Algebraic Topology - MATH0023

Based on lectures by Prof FEA Johnson
Notes taken by Imran Radzi

Notes based on the Autumn 2021 Algebraic Topology lectures by Prof FEA Johnson.

Contents

1 Simplicial complexes

Definition. A *simplicial complex* X is a pair $(V_X, \mathcal{S}_X)$ where V_X denotes the vertex set of X and $\mathcal{S}_X$ is the set of *finite, non-empty* subsetse of V_X satisfying

1. $\forall v \in V_X$, then $\{v\} \in \mathcal{S}_X$
2. If $\sigma \in \mathcal{S}_X$, $\tau \subset \sigma$, $\tau \neq \emptyset$, then $\tau \in \mathcal{S}_X$.

$\mathcal{S}_X$ is called the set of *simplices* of X.

Examples.

A standard 1-simplex, denoted by Δ^1 is simply the line segment (or usually denoted by I).

$$V_{\Delta^1} = \{0, 1\}$$

$$\mathcal{S}_{\Delta^1} = \{\{0\}, \{1\}, \{0, 1\}\}$$

A standard 2-simplex, denoted by Δ^2 is the equilateral triangle.

$$V_{\Delta^2} = \{0, 1, 2\}$$

$$\mathcal{S}_{\Delta^2} = \{\{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$$

In general, the *standard n-simplex* Δ^n, is $\Delta^n = (V_{\Delta^n}, \mathcal{S}_{\Delta^n})$ where

$$V_{\Delta^n} = \{0, 1, \ldots, n\}$$

$$\mathcal{S}_{\Delta^n} = \{\alpha : \alpha \subset \{0, \ldots, n\},\ \alpha \neq \emptyset\}$$