CSE4203: Computer Graphics
Chapter – 4 (part - C)
Ray Tracing

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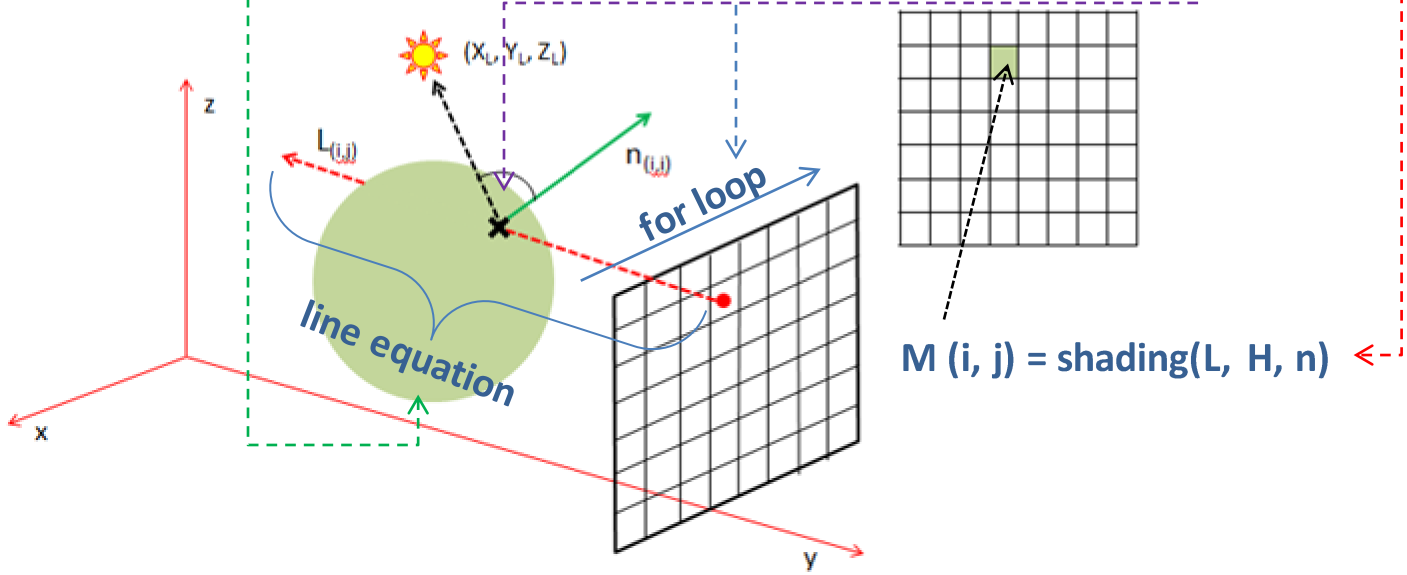
Outline

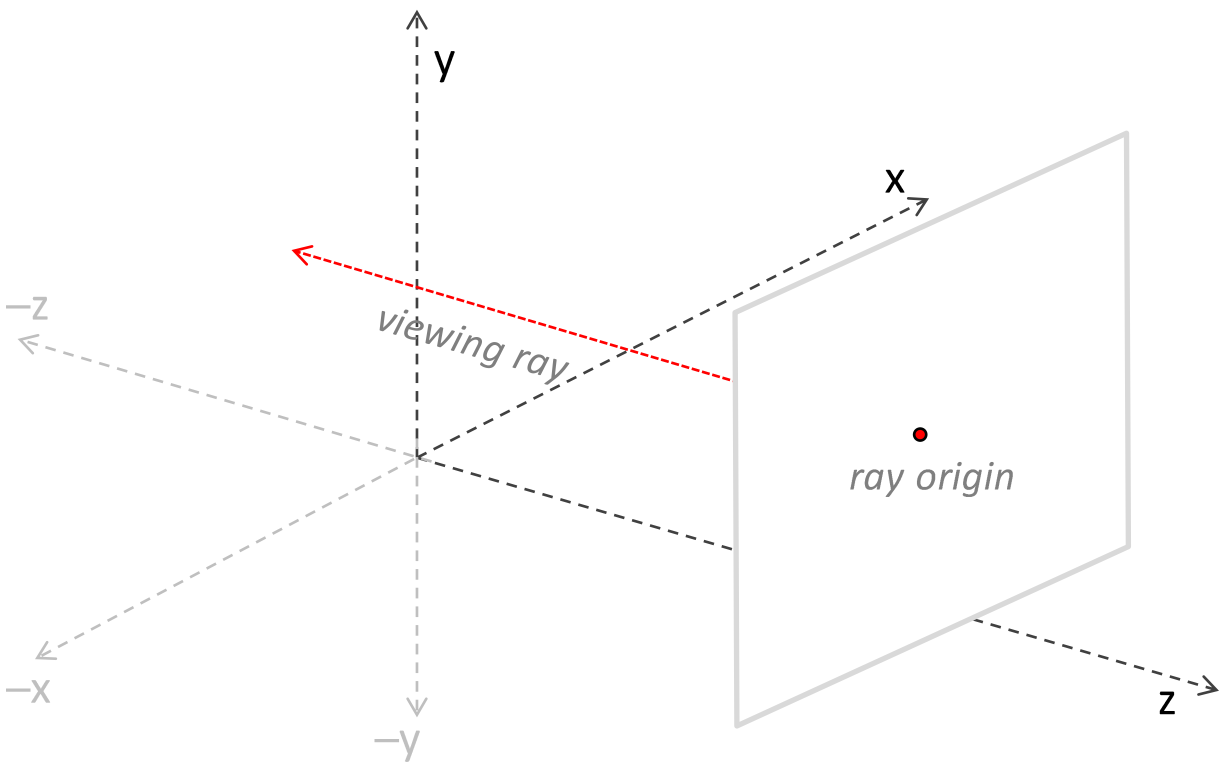
– Ray-tracing
– Camera Frame
– Image Plane and Raster Image
– Computing Viewing Rays
– Ray-sphere Intersection
– Shading

Ray-Tracing Algorithm

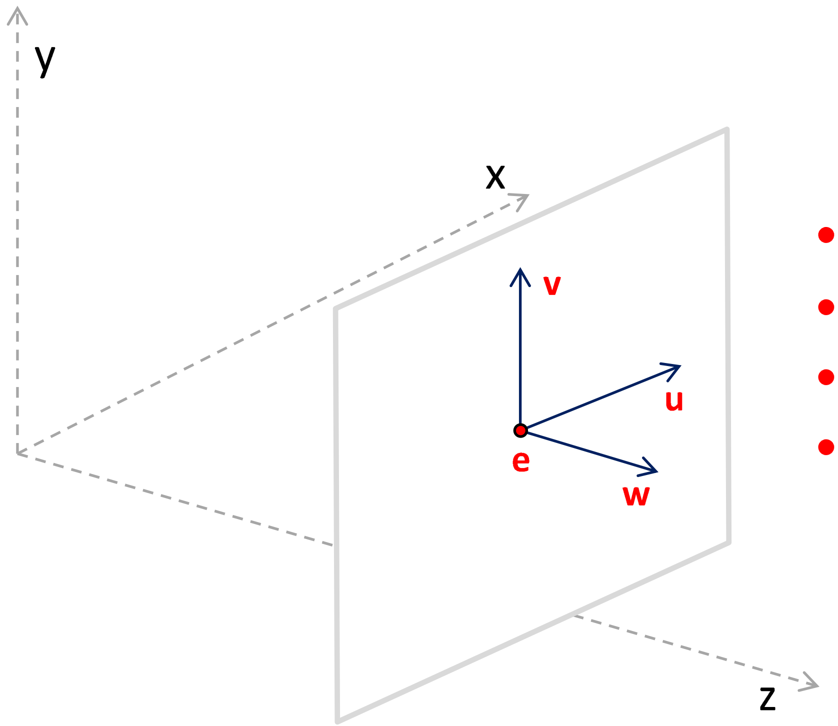
- **for each pixel do:**
  - compute viewing ray
  - find first object hit by ray and its surface normal **n**
  - set pixel color computed from hit point, light, and **n**



Camera Frame (1/11)



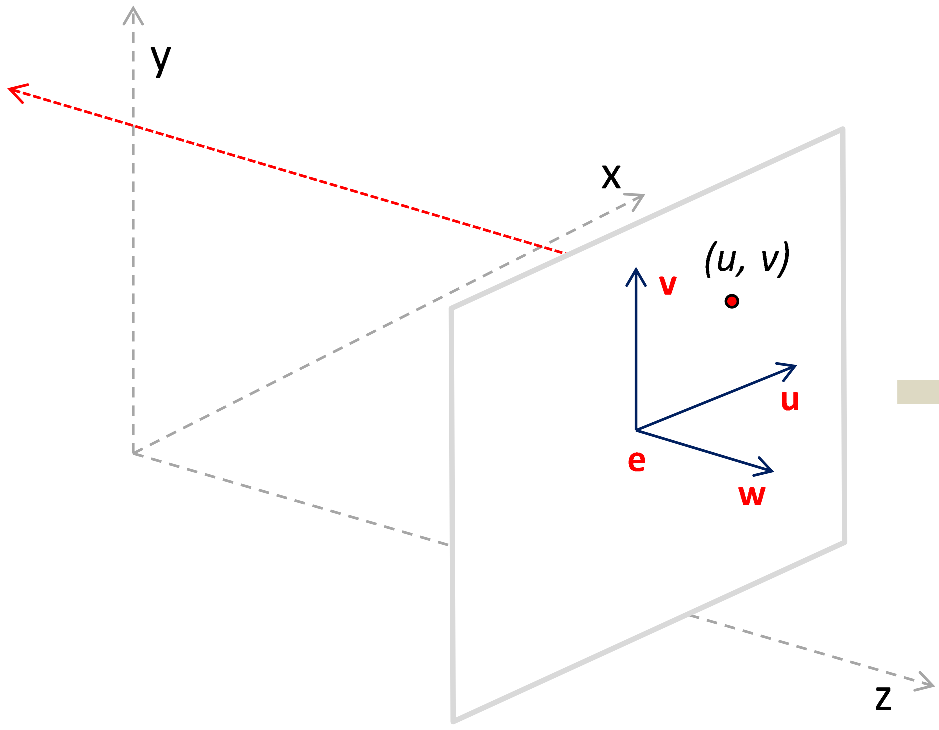
Camera Frame (2/11)

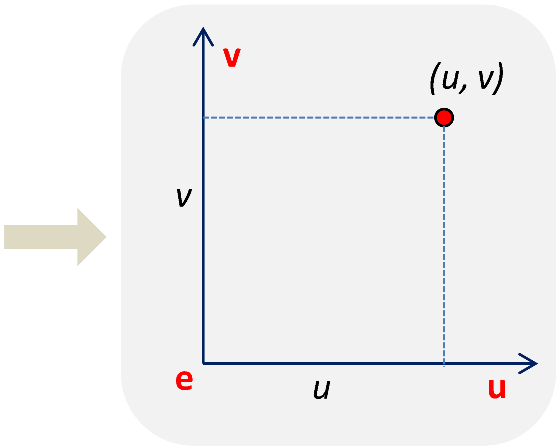


- **e**: *viewpoint*
- **u**: *right*
- **v**: *up*
- **w**: *back*

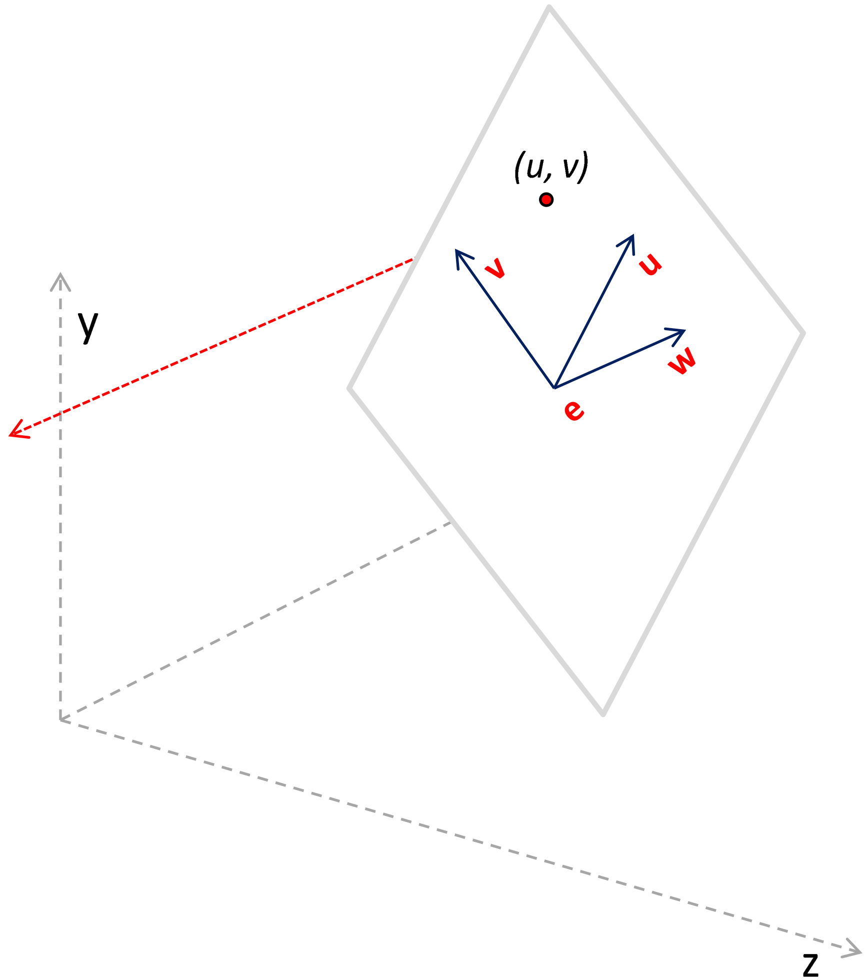
Camera Frame (3/11)

- ray origin = $\mathbf{e} + u\,\mathbf{u} + v\,\mathbf{v}$
  - ray direction = $-\mathbf{w}$

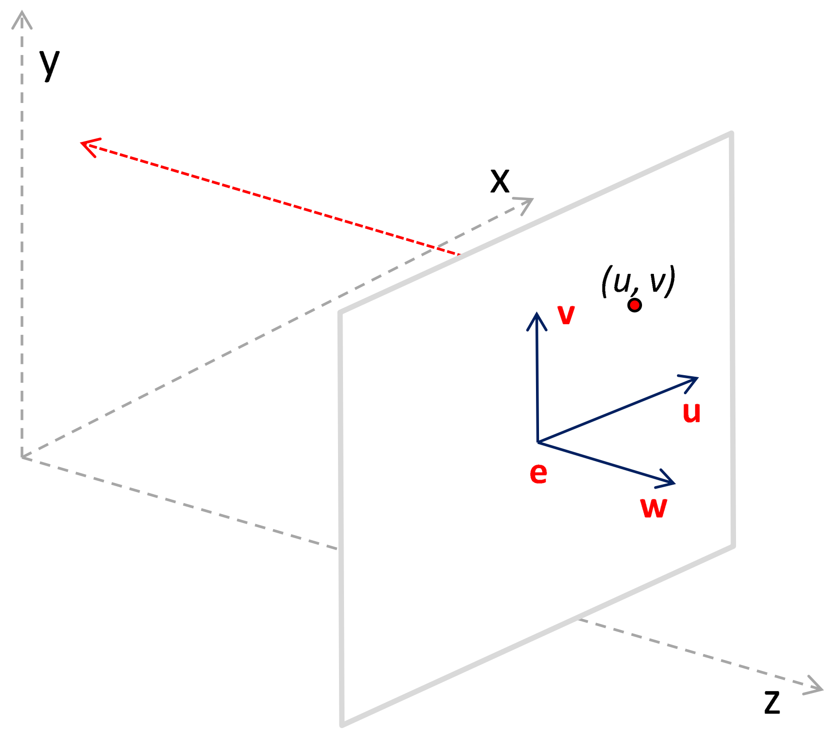




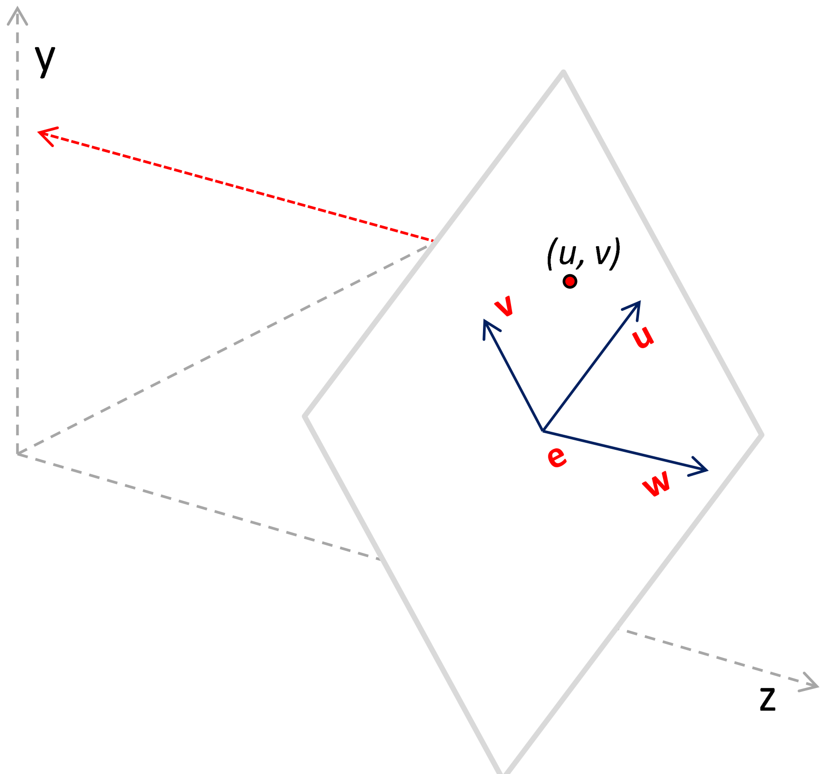
Camera Frame (4/11)



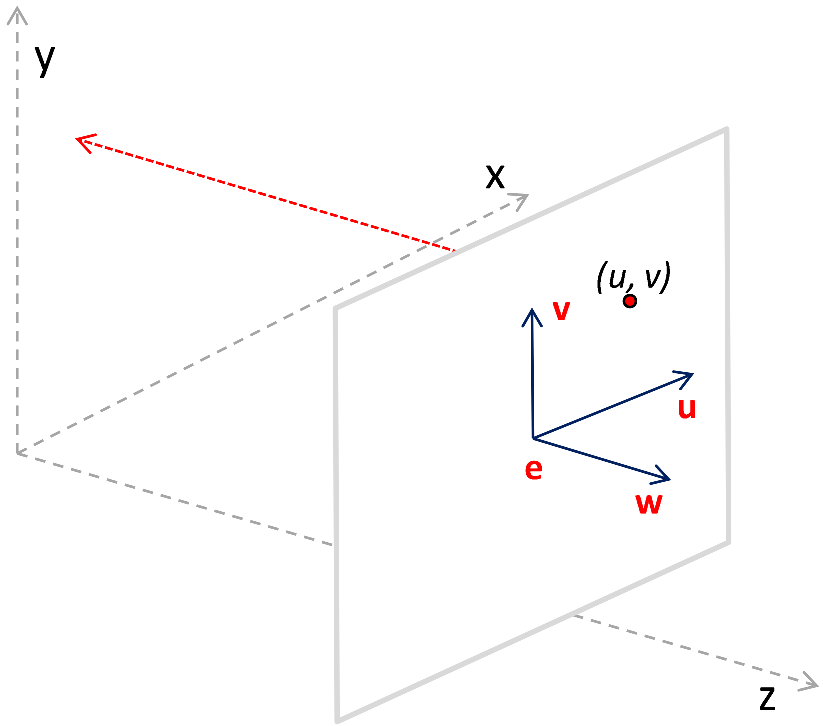
Camera Frame (5/11)



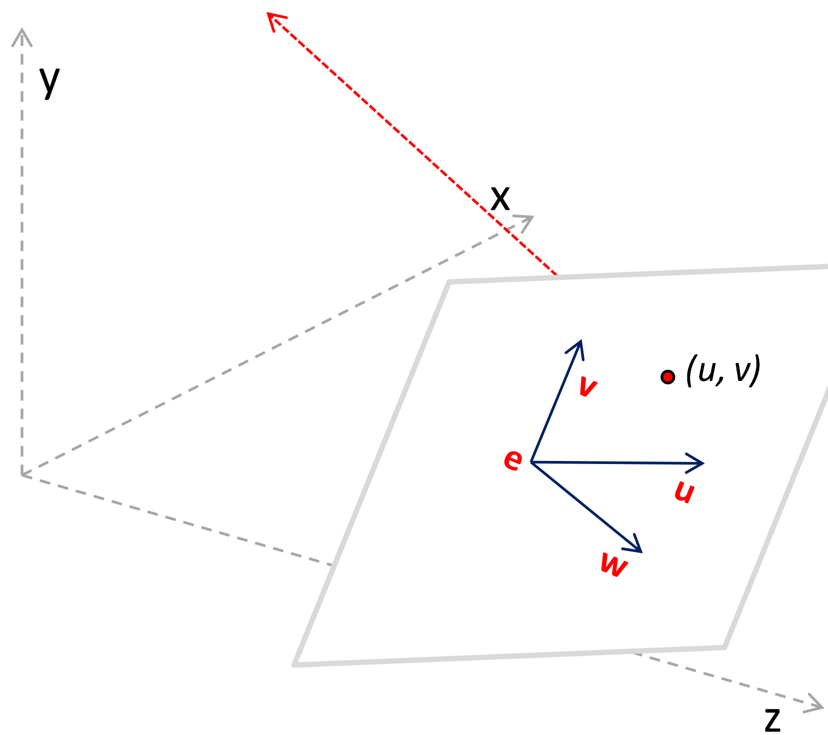
Camera Frame (6/11)



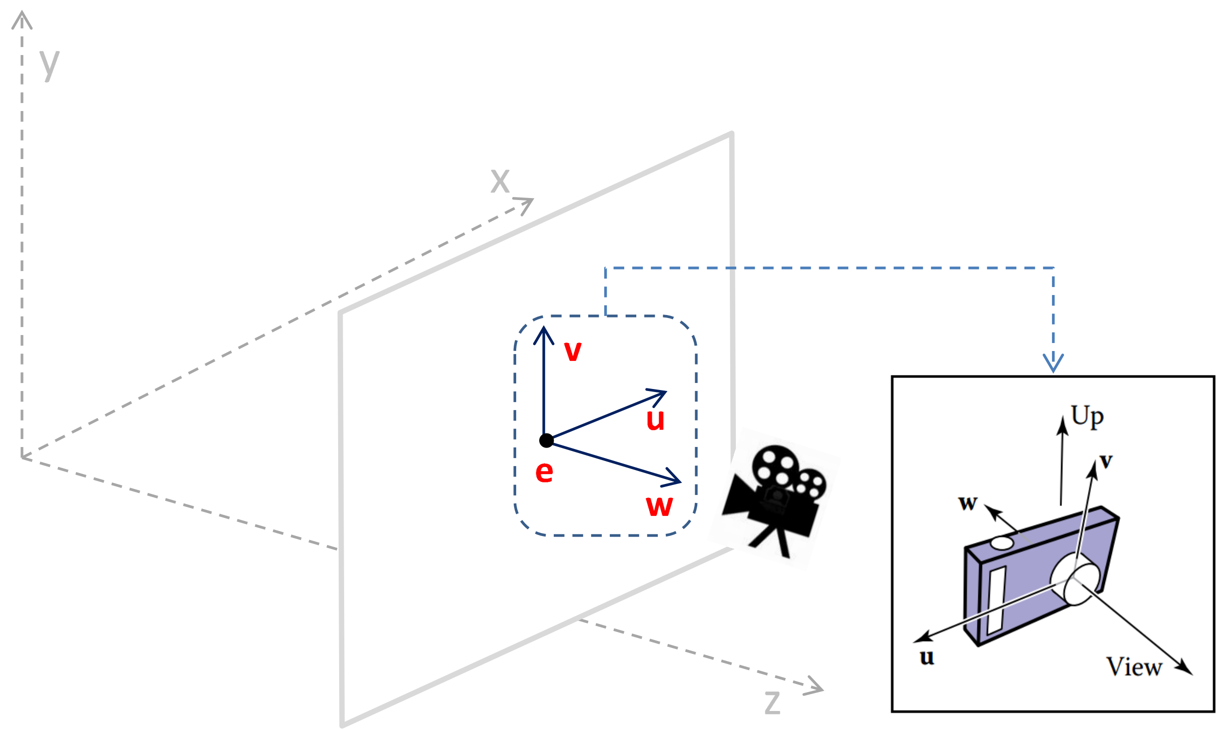
Camera Frame (7/11)



Camera Frame (8/11)

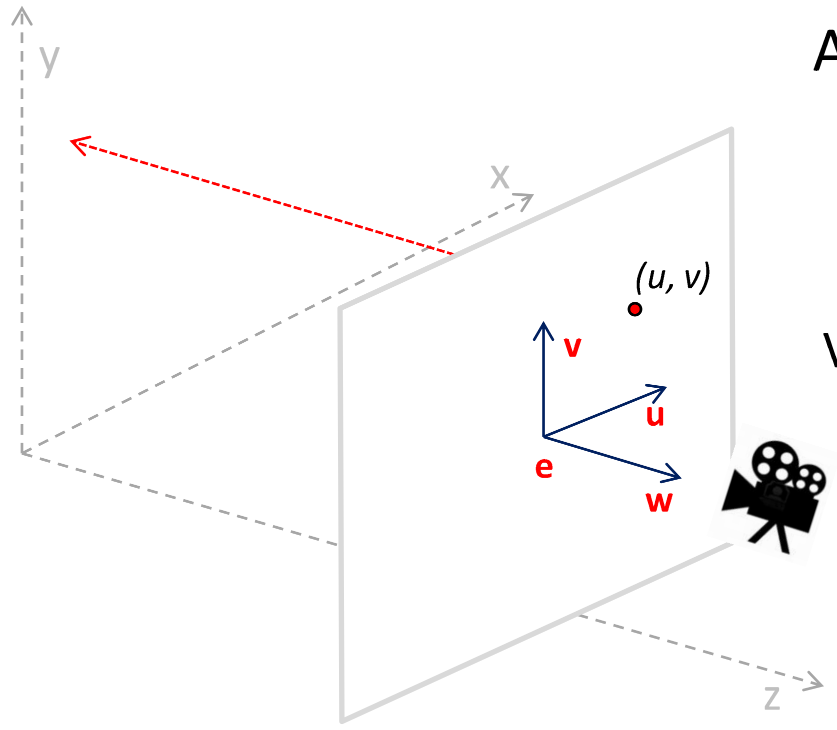


Camera Frame (9/11)



Camera Frame (10/11)

Camera frame: (Camera coordinate)



e: viewpoint

And 3 basis vectors:

- **u**: right
- **v**: up
- **w**: back

View direction **(−w)**

Camera Frame (11/11)

Orthographic:

- ray direction = −**w**
- ray origin = **e** + u **u** + v **v**

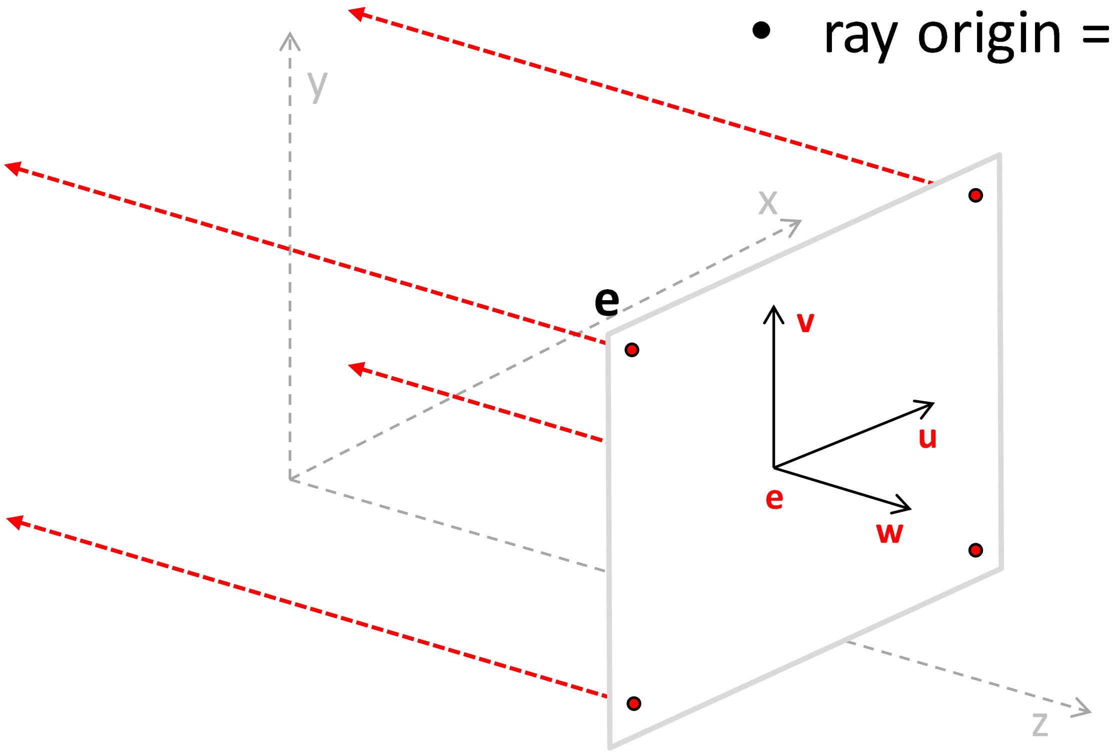


Image Plane (1/4)

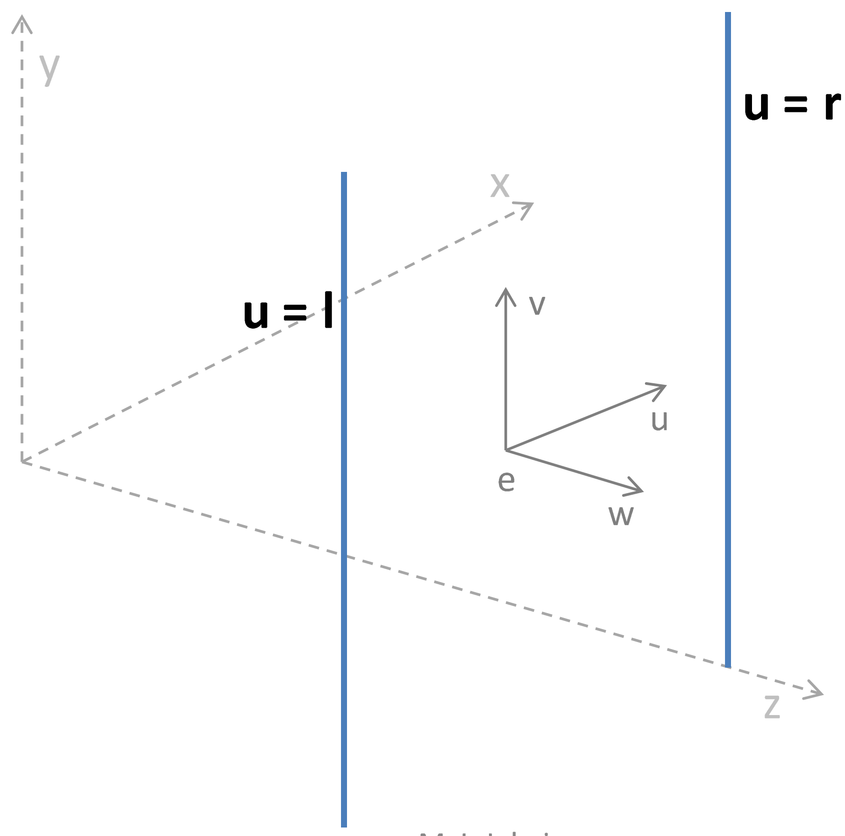


Image Plane (2/4)

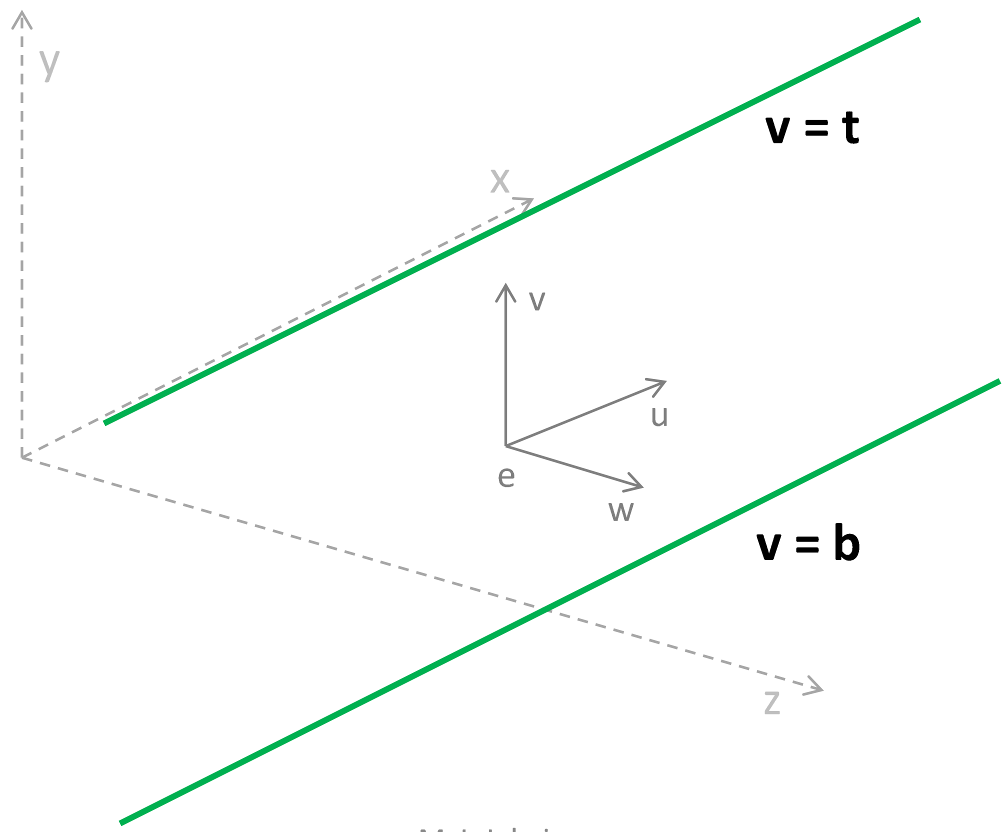


Image Plane (3/4)

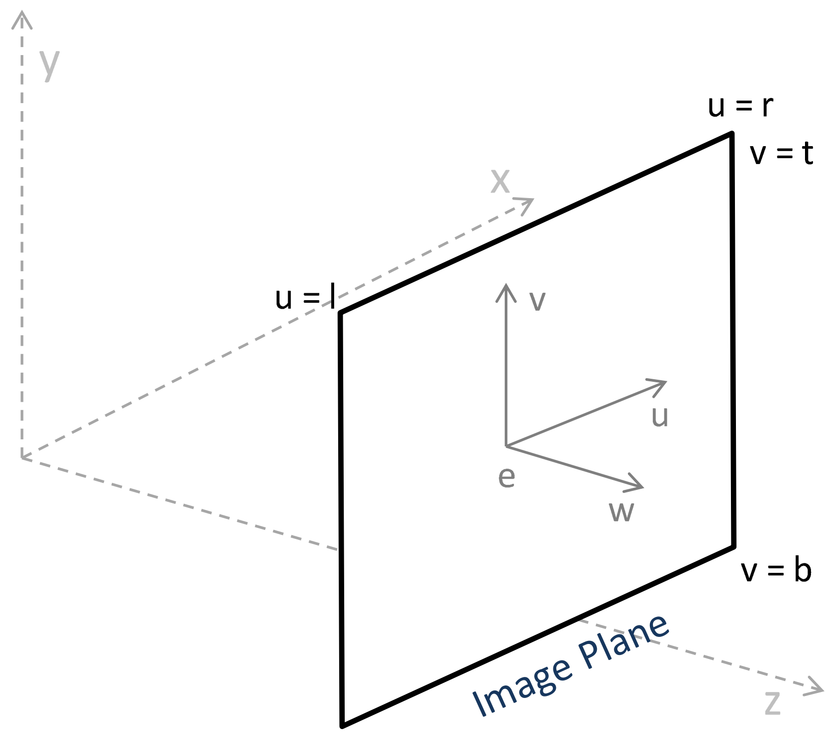
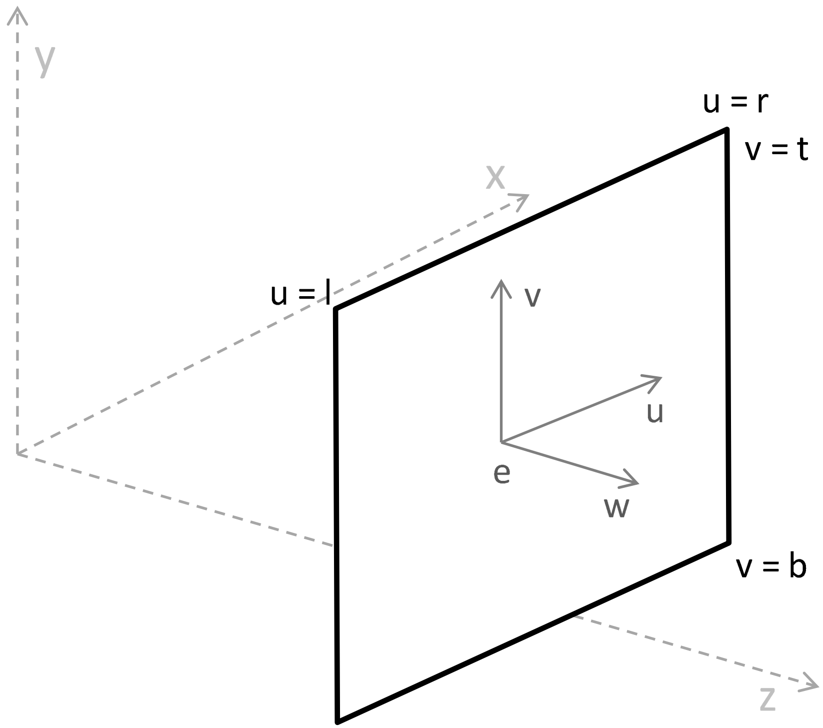
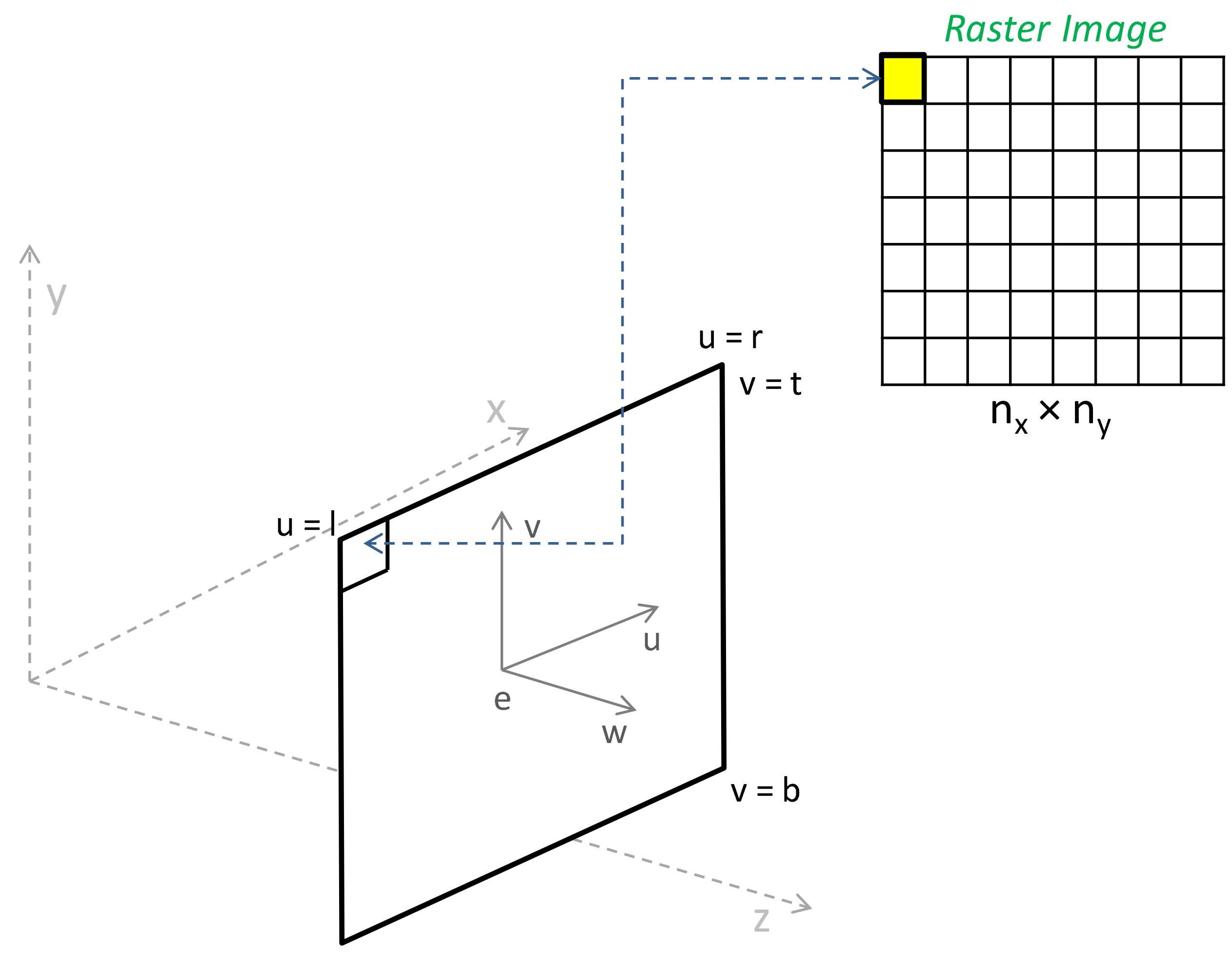


Image Plane (4/4)

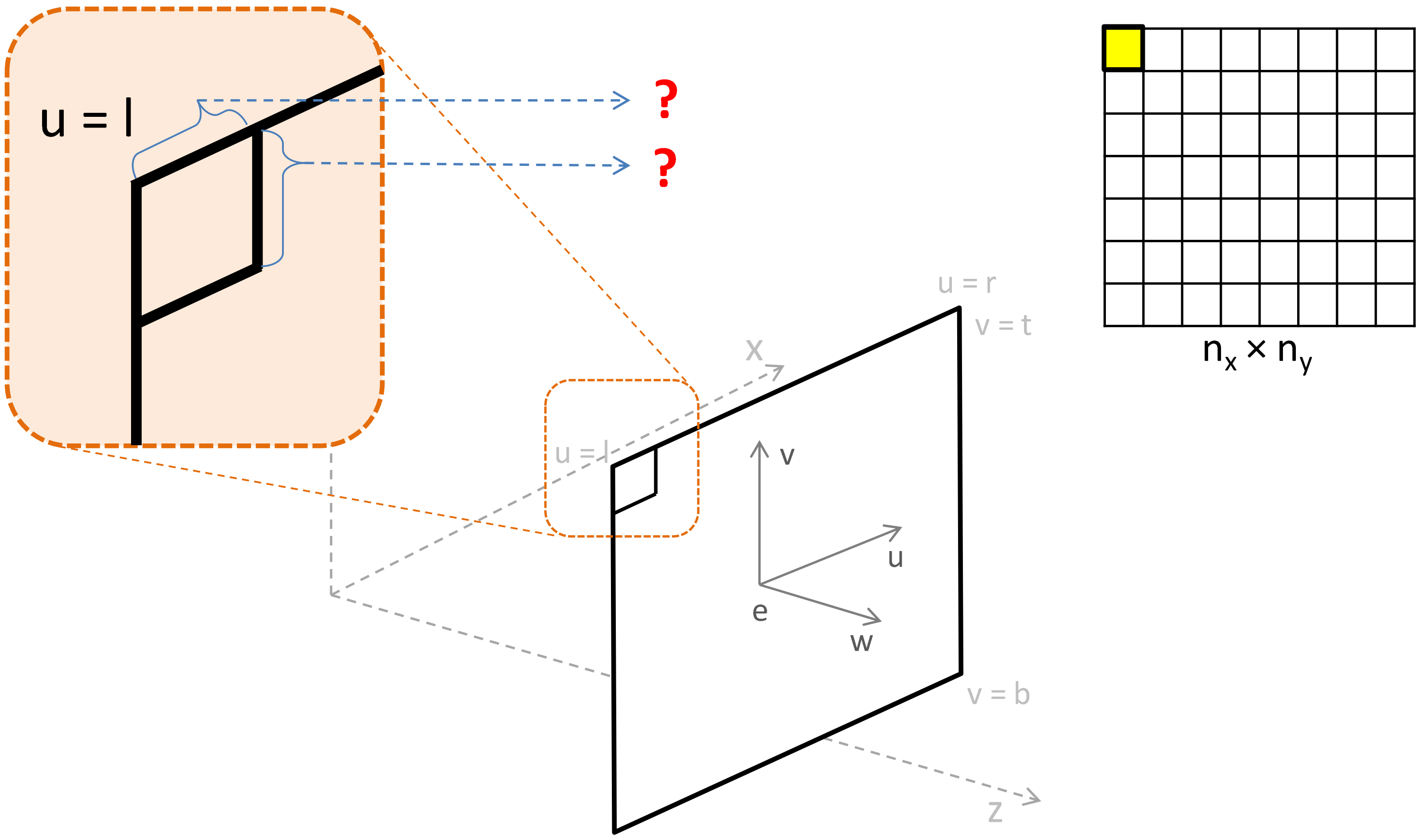
Q: determine the area of the image plane in terms of *l, r, t and b.*



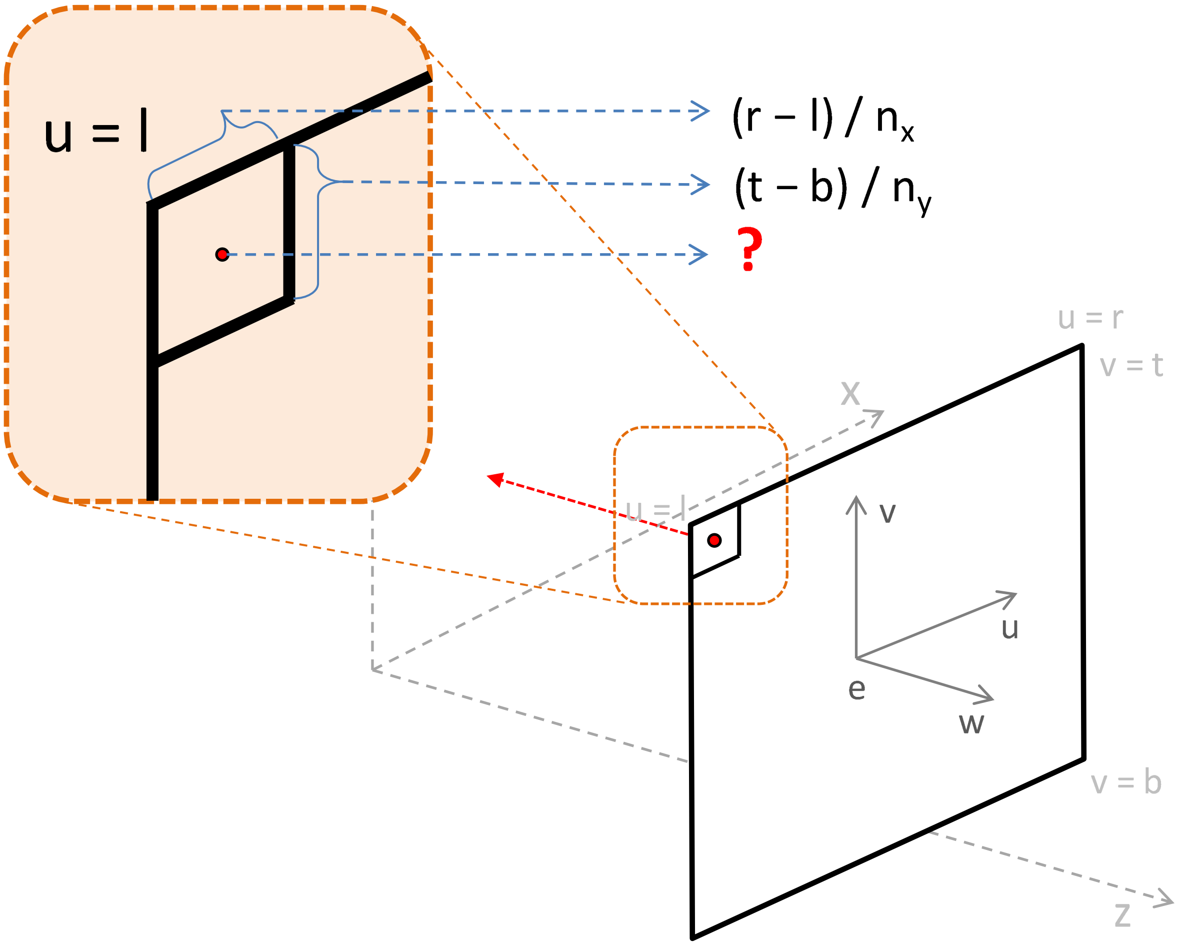
Raster Image ↔ *Image Plane* (1/8)

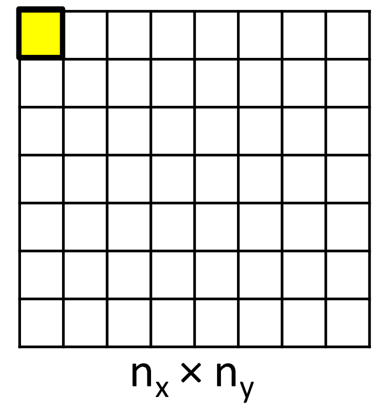


Raster Image ↔ Image Plane (2/8)

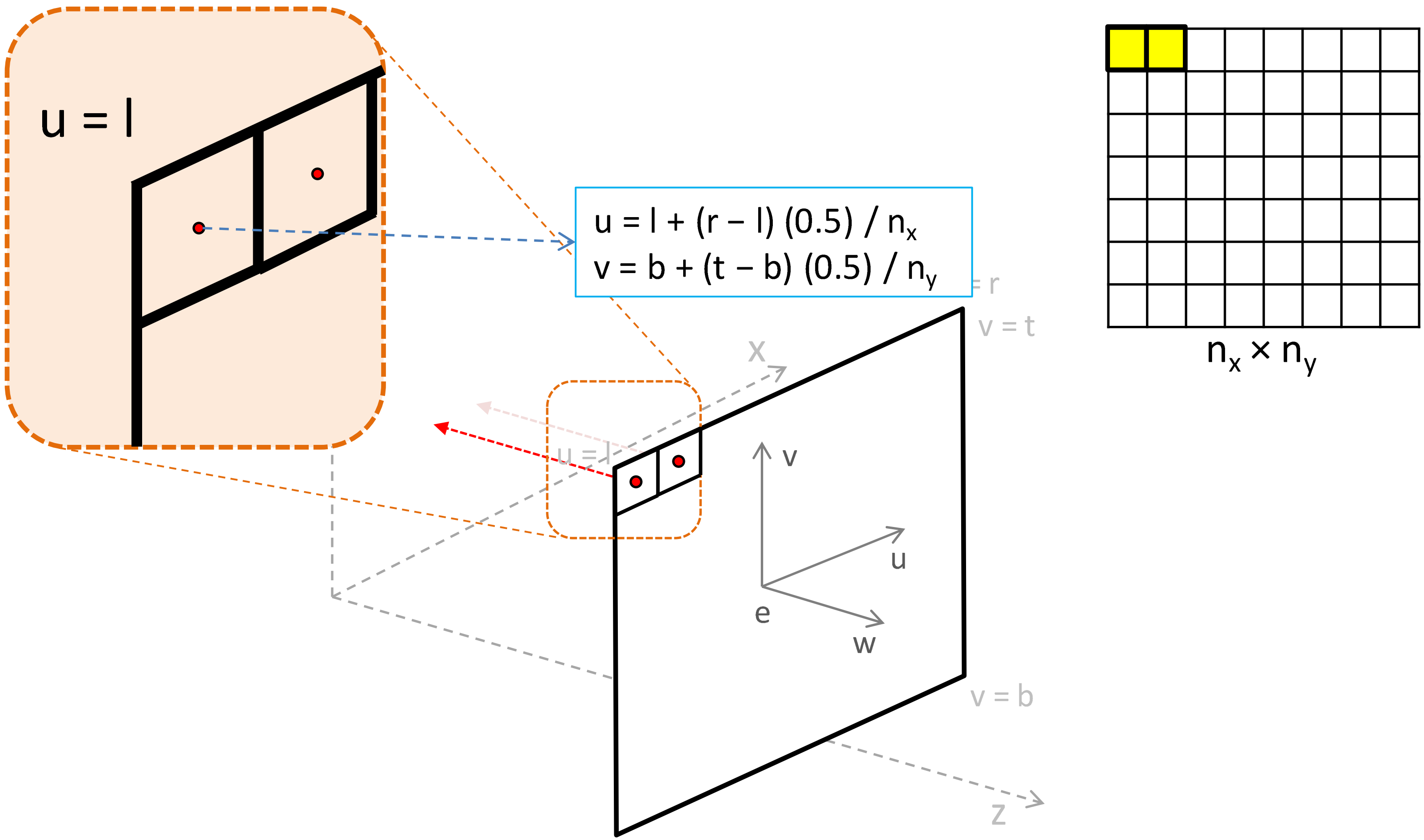


Raster Image ↔ Image Plane (3/8)

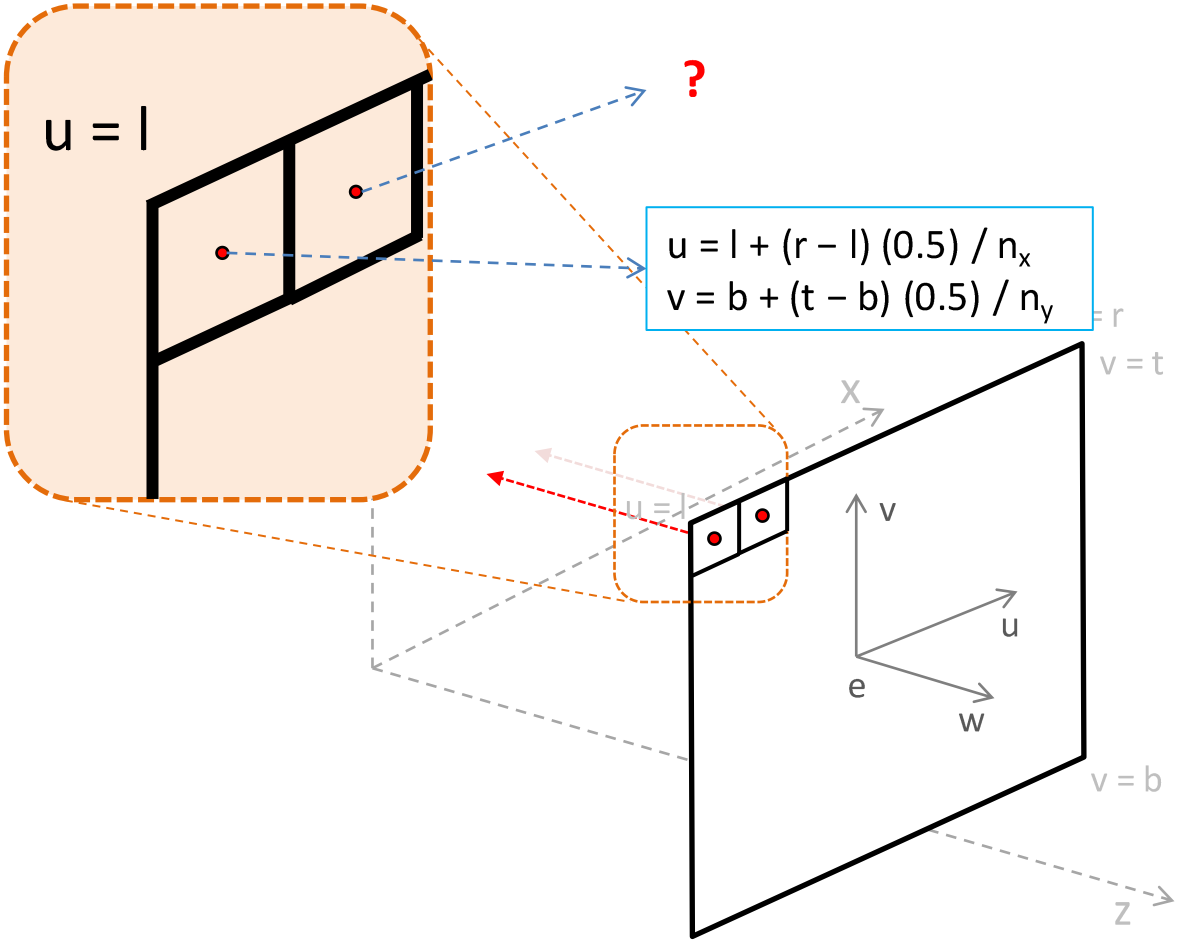




Raster Image ↔ Image Plane (4/8)

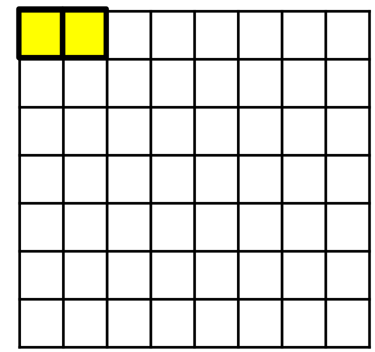


Raster Image ↔ Image Plane (5/8)

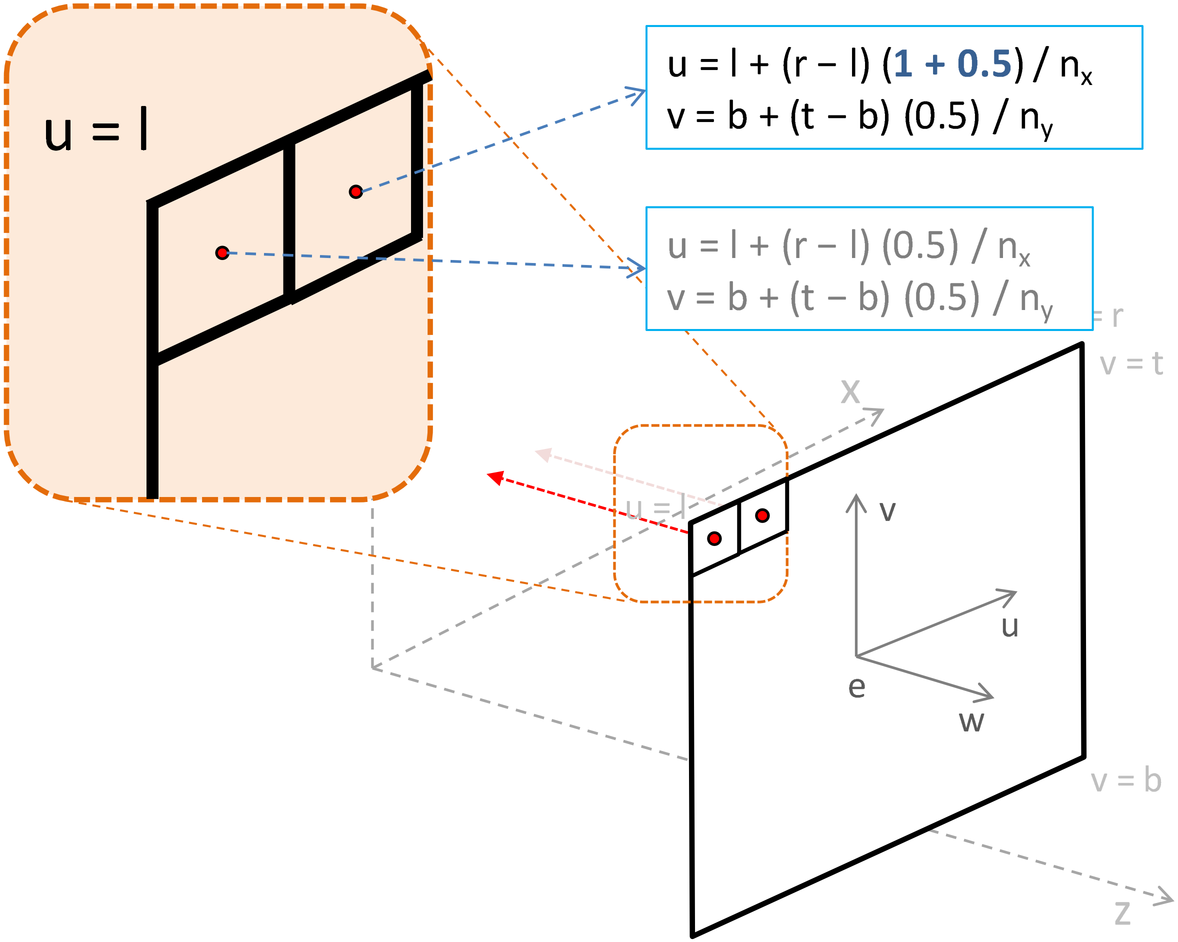


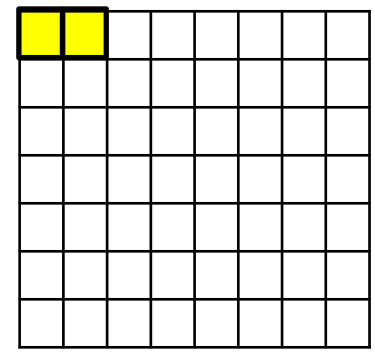
$u = l + (r - l)\,(0.5) / n_x$

$v = b + (t - b)\,(0.5) / n_y$

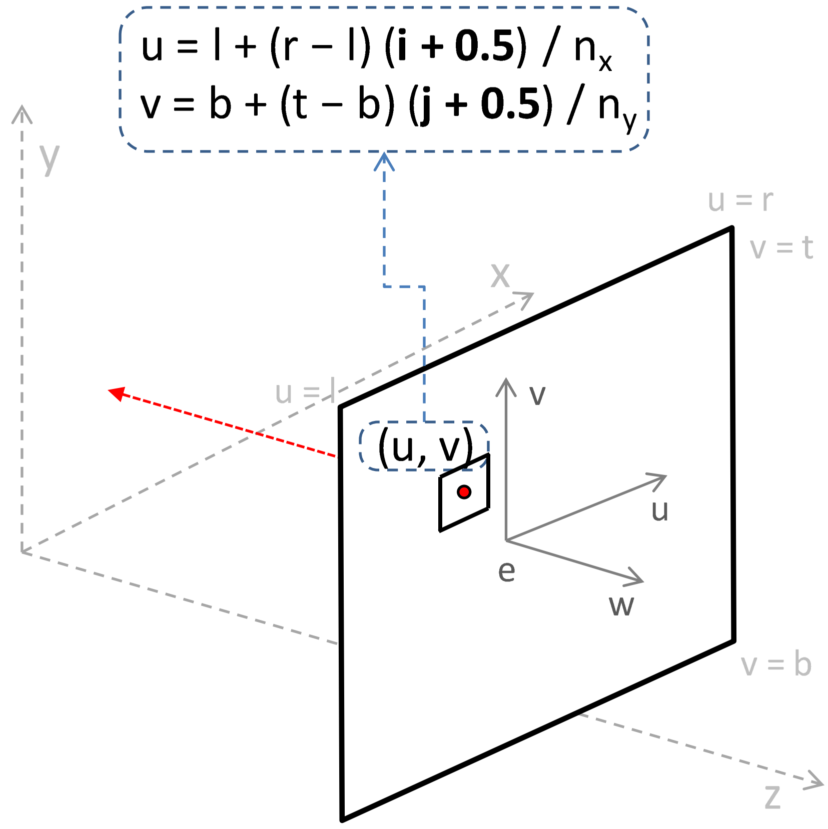


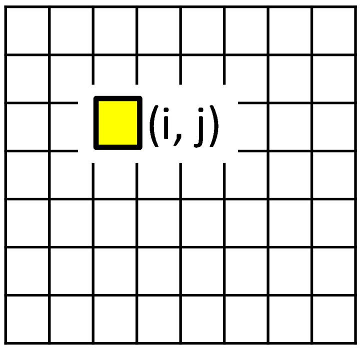
Raster Image ↔ Image Plane (6/8)





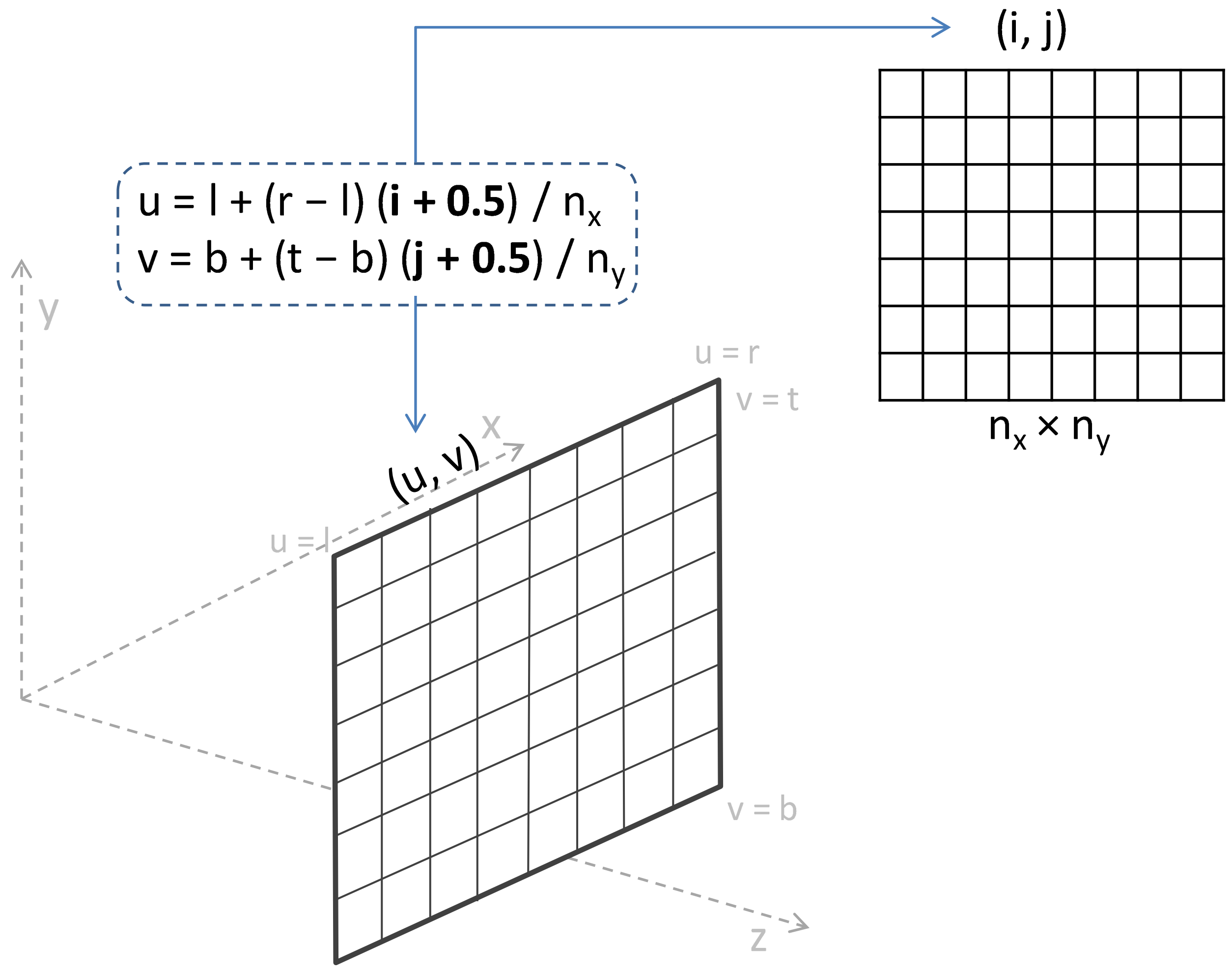
Raster Image ↔Image Plane (7/8)



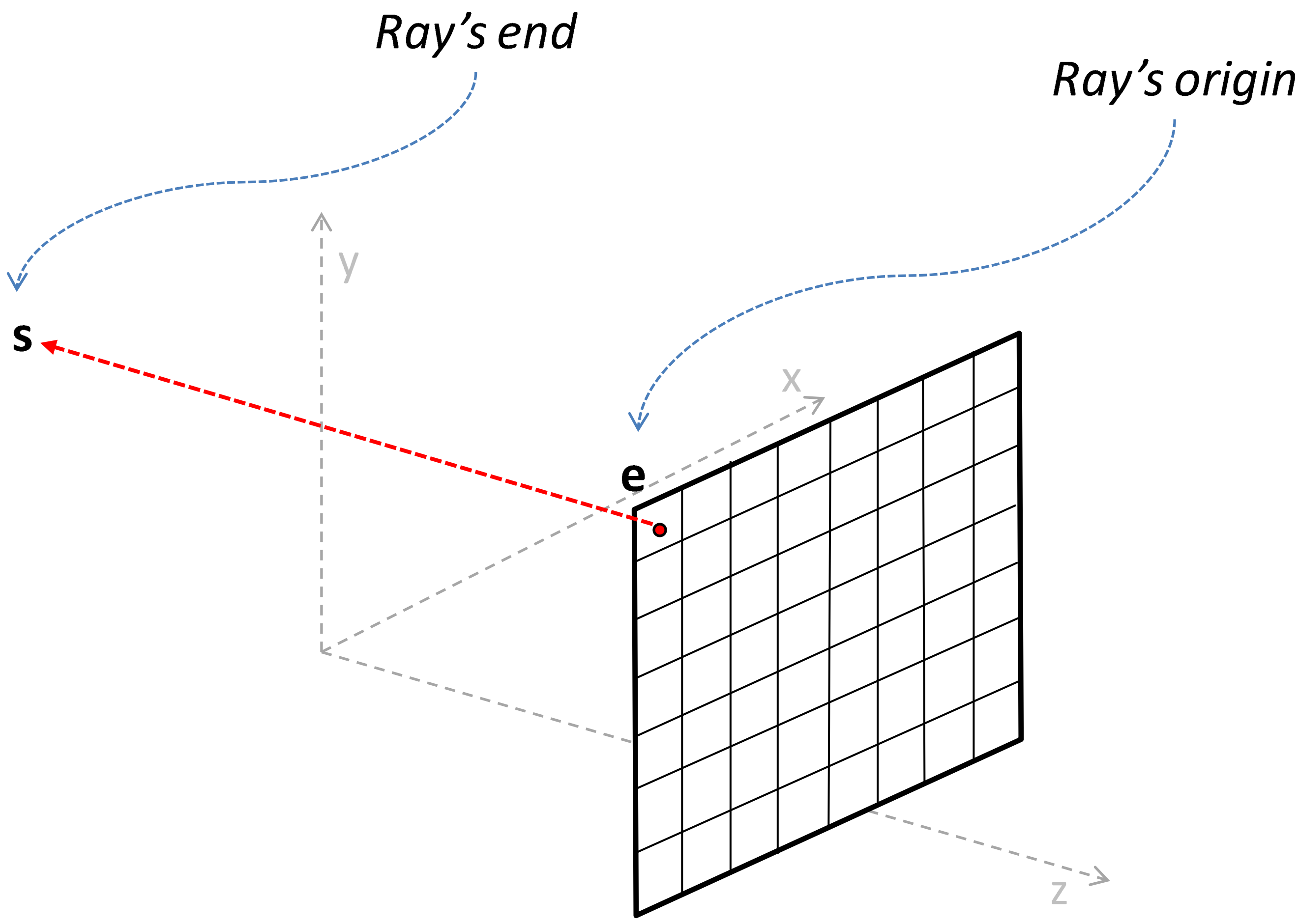


$n_x \times n_y$

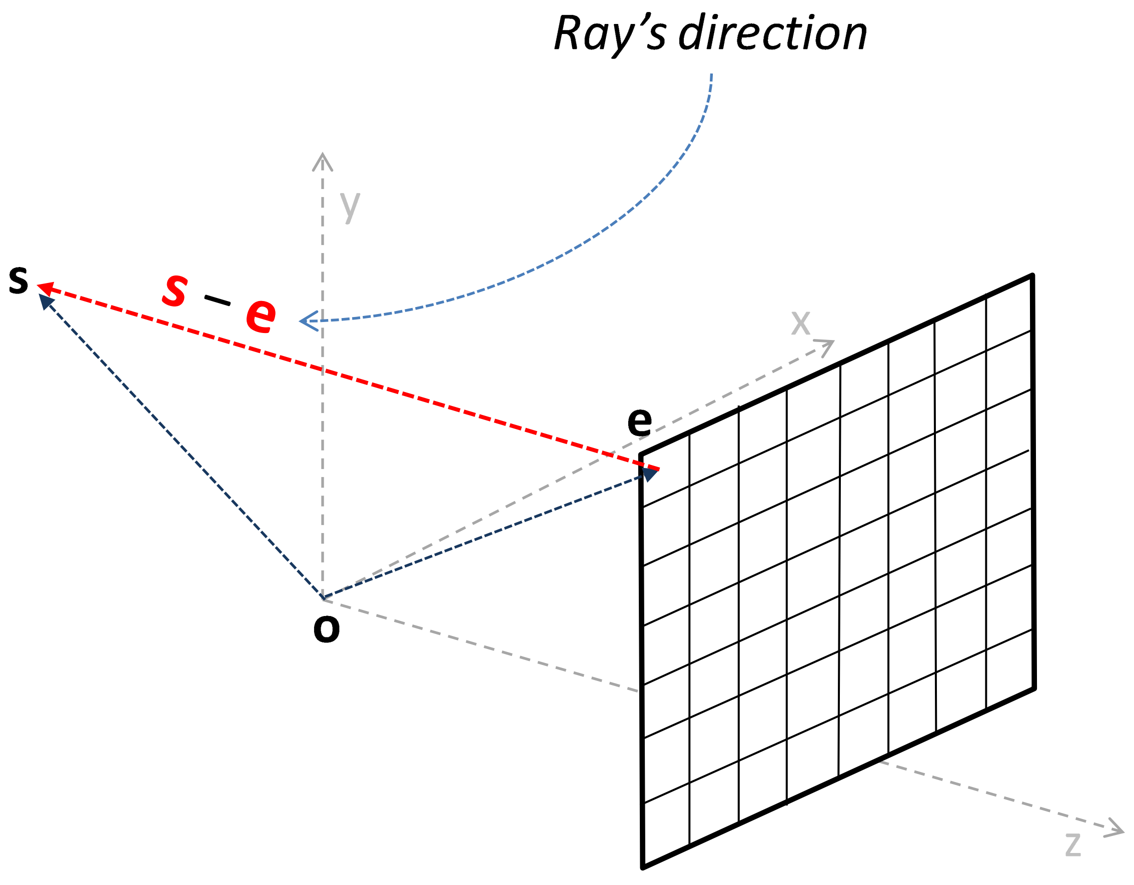
Raster Image ↔ Image Plane (8/8)



Computing Viewing Rays (1/4)

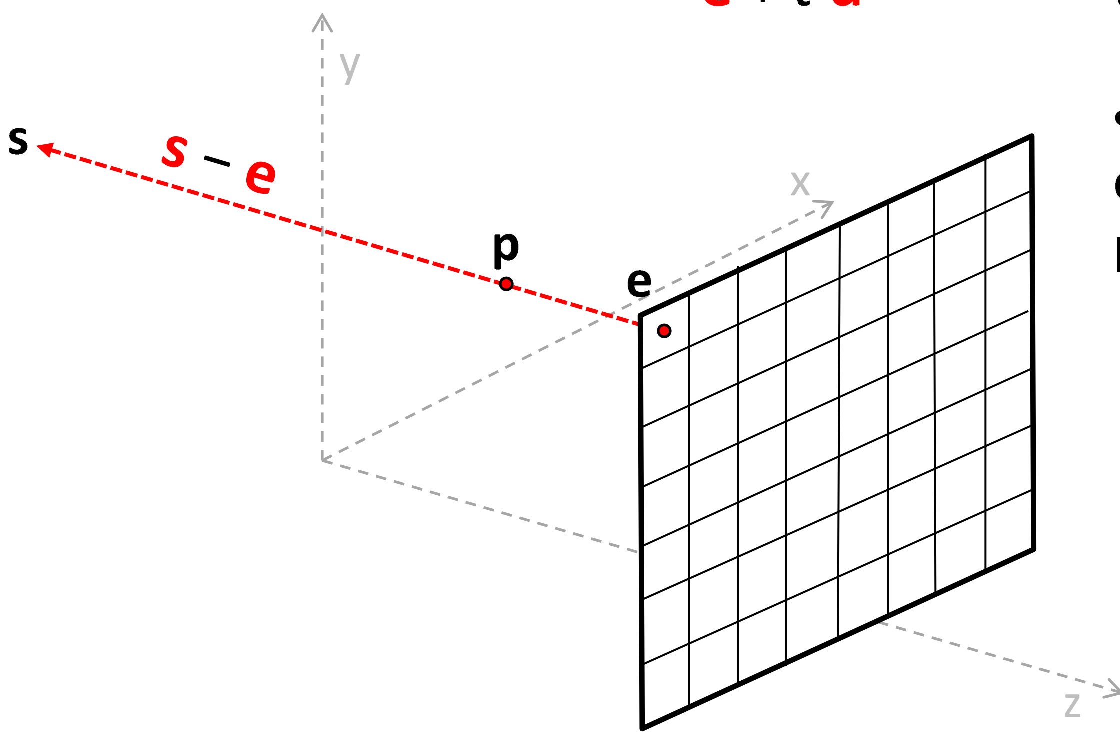


Computing Viewing Rays (2/4)



Computing Viewing Rays (3/4)

$$\mathbf{p} = \mathbf{e} + t\,(\mathbf{s} - \mathbf{e}) = \mathbf{e} + t\,\mathbf{d}$$

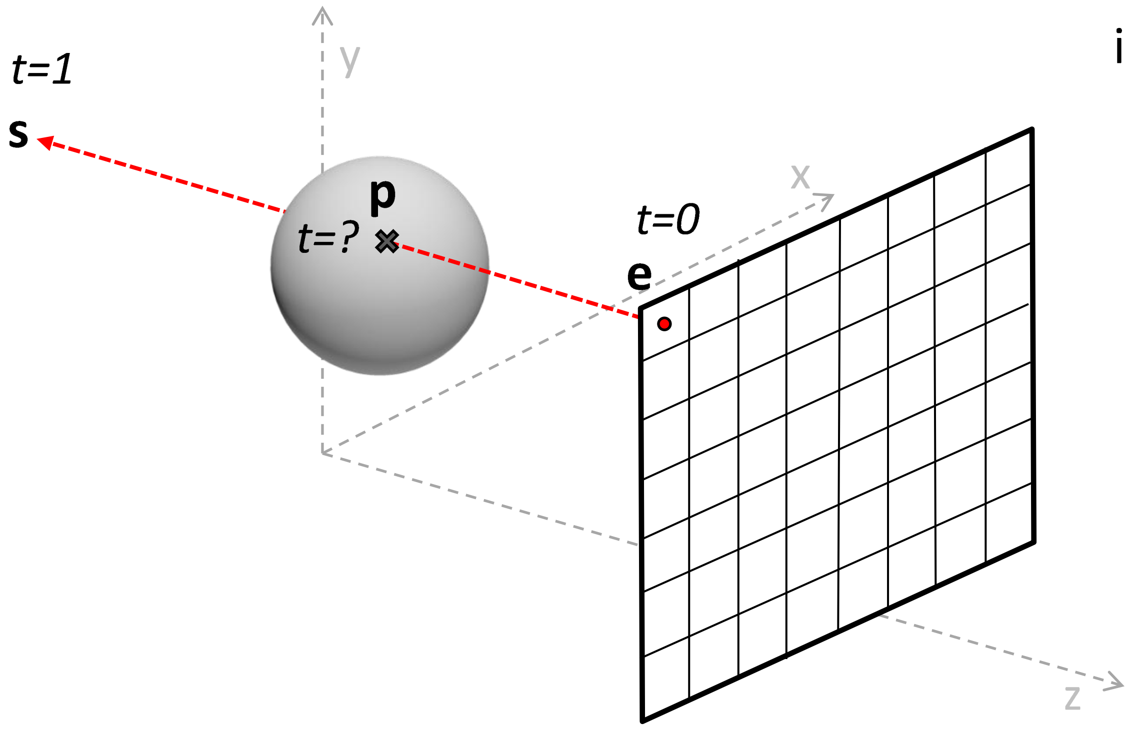


- Advancing from **e** along the vector **(s – e)**
- With a fractional distance ***t*** to find the point **p**

Computing Viewing Rays (4/4)

$$\mathbf{p} = \mathbf{e} + t\,(\mathbf{s} - \mathbf{e})$$
$$= \mathbf{e} + t\,\mathbf{d}$$

- We can use ***t*** to determine the intersection point **p**

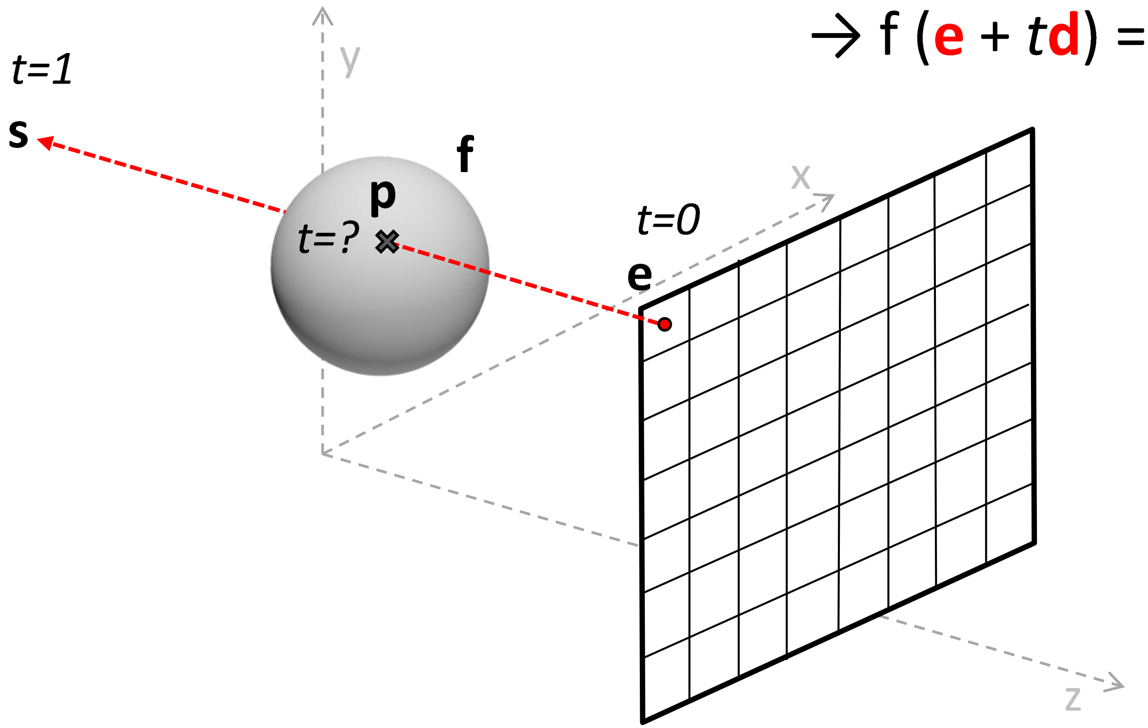


Ray - Sphere Intersection (1/8)

We have, $\mathbf{p} = \mathbf{e} + t\,(\mathbf{s} - \mathbf{e}) = \mathbf{e} + t\,\mathbf{d}$

$\rightarrow$ $f(\mathbf{p}) = 0$

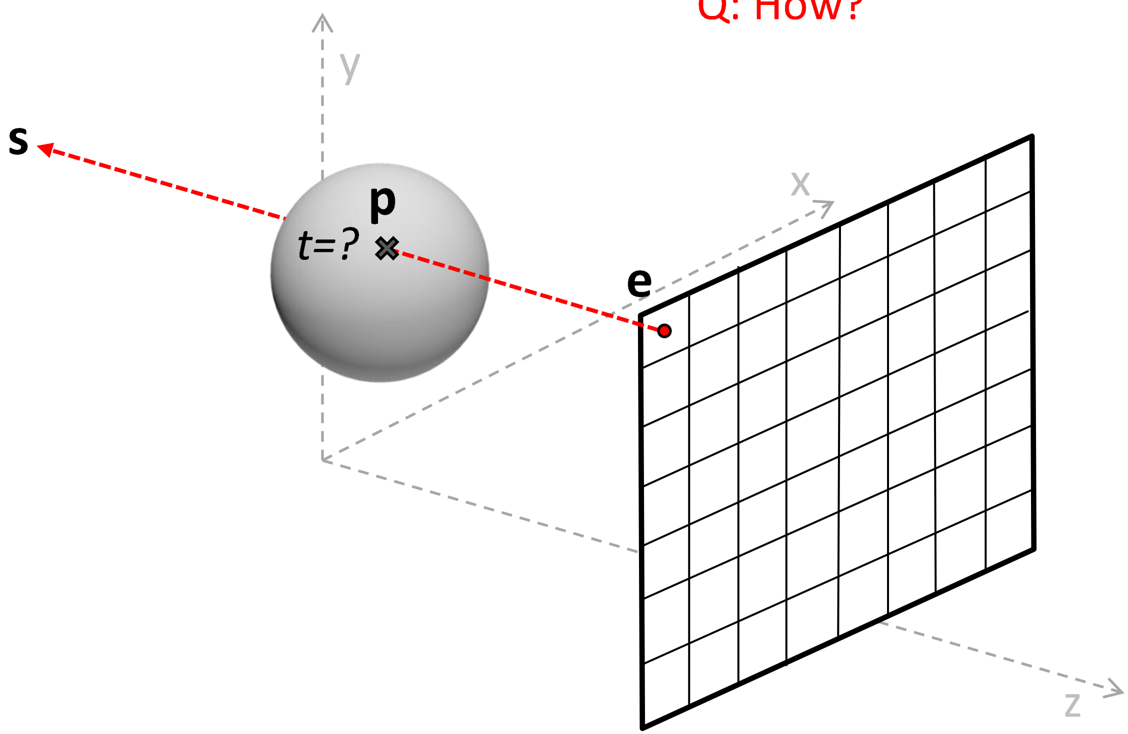
$\rightarrow$ $f(\mathbf{e} + t\mathbf{d}) = 0$



Ray - Sphere Intersection (2/8)

$\rightarrow$ $(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 - R^2 = 0$

$\rightarrow$ $\underbrace{(\mathbf{p} - \mathbf{c}) \,.\, (\mathbf{p} - \mathbf{c}) - R^2 = 0}_{\text{Q: How?}}$



Ray - Sphere Intersection (3/8)

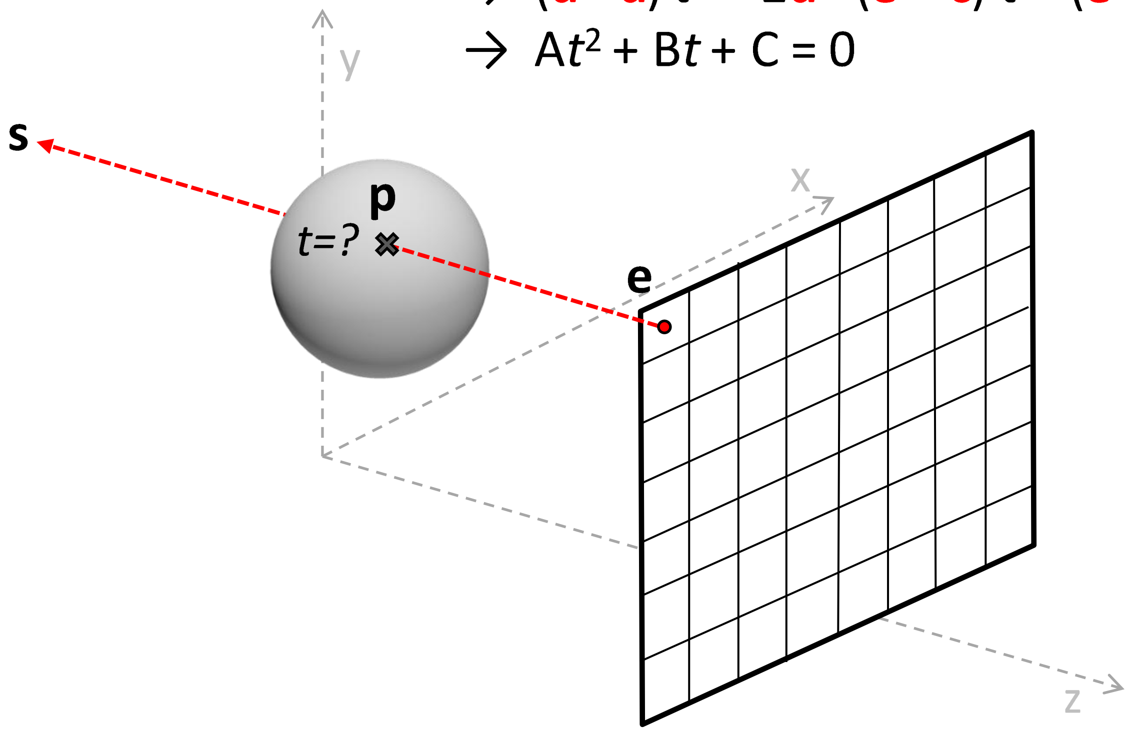
→ $(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 - R^2 = 0$

→ $(\mathbf{p} - \mathbf{c}) . (\mathbf{p} - \mathbf{c}) - R^2 = 0$

→ $(\mathbf{e} + t\mathbf{d} - \mathbf{c}) \cdot (\mathbf{e} + t\mathbf{d} - \mathbf{c}) - R^2 = 0$

→ $(\mathbf{d} \cdot \mathbf{d})\, t^2 + 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{c})\, t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - R^2 = 0$

→ $At^2 + Bt + C = 0$



Ray - Sphere Intersection (4/8)

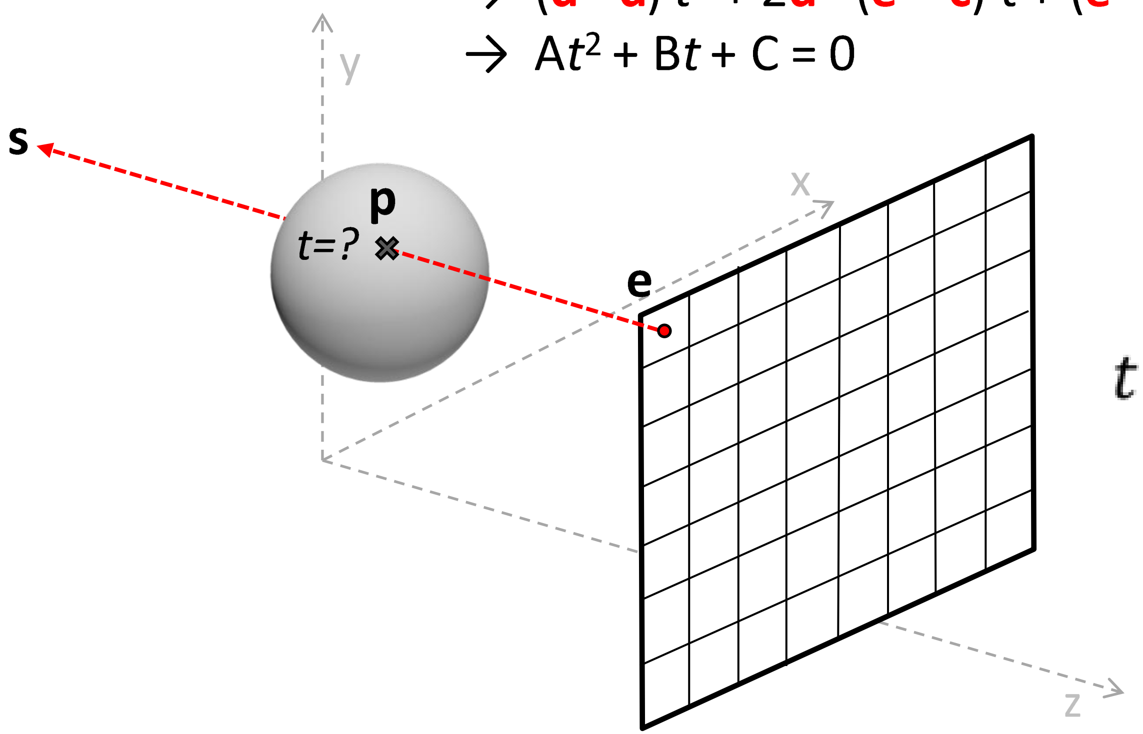
$\rightarrow$ $(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 - R^2 = 0$

$\rightarrow$ $(\mathbf{p} - \mathbf{c}) \,.\, (\mathbf{p} - \mathbf{c}) - R^2 = 0$

$\rightarrow$ $(\mathbf{e} + t\mathbf{d} - \mathbf{c}) \cdot (\mathbf{e} + t\mathbf{d} - \mathbf{c}) - R^2 = 0$

$\rightarrow$ $(\mathbf{d} \cdot \mathbf{d})\, t^2 + 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{c})\, t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - R^2 = 0$

$\rightarrow$ $At^2 + Bt + C = 0$



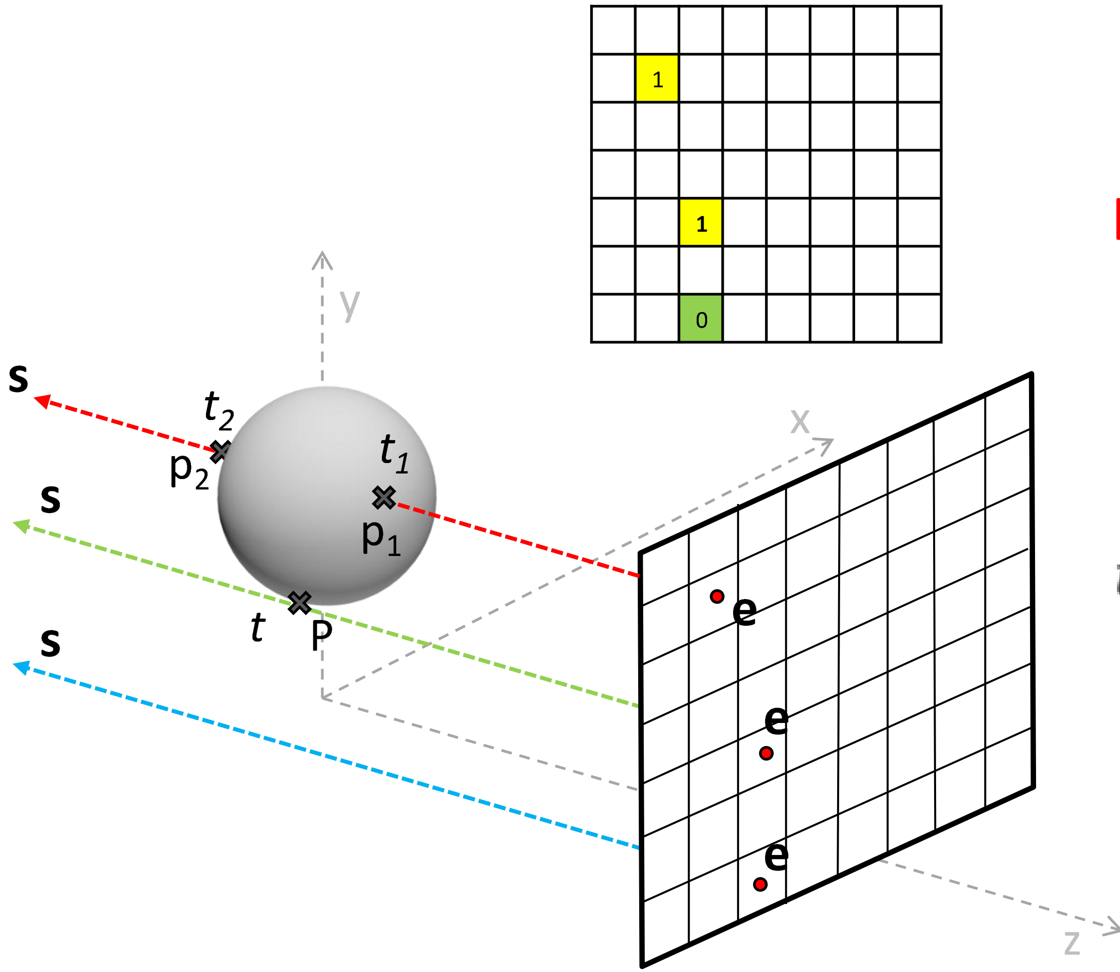
$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Ray - Sphere Intersection (5/8)

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

- ***B^2 – 4AC***, is called the discriminant and if it is –
  - **negative:** its square root is imaginary and the line and sphere **do not intersect**.
  - **positive**: there are two solutions –
    - one solution where the ray **enters** the sphere.
    - one where it **leaves**.
  - **zero:** the ray grazes the sphere, touching it at exactly **one point**.

Ray - Sphere Intersection (6/8)



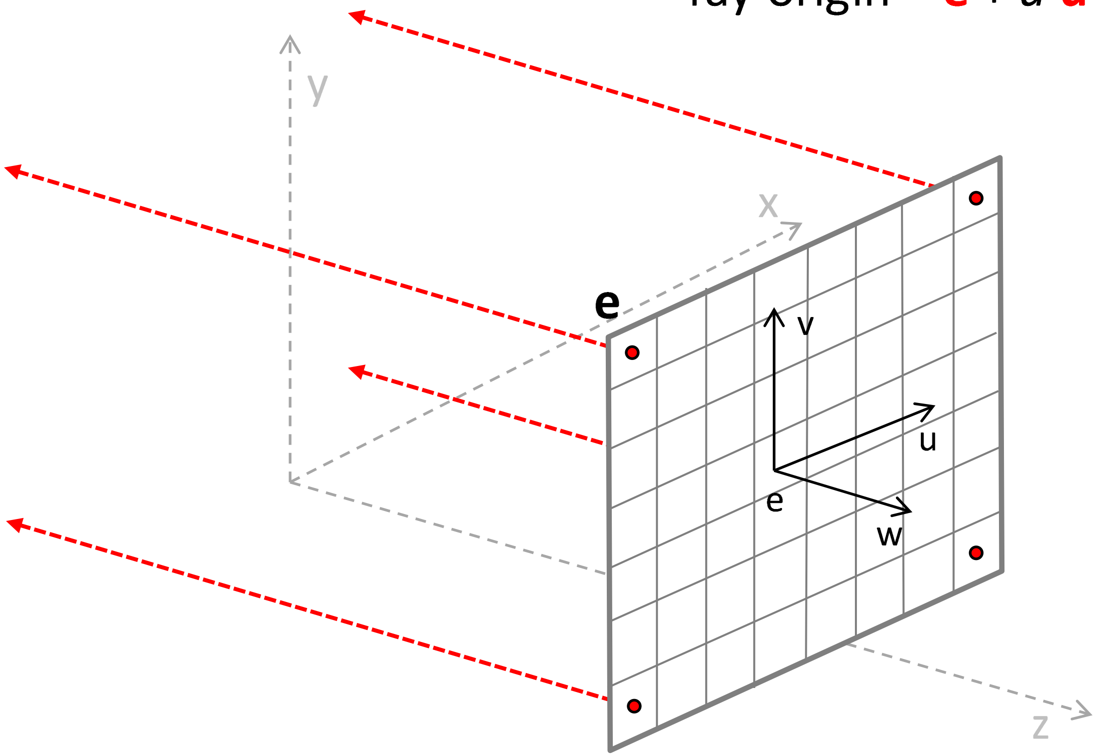
$$\mathbf{p} = \mathbf{e} + t\,(\mathbf{s} - \mathbf{e})$$
$$= \mathbf{e} + t\,\mathbf{d}$$

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Ray - Sphere Intersection (7/8)

Orthographic:

- ray direction = $-\mathbf{w}$
- ray origin = $\mathbf{e} + u\,\mathbf{u} + v\,\mathbf{v}$

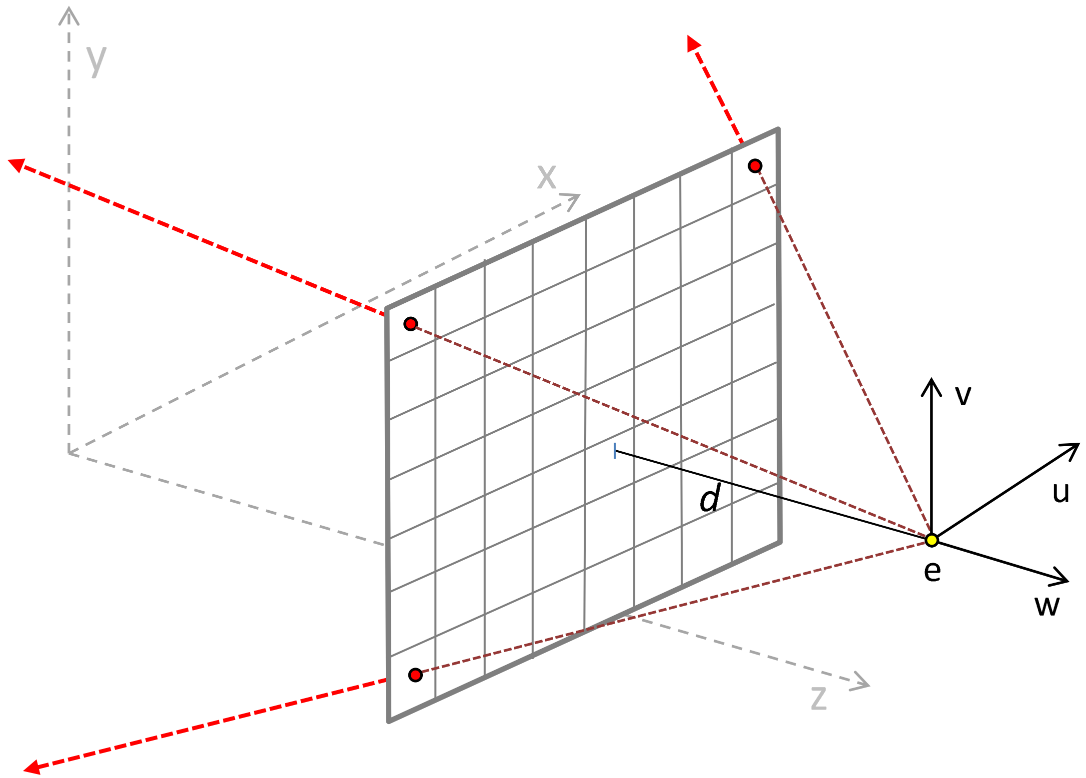


Ray - Sphere Intersection (8/8)

Perspective:

- ray direction = $-d\ \mathbf{w} + u\ \mathbf{u} + v\ \mathbf{v}$
- ray origin = $\mathbf{e}$

Q: why?

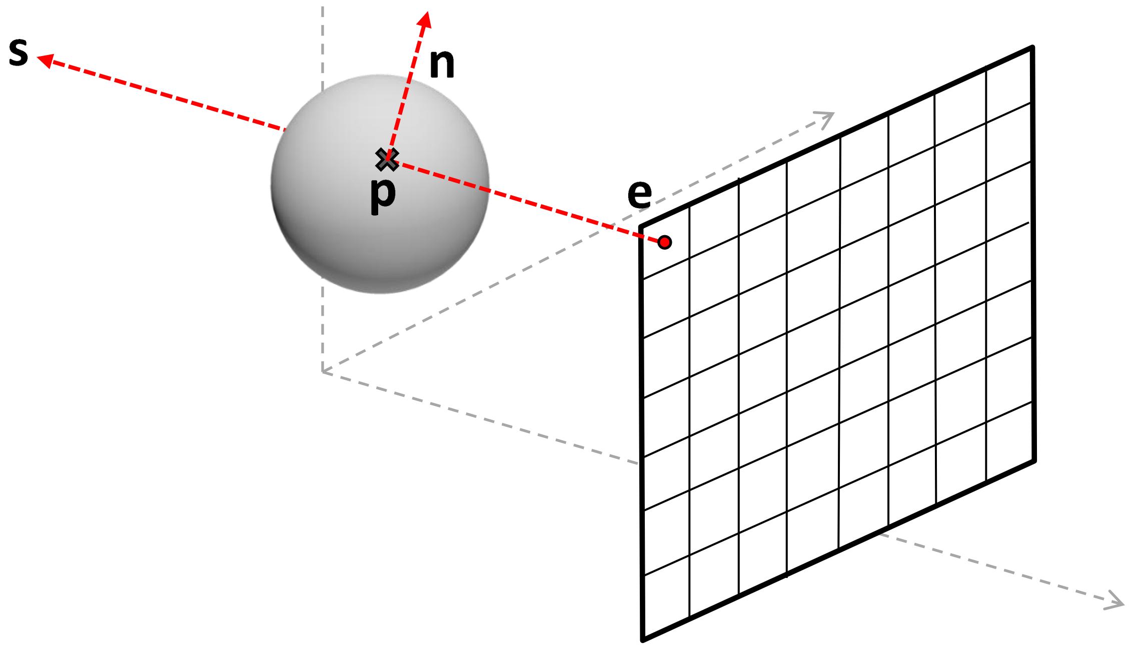


Shading (1/3)

Normal vector at point **p**:

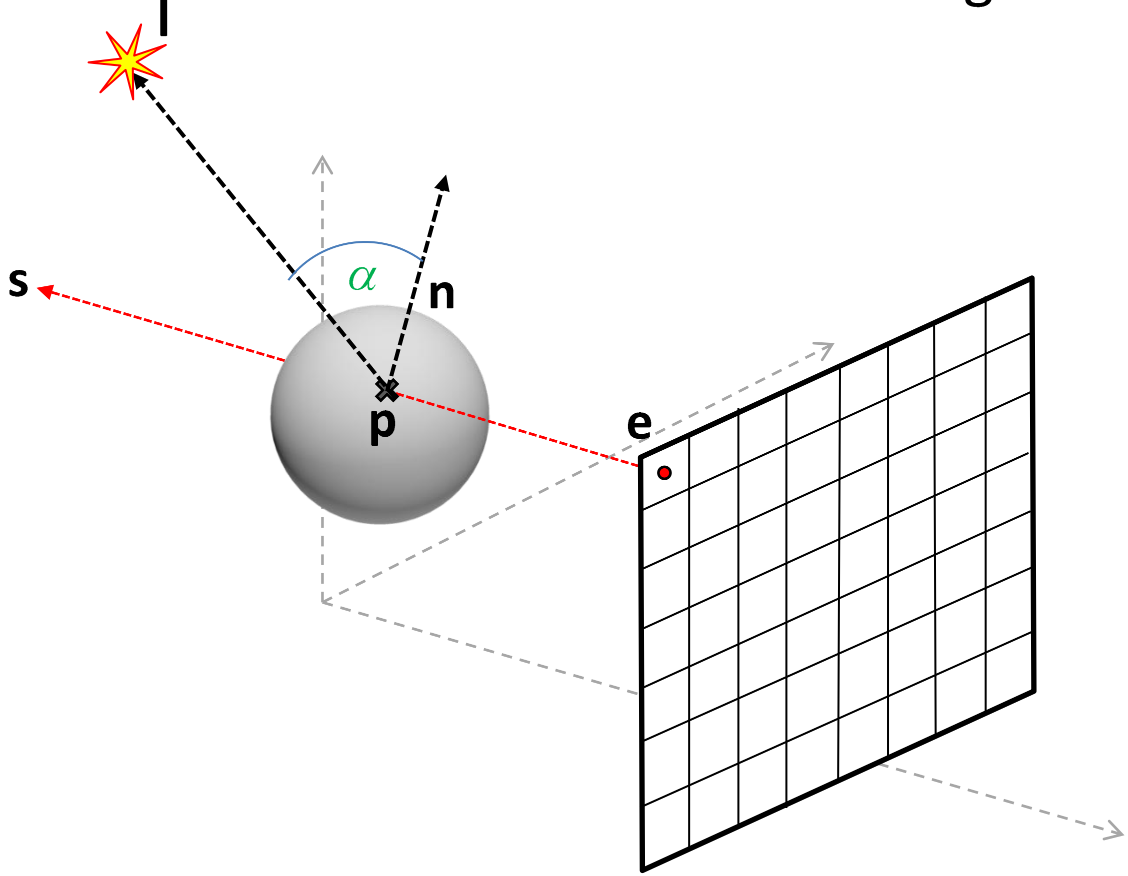
- gradient **n** = 2 (**p** − **c**).
- unit normal is (**p** − c)/R.

[See section 2.5.4]



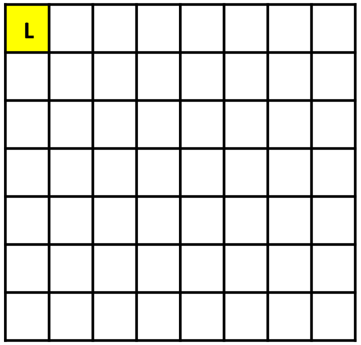
Shading (2/3)

Lambertian Shading: $L = k_d\, P\, max\,(0, \mathbf{n} \cdot \mathbf{l})$



where,

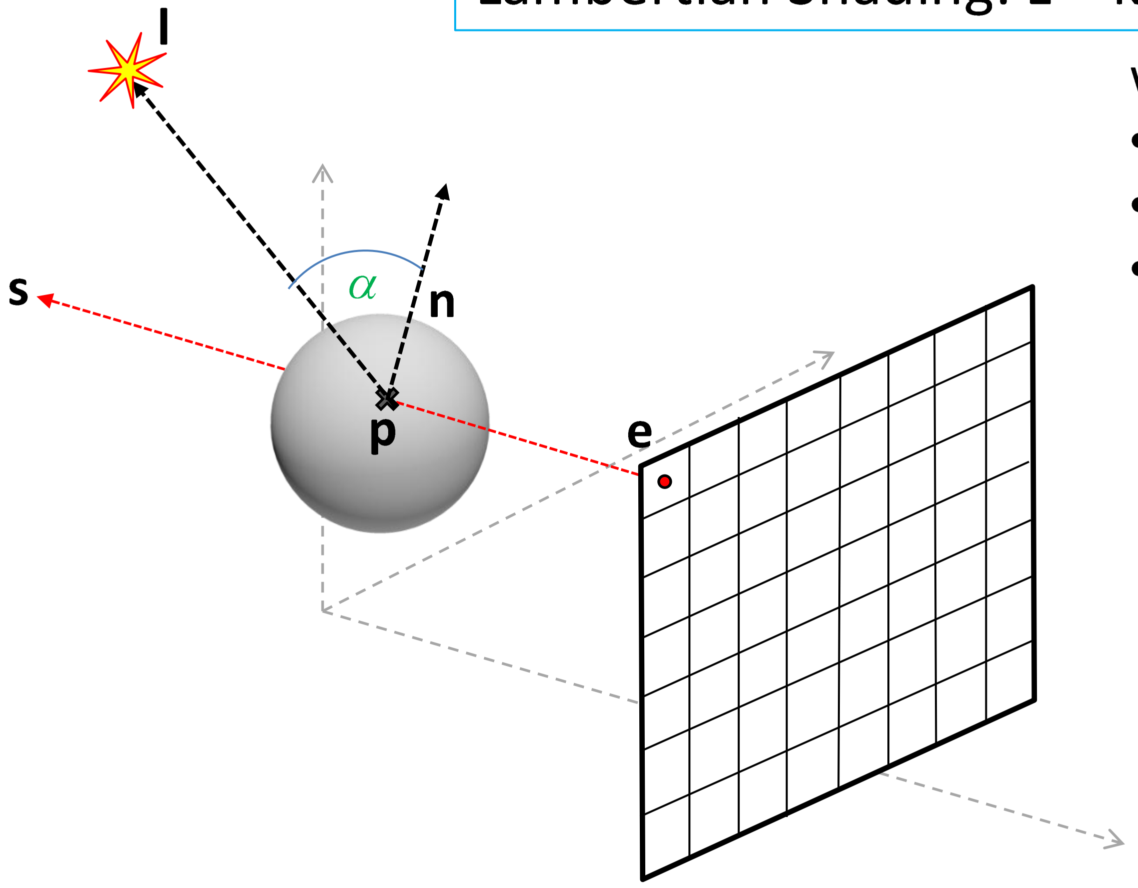
- L = pixel color
- k_d = surface color
- P = intensity of the light source.



Shading (3/3)

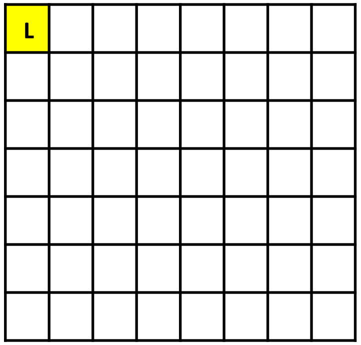
Q: Are we considering angle in this formula? If yes – how?

Lambertian Shading: $L = k_d\, P\, max\, (0, \mathbf{n} \cdot \mathbf{l})$



where,

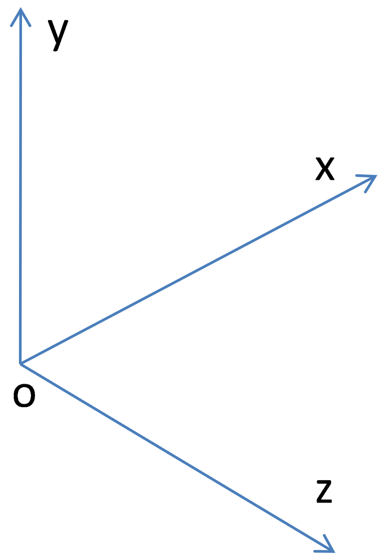
- L = pixel color
- k_d = surface color
- P = intensity of the light source.



Additional Reading

- 4.6: A Ray-Tracing Program

Practice Problem (1/2)



Camera frame *(orthographic)***:**

- $\mathbf{e} = [4, 4, 6]$; $\mathbf{u} = [1, 0, 0]$; $\mathbf{v} = [0, 1, 0]$; $\mathbf{w} = [0, 0, 1]$
  - Plot the camera frame on the given axis.

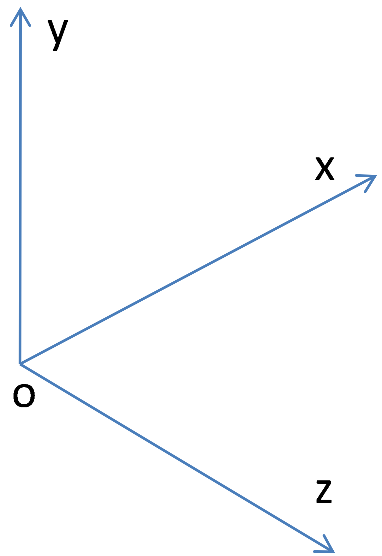
Viewing Ray:

- $ray_1.origin = \mathbf{e} + 2\mathbf{u} + 2\mathbf{v}$; $ray_1.end = [6, 6, 0]$
- $ray_2.origin = \mathbf{e} - 1\mathbf{u} + 1\mathbf{v}$; $ray_2.end = [4, 4, 0]$
  - Plot the origins for ray_1 and ray_2.

Sphere: $f(x, y, z) = x^2 + y^2 + z^2 - (4)^2 = 0$

1. What are the intersecting points for ray_1 and ray_2?
2. Plot the intersecting points.

Practice Problem (2/2)



Camera frame *(orthographic)***:**

- **e** = [4, 4, 8]; **u** = [1, 0, 0]; **v** = [0, 1, 0]; **w** = [0, 1, 0]

Image Plane:

- left: u = - 5; right: u = 5; top: v = 4; bottom: v = -4

1. Plot the image plane on the given axis.
2. For a *10 x 10* image matrix M, what is the position on the image plane for the ray origin at *M (4,3)?*
3. Will it intersect $f(x, y, z) = x^2 + y^2 + z^2 - 5^2 = 0$?

Exercise

- Textbook exercise
  - no: 1