CSE-2202: Assignment on Euler Circuit

Imran Zahid

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Statement. *If all the nodes in an undirected, connected graph has even degree, then the graph has a Euler Circuit.*

Proof. Let G be an undirected, connected graph of which each vertex is of even degree.

Proof by Contradiction: Let us take a path $P(u, ..., v)$ which is the largest possible path of G that visits every vertex at least once and doesn't traverse an edge more than once. Since u is of even degree, and each vertex adjacent to u is in the path P, every edge joining u and its adjacent vertices must be in P. As u is the starting point, the only way for all of it's adjacent edges to be traversed is if P ends in u. Therefore $u = v$ and P is a circuit.

Let us assume that P is not an euler circuit, that is, it does not traverse every edge in the graph G. Then, there exists an edge (a, b) which is not a part of P. Since G is a connected graph, b must connect to some vertex in the path, w. Since P is a circuit, there will exist some path $(a, b..., w, ...u, ...w)$ and since P could be written as $(w, ...u, ...w)$ as w is a part of P, this means that our new path is larger than P. This contradicts our statement that it is the largest possible path of G.

So the largest possible path of any undirected, connected graph G that visits every vertex at least once and doesn't traverse an edge more than once, must traverse every edge in the graph. Thus, the graph will have an Euler Circuit. $\blacksquare$