

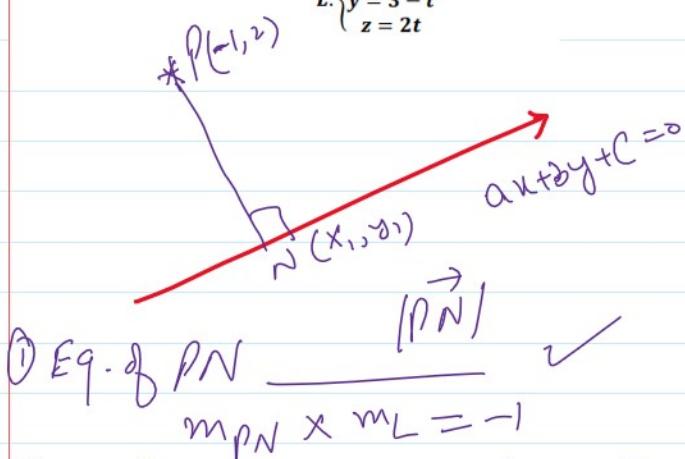
## Lecture # 7,8 (distance of a point from a vector)

Wednesday, October 15, 2025 10:07 AM

### Example:

Find the distance from the point  $S(1, 1, 5)$  to the line:

$$L: \begin{cases} x = 1 + t \\ y = 3 - t \\ z = 2t \end{cases}$$



③  $PN = \sqrt{(x_1 - 1)^2 + (y_1 - 2)^2}$

vector S point

I  $\vec{PN} = \vec{ON} - \vec{OP}$

$$\vec{PN} = \begin{pmatrix} x_0 + td_1 \\ y_0 + td_2 \\ z_0 + td_3 \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$$

$$\vec{PN} = \begin{pmatrix} (x_0 - x_1) + td_1 \\ (y_0 - y_1) + td_2 \\ (z_0 - z_1) + td_3 \end{pmatrix}$$

You will get

value of  $t$

### Equation of line

(i) cartesian form  $\Rightarrow y - y_1 = m(x - x_1)$

(ii) vector form  $\vec{r} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + t \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$

### Parametric form

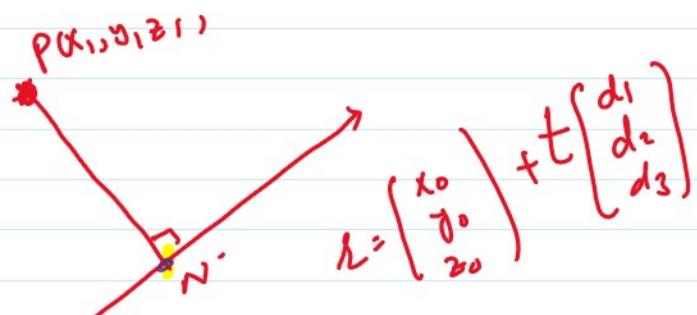
$$x = x_0 + t d_1$$

$$y = y_0 + t d_2$$

$$z = z_0 + t d_3$$

(iii)  $\boxed{\frac{x - x_0}{d_1} = \frac{y - y_0}{d_2} = \frac{z - z_0}{d_3} = (t)}$

Symmetric (Cartesian form)



$$\vec{ON} = \begin{pmatrix} x_0 + t d_1 \\ y_0 + t d_2 \\ z_0 + t d_3 \end{pmatrix}$$

$$\vec{PN} \cdot \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} (x_0 - x_1) + td_1 \\ (y_0 - y_1) + td_2 \\ (z_0 - z_1) + td_3 \end{pmatrix} \cdot \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = 0$$

$S(1, 1, 5)$

$$\begin{pmatrix} 1+t \\ 3-t \\ 2t \end{pmatrix}$$

### Example:

Find the distance from the point  $S(1, 1, 5)$  to the line:

$$L: \begin{cases} x = 1 + t \\ y = 3 - t \\ z = 2t \end{cases}$$

$$\vec{SN} = \vec{ON} - \vec{OS}$$

$$\vec{ON} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$$

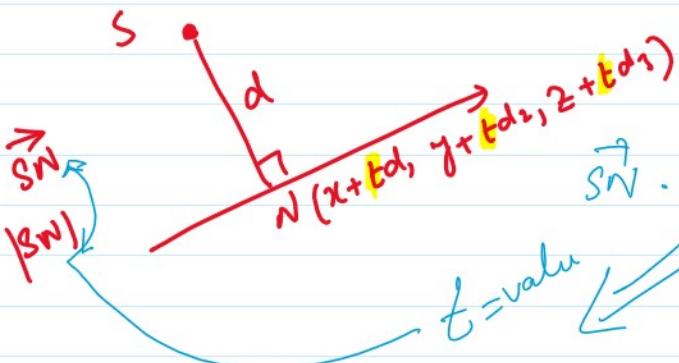
. . .

$$\vec{SN} = \vec{ON} - \vec{OS}$$

$$= \begin{pmatrix} 1+t \\ 3-t \\ 2+2t \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$$

$$\vec{SN} = \begin{pmatrix} t \\ 2-t \\ -5+2t \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

$$|\vec{SN}| = \sqrt{4+0+1} = \sqrt{5}$$



$$\vec{ON} = \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$$

$$\vec{d}_L = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$\vec{SN} \cdot \vec{d}_L = 0$$

$$\begin{pmatrix} t \\ 2-t \\ -5+2t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = 0$$

$$t - 2 + t - 10 + 4t = 0$$

$$6t - 12 = 0$$

$$6t = 12$$

$$t = 2$$

### Example:

Find the distance from the point  $S(1, 1, 5)$  to the line:

$$L: \begin{cases} x = 1 + t \\ y = 3 - t \\ z = 2t \end{cases}$$

### Solution:

Since the given point is  $S(x_0, y_0, z_0) = P_0(1, 1, 5)$ .

And the given parametric equations of line are

$$\begin{aligned} x &= x_1 + at = 1 + t \\ y &= y_1 + bt = 3 - t \\ z &= z_1 + ct = 0 + 2t \end{aligned}$$

This means that  $P_1(x_1, y_1, z_1) = P_1(1, 3, 0)$  and  $\vec{v} = \langle a, b, c \rangle = \langle 1, -1, 2 \rangle$ .

Thus, the vector parallel to the line  $L$  is

$$\vec{v} = \hat{i} - \hat{j} + 2\hat{k} \quad (\text{direction of line})$$

$$\vec{u} = \vec{P_1 P_0}$$

$$\vec{P_L P_0} = \vec{u}$$

The Line passes through the point  $P_1(1, 3, 0)$

$$\vec{u} = \vec{P_1 P_0} = \vec{OP_0} - \vec{OP_1}$$

$$\vec{u} = \vec{OP_0} = (\hat{i} + \hat{j} + 5\hat{k}) - (\hat{i} + 3\hat{j} + 0\hat{k})$$

$$\vec{u} = (1-1)\hat{i} + (1-3)\hat{j} + (5-0)\hat{k}$$

$$\boxed{\vec{u} = 0\hat{i} - 2\hat{j} + 5\hat{k}}$$

Now,

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & 5 \\ 1 & -1 & 2 \end{vmatrix}$$

$$\vec{u} \times \vec{v} = \hat{i} \begin{vmatrix} -2 & 5 \\ -1 & 2 \end{vmatrix} - \hat{j} \begin{vmatrix} 0 & 5 \\ 1 & 2 \end{vmatrix} + \hat{k} \begin{vmatrix} 0 & -2 \\ 1 & -1 \end{vmatrix}$$

Putting values, we have

$$d = \frac{\sqrt{30}}{\sqrt{6}} = \sqrt{\frac{30}{6}} = \sqrt{5}$$

$$d = \sqrt{5} = 2.24$$

Hence the distance of a point  $P_1$  to the line  $L$  is **2.24** units.

$$L = \left( \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + t \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} \right) \quad P(x_0, y_0, z_0)$$

$$\vec{u} \times \vec{v} = \hat{i}(-4+5) - \hat{j}(0-5) + \hat{k}(0+2)$$

$$\vec{u} \times \vec{v} = \hat{i} + 5\hat{j} + 2\hat{k}$$

Taking magnitude of  $\vec{u} \times \vec{v}$ , we have

$$|\vec{u} \times \vec{v}| = \sqrt{(1)^2 + (5)^2 + (2)^2}$$

$$|\vec{u} \times \vec{v}| = \sqrt{30}$$

Similarly, taking magnitude of vector  $\vec{v}$ , we have

$$|\vec{v}| = \sqrt{(1)^2 + (-1)^2 + (2)^2} = \sqrt{6}$$

Now, apply the formula, we have

$$|\vec{v}| = \sqrt{(1)^2 + (-1)^2 + (2)^2} = \sqrt{6}$$

Now, apply the formula, we have

$$d = \frac{|\vec{u} \times \vec{v}|}{|\vec{v}|}$$

$$\vec{U} \times \vec{V} = \begin{vmatrix} i & j & k \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ d_1 & d_2 & d_3 \end{vmatrix}$$

$$\frac{|\vec{U} \times \vec{V}|}{|\vec{V}|} = d$$

$$\vec{U} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + t \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

$$\vec{V} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

$$\vec{U} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} - \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_1 - x_0 \\ y_1 - y_0 \\ z_1 - z_0 \end{pmatrix}$$

Q Find The distance of a Point  $P(1, 2, 3)$  from a Line passing Through

$$A(-1, -2, 4)$$

$$B(5, 6, 7)$$

$$\vec{OP} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\textcircled{1} \quad \vec{U} = \vec{OA} + t \vec{AB}$$

$$= \begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix}$$

$$\text{Formula} \quad d = \frac{|\vec{U} \times \vec{V}|}{|\vec{V}|}$$

$$\vec{V} = \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix}$$

$$\vec{U} \times \vec{V} = \begin{pmatrix} i & j & k \\ 2 & 4 & -1 \\ 6 & 8 & 3 \end{pmatrix}$$

$$= i(20) - j(12) + k(16 - 24) = i(-20) + k(8)$$

$$|\vec{U} \times \vec{V}| = \sqrt{400 + 144 + 64} = \sqrt{608}$$

$$|\vec{V}| = \sqrt{36 + 64 + 9} = \sqrt{109}$$

$$\frac{\sqrt{608}}{\sqrt{109}} = \sqrt{\frac{608}{109}} = 2.36$$

$\rightarrow$   $\rightarrow$   $\dots \rightarrow$

$$P(1, 2, 3) \rightarrow$$

$$\textcircled{1} \quad \vec{z} = \vec{OA} + t \vec{AB}$$

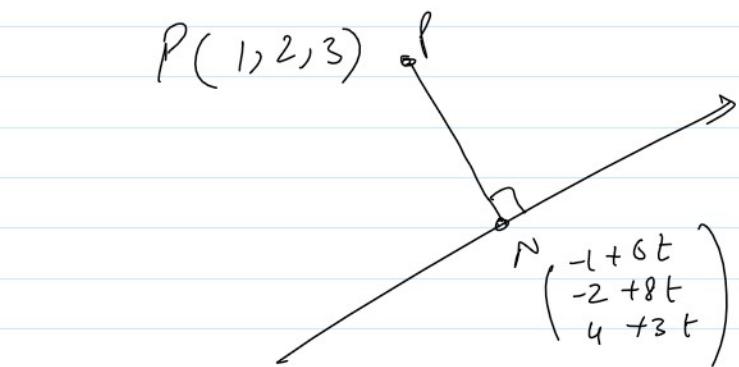
$$= \begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix}$$

$$\vec{PN} = \vec{ON} - \vec{OP}$$

$$= \begin{pmatrix} -1+6t \\ -2+8t \\ 4+3t \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\vec{PN} = \begin{pmatrix} -2+6t \\ -4+8t \\ 1+3t \end{pmatrix}$$

$$\vec{PN} = \begin{pmatrix} -2 + 6\left(\frac{41}{109}\right) \\ -4 + 8\left(\frac{41}{109}\right) \\ 1 + 3\left(\frac{41}{109}\right) \end{pmatrix}$$



$$\vec{PN} \cdot d_L = 0$$

$$d_L = \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix}$$

$$\frac{6(-2+6t) + 8(-4+8t) + 3(1+3t)}{-12+36t - 32+64t + 3+9t} = 0$$

$$109t - 41 = 0$$

$$t = \frac{41}{109}$$

$$= \begin{pmatrix} -2 + \frac{246}{109} \\ -4 + \frac{328}{109} \\ 1 + \frac{123}{109} \end{pmatrix} = \begin{pmatrix} \frac{-218+246}{109} \\ \frac{-436+328}{109} \\ \frac{109+123}{109} \end{pmatrix}$$

$$\vec{PN} = \begin{pmatrix} 28/109 \\ -108/109 \\ 232/109 \end{pmatrix} =$$

$$|\vec{PN}| = \sqrt{\left(\frac{28}{109}\right)^2 + \left(\frac{-108}{109}\right)^2 + \left(\frac{232}{109}\right)^2}$$

$$= 2.36$$

$$\begin{matrix} & \overset{0 \ 8 \ 9}{\overbrace{|}} \\ (0, 8, 9) & A \end{matrix} \quad \begin{matrix} & \overset{1 \ 9 \ 8}{\overbrace{|}} \\ (1, 9, 8) & B \end{matrix}$$

a b c d  
(a, b, c), (d, c, b)