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Rajshahi University of Engineering and Technology

Dept. of Electrical & Computer Engineering

Course Title:

Control System and Robotics Sessional

Course No:

MTE 4118

Lab Report 2

Submitted To:	Submitted By:
Md. Sakib Hassan Chowdhury	Dibakar Roy
Lecturer	Roll: 2010056
Department of MTE, RUET	Session: 2020-21

Experiment No: 02
Experiment Name: Experiment on Controllability and Observability of a System

Objectives:

- To understand the concepts of controllability and observability in state-space representation.
- To verify controllability and observability of a given system using MATLAB.
- To analyze the system behavior using controllability and observability matrices.

Theory:
In control systems, controllability and observability are fundamental properties of dynamic systems:

- Controllability determines whether it is possible to move the system from any initial state to any final state within finite time using suitable control inputs. A system is said to be controllable at time t_0 if it is possible by means of an unconstrained control vector to transfer the system from any initial state $x(t_0)$ to any other state in a finite interval of time [1]. A system described by (A, B) is controllable if the controllability matrix $C = [B\ A^1B\ A^2B \ldots A^{n-1}B]$ has full rank n (where n is the number of states).
- Observability determines whether it is possible to determine the internal states of the system from its outputs over time. A system is said to be observable at time t_0 if, with the system in state $x(t_0)$,it is possible to determine this state from the observation of the output over a finite time interval [1]. A system described by (A, C) is observable if the observability matrix

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \qquad \text{has full rank } n.$$

These concepts help determine the feasibility of control and state estimation in real-world applications like robotics, aerospace systems, and automation.

Working Principle (MATLAB Code):

```
clc

clear all

% Define your state-space matrices
A = [2 0 1; 0 0 5; 6 2 0];
B = [0; 5; 6];
C = [0 5 6];
D = 0;  % Define D to avoid error in ss()

% Create state-space system
sys = ss(A, B, C, D);

% Check controllability
Co = ctrb(A, B);
rank_Co = rank(Co);
n = size(A, 1);  % Number of states
```

```
fprintf('Rank of controllability matrix: %d\n', rank_Co);
if rank_Co == n
    disp('The system is CONTROLLABLE.');
else
    disp('The system is NOT CONTROLLABLE.');
end

% Check observability
Ob = obsv(A, C);
rank_Ob = rank(Ob);

fprintf('Rank of observability matrix: %d\n', rank_Ob);
if rank_Ob == n
    disp('The system is OBSERVABLE.');
else
    disp('The system is NOT OBSERVABLE.');
end
```

Result:

```
Command Window
New to MATLAB? See resources for Getting Started.
  Rank of controllability matrix: 3
  The system is CONTROLLABLE.
  Rank of observability matrix: 3
  The system is OBSERVABLE.
fx >>
```

Controllability Matrix Rank: 3

Observability Matrix Rank: 3

Since the ranks are equal to the number of states (3), the system is:

- CONTROLLABLE
- OBSERVABLE

Discussion:

The experiment demonstrates the use of MATLAB to assess the internal properties of a dynamic system modeled in state-space form. The controllability matrix was computed using the ctrb() [2] function and found to have full rank, confirming that the system's states can be fully controlled via the input. Similarly, the observability matrix using obsv() [3] also had full rank, indicating that the internal state variables can be reconstructed from the system's outputs. These properties are crucial for designing effective controllers and observers in practical systems such as autonomous robots or industrial control loops.

Conclusion:

The system under consideration was found to be both controllable and observable, satisfying essential criteria for the design of state feedback controllers and observers. MATLAB provides a robust platform for verifying these properties efficiently using built-in functions and state-space representations.

References

[1] K. Ogata, Modern Control Engineering, 5th ed., Upper Saddle River, NJ, USA: Prentice Hall, 2010.

[2] "MATLAB," [Online]. Available: https://www.mathworks.com/help/control/ref/statespacemodel.ctrb.html. [Accessed 7 August 2025].

[3] "MATLAB," [Online]. Available: https://www.mathworks.com/help/control/ref/statespacemodel.obsv.html. [Accessed 7 August 2025].

Controllability and Observability of a System

Given System

The system is defined by the following matrices:

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 5 \\ 6 & 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 5 \\ 6 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 5 & 6 \end{bmatrix}$$

Controllability Matrix $\mathcal{Q}_c$

The controllability matrix is defined as:

$$\mathcal{Q}_c = [B \quad AB \quad A^2B]$$

First, compute:

$$AB = A \cdot B = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 5 \\ 6 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 30 \\ 10 \end{bmatrix}$$

$$A^2B = A \cdot (AB) = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 5 \\ 6 & 2 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 30 \\ 10 \end{bmatrix} = \begin{bmatrix} 22 \\ 50 \\ 96 \end{bmatrix}$$

Thus:

$$\mathcal{Q}_c = \begin{bmatrix} 0 & 6 & 22 \\ 5 & 30 & 50 \\ 6 & 10 & 96 \end{bmatrix}$$

Determinant of $\mathcal{Q}_c$

$$\det(\mathcal{Q}_c) = \begin{vmatrix} 0 & 6 & 22 \\ 5 & 30 & 50 \\ 6 & 10 & 96 \end{vmatrix} = -3940 \neq 0$$

Hence, $\text{rank}(\mathcal{Q}_c) = 3 = \text{n}$, the system is completely controllable.

Observability Matrix $\mathcal{Q}_o$

$$\mathcal{Q}_o = \begin{bmatrix} C^T & A^T C^T & (A^T)^2 C^T \end{bmatrix}$$

$$\mathcal{C}^T = \begin{bmatrix} 0 \\ 5 \\ 6 \end{bmatrix}$$

First, compute:

$$A^T C^T = A^T \cdot C^T = \begin{bmatrix} 2 & 0 & 6 \\ 0 & 0 & 2 \\ 1 & 5 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 36 \\ 12 \\ 25 \end{bmatrix}$$

$$(A^T)^2 C^T = A^T \cdot (A^T C^T) = \begin{bmatrix} 2 & 0 & 6 \\ 0 & 0 & 2 \\ 1 & 5 & 0 \end{bmatrix} \begin{bmatrix} 36 \\ 12 \\ 25 \end{bmatrix} = \begin{bmatrix} 222 \\ 50 \\ 96 \end{bmatrix}$$

Thus:

$$\mathcal{Q}_o = \begin{bmatrix} 0 & 36 & 222 \\ 5 & 12 & 50 \\ 6 & 25 & 96 \end{bmatrix}$$

Determinant of $\mathcal{Q}_o$

$$\det(\mathcal{Q}_o) = \begin{bmatrix} 0 & 36 & 222 \\ 5 & 12 & 50 \\ 6 & 25 & 96 \end{bmatrix} = 5286 \neq 0$$

Hence, rank($\mathcal{Q}_o$) = 3 = n, the system is completely observable.

Conclusion

Since both the controllability matrix $\mathcal{Q}_c$ and observability matrix $\mathcal{Q}_o$ have full rank (3), the system is:

- Completely Controllable
- Completely Observable