Lecture # 4,5

Wednesday, October 8, 2025 10:13 AM

Vector Projection Excluded

$\vec{V} \cdot \vec{V} = |V||V| \cos\theta$

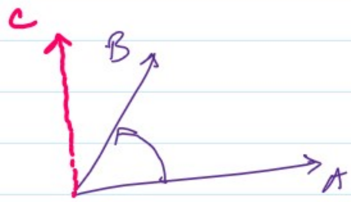
$= |V||V| = |V|^2$

Cross Product

$A \times B \Rightarrow$ vector

$\vec{C}$ is a vector $\perp$cular to A & B

$\bar{C} = \bar{A} \times \bar{B}$



$A \cdot B =$ scalar

$A \times B =$ vector

* $\vec{n}$ is a Normal vector to $\vec{A}$ & $\vec{B}$ where vector A & B forms a plane.

$$\vec{n} = \bar{A} \times \bar{B} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b_1)$$

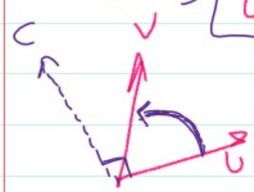
$$\vec{A} \times \vec{B} = \boxed{|A||B| \sin\theta} \cdot \hat{n}$$

Example 1:

- Find the cross product of $\vec{u} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{v} = 3\hat{\imath} + \hat{k}$.
- Check that the cross product $\vec{u} \times \vec{v}$ is perpendicular to $\vec{u}$ or not.
- Check that the cross product $\vec{u} \times \vec{v}$ is perpendicular to $\vec{v}$ or not.

$$\vec{C} = \boxed{\vec{u} \times \vec{v}} = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ 3 & 0 & 1 \end{vmatrix} = i(1) - j(2 - (-6)) + k(0 - 3)$$

$$= i - 8j - 3k$$



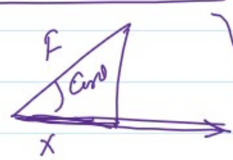
$\boxed{a \times b = |a||b| \sin\theta \cdot \hat{n}}$

$\vec{u} \cdot \vec{c} = 0$

$$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -8 \\ -3 \end{pmatrix} = 2 - 8 + 6 = 0$$

$\vec{v} \cdot \vec{c} = 0$

$$\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -8 \\ -3 \end{pmatrix} = 3 + 0 - 3 = 0$$



$\frac{x}{F} = \cos\theta$

$x = F\cos\theta$ $(F\cos\theta)(d)$

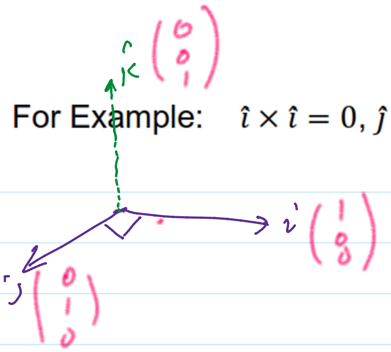
Definition: Parallel vector

Two non-zero vectors $\vec{u}$ and $\vec{v}$ are **parallel** if and only

$$\vec{u} \times \vec{v} = \mathbf{0}$$

$i \times i = |i||i| \sin 0 \cdot \hat{n}$
$= 0$

$i \cdot i = j \cdot j = k \cdot k = 1$
$i \cdot j = j \cdot k = k \cdot i = i \cdot k = 0$

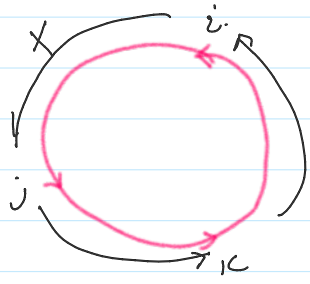


For Example: $\hat{\imath} \times \hat{\imath} = 0,\ \hat{\jmath} \times \hat{\jmath} = 0,\ \hat{k} \times \hat{k} = 0$

$i \times i = j \times j = \hat{k} \times \hat{k} = 0$

$i \times j = |i||j| \sin 90 \ \hat{n}$
$i \times j = \hat{n}$

$i \times j = k \Rightarrow j \times i = -k$
$j \times k = i \qquad k \times j = -i$
$k \times i = j \qquad i \times k = -j$



Rotation in anti-clockwise is +ve

1) $\vec{u} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$
$\quad \vec{v} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

2) $\vec{u} = -3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$
$\quad \vec{v} = \hat{\imath} - 3\hat{\jmath} - \hat{k}$

3) $\vec{u} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$
$\quad \vec{v} = -6\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$= i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1)$
$+ k(a_1 b_2 - a_2 b_1)$

Equation of line — atleast one point & slope = gradiant → direction

$m = \frac{Rise}{Run} = \frac{7-(-2)}{4-1}$

(x_0, y_0): $(1, -2)$, $(4, 7)$ → $y - 7 = 3(x-4)$
$y - 7 = 3x - 12$
$y = 3x - 12 + 7$

$m = \frac{7-(-2)}{4-1} = \frac{9}{3} = 3$

$\boxed{y = 3x - 5}$

$\vec{i} : \begin{pmatrix}0\\-5\end{pmatrix}$

$y + 2 = 3(x - 1)$ → $y - y_0 = m(x - x_0)$

$$y + 2 = 3(x-1) \rightarrow y - y_0 = m(x - x_0)$$

$$y = 3x - 3 - 2$$

$$\boxed{y = 3x - 5}$$

$3(4) - 5 = y = 7$

$r : \begin{pmatrix} 0 \\ -5 \end{pmatrix}$

$\boxed{x = 0,\ y = -5}$

$x = 1,\ y = -2$

$x = 2$

$x = -1,\ -8$

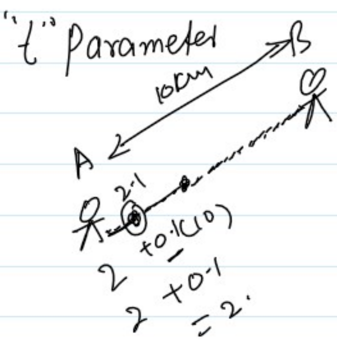
vector Equation of a Line

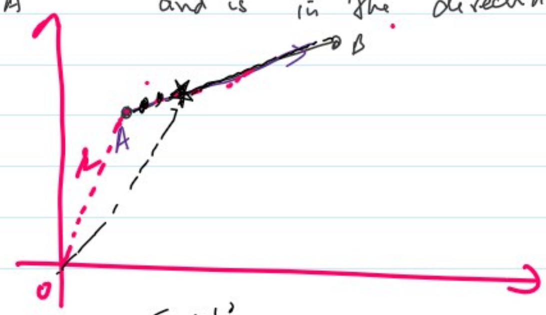
Passing Through "A" $A(1,-2)$, and is in the direction of $\bar{AB}$ $B(4,7)$

$$\vec{r} = \vec{OA} + t\,\vec{AB}$$

$\vec{r}$ = Position of a vector from origin

"t" Parameter





$\bar{OA} + t\omega$

$$\vec{OA} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$t = 0$

$x = -1$

$y = 2$

$$\vec{r} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} + t\begin{pmatrix} 3 \\ 9 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} + t\begin{pmatrix} 3 \\ 9 \end{pmatrix} \Rightarrow \begin{cases} x = 1 + 3t \\ y = -2 + 9t \end{cases}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 3t \\ 9t \end{pmatrix} =$$

$t = 0,\ x = 1,\ y = -2$

$t = 1,\ x = 4,\ y = 7$

Parameteric form

$$x = 1 + 3t, \qquad y = -2 + 9t$$

$$\frac{x-1}{3} = t, \qquad \frac{y+2}{9} = t$$

$$\frac{x-1}{3} = \frac{y+2}{9} = t$$

What is a Parametric Equation of a Line Passing Throug $A(x_0, y_0, z_0)$ to $B(x_1, y_1, z_1)$

$$\vec{r} = \vec{OA} + t\,\vec{AB} \rightarrow \text{vector Equation of a Line in 3D}$$

$r = OA + t\,AB \rightarrow$ vector equation in 3D

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + t \begin{pmatrix} x_1 - x_0 \\ y_1 - y_0 \\ z_1 - z_0 \end{pmatrix} \Rightarrow \left.\begin{aligned} x &= x_0 + t(x_1 - x_0) \\ y &= y_0 + t(y_1 - y_0) \\ z &= z_0 + t(z_1 - z_0) \end{aligned}\right] \text{Parameter}$$

$$\frac{x - x_0}{\boxed{x_1 - x_0}} = \frac{y - y_0}{\boxed{y_1 - y_0}} = \frac{z - z_0}{\boxed{z_1 - z_0}} = (t) \quad \text{Symmetric form}$$

Q#1 Form a vector Equation whose Parametric Equation is

$$\frac{x-2}{3} = \frac{y-1}{2} = \frac{z+1}{5} = t$$

$$l = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$$

(11)
$$\frac{2x-3}{5} = \frac{y+\frac{1}{2}}{3} = \frac{4-z}{2} = t$$

$$\frac{x-x_0}{d_1} = \frac{y-y_0}{d_2} = \frac{z-z_0}{d_3} = t$$

$$\frac{2\left(x - \frac{3}{2}\right)}{5} = \frac{y - \left(-\frac{1}{2}\right)}{3} = \frac{-(z-4)}{2} = t$$

$$\frac{x - \frac{3}{2}}{\frac{5}{2}} = \frac{y - \left(-\frac{1}{2}\right)}{3} = \frac{z-4}{-2} = t$$

direction or direction ratios

$$\left(\frac{5}{2} : 3 : -2\right) \times 2$$

$$5 : 6 : -4$$

$$\vec{l} = \begin{pmatrix} \frac{3}{2} \\ -\frac{1}{2} \\ 4 \end{pmatrix} + t \begin{pmatrix} 5 \\ 6 \\ -4 \end{pmatrix} \rightarrow \text{vector}$$

Equation of Line for (1) to (5)