402.0

1. Consider the system of 2x2 equations x + 2y = 5 and 3x - 5y = -7. Solve the system of equations using inverse of a matrix.

Answer:

$$x + 2y = 5$$
$$3x - 5y = -7$$

This can be written in the form of :-

$$Ax = b$$

$$\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \end{bmatrix}$$

$$x = A^{-1}b$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{11}\begin{bmatrix} -5 & -3 \\ -2 & 1 \end{bmatrix}^T \begin{bmatrix} 5 \\ -7 \end{bmatrix} \Rightarrow \begin{bmatrix} 5/11 & 2/11 \\ 3/11 & -1/11 \end{bmatrix}\begin{bmatrix} 5 \\ -7 \end{bmatrix}$$

$$= \begin{bmatrix} -25 & 21 \\ 10 & -7 \end{bmatrix} = \begin{bmatrix} 4 \\ -17 \end{bmatrix} \begin{bmatrix} 25/11 & -14/11 \\ 25/11 \end{bmatrix}$$

$$\begin{bmatrix} 11/11 \\ 22/11 \end{bmatrix} = \begin{bmatrix} 4/4 \\ 7/4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

which can be verified as:-

$$3(x + 2y = 5) = 3x + 6y = 15$$
$$3x - 5y = -7 \Rightarrow 3x - 5y = -7$$
$$\underline{\qquad - \qquad + \qquad + \qquad}$$
$$11y = 22$$
$$y = 2$$

$$x + 4 = 5 \Rightarrow x = 1$$