

Lecture # 13,14 (Assignment Practice)

Wednesday, October 29, 2025 10:20 AM

- 4 The points A and B have position vectors, relative to the origin O , given by

$$\vec{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \vec{OB} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$$

The line l has vector equation

$$t=2$$

$$\mathbf{r} = (1-2t)\mathbf{i} + (5+t)\mathbf{j} + (2-t)\mathbf{k}$$

$$\vec{OB} - \vec{OA} = \vec{AB}$$

$$t = \frac{1}{3}, \frac{1+2}{3}, \frac{5-1}{3}$$

$$\frac{5}{3}$$

$$\frac{14}{3}$$

2-

- (i) Show that l does not intersect the line passing through A and B . [4]

- (ii) The point P lies on l and is such that angle PAB is equal to 60° . Given that the position vector of P is $(1-2t)\mathbf{i} + (5+t)\mathbf{j} + (2-t)\mathbf{k}$, show that $3t^2 + 7t + 2 = 0$. Hence find the only possible position vector of P . [6]

① Eq. of Line AB $\mathbf{r} = \vec{p} + s\vec{d}$

$$\mathbf{r}_2 = \begin{pmatrix} 1-2t \\ 5+t \\ 2-t \end{pmatrix}$$

$$\begin{aligned} \mathbf{r}_1 &= \vec{OA} + s\vec{AB} \\ &= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+s \\ 2-s \\ 3+s \end{pmatrix} \end{aligned}$$

$$\boxed{\mathbf{r}_1 = \mathbf{r}_2}$$

$$x: \underline{1-2t} = \underline{1+s} \quad \text{---(1)}$$

$$y: \underline{5+t} = \underline{2-s} \quad \text{---(2)}$$

$$z: \underline{2-t} = \underline{3} \quad \text{---(3)}$$

$$2-3 = \boxed{-1 \neq 3}$$

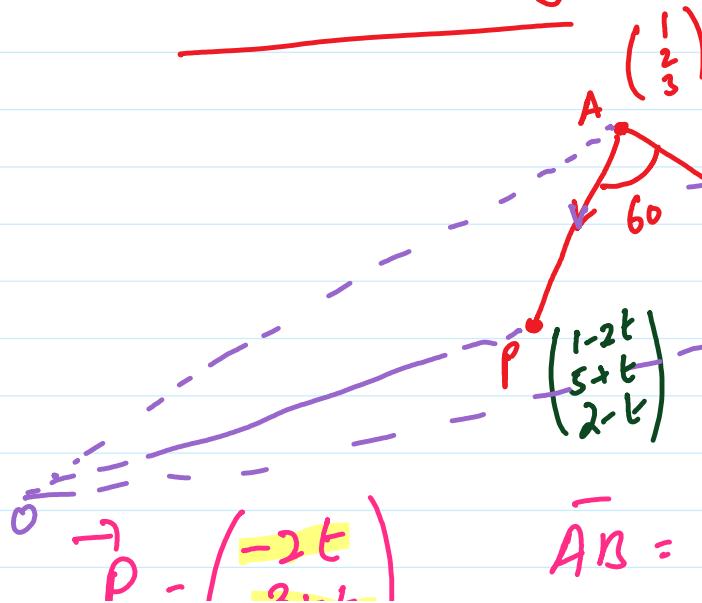
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$$6-t = 3$$

$$\boxed{t=3}$$

Not Intersecting

b)



$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} &= |\mathbf{a}| |\mathbf{b}| \cos \theta \\ \vec{AP} \cdot \vec{AB} &= |\vec{AP}| |\vec{AB}| \cos 60^\circ \end{aligned}$$

$$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$$3t^2 + 7t + 2 = 0$$

$$\vec{AB} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad \left| \begin{array}{c} \vec{AP} \cdot \vec{AB} \\ -2t-3-t \end{array} \right.$$

$$\vec{AP} = \begin{pmatrix} -2t \\ 3+t \\ -1-t \end{pmatrix}$$

$$\vec{AB} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$$\vec{AP} \cdot \vec{AB} = -2t - 3 - t = -3t - 3$$

$$|\vec{AP}| = \sqrt{4t^2 + (3+t)^2 + (1+t)^2}; |\vec{AB}| = \sqrt{1+1}$$

cos 60°

$$\frac{-3t - 3}{\left[-6(1+t) \right]^2} = \frac{\sqrt{4t^2 + 9 + t^2 + 6t + 1 + 2t + t^2}}{\sqrt{(6t^2 + 8t + 10)}^2} \cdot \sqrt{2} \cdot \frac{1}{2}$$

$$18 \cancel{36} (1+t)^2 = (6t^2 + 8t + 10)^2$$

$$18(1+t^2 + 2t) = 6t^2 + 8t + 10$$

$$18 + 18t^2 + 36t - 6t^2 - 8t - 10 = 0$$

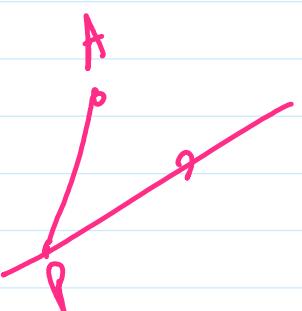
$$12t^2 + 28t + 8 = 0 \quad \div \text{by } 4$$

$$3t^2 + 7t + 2 = 0$$

$$\underline{3t^2 + 6t + t + 2} = 0$$

$$3t(t+2) + 1(t+2) = 0$$

$$t = -2, \quad t = -\frac{1}{3}$$



$$t = -2, \quad \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1-2t \\ 5+t \\ 2-t \end{pmatrix}$$

$$\begin{pmatrix} 5/3 \\ 14/3 \end{pmatrix}$$

$\left(\begin{array}{c} \frac{14}{3} \\ 7/3 \end{array} \right)$

Angle between lines

$$d_1 \cdot d_2 = |d_1| |d_2| \cos \theta$$

- 6 With respect to the origin O , the points A and B have position vectors given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$. The point P lies on the line AB and OP is perpendicular to AB .

(i) Find a vector equation for the line AB . $\rightarrow r = \overrightarrow{OA} + t \overrightarrow{AB}$

[1]

(ii) Find the position vector of P .

[4]

Known

$$\overrightarrow{OP} = \begin{pmatrix} 1 - \frac{1}{3} \\ 2 - \frac{1}{3} \\ 2 + \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 2/3 \\ 5/3 \\ 7/3 \end{pmatrix}$$

Wanted

$$\overrightarrow{r} = \begin{pmatrix} 1+2t \\ 2+2t \\ 2-2t \end{pmatrix}$$

$$\overrightarrow{OP} \cdot \overrightarrow{d}_{AB} = 0$$

$$\begin{pmatrix} 1+2t \\ 2+2t \\ 2-2t \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix} = 0$$

$$2+4t+4+4t-4+4t=0$$

$$12t+2=0$$

$$t = -\frac{1}{6}$$

- 8 With respect to the origin O , the position vectors of two points A and B are given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$. The point P lies on the line through A and B , and $\overrightarrow{AP} = \lambda \overrightarrow{AB}$.

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$

(i) Show that $\overrightarrow{OP} = (1+2\lambda)\mathbf{i} + (2+2\lambda)\mathbf{j} + (2-2\lambda)\mathbf{k}$.

[2]

(ii) By equating expressions for $\cos AOP$ and $\cos BOP$ in terms of λ , find the value of λ for which OP bisects the angle AOB .

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(iii) When λ has this value, verify that $\overrightarrow{AP} : \overrightarrow{PB} = \overrightarrow{OA} : \overrightarrow{OB}$.

[1]

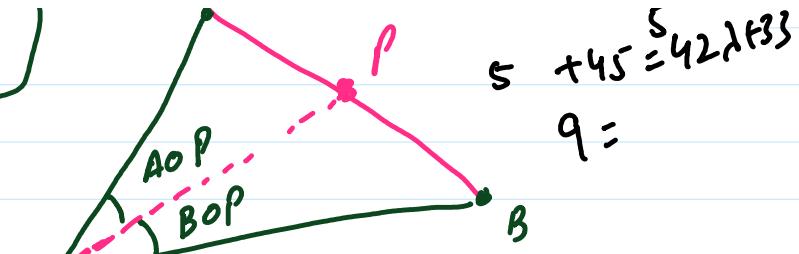
$\boxed{\cos AOP = \cos BOP}$

$$5 + 45 = 42\lambda + 33$$

$$\boxed{\cos AOP = \cos BOP}$$

$$\vec{OA} \cdot \vec{OP} = |\vec{OA}| |\vec{OP}| \cos AOP$$

$$\frac{\vec{OA} \cdot \vec{OP}}{|\vec{OA}| |\vec{OP}|} = \cos AOP$$



$$s + 45 = 42\lambda^3$$

$$q =$$

$$\vec{OB} \cdot \vec{OP} = |\vec{OB}| |\vec{OP}| \cos BOP$$

$$\frac{\vec{OB} \cdot \vec{OP}}{|\vec{OB}| |\vec{OP}|} = \cos BOP$$

$$\frac{\vec{OA} \cdot \vec{OP}}{|\vec{OA}| |\vec{OP}|} = \frac{\vec{OB} \cdot \vec{OP}}{|\vec{OB}| |\vec{OP}|}$$

$$\vec{OA} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$$\vec{OB} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$$

$$\vec{AP} = \begin{pmatrix} 3/4 \\ 3/4 \\ -3/4 \end{pmatrix}$$

$$|AP| = \sqrt{\frac{27}{16}} = \frac{\sqrt{27}}{4}$$

$$\frac{9+2\lambda}{3} = \frac{11+14\lambda}{5}$$

$$\text{Ansatz: } \frac{3}{8}$$

$$\vec{BP} = \begin{pmatrix} 1/4 - 3 \\ 11/4 - 4 \\ 5/4 \end{pmatrix} = \begin{pmatrix} -5/4 \\ -5/4 \\ 5/4 \end{pmatrix}$$

$$\frac{\sqrt{27}}{4} : \frac{\sqrt{75}}{4}$$

$$\frac{\sqrt{27}}{\sqrt{9 \times 3}} : \frac{\sqrt{75}}{\sqrt{25 \times 3}}$$

$$5(9+2\lambda) = 3(11+14\lambda)$$

$$45+10\lambda = 33+42\lambda$$

$$12 = 32\lambda$$

$$|\vec{BP}| = \frac{\sqrt{75}}{4}$$

3:5

3:5

$$\begin{array}{rcl} \sqrt{27} & = & \sqrt{3} \\ \sqrt{9x^3} & = & \sqrt{27x^3} \\ 3\sqrt{3} & = & 3\sqrt{3} \end{array}$$

- 10 The lines l and m have equations $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} - \mathbf{k})$ respectively, where a and b are constants.

- (i) Given that l and m intersect, show that

$$2a - b = 4. \quad [4]$$

- (ii) Given also that l and m are perpendicular, find the values of a and b . [4]

- (iii) When a and b have these values, find the position vector of the point of intersection of l and m . [2]