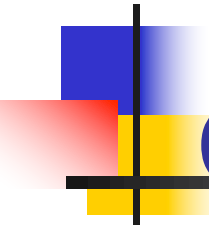
CMSC 510
Regularization Methods for Machine Learning

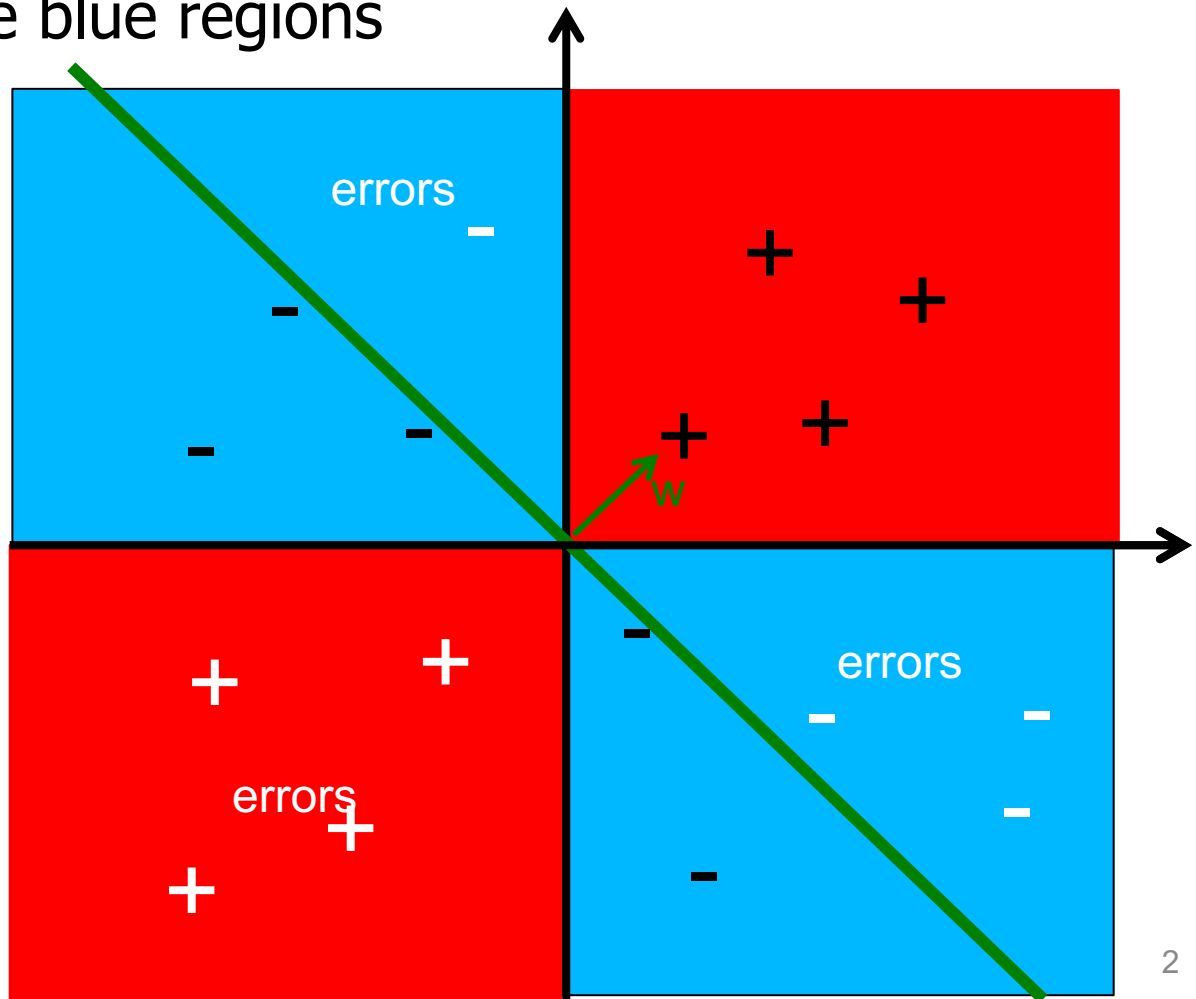


Classification via nonlinear functions

Instructor:
Dr. Tom Arodz

Nonlinear classification

- XOR problem
  - samples in the red regions have +1 class
  - Samples in the blue regions have -1 class
  - Linear decision will have 50% error



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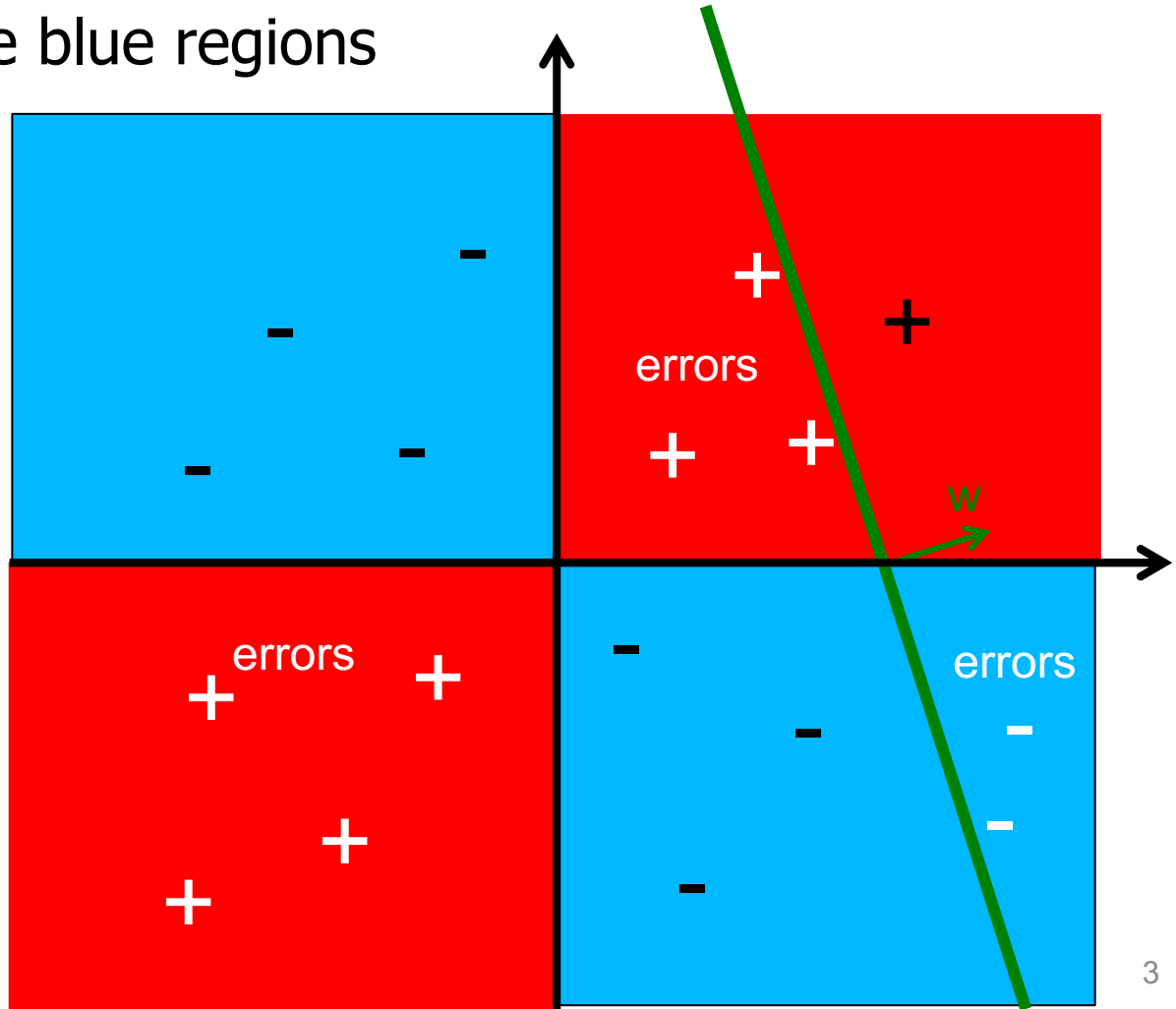

Nonlinear classification

- Circle problem
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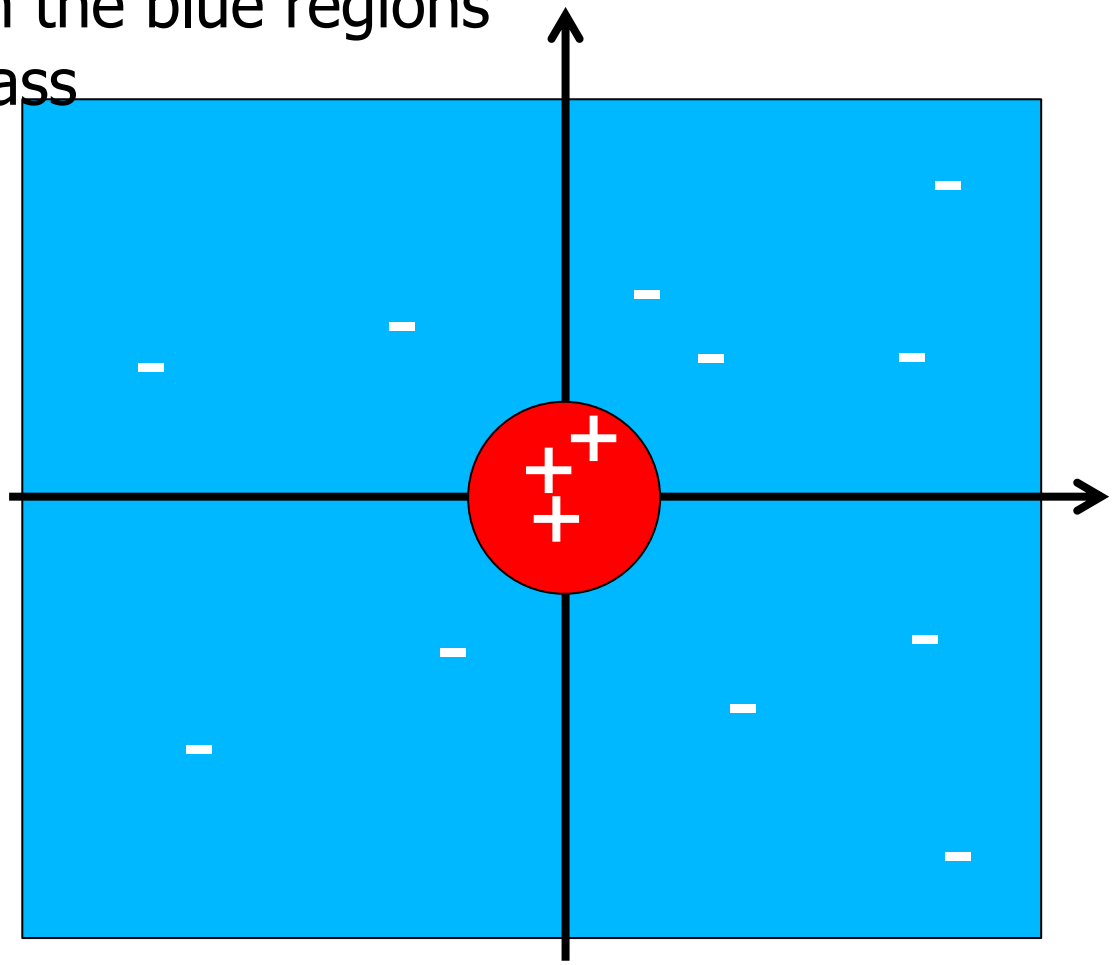

Nonlinear classification

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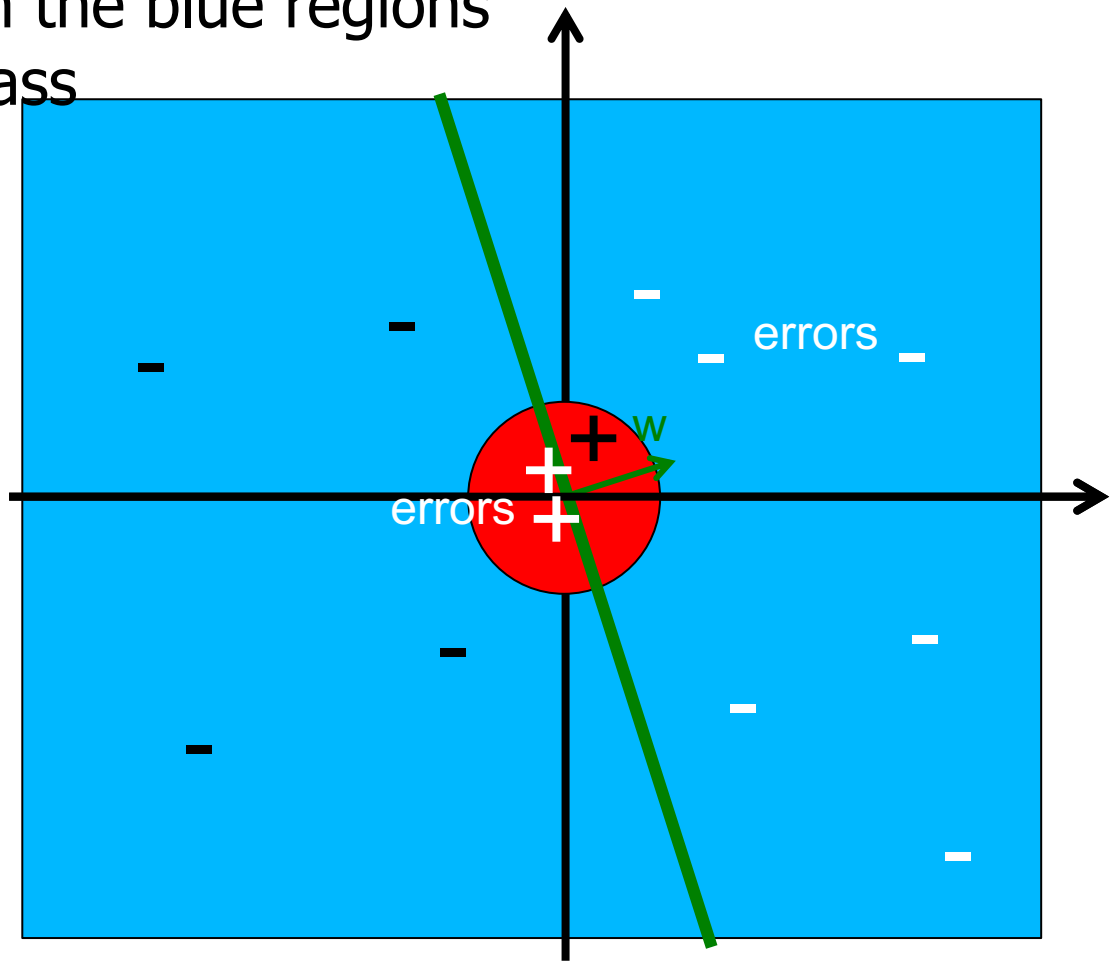
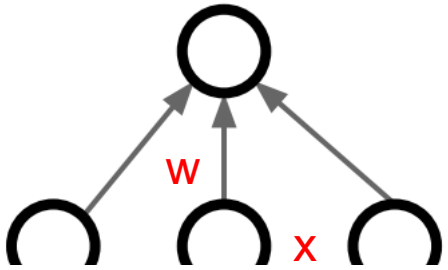


Nonlinear models

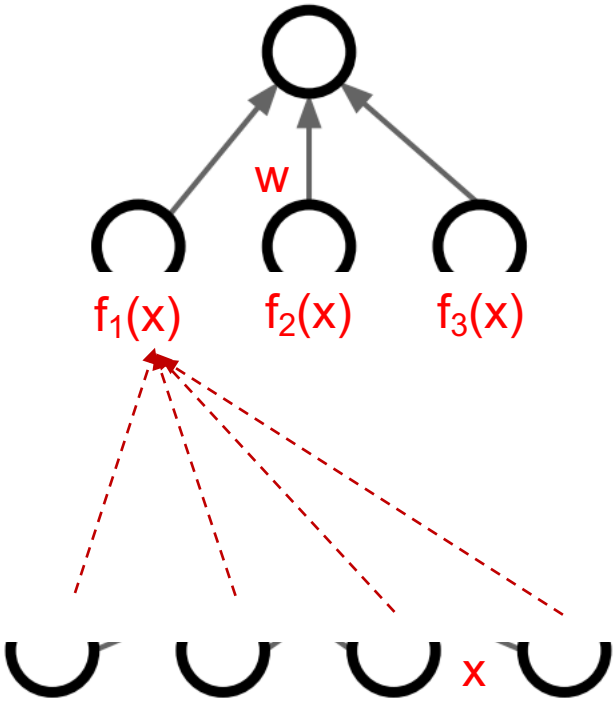
- Linear models work directly on input features x



- Nonlinear models can be seen as a linear model operating on nonlinear functions f_i of features

- instead of $h(x)=\sum_j w_j x_j$

  we have $h(x)=\sum_k w_k f_k(x)$
  - We do not use the features x directly,
  - but process them using some function f_k



Nonlinear classification

instead of $h(x)=\sum_j w_j x_j$ we can have $h(x)=\sum_k w_k f_k(x)$

- Sum of polynomial terms

  $h(x)=w_1 x_1^2+w_2 x_1 x_2+w_3 x_2^2$

  - Training = finding good w_i's
- Sum of Gaussians of the form

  $h(x)=\Sigma_i w_i \exp(-(x-m_i)^2)$

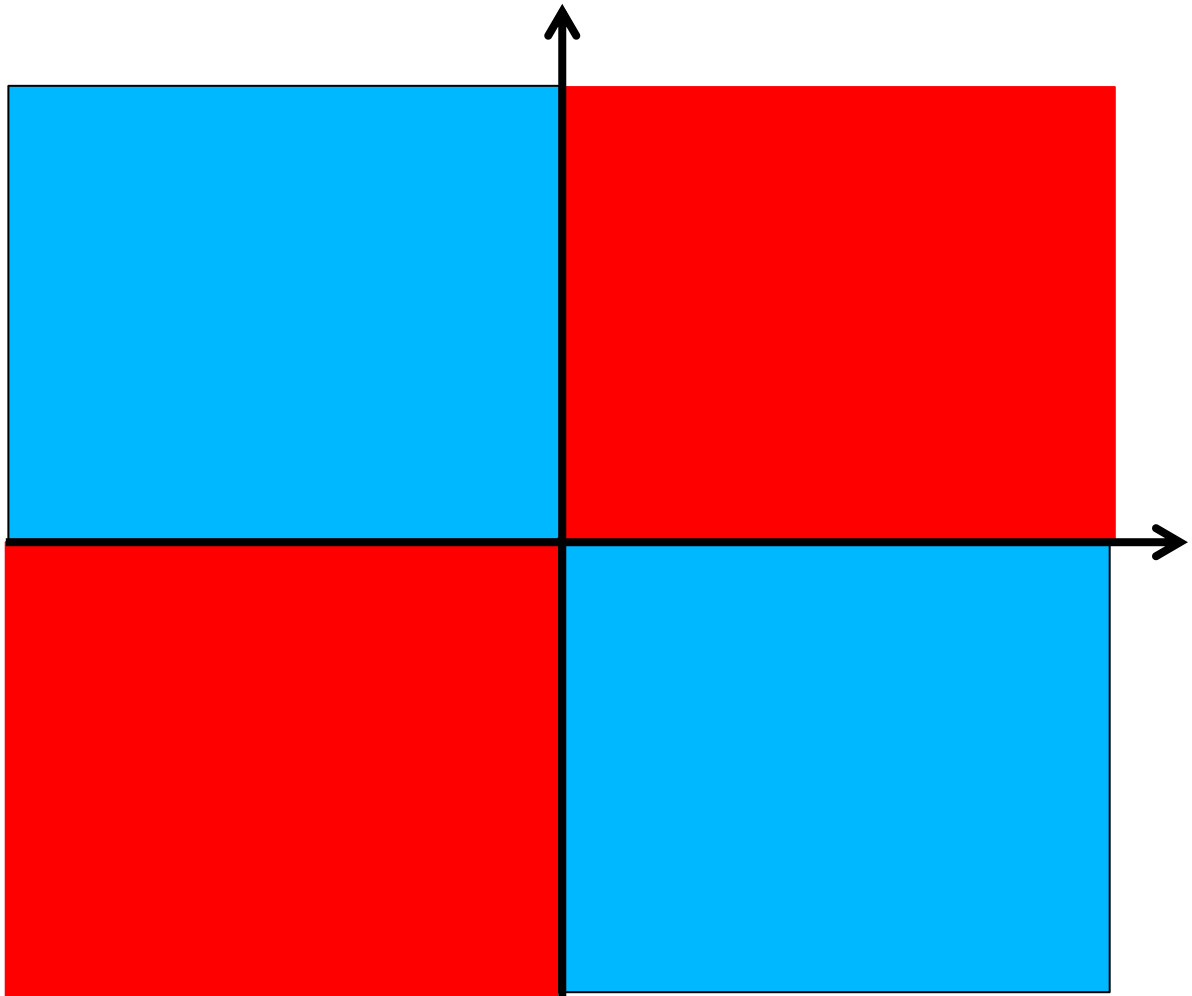
  (ignoring normalization constant, and covariance for now)

  - Training = finding good w_i's and m_i's

- We have a non-linear h(x), that will lead to nonlinear boundary h(x)=0

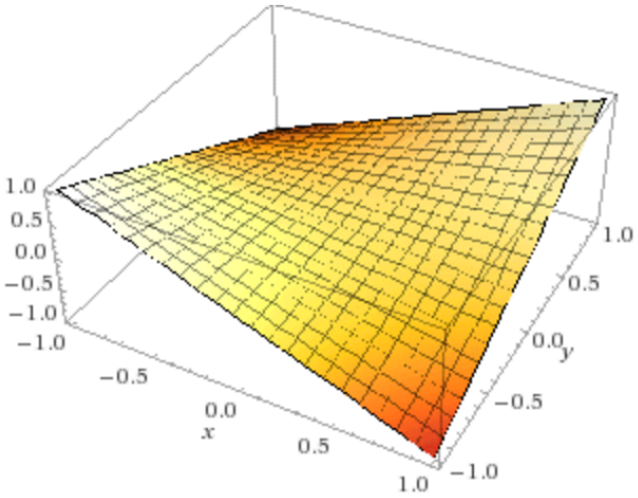
Nonlinear classification

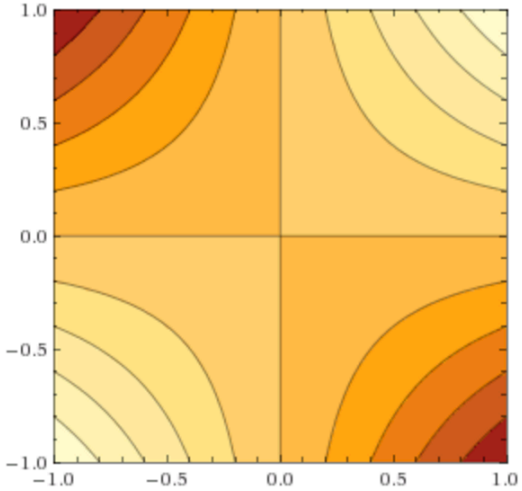
- XOR problem
  - How to solve it using polynomials?



Nonlinear classification

- XOR problem
  - $h(x)=w_1x_1^2+w_2x_1x_2+w_3x_2^2$
- $h(x)=x_1x_2$
  - Works! Positive for red, negative for blue!
  - $h(x)=x_1x_2=1/2\ (ax)^2 - 1/2(bx)^2 - 1/2(cx)^2$
  - Solution: a=[1,1] b=[1,0] c=[0,1]

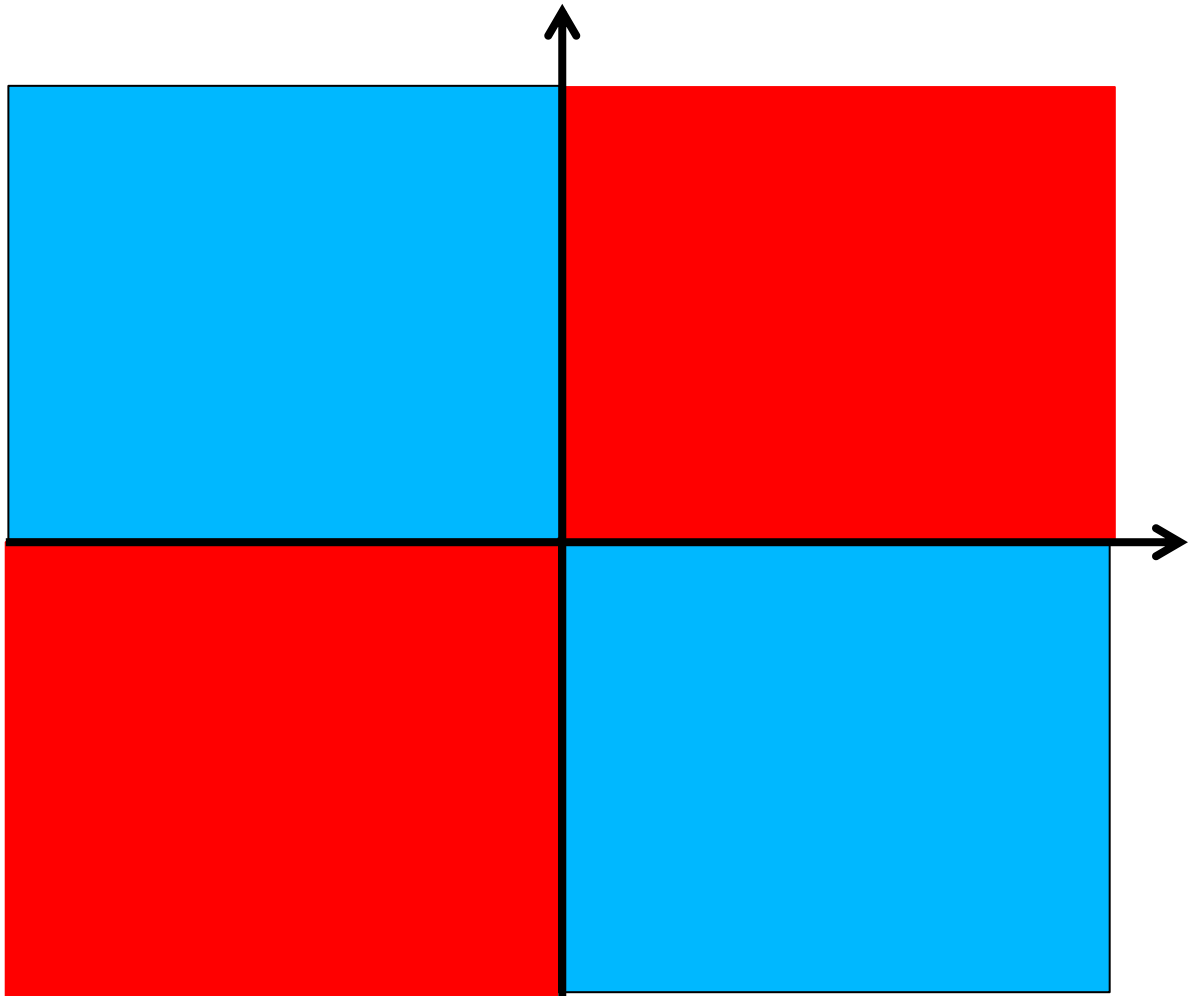




- How can computer find that solution?

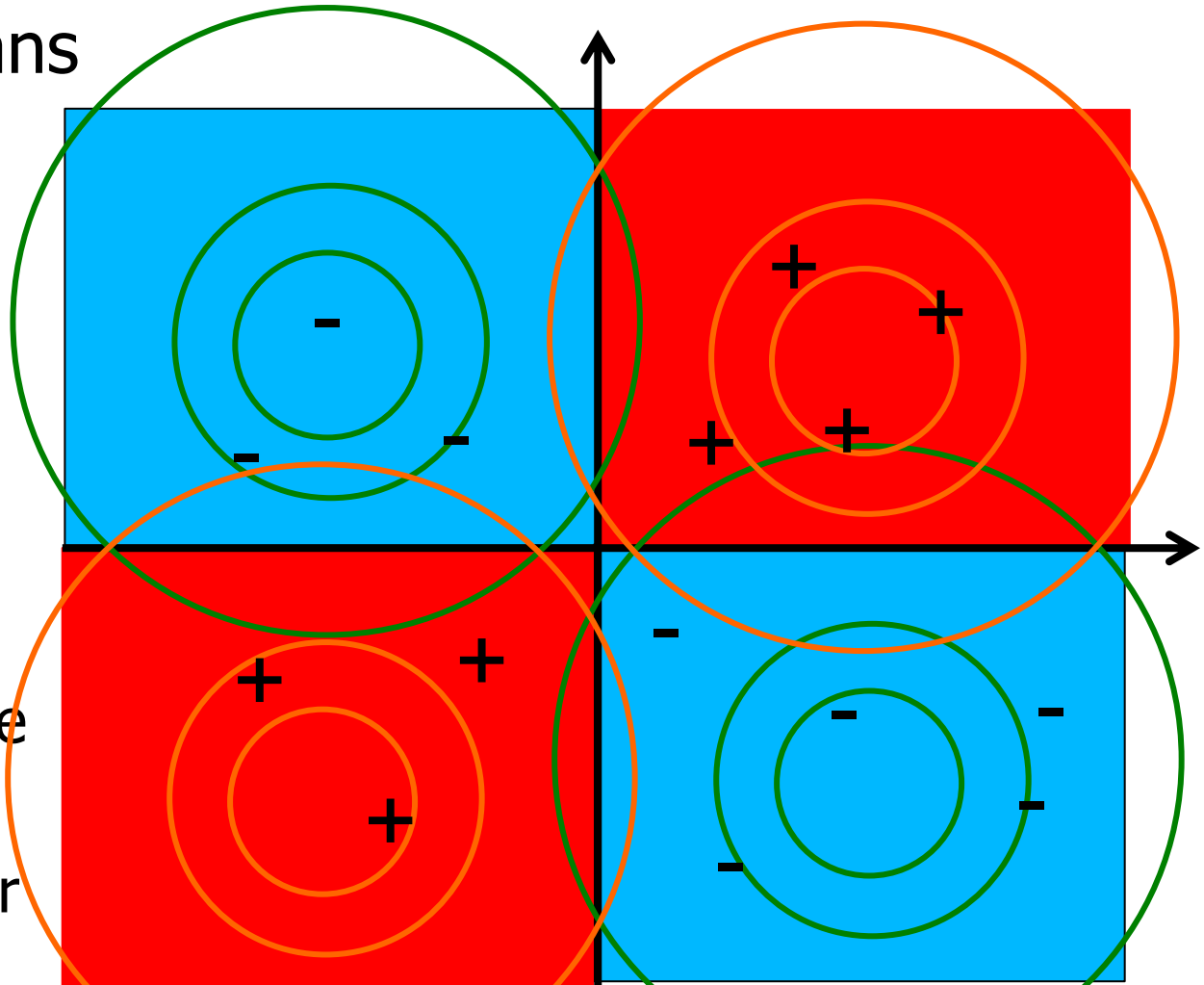
Nonlinear classification

- XOR problem
  - How to solve it using Gaussians?



Nonlinear classification

- XOR problem
  - $h(x)=\Sigma_i w_i \exp(-(x-m_i)^2)$
- Fix 4 Gaussians with means at [1,1], [-1,-1] [-1,1],[1,-1] c=+1 or -1
  - Works! Positive for red, negative for blue!
- How can computer find that solution?



Nonlinear classification

- XOR problem
  - $h(x)=\Sigma_i w_i \exp(-(x-m_i)^2$
  - Fix 4 Gaussians
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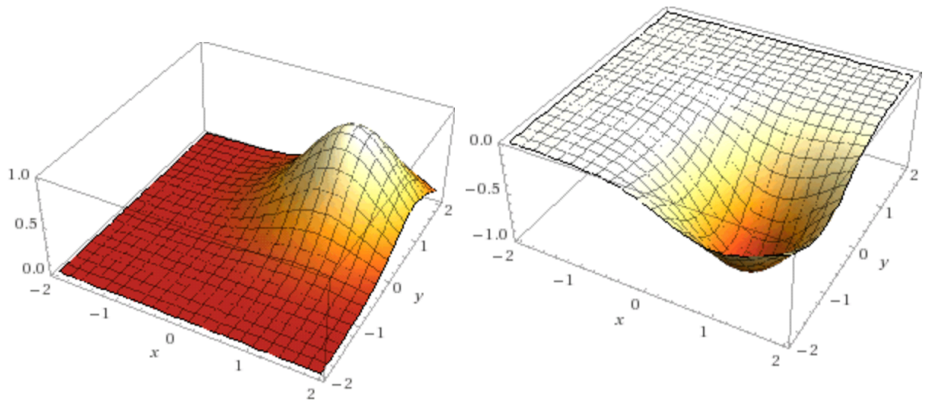
    - Works!



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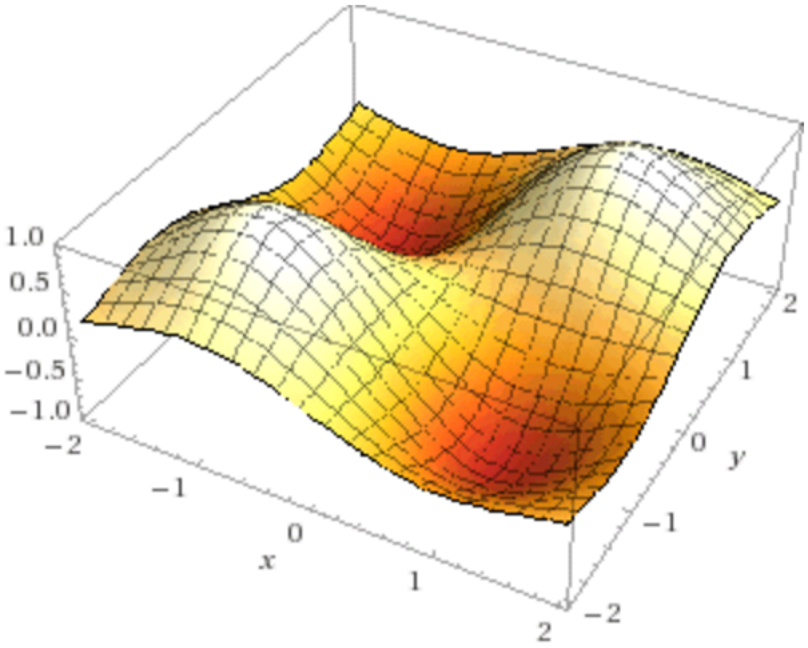
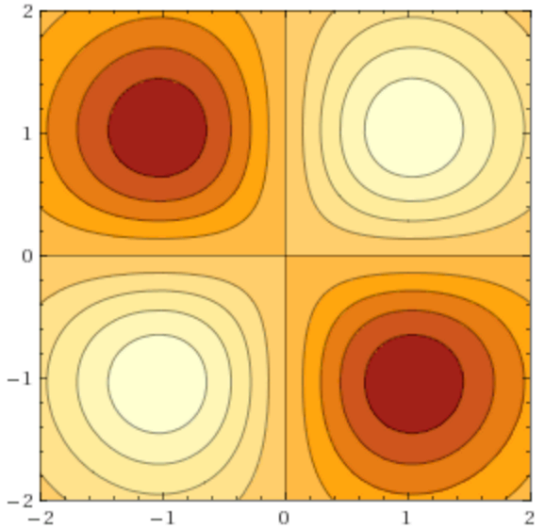
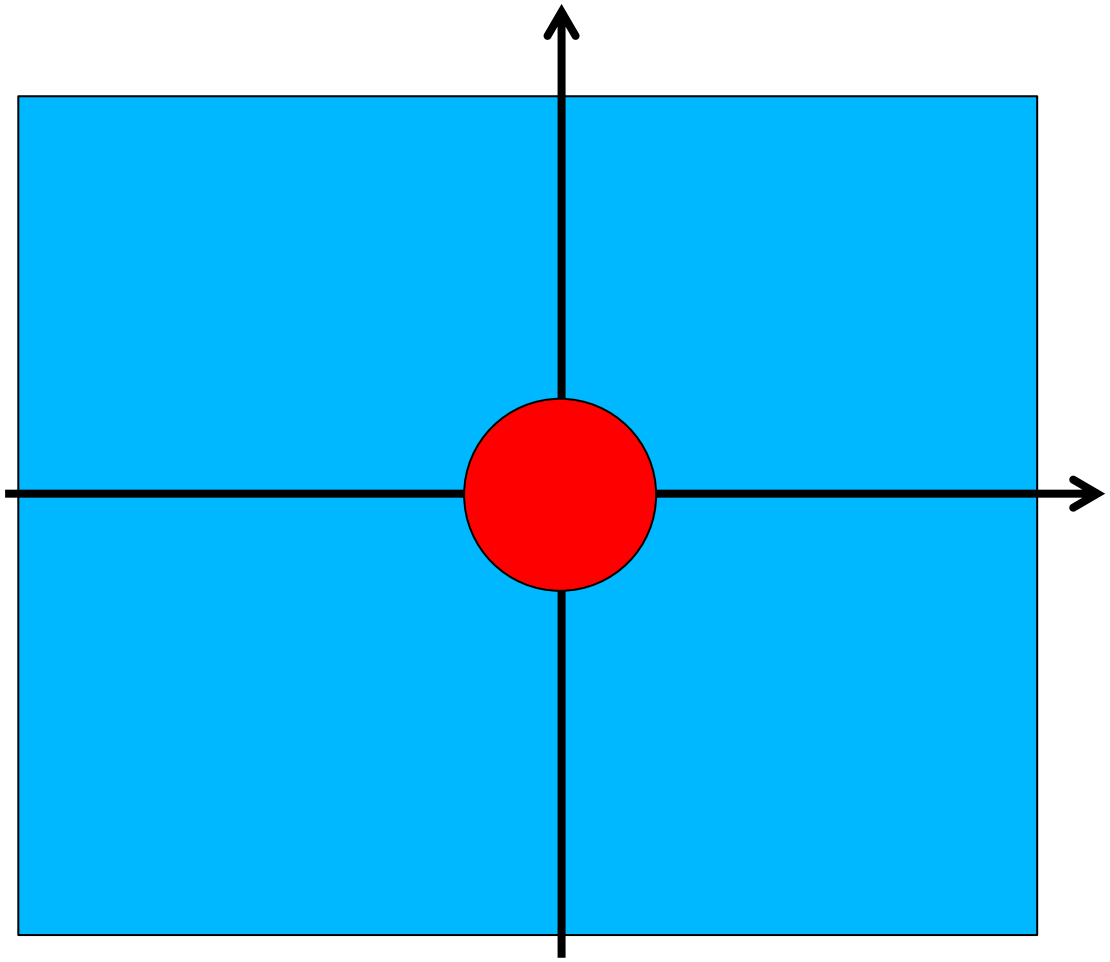




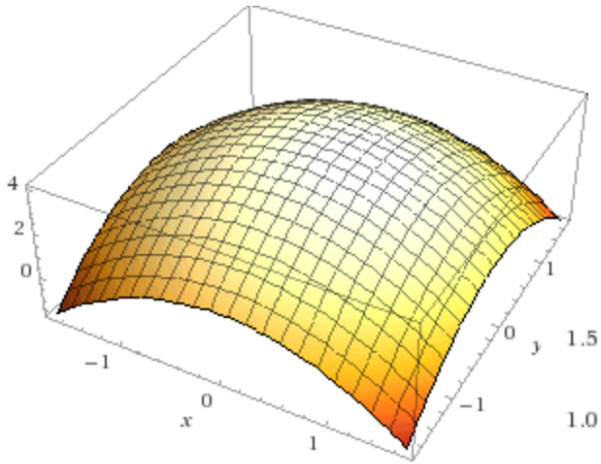
Nonlinear classification

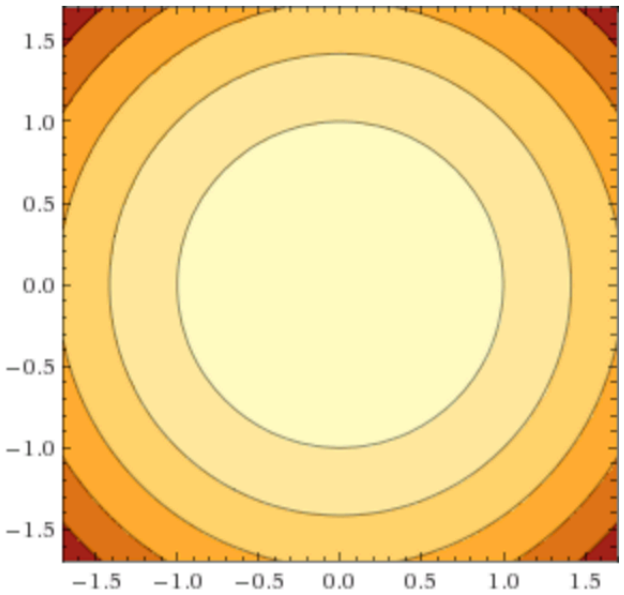
- Circle problem
  - How to solve it using polynomials?



Nonlinear classification

- Circle problem
  - $h(x)=w_1x_1^2+w_2x_1x_2+w_3x_2^2$
- $h(x) = -x_1^2 - x_2^2 + r^2$
  - $h(x)= (ax)^2 + (bx)^2$
  - a=-[1,0]
    b=-[0,1]





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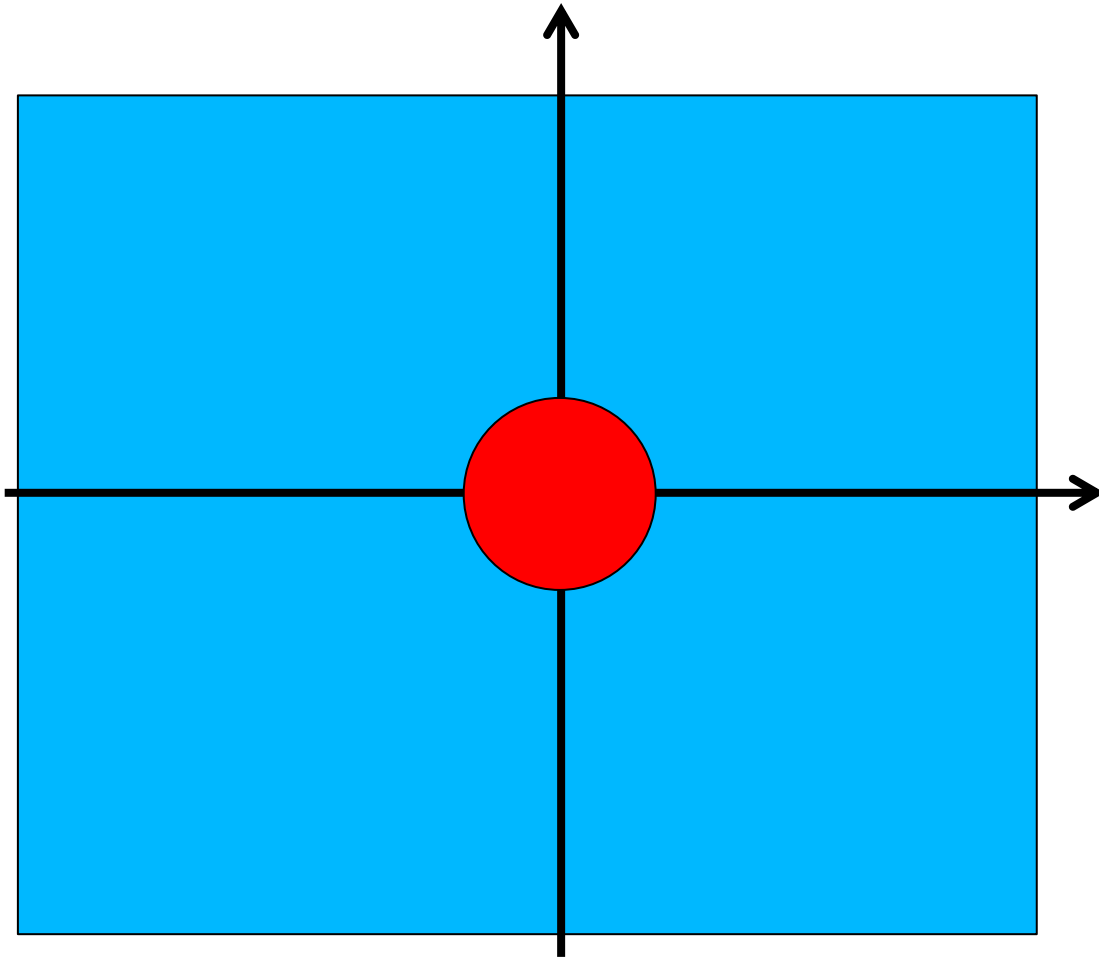
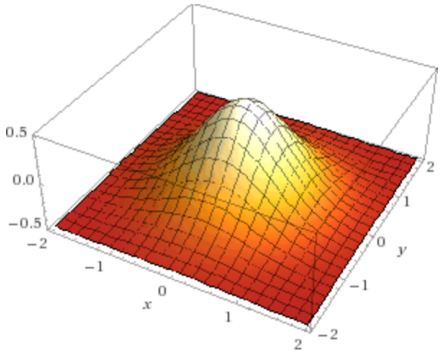
Nonlinear classification

- Circle problem
  - How to solve it using Gaussians?



Nonlinear classification

- Circle problem
  - How to solve it using Gaussians?



- Just place one Gaussian at the center of the red circle!
- If Gaussian value falls below a certain threshold, predict "-1"