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Q1. Test cases:

Input 1: \rightarrow Output: 5

$A = [1, 2, 3]$

Input 2: \rightarrow Output 2:

$A = [2, 2]$

Problem statement:

We define $f(x, y)$ as: number of different corresponding bits in the binary representation of x and y .

For example, $f(2, 7) = 2$, since the binary representation of 2 and 7 are 010 and 111 respectively. The first and third bit differs, so $f(2, 7) = 2$.

You are given an array of N positive integers, $A_1, A_2, A_3, \dots, A_n$. Find the sum of $f(A_i, A_j)$ for all pairs (i, j) such that $1 \leq i, j \leq N$. Return the answer modulo $10^9 + 7$.

Constraint:

$1 \leq N \leq 10^5$

$1 \leq A[i] \leq 2^{31}$

Soln: Approach:

Count total different bits for all ordered pairs (i, j)

As in problem statement, $f(2, 7) :$

$2 \rightarrow 010$	<u>Other bit</u>	<u>1st bit</u>	<u>2nd bit</u>
$7 \rightarrow 111$	\vdots	$2 \rightarrow 0 \neq 1$ $7 \rightarrow 1$	$2 \rightarrow 0 \neq 1$ $7 \rightarrow 1$

Total different bits $f(2, 7) = 2$

Bitwise intuition:

Instead of comparing every pair (which would be too slow), we solve it by bit bit.

For a fixed bit position k :

- $\text{count}1 = \text{no. of } \text{set} \text{ having bit } k \text{ set} = 1$
- $\text{count}0 = \text{no. of } \text{set} \text{ having bit } k \text{ unset}$

and for this bit:

- Every pair where one number has bit = 1 (set) and the other has bit = 0 (unset)
- contributes 1 to answer.

Number of such ordered pairs:

$$\text{count}1 \times \text{count}0$$

But since (i, j) and (j, i) both counted:

$$\text{contribution} = 2 \times \text{count}1 \times \text{count}0$$

C++ Implementation:

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
#define MOD 1000000007;
```

```
long long solve (vector<int> &A) {
```

```
    long long N = A.size();
```

```
    long long ans = 0;
```

```
    for (int i = 0; i < 31; i++) {
```

```
        long long count1 = 0;
```

```
        for (int j = 0; j < N; j++) {
```

```
            if ((A[j] & (1LL << i)) != 0)
```

```
                count1++;
```

```
}
```

```
    long long count0 = N - count1;
```

```
    long long contribution = ((2LL * count1 % MOD * count0 % MOD) % MOD);
```

```
    ans = (ans + contribution) % MOD;
```

```
    return ans;
```



```

int main() {
    ios::sync_with_stdio(false);
    cin.tie(NULL); int N; cin >> N;
    vector<int> A(N);
    for (int i=0; i<N; i++) {
        cin >> A[i];
    }
    cout << value(A) << "\n";
    return 0;
}

```

Time complexity :

$$O(31 \times N) = O(N)$$

dry run for ~~A[0..3]~~ A = [2, 3]

$$2 \rightarrow 10$$

$$3 \rightarrow 11$$

<u>bit 0</u>	2 2 → 10	3 3 → 11	$\text{count } 1 = 1$ $\text{count } 0 = 1$
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$$\begin{aligned} \text{contribution} &= 2 \times \text{count } 1 \times \text{count } 0 \\ &= 2 \times 1 \times 1 = 2 \end{aligned}$$

<u>bit 1</u>	2 2 → 10	3 3 → 11	$\text{count } 1 = 2$ $\text{count } 0 = 0$
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$$\text{contribution} = 2 \times 2 \times 0 = 0$$

Higher bits 2 to 30.

$$\begin{aligned} \text{count } 1 &= 0 \\ \text{count } 0 &= \dots \end{aligned}$$

$$\text{contribution} = 2 \times 0 \times \dots = 0$$

$$\begin{aligned} \text{ans} &= 2 + 0 \\ &= 2 \quad \checkmark \end{aligned}$$