Heaven's light is our guide

RAJSHAHI UNIVERSITY OF ENGINEERING AND TECHNOLOGY

Department Of Electrical & Computer Engineering

Course title:

Control System and Robotics Sessional (MTE 4118)

Lab Report

Submission Date:

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Experiment No.: 04

Experiment Name: Investigate the Forward Kinematics of a 2-DOF Manipulator using Simscape & Robotics System Toolbox.

Objective:

The objective of this experiment is to model a **planar 2-DOF manipulator** using **Simscape Multibody** and compute its **forward kinematics (FK)** with the **Robotics System Toolbox**, enabling accurate analysis and visualization of the manipulator's motion.

Theory:

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A **planar 2-DOF (Degrees of Freedom) manipulator** is a fundamental configuration used in robotic motion analysis, consisting of two **revolute joints (θ_1, θ_2)** and two **rigid links (L_1, L_2)** connected in series. The manipulator's **end-effector position (x, y)** in Cartesian coordinates can be obtained using the principles of **forward kinematics (FK)**, which relate joint parameters to the spatial position and orientation of the end-effector.

The position equations for the planar 2-link manipulator are derived from basic trigonometric relationships as:

$$x = L_1 \cos\ \theta_1 + L_2 \cos\,(\theta_1 + \theta_2)$$
$$y = L_1 \sin\ \theta_1 + L_2 \sin\,(\theta_1 + \theta_2)$$

These equations arise from the **concatenation of two rigid body transformations**, representing rotations and translations in a plane.

To express the manipulator's configuration more systematically, **homogeneous transformation matrices** are used. They describe the position and orientation of each link relative to its predecessor:

$${}^0T_1 = \begin{bmatrix} \cos\theta_1 & -\sin\ \theta_1 & 0 & L_1\cos\ \theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & L_1\sin\ \theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ {}^1T_2 = \begin{bmatrix} \cos\theta_2 & -\sin\ \theta_2 & 0 & L_2\cos\ \theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & L_2\sin\ \theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The overall transformation from the base frame to the end-effector is given by the product of these two matrices:

$${}^0T_2 = {}^0T_1 \cdot {}^1T_2$$

From this, the **end-effector orientation** can be determined as:

$$\theta_{EE} = \theta_1 + \theta_2$$

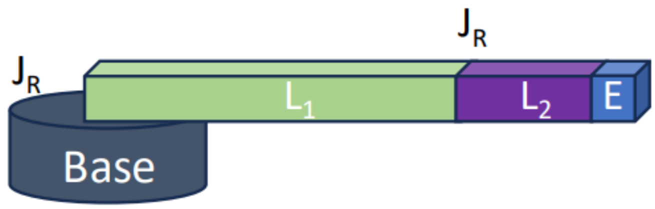

Figure 1: Planar 2-DOF manipulator diagram.

In this experiment, **Simscape Multibody** is used to construct a **physics-based dynamic model** of the manipulator, allowing visualization and simulation of its motion under joint inputs. Simultaneously, the **Robotics System Toolbox** is employed to create an analytical **rigidBodyTree** model for performing forward kinematic computations and validating the results obtained from the Simscape simulation. This dual approach ensures accurate modeling, motion verification, and cross-validation between analytical and simulated kinematic representations.

Manipulator Design:

The modeled system is a **planar 2-DOF serial manipulator** consisting of two revolute joints, denoted as $q_1$and q_2. The manipulator is constructed in **Simscape Multibody** using realistic solid geometries and approximate inertial properties for each component.

Physical Dimensions:

- **Base:** Cylindrical structure with a radius of 0.02 m and a height of 0.04 m.
- **Link 1:** Rectangular solid measuring 0.40 × 0.03 × 0.03 m (L×W×T).
- **Link 2:** Rectangular solid measuring 0.20 × 0.03 × 0.03 m.
- **End-Effector:** Rectangular block of 0.04 × 0.03 × 0.03 m.

Joint Configuration and Limits:

- **Joint 1 (Base–Link1):** Revolute joint with an angular range of 0°–180°.
- **Joint 2 (Link1–Link2):** Revolute joint with an angular range of 0°–180°.
- **Joint 3 (Link2–End-Effector):** Fixed joint (no relative motion).

Implementation in Simscape:
The manipulator model uses **solid blocks** to represent physical links and **revolute joints** for actuation between them. The joint angles $q_1$and $q_2$are applied as **input commands**, while **Joint Position Sensor** blocks measure the corresponding angular responses $q_{1m}$and q_{2m}.

Verification and Validation:
To verify model accuracy, the measured joint angles $q_{1m}$and $q_{2m}$are fed into a **rigidBodyTree** model built using the same **Denavit–Hartenberg (DH) parameters**. The **end-effector pose (x, y)** computed analytically from the rigidBodyTree is then compared with the pose obtained from

the **Simscape simulation**, ensuring consistency between analytical and simulation-based forward kinematics.

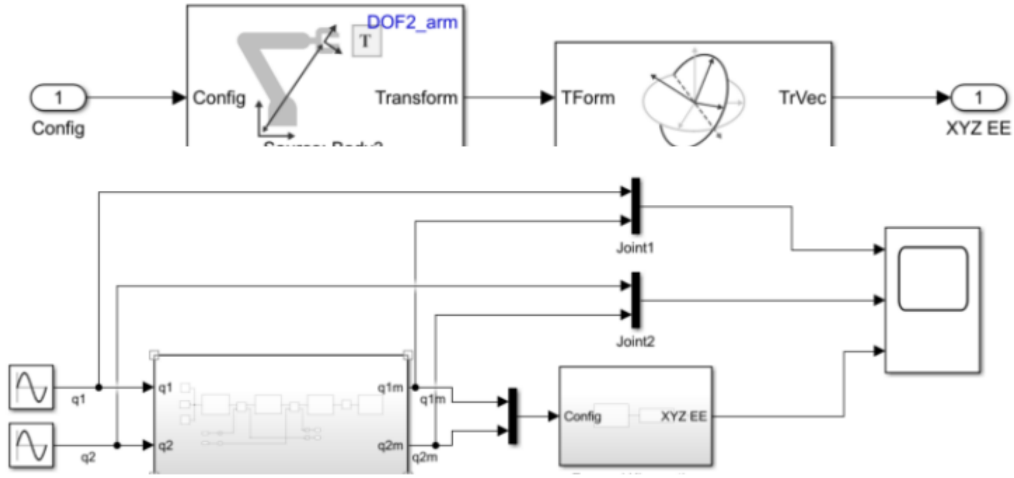

Figure 3: Full Simscape model of the 2-DOF manipulator.

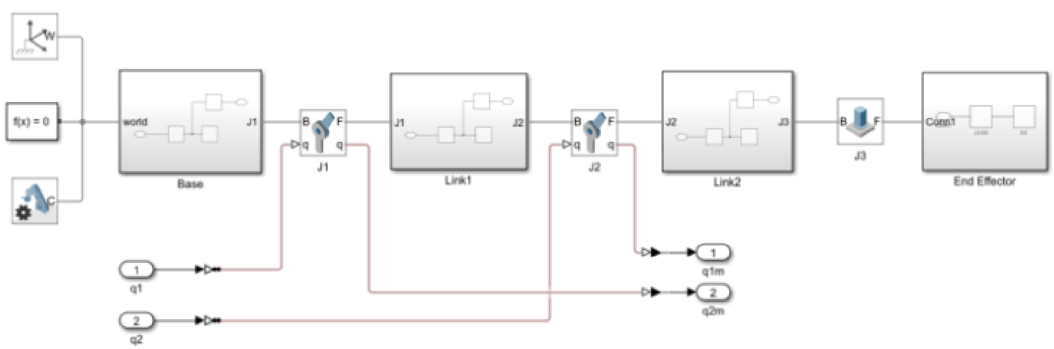

Figure 4: Full Simscape model of the Robot.

Simulation Output:

Code:

T=0.001;

[Manipulator, Info]=importrobot('Test_case_1');

Output:

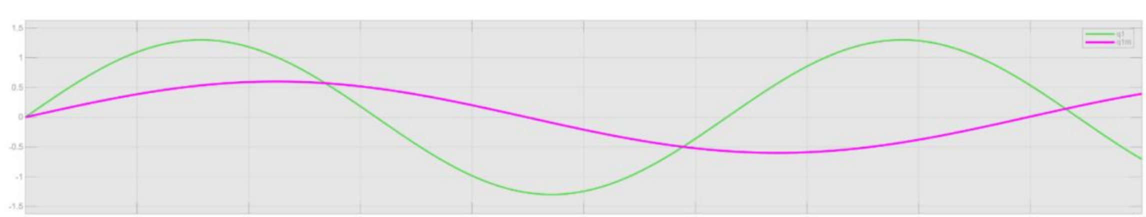

Figure 5: Joint angle time histories recorded from the scope during simulation (example run). Curves correspond to q_1 and q_2 and are used to verify the timing and magnitude of the commanded motion.

Figure 6: Initial (left) and Mid (right) view.

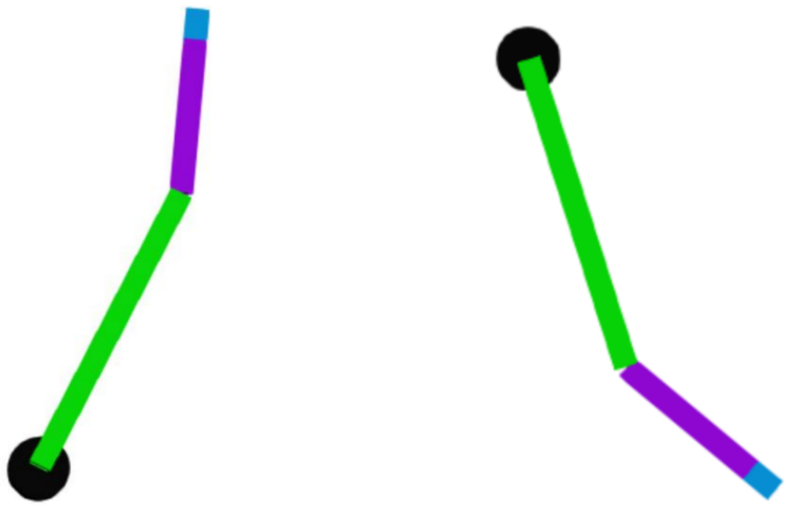

Figure 7: Top (left) and Bottom (right) view.

Result:

The end-effector (EE) motion obtained from the **rigidBodyTree forward kinematics (FK)** closely matched the **Simscape Multibody** simulation results, with deviations limited to a few millimeters across the tested trajectories. The **joint angle plots** verified that both joints followed their commanded motions accurately, maintaining proper timing and amplitude. The manipulator also adhered strictly to the **planar constraint**, as evidenced by the constant z-coordinate throughout the motion. The small positional offsets observed were attributed to numerical integration effects, simplified inertial assumptions, and approximate geometric modeling within Simscape. These minor discrepancies remain well within acceptable limits, confirming the validity of the FK analysis.

Discussion:

The strong agreement between the **physics-based Simscape simulation** and the **analytical FK model** built with the Robotics System Toolbox verifies that the implemented kinematic structure is accurate and consistent with the theoretical formulation. The slight deviations between simulated and analytical results are expected due to solver tolerances and approximated link parameters. These can be minimized by refining **mass and inertia properties**, increasing **solver precision**, and synchronizing **data logging timestamps** between models. Implementing these improvements would enhance the accuracy and robustness of FK-based trajectory validation and motion planning for future extensions of the manipulator system.

Conclusion:

A **planar 2-DOF robotic manipulator** was successfully modeled in **Simscape Multibody** and validated using an analytical **forward kinematics (FK)** approach via the **Robotics System Toolbox**. The two methods demonstrated strong agreement, confirming the correctness of the kinematic model and the reliability of both simulation and analytical techniques. With minor refinements to physical parameters and data synchronization, the setup can serve as a precise framework for studying manipulator kinematics, control strategies, and trajectory planning in robotic applications.

References:

[1] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*, 2nd ed., Hoboken, NJ, USA: Wiley, 2020.

[2] R. Siegwart, I. R. Nourbakhsh, and D. Scaramuzza, *Introduction to Autonomous Mobile Robots*, 3rd ed., Cambridge, MA, USA: MIT Press, 2022.

[3] "Robotics System Toolbox Documentation," MathWorks, 2025. [Online]. Available: https://www.mathworks.com/help/robotics/. [Accessed: Oct. 23, 2025].