}{\sigma_m} = \frac{0.23 - 0.07}{0.32} = 0.5$

(b)

(i)

If $\mu = 0.39$, and $0.39 = 0.07 + 0.5\sigma$, then $\sigma = 0.64$

(ii)

Honestly I'll use it to pay my tuition. But answering the question, let α be the proportion invested in market portfolio. $\alpha = \frac{\sigma}{\sigma_m} = \frac{0.64}{0.32} = 2$. Then, weight of risk free assets is -1. Short \$1000 risk free, long \$2000 risky portfolio.

(c)

In this case, $\alpha = 0.7$. Given that

$$\mu = \mu_f + \alpha(\mu_m - \mu_f) = 0.182$$

You should expect to have $1000 * (1 + 0.182) = \$1182$

Additional Problem A2

(a)

$$w_A = \frac{100 * 1.5}{100 * 1.5 + 150 * 2} = \frac{1}{3}$$

$$w_B = 1 - \frac{1}{3} = \frac{2}{3}$$

$$r_m = \frac{1}{3}0.15 + \frac{2}{3}0.12 = 0.13$$

(b)

$$\sigma_m^2 = w_A^2 0.15^2 + (1 - w_A)^2 0.09^2 + 2w_A(1 - w_A) * 0.15 * 0.09 * \frac{1}{3} = 0.0081$$

Then, $\sigma_m = 0.09$.

(c)

$$\begin{aligned}
\beta_A &= \frac{\sigma_{A,m}}{\sigma_m^2} \\
&= \frac{\frac{1}{3}\sigma_A^2 + \frac{2}{3}\sigma_{A,B}}{\sigma_m^2} \\
&= \frac{\frac{1}{3}\sigma_A^2 + \frac{2}{3}\sigma_A\sigma_B\rho_{AB}}{\sigma_m^2} \\
&= 1.296296
\end{aligned}$$

(d)

$$r_A = r_f + \beta_A(r_m - r_f)$$

It's late and I can't do math anymore, so...

```
> f = function(x) {x + 1.296296*(0.13-x) - 0.15}
> uniroot(f, c(-1000, 1000))
$root
[1] 0.06249993

$f.root
[1] -2.939315e-14

$iter
[1] 2

$init.it
[1] NA

$estim.prec
[1] 6.103516e-05
```

$$r_f = 0.06249993$$

I initially thought we had to calculate the tangency portfolio on our own. If this is the case, please use this answer instead:

To calculate the tangency portfolio, let w_A and w_B be weights of A and B in the tangency portfolio. $w_B = 1 - w_A$.

$$r_m = w_A 0.15 + (1 - w_A) 0.12$$

and

$$\sigma_m^2 = w_A^2 0.15^2 + (1 - w_A)^2 0.09^2 + 2 w_A (1 - w_A) * 0.15 * 0.09 * \frac{1}{3}$$

w_A of the tangency portfolio is the one that minimizes σ_m^2.

```
> sigmam2 = function(w) {w^2 * 0.15^2 + (1-w)^2 * 0.09^2 + 2*w*(1-w)*0.15*0.09/3}
> optimize(sigmam2, interval=c(-100,100), maximum=FALSE)
$minimum
[1] 0.1666667

$objective
[1] 0.0075
```

$w_A = 0.1666667$ with an accompanying $\sigma_m^2 = 0.0075$

$$r_m = w_A 0.15 + (1 - w_A) 0.12 = 0.125$$

As calculated earlier, $\sigma_m^2 = 0.0075$. Hence,

$$\sigma_m = \sqrt{w_A^2 0.15^2 + (1 - w_A)^2 0.09^2 + 2 w_A (1 - w_A) * 0.15 * 0.09 * \frac{1}{3}} = 0.08660254$$