Lecture # 12 (Intersection of Planes)

Thursday, October 23, 2025 11:14 AM

① vector Equation of line $P(1,-2,3)$, $Q(2,3,5)$

$\vec{r} = \vec{OP} + t\,\vec{PQ}$ $\quad r = \begin{pmatrix} +1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$

$x = 1+t$
$y = -2+5t$ $\Rightarrow$ Parameter
$z = 3+2t$

$$\boxed{\frac{x-1}{1} = \frac{y+2}{5} = \frac{z-3}{2} = t}$$

Cartisian form or symmetric

② distance of Point from line

③ // Lines $\frac{d_1}{d_2} = \frac{e_1}{e_2} = \frac{f_1}{f_2} = K$ $\quad p + t(d_1, e_1, f_1)$

$q + s(d_2, e_2, f_2)$

④ Intersecting lines L_1 $\quad L_2$

$2+3t = 1-s$ — ①
$1+2t = -2+s$ — ②
$2+t = 3+2s$ — ③

① + ②: $3+5t = -1$
$5t = -4$
$t = -\frac{4}{5}$ $\quad s = \frac{7}{5}$

$1 + \frac{-8}{5} = -2+s$

$3 - \frac{8}{5} = s$

⑤ Angle b/w Intersecting Line d_1 & d_2 direction

$d_1 \cdot d_2 = |d_1||d_2|\cos\theta$

⑥ Equation of plane

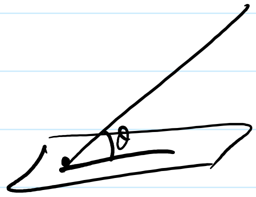
$\vec{r}\cdot\vec{n} = a\cdot n$ $\quad ax+by+cz = d$

$\vec{n} = \underline{AB} \times \underline{AC}$ $\quad a = \vec{OA}$

⑦ Line & Plane

$x = 1+2t$
$y = 2+3t$ $\quad 2x-3y+4z = 10$
$z = -1+2t$ $\quad 2+4t-6-9t-4+8t = 10$

$3t = 18$
$t = 6$



$x = 13$
$y = 20$ $\quad (13, 20, 11)$
$z = 11$

⑧ Angle b/w Line & Plane $\quad$ direction of line $= d_1$

Normal of Plane $= n_1$

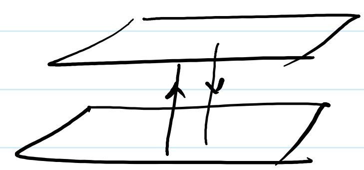
Normal of Plane $:= n_1$

$$d_1 \cdot n_1 = |d_1||n_1| \sin\theta$$

(9) $P(x_1, y_1, z_1)$ $\quad ax + by + cz + d = 0$ $\quad$ distance of Point from Plane

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$$

(10) If Two Planes are // –



$ax + by + cz = d_1$

$ax + by + cz = d_2$

$$D = \frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$$

(11) Angle b/w Planes = Angle b/w Normals

$$n_1 \cdot n_2 = |n_1||n_2| \cos\theta$$

(12) Intersection of Planes $\quad$ They generate Line

$2x + y - 3z = 10$ — (1) $\longrightarrow$ $\dfrac{x-x_0}{a} = \dfrac{y-y_0}{b} = \dfrac{z-z_0}{c}$

$x + 2y + z = 15$ — (2)

(1) $x = f(y)$

$x = f(z)$

(1) $X \to Z$

$2x + y - 3z = 10$

$3x + 6y + 3z = 45$

$5x + 7y = 55$

$5x = -7y + 55$

$x = \dfrac{-7y+55}{5}$

$4x + 2y - 6z = 20$

$-x \ +2y \ +z = -15$

$3x - 7z = 5$

$3x = 7z + 5$

$x = \dfrac{7z+5}{3}$

$$x = \frac{-7y+55}{5} = \frac{7z+5}{3}$$

$$\frac{x-0}{1} = \frac{-7\left(y - \frac{55}{7}\right)}{5} = \frac{7\left(z + \frac{5}{7}\right)}{3}$$

$$\frac{x-0}{1} = \frac{y - \frac{55}{7}}{\frac{5}{-7}} = \frac{z - \left(-\frac{5}{7}\right)}{3/7}$$

$d \left(1 : -\frac{5}{7} : \frac{3}{7}\right) \times 7$

$$\frac{x-0}{1} = \frac{\quad}{\frac{5}{-7}} = \frac{\quad}{3/7}$$

$7 : -5 : 3$

$$\bar{OP} \begin{pmatrix} 0 \\ 55/7 \\ -5/7 \end{pmatrix} + t \begin{pmatrix} 7 \\ -5 \\ 3 \end{pmatrix}$$

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$