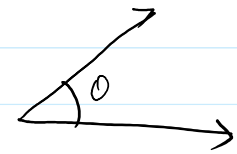
Two Lines

Angle b/w 2 - Lines

$a \cdot b = |a||b| \cos\theta$



$l_1 = \begin{pmatrix} 2-3t \\ 1+2t \\ -t \end{pmatrix}$ $l_2 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + s \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

$l_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$

$d_1 = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ $d_2 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

$d_1 \cdot d_2 = (\sqrt{9+4+1})(\sqrt{4+9+1}) \cos\theta$

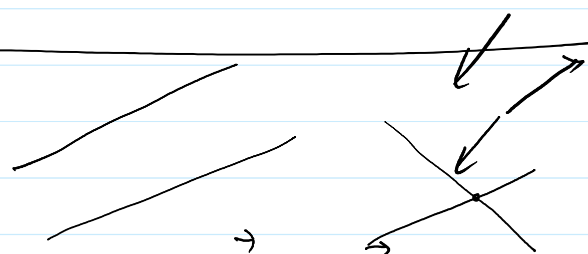
$-\cancel{6}+\cancel{6}-1 = (\sqrt{14})(\sqrt{14}) \cos\theta$

$\cos\theta = \frac{-1}{14}$

$\theta = \cos^{-1}\left(-\frac{1}{14}\right)$

---

① Parallel Lines



$l_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + t \begin{pmatrix} a \\ b \\ c \end{pmatrix}$, $l_2 = \begin{pmatrix} p_1 \\ q_1 \\ s_1 \end{pmatrix} + \lambda \begin{pmatrix} e \\ f \\ g \end{pmatrix}$

$\vec{a} = K\vec{b}$

If $\frac{a}{e} = \frac{b}{f} = \frac{c}{g} = K$

AB & AC are //

AB & CD are //

$l_1 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$, $l_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + s \begin{pmatrix} 4 \\ -2 \\ -6 \end{pmatrix}$

$\frac{-2}{4} = -\frac{1}{2}$, $\frac{1}{-2} = -\frac{1}{2}$, $\frac{3}{-6} = -\frac{1}{2}$

Q $l_1 = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ $l_2 = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} + s \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$l_1 = l_2$

x: $-1-2t = -2+s$ — ①

y: $2+t = 1-2s$ — ②

$x:\quad -1-2t = -2+s \quad — (1)$

$y:\quad 2+t = 1-2s \quad — (2)$

$z:\quad 3+t = -1+s \quad — (3)$

(1) + 2 × (2)

$$\begin{aligned} -1-\cancel{2t} &= -\cancel{2}+s \\ 4+\cancel{2t} &= \cancel{2}-4s \\ \hline 3 &= -3s \end{aligned}$$

$\boxed{s=-1}$ $\boxed{t=1}$

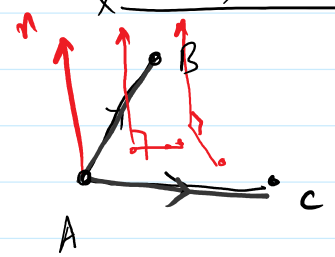
in (1): $-1-2t = -2-1$
$= -3+1$
$-2t = -2$

Eq(3) $\Rightarrow$ $3+1 = -1-1$

$4 \neq -2$

Lines are Not Parallel & Skewed (Not Intersecting)

Equation of Plane



(i) A plane is specified by atleast 3- NON Colinear Points

(ii) One Normal of plane which is Perpendicular to all Points

Vector Equation of Plane

$$\vec{r} = \vec{OA} + t\,\vec{AB} + s\,\vec{AC}$$

A(1, -1, 1)
B(2, 2, -3)
C(3, -1, 2)

$$\vec{r} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} + s\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$ —— "vector Equation"

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1+t+2s \\ -1+3t+0s \\ 1-4t+s \end{pmatrix} \Rightarrow \left.\begin{aligned} x &= 1+t+2s \quad -(i) \\ y &= -1+3t+0s \quad -(ii) \\ z &= 1-4t+s \quad -(iii) \end{aligned}\right\}$$ Parametric form

(i) & (ii) $t \to$ Removed $\quad 3x = 3 + \cancel{3t} + 6s$

(i) & (ii) $t \to$ Removed

$$3x = 3 + \cancel{3t} + 6s$$
$$y = -1 + \cancel{3t} + 0s$$
$$\boxed{3x - y = 4 + 6s} \quad \text{— (iv)}$$

(i) & (iii)

$$4x = 4 + \cancel{4t} + 8s$$
$$z = 1 - \cancel{4t} + s$$
$$\boxed{4x + z = 5 + 9s} \longrightarrow \text{(v)}$$

$\times 3$

$$9x - 3y = 12 + 18\cancel{s}$$

$2\times$

$$8x + 2z = 10 + \cancel{18s}$$
$$\boxed{x - 3y - 2z = 2}$$

Cartisian form

$$\boxed{ax + by + cz = d}$$

IInd Method

$$\vec{AB} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} \qquad \vec{AC} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 1 & 3 & -4 \\ 2 & 0 & 1 \end{vmatrix}$$
$$= i(3) - j(9) + k(-6)$$
$$= 3i - 9j - 6k$$
$$= \begin{pmatrix} 3 \\ -9 \\ -6 \end{pmatrix} \approx \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}$$

$$\vec{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$r \cdot \vec{n} = a \cdot n$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}$$

$$\boxed{\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}}$$

Equation of plane

$$x - 3y - 2z = 1 + 3 - 2$$
$$\boxed{x - 3y - 2z = 2}$$

$$\boxed{A(x - x_0) + B(y - y_0) + C(z - z_0) = 0}$$

(i) $r \cdot \vec{n} = a \cdot \vec{n}$

find Equation of Plane

find Equation of Plane Containing Two Lines

$$l_1 = \begin{pmatrix} 2-t \\ 1+t \\ 3-t \end{pmatrix} \qquad l_2 = \begin{pmatrix} 5+2s \\ 3-s \\ 4-3s \end{pmatrix}$$

$$\vec{n} = \begin{vmatrix} i & j & k \\ -1 & 1 & -1 \\ 2 & -1 & -3 \end{vmatrix} = i(-2) - j(3+2) + k(1-2)$$

$$= -4i \cancel{+} 5j - k$$

$$4i + 5j + k$$

$$\vec{n} = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} ✓$$

$t = -1,$

$a : (3, 0, 4)$

$$4x+5y+1z = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix}$$

$$\boxed{4x+5y+z = 16}$$

$$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \quad \cancel{4+5}$$

$s = -1 \qquad a : (3, 4, 7)$

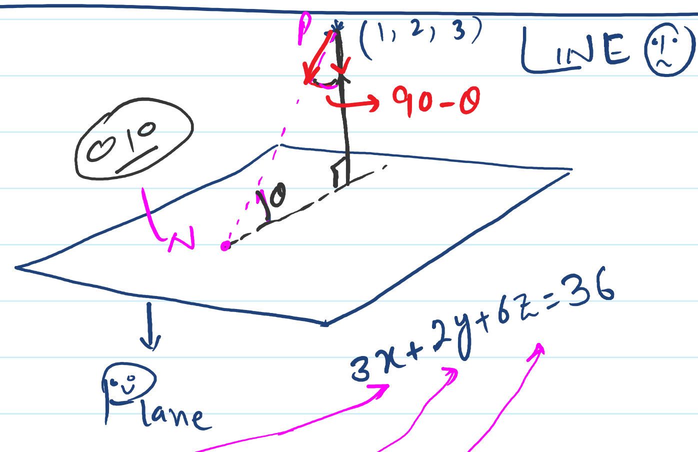
$$2x+5y+z = \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix}$$

$$= 12+20+7$$

$$40$$

$$\boxed{2x+5y+z = 33}$$

Equation of Line PN



$\vec{N}\ (0, 0, 6)$

$$\vec{PN} : \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$$

$$\vec{l} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$$

$x = 1-t$

$y = 2-2t$

$z = 3+3t$

$$3x+2y+6z = 36$$

$$3(1-t) + 2(2-2t) + 6(3+3t) = 36$$

$$3-3t + 4-4t + 18 + 18t = 36$$

$$11t = 36-25$$

$$\boxed{t = 1}$$

$$t = 1, \quad \vec{ON} : \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$$

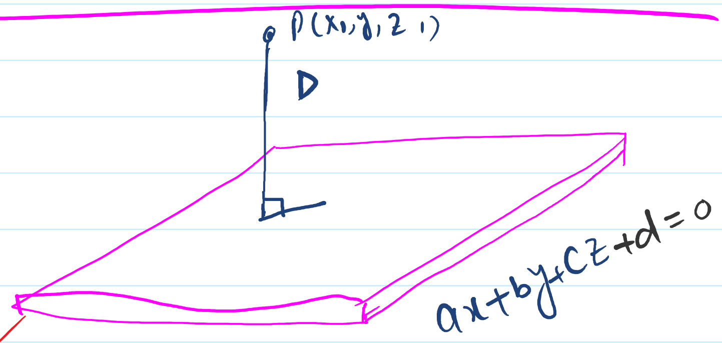
Angle b/w Line & Plane

$$d_\ell \cdot \vec{n} = |d_\ell||\vec{n}| \cos(90-\theta) \qquad \cos(90-\theta) = \sin\theta$$

$$\boxed{d_\ell \cdot \vec{n} = |d_\ell||\vec{n}| \sin\theta}$$

$\theta \Rightarrow$ Angle b/w line & plane

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$$



$$2x + 6y + 3z - 19 = 0$$

$$(1, 2, 3)$$

$$D = \frac{|2x_1 + 6y_1 + 3z_1 - 19|}{\sqrt{2^2+6^2+3^2}} \Rightarrow \frac{|2(1) + 6(2) + 3(3) - 19|}{\sqrt{4+36+9}}$$

$$= \frac{|2+12+9-19|}{7} = \frac{4}{7}$$

Distance b/w 2 - Parallel planes

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = K$$

I make co-efficient of x, y, z, same

$$ax + by + cz = K_1$$

$$ax + by + cz = K_2$$

$$D = \frac{|K_1 - K_2|}{\sqrt{a^2+b^2+c^2}}$$

$$(2x - 3y + 5z = 20) \times 2 \Rightarrow \begin{array}{l} 4x - 6y + 10z = 40 \\ 4x - 6y + 10z = 60 \end{array}$$

$$4x - 6y + 10z = 60$$

$$|60 - 40| \qquad 20$$

$$4x - 6y + 10z = 60$$

$$D = \frac{|60-40|}{\sqrt{4^2+6^2+10^2}} = \frac{20}{\sqrt{152}}$$