Statistical Finance (STAT W4290)

Assignment 4

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Rupert Problem 3

$$1200 = \frac{40}{r}\left(1 - \frac{1}{(1+r)^{30}}\right) + \frac{1000}{(1+r)^{30}}$$

Solving for r, we get $r = 0.03241618$

```
> zero = function(r) {(40/r)*(1-1/(1+r)^60) + 1000/(1+r)^60 - 1200}
> uniroot(zero, c(0.03, 0.04), maxiter=100)
$root
[1] 0.03241618

$f.root
[1] -0.5497406

$iter
[1] 3

$init.it
[1] NA

$estim.prec
[1] 6.103516e-05
```

Rupert Problem 4

$$9800 = \frac{280}{\frac{r}{2}}\left(1 - \frac{1}{(1+\frac{r}{2})^{8*2}}\right) + \frac{10000}{(1+\frac{r}{2})^{8*2}}$$

Solving for r, we get

```
> zero = function(r) {(280/(r/2)) * (1-1/(1+(r/2))^(8*2)) +
    10000/(1+(r/2))^(8*2) - 9800}
> uniroot(zero, c(0.05, 0.06), maxiter=100)
$root
[1] 0.05920323

$f.root
[1] -1.794659

$iter
[1] 2

$init.it
[1] NA

$estim.prec
[1] 6.103516e-05
```

Rupert Exercise 1

$$y(t) = \frac{1}{t}\int_0^t r(s)\,ds$$

(a)

Setting $t = 20$,

$$\begin{aligned} y(20) &= \frac{1}{20}\int_0^t 0.028 + 0.00042s\,ds \\ &= \frac{1}{20}(0.028 * 20 + 0.00021 * (20^2)) \\ &= 0.0322 \end{aligned}$$

(b)

$$y(15) = 0.03115$$

```
> r = function(t) {0.028 + 0.00042*t}
> y = integrate(r, 0, 15)$value / 15
> y
[1] 0.03115
```

Then the price of the bond is

$$P = \frac{1000}{(1 + 0.03115)^{15}} = 631.2077$$

Rupert Exercise 3

$$y(t) = \frac{1}{t} \int_0^t r(s)\, ds$$

(a)

Setting $t = 5$, $y(5) = 0.03616667$

```
> r = function(t){0.032+0.001*t+0.0002*t^2}
> y = integrate(r, 0, 5)$value / 5
> y
[1] 0.03616667
```

(b)

Assume bond has par \$100, then

$$P = \frac{100}{(1 + 0.03616667)^5} = 83.72438$$

Ruppert Exercise 5

Now the sum of a geometric series $C + Cm + Cm^2 + ... = \frac{C}{1-m}$. If $m = \frac{1}{1-r}$, then $C + Cm + Cm^2 + ... = \frac{C(1+r)}{r}$. If the cash flow C begins a year later, we discount the entire amount by $(1+r)$ thereby making the present value of the sum $\frac{C}{r}$.

Then,

$$
\begin{aligned}
&\sum_{t=1}^{2T} \frac{C}{(1+r)^t} + \frac{\text{PAR}}{(1+r)^{2T}} \\
&= \sum_{t=1}^{\infty} \frac{C}{(1+r)^t} - \sum_{t=2T+1}^{\infty} \frac{C}{(1+r)^t} + \frac{\text{PAR}}{(1+r)^{2T}} \\
&= \frac{C}{r} - \frac{C}{r(1+r)^{2T}} + \frac{\text{PAR}}{(1+r)^{2T}} \\
&= \frac{C}{r} + \left(\text{PAR} - \frac{C}{r}\right)(1+r)^{-2T}
\end{aligned}
$$

Ruppert Exercise 7

(a)

Given that

$$
\begin{aligned}
818 &= \frac{1000}{\exp 5r} \\
r &= \frac{\log \frac{1000}{818}}{5} && = 0.04017859
\end{aligned}
$$

This means that $r = 0.04017859$

(b)

$$P = \frac{1000}{\exp 4 * 0.042} = 845.3538$$

(c)

$$R_2 = \frac{P_2}{P_1} - 1 = \frac{845.3538}{818} - 1 = 0.03343985$$

Ruppert Exercise 8

(a)

$$P = \frac{22}{\frac{0.04}{2}}\left(1 - \frac{1}{1.02^{20}}\right) + \frac{1000}{1.02^{20}} = 1032.703$$

(b)

Coupon rate is $\frac{22}{1000} = 0.022$. Coupon rate is higher than yield rate of 0.02, hence selling above par.

Ruppert Exercise 9

(a)

$$1050 = \frac{24}{\frac{r}{2}}\left(1 - \frac{1}{\left(1+\frac{r}{2}\right)^{14}}\right) + \frac{1000}{\left(1+\frac{r}{2}\right)^{14}}$$

Solving for r, we get $r = 0.03975274$

```
> zero = function(r) {(24/(r/2))*(1-1/(1+r/2)^14) + 1000/(1+
   r/2)^14 - 1050}
> uniroot(zero, c(0.001, 1))
$root
[1] 0.03975274

$f.root
[1] -0.03484846

$iter
[1] 5

$init.it
[1] NA

```

```
$estim.prec
[1] 6.103516e-05
```

(b)

Current price is \$1050. Coupon payment is \$24. Current yield is $2 * \frac{24}{1050} = 0.04571429$

(c)

Yield to maturity is less than current yield. This is because the bond is selling at above par (price > face value), hence the eventual face value principal payment would be discounted at a level steeper than the current yield, thereby resulting in a lower yield to maturity.

Ruppert Exercise 14

(a)

$$Y_T = 0.04 + 0.001T$$

Then,

$$P = \frac{1000}{(1 + Y_T(10))^{10}} = 613.9133$$

(b)

$$P = \frac{1000}{(1 + Y_T(9))^{9}} = 639.1099$$

Then, return R_2 is

$$R_2 = \frac{P_2}{P_1} - 1 = \frac{639.1099}{613.9133} - 1 = 0.0410426$$

Ruppert Exercise 16

(a)

To find the 5 year yield to maturity,

$$y(t) = \int_0^t r(s)\,ds = \int_0^t 0.03 + 0.001s + 0.0002s^2\,ds$$

Then, $y(5) = 0.03416667$

```
> r = function(s) {0.03 + 0.001*s + 0.0002*s^2}
> integrate(r, 0, 5)$value/5
[1] 0.03416667
```

(b)

Assume face value of $100,

$$P = \frac{100}{(1 + 0.03416667)^5} = 84.53709$$

Ruppert Exercise 20

(a)

To find the 4 year yield to maturity,

$$y(t) = \int_0^t r(s)\,ds = \int_0^t 0.022 + 0.005s - 0.004s^2 + 0.0003s^3\,ds$$

Then,

$$P = \sum_{t=1}^{8} \frac{21}{\left(1 + y\left(\frac{t}{2}\right)\right)^t} + \frac{1000}{\left(1 + y\left(\frac{8}{2}\right)\right)^8}$$

```
> r = function(s) {0.022 + 0.005*s - 0.004*s^2 + 0.0003*s^3}
> y = function(t) {integrate(r, 0, t)$value/t}
> yields = sapply(n/2, y)
> yields
[1] 0.02292604 0.02324167 0.02300312 0.02226667 0.02108854
    0.01952500 0.01763229 0.01546667
> pv_coupon = mapply(function(x, yield) {21/(1+yield)^x}, n,
    yields, SIMPLIFY=TRUE)
> pv_coupon
[1] 20.52934 20.05686 19.61500 19.22926 18.91918 18.69959
    18.58160 18.57351
> pv_face = 1000/(1+y(4))^8
> sum(pv_coupon) + pv_face
[1] 1038.657
```

The price of the bond is \$1038.657.

(b)

Assume that we are asking for **Macaulay duration** (not modified duration)

$$D = \frac{\sum_{t=1}^{8}\left[\frac{t}{2}\left(\frac{21}{\left(1+y\left(\frac{t}{2}\right)\right)^t}\right)\right] + 4 * \frac{1000}{(1+y(4))^8}}{P}$$

```
> coupons = sapply(n, function(x) {(x/2) * (21/(1+y(x/2))^x)
    })
> coupons
[1] 10.26467 20.05686 29.42251 38.45851 47.29795 56.09876
    65.03560 74.29403
> sum(coupons) + 4*pv_face
[1] 3878.74
> (sum(coupons) + 4*pv_face)/(sum(pv_coupon) + pv_face)
[1] 3.73438
```

The duration is 3.73438