Topics:

- **Vectors equation of line in 3D**
- **Distance from a point to a line in 3D**

Food for thought

- What is x-intercept?
- What is y-intercept?

Line in 2D (plane) and in 3D (Space)

A Line in plane (2D):

A **line** in the $\boldsymbol{xy}$**-plane** is determined when $\boldsymbol{y}$**-coordinate of a point (that is y-intercept)** on the line and the **direction of the line** (its slope or angle of inclination) are **given**. The **equation of the line** can then be written using the **point-slope form**.

Recall: Point-slope form:

For any point $(\boldsymbol{x_0}, \boldsymbol{y_0})$ on the line having slope $\boldsymbol{m}$ is given by the equation

$$\boxed{\boldsymbol{y - y_0 = m(x - x_0)}}$$

If we fix the point, to be any point on the $\boldsymbol{y}$-axis ($\boldsymbol{y}$-intercept), then $\boldsymbol{x_0 = 0}$ and $\boldsymbol{y_0 = c}$, $\forall\ \boldsymbol{c} \in \mathbb{R}$ and above equation becomes

$$\boldsymbol{y - c = mx}$$

or

$$\boxed{\boldsymbol{y = mx + c}}$$

A Line in a Space (3D):

A line $\boldsymbol{L}$ in **three-dimensional space** is determined when we know a point $\boldsymbol{P_0}(\boldsymbol{x_0}, \boldsymbol{y_0}, \boldsymbol{z_0})$ on $\boldsymbol{L}$ and the direction of line $\boldsymbol{L}$. In three dimensions, the direction of a line is conveniently described by a vector, so we let $\vec{\boldsymbol{v}}$ be a vector parallel to the line $\boldsymbol{L}$.

Vector Equation for a Line in space:

Let $\boldsymbol{P}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ be an arbitrary point on $\boldsymbol{L}$ and let $\overrightarrow{\boldsymbol{r_0}}$ and $\vec{\boldsymbol{r}}$ be the position vectors of $\boldsymbol{P_0}$ and $\boldsymbol{P}$ (that is, they have representations $\overrightarrow{\boldsymbol{OP_0}}$ and $\overrightarrow{\boldsymbol{OP}}$). If $\vec{\boldsymbol{u}}$ is the vector with representation $\overrightarrow{\boldsymbol{P_0P}}$, as in **Figure 1**, then the Triangle Law for vector addition gives

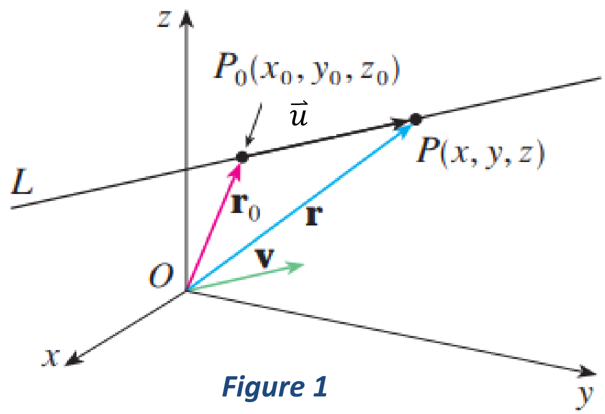


Figure 1

$$\vec{\boldsymbol{r}} = \overrightarrow{\boldsymbol{r_0}} + \vec{\boldsymbol{u}}$$

But, since $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$ are parallel vectors, there is a scalar $\boldsymbol{t} \in (-\infty, +\infty)$ such that $\vec{\boldsymbol{u}} = \boldsymbol{t}\vec{\boldsymbol{v}}$. Thus

$$\boxed{\vec{\boldsymbol{r}} = \overrightarrow{\boldsymbol{r_0}} + \boldsymbol{t}\vec{\boldsymbol{v}} \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (\mathbf{1})}$$

which is a **vector equation** of a line $\boldsymbol{L}$.

Each value of the parameter $\boldsymbol{t}$ gives the position vector $\vec{\boldsymbol{r}}$ of a point on $\boldsymbol{L}$. In other words, as $\boldsymbol{t}$ varies, the line is traced out by the tip of the vector $\vec{\boldsymbol{r}}$. As **Figure 2** indicates, positive values of $\boldsymbol{t}$ correspond to points on $\boldsymbol{L}$ that lie on one side of $\boldsymbol{P_0}$.

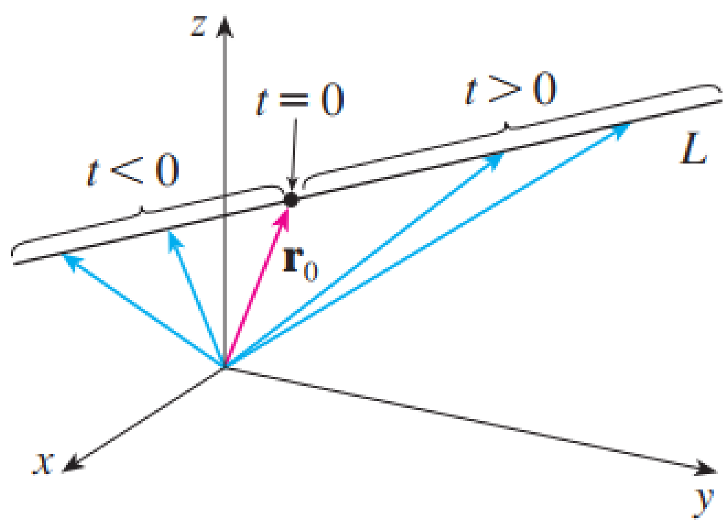


Figure 2

If the vector $\vec{v}$ that gives the direction of the line L is written in component form as $\vec{v} =< a, b, c >$, then we have $t\vec{v} =< ta, tb, tc >$.

We can also write $\vec{r} =< x, y, z >$and $\vec{r_0} =< x_0, y_0, z_0 >$, so the vector equation becomes

$$< x, y, z > = < x_0 + ta,\ y_0 + tb,\ z_0 + tc >$$

Two vectors are **equal** if and only if **corresponding components** are **equal**. Therefore, we have the **three scalar equations**:

$$\boxed{x = x_0 + ta, \qquad y = y_0 + tb, \qquad z = z_0 + tc \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (2)}$$

where $t \in \mathbb{R}$.

These equations are called **parametric equations** of the line L through the point $P_0(x_0, y_0, z_0)$ and parallel to the vector $\vec{v} =< a, b, c >$. Each value of the parameter t gives a point (x, y, z) on L.

<u>Parametric Equation for a Line in space</u>

Parametric equations for a line through the point $P_0(x_0, y_0, z_0)$ and parallel to the direction vector $\vec{v} =< a, b, c >$ **are**

$$x = x_0 + ta$$

$$y = y_0 + tb$$

$$z = z_0 + tc$$

Note:

Two vectors $\vec{u}$ and $\vec{v}$ are parallel if $\vec{u} = t\,\vec{v}$, where t is scalar. We understand it with the help of an example given below.

$$\vec{u} = < 1, 2, 3 >$$

$$\vec{v} = < 2, 4, 6 >$$

Then

$$\vec{v} = 2 < 1, 2, 3 >$$

$$\vec{v} = 2\vec{u}$$

This implies that $\vec{\boldsymbol{u}}$ & $\vec{\boldsymbol{v}}$ are parallel.

Example 1:

Find **parametric equation** & **vector equation** for the line through the point $(\mathbf{-2,0,4})$ and parallel to the vector

$$\vec{\boldsymbol{v}} = \mathbf{2}\vec{\boldsymbol{\imath}} + \mathbf{4}\vec{\boldsymbol{\jmath}} - \mathbf{2}\vec{\boldsymbol{k}}$$

Solution:

Since the given point is

$$P = (x_0, y_0, z_0) = (-2,0,4)$$

and the vector is

$$\vec{v} = \mathrm{a}\vec{\imath} + \mathrm{b}\vec{\jmath} + \mathrm{c}\vec{k} = v_1\vec{\imath} + v_2\vec{\jmath} + v_3\vec{k} = 2\vec{\imath} + 4\vec{\jmath} - 2\vec{k}$$

Here we have

$$x_0 = -2, \qquad y_0 = 0, \qquad z_0 = 4$$

$$a = 2, \qquad b = 4, \qquad c = -2$$

<u>**Parametric equation of line in space:**</u>

$$x = x_o + ta = -2 + 2t$$

$$y = y_o + tb = 0 + 4t = 4t$$

$$z = z_o + tc = 4 - 2t$$

That is,

$$x = -2 + 2t$$

$$y = 4t$$

$$z = 4 - 2t$$

where $-\infty < \boldsymbol{t} < \infty$.

Vector Equation of Line in space:

$$\vec{r} = \vec{r}_0 + t\,\vec{v}$$

$$x\vec{\imath} + y\vec{\jmath} + z\vec{k} = (-2\vec{\imath} + 0\vec{\jmath} + 4\vec{k}) + t\,(2\vec{\imath} + 4\vec{\jmath} - 2\vec{k})\ ; \quad -\infty < t < \infty$$

$$x\vec{\imath} + y\vec{\jmath} + z\vec{k} = (-2\vec{\imath} + 0\vec{\jmath} + 4\,\vec{k}) + (2t\,\vec{\imath} + 4t\,\vec{\jmath} - 2t\,\vec{k})$$

$$x\vec{\imath} + y\vec{\jmath} + z\vec{k} = (-2 + 2t)\,\vec{\imath} + (0 + 4t)\vec{\jmath} + (4 - 2t)\,\vec{k}$$

$$x\vec{\imath} + y\vec{\jmath} + z\vec{k} = (-2 + 2t)\,\vec{\imath} + (4t)\vec{\jmath} + (4 - 2t)\,\vec{k}$$

Example 2:

Find the **parametric equation** of the line passing through origin and parallel to the vector

$$\vec{v} = 2\hat{\jmath} + \hat{k}$$

Solution:

Given that

$$\text{Origin} = P = (0,0,0) = P(x_0, y_0, z_0)$$

$$\vec{v} = 0\hat{\imath} + 2\hat{\jmath} + \hat{k} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$$

Then the parametric equations of line are

$$x = x_0 + at = 0 + 0t = 0$$

$$y = y_0 + bt = 0 + 2t = 2t$$

$$z = z_0 + ct = 0 + t = t$$

Then the required parametric equations are

$$\boldsymbol{x = 0}$$

$$\boldsymbol{y = 2t}$$

$$\boldsymbol{z = t}$$

Example 3:

Find the **parametric equation** and **vector equation** of the line through $\boldsymbol{P(-3,2,-3)}$ and $\boldsymbol{Q(1,-1,4)}$.

Solution:

$$P=(x_0,y_0,z_0)=(-3,2,-3)$$

$$Q=(x_1,y_1,z_1)=(1,-1,4)$$

The vector parallel to line is

$$\vec{v}=\overrightarrow{PQ}=<x_1-x_0,\quad y_1-y_0,\quad z_1-z_0>$$

$$\vec{v}=\overrightarrow{PQ}=<1-(-3),\quad -1-2,\quad 4-(-3)>$$

$$\vec{v}=\overrightarrow{PQ}=<4,-3,7>=<a,b,c>$$

We can write the vector in standard form as

$$\boldsymbol{\vec{v}=a\vec{i}+b\vec{j}+c\vec{k}=4\vec{i}-3\vec{j}+7\vec{k}}$$

<u>Parametric equation of a line in space:</u>

$$x=x_o+t\,v_1=-3+4t$$

$$y=y_o+t\,v_2=2-3t$$

$$z=z_o+t\,v_3=-3+7t$$

where $-\infty<t<\infty$.

<u>Vector equation for a line in space:</u>

$$xi+yj+zk=(-3i+2j-3k)+t\,(4i-3j+7k)\,,-\infty<t<\infty$$

$$xi+yj+zk=(-3i+2j-3k)+(4t\,i-3t\,j+7t\,k)$$

$$\boldsymbol{xi+yj+zk=(-3+4t)\,i+(2-3t)j+(-3+7t)k}$$

Do yourself Exercise 12.5 (Textbook: Thomas Calculus 11th Edition):

Q # 1 to 20.

Example 4:

Find the **parametric equation** of the line passing through the point $(3, -2, 1)$ and parallel to the line having parametric equations

$$x = 1 + 2t$$

$$y = 2 - t$$

$$z = 3t$$

Solution:

Since we have given the parametric equations as

$$x = 1 + 2t = x_0 + at$$

$$y = 2 - t = y_0 + bt$$

$$z = 0 + 3t = z_0 + ct$$

This implies that

$$a = 2, \qquad b = -1, \qquad c = 3$$

which are the components of a vector $\boldsymbol{v}$**, given as**

$$\boldsymbol{v} =< \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} >=< \boldsymbol{2}, -\boldsymbol{1}, \boldsymbol{3} >$$

The given point is

$$P = (3, -2, 1) = P(x_0, y_0, z_0)$$

$$\boldsymbol{v} =< \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \geq< \boldsymbol{2}, -\boldsymbol{1}, \boldsymbol{3} >$$

That is,

$$x_0 = 3, \qquad y_0 = -2, \qquad z_0 = 1$$

$$a = 2, \qquad b = -1, \qquad c = 3$$

Then the **parametric equation** of the line passing through the point $P(3, -2, 1)$ and parallel to the line with the given parametric equations are given below:

$$x = x_0 + at$$

$$y = y_0 + bt$$

$$z = z_0 + ct$$

Putting the values, we have

$$\boldsymbol{x = 3 + 2t}$$

$$\boldsymbol{y = -2 - t}$$

$$\boldsymbol{z = 1 + 3t}$$

which are the required parametric equations of the line.

Example 5:

Find the parametric equations of the line passing through the point $(2, 3, 0)$, and perpendicular to the vectors $\vec{u} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{v} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$.

Solution:

Since the line is perpendicular to the vectors $\vec{u}$ and $\vec{v}$, therefore using the cross product, we have

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix}$$

$$\vec{u} \times \vec{v} = \hat{\imath}(10 - 12) - \hat{\jmath}(5 - 9) + \hat{k}(4 - 6)$$

$$\vec{u} \times \vec{v} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k} = a\hat{\imath} + b\hat{\jmath} - c\hat{k}$$

The given point is

$$P = (2,3,0) = P(x_0, y_0, z_0)$$

Then the parametric equations of line are

$$x = x_0 + at = 2 - 2t$$

$$y = y_0 + bt = 3 + 4t$$

$$z = z_0 + ct = 0 - 2t$$

Then the required parametric equations are

$$x = 2 - 2t$$

$$y = 3 + 4t$$

$$z = -2t$$

Practice questions:

(Textbook: Thomas Calculus 11th Edition) Ex. 12.5: 1-7, 10.

Parameterization of a Line segment

Example:

Parameterize the line segment joining the points $\boldsymbol{P}\ (-\mathbf{3}, \mathbf{2}, -\mathbf{3})$ to $\boldsymbol{Q}\ (\mathbf{1}, -\mathbf{1}, \mathbf{4})$.

Solution:

First of all, we will find the parametric equation of the line through the points $\boldsymbol{P}\ (-\mathbf{3}, \mathbf{2}, -\mathbf{3})$ and $\boldsymbol{Q}\ (\mathbf{1}, -\mathbf{1}, \mathbf{4})$ and then restrict the domain of parameter $\boldsymbol{t}$ to obtain the parametric equation of the line segment from $\boldsymbol{P}$ to $\boldsymbol{Q}$.

Step 1: (Equation of line)

$$\vec{v} = \overrightarrow{PQ} = (1+3)\hat{\imath} + (-1-2)\hat{\jmath} + (4+3)\hat{k}$$

$$\boldsymbol{\vec{v} = 4\hat{\imath} - 3\hat{\jmath} + 7\hat{k} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}}$$

Let us consider the point $\boldsymbol{P}\ (-\mathbf{3}, \mathbf{2}, -\mathbf{3}) = \boldsymbol{P}(\boldsymbol{x_0}, \boldsymbol{y_0}, \boldsymbol{z_0})$. **(NOTE: We can also consider point Q here).** Then the parametric equations of line are

$$\begin{aligned}\boldsymbol{x} &= \boldsymbol{x_0} + \boldsymbol{at} = -\mathbf{3} + \mathbf{4}\boldsymbol{t}\\ \boldsymbol{y} &= \boldsymbol{y_0} + \boldsymbol{bt} = \mathbf{2} - \mathbf{3}\boldsymbol{t}\\ \boldsymbol{z} &= \boldsymbol{z_0} + \boldsymbol{ct} = -\mathbf{3} + \mathbf{7}\boldsymbol{t}\end{aligned}$$

Step 2: (Line segment)

In order to find the value of t for which an arbitrary point $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ of the line is at $\boldsymbol{P}\ (-\mathbf{3}, \mathbf{2}, -\mathbf{3})$, we solve the equation

$$\left.\begin{array}{l}-3 = -3 + 4t\\ 2 = 2 - 3t\\ -3 = -3 + 7t\end{array}\right\} \Rightarrow t = 0$$

Similarly, when $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ is at $\boldsymbol{Q}\ (\mathbf{1}, -\mathbf{1}, \mathbf{4})$**,** we solve

$$\left.\begin{array}{l}1 = -3 + 4t\\ -1 = 2 - 3t\\ 4 = -3 + 7t\end{array}\right\} \Rightarrow t = 1$$

So, the parametric equation of the line segment is

$$x = -3 + 4t$$

$$y = 2 - 3t$$

$$z = -3 + 7t; \quad 0 \leq t \leq 1$$

Question 19: Find the parametric equations of the line segment joining the points $\boldsymbol{P(-2, 0, 2)}$ and $\boldsymbol{Q(0, 2, 0)}$.

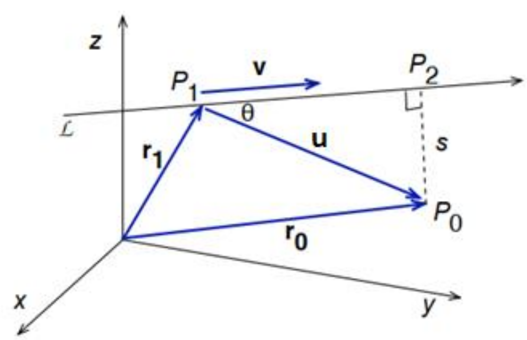
Practice Questions

(Textbook: Thomas Calculus 11th Edition) Ex. 12.5: 13-20.

The Distance from a Point to a Line in Space

Let $\boldsymbol{L}$ be a line and let $\vec{\boldsymbol{v}}$ be a vector parallel to the line $\boldsymbol{L}$. This means that $\vec{\boldsymbol{v}} || \boldsymbol{L}$.

Let $\boldsymbol{P_0}$ be a point in space from which we need to measure the distance to the line $\boldsymbol{L}$, i.e., we need to measure the distance between $\boldsymbol{P_0}$ and $\boldsymbol{L}$.



We consider an arbitrary point $\boldsymbol{P_1}$ on the line $\boldsymbol{L}$ assuming it might give us a distance from the point $\boldsymbol{P_0}$.

Considering $\boldsymbol{P_0}$ and $\boldsymbol{P_1}$, we draw their **position vectors** $\vec{\boldsymbol{r}}_0$ and $\vec{\boldsymbol{r}}_1$ respectively from the **origin**.

We draw a vector $\vec{\boldsymbol{u}}$ at an angle $\boldsymbol{\theta}$ from $\boldsymbol{P_1}$ to $\boldsymbol{P_0}$ such that by **head to tail rule** we have,

$$\vec{\boldsymbol{r}}_1 + \vec{\boldsymbol{u}} = \vec{\boldsymbol{r}}_0$$

This implies that

$$\vec{\boldsymbol{u}} = \vec{\boldsymbol{r}}_0 - \vec{\boldsymbol{r}}_1.$$

Now logically the shortest distance of a point from a line is the length of the perpendicular drawn from the point to the line.

Let us consider a point $\boldsymbol{P_2}$ on the line $\boldsymbol{L}$**,** such that $\boldsymbol{P_0P_2}$ is perpendicular to the line $\boldsymbol{L}$**.** i.e., $\boldsymbol{P_0P_2} \perp \boldsymbol{L}$**.** We say that $\overline{\boldsymbol{P_0P_2}} = s$.

Distance is a scalar quantity. From the figure, we can write

$$\boldsymbol{sin\ \theta = \frac{Perpandicular}{Hypotenuse} = \frac{s}{|\overrightarrow{r_0} - \overrightarrow{r_1}|}}$$

$$\boldsymbol{sin\ \theta = \frac{s}{|\vec{u}|}}$$

$$\boldsymbol{|\vec{u}|\ sin\ \theta = s}$$

$$\boldsymbol{s = |\vec{u}|\ sin\ \theta}$$

Here $\boldsymbol{\theta}$ is the angle between $\boldsymbol{\vec{u}}$ and $\boldsymbol{\vec{v}}$**.** $\boldsymbol{P_0P_2} \perp \boldsymbol{L}$ and $\boldsymbol{\vec{v}}||\boldsymbol{L}$ implies that $\boldsymbol{P_0P_2} \perp \boldsymbol{\vec{v}}$**.**

Now considering $\boldsymbol{\vec{v}}$ and $\boldsymbol{\vec{u}}$, we have

$$\boldsymbol{\vec{u} \times \vec{v} = |\vec{u}|\ |\vec{v}|\ \ \mathbf{sin}\,\theta}$$

We can rearrange as

$$\boldsymbol{\vec{u} \times \vec{v} = |\vec{v}||\vec{u}|\ sin\ \theta}$$

We know $\boldsymbol{s = |\vec{u}| sin\ \theta}$

$$\boldsymbol{\vec{u} \times \vec{v} = |\vec{v}|\ s}$$

or

$$\boldsymbol{\vec{u} \times \vec{v} = s\ |\vec{v}|}$$

Since distance is a scalar quantity thus, we need to take the magnitude of the cross product to balance the equation.

$$\boldsymbol{|\vec{u} \times \vec{v}| = s\ |\vec{v}|}$$

$$\boldsymbol{\frac{|\vec{u} \times \vec{v}|}{|\vec{v}|} = s}$$

This implies that the distance between the line $\boldsymbol{L}$ and a point can be computed by the formula,

$$\boldsymbol{s} = \frac{|\vec{\boldsymbol{u}} \times \vec{\boldsymbol{v}}|}{|\vec{\boldsymbol{v}}|}$$

where $\vec{\boldsymbol{u}} = \overrightarrow{\boldsymbol{r_0}} - \overrightarrow{\boldsymbol{r_1}}$

Example:

Find the distance from the point $\boldsymbol{S}\ (\mathbf{1}, \mathbf{1}, \mathbf{5})$ to the line:

$$\boldsymbol{L}: \begin{cases} \boldsymbol{x} = \mathbf{1} + \boldsymbol{t} \\ \boldsymbol{y} = \mathbf{3} - \boldsymbol{t} \\ \boldsymbol{z} = \mathbf{2}\boldsymbol{t} \end{cases}$$

Solution:

Since the given point is $\boldsymbol{S}(\boldsymbol{x_0}, \boldsymbol{y_0}, \boldsymbol{z_0}) = \boldsymbol{P_0}(\mathbf{1}, \mathbf{1}, \mathbf{5})$.

And the given parametric equations of line are

$$\boldsymbol{x} = \boldsymbol{x_1} + \boldsymbol{at} = \mathbf{1} + \boldsymbol{t}$$
$$\boldsymbol{y} = \boldsymbol{y_1} + \boldsymbol{bt} = \mathbf{3} - \boldsymbol{t}$$
$$\boldsymbol{z} = \boldsymbol{z_1} + \boldsymbol{ct} = \mathbf{0} + \mathbf{2}\boldsymbol{t}$$

This means that $\boldsymbol{P_1}(\boldsymbol{x_1}, \boldsymbol{y_1}, \boldsymbol{z_1}) = \boldsymbol{P_1}(\mathbf{1}, \mathbf{3}, \mathbf{0})$ and $\vec{\boldsymbol{v}} =< \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} > =< \mathbf{1}, -\mathbf{1}, \mathbf{2} >$. Thus, the vector parallel to the line $\boldsymbol{L}$ is

$$\vec{\boldsymbol{v}} = \hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}} + \mathbf{2}\hat{\boldsymbol{k}}$$

The Line passes through the point $\boldsymbol{P_1}\ (\mathbf{1}, \mathbf{3}, \mathbf{0})$

$$\vec{u} = \overrightarrow{P_1P_0} = \overrightarrow{OP_0} - \overrightarrow{OP_1}$$

$$\vec{u} = \overrightarrow{P_1P_0} = (\hat{\imath} + \hat{\jmath} + 5\hat{k}) - (\hat{\imath} + 3\hat{\jmath} + 0\,\hat{k})$$

$$\vec{u} = (1-1)\hat{\imath} + (1-3)\hat{\jmath} + (5-0)\hat{k}$$

$$\vec{u} = 0\hat{\imath} - 2\hat{\jmath} + 5\hat{k}.$$

Now,

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & -2 & 5 \\ 1 & -1 & 2 \end{vmatrix}$$

$$\vec{u} \times \vec{v} = \hat{\imath} \begin{vmatrix} -2 & 5 \\ -1 & 2 \end{vmatrix} - \hat{\jmath} \begin{vmatrix} 0 & 5 \\ 1 & 2 \end{vmatrix} + \hat{k} \begin{vmatrix} 0 & -2 \\ 1 & -1 \end{vmatrix}$$

$$\vec{u} \times \vec{v} = \hat{\imath}(-4+5) - \hat{\jmath}(0-5) + \hat{k}(0+2)$$

$$\vec{u} \times \vec{v} = \hat{\imath} + 5\hat{\jmath} + 2\hat{k}$$

Taking magnitude of $\vec{\boldsymbol{u}} \times \vec{\boldsymbol{v}}$, we have

$$|\vec{u} \times \vec{v}| = \sqrt{(1)^2 + (5)^2 + (2)^2}$$

$$|\vec{u} \times \vec{v}| = \sqrt{30}$$

Similarly, taking magnitude of vector $\vec{\boldsymbol{v}}$, we have

$$|\vec{v}| = \sqrt{(1)^2 + (-1)^2 + (2)^2} = \sqrt{6}$$

Now, apply the formula, we have

$$d = \frac{|\vec{u} \times \vec{v}|}{|\vec{v}|}$$

Putting values, we have

$$d = \frac{\sqrt{30}}{\sqrt{6}} = \sqrt{\frac{30}{6}} = \sqrt{5}$$

$$d = \sqrt{5} = 2.24$$

Hence the distance of a point $\boldsymbol{P_1}$ to the line $\boldsymbol{L}$ is **2.24** units.

Practice Questions:

Textbook: Thomas Calculus 11th Edition Ex. 12.5: 33-38

In Exercises 33–38, find the distance from the point to the line.

33. $(0, 0, 12)$; $x = 4t$, $y = -2t$, $z = 2t$

34. $(0, 0, 0)$; $x = 5 + 3t$, $y = 5 + 4t$, $z = -3 - 5t$

35. $(2, 1, 3)$; $x = 2 + 2t$, $y = 1 + 6t$, $z = 3$

36. $(2, 1, -1)$; $x = 2t$, $y = 1 + 2t$, $z = 2t$

37. $(3, -1, 4)$; $x = 4 - t$, $y = 3 + 2t$, $z = -5 + 3t$

38. $(-1, 4, 3)$; $x = 10 + 4t$, $y = -3$, $z = 4t$