Technical Analysis Report – Group 21B

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1. Model design

A model was written in MATLAB to simulate the motion of the roof. Using a simple ODE (see equation 1), the system was approximated by a mass attached to a pivot by a rigid, massless rod, as shown in fig. 1.

$$\ddot{\theta}mr^2 = \sum T \quad (1)$$

By solving this equation across the motion of the system, the angular speed and acceleration of the centre of mass of the roof can be found. The most obvious limitation of this model is the use of a constant radius, which does not accurately represent the real situation of a roof opening and closing; it is clear from the motion of existing systems that the centre of mass of the system is dynamic, and should be modelled as such.

The system was initially designed in Linkage (Rector, 2022), which allowed the motion paths of each pivot to be exported as a table. This offered an accurate, if inefficient, solution to the radius problem: by calculating the centre of mass of the system at each point in its motion (based on a constant mass per length for all members), the current angle at each step in the ODE solver could be mapped to the nearest value for centre of mass radius in the table of calculated radii and positions. This method produced an accurate approximation to the true radius, limited only by the resolution of the data produced by Linkage (in this case, the output was 411 position values across 2.08 radians). This data is plotted in fig. 2.

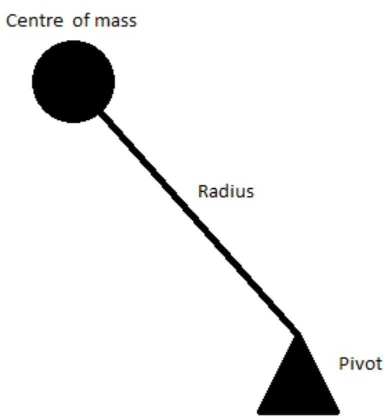

Figure 1 - basic approximation of mechanism

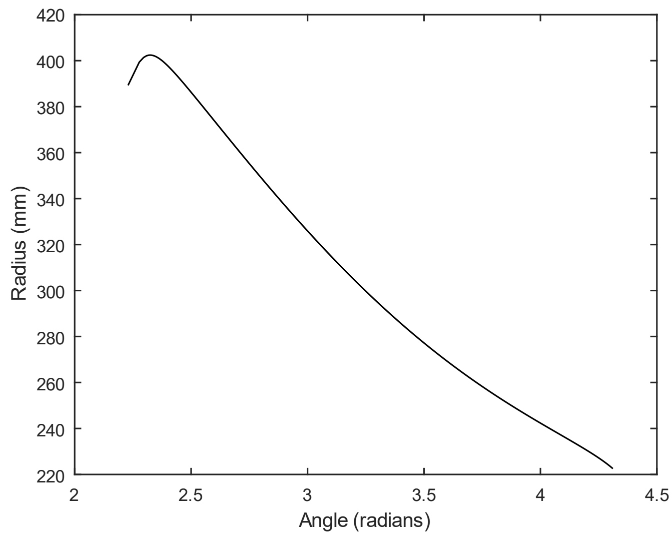

Figure 2 - plot of centre of mass radius against angle

With this in place the first torque elements could be introduced to the differential equation, namely those induced by gravity and the motor (equations 2 and 3 respectively).

$$T_g = mgrcos(\frac{\pi}{2} - \theta) \quad (2)$$

Where T_g is torque due to gravity, and θ is measured as positive clockwise from the y-axis.

$$T_m = VR(-k\dot{\theta} \times VR + dT_s) \quad (3)$$

Where T_m is motor torque, VR is velocity ratio (from gearbox), k is a constant for the motor (see equation 4), T_s is stall torque (from catalogue), and d is a direction constant (+1/-1 depending on direction of motor operation).

$$k = \frac{T_s}{\omega_{no\ load}} \qquad (4)$$

Where $\omega_{no\ load}$ is the unloaded angular speed of the motor in radians.

2. Damping & springs

With these conditions in place, and using an estimated mass of 20 kg, an initial simulation of the system's motion could be conducted. A plot of the angular position over time for three candidate velocity ratios is shown in fig. 3[1]. The key takeaway from these data was that the roof was moving far too quickly than safety would allow, and that therefore some form of damping would be necessary.

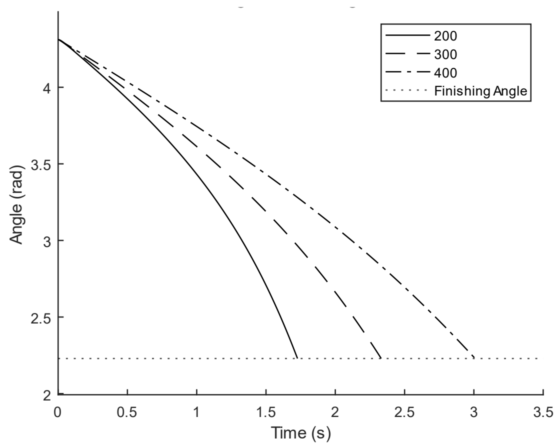

Figure 3 - deploy including only gravity and motor torques

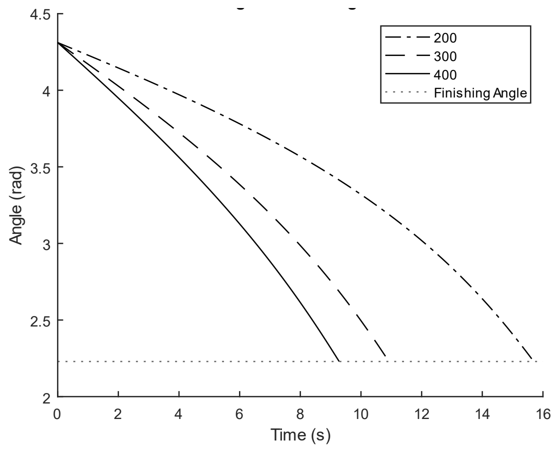

Figure 4 - deploy with damping constant c = 500

It was decided that a rotational damper would be used to reduce the speed of the roof's motion, initially simplified as shown in equation 5. For a target time of 10-15s, a value of around 500 Ns/rad was found to be appropriate for the damping constant. Fig. 4 shows the simulation from fig. 3 including the simple damping model.

$$T_d = c\dot{\theta} \qquad (5)$$

Where T_d is torque due to damping and c is damping constant.

With the system taking a more appropriate length of time to deploy, a new problem was identified. Since the mechanism is moving fastest at the end of its motion (gravity assists beyond vertical in both directions), it will impact either the top of the windshield or the chassis with considerable force. While this may not prove seriously problematic it is inexpensive to install a pair of springs, one for either end of the motion, and avoid the issue altogether. Simulating these springs is easy in principle, using equation 6, but fairly challenging in practice.

$$T_s = r_s k x \qquad (6)$$

Where r_s is the perpendicular distance from the vector of the spring's applied force to the pivot, k is the spring constant, and x is the linear displacement of the spring.

[1] All plots are of the roof deploying to aid comparison

Using this spring model, it was possible to position the spring in space on Linkage and then simulate its torque using its key characteristics (chassis attachment position, attachment position on mechanism, spring constant, extended/compressed length, and rest length). A combination of one compression and one extension spring positioned roughly as shown in fig. 5 (such that each is only in contact with the mechanism during half of its motion) was found to be effective at slowing the mechanism down at either end of its motion. An unintended consequence of this was that the mechanism was sped up considerably at the beginning of its motion, resulting in a near-constant angular velocity. Fig. 6 shows the resulting simulation.

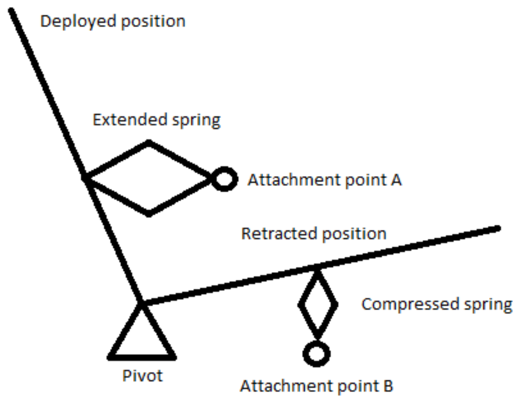

Figure 5 - spring positions

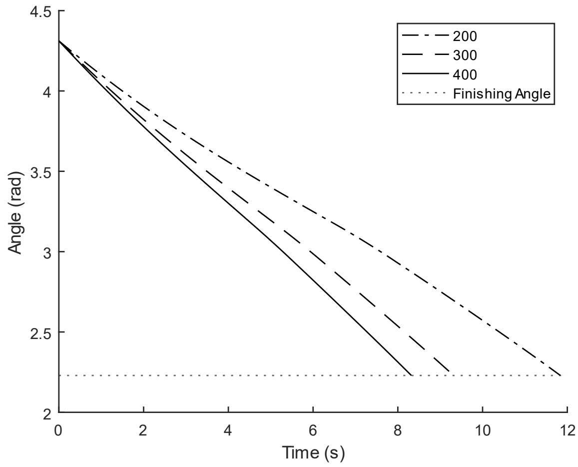

Figure 6 - deploy including spring simulation

Note that while fig. 5 uses simplified notation, extensions were calculated according to the physical motion of the attachment point on the mechanism.

3. Aerodynamic effects

The final major factor considered was the effect of wind. It is possible to conduct detailed aerodynamics simulations in MATLAB, but after testing a simplified model (equation 7) it was decided that the impact was not major enough to warrant spending an extended period solving fully. Fig. 7 shows the same deployment simulation including a headwind of 30kph (the specified maximum wind speed in our PDS).

$$T_{aero} = PAr_e \qquad (7)$$

Where T_{aero} is the torque due to aerodynamic forces, P is the pressure on the roof due to the wind (approximated as $0.5 * 1.225 * v_{wind}{}^2$), A is the exposed area of the roof, and r_e is the perpendicular distance to the vector of the resultant force applied by the wind.

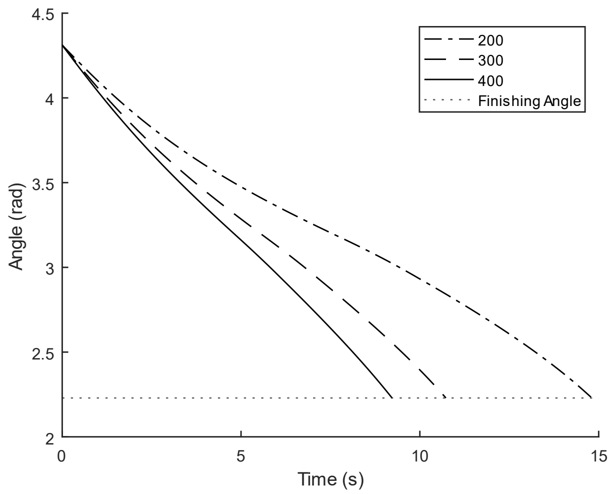

Figure 7 - plot including aerodynamics

4. Evaluation

On reflection, the biggest weaknesses in the model are its approximations to damping and aerodynamic torques; however, the improvement to the model's accuracy would not have justified the necessary additional time investment, especially as the fundamental approximation of the system to a simple mass and pivot arrangement likely introduces a much higher degree of inaccuracy. Other than these, it would have been informative to include a term considering inefficiencies within the mechanism (caused by fixings and manufacturing imperfections) had more time been available. The model provides a useful approximation to the mechanism's real behaviour, but prototyping is highly recommended before taking the design to production.

References

Rector, D., 2022. *Linkage Mechanism Designer and Simulator*. [Online]
Available at: https://blog.rectorsquid.com/linkage-mechanism-designer-and-simulator/