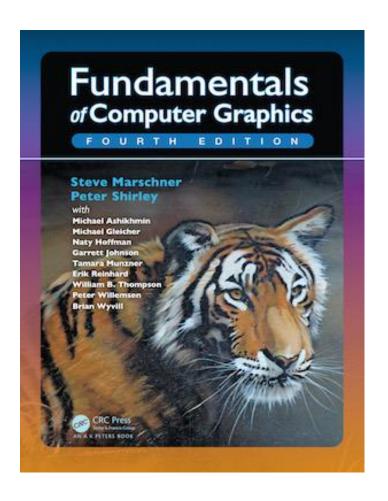
CSE4203: Computer Graphics Chapter – 8 (part - C) Graphics Pipeline

Mohammad Imrul Jubair

Outline

- Barycentric Interpolation
- Rasterizing a triangle
- Clipping
- Operations before and after rasterization

Credit



CS4620: Introduction to Computer Graphics

Cornell University

Instructor: Steve Marschner

http://www.cs.cornell.edu/courses/cs46

20/2019fa/

Barycentric Interpolation (1/4)

Barycentric coordinates:

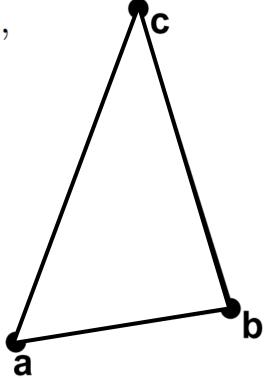
$$\mathbf{p}(\alpha, \beta, \gamma) = \alpha \mathbf{a} + \beta \mathbf{b} + \gamma \mathbf{c},$$

$$\alpha + \beta + \gamma = 1.$$

$$0 < \alpha < 1$$
,

$$0 < \beta < 1$$
,

$$0 < \gamma < 1$$
.

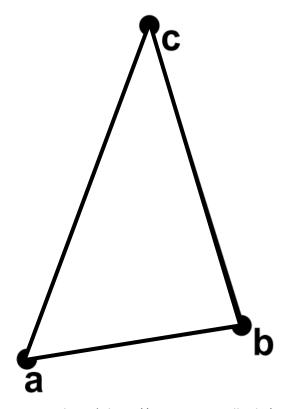


Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Barycentric Interpolation (2/4)

Barycentric coordinates:

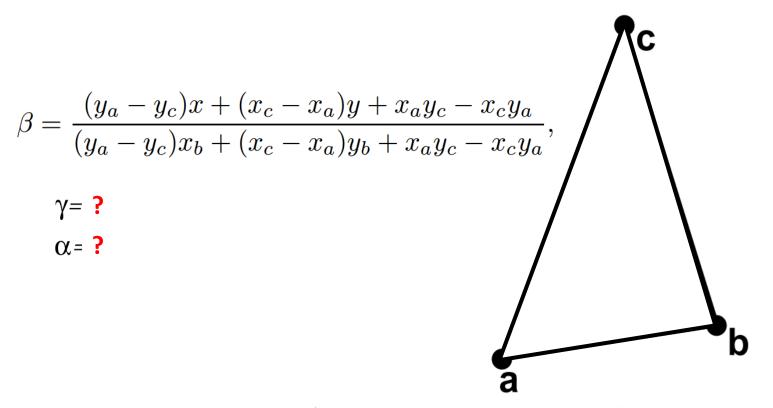
$$\beta = \frac{f_{ac}(x,y)}{f_{ac}(x_b, y_b)}$$



Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Barycentric Interpolation (3/4)

Barycentric coordinates:



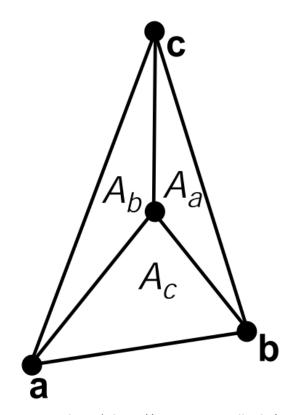
Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

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Barycentric Interpolation (4/4)

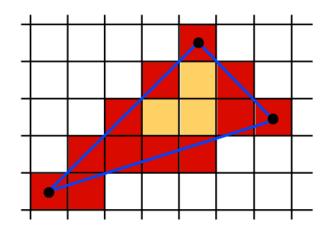
Barycentric coordinates:

$$\alpha = A_a / A$$
 $\beta = A_b / A$
 $\gamma = A_c / A$

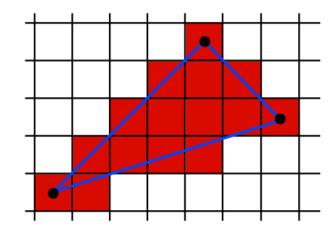


Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Triangle Rasterization (1/7)



Use Midpoint Algorithm for edges and fill in?



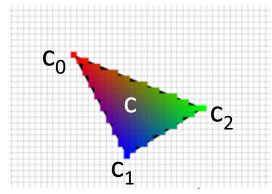
Use an approach based on barycentric coordinates

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Triangle Rasterization (2/7)

• If the vertices have colors c_0 , c_1 , and c_2 , the color at a point in the triangle with *Barycentric coordinates* (α, β, γ) is:

$$\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$$



This type of interpolation of color is known in graphics as Gouraud interpolation

M. I. Jubair

Triangle Rasterization (3/7)

 $\begin{array}{l} \text{ for all } x \text{ do} \\ \text{ for all } y \text{ do} \\ \text{ compute } (\alpha,\beta,\gamma) \text{ for } (x,y) \\ \text{ if } (\alpha \in [0,1] \text{ and } \beta \in [0,1] \text{ and } \gamma \in [0,1]) \text{ then } \\ \mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2 \\ \text{ drawpixel } (x,y) \text{ with color } \mathbf{c} \end{array}$



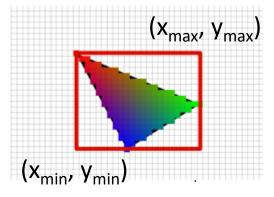
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Triangle Rasterization (4/7)

for
$$y = y_{\min}$$
 to y_{\max} do
for $x = x_{\min}$ to x_{\max} do

compute (α, β, γ) for (x, y)

if
$$(\alpha > 0 \text{ and } \beta > 0 \text{ and } \gamma > 0)$$
 then $\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$ drawpixel (x, y) with color \mathbf{c}



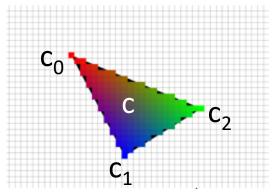
Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Triangle Rasterization (5/7)

for
$$y=y_{\min}$$
 to y_{\max} do
$$\alpha=f_{12}(x,y)/f_{12}(x_0,y_0)$$

$$\beta=f_{20}(x,y)/f_{20}(x_1,y_1)$$

$$\gamma=f_{01}(x,y)/f_{01}(x_2,y_2)$$
 if $(\alpha>0$ and $\beta>0$ and $\gamma>0$) then
$$\mathbf{c}=\alpha\mathbf{c}_0+\beta\mathbf{c}_1+\gamma\mathbf{c}_2$$
 drawpixel (x,y) with color \mathbf{c}



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Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Triangle Rasterization (6/7)

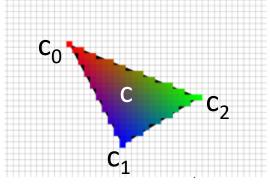
for
$$y = y_{\min}$$
 to y_{\max} do

for $x = x_{\min}$ to x_{\max} do

$$for $x = x_{\min}$ to x_{\max} do$$

$$\begin{cases}
\alpha = f_{12}(x, y) / f_{12}(x_0, y_0) \\
\beta = f_{20}(x, y) / f_{20}(x_1, y_1) \\
\gamma = f_{01}(x, y) / f_{01}(x_2, y_2)
\end{cases}$$

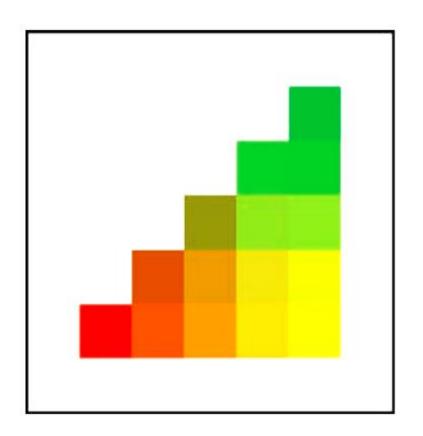
if
$$(\alpha > 0 \text{ and } \beta > 0 \text{ and } \gamma > 0)$$
 then $\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$ drawpixel (x, y) with color \mathbf{c}

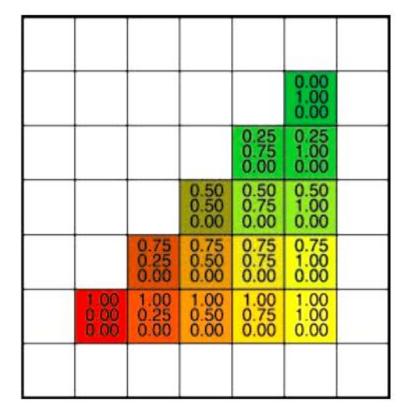


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Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Triangle Rasterization (7/7)





Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

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Clipping

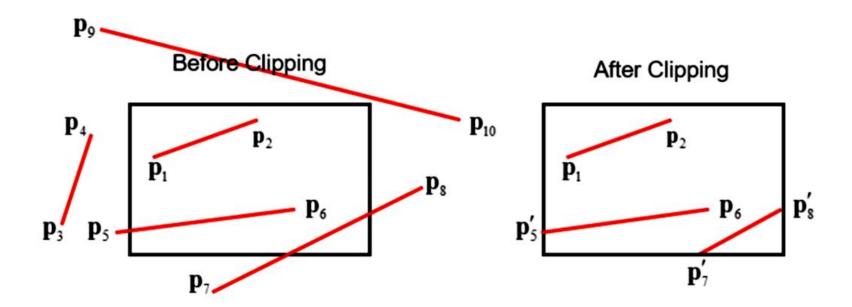
Clipping (2/2)

- Clipping is a method to selectively enable or disable rendering operations within a defined region of interest.
 - The primary use of clipping is to remove objects, lines, or line segments that are outside the viewing pane.

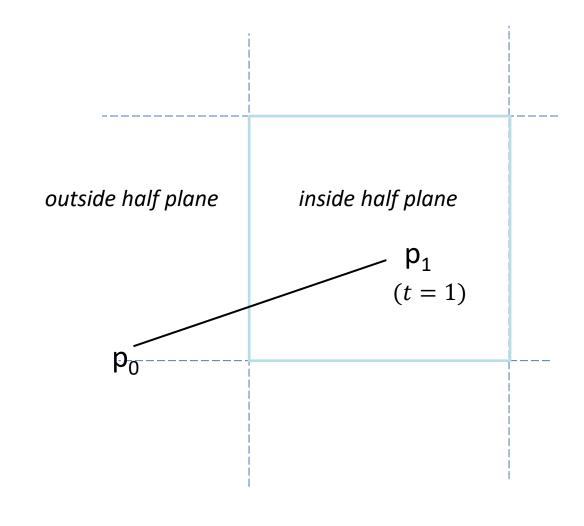
Line Clipping (2/2)

We must clip against a plane.

• Cyrus-Beck Parametric Line Clipping Algorithm



Inside/ outside of Half Plane (1/1)



Parametric Eq. of a line (1/2)

$$p(t) = p_0 + t(p_1 - p_0)$$

$$(t = 0)$$

$$p_0$$

$$(t = 1)$$

Parametric Eq. of a line (2/2)

$$p(t) = p_0 + t(p_1 - p_0)$$

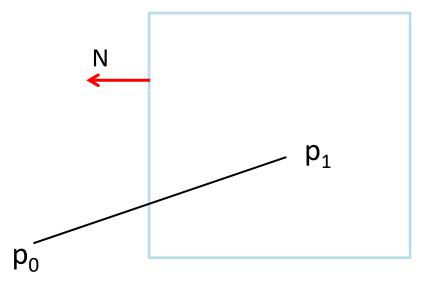
$$(t = 0)$$

$$p(t)$$

$$(t = 1)$$

Edge-line Intersection (1/7)

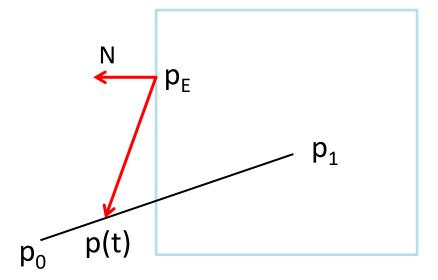
N =outward normal to the edge E



Edge-line Intersection (2/7)

N = outward normal to the edge E p_E = any point to the edge E

 $[p(t) - p_E]$ = vector from p_E to p(t)



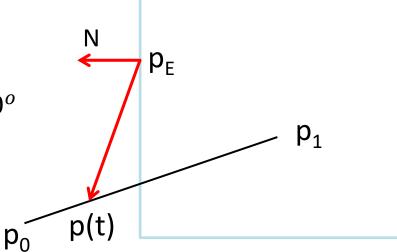
Edge-line Intersection (3/7)

N = outward normal to the edge E p_E = any point to the edge E

$$[p(t) - p_E]$$
 = vector from p_E to $p(t)$

$$N.[p(t) - p_E] > 0$$

• Angel between N and $[p(t) - p_E] < 90^o$



Edge-line Intersection (4/7)

N = outward normal to the edge E p_E = any point to the edge E

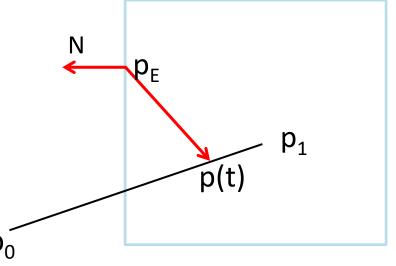
$$[p(t) - p_E]$$
 = vector from p_E to $p(t)$

$$N.[p(t) - p_E] > 0$$

• Angel between N and $[p(t) - p_E] < 90^o$

$$N.[p(t) - p_E] < 0$$

• Angel between N and $[p(t) - p_E] > 90^o$



Edge-line Intersection (5/7)

N = outward normal to the edge E p_E = any point to the edge E

$$[p(t) - p_E]$$
 = vector from p_E to $p(t)$

$$N.[p(t) - p_E] > 0$$

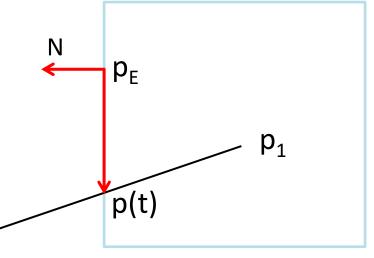
• Angel between N and $[p(t) - p_E] < 90^o$

$$N.[p(t) - p_E] < 0$$

• Angel between N and $[p(t) - p_E] > 90^o$

$$N.[p(t) - p_E] = 0$$

• Angel between N and $[p(t) - p_E] = 90^o$



Edge-line Intersection (6/7)

For intersection, **N**. $[p(t) - p_e] = 0 \dots \dots (1)$

we know,
$$p(t) = p_0 + t(p_1 - p_0)$$

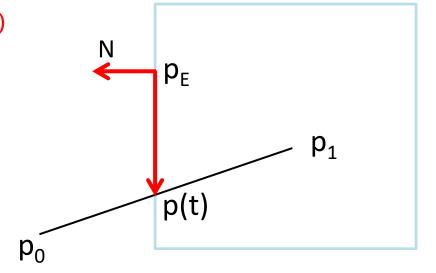
Putting into Eq.(1):

$$N.[p_0 + t(p_1 - p_0) - p_E] = 0$$

$$t = \frac{N.[p_0 - p_E]}{-N.[p_1 - p_0]}$$

$$t = \frac{N.[p_0 - p_E]}{-N.D}$$

where, $D = p_1 - p_0$



Edge-line Intersection (7/7)

Therefore, edge and line are intersected at -

$$t=rac{N.\left[p_{0}-p_{E}
ight]}{-N.D}$$
 where, $D=p_{1}-p_{0}$

Check for Nonzero (1/2)

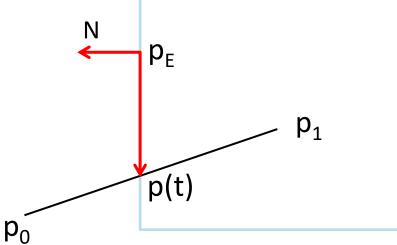
Therefore, edge and line are intersected at –

$$t = \frac{N.[p_0 - p_E]}{-N.D}$$
 where, $D = p_1 - p_0$

However, N. D can not be zero.

We need to check –

- $N \neq 0$ (by mistake, normal should not be 0)
- $D \neq 0$ (means what?)
- $N.D \neq 0$ (means what?)



Check for Nonzero (2/2)

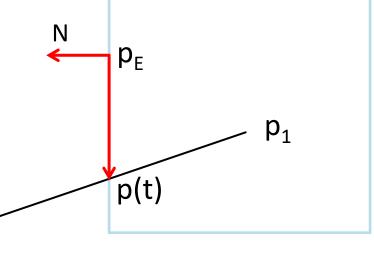
Therefore, edge and line are intersected at –

$$t = \frac{N.[p_0 - p_E]}{-N.D}$$
 where, $D = p_1 - p_0$

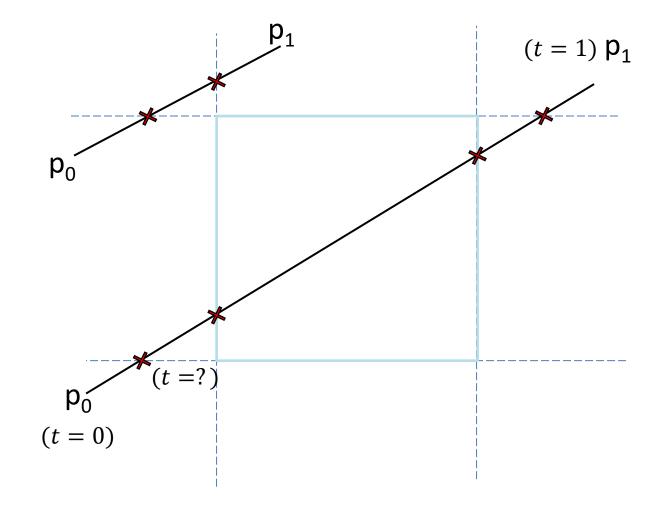
However, N. D can not be zero.

We need to check –

- $N \neq 0$ (by mistake, normal should not be 0)
- $\boldsymbol{D} \neq \boldsymbol{0}$ (that is $p_1 \neq p_0$ for a line)
- N. D ≠ 0 (line and the normal are not perpendicular; line and edge are parallel)

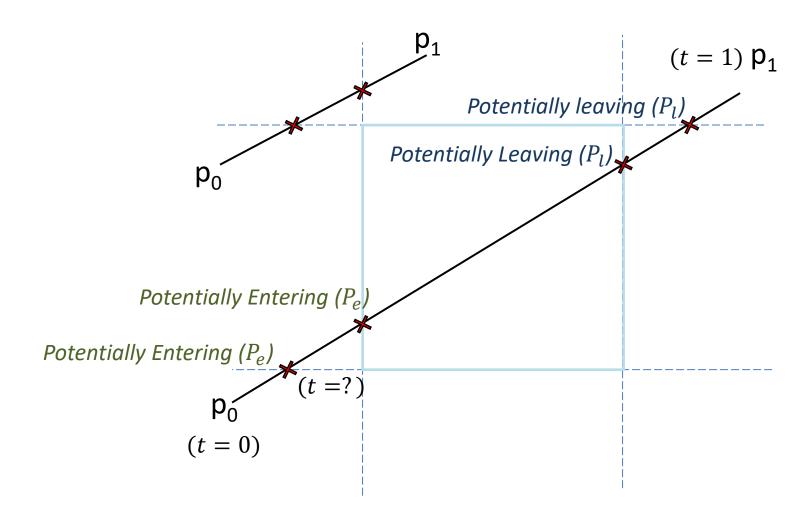


Inside/ outside Half Plane (1/1)



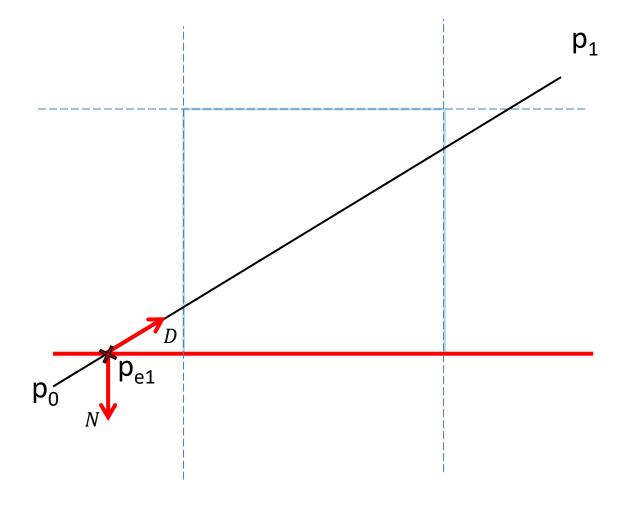
 $t = \frac{N.[p_0 - p_E]}{-N.D}$ Only this formula is not enough! Why?

Potentially Entering/Leaving (1/1)

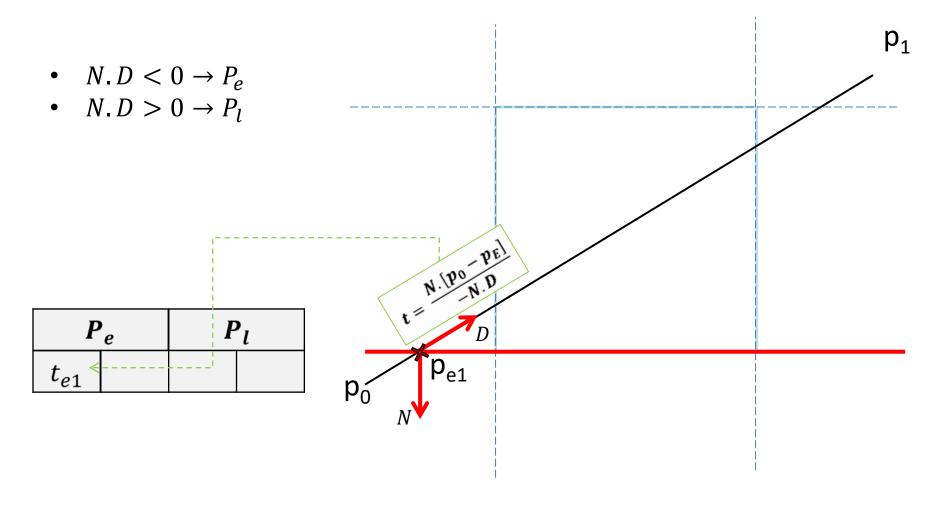


True Clipping Intersection (1/12)

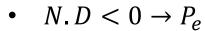
- $N.D < 0 \rightarrow P_e$
- $N.D > 0 \rightarrow P_L$



True Clipping Intersection (2/12)

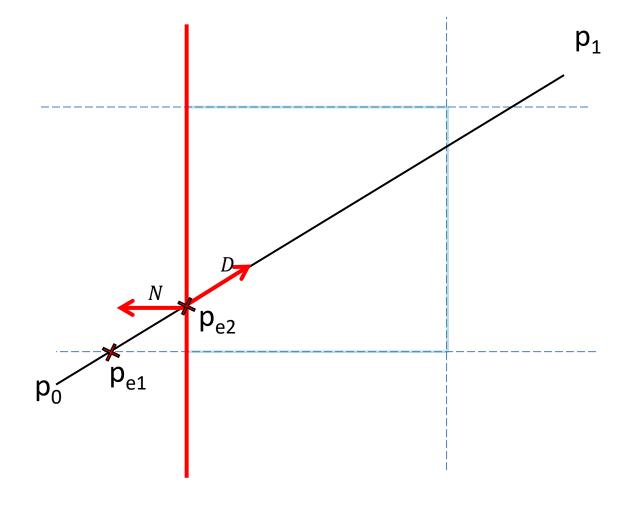


True Clipping Intersection (3/12)

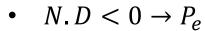


•
$$N.D > 0 \rightarrow P_l$$

P_e		P_l	
t_{e1}	t_{e2}		

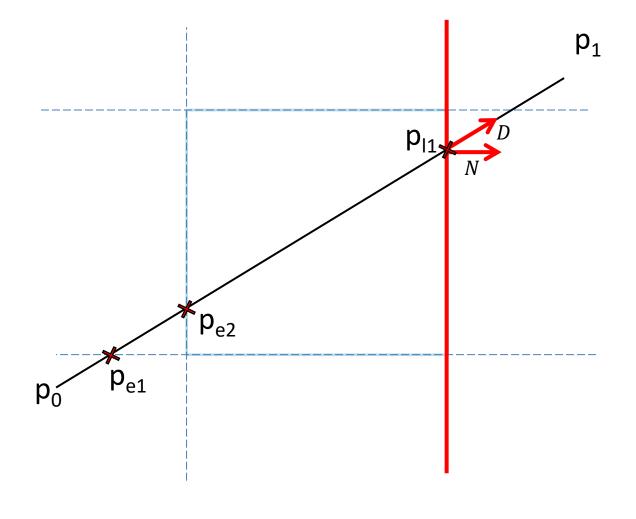


True Clipping Intersection (4/12)

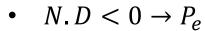


•
$$N.D > 0 \rightarrow P_l$$

P_e		P_l	
t_{e1}	t_{e2}	t_{l1}	

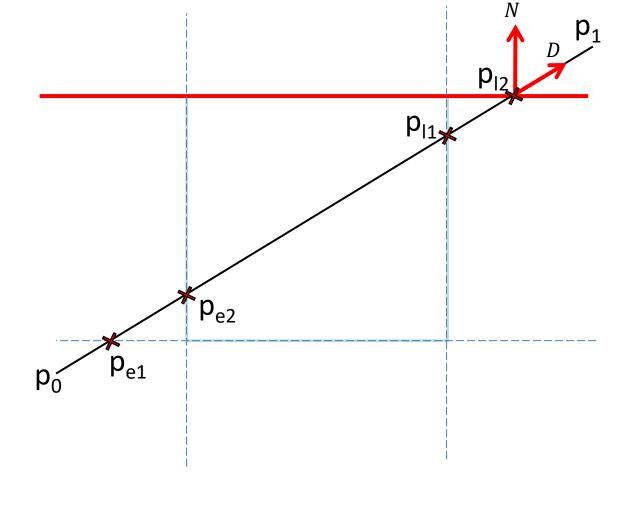


True Clipping Intersection (5/12)

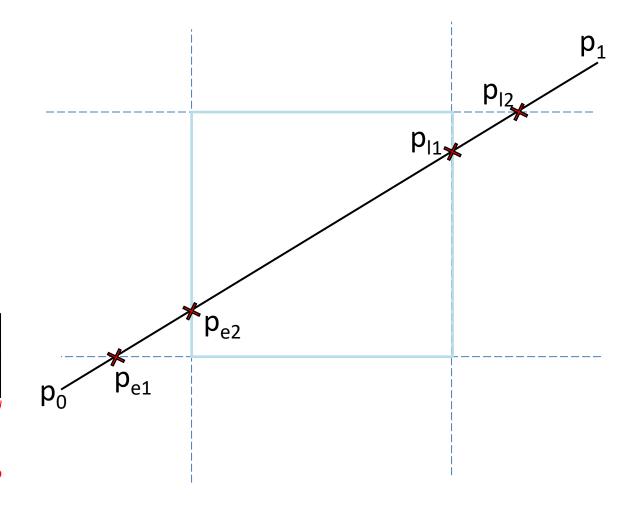


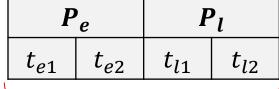
•
$$N.D > 0 \rightarrow P_l$$

P_e		P_l	
t_{e1}	t_{e2}	t_{l1}	t_{l2}



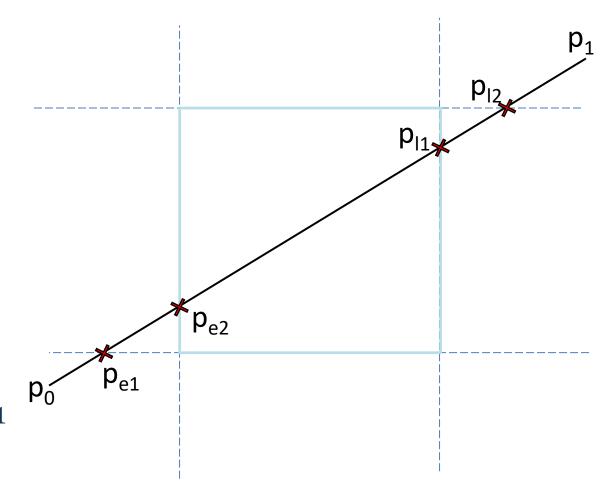
True Clipping Intersection (6/12)





Are they in order?
Ascending or descending?

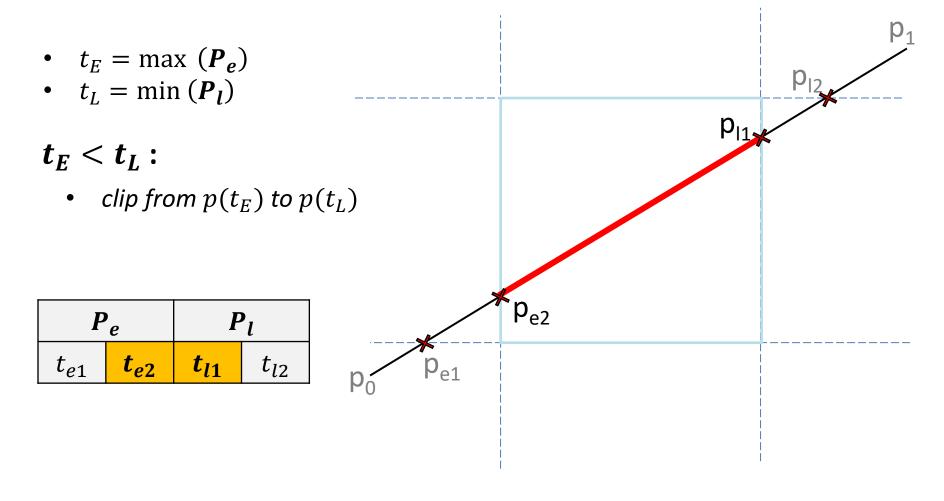
True Clipping Intersection (7/12)



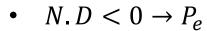
P_e		P_l		
t_{e1}	t_{e2}	t_{l1}	t_{l2}	

 $0 < t_{e1} < t_{e2} < t_{l1} < t_{l2} < 1$

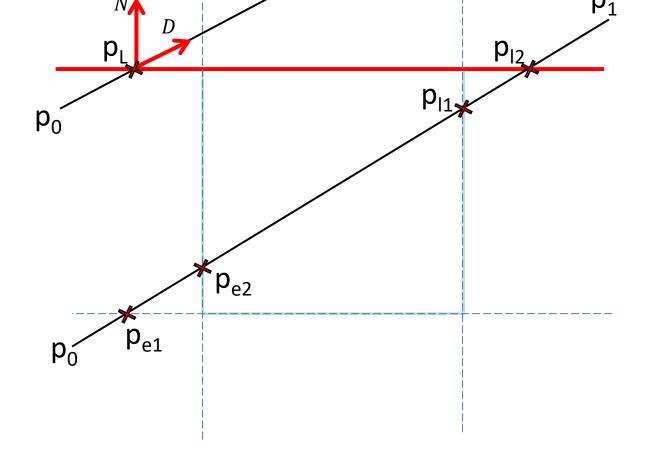
True Clipping Intersection (8/12)



True Clipping Intersection (9/12)



•
$$N.D > 0 \rightarrow P_l$$

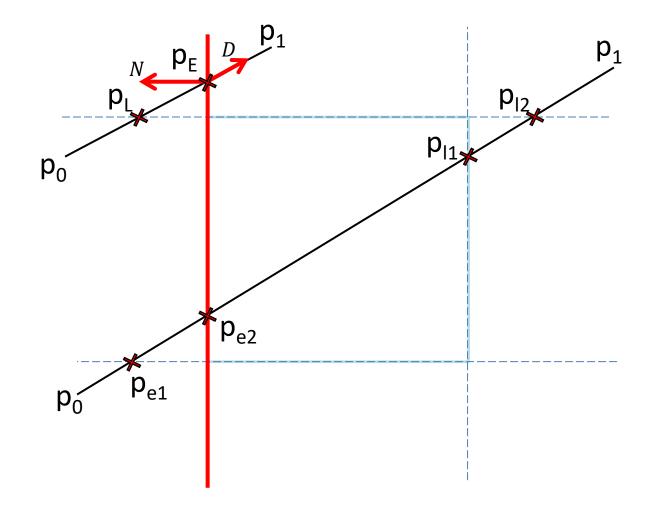


P_e	P_l		
	t_l		

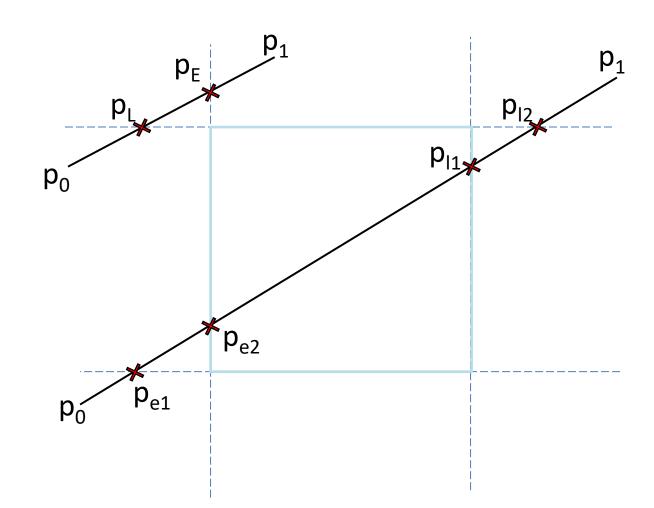
True Clipping Intersection (10/12)

- $N.D < 0 \rightarrow P_e$
- $N.D > 0 \rightarrow P_l$

P_e	P_l		
t_e	t_l		



True Clipping Intersection (11/12)



P_e	P_{l}	
t_e	t_l	

$$1 > t_e > t_l > 0$$

True Clipping Intersection (12/12)

•
$$t_E = \max(\boldsymbol{P_e})$$

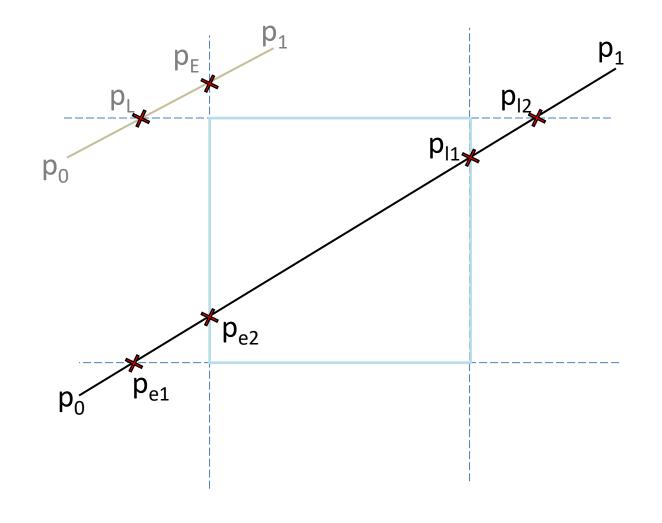
•
$$t_L = \min(\boldsymbol{P_l})$$

But this time,

$$t_E > t_L$$
:

• Reject the line

P_e	P_l	
t_e	t_l	



Cyrus-Beck Algorithm (1/1)

```
precalculate N_i and select a P_{E_i} for each edge;
for each line segment to be clipped
  if P_1 = P_0 then
         line is degenerate so clip as a point;
   else
         begin
            t_{\rm E} = 0; t_{\rm L} = 1;
            for each clip edge
               if Ni \cdot D \neq 0 then {Ignore edges parallel to line}
                   begin
                      calculate t; {of line \cap clip edge}
                      use sign of N_i \cdot D to categorize as PE or PL;
                      if PE then t_E = \max(t_E, t);
                      if PL then t_{\rm L} = \min(t_{\rm L}, t)
                   end
            if t_E > t_L then
               return nil
            else
               return P (t_{\rm F}) and P (t_{\rm I}) as true clip intersections
         end {else}
```

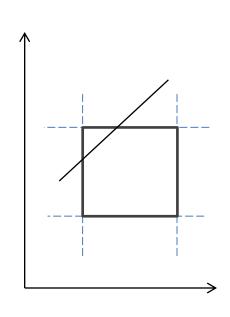
Known Cases (1/1)

•
$$D = P_1 - P_0 = (x_1 - x_0, y_1 - y_0)$$

 P_{Ei} as an arbitrary point on the clip edge; it's a free variable and drops out

Calculations for Parametric Line Clipping Algorithm

Clip Edge _i	Normal N _i	P_{E_i}	P_o – P_{E_i}	$t = \frac{N_i \cdot (P_0 - P_{E_i})}{-N_i \cdot D}$	
left: $x = x_{min}$	(-1, 0)	(x_{min}, y)	$(x_0 - x_{min}, y_0 - y)$	$\frac{-(x_o - x_{min})}{(x_1 - x_o)}$	
right: $x = x_{max}$	(1,0)	(x_{max}, y)	$(x_0 - x_{max}, y_0 - y)$	$\frac{(x_0 - x_{max})}{-(x_1 - x_0)}$	
bottom: $y = y_{min}$	(0, -1)	(x, y_{min})	$(x_0 - x, y_0 - y_{min})$	$\frac{-(y_0 - y_{min})}{(y_1 - y_0)}$	
top: $y = y_{max}$	(0, 1)	(x, y_{max})	$(x_0 - x, y_0 - y_{max})$	$\frac{(y_0 - y_{max})}{-(y_1 - y_0)}$	

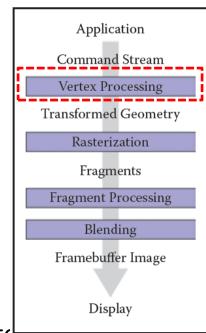


Operations Before and After Rasterization

Before Rasterization (1/1)

Before a primitive can be rasterized:

- The vertices must be in screen:
 - Modeling
 - **V**iewing
 - Projection transformations
 - Original coordinates → screen space
- Attributes that are supposed to be interpolated must be known.
 - colors, surface normals, or texture coordinates, is transformed as needed.
- Done in Vertex Processing stage

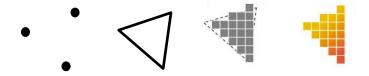


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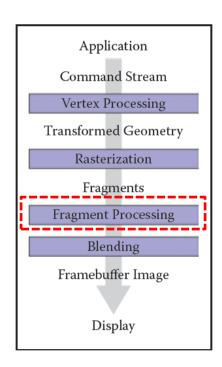
After Rasterization (1/1)

After a primitive can be rasterized:

 Computing a color and depth for each fragment (i.e. Shading).



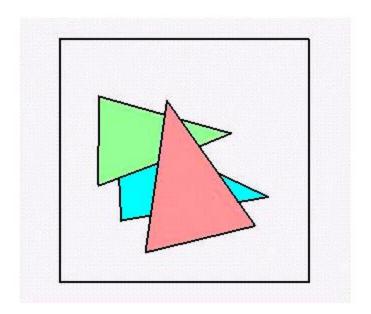
- Performing blending phase.
 - combines the fragments that overlapped.
 - compute the final color.
- Done in Fragment Processing stage



Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

A Minimal 3D Pipeline (2/16)

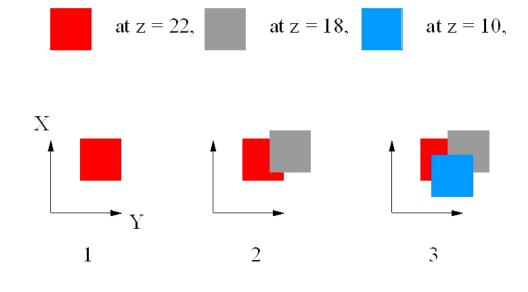
Main challenge is – occlusion.



A Minimal 3D Pipeline (3/16)

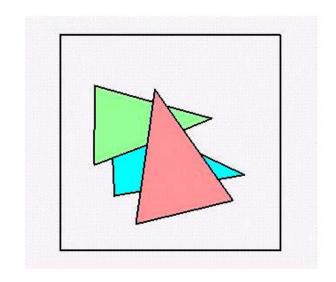
Painter's Algorithm

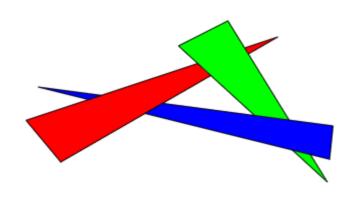
- Sort surfaces/ polygons by their depth (z values)
- Draw objects in order (farthest to closest)



A Minimal 3D Pipeline (4/16)

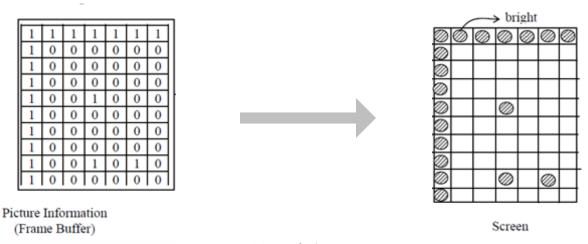
- Painter's Algorithm
 - Disadvantage:
 - Sometimes it is difficult to sort





A Minimal 3D Pipeline (6/16)

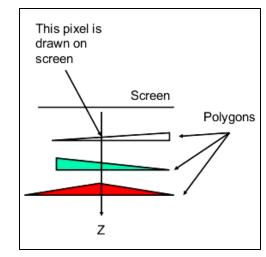
- A **frame buffer** is a portion of memory (RAM) containing a bitmap that drives a video display.
 - It is a memory buffer containing a complete frame of data

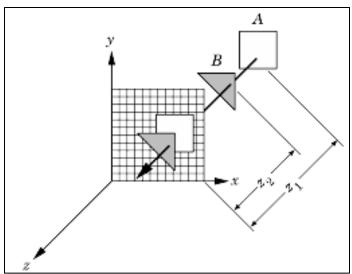


A Minimal 3D Pipeline (7/16)

Z-buffer Algorithm:

- At each pixel we keep track
 of the distance to the closest
 surface that has been drawn
 so far
 - we throw away fragments that are farther away than that distance.





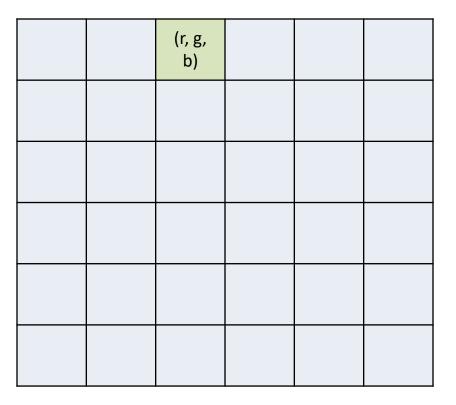
A Minimal 3D Pipeline (8/16)

Z-buffer Algorithm:

- Implementation:
 - Red, green, and blue color values (frame buffer) + depth, or z-value (z-buffer).
 - {(r, g,b), z}

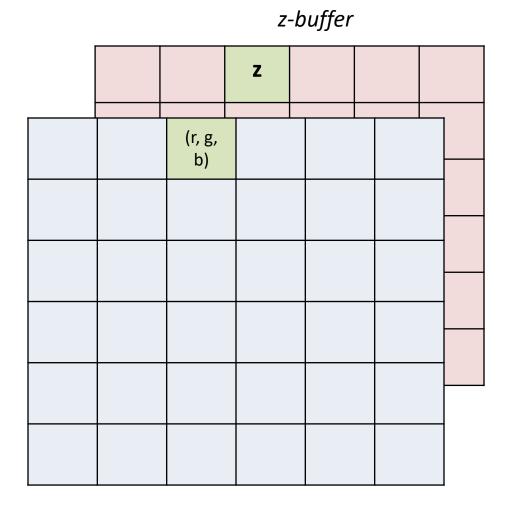
A Minimal 3D Pipeline (9/16)

Z-buffer Algorithm:



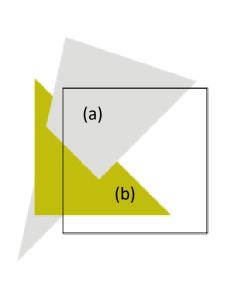
A Minimal 3D Pipeline (10/16)

Z-buffer Algorithm:



A Minimal 3D Pipeline (11/16)

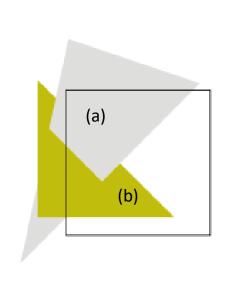
Z-buffer Algorithm:

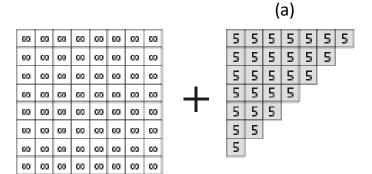


0/0	00	00	00	00	00	00	00
00	00	00	00	00	00	00	00
00	00	00	00	00	00	00	00
00	60	00	00	00	00	00	00
00	00	00	00	00	00	00	00
00	00	00	00	00	00	00	00
0/0	00	00	00	00	00	00	00
00	00	00	00	00	00	00	00

A Minimal 3D Pipeline (12/16)

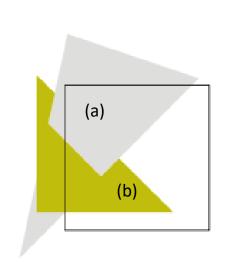
Z-buffer Algorithm:

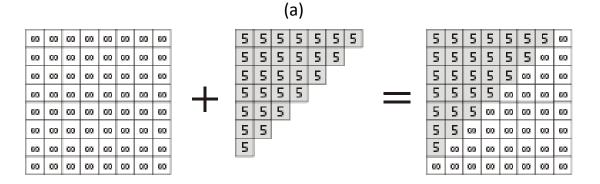




A Minimal 3D Pipeline (13/16)

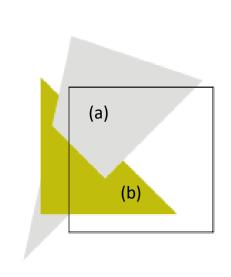
Z-buffer Algorithm:

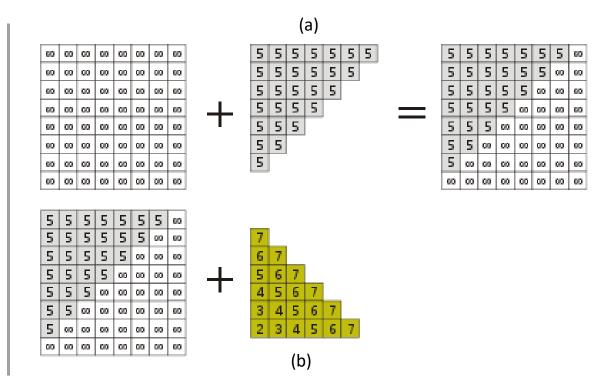




A Minimal 3D Pipeline (14/16)

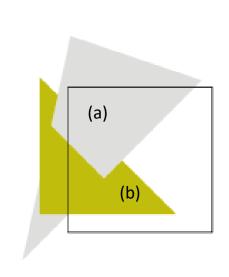
Z-buffer Algorithm:

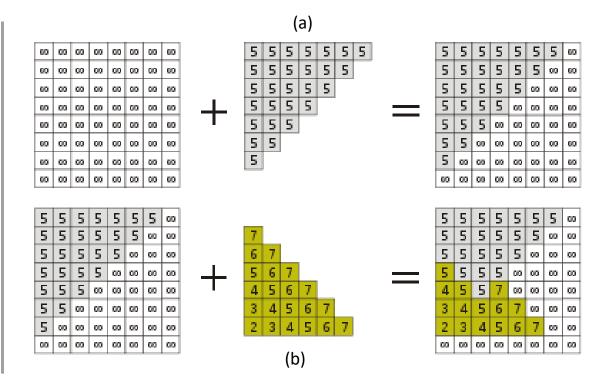




A Minimal 3D Pipeline (15/16)

Z-buffer Algorithm:

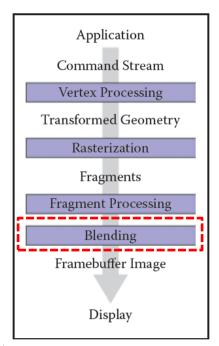




A Minimal 3D Pipeline (16/16)

Z-buffer Algorithm:

• Done in the *fragment blending phase*.

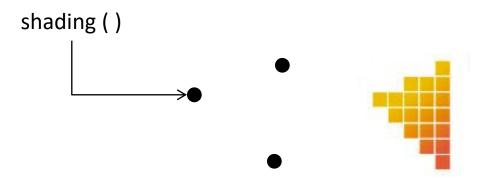


Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Per-vertex Shading (2/3)

Gouraud Shading

- Only shading equation on each vertex.
 - Normal at each vertices
- Then interpolated.



Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Per-vertex Shading (3/3)

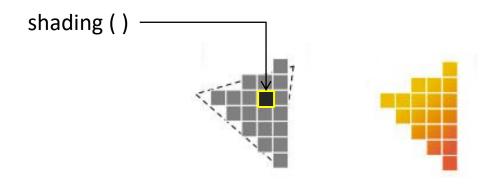
Disadvantage:

- it cannot produce any details in the shading that are smaller than the primitives used to draw the surface.
 - Because it only computes shading once for each vertex and never in between vertices.
 - (see example from the text book)

Per-fragment Shading (1/2)

Phong Shading.

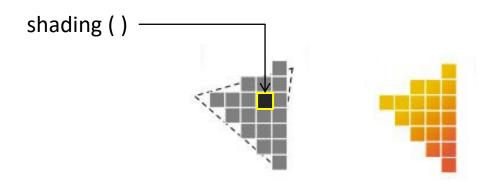
- Only shading equation on each fragment.
 - Normal at each fragment
- vertex stage must help the fragment stage to prepare the data.



Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Per-fragment Shading (2/2)

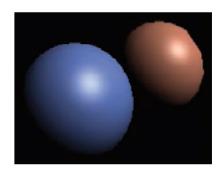
- Phong Shading.
 - Only shading equation on each fragment.
 - Normal at each fragment
 - Q: Name another per-fragment technique.



Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

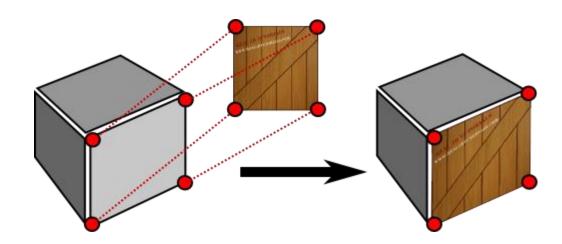
Per-vertex vs. Per-fragment Shading (1/1)





Texture Mapping (1/3)

- During shading, we read one of the color values from a texture.
 - instead of using the attribute values (colors) that are attached to the geometry.



Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

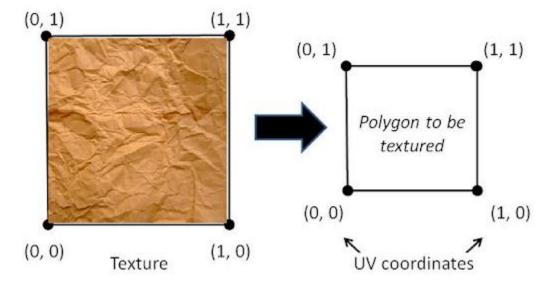
Texture Mapping (2/3)

Texture lookup:

- specifies a *texture coordinate*
 - a point in the domain of the texture, and the texture-mapping.

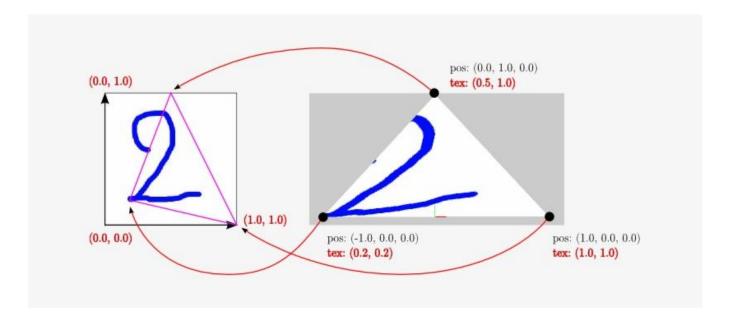
Texture Mapping (3/3)

- XY coordinate ← UV coordinate
 - Example: Quad



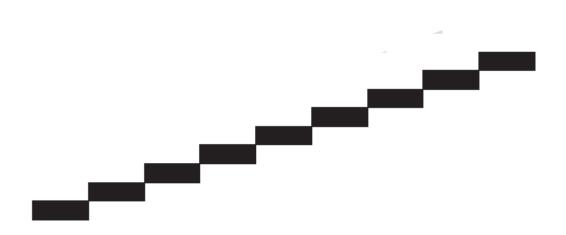
Texture Mapping (3/3)

- XY coordinate ←→ UV coordinate
 - Example: triangle



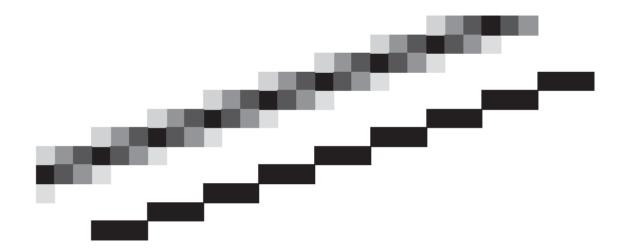
Anti-aliasing (1/6)

Aliasing



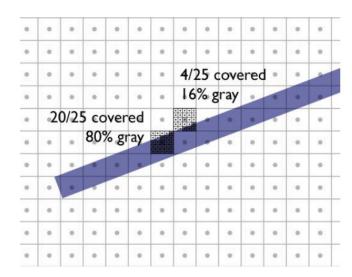
Anti-aliasing (2/6)

Anti-aliasing



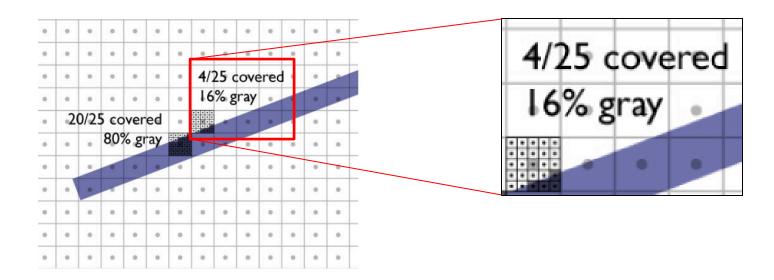
Anti-aliasing (3/6)

- Anti-aliasing:
 - Box filtering by supersampling



Anti-aliasing (4/6)

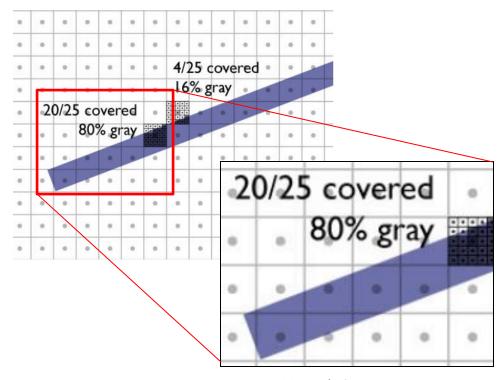
- Anti-aliasing:
 - Box filtering by supersampling



Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Anti-aliasing (5/6)

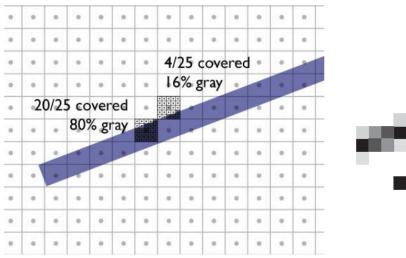
- Anti-aliasing:
 - Box filtering by supersampling

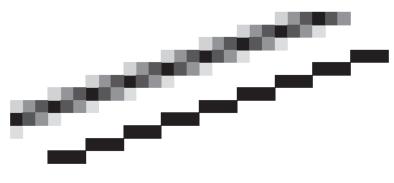


Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/cours es/cs4620/2019fa/

Anti-aliasing (6/6)

- Anti-aliasing:
 - Box filtering by supersampling

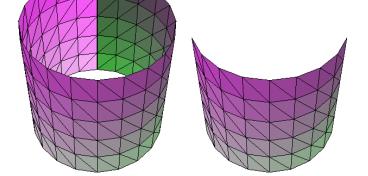




Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Backface Culling (1/3)

- Removal of primitives facing away from the camera.
 - Polygons that face away from the eye are certain to be overdrawn by polygons that face the eye.
 - Those polygons can be culled before the pipeline even starts.



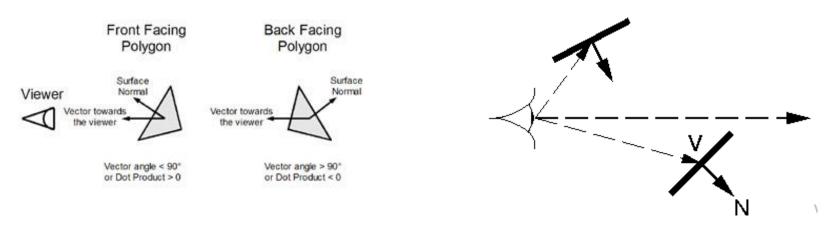
Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

M. I. Jubair

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Backface Culling (2/3)

- If polygon normal is facing away from the viewer then it is "backfacing".
 - For solid objects, polygon will not be seen.
- Thus, if N.V > 0, then cull polygon.
 - V is vector from eye to point on polygon



Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

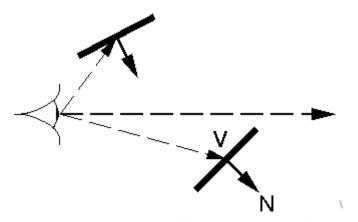
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Backface Culling (3/3)

- If polygon normal is facing away from the viewer then it is "backfacing".
 - For solid objects, polygon will not be seen.
- Thus, if N.V > 0, then cull polygon.
 - V is vector from eye to point on polygon

Q: Disadvantage?



Credit: Fundamentals of Computer Graphics 3rd Edition by Peter Shirley, Steve Marschner | http://www.cs.cornell.edu/courses/cs4620/2019fa/

Practice Problem

- Verify Cyrus-Beck line clipping algorithm for different condition.
- Take three vertices of a triangle, choose two points, P and Q, such that they stay inside and outside the triangle respectively.
 - Apply barycentric interpolation and verify that P lies inside and Q lies outside the triangle.