Outline

- Why FOL?
- Syntax and semantics of FOL
- Using FOL
- Wumpus world in FOL
- Inference in FOL

Pros and Cons of Pro. Logic

- ☺ Propositional logic is declarative
- ☺ Propositional logic allows partial/disjunctive/negated information
  - unlike most data structures and databases, which are domain-specific

- ☺ Propositional logic is compositional
  - meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$

- ☺ Meaning in propositional logic is context-independent
  - (unlike natural language, where meaning depends on context)

- ☹ Propositional logic has very limited expressive power
  - (unlike natural language)
  - E.g., cannot say "pits cause breezes in adjacent squares‟
    - except by writing one sentence for each square

Our Approach

- Adopt the foundation of propositional logic – a declarative, compositional semantics that is context-independent and unambiguous – but with more expressive power, borrowing ideas from natural language while avoiding its drawbacks

- Important elements in natural language:
  - Objects (squares, pits, wumpuses)
  - Relations among objects (is adjacent to, is bigger than) or *unary relations* or properties (is red, round)
  - Functions (father of, best friend of)
  - **First-order logic (FOL)** is built around the above 3 elements

Identify Objects, Relations, Functions

- One plus two equals three.
- Squares neighboring the wumpus are smelly.
- Evil King John ruled England in 1200.

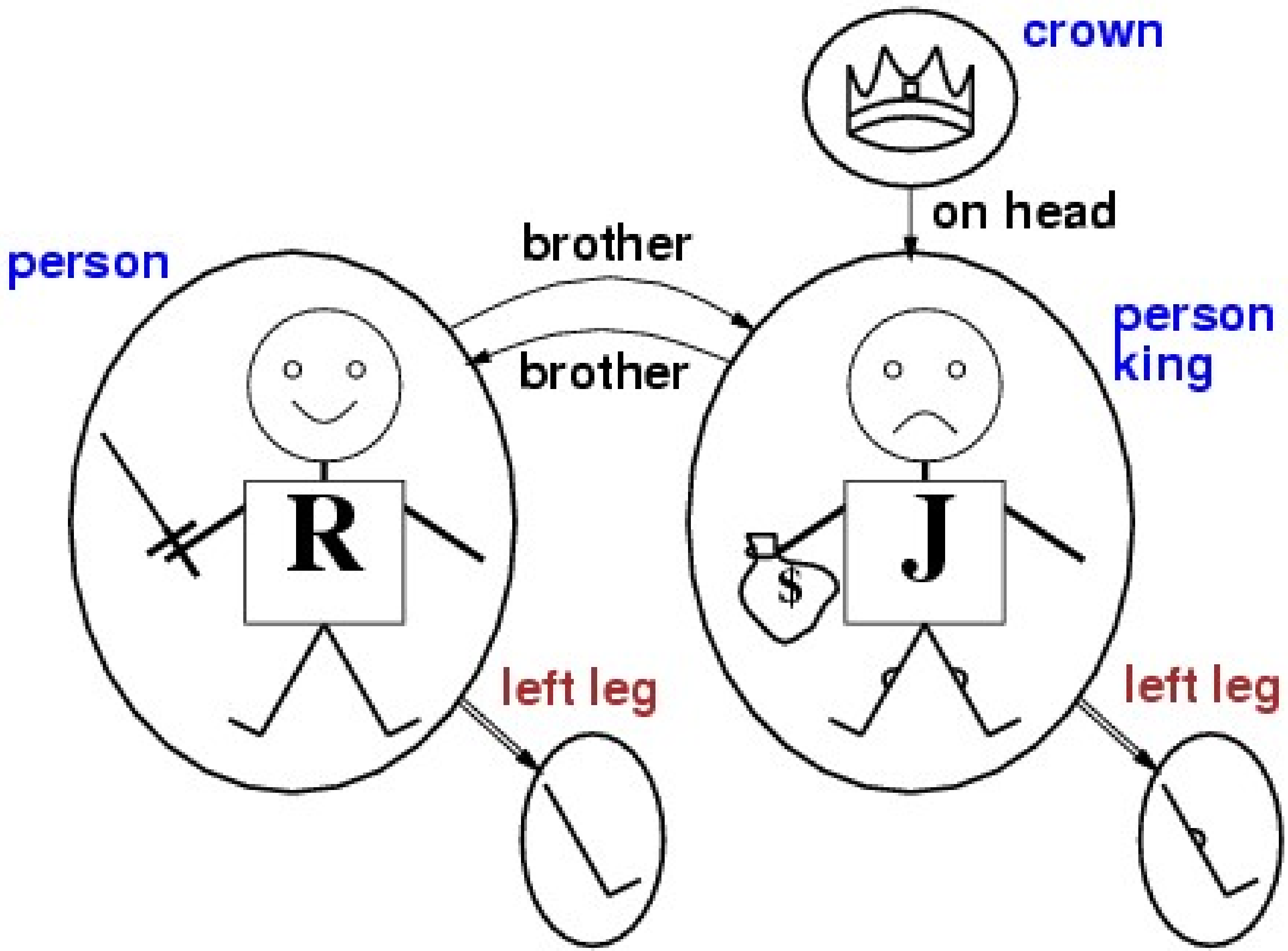
Prop. Logic vs. FOL

- Propositional logic assumes that there are facts that either hold or do not hold
- FOL assumes more: the world consists of objects with certain relations among them that do or do not hold
- Other special-purpose logics:
  - Temporal logic: facts hold at particular times / T,F,U
    - E.g., “I am always hungry”, “I will eventually be hungry”
  - Higher-order logic: views the relations and functions in FOL as objects themselves / T,F,U
  - Probability theory: facts / degree of belief [0, 1]
  - Fuzzy logic: facts with degree of truth [0, 1] / known interval values
    - E.g, “the temperature is very hot, hot, normal, cold, very cold”

First-Order Logic (FOL)

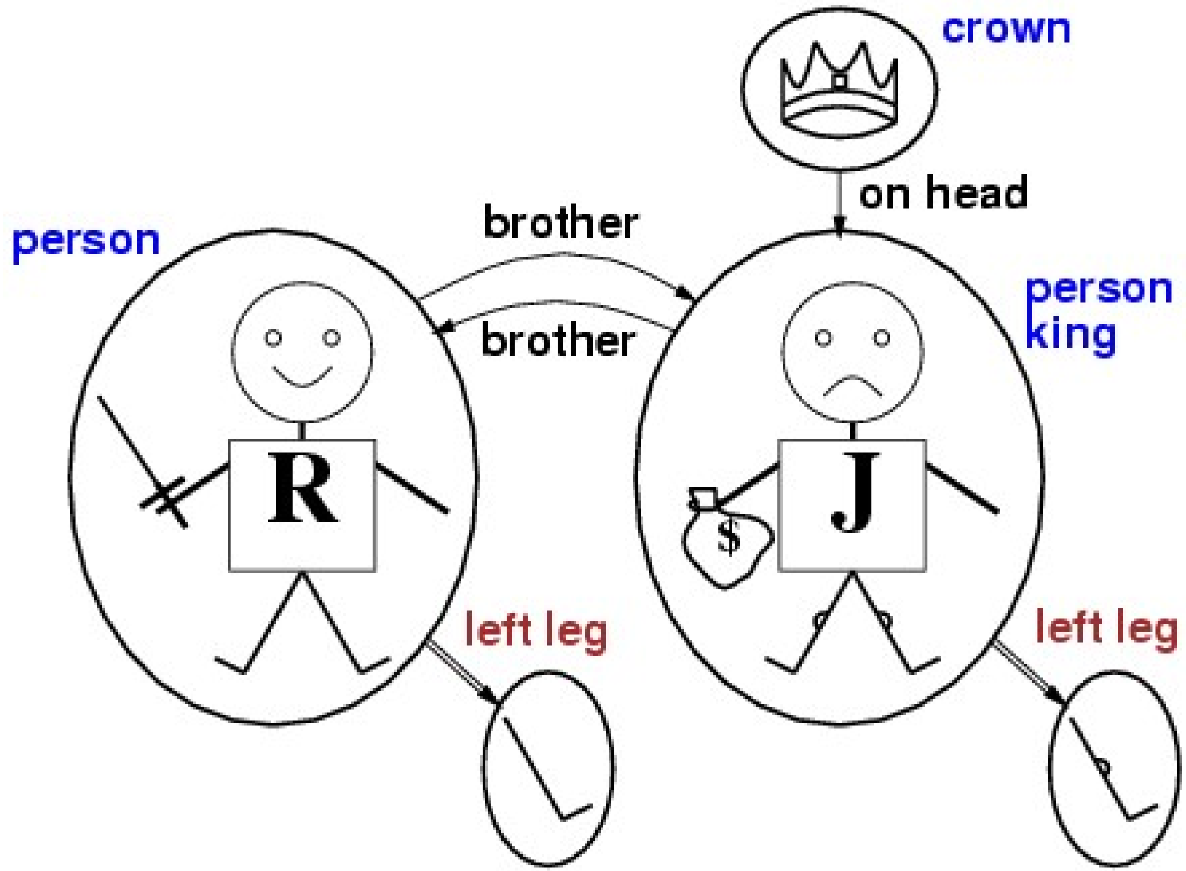
- Whereas propositional logic assumes the world contains facts
- First-order logic (like natural language) assumes the world contains
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

Models for FOL: Example



Example

- Five objects
- Two binary relations
- Three unary relations
- One unary function



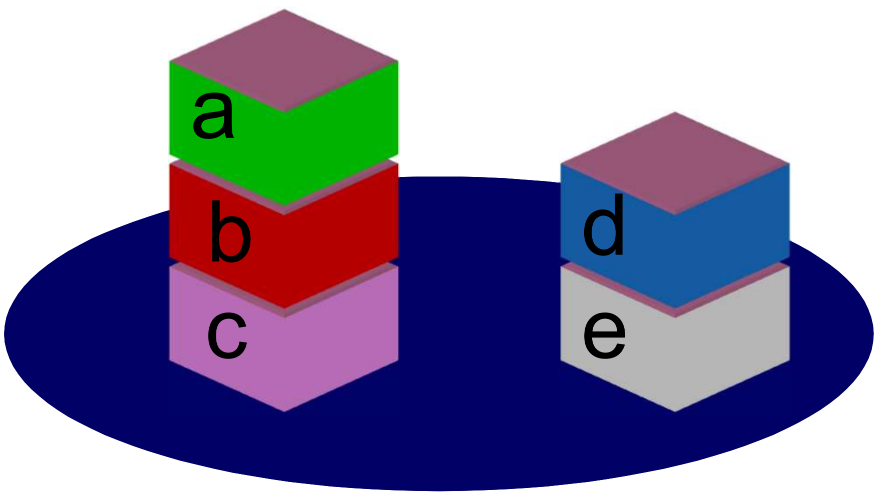
Syntax of FOL: Basic Elements

- Basic symbols: objects (constant symbols), relations (predicate symbols), and functions (functional symbols).
- Constants King John, 2, Wumpus...
- Predicates Brother, >,...
- Functions Sqrt, LeftLegOf,...
- Variables x, y, a, b,...
- Connectives $\neg, \Rightarrow, \wedge, \vee, \Leftrightarrow$
- Equality =
- Quantifiers $\forall, \exists$
- A legitimate expression of predicate calculus is called well-formed formula (wff), or simply, sentence

Relations (Predicates)

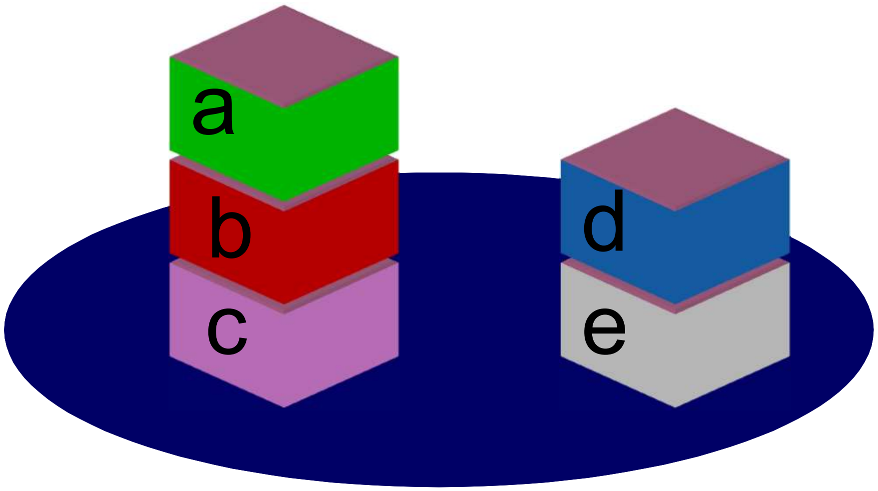
- A relation is the set of tuples of objects
  - Brotherhood relationship {<Richard, John>, <John, Richard>}
  - Unary relation, binary relation, …

- Example: set of blocks {a, b, c, d, e}
- The "On" relation includes:
  - On = {<a,b>, <b,c>, <d,e>}
  - the predicate On(A,B) can be interpreted as <a,b> ∈ On.



Functions

- In English, we use “King John’s left leg” rather than giving a name to his leg, where we use “function symbol”
- hat(c) = b
- hat(b) = a
- hat(d) is not defined



Terms and Atomic Sentences

Atomic sentence	=	*predicate* ($term_1$,...,$term_n$) or $term_1$ = $term_2$
Term	=	*function* ($term_1$,...,$term_n$) or *constant* or *variable*

- Atomic sentence states facts
- Term refers to an object
- For example:
  - *Brother*(*KingJohn*,*RichardTheLionheart*)
  - *Length*(*LeftLegOf*(*Richard*))
  - *Married*(*Father*(*Richard*), *Mother*(*John*))

Composite Sentences

- Complex sentences are made from atomic sentences using connectives
- $\neg S$, $S_1 \wedge S_2$, $S_1 \vee S_2$, $S_1 \Rightarrow S_2$, $S_1 \Leftrightarrow S_2$

For example:

$$Sibling(John, Richard) \Rightarrow Sibling(Richard, John)$$

$$\neg Brother(LeftLeg(Richard), John)$$

$$King(Richard) \vee King(John)$$

$$\neg King(Richard) \Rightarrow King(John)$$

Intended Interpretation

- The semantics relate sentences to models to determine truth
- Interpretation specifies exactly which objects, relations and functions are referred to by the constant, predicate, and function symbols
  - *Richard* refers to Richard the Lionheart and *John* refers to the evil King John
  - *Brother* refers to the brotherhood relation; *Crown* refer to the set of objects that are crowns
  - *LeftLeg* refers to the “left leg” function
  - There are many other possible interpretations that relate symbols to model; *Richard* refers to the crown
  - If there are 5 objects in the model, how many possible interpretations are there for two symbols *Richard* and *John*?

Intended Interpretation (Con't)

- The truth of any sentence is determined by a model and an interpretation for the sentence's model

- The entailment and validity are defined in terms of all possible models and all possible interpretations

- The number of domain elements in each model may be unbounded; thus the number of possible models is unbounded

- Checking entailment by enumeration of all possible models is **NOT** doable for FOL

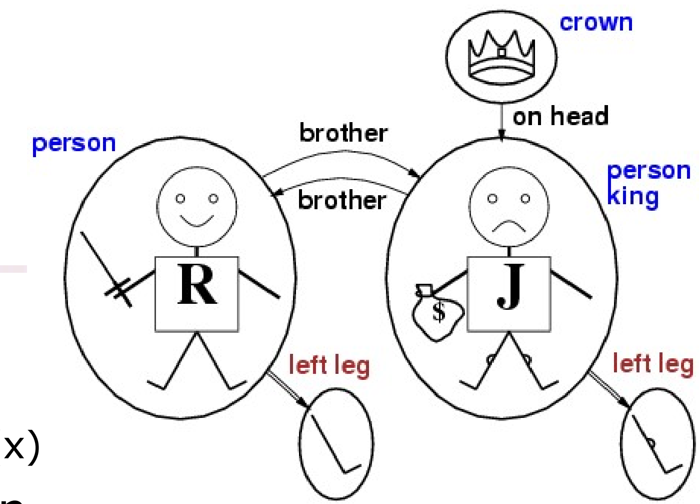
In General

- Sentences are true with respect to a model and an interpretation
- Model contains objects (domain elements) and relations among them
- Interpretation specifies referents for

constant symbols	→	objects
predicate symbols	→	relations
function symbols	→	functional relations

- Once we have a logic that allows objects, it is natural to want to express properties of entire collections of objects, instead of enumerating the objects by name → Quantifiers

Universal Quantifier



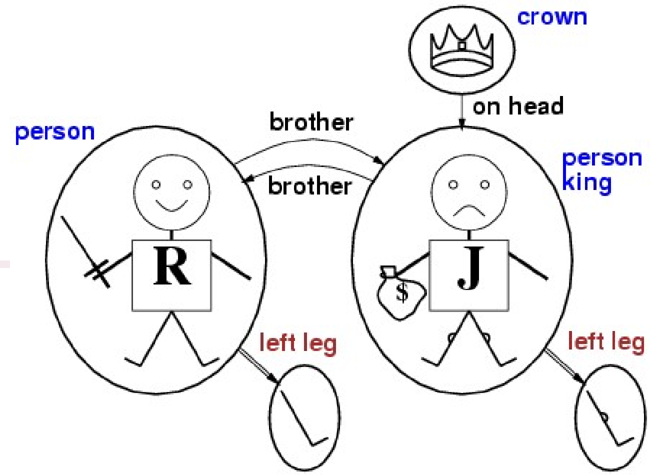
- Universal quantification ($\forall$)
  - "All kings are person": $\forall x\ King(x) \Rightarrow Person(x)$
  - For all x, if x is a king, then x is a person

- In general, $\forall x$ P is true in a given model under a given interpretation if P is true in all possible extended interpretations

- In the above example, x could be one of the following:
  - Richard, John, Richard's left leg, John's left leg, Crown
  - 5 extended interpretations

- A common mistake: $\forall x\ (King(x)\ \wedge\ Person(x))$

Existential Quantifier



- Existential quantification (∃)
  - $\exists x$ Crown(x) $\wedge$ OnHead(x, John)
  - There is an x such that x is a crown and x is on the John's head
- In general, $\exists x$ P is true in a given model under a given interpretation if P is true in at least one extended interpretation that assigns x to a domain element
- In the above example, "Crown(crown) $\wedge$ OnHead(crown, John)" is true
- Common mistake: $\exists x$ Crown(x) $\Rightarrow$ OnHead(x, John)

Nested Quantifiers

- Nested quantifiers
  - $\forall x\ \forall y\ [Brother(x, y) \Rightarrow Sibling(x, y)]$
  - $\forall x\ \exists y\ Loves(x, y)$
  - $\exists y\ \forall x\ Loves(x, y)$
  - $\exists x\ \forall y$ is not the same as $\forall y\ \exists x$
  - The order of quantification is important

- Quantifier duality: each can be expressed using the other
  - $\forall x\ Likes(x, IceCream)\quad \neg\exists x\ \neg Likes(x, IceCream)$
  - $\exists x\ Likes(x, Broccoli)\quad \neg\forall x\ \neg Likes(x, Broccoli)$

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object
  - Can be used to state facts about a given function
    - E.g., Father(John) = Henry
  - Can be used with negation to insist that two terms are not the same object
  - E.g., definition of *Sibling* in terms of *Parent*:
  - $\forall x,y\ Sibling(x,y) \Leftrightarrow [\neg(x = y) \wedge \exists m,f\ \neg (m = f) \wedge Parent(m,x) \wedge Parent(f,x) \wedge Parent(m,y) \wedge Parent(f,y)]$

Using FOL

- Sentences are added to a KB using TELL, which is called assertions
- Questions asked using ASK are called queries
- Any query that is logically entailed by the KB should be answered affirmatively
- The standard form for an answer is a substitution or binding list, which is a set of variable/term pairs

$$TELL(KB,\ King(John))$$

$$TELL(KB,\ \forall x\ King(x) \Rightarrow Person(x))$$

$$ASK(KB,\ King(John))$$

$$ASK(KB,\ Person(John))$$

$$ASK(KB,\ \exists x\ Person(x)) \qquad answer: \{x / John\}$$

Example: The Kinship Domain

- An example KB includes things like:
  - Fact:
    - “Elizabeth is the mother of Charles”
    - “Charles is the father of William”
  - Rules:
    - One’s grandmother is the mother of one’s parent”
- <u>Object</u>: people
- <u>Unary predicate</u>: Male, Female
- <u>Binary predicate</u>: Son, Spouse, Wife, Husband, Grandparent, Grandchild, Cousin, Aunt, and Uncle
- <u>Function</u>: Mother, Father

Example: The Kinship Domain

$$One's\ mom\ is\ one's\ female\ parent.$$

$$\forall m,c\ Mother(c)=m \Leftrightarrow Female(m) \wedge Parent(m,c)$$

$$One's\ husband\ is\ one's\ male\ spouse.$$

$$\forall w,h\ Husband(h,w) \Leftrightarrow Male(h) \wedge Spouse(h,w)$$

$$Paren\ and\ child\ are\ inverse\ relations.$$

$$\forall p,c\ Parent(p,c) \Leftrightarrow Child(c,p)$$

$$A\ grandparent\ is\ a\ parent\ of\ one's\ parent.$$

$$\forall g,c\ Grandparent(g,c) \Leftrightarrow \exists p\ Parent(g,p) \wedge Parent(p,c)$$

Practice:

Male and female are disjoint categories

A sibling is another child of one’s parents

Answer

- Male and female are disjoint categories:
  - $\forall x\ Male(x) \Leftrightarrow \neg Female(x)$

- A sibling is another child of one's parent:
  - $\forall x,y\ Sibling(x, y) \Leftrightarrow x \neq y \wedge \exists p\ Parent(p,x) \wedge Parent(p, y)$

The Wumpus World

- A percept is a binary predicate:
  - Percept([Stench, Breeze, Glitter, None, None], 5)
- Actions are logical terms:
  - Turn(Right), Turn(Left), Forward, Shoot, Grab, Release, Climb
- Query:
  - $\exists$a BestAction(a, 5)
  - Answer: {a/Grab}
- Perception: percept implies facts:
  - $\forall$ t, s, b, m, c Percept([s, b, Glitter, m, c], t) $\Rightarrow$ Glitter(t)
- Reflex behavior:
  - $\forall$ t Glitter(t) $\Rightarrow$ BestAction(Grab, t)

The Wumpus World

- The Environment:
  - Objects: squares, pits and the wumpus
  - $\forall x,y,a,b$ *Adjacent*([x,y],[a,b]) $\Leftrightarrow$

  $[a,b] \in \{[x+1,y], [x-1,y],[x,y+1],[x,y-1]\}$
  - A unary predicate *Pit*(x)
  - $\forall s,t$ *At*(Agent,s,t) $\wedge$ *Breeze*(t) $\Rightarrow$ *Breezy*(s), the agent infers properties of the square from properties of its current percept
  - Breezy() has no time argument
  - Having discovered which places are breezy or smelly, not breezy or not smelly, the agent can deduce where the pits are and where the wumpus is

Diagnostic Rules

- Diagnostic rule: lead from observed effects to hidden causes
  - If the square is breezy then adjacent square(s) must contain a pit

    $\forall s \; Breezy(s) \Rightarrow \exists r \; Adjacent(r,s) \wedge Pit(r)$
  - If the square is not breezy, no adjacent square contains a pit

    $\forall s \; \neg Breezy(s) \Rightarrow \neg \; (\exists r \; Adjacent(r,s) \wedge Pit(r))$
  - Combining both:

    $\forall s \; Breezy(s) \Leftrightarrow \exists r \; Adjacent(r,s) \wedge Pit(r)$

Causal Rules

- Causal rule: some hidden property of the world causes certain percept to be generated
  - A pit causes all adjacent squares to be breezy

    $\forall r\ Pit(r) \Rightarrow [\forall s\ Adjacent(r,s) \Rightarrow Breezy(s)]$
  - If all squares adjacent to a given square are pitless, the square will not be breezy

    $\forall s\ [\forall r\ [Adjacent(r,s) \Rightarrow \neg Pit(r)] \Rightarrow \neg Breezy(s)$

Summary

- First-order logic:
  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, equality, quantifiers
- Increased expressive power: sufficient to define wumpus world

Exercises

- Represent the following sentences in FOL
  - Some students took French in spring 2001.
  - Every student who takes French passes it.
  - Only one student took Greek in spring 2001.
  - The best score in Greek is always higher than the best score in French.
- Let the basic vocabulary be as follows:

$Student(x)$

$Takes(x, c, s)$: student x takes course c in semester s;

$Passes(x, c, s)$: student x passes course c in semester s;

$Score(x, c, s)$: the score obtained by student x in course c in semester s;

$x > y$: x is greater than y;

F and G: specific French and Greek courses (one could also interpret these sentences as referring to *any* such course, in which case one could use a predicate $Subject(c, f)$ meaning that the subject of course c is field f;

Inference in FOL

- Two ideas:
  - convert the KB to propositional logic and use propositional inference
  - a shortcut that manipulates on first-order sentences directly (resolution, will not be introduced here)

Universal Instantiation

- Universal instantiation
  - infer any sentence by substituting a ground term (a term without variables) for the variable
  - $$\frac{\forall v\ \alpha}{\text{Subst}(\{v/g\},\ \alpha)}$$
  - for any variable v and ground term g
- Examples
  - $\forall x$ *King*(x) $\wedge$ *Greedy*(x) $\Rightarrow$ *Evil*(x) yields:
  - *King*(*John*) $\wedge$ *Greedy*(*John*) $\Rightarrow$ *Evil*(*John*)
  - *King*(*Richard*) $\wedge$ *Greedy*(*Richard*) $\Rightarrow$ *Evil*(*Richard*)
  - *King*(*Father*(*John*)) $\wedge$ *Greedy*(*Father*(*John*)) $\Rightarrow$ *Evil*(*Father*(*John*))

Existential Instantiation

- Existential instantiation
  - For any sentence α, variable v, and constant symbol k that does NOT appear elsewhere in the knowledge base, replace v with k

$$\frac{\exists v\ \alpha}{\text{Subst}(\{v/k\}, \alpha)}$$

- E.g., $\exists x\ Crown(x) \land OnHead(x, John)$ yields:

$$Crown(C_1) \land OnHead(C_1, John)$$

provided C_1 is a new constant symbol, called a Skolem constant

Exercise

- Suppose a knowledge base contains just one sentence, $\exists x$ AsHighAs(x, Everest). Which of the following are legitimate results of applying Existential Instantiation?
  - AsHighAs(Everest, Everest)
  - AsHighAs(Kilimanjaro, Everest)
  - AsHighAs(Kilimajaro, Everest) and AsHighAs(BenNevis, Everest)

Reduction to Propositional Inference

Suppose the KB contains just the following:

$\forall x\ King(x) \wedge Greedy(x) \Rightarrow Evil(x)$
King(John)
Greedy(John)
Brother(Richard,John)

- Instantiating the universal sentence in all possible ways, we have:
  King(John) $\wedge$ *Greedy(John)* $\Rightarrow$ *Evil(John)*
  King(Richard) $\wedge$ *Greedy(Richard)* $\Rightarrow$ *Evil(Richard)*
  King(John)
  Greedy(John)
  Brother(Richard,John)

- The new KB is propositionalized: proposition symbols are
- *King(John), Greedy(John), Evil(John), King(Richard)*, etc.

- What conclusion can you get?

Reduction Cont'd.

- Every FOL KB can be propositionalized so as to preserve entailment

- (A ground sentence is entailed by new KB iff entailed by original KB)

- Idea: propositionalize KB and query, apply resolution, return result

Problems with Propositionalization

- Propositionalization seems to generate lots of irrelevant sentences

- E.g., from:
  $\forall x$ *King*(x) $\wedge$ *Greedy*(x) $\Rightarrow$ *Evil*(x)
  King(John)
  $\forall y$ *Greedy*(y)
  Brother(Richard,John)

- It seems obvious that *Evil*(*John*) is true, but propositionalization produces lots of facts such as *Greedy*(*Richard*) that are irrelevant

- With p k-ary predicates and n constants, there are $p \cdot n^k$ instantiations

Problems with Propositionalization

- When the KB includes a function symbol, the set of possible ground term substitutions is infinite
  - Father(Father(…(John)…))
- Theorem:
  - If a sentence is entailed by the original, first-order KB, then there is a proof involving just a finite subset of the propositionalized KB
  - First instantiation with constant symbols
  - Then terms with depth 1 (Father(John))
  - Then terms with depth 2 …
  - Until the sentence is entailed

Unification and Lifting

Propositionalization approach is rather inefficient

A First-Order Inference Rule

- If there is some substitution θ that makes the premise of the implication identical to sentences already in the KB, then we assert the conclusion of the implication, after applying θ

$\forall x$ *King*(x) $\wedge$ *Greedy*(x) $\Rightarrow$ *Evil*(x)

King(John)

Greedy(John)

What's θ here?

- Sometimes, we need to do is find a substitution θ both for the variables in the implication and in the sentence to be matched

$\forall x$ *King*(x) $\wedge$ *Greedy*(x) $\Rightarrow$ *Evil*(x)

King(John)

$\forall y$ *Greedy*(y)

Brother(Richard,John)

What's θ here?

Generalized Modus Ponens

- For atomic sentences p_i, p_i', and q, where there is a substitution θ such that Subst(θ, p_i') = Subst(θ, p_i), for all i:

$$\frac{p_1', p_2', \ldots, p_n', (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{\text{Subst}(\theta, q)}$$

p_1' is *King*(*John*) p_1 is *King*(*x*)

p_2' is *Greedy*(*y*) p_2 is *Greedy*(*x*)

θ is {x/John,y/John} q is *Evil*(*x*)

Subst(θ, q) is *Evil*(*John*)

Can be used with KB of definite clauses (exactly one positive literal)

Unification

- GMP is a lifted version of Modus Ponens – raises Modus Ponens from propositional to first-order logic
- What's the key advantage of GMP over propositionalization?
- Lifted inference rules require finding substitutions that make different logical expressions look identical
- This process is called **unification** and is a key component of all first-order inference algorithms

Unification

- The unify algorithm takes two sentences and returns a unifier for them if one exists:
  - UNIFY(p, q) = θ where Subst(θ, p) = Subst(θ, q)

p	q	θ
Knows(John,x)	Knows(John,Jane)	
Knows(John,x)	Knows(y,OJ)	
Knows(John,x)	Knows(y,Mother(y))	
Knows(John,x)	Knows(x,OJ)	

Unification

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  - UNIFY(p, q) = θ where Subst(θ, p) = Subst(θ, q)

p	q	θ
Knows(John,x)	Knows(John,Jane)	{x/Jane}
Knows(John,x)	Knows(y,OJ)	
Knows(John,x)	Knows(y,Mother(y))	
Knows(John,x)	Knows(x,OJ)	

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p	q	θ
Knows(John,x)	Knows(John,Jane)	{x/Jane}
Knows(John,x)	Knows(y,OJ)	{x/OJ,y/John}
Knows(John,x)	Knows(y,Mother(y))	
Knows(John,x)	Knows(x,OJ)	

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Knows(John,x)	Knows(John,Jane)	{x/Jane}
Knows(John,x)	Knows(y,OJ)	{x/OJ,y/John}
Knows(John,x)	Knows(y,Mother(y))	{y/John,x/Mother(John)}
Knows(John,x)	Knows(x,OJ)	

Unification

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p	q	θ
Knows(John,x)	Knows(John,Jane)	{x/Jane}
Knows(John,x)	Knows(y,OJ)	{x/OJ,y/John}
Knows(John,x)	Knows(y,Mother(y))	{y/John,x/Mother(John)}
Knows(John,x)	Knows(x,OJ)	{fail}

Standardizing apart: renaming variables to avoid name clashes

UNIFY(Knows(John, x), Knows(z, OJ)) = {x/OJ, z/John}

Unification Contd.

- To unify *Knows(John, x)* and *Knows(y, z)*,
  $\theta = \{ y/\text{John}, x/z \}$ or $\theta = \{ y/\text{John}, x/\text{John}, z/\text{John}\}$

- The first unifier is more general than the second
- There is a single most general unifier (MGU)
- MGU = $\{y/\text{John}, x/z\}$

Exercise

- For each pair of atomic sentences, give the most general unifier if it exists:
  - P(A, B, B), P(x, y, z)
  - Q(y, G(A, B)), Q(G(x, x), y)
  - Older(Father(y), y), Older(Father(x), John)
  - Knows(Father(y), y), Knows(x, x)

Example Knowledge Base

- The law says that it is a crime for an *American* to sell weapons to hostile nations. The country *Nono*, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel *West*, who is American.
- Prove that Colonel West is a criminal
  - American(x): x is an American
  - Weapon(x): x is a weapon
  - Hostile(x): x is a hostile nation
  - Criminal(x): x is a criminal
  - Missile(x): x is a missile
  - Owns(x, y): x owns y
  - Sells(x, y, z): x sells y to z
  - Enemy(x, y): x is an enemy of y
  - Constants: America, Nono, West

Example Knowledge Base

First-Order Definite Clauses

... it is a crime for an American to sell weapons to hostile nations:

American(x) $\wedge$ *Weapon(y)* $\wedge$ *Sells(x,y,z)* $\wedge$ *Hostile(z)* $\Rightarrow$ *Criminal(x)*

Nono … has some missiles, i.e., $\exists x$ Owns(Nono,x) $\wedge$ Missile(x):

Owns(Nono,M_1*) and Missile(*M_1*)*

… all of its missiles were sold to it by Colonel West

Missile(x) $\wedge$ *Owns(Nono, x)* $\Rightarrow$ *Sells(West, x,Nono)*

Missiles are weapons:

Missile(x) $\Rightarrow$ *Weapon(x)*

An enemy of America counts as "hostile":

Enemy(x, America) $\Rightarrow$ *Hostile(x)*

West, who is American …

American(West)

The country Nono, an enemy of America …

Enemy(Nono,America)

Forward Chaining

- Starting from known facts, it triggers all the rules whose premises are satisfied, adding their conclusions to the known facts. This process will continue until query is answered.

- When a new fact P is added to the KB:
  - For each rule such that P unifies with a premise
    - if the other premises are already known
    - then add the conclusion to the KB and continue chaining
- Forward chaining is data-driven:
  - inferring properties and categories from percepts

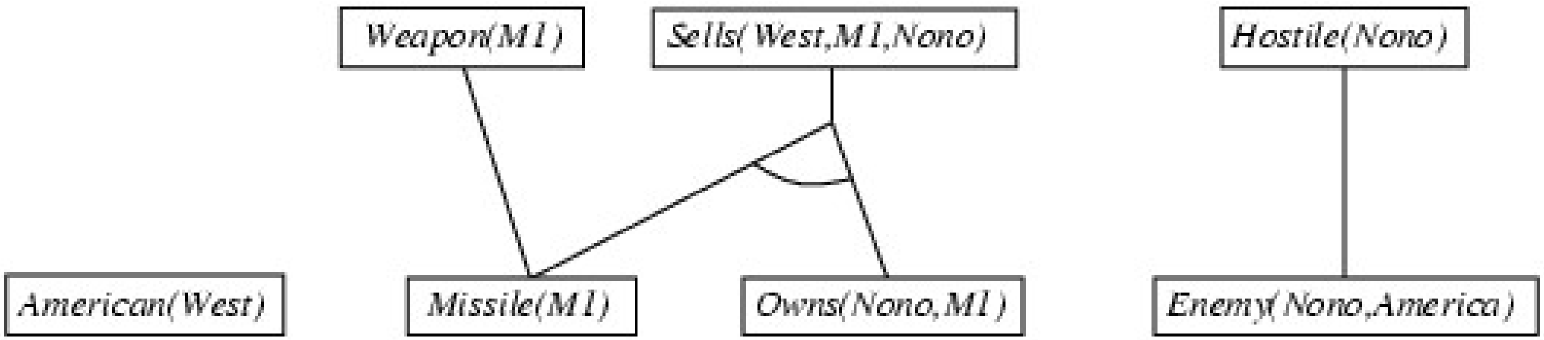
Forward Chaining Algorithm

function FOL-FC-ASK(KB, α) **returns** a substitution or *false*

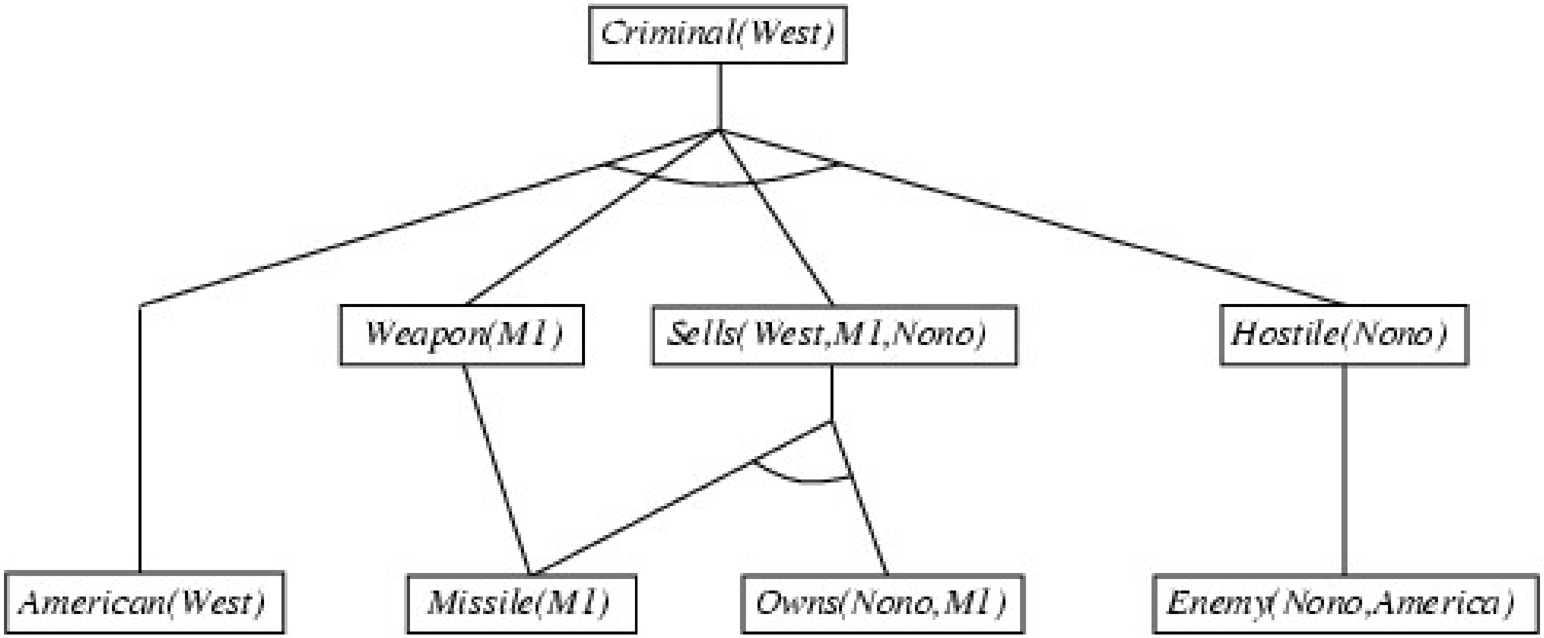
repeat until *new* is empty
    $new \leftarrow \{\}$
    for each sentence r **in** KB **do**
        $(p_1 \wedge \ldots \wedge p_n \Rightarrow q) \leftarrow$ STANDARDIZE-APART(r)
        for each θ such that $(p_1 \wedge \ldots \wedge p_n)\theta = (p'_1 \wedge \ldots \wedge p'_n)\theta$
                for some $p'_1, \ldots, p'_n$ in KB
            $q' \leftarrow$ SUBST(θ, q)
            if q' is not a renaming of a sentence already in KB or *new* **then do**
                add q' to *new*
                $\phi \leftarrow$ UNIFY(q', α)
                if ϕ is not *fail* **then return** ϕ
    add *new* to KB
return *false*

Forward Chaining Proof

Forward Chaining Proof



Forward Chaining Proof



Analysis of Forward Chaining

- A fixed point of the inference process can be reached
- The algorithm is sound and complete
- Its efficiency can be improved

FC Algorithm Analysis

```
function FOL-FC-ASK(KB, α) returns a substitution or false

  repeat until new is empty
      new ← { }
      for each sentence r in KB do
          ( p_1 ∧ ... ∧ p_n ⇒ q) ← STANDARDIZE-APART(r)
          for each θ such that (p_1 ∧ ... ∧ p_n)θ = (p'_1 ∧ ... ∧ p'_n)θ
                      for some p'_1, ..., p'_n in KB
              q' ← SUBST(θ, q)
              if q' is not a renaming of a sentence already in KB or new then do
                  add q' to new
                  φ ← UNIFY(q', α)
                  if φ is not fail then return φ
      add new to KB
  return false
```

Sources of Complexity

- 1. “Inner loop” involves finding all possible unifiers such that the premise of a rule unifies with facts in the KB
  - Often called “pattern matching”, very expensive
- 2. The algorithm rechecks every rule on every iteration to see whether its premises are met
- 3. It might generate many facts that are irrelevant to the goal

Matching Rules Against Known Facts

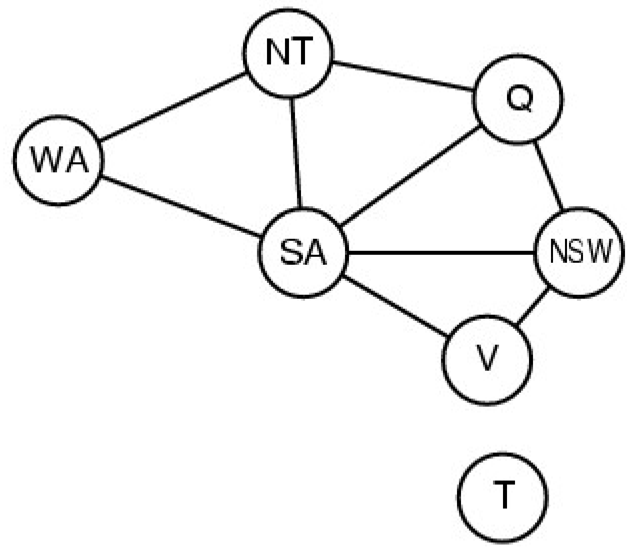
- Consider a rule:
  - *Owns(Nono, x)$\wedge$Missile(x) $\Rightarrow$ Sells(West, x, Nono)*
  - We find all the objects owned by Nono in constant time per object;
  - Then, for each object, we check whether it's a missile.
  - If more objects and very few missiles ➔ inefficient
  - Conjunct ordering problem: NP-hard
  - Heuristics?

Pattern Matching and CSP

- The most constrained variable heuristic used for CSPs would suggest ordering the conjuncts to look for missiles first if there are fewer missiles than objects owned by Nono
- We can actually express every finite-domain CSP as a single definite clause together with some associated ground facts

Diff(wa, nt) $\wedge$ Diff(wa, sa) $\wedge$
Diff(nt, q) $\wedge$ Diff(nt, sa) $\wedge$
Diff(nsw, v) $\wedge$ Diff(nsw, v) $\wedge$
Diff(v, sa) $\rightarrow$ Colorable()

Diff(R, B) Diff(R, G)
Diff(G, R) Diff(G, B)
Diff(B, R) Diff(B, G)



Incremental Forward Chaining

- Observation:
  - Every new fact inferred on iteration t must be derived from at least one new fact inferred on iteration t-1

- Modification:
  - At iteration t, we check a rule only if its premise includes a conjunct p_i that unifies with a fact p_i' newly inferred at iteration t-1
  - Many real systems operate in an "update" mode wherein forward chaining occurs in response to each new fact that is TOLD to the system

Irrelevant Facts

- FC makes all allowable inferences based on known facts, even if they are irrelevant to the goal at hand
- Similar to FC in propositional context

- Solution?

Backward Chaining

- $p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q$
- When a query q is examined:
  - if a matching fact q' is known, return the unifier
  - for each rule whose consequent q' matches q
  - attempt to prove each premise of the rule by backward chaining

- Backward chaining is the basis for "logic programming," e.g., Prolog

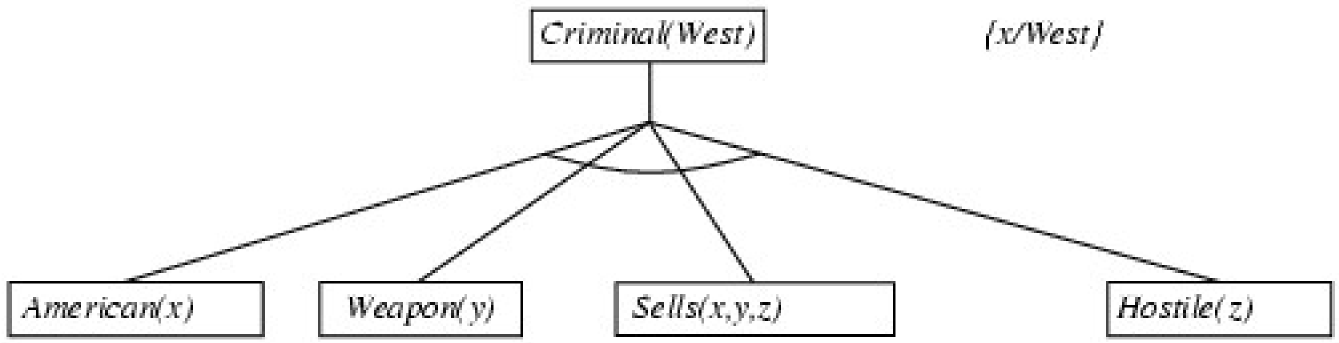
Backward Chaining Algorithm

function FOL-BC-ASK(KB, goals, θ) **returns** a set of substitutions
  inputs: KB, a knowledge base
    $goals$, a list of conjuncts forming a query
    θ, the current substitution, initially the empty substitution $\{\,\}$
  local variables: ans, a set of substitutions, initially empty

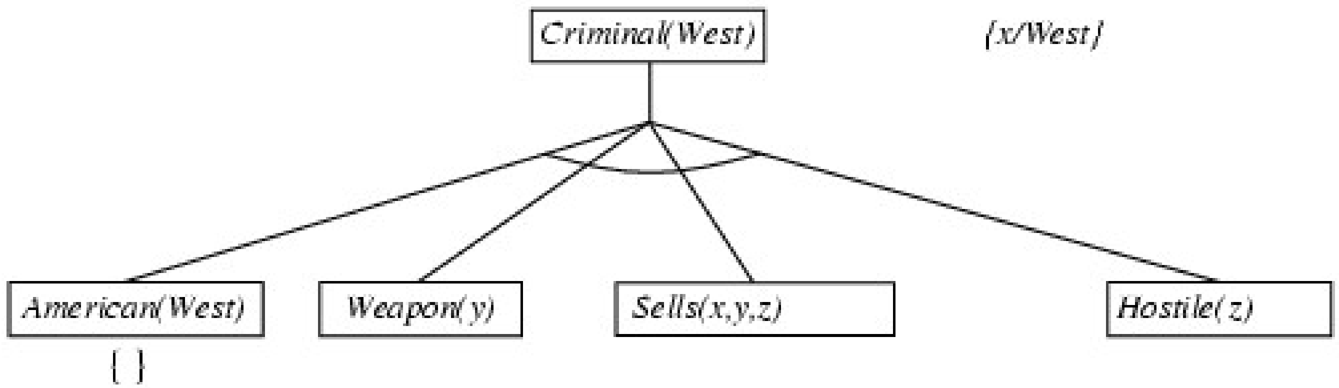
  if $goals$ is empty **then return** $\{\theta\}$
  $q' \leftarrow \text{SUBST}(\theta, \text{FIRST}(goals))$
  for each r **in** KB where $\text{STANDARDIZE-APART}(r) = (\, p_1 \ \wedge \ \ldots \ \wedge \ p_n \ \Rightarrow \ q)$
    and $\theta' \leftarrow \text{UNIFY}(q, q')$ succeeds
   $ans \leftarrow \text{FOL-BC-ASK}(KB, [p_1, \ldots, p_n | \text{REST}(goals)], \text{COMPOSE}(\theta, \theta')) \cup ans$
  return ans

Backward Chaining Example

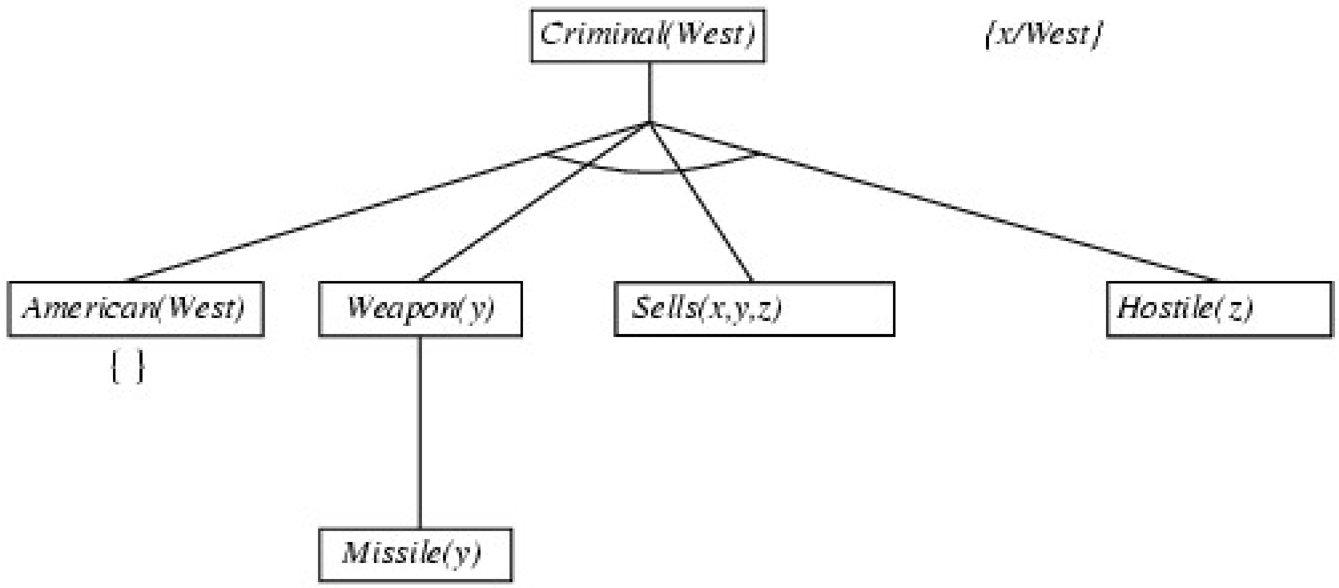
Backward Chaining Example



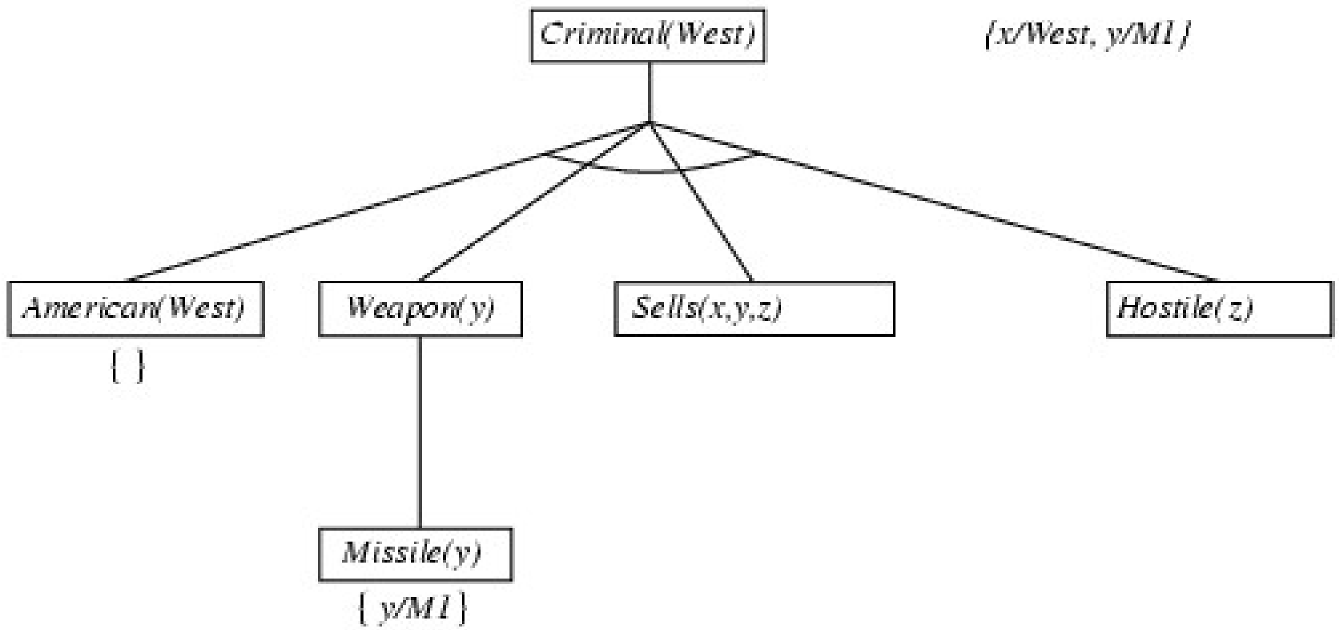
Backward Chaining Example



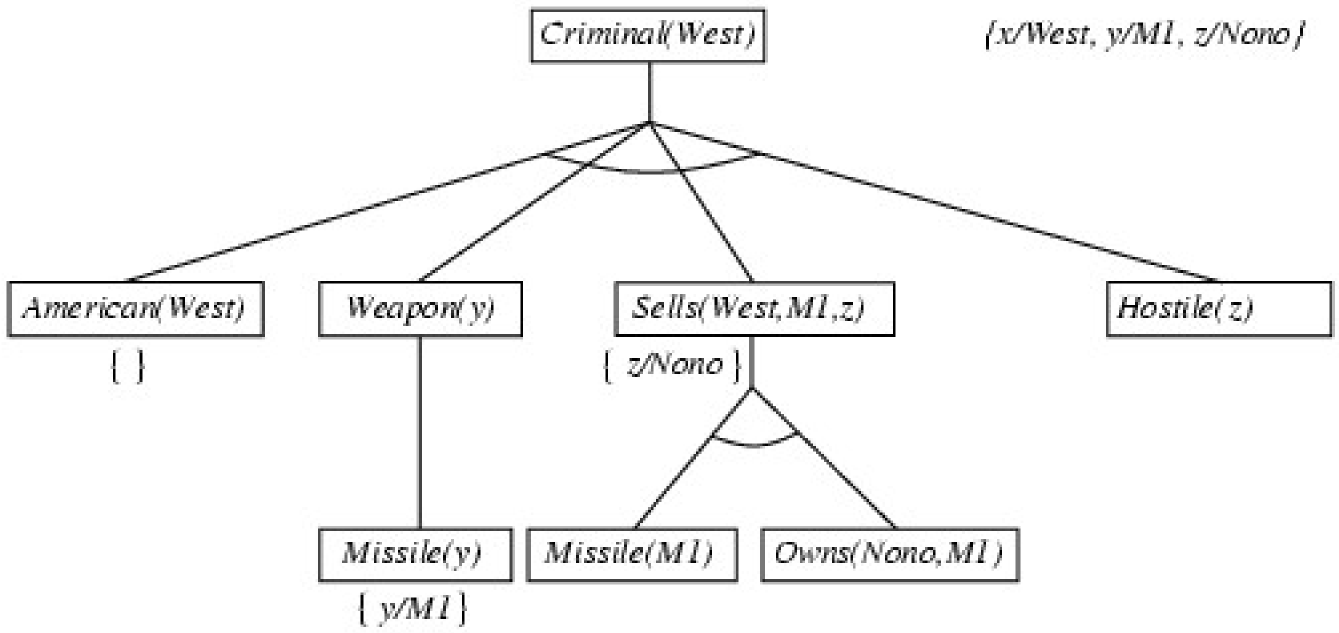
Backward Chaining Example



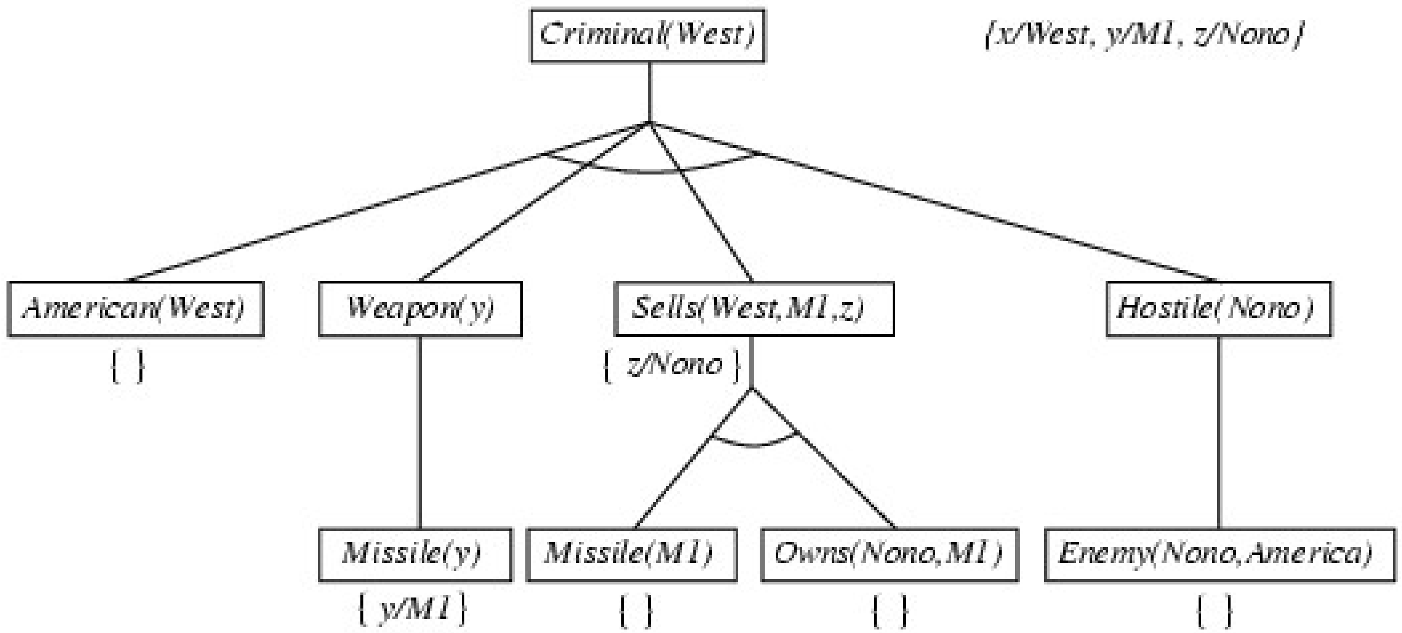
Backward Chaining Example



Backward Chaining Example



Backward Chaining Example



Analysis of Backward Chaining

- Depth-first search: space is linear in size of proof
- Repeated states and incompleteness
  - can be fixed by checking current goal against every goal on stack
- Inefficient due to repeated subgoals
  - can be fixed by caching previous results