Week 1: Probability and Statistics

❖ *Basic Definition and Main Branches of Statistics:*

Statistics is the science of conducting studies to collect, organize, summarize, analyze, and draw conclusions from data.

Descriptive statistics consists of the collection, organization, summarization, and presentation of data.

In *descriptive statistics* the statistician tries to describe a situation. Consider the national census conducted by the U.S. government every 10 years. Results of this census give you the average age, income, and other characteristics of the U.S. population. To obtain this information, the Census Bureau must have some means to collect relevant data. Once data are collected, the bureau must organize and summarize them. Finally, the bureau needs a means of presenting the data in some meaningful form, such as charts, graphs, or tables.

The second area of statistics is called *inferential statistics.*

Inferential statistics consists of generalizing from samples to populations, performing estimations and hypothesis tests, determining relationships among variables, and making predictions.

Here, the statistician tries to make inferences from *samples* to *populations.* Inferential statistics uses **probability,** i.e., the chance of an event occurring. You may be familiar with the concepts of probability through various forms of gambling. If you play cards, dice, bingo, or lotteries, you win or lose according to the laws of probability. Probability theory is also used in the insurance industry and other areas.

❖ *Key Terms of Statistics:*

A **population** consists of all subjects (human or otherwise) that are being studied.

Most of the time, due to the expense, time, size of population, medical concerns, etc., it is not possible to use the entire population for a statistical study; therefore, researchers use samples.

A **sample** is a group of subjects selected from a population.

If the subjects of a sample are properly selected, most of the time they should possess the same or similar characteristics as the subjects in the population. The techniques

Parameter: The numerical results conducted from population are known as parameters.

Statistic: The numerical results obtained from sample data are known as statistic.

Variables and Types of Data

As stated in Section 1–1, statisticians gain information about a particular situation by collecting data for random variables. This section will explore in greater detail the nature of variables and types of data.

Variables can be classified as qualitative or quantitative. **Qualitative variables** are variables that can be placed into distinct categories, according to some characteristic or attribute. For example, if subjects are classified according to gender (male or female), then the variable *gender* is qualitative. Other examples of qualitative variables are religious preference and geographic locations.

Quantitative variables are numerical and can be ordered or ranked. For example, the variable *age* is numerical, and people can be ranked in order according to the value of their ages. Other examples of quantitative variables are heights, weights, and body temperatures.

Quantitative variables can be further classified into two groups: discrete and continuous. *Discrete variables* can be assigned values such as 0, 1, 2, 3 and are said to be countable. Examples of discrete variables are the number of children in a family, the number of students in a classroom, and the number of calls received by a switchboard operator each day for a month.

Discrete variables assume values that can be counted.

Continuous variables, by comparison, can assume an infinite number of values in an interval between any two specific values. Temperature, for example, is a continuous variable, since the variable can assume an infinite number of values between any two given temperatures.

Continuous variables can assume an infinite number of values between any two specific values. They are obtained by measuring. They often include fractions and decimals.

❖ *Levels of Measurement:*

What is a *nominal scale?*	A scale that categorizes items
What is an *ordinal scale?*	A scale that categorizes and rank orders items
What is an *interval scale?*	A scale that categorizes and rank orders items, and has equal intervals
What is a *ratio scale?*	A scale that categorizes and rank orders items, has equal intervals, and a zero that means the absence or none of the thing being measured

Table 1–2 Examples of Measurement Scales

Nominal-level data	Ordinal-level data	Interval-level data	Ratio-level data
Zip code	Grade (A, B, C, D, F)	SAT score	Height
Gender (male, female)	Judging (first place, second place, etc.)	IQ	Weight
Eye color (blue, brown, green, hazel)	Rating scale (poor, good, excellent)	Temperature	Time
Political affiliation	Ranking of tennis players		Salary
Religious affiliation			Age
Major field (mathematics, computers, etc.)			
Nationality			

❖ *Frequency and Frequency tables:*

Introduction

When conducting a statistical study, the researcher must gather data for the particular variable under study. For example, if a researcher wishes to study the number of people who were bitten by poisonous snakes in a specific geographic area over the past several years, he or she has to gather the data from various doctors, hospitals, or health departments.

To describe situations, draw conclusions, or make inferences about events, the researcher must organize the data in some meaningful way. The most convenient method of organizing data is to construct a *frequency distribution.*

After organizing the data, the researcher must present them so they can be understood by those who will benefit from reading the study. The most useful method of presenting the data is by constructing *statistical charts* and *graphs.* There are many different types of charts and graphs, and each one has a specific purpose.

2–1

Organizing Data

Wealthy People

Objective 1

Organize data using a frequency distribution.

Suppose a researcher wished to do a study on the ages of the top 50 wealthiest people in the world. The researcher first would have to get the data on the ages of the people. In this case, these ages are listed in *Forbes Magazine*. When the data are in original form, they are called **raw data** and are listed next.

49	57	38	73	81
74	59	76	65	69
54	56	69	68	78
65	85	49	69	61
48	81	68	37	43
78	82	43	64	67
52	56	81	77	79
85	40	85	59	80
60	71	57	61	69
61	83	90	87	74

Since little information can be obtained from looking at raw data, the researcher organizes the data into what is called a *frequency distribution.* A frequency distribution consists of *classes* and their corresponding *frequencies.* Each raw data value is placed into a quantitative or qualitative category called a **class.** The **frequency** of a class then is the number of data values contained in a specific class. A frequency distribution is shown for the preceding data set.

Class limits	Tally	Frequency
35–41	///	3
42–48	///	3
49–55	////	4
56–62	𝍸 𝍸	10
63–69	𝍸 𝍸	10
70–76	𝍸	5
77–83	𝍸 𝍸	10
84–90	𝍸	5
	Total	50

Now some general observations can be made from looking at the frequency distribution. For example, it can be stated that the majority of the wealthy people in the study are over 55 years old.

Unusual Stat

Of Americans 50 years old and over, 23% think their greatest achievements are still ahead of them.

A **frequency distribution** is the organization of raw data in table form, using classes and frequencies.

The classes in this distribution are 35–41, 42–48, etc. These values are called *class limits.* The data values 35, 36, 37, 38, 39, 40, 41 can be tallied in the first class; 42, 43, 44, 45, 46, 47, 48 in the second class; and so on.

Categorical Frequency Distributions

The **categorical frequency distribution** is used for data that can be placed in specific categories, such as nominal- or ordinal-level data. For example, data such as political affiliation, religious affiliation, or major field of study would use categorical frequency distributions.

Example 2–1

Distribution of Blood Types

Twenty-five army inductees were given a blood test to determine their blood type. The data set is

A	B	B	AB	O
O	O	B	AB	B
B	B	O	A	O
A	O	O	O	AB
AB	A	O	B	A

Construct a frequency distribution for the data.

Solution

Since the data are categorical, discrete classes can be used. There are four blood types: A, B, O, and AB. These types will be used as the classes for the distribution.

The procedure for constructing a frequency distribution for categorical data is given next.

Step 1 Make a table as shown.

A Class	B Tally	C Frequency	D Percent
A			
B			
O			
AB			

Step 2 Tally the data and place the results in column B.

Step 3 Count the tallies and place the results in column C.

Step 4 Find the percentage of values in each class by using the formula

$$\% = \frac{f}{n} \cdot 100\%$$

where $f =$ frequency of the class and $n =$ total number of values. For example, in the class of type A blood, the percentage is

$$\% = \frac{5}{25} \cdot 100\% = 20\%$$

Percentages are not normally part of a frequency distribution, but they can be added since they are used in certain types of graphs such as pie graphs. Also, the decimal equivalent of a percent is called a *relative frequency.*

Step 5 Find the totals for columns C (frequency) and D (percent). The completed table is shown.

A Class	B Tally	C Frequency	D Percent
A	𝍸	5	20
B	𝍸 //	7	28
O	𝍸 ////	9	36
AB	////	4	16
		Total 25	100

For the sample, more people have type O blood than any other type.

Grouped Frequency Distributions

When the range of the data is large, the data must be grouped into classes that are more than one unit in width, in what is called a **grouped frequency distribution.** For example, a distribution of the number of hours that boat batteries lasted is the following.

Unusual Stat

Six percent of Americans say they find life dull.

Class limits	Class boundaries	Tally	Frequency
24–30	23.5–30.5	///	3
31–37	30.5–37.5	/	1
38–44	37.5–44.5	𝍸	5
45–51	44.5–51.5	𝍸 ////	9
52–58	51.5–58.5	𝍸 /	6
59–65	58.5–65.5	/	1
			25