Assignment
on
Euler circuit

(CSE-2202: Design and Analysis of Algorithms-I)

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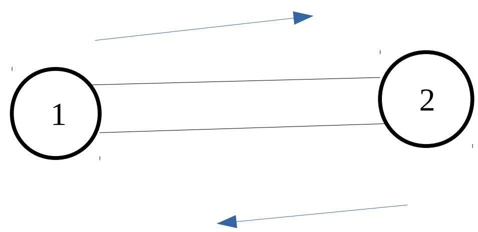
Statement: If all the nodes in an undirected, connected graph has even degree, then the graph has a Euler Circuit.

Euler path is a trail in a finite graph that visits every edge exactly once (allowing vertices to be revisited) and Euler circuit is an Euler path that starts and ends on the same vertex.

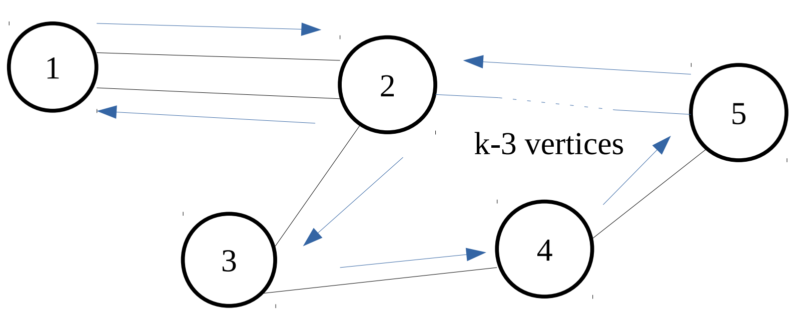
Proof by induction: Let, X be an undirected connected graph with even degree.

Basis step: Let, in the graph X the number of vertices of X be 2 and edge be 2, which results in an even number of edges connecting both the vertices.



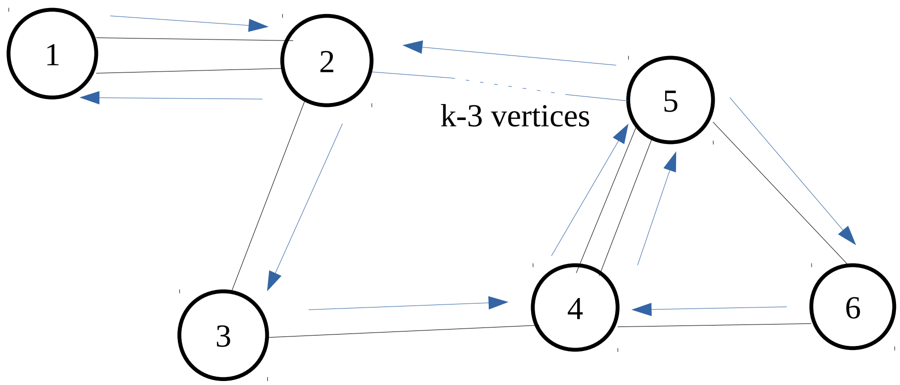
This graph is connected and each of this vertices has even degree. The largest path here is (1,2,1) or (2,1,2) which by definition is an euler circuit as all the edges are traversed and it starts and ends on the same vertex.

Inductive step: We have to show that for any k ≥ *2*, if *P*(*k*) holds, then P(k+1) also holds. Let us add a random number of vertices and even number of edges connecting them to form a connected graph in graph X.



Here, we can see that each of the vertices has even degree. The largest path here that connects all the edges is (1,2,3,4,5,k-3(start),……,k-3(end),2,1). We can see that the path visits each edge only once and starts and end at the same vertex. Thus, by the definition of Euler circuit, this is an euler circuit. Thus we can say that P(k) holds.

Again, let the number of vertices be (k+1) in graph X. We will add some edges to ensure each vertex has even degree.



Here, the path that traverses all the edges be (1,2,3,4,5,6,4,5,k-3(start),……,k-3(end),2,1). No edge is visited twice but all the edges are traversed. So, it can be said that its a Euler circuit.

We can see that the given statement is true for both k and k+1. So, it is bound to be true for any arbitrary number of vertices. So we can say that the statement is true.