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@imranm3

DERIVATION

Deriving the formula for equation:

$$g(x_i, y_i) = \nabla_w L$$

Given polynomial equation:

$$h(x, w) = \sum_{j=1}^{n} w_j x^j$$

Give loss function equation:

$$L(h(x), y) = (h(x) - y)^2$$

For n = 0,

$$h(x, w) = w_0$$

$$L(h(x, w), y) = (w_0 - y)^2$$

$$\frac{\delta L}{\delta w_0} = 2(w_0 - y) \tag{1}$$

For n = 1,

$$h(x, w) = w_0 + w_1 x$$

$$L(h(x, w), y) = ((w_0 + w_1 x) - y)^2$$

$$\frac{\delta L}{\delta w_1} = 2x((w_0 + w_1 x) - y) \tag{2}$$

For n = 2,

$$h(x, w) = w_0 + w_1 x + w_2 x^2$$

$$L(h(x, w), y) = ((w_0 + w_1 x + w_2 x^2) - y)^2$$

$$\frac{\delta L}{\delta w_2} = 2x^2((w_0 + w_1 x + w_2 x^2) - y) \tag{3}$$

For n = 3,

$$h(x, w) = w_0 + w_1 x + w_2 x^2 + w_3 x^3$$

$$L(h(x, w), y) = ((w_0 + w_1 x + w_2 x^2 + w_3 x^3) - y)^2$$

$$\frac{\delta L}{\delta w_3} = 2x^3((w_0 + w_1 x + w_2 x^2 + w_3 x^3) - y) \tag{4}$$

For n = 4,

$$h(x, w) = w_0 + w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4$$

$$L(h(x, w), y) = ((w_0 + w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4) - y)^2$$

$$\frac{\delta L}{\delta w_4} = 2x^4((w_0 + w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4) - y) \tag{5}$$

For n = 5,

$$h(x, w) = w_0 + w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4 + w_5 x^5$$

$$L(h(x, w), y) = ((w_0 + w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4 + w_5 x^5) - y)^2$$

$$\frac{\delta L}{\delta w_5} = 2x^5((w_0 + w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4 + w_5 x^5) - y) \tag{6}$$

The gradient is the vector of partial derivatives, that is,

$$g(x_i, y_i) = \left[\frac{\delta L}{\delta w_0}, \frac{\delta L}{\delta w_1}, ..., \frac{\delta L}{\delta w_j} \right]$$

where j = 0, 1, .., polynomial degree.

For each polynomial degree, calculated mean squared error, gamma and number of iteration is shown on table 1.

Degree	Gamma	Number of iterations	Mean squared error
1	0.001	5081	1.5010869147046082
2	0.001	30096	0.8903534048827567
3	0.001	35893	0.5875820379805796
4	0.00008	5702507	0.0011639665990301783
5	0.00008	5671070	0.0011994142172381424

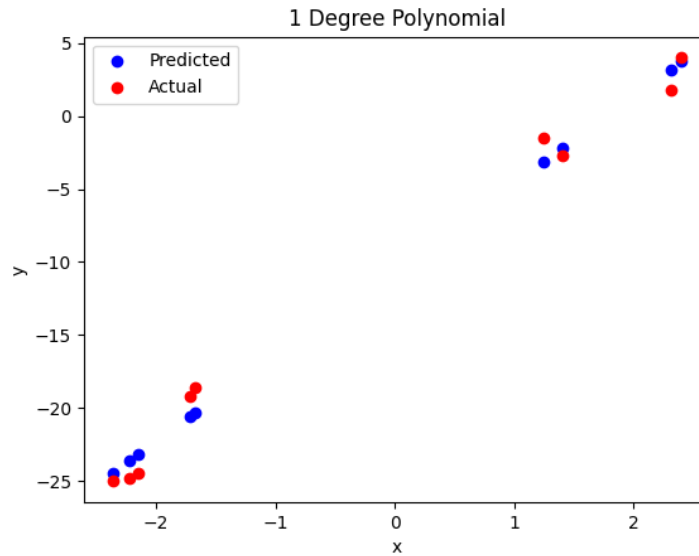Table 1: Polynomial degree and respective values
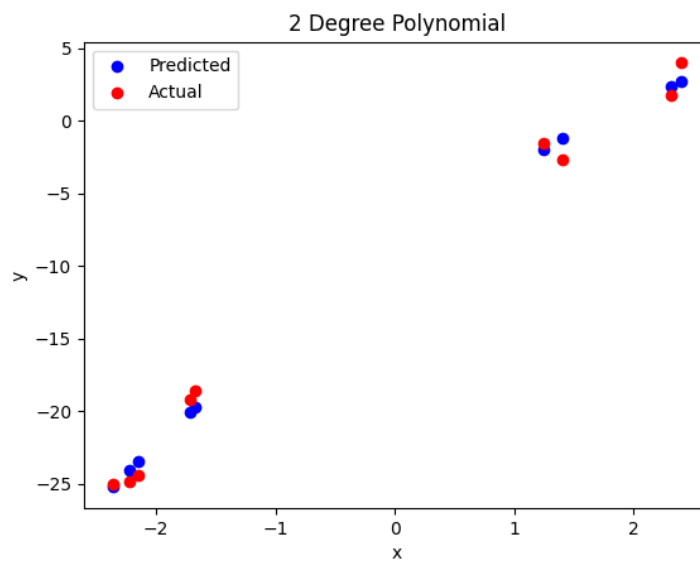
PLOTS

Figure 1: Polynomial Degree 1

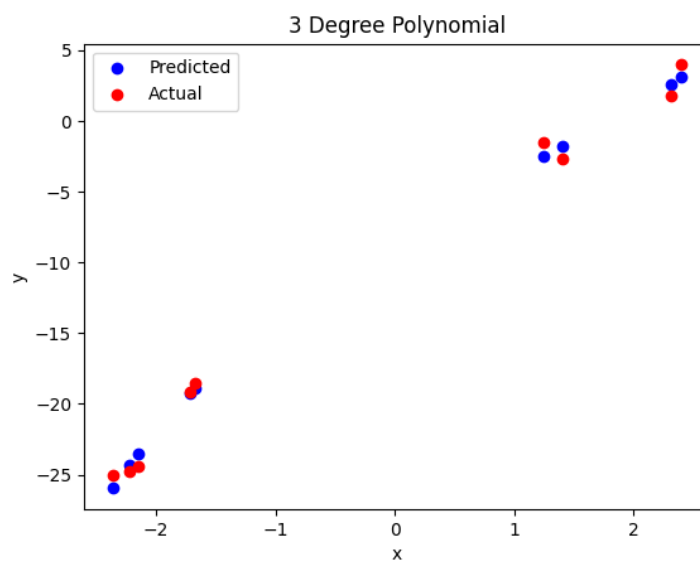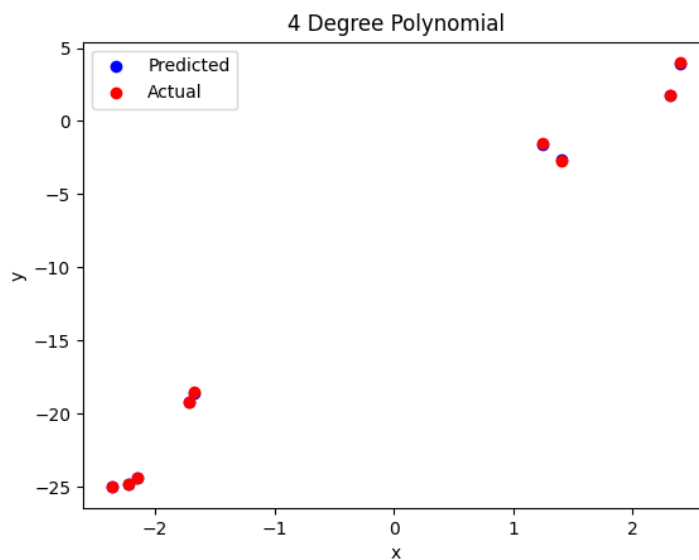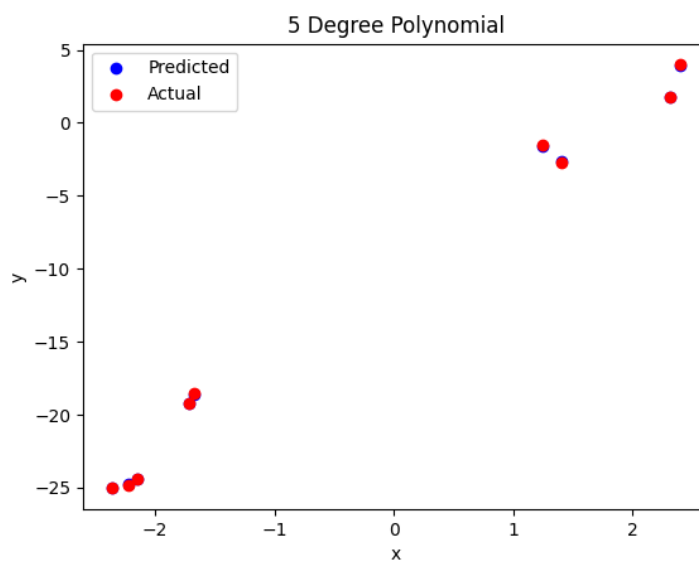Figure 2: Polynomial Degree 2

Figure 3: Polynomial Degree 3

Figure 4: Polynomial Degree 4

Figure 5: Polynomial Degree 5