lecture #6

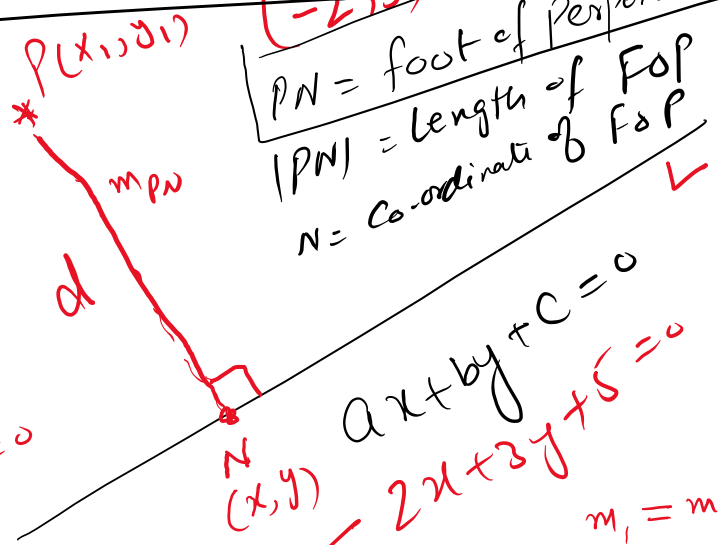
distance of a point from a Line

$(-2, 3)$

PN = foot of perpendicular $\boxed{FOP}$

$|PN|$ = Length of FOP

N = Co-ordinate of FOP



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$|PN| \Rightarrow$ distance

N: Line PN; $\boxed{ax+by+c=0} \Rightarrow 2x+3y+5=0$

To find out Equation of PN

$$y - y_1 = m(x - x_1)$$

$$y - 3 = m(x+2)$$

$m_1 = m_2$

$m_1 \times m_2 = -1$

$m_{PN} \times \boxed{m_L} = -1$

$2x+3y+5=0$

$y = mx + c$

$3y = -2x - 5$

$y = -\frac{2}{3}x - \frac{5}{3}$

$m_{PN} \times -\frac{2}{3} = -1$

$m_{PN} = \frac{3}{2}$

$$y - 3 = \frac{3}{2}(x+2)$$

$$2y - 6 = 3x + 6$$

$-3x + 2y = 12$ Eq PN

$2x + 3y = -5 \to L$

$$-6x + 4y = 24$$

$$6x + 9y = -15$$

$$13y = 9$$

$$y = \frac{9}{13}$$

$$-3x + 2\left(\frac{9}{13}\right) = 12 \Rightarrow 3x = \frac{18}{13} - 12$$

$$3x = \frac{18 - 156}{13}$$

$$3x = \frac{138}{13}$$

$$x = -\frac{\cancel{138}^{46}}{13} \times \frac{1}{\cancel{3}}$$

$N\left(-\frac{46}{13}, \frac{9}{13}\right)$

$P(-2, 3)$

$$\text{distance} = |PN| = \sqrt{\left(-\frac{46}{13}+2\right)^2 + \left(\frac{9}{13}-3\right)^2}$$

$$= \sqrt{\frac{20^2}{13^2} + \frac{30^2}{13^2}} = \frac{\sqrt{400+900}}{13} = \frac{\sqrt{1300}}{13}$$

$$= \frac{\sqrt{13}\,(10)}{13} = \frac{10}{\sqrt{13}}$$

IInd method

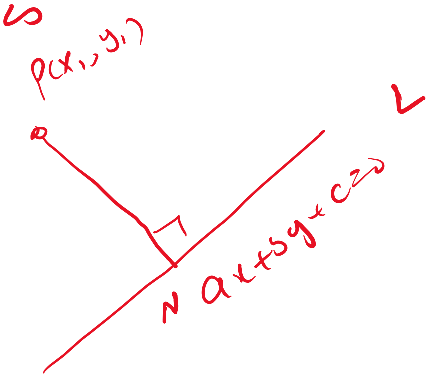
$2x+3y+5=0 \qquad P(-2,3)$

$ax+by+c=0 \quad ; \quad P(x_1, y_1)$

$$d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \frac{|2x_1+3y_1+5|}{\sqrt{2^2+3^2}} = \frac{|2(-2)+3(3)+5|}{\sqrt{4+9}} = \frac{|-4+9+5|}{\sqrt{13}} = \frac{10}{\sqrt{13}}$$

Prove that distance of a point $P(x_1, y_1)$ from a line $L: ax+by+c=0$ is

$$d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$



(1) Slope of $L \Rightarrow$ $by = -ax - c$

$y = -\frac{a}{b}x - \frac{c}{b}$

$m_L = -\frac{a}{b} \quad ; \quad m_{PN} = \frac{b}{a}$

PN Lin
$y - y_1 = \frac{b}{a}(x - x_1)$

$ay - ay_1 = bx - bx_1$

$-bx + ay = \boxed{ay_1 - bx_1} \Rightarrow d = ay_1 - bx_1$

3

$$\begin{cases} -bx + ay = d & \text{— PN} \\ ax + by = -c & \text{L} \end{cases}$$

find x & y

$N(x, y)$

$P(x_1, y_1)$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

over to you

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

These are parallel

when $\frac{a_1}{a_2} = \frac{b_1}{b_2} = K$

L_1

L_2

$2x - 3y = 10$

$-4x + 6y = 26$

$-\frac{2}{4} = -\frac{1}{2} = -\frac{3}{6}$

$\frac{10}{26} \neq -\frac{1}{2}$

$\times 2 \,(2x - 3y = 10$

$4x - 6y = 20$

$-4x + 6y = 26$

$0 = 46$

2nd example

$2x - 3y = 10$

$(-4x + 6y = -20) \div -2$

$-\frac{1}{2}$ $\quad -\frac{1}{2}$ $\quad = -\frac{1}{2}$

$+2x - 3y = 10$

$-2x - 3y = 10$

$0 = 0$

$d = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2}}$

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = K$

They are //

$\frac{c_1}{c_2} = K$ colinear

$\frac{c_1}{c_2} \neq K$ Non-colin