

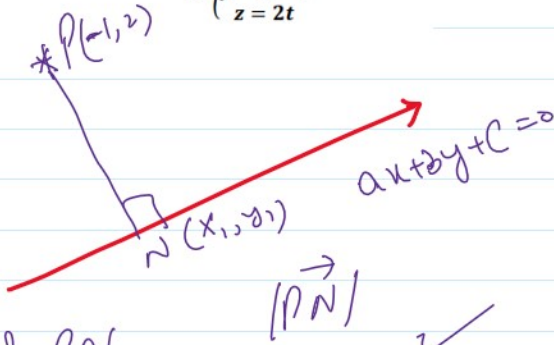
Lecture # 7,8 (distance of a point from a vector)

Wednesday, October 15, 2025 10:07 AM

Example:

Find the distance from the point $S(1, 1, 5)$ to the line:

$$L: \begin{cases} x = 1 + t \\ y = 3 - t \\ z = 2t \end{cases}$$



① Eq. of PN $\frac{m_{PN} \times m_L = -1}$

② Solve PN & $ax+by+c=0$ to $N(x_1, y_1)$

③ $PN = \sqrt{(x_1 - 1)^2 + (y_1 - 1)^2 + (z_1 - 5)^2}$

Vector & Point

I $\vec{PN} = \vec{ON} - \vec{OP}$

$$\vec{PN} = \begin{pmatrix} x_0 + td_1 \\ y_0 + td_2 \\ z_0 + td_3 \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$$

$$\vec{PN} = \begin{pmatrix} (x_0 - x_1) + td_1 \\ (y_0 - y_1) + td_2 \\ (z_0 - z_1) + td_3 \end{pmatrix}$$

you will get value of t

$$\begin{pmatrix} (x_0 - x_1) + td_1 \\ (y_0 - y_1) + td_2 \\ (z_0 - z_1) + td_3 \end{pmatrix} \cdot \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = 0$$

Example:

Find the distance from the point $S(1, 1, 5)$ to the line:

$$L: \begin{cases} x = 1 + t \\ y = 3 - t \\ z = 2t \end{cases}$$

$\vec{SN} = \vec{ON} - \vec{OS}$

$$\vec{ON} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

$$\vec{OS} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$$

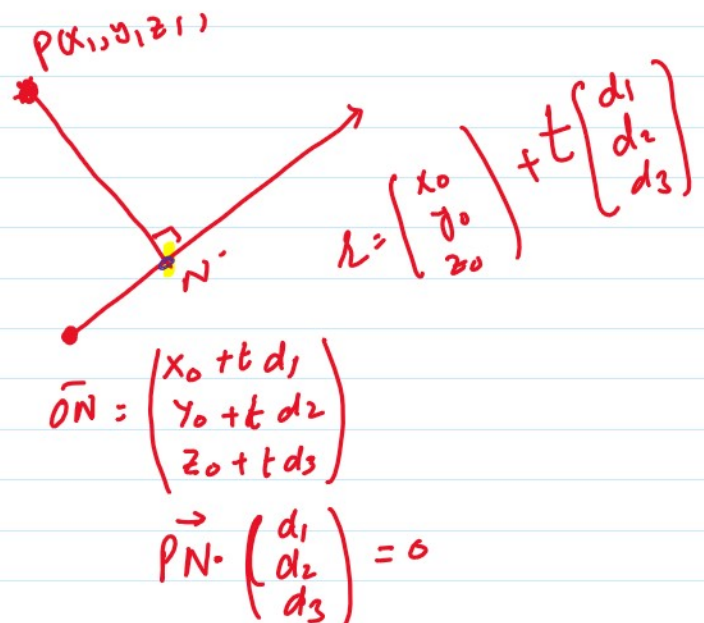
Equation of Line

(i) cartesian form $\Rightarrow y - y_1 = m(x - x_1)$

(ii) vector form $\vec{r} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + t \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$

(ii) Parametric form
 $x = x_0 + td_1$
 $y = y_0 + td_2$
 $z = z_0 + td_3$

(iii) $\frac{x - x_0}{d_1} = \frac{y - y_0}{d_2} = \frac{z - z_0}{d_3} = t$
Symmetric (Cartesian form)



$$\vec{SN} = \vec{ON} - \vec{OS}$$

$$= \begin{pmatrix} 1+t \\ 3-t \\ -5+2t \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$$

$$\vec{SN} = \begin{pmatrix} t \\ 2-t \\ -5+2t \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

$$|\vec{SN}| = \sqrt{4+0+1} = \sqrt{5}$$

$$\vec{ON} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$d_L = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$\vec{SN} \cdot d_L = 0$$

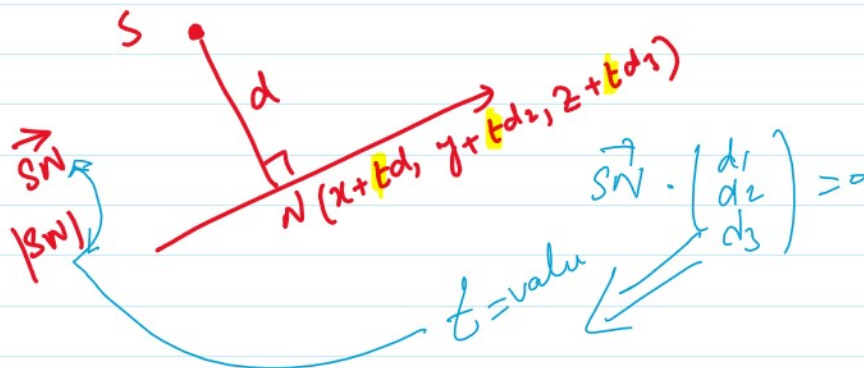
$$\begin{pmatrix} t \\ 2-t \\ -5+2t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = 0$$

$$t - 2 + t - 10 + 4t = 0$$

$$6t - 12 = 0$$

$$6t = 12$$

$$t = 2$$



Example:

Find the distance from the point $S(1, 1, 5)$ to the line:

$$L: \begin{cases} x = 1+t \\ y = 3-t \\ z = 2t \end{cases}$$

Solution:

Since the given point is $S(x_0, y_0, z_0) = P_0(1, 1, 5)$.

And the given parametric equations of line are

$$\begin{aligned} x &= x_1 + at = 1 + t \\ y &= y_1 + bt = 3 - t \\ z &= z_1 + ct = 0 + 2t \end{aligned}$$

This means that $P_1(x_1, y_1, z_1) = P_1(1, 3, 0)$ and $\vec{v} = \langle a, b, c \rangle = \langle 1, -1, 2 \rangle$.

Thus, the vector parallel to the line L is

$$\vec{v} = \hat{i} - \hat{j} + 2\hat{k} \quad (\text{direction of line})$$

The Line passes through the point $P_1(1, 3, 0)$

$$\vec{u} = \vec{P_1P_0} = \vec{OP_0} - \vec{OP_1}$$

$$\vec{u} = \vec{P_1P_0} = (\hat{i} + \hat{j} + 5\hat{k}) - (\hat{i} + 3\hat{j} + 0\hat{k})$$

$$\vec{u} = (1-1)\hat{i} + (1-3)\hat{j} + (5-0)\hat{k}$$

$$\vec{u} = 0\hat{i} - 2\hat{j} + 5\hat{k}$$

Now,

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & 5 \\ 1 & -1 & 2 \end{vmatrix}$$

$$\vec{u} \times \vec{v} = \hat{i} \begin{vmatrix} -2 & 5 \\ -1 & 2 \end{vmatrix} - \hat{j} \begin{vmatrix} 0 & 5 \\ 1 & 2 \end{vmatrix} + \hat{k} \begin{vmatrix} 0 & -2 \\ 1 & -1 \end{vmatrix}$$

$$\vec{u} \times \vec{v} = \hat{i}(-4+5) - \hat{j}(0-5) + \hat{k}(0+2)$$

$$\vec{u} \times \vec{v} = \hat{i} + 5\hat{j} + 2\hat{k}$$

Taking magnitude of $\vec{u} \times \vec{v}$, we have

$$|\vec{u} \times \vec{v}| = \sqrt{(1)^2 + (5)^2 + (2)^2}$$

$$|\vec{u} \times \vec{v}| = \sqrt{30} \quad \checkmark$$

Similarly, taking magnitude of vector \vec{v} , we have

$$|\vec{v}| = \sqrt{(1)^2 + (-1)^2 + (2)^2} = \sqrt{6}$$

Now, apply the formula, we have

Putting values, we have

$$d = \frac{\sqrt{30}}{\sqrt{6}} = \sqrt{\frac{30}{6}} = \sqrt{5}$$

$$d = \sqrt{5} = 2.24$$

Hence the distance of a point P_1 to the line L is 2.24 units.

$$L = \left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + t \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \right) \quad P(x_0, y_0, z_0)$$

$$|\vec{v}| = \sqrt{(1)^2 + (-1)^2 + (2)^2} = \sqrt{6}$$

Now, apply the formula, we have

$$d = \frac{|\vec{u} \times \vec{v}|}{|\vec{v}|}$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ d_1 & d_2 & d_3 \end{vmatrix}$$

$$\frac{|\vec{u} \times \vec{v}|}{|\vec{v}|} = d$$

$$r = \begin{pmatrix} x_1 + t d_1 \\ y_1 + t d_2 \\ z_1 + t d_3 \end{pmatrix}$$

$$\vec{v} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} - \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_1 - x_0 \\ y_1 - y_0 \\ z_1 - z_0 \end{pmatrix}$$

Q Find The distance of a Point $P(1, 2, 3)$ from a line passing through $A(-1, -2, 4)$ $B(5, 6, 7)$

$$\vec{OP} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{aligned} \textcircled{1} \vec{r} &= \vec{OA} + t \vec{AB} \\ &= \begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix} \end{aligned}$$

$$\vec{u} = \vec{P} - \vec{A}$$

$$\vec{u} = \begin{pmatrix} P_x - A_x \\ P_y - A_y \\ P_z - A_z \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$$

Formula $d = \frac{|\vec{u} \times \vec{v}|}{|\vec{v}|}$

$$\vec{v} = \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix}$$

$$\vec{u} \times \vec{v} = \begin{pmatrix} 20 \\ -12 \\ -8 \end{pmatrix}$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & -1 \\ 6 & 8 & 3 \end{vmatrix} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -4 & 1 \\ 6 & 8 & 3 \end{vmatrix} = \hat{i}(-20) - \hat{j}(-12) + \hat{k}(16-24) = -20\hat{i} + 12\hat{j} - 8\hat{k}$$

$$|\vec{u} \times \vec{v}| = \sqrt{400 + 144 + 64} = \sqrt{608}$$

$$= \sqrt{608} = 24.65$$

$$|\vec{v}| = \sqrt{36 + 64 + 9} = \sqrt{109}$$

$$\frac{\sqrt{608}}{\sqrt{109}} = \sqrt{\frac{608}{109}} = 2.36$$

$P(1, 2, 3)$

$$\textcircled{1} \vec{r} = \vec{OA} + t \vec{AB}$$

$$= \begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix}$$

$$\vec{PN} = \vec{ON} - \vec{OP}$$

$$= \begin{pmatrix} -1+6t \\ -2+8t \\ 4+3t \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\vec{PN} = \begin{pmatrix} -2+6t \\ -4+8t \\ 1+3t \end{pmatrix}$$

$$\vec{PN} = \begin{pmatrix} -2 + 6(\frac{41}{109}) \\ -4 + 8(\frac{41}{109}) \\ 1 + 3(\frac{41}{109}) \end{pmatrix}$$

$$6(-2+6t) + 8(-4+8t) + 3(1+3t) = 0$$

$$-12+36t - 32+64t + 3+9t = 0$$

$$109t - 41 = 0$$

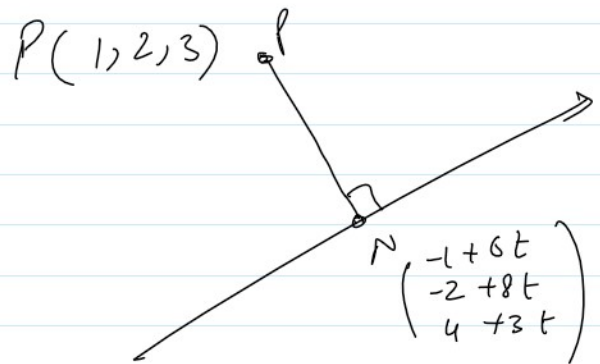
$$t = \frac{41}{109}$$

$$= \begin{pmatrix} -2 + \frac{246}{109} \\ -4 + \frac{328}{109} \\ 1 + \frac{123}{109} \end{pmatrix} = \begin{pmatrix} \frac{-218+246}{109} \\ \frac{-438+328}{109} \\ \frac{109+123}{109} \end{pmatrix}$$

$$\vec{PN} = \begin{pmatrix} 28/109 \\ -108/109 \\ 232/109 \end{pmatrix}$$

$$|\vec{PN}| = \sqrt{\left(\frac{28}{109}\right)^2 + \left(\frac{-108}{109}\right)^2 + \left(\frac{232}{109}\right)^2}$$

$$= 2.36$$



$$\vec{PN} \cdot \vec{d_L} = 0 \quad \vec{d_L} = \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix}$$

$$\begin{matrix} \text{0 8 9} \\ \hline \text{(0, 8, 9)} & \text{(1, 9, 8)} \\ \text{A} & \text{B} \end{matrix}$$

$$a b c d$$

$$(a, b, c), (d, c, b)$$