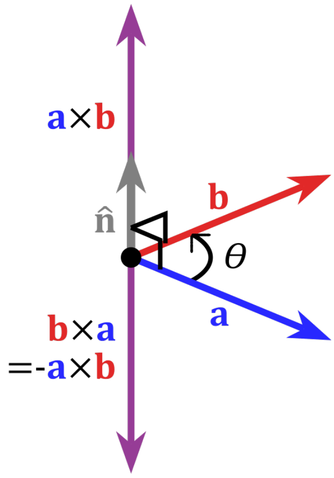
CMPUT 350 Lab 8 Prep Problems

Geometry Review

3D Vector Cross-Product



for $a, b \in \mathbb{R}^2$:

$$a \times b = ||a|| \cdot ||b|| \cdot n(a,b) \cdot \sin\left(\theta(a,b)\right),$$

where

- $||v||$: length of vector v
- $n(a,b)$: unit vector perpendicular to a and b (*normal vector*) using the right-hand rule
- $\theta(a,b)$: angle turning a into b (counter-clockwise)

When a, b are in the (x,y) plane ($a_z = b_z = 0$), n faces upwards if and only if b is in left-plane w.r.t (with respect to) a.

More generally, the z component of $a \times b$ tells us how b relates to a:

- $z > 0$: b in the left half-plane w.r.t. a
- $z < 0$: b in the right half-plane w.r.t. a
- $z = 0$: a, b collinear

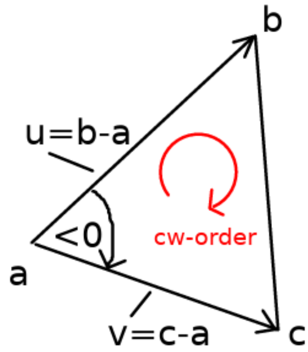
$a \times b$ can be computed by evaluating the following 3×3 determinant:

$$\begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = i \cdot \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - j \cdot \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + k \cdot \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix},$$

where i, j, k are the unit vectors in x, y, z directions respectively, and $|m|$ denotes the determinant of matrix m. In particular,

$$(a \times b)_z = \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} = a_x \cdot b_y - a_y \cdot b_x$$

Orientation Test



To check whether 3 points are in clockwise order we can use the cross-product:

$a, b, c \in \mathbb{R}^2$ are in clockwise (cw) order if and only if for vectors $u = b - a$ and $v = c - a$, v lies in the right half-plane w.r.t. u, i.e.,

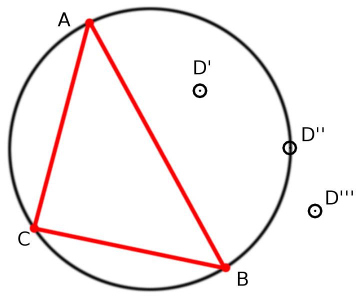
$$u_x \cdot v_y - v_x \cdot u_y < 0$$

Likewise, $a, b, c \in \mathbb{R}^2$ are in counter-clockwise (Ccw) order if and only if

$$u_x \cdot v_y - v_x \cdot u_y > 0$$

If $u_x \cdot v_y - v_x \cdot u_y = 0$, the three points are *collinear*, i.e., they lie on a line.

In-Circle Test



It can be shown that $d \in \mathbb{R}^2$ lies inside, on, or outside the circle defined by $a, b, c \in \mathbb{R}^2$ given in cw order, if the following 4x4 determinant

$$\begin{vmatrix} a_x & a_y & a_x^2 + a_y^2 & 1 \\ b_x & b_y & b_x^2 + b_y^2 & 1 \\ c_x & c_y & c_x^2 + c_y^2 & 1 \\ d_x & d_y & d_x^2 + d_y^2 & 1 \end{vmatrix}$$

is < 0, $= 0$, > 0 respectively.

If a, b, c are in ccw order, the signes are reversed.

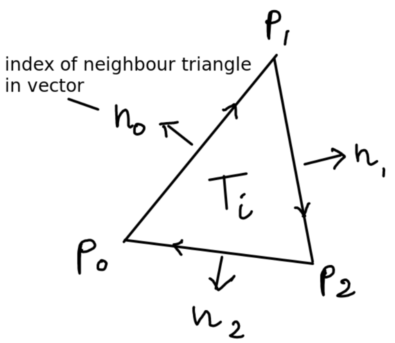
Thus, in-circle tests are basic polynomial computations, which are exact when using rational arithmetic. No square roots or trigonometric functions are required!

This is useful for detecting beneficial edge flips when generating Delaunay triangulations.

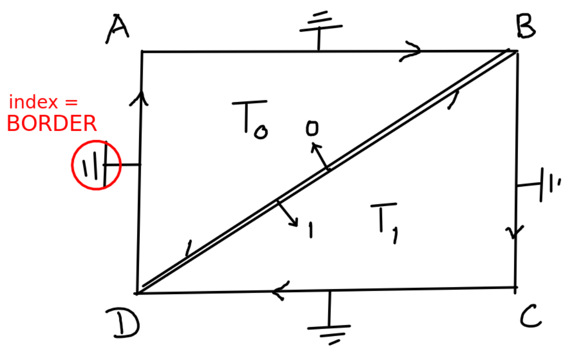
Triangle Mesh Representation

- a triangle mesh is an unordered vector of triangles
- triangles are made of points and indexes of neighbouring triangles in the vector
- all triangles in a mesh are either oriented cw or ccw
- all triangles are contained in a bounding rectangle
- outside neighbour index = BORDER (a large constant)

Triangle in cw order



Triangulation of bounding rectangle



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1. Look at `Triang.h` and `Triang.cpp`. Implement the tests for functions `orientation_test` and `in_circle` in `test.cpp`

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2. Implement and test function `rectangle_test` in `test.cpp`