Lecture# 18 (Derivative of vector valued function)

Thursday, November 6, 2025 11:11 AM

Derivative of Position = velocity

$\vec{r}(t) = \langle f_1(t); f_2(t), f_3(t) \rangle$

$\frac{d\vec{r}}{dt} = \langle f_1'(t), f_2'(t), f_3'(t) \rangle$

Rule

(1) $\frac{d}{dt}\left(c\,\vec{r}(t)\right) = c \cdot \frac{d}{dt}\vec{r}(t) = c\,r'(t)$

(2) $\frac{d}{dt}\left(u(t)\,\vec{r}(t)\right) = u \cdot r'(t) + u'\,r(t)$ - Product Rule

(3) $\frac{d}{dt}\left(u(t) \cdot \vec{r}(t)\right) = u(t) \cdot \vec{r}'(t) + u'(t) \cdot \vec{r}(t)$ dot Product

(4) $\frac{d}{dt}\left(u(t) \times \vec{r}(t)\right) = u(t) \times \vec{r}'(t) + u'(t) \times \vec{r}(t)$ cross Product

Properties of the Derivative of Vector-Valued Functions

Let **r** and **u** be differentiable vector-valued functions of *t*, let *f* be a differentiable real-valued function of *t*, a scalar.

i.	$\frac{d}{dt}[c\mathbf{r}(t)]$	$=$	$c\mathbf{r}'(t)$	Scalar multiple
ii.	$\frac{d}{dt}[\mathbf{r}(t) \pm \mathbf{u}(t)]$	$=$	$\mathbf{r}'(t) \pm \mathbf{u}'(t)$	Sum and difference
iii.	$\frac{d}{dt}[f(t)\mathbf{u}(t)]$	$=$	$f'(t)\mathbf{u}(t) + f(t)\mathbf{u}'(t)$	Scalar product
iv.	$\frac{d}{dt}[\mathbf{r}(t) \cdot \mathbf{u}(t)]$	$=$	$\mathbf{r}'(t) \cdot \mathbf{u}(t) + \mathbf{r}(t) \cdot \mathbf{u}'(t)$	Dot product
v.	$\frac{d}{dt}[\mathbf{r}(t) \times \mathbf{u}(t)]$	$=$	$\mathbf{r}'(t) \times \mathbf{u}(t) + \mathbf{r}(t) \times \mathbf{u}'(t)$	Cross product
vi.	$\frac{d}{dt}[\mathbf{r}(f(t))]$	$=$	$\mathbf{r}'(f(t)) \cdot f'(t)$	Chain rule
vii.	If $\mathbf{r}(t) \cdot \mathbf{r}(t)$	$=$	c, then $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 0$.	

$(x^2+3x)^5 = 5(x^2+3x)^4(2x+3)$

- $\frac{d}{dt}[\sin(at)] = a \times \cos(at)$ $\qquad \frac{d}{dt}[\sin(f(t))] = \cos f(t) \cdot f'(t)$
- $\frac{d}{dt}[cos(at)] = -a \times \sin(at)$

a. We have $\mathbf{r}'(t) = 6\mathbf{i} + (8t+2)\,\mathbf{j} + 5\mathbf{k}$ and $\mathbf{u}'(t) = 2t\mathbf{i} + 2\mathbf{j} + (3t^2-3)\,\mathbf{k}$. Therefore, according to property iv.:

$$\begin{aligned}\frac{d}{dt}[\mathbf{r}(t)\cdot\mathbf{u}(t)] &= \mathbf{r}'(t)\cdot\mathbf{u}(t) + \mathbf{r}(t)\cdot\mathbf{u}'(t)\\ &= (6\mathbf{i} + (8t+2)\,\mathbf{j} + 5\mathbf{k})\cdot((t^2-3)\,\mathbf{i} + (2t+4)\,\mathbf{j} + (t^3-3t)\,\mathbf{k})\\ &\quad + ((6t+8)\,\mathbf{i} + (4t^2+2t-3)\,\mathbf{j} + 5t\mathbf{k})\cdot(2t\mathbf{i} + 2\mathbf{j} + (3t^2-3)\,\mathbf{k})\\ &= 6(t^2-3) + (8t+2)(2t+4) + 5(t^3-3t)\\ &\quad + 2t(6t+8) + 2(4t^2+2t-3) + 5t(3t^2-3)\\ &= 20t^3 + 42t^2 + 26t - 16.\end{aligned}$$

$$\begin{pmatrix}6\\8t+2\\5k\end{pmatrix}$$

$$\begin{cases}\mathbf{r}(t) = (6t+8)\,\mathbf{i} + (4t^2+2t-3)\,\mathbf{j} + 5t\mathbf{k}\\ \mathbf{u}(t) = (t^2-3)\,\mathbf{i} + (2t+4)\,\mathbf{j} + (t^3-3t)\,\mathbf{k},\end{cases}$$

; $r' = 6i + (8t+2)j + 5k$

$u' = 2ti + 2j + (3t^2-3)k$

$\frac{d}{dt}(\vec{r}\cdot\vec{u}) = \vec{r}\cdot u' + r'\cdot u$

do your self

$$\vec{r}\cdot u' = \begin{pmatrix}6t+8\\4t^2+2t-3\\5t\end{pmatrix}\cdot\begin{pmatrix}2t\\2\\3t^2-3\end{pmatrix}$$

$$\begin{aligned}&= 2t(6t+8t) + 2(4t^2+2t-3) + 5t(3t^2-3)\\ &= 12t^2 + 16t + 8t^2 + 4t - 6 + 15t^3 - 15t\\ &= 15t^3 + 20t^2 + 5t - 6\end{aligned}$$

$\frac{d}{dt}(\vec{u}\times\bar{r}) = \boxed{u\times r'} + \boxed{u'\times r}$ property of derivative

1 Direct

$$\text{Calcular } \vec{u}\times\bar{r} = \begin{vmatrix} i & j & k\\ u_1 & u_2 & u_3\\ r_1 & r_2 & r_3\end{vmatrix}$$

$= i(u_2 r_3 - u_3 r_2) - j(u_1 r_3 - r_1 u_3) + k(u_1 r_2 - r_1 u_2)$

$\frac{d}{dt}(u\times\hat{r})$