Heaven's light is our guide

RAJSHAHI UNIVERSITY OF ENGINEERING AND TECHNOLOGY

Department Of Electrical & Computer Engineering

Course title:

Digital Signal Processing Sessional (ECE 4124)

Lab Report

Submission Date:

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Experiment No. 6

Experiment Name: Design and Analysis of a Low-Pass FIR and IIR Filter and Comparison of Their Frequency Responses

Theory:

In **Digital Signal Processing (DSP)**, filters are essential tools used to extract, suppress, or modify specific frequency components of a discrete-time signal. Among various types, the **low-pass filter (LPF)** is one of the most fundamental. It allows frequencies below a specified **cutoff frequency (f_a)** to pass while attenuating higher frequencies. Such filters are crucial in applications like noise reduction, signal smoothing, and data reconstruction.

Filters in DSP are broadly categorized into two types — **Finite Impulse Response (FIR)** and **Infinite Impulse Response (IIR)** — based on the duration of their impulse responses and structural characteristics.

1. FIR Filter (Finite Impulse Response):

An FIR filter produces an output that depends only on the current and a finite number of past input samples. Its impulse response settles to zero after a finite time. The general difference equation is:

$$y[n] = \sum_{k=0}^{N-1} b_k \, x[n-k]$$

where b_k are the filter coefficients and N is the filter order.

Key Characteristics:

- Always **stable** since it has no feedback component.
- Can be designed to exhibit a **linear phase response**, preserving the shape of time-domain waveforms—important for audio, biomedical, and communication systems.
- Requires higher order for sharp transitions, leading to increased computational load.

2. IIR Filter (Infinite Impulse Response):

An IIR filter has a recursive structure where the output depends on both past inputs and past outputs, leading to an impulse response of theoretically infinite duration. Its general difference equation is:

$$y[n] = \sum_{k=0}^{M} b_k \, x[n-k] - \sum_{l=1}^{N} a_l \, y[n-l]$$

Key Characteristics:

- More **computationally efficient** than FIR filters, achieving sharp cutoff characteristics with lower filter order.
- May exhibit **non-linear phase response**, which can distort time-domain signals.
- **Potentially unstable** if pole locations are not carefully chosen within the unit circle in the z-plane.

3. Frequency Response:

The frequency response of a discrete-time filter describes how each frequency component of the input signal is modified. It is derived from the filter's **transfer function** $H(e^{j\omega})$, with the magnitude response expressed as:

$$| H(e^{j\omega}) |= \sqrt{[\mathrm{Re}(H)]^2 + [\mathrm{Im}(H)]^2}$$

This represents the amplitude gain across different frequencies, allowing visualization of how effectively the filter passes or attenuates components.

4. Design Specification:

For comparative analysis, both **FIR** and **IIR** filters are designed with identical parameters:

- **Cutoff Frequency (f_a):** 100 Hz
- **Sampling Frequency (f_s):** 1000 Hz

This ensures a fair comparison of their **magnitude responses**, stability, computational complexity, and phase characteristics. Through such analysis, the trade-offs between **stability, accuracy, and efficiency** in filter design can be clearly demonstrated.

Required Tools:

1. Python
2. VS Code
3. MS Office

Code:

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import butter,
firwin, freqz, lfilter

# Filter and Signal Parameters
fs = 1000           # Sampling
frequency (Hz)
fcut = 100          # Cutoff
frequency (Hz)
n = 50              # FIR filter
order
t = np.arange(0, 1, 1/fs)  # 1-
second time vector

# FIR Filter Design
b_fir = firwin(numtaps=n+1,
cutoff=fcut, fs=fs)
w_fir, H_fir = freqz(b_fir,
worN=1024, fs=fs)

# IIR Filter Design (Butterworth)
b_iir, a_iir = butter(4,
fcut/(fs/2), btype='low')
w_iir, H_iir = freqz(b_iir, a_iir,
worN=1024, fs=fs)

# Generate Input Signal
x = np.sin(2*np.pi*50*t) +
0.5*np.random.randn(len(t))  # 50
Hz + noise

# Apply Filters
y_fir = lfilter(b_fir, 1, x)
y_iir = lfilter(b_iir, a_iir, x)

# Plot Results
plt.figure(figsize=(10, 10))
```

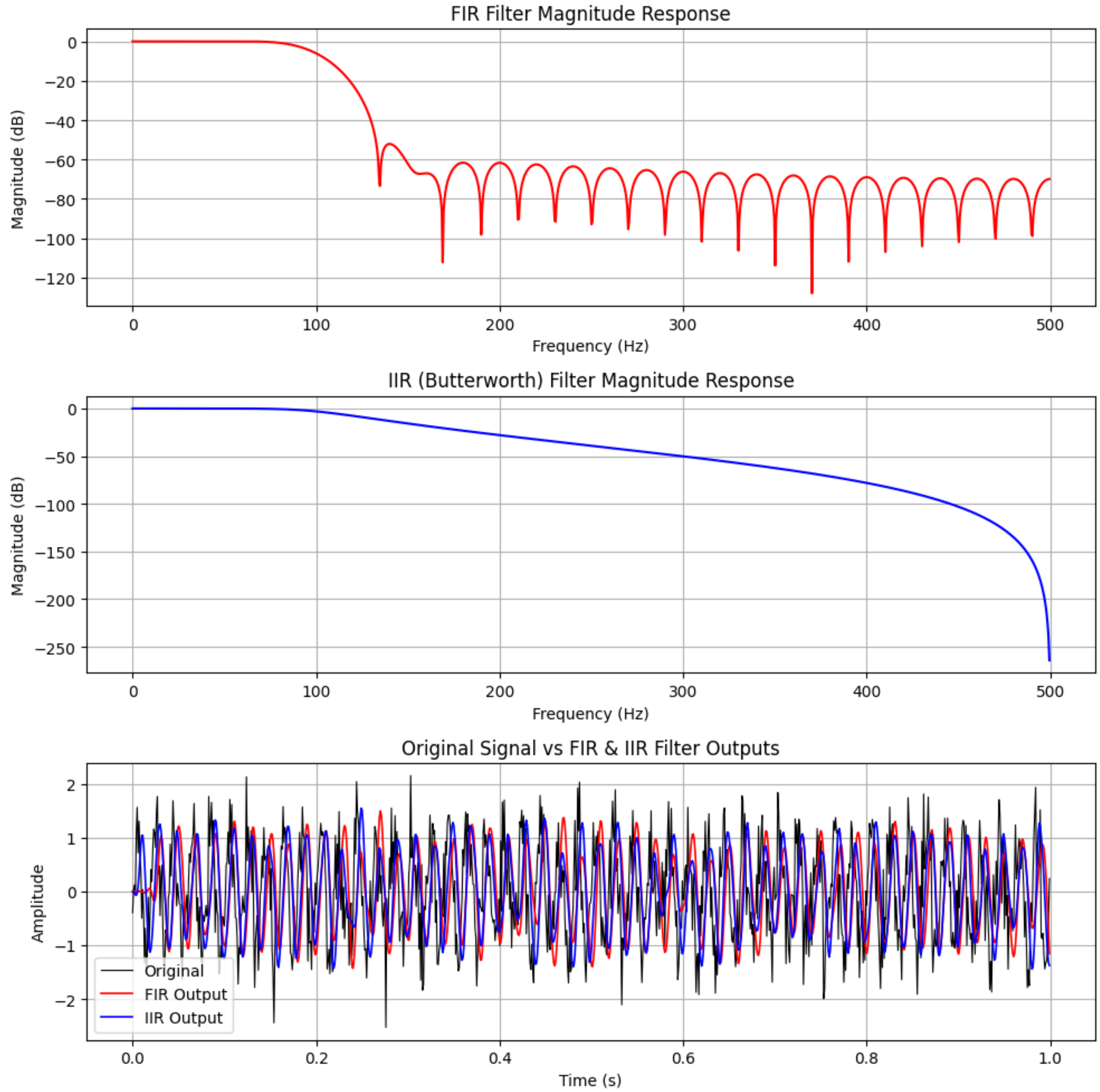
```python
# --- Subplot 1: FIR Frequency
Response ---
plt.subplot(3, 1, 1)
plt.plot(w_fir,
20*np.log10(abs(H_fir)), 'r',
linewidth=1.5)
plt.title('FIR Filter Magnitude
Response')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Magnitude (dB)')
plt.grid(True)

# --- Subplot 2: IIR Frequency
Response ---
plt.subplot(3, 1, 2)
plt.plot(w_iir,
20*np.log10(abs(H_iir)), 'b',
linewidth=1.5)
plt.title('IIR (Butterworth) Filter
Magnitude Response')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Magnitude (dB)')
plt.grid(True)

# --- Subplot 3: Time-Domain
Comparison ---
plt.subplot(3, 1, 3)
plt.plot(t, x, 'k', linewidth=0.8,
label='Original')
plt.plot(t, y_fir, 'r',
linewidth=1.2, label='FIR Output')
plt.plot(t, y_iir, 'b',
linewidth=1.2, label='IIR Output')
plt.title('Original Signal vs FIR &
IIR Filter Outputs')
plt.xlabel('Time (s)')
plt.ylabel('Amplitude')
plt.legend()
plt.grid(True)

plt.tight_layout()
plt.show()
```

Output:



Result:
Both FIR and IIR filters effectively removed noise components above the 100 Hz cutoff frequency while preserving the low-frequency (≈ 50 Hz) signal. The **FIR filter** exhibited a linear phase response but had a wider transition band and minor stopband ripples. The **IIR (Butterworth) filter** achieved a sharper cutoff and smoother magnitude response with fewer coefficients, though it introduced slight phase distortion due to its recursive structure. Overall, both filters demonstrated efficient low-pass filtering performance.

Discussion:

The experiment highlights clear distinctions between FIR and IIR filter characteristics. The **FIR filter**, designed using the window method, ensured linear phase accuracy, making it suitable for audio and data-sensitive applications, but required higher order for sharper transitions. Conversely, the **IIR filter** provided a steeper roll-off with lower computational demand, at the expense of phase linearity. The comparative results confirmed that FIR filters are more phase-accurate but computationally intensive, while IIR filters are more efficient but exhibit phase distortion, reflecting the trade-off between precision and efficiency in DSP design.

Conclusion:

This study successfully demonstrated the design and comparison of FIR and IIR low-pass filters. Both effectively suppressed high-frequency noise and retained the desired low-frequency content. The **FIR filter** offered stable, linear-phase performance, whereas the **IIR (Butterworth) filter** achieved better frequency selectivity with reduced computational cost. The results emphasize that the choice between FIR and IIR designs depends on application requirements—balancing **accuracy, stability, and computational efficiency** in practical DSP systems such as audio processing, signal denoising, and biomedical analysis.

References:

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[4] S. K. Mitra, *Digital Signal Processing: A Computer-Based Approach*, 4th ed., McGraw-Hill, 2011.