Composition of coplanar Non-concurrent Force System:

- If two or more forces are acting in a single plane, but not passing through the single point, such a force system is known as coplanar non-concurrent force system.
- In a coplanar non-concurrent force system, we can calculate the magnitude, direction & position of the resultant as follows

Magnitude of resultant, $R=\sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

Direction of resultant, $\theta = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

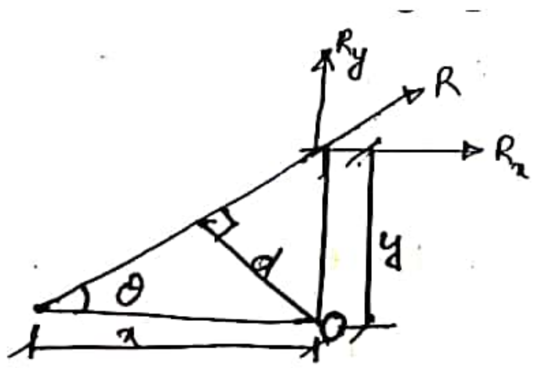
Position of resultant: The position of the resultant means the calculation of d or 'x' & 'y' intercepts as shown in figure.

$\sum$M= R x d = algebraic sum of moments of number of forces about that point (Moment centre)

$d = \frac{\sum M}{R}$

x- Intercept: $x = \left|\frac{\sum M}{\sum Fy}\right|$

y- Intercept: $y = \left|\frac{\sum M}{\sum Fx}\right|$



Composition of coplanar Non-concurrent Force System: (Problems procedure)

Steps:

1. In a Composition of coplanar Non-concurrent Force System, the magnitude, direction and position of resultant can be determined.
2. Calculate the algebraic sum of all the forces acting in x-direction (i.e $\sum Fx$) and also in the y-direction (i.e $\sum Fy$).
3. Determine the magnitude of the resultant using the formula, $R=\sqrt{(\sum Fx)^2 + (\sum Fy)^2}$
4. Determine the direction of the resultant using the formula, $\theta = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$
5. The position of resultant can be determined by using the Varignon's theorem or using the formula.

$$d=\left|\frac{\sum M}{R}\right|$$

$$\text{x- Intercept: } x=\left|\frac{\sum M}{\sum Fy}\right|$$

$$\text{y- Intercept: } y=\left|\frac{\sum M}{\sum Fx}\right|$$

Virignon's Theorem: (Principle of Moments):

Statement: The algebraic sum of the moments of individual forces of a force system about a point is equal to the moment of their resultant about the same point.

Let 'R' be the resultant of forces P_1 & P_2 and 'B' be the moment centre.

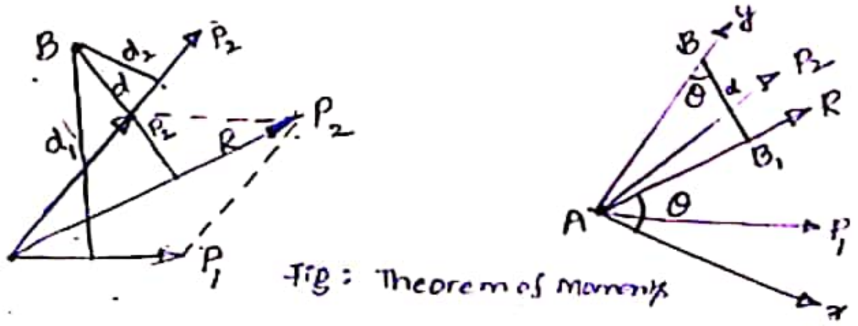


Fig: Theorem of Moments

Let d, d_1 & d_2 be the moment arms of forces R, P_1 & P_2 respectively, from the moment centre 'B'

We have to prove that

$$R.\, d = P_1\, d_1 + P_2\, d_2$$

Proof: Join AB and consider it as the y-axis and draw the x-axis at right angles to it at 'A'

Let 'θ' be the angle made by 'R' with the x-axis and note that the same angle is formed with the y-axis by the perpendicular to 'R' from 'B' and note as 'B_1'

W.K.T $\qquad R.\, d = R \times AB \cos\theta$

$$R.\, d = AB \times R \cos\theta$$

$$R.\, d = AB \times R_x \text{ --------------- } (1)$$

Where R_x is component of R in x – direction

Similarly P_{1x} & P_{2x} are the components of P_1 & P_2 in the x-directions respectively, then $P_1 . d_1 = AB \times P_{1x}$------------------- (2)

$P_2 . d_2 = AB \times P_{2x}$ ------------------- (3)

Adding equation (2) & (3), we get

$P_1 . d_1 + P_2 . d_2 = AB (P_{1x} + P_{2x})$

$P_1 . d_1 + P_2 . d_2 = AB (R_x)$ ------------------- (4)

Therefore the sum of x - components of the individual forces is equal to the x - component of the resultant 'R'

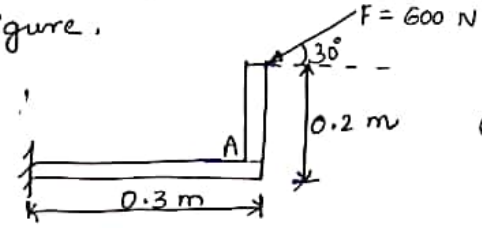
From equation (1) & (4)

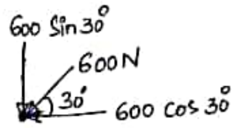
$R . d = P_1 . d_1 + P_2 . d_2$

Numericals

(9)

(P) Find the moment of the force F = 600 N about 'A' as shown in figure.

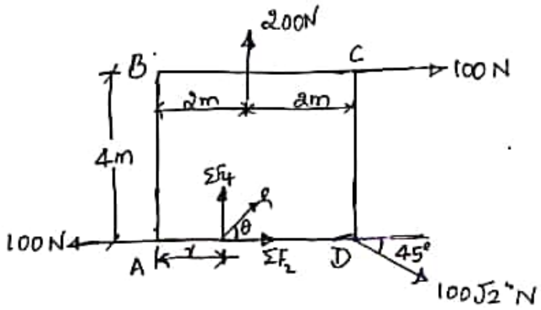




Sol: Moment of force about 'A', $M_A = -600\cos 30^\circ \times 0.2 + 600 \sin 30^\circ \times 0.3$

$$\boxed{M_A = -13.923 \text{ N-m}}$$

(P) For the nonconcurrent coplanar system shown in figure, determine the magnitude, direction and position of the resultant force w.r.t. 'A'.



Sol: Magnitude ΣF_x & ΣF_y

$$\Sigma F_x = -100 + 100 + 100\sqrt{2}\cos 45^\circ = 100 \text{ N}$$

$$\Sigma F_y = +200 - 100\sqrt{2}\sin 45^\circ = 100 \text{ N}$$

Resultant $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$$R = \sqrt{100^2 + 100^2}$$

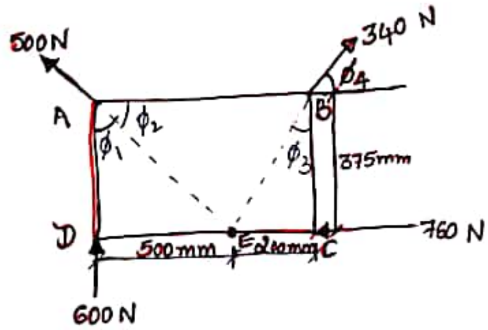
$$R = 100\sqrt{2} \text{ N} = 141.421 \text{ N}.$$

Direction (θ): $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{100}{100} = 1 \Rightarrow \theta = 45^\circ$

$|\Sigma F_x|$ $|824264|$

(2) a) Four forces act on a 700 mm x 375 mm plate as shown in figure. Find the resultant of these forces, and (10)

b) Locate the point where the line of action of the resultant intersects the edge AB of the plate.



Sol: a) $\tan \phi_1 = \frac{500}{375} \Rightarrow \phi_1 = 53.13°$

$\phi_1 + \phi_2 = 90° \Rightarrow \phi_2 = 90 - \phi_1$

$= 90 - 53.13°$

$= 36.87°$

$\tan \phi_3 = \frac{200}{375} \Rightarrow \phi_3 = 28.07°$

$\tan \phi_4 = 90° - 28.07 = 61.93°$

$\Sigma F_x = -760 + 340 \cos 61.93° - 500 \cos 36.87° = -1000$ N

$\Sigma F_y = 600 + 500 \sin 36.87° + 340 \sin 61.93° = 1200$ N

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{(-1000)^2 + (1200)^2} = 1562.05$ N

$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{1200}{-1000}\right) = -50.19°$

b) x - intercept $= \left|\frac{\Sigma M_A}{\Sigma F_y}\right|$

$\Sigma M_A = 760 \times 375 - 340 \sin 61.928° \times 700 = 74999.0$ N-m

$\therefore$ x - intercept $= \left|\frac{74999.0}{1200}\right| = 62.49$ mm.

EQUILIBRIUM OF COPLANAR FORCES

Definition of Static Equilibrium : When a Stationary body is subjected to external forces and if the body remains in the state of rest under the action of forces, it is said to be in "equilibrium".

- **Principle of Equilibrium :-** According to this principle, 'A body is said to be in equilibrium if the algebraic sum of all forces acting on a body is zero, and also if the algebraic sum of moments of forces about any fixed point is zero

i.e., $\Sigma F = 0$

(or) $\Sigma F_x = 0 \; ; \; \Sigma F_y = 0 , \; \Sigma M = 0$

A body is said to be in equilibrium if there is no translation and no rotation of the body under the application of external forces.

> **Equilibrant :**

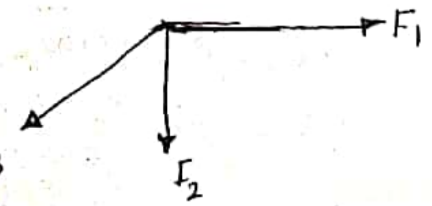
Sometimes, the resultant of the force system is not equal to zero. That means the body is not in equilibrium.

The force which is required to keep the body in equilibrium, is known as "Equilibrant."

> **Conditions of Static Equilibrium for different Coplanar Force Syste**

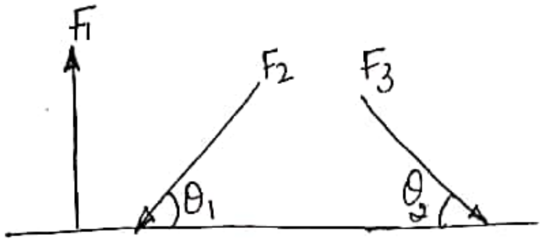
1. Coplanar concurrent force system

$\Sigma F_x = 0 \; , \; \Sigma F_y = 0$ (moment is already zero)



2. Coplanar non-current Force System

$$\Sigma F_x = 0 \; ; \; \Sigma F_y = 0 \; , \; \Sigma M = 0$$



3. Parellel Force System

$$\Sigma F = 0 \; , \; \Sigma M = 0$$

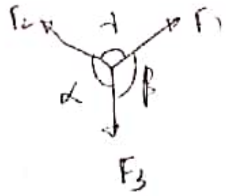
4. Non-coplanar Force System

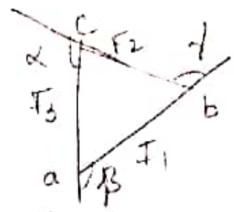
$$\Sigma F_x = 0 \; , \; \Sigma F_y = 0 \; , \; \Sigma F_z = 0 \; , \; \Sigma M = 0$$

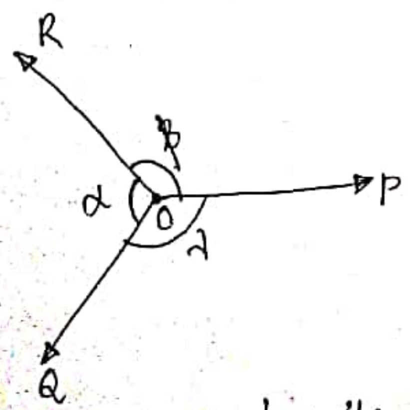
> LAMI'S THEOREM

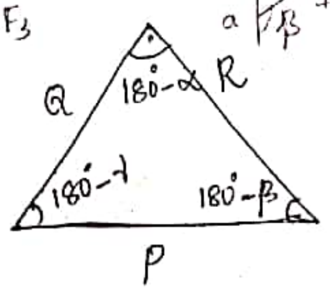
This theorem states that, 'if three forces acting at a point are in equilibrium, then each force is directly proportional to the sine of angle between the other two forces.

Let P, Q, R be the three forces acting at a point 'O' and let α, β, γ be the angles between R and Q, P and R, P & Q respectively.









Lami's Theorem

ab = F_1
bc = F_2
ca = F_3

Applying sine rule of triangle ABC

$$\frac{ab}{\sin(180-\alpha)} = \frac{bc}{\sin(180-\beta)} = \frac{ca}{\sin(180-\gamma)}$$

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

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Amaranatha G A

Using Lami's Theorem, we have

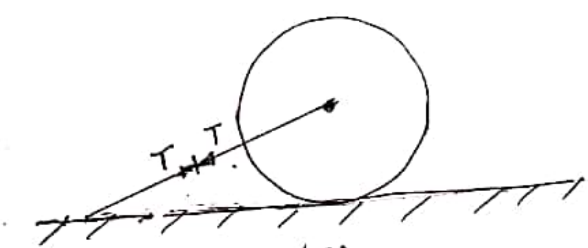
$$\frac{P}{\sin(180^\circ - \alpha)} = \frac{Q}{\sin(180^\circ - \beta)} = \frac{R}{\sin(180^\circ - \gamma)}$$

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

It is possible to apply the Lami's theorem, if only three forces are acting on a particle (or) at a point.

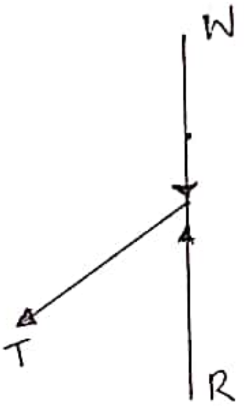
> **Concept of Free Body Diagram : (FBD)**

A free body diagram is the diagram which represents the various forces acting on the body.



(a)

Spherical ball (on a horizantal plane)



(b) FBD

Let us consider a spherical ball of mass 'm', placed on a horizantal plane and tied to the plane by a string as shown in figure(a) above.

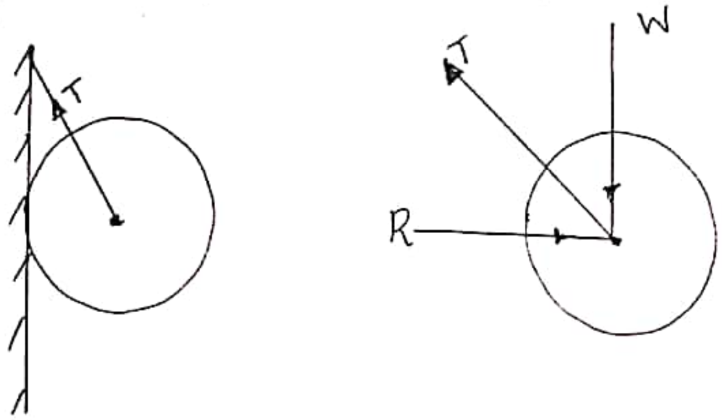
Figure (b) shows the free body diagram of the spherical ball subjected to various forces like

i) self weight, W, always acting vertically downwards

ii) Normal reaction, R, always acting perpendicular to the plane under consideration.

iii) Tension 'T' in the spring.

Eg: 2 A Spherical ball supported by a string and resisting against a wall, is shown together with its free body diagram.



> **Tips to solve the coplanar concurrent Force System:**

1. In the coplanar concurrent force system, two conditions of equilibrium can be applied, namely

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

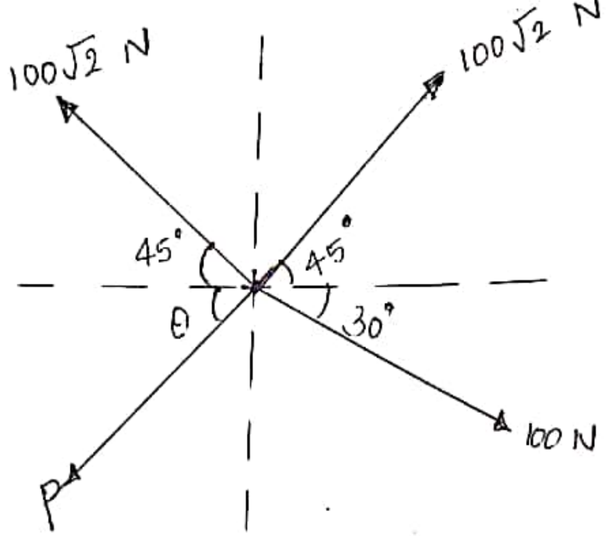
2. Analyze the given problem by applying the above conditions of equilibrium (or) by applying the Lami's Theorem.

3. Lami's theorem can be applied if only three forces are acting at a point.

i.e $$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Numericals

Determine the magnitude and direction of force 'P', which keeps the concurrent system in equilibrium as shown in figure.



Sol: Apply two conditions of equilibrium to calculate the magnitude and direction of the unknown force.

•) $\Sigma F_x = 0$

$$-P\cos\theta - 100\sqrt{2}\cos 45^\circ + 100\sqrt{2}\cos 45^\circ + 100\cos 30^\circ = 0$$

$$P\cos\theta = 100\cos 30^\circ$$

$$P\cos\theta = 86.603 \longrightarrow (1)$$

•) $\Sigma F_y = 0$

$$-P\sin\theta + 100\sqrt{2}\sin 45^\circ + 100\sqrt{2}\sin 45^\circ - 100\sin 30^\circ = 0$$

$$P\sin\theta = 150 \longrightarrow (2)$$

Dividing (2) by (1)

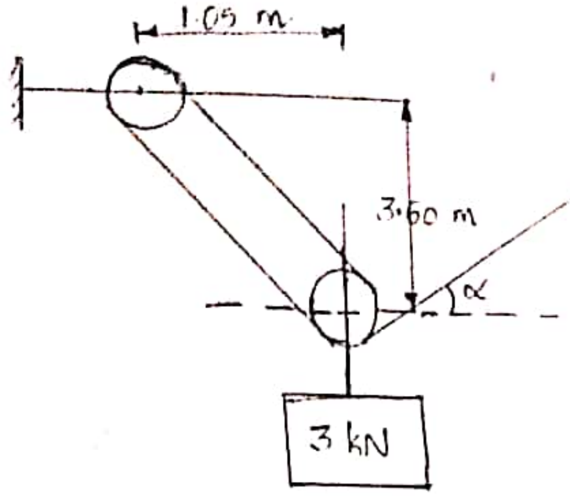
$$\frac{P\sin\theta}{P\cos\theta} = \frac{150}{86.603}$$

$$\tan\theta = \frac{150}{86.603}$$

$$\boxed{\theta = 60^\circ}$$

$$P = \frac{150}{\sin\theta} = \frac{150 \times 2}{\sqrt{3}} = 173.2\ N$$

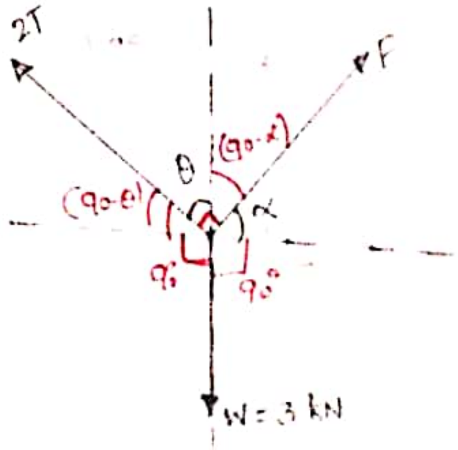
(P) A 3 kN crate is supported by the rope and pulley arrangement shown in figure. Determine the magnitude and direction of the minimum force 'F' that should be exerted at the free end of the rope.



Sol:

$$\theta = \tan^{-1}\left(\frac{1.05}{3.60}\right) = 16.26°$$

According to Lami's Theorem,



$$\frac{F}{\sin(180° - \theta)} = \frac{2T}{\sin(90+\alpha)} = \frac{W}{\sin(\theta+90-\alpha)}$$

$$\frac{F}{\sin\theta} = \frac{3}{\sin(\theta + 90° - \alpha)}$$

$$\frac{F}{\sin 16.26°} = \frac{3}{\sin(\theta + 90° - \alpha)}$$

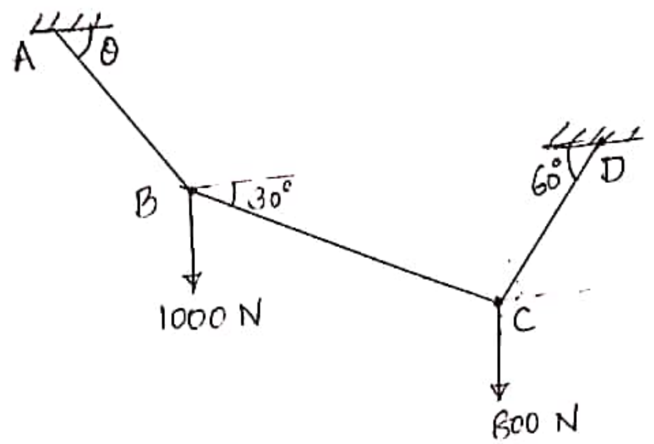
$\sin(\theta + 90° - \alpha)$ is maximum when $\sin(\theta + 90° - \alpha) = \sin 90°$

$$\theta = \alpha$$

$$\frac{F}{\sin 16.26} = 3$$

$$F = 3 \times \sin 16.26° = 0.839\ kN.$$

Compute the tensions in the strings AB, BC & CD as shown in the figure.



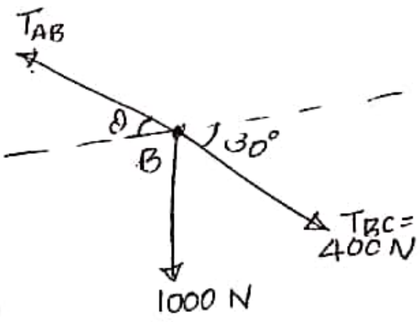
Sol: Consider FBD at 'C'

Using Lami's Theorem, we have

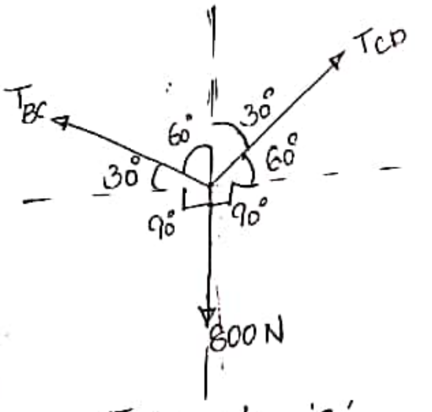
$$\frac{T_{CD}}{\sin 120^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{800}{\sin 90^\circ}$$

$\therefore \quad T_{BC} = 400 \text{ N}$

$T_{CD} = 692.82 \text{ N}$



FBD at 'B'



FBD at 'C'

Consider FBD at 'B', Using conditions of equilibrium,

We have $\Sigma F_x = 0$

$-T_{AB} \cos\theta + 400 \cos 30^\circ = 0$

$T_{AB} \cos\theta = 346.41 \longrightarrow (1)$

$\Sigma F_y = 0$

$T_{AB} \sin\theta - 400 \sin 30^\circ - 1000 = 0$

$T_{AB} \sin\theta = 1200 \longrightarrow (2)$

Dividing (2) by (1)

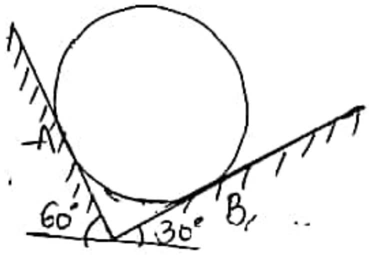
$$\frac{T_{AB} \sin\theta}{T_{AB} \cos\theta} = \frac{1200}{346.41}$$

$$\theta = 73.90^\circ$$

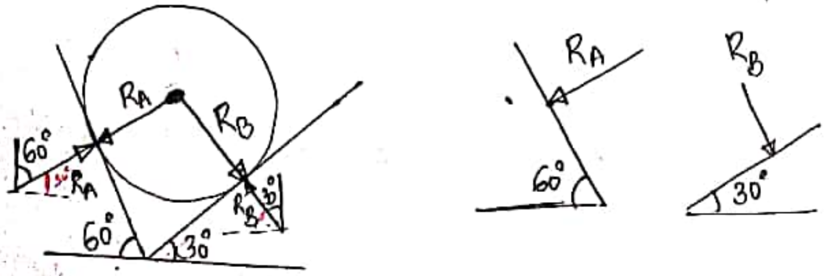
Substituting the value of 'θ' in (1), we get

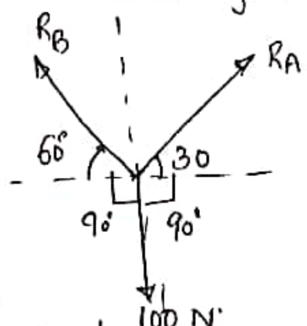
$$\boxed{T_{AB} = 1249.158 \text{ N}}$$

(P) A sphere weighing 100 N is fitted in a right-angled notch as shown in figure. If all contact surfaces are smooth, determine the reaction at contact surfaces.



Sol: At contact points, reaction is developed which is perpendicular to each plane.





Suppose two planes are perpendicular to each other, if one plane makes an angle of 'θ' with the horizontal, the perpendicular plane makes an angle 'θ' with the vertical.

Applying Lami's Theorem, we have

$$\frac{R_B}{\sin 120^\circ} = \frac{R_A}{\sin 150^\circ} = \frac{100}{\sin 90^\circ}$$

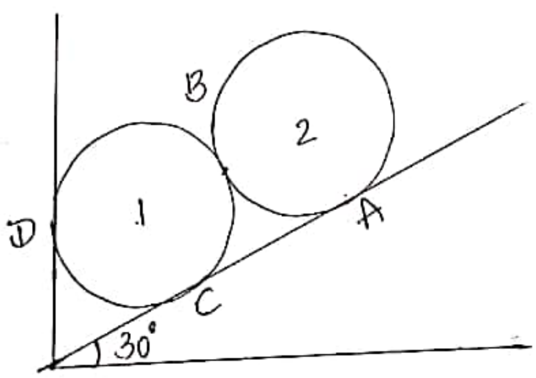
$$R_B = \frac{\sin 120^\circ \times 100}{\sin 90^\circ}$$

$$\boxed{R_B = 86.602 \text{ N}}$$

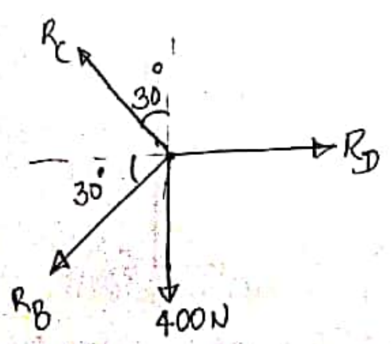
$$R_A = \frac{\sin 150^\circ \times 100}{\sin 90^\circ}$$

$$\boxed{R_A = 50 \text{ N}}$$

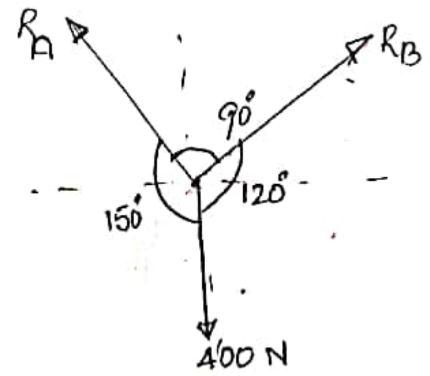
(P) Two Identical Rollers, each weighing 400 N are placed in a trench as shown in figure. Assuming that all contact surfaces are smooth, determine the reactions at contact points A, B, C and D.



Sol: FBD of Sphere 1



FBD of Sphere 2



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Sol Consider the FBD of Sphere 2

Applying Lami's Theorem

$$\frac{400}{\sin 90^\circ} = \frac{R_A}{\sin 120^\circ} = \frac{R_B}{\sin 150^\circ}$$

$$R_B = \frac{400 \times \sin 150^\circ}{\sin 90^\circ} = 200\ N$$

$$R_A = \frac{400 \times \sin 120^\circ}{\sin 90^\circ} = 346.410\ N$$

From the FBD of Sphere 1

- $\Sigma F_y = 0$

$$R_c \cos 30^\circ - 400 - 200 \sin 30^\circ = 0$$

$$R_C = 577.650\ N$$

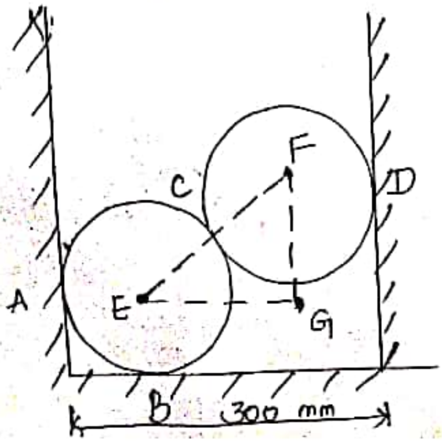
- $\Sigma F_x = 0$

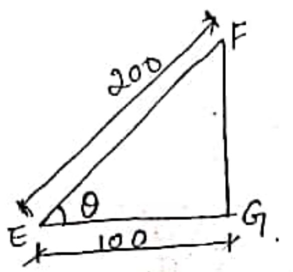
$$-R_C \sin 30^\circ \cancel{60^\circ} + R_D - R_B \cos 30^\circ = 0$$

$$577.850 \times \sin 30^\circ + 200 \cos 30^\circ = R_D$$

$$R_D = 461.880\ N.$$

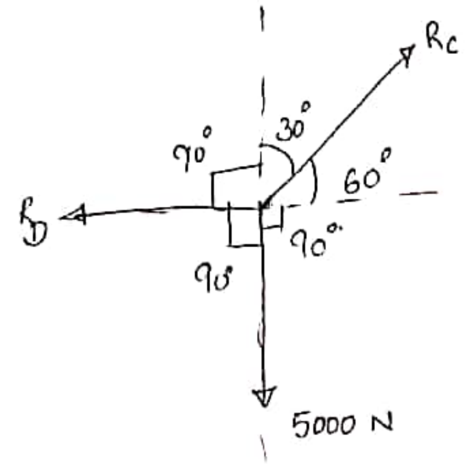
(P) Two spheres each of radius 100 mm and weight 5 kN are in a rectangular box as shown in figure. Calculate the reactions at all points of contact.





$\cos\theta = 100/200 \Rightarrow \theta = 60^\circ$

Consider the FBD of sphere 2



Using Lami's Theorem, we have

$$\frac{R_D}{\sin 150^\circ} = \frac{R_C}{\sin 90^\circ} = \frac{5000}{\sin 120^\circ}$$

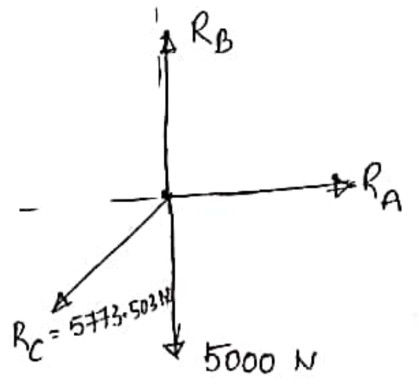
$$\frac{R_D}{\sin 150} = \frac{5000}{\sin 120^\circ}$$

$$R_D = 2886.751 \text{ N}$$

Now, $\frac{R_C}{\sin 90^\circ} = \frac{5000}{\sin 120^\circ}$

$$R_C = 5773.503 \text{ N}$$

Now, consider the FBD of sphere 1



Using equilibrium conditions, we have

$$\Sigma F_x = 0$$

$$R_A - 5773.503 \cos 60^\circ = 0$$

$$R_A = 2886.751 \text{ N}$$

$$\Sigma F_y = 0$$

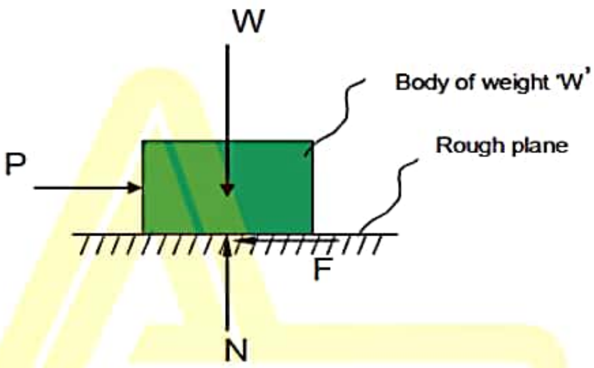
$$R_B - 5000 - 5773.503 \sin 60^\circ = 0$$

$$R_B = 10000 \text{ N}.$$

FRICTION

Whenever a body moves or tends to move over another surface or body, a force which opposes the motion of the body is developed tangentially **at the surface of contact**, such on opposing force developed is called friction or frictional resistance**.** The frictional resistance is developed due to the interlocking of the surface irregularities at the contact surface b/w two bodies.

Consider a body weighing W resting on a rough plane & subjected to a force P to displace the body.



Where,

P = Applied force.

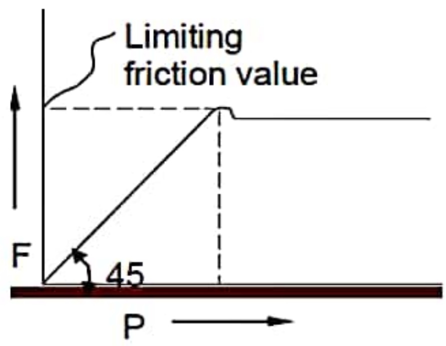
N = Normal reaction from rough surface.

F = Frictional resistance.

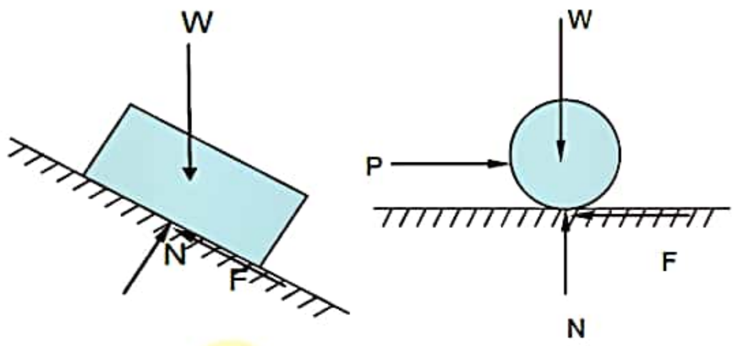
W = Weight of the body.

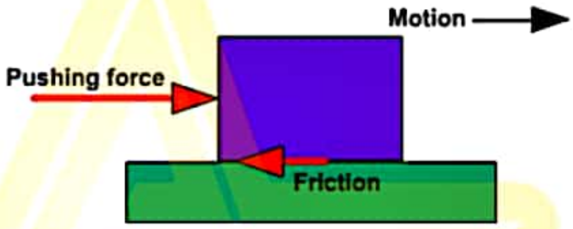
The body can start moving or slide over the plane if the **force P overcomes the frictional force.**

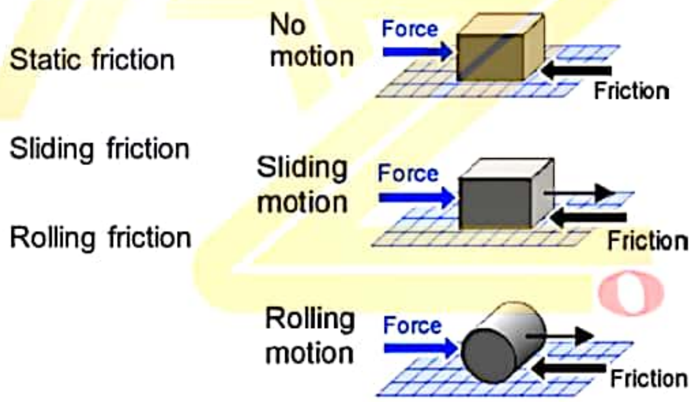
The frictional resistance developed is proportional to the magnitude of the applied force which is responsible for causing motion up to a certain limit.



From the above graph we see that as P increases, F also increases. However F cannot increase beyond a certain limit. Beyond this limit (Limiting friction value) the frictional resistance becomes constant for any value of applied force. If the magnitude of the applied force is less than the limiting friction value, the body remains at rest or in equilibrium. If the magnitude of the applied force is greater than the limiting friction value the body starts moving over the surface.







TYPES OF FRICTION

Based on the surface of contact frictional force is broadly classified as Dry Friction and Fluid Friction.

1) Dry Friction: it is the frictional force developed between two bodies when they are sliding one over the other. This is also called as Coloumb's Dry Friction.

Dry Friction further divided into two types:

(a) Solid/ Sliding Friction: It is the friction developed between two bodies having relative motion, as they are sliding one over the other. This friction force is greater to rolling friction in magnitude.

(b) Rolling friction: It is the friction developed between the two bodies when they are rolling one over the other the magnitude of rolling friction is always less than sliding friction

.Eg:Riding a bicycle

2) Fluid Friction: It is the frictional force developed between bodes because of the fluid introduced between them.

Fluid Friction further divided into two types:

(a) Viscous friction: The friction action on the body when the contact surface are completely separated by lubricant is called Viscous or Film Friction.

(b) Non-Viscous friction: The friction acting on the body when contact surface are lubricated with extremely thin layer of lubricant is called Non-Viscous Friction.

3) Static friction: It is the friction experienced between two bodies when both bodies are at rest.

4) Limiting Friction: the friction acting on a body which is just on the point or verge of sliding is called Limiting Friction.

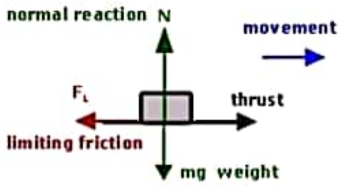
5) Dynamical Friction: The friction acting on a body which is actually in motion is called Dynamical Friction or Kinetic Friction. Eg: Slipping on wet floor.

LIMITING FRICTION

The self-adjusting opposite and resisting friction F which opposes the sliding motion of one body over another, has a limiting value and if the applied force exceeds this values, the body begins to move. This limiting values of the force is called limiting friction and it this stage the body is in limiting equilibrium and just on the verge of motion.

Limiting Friction

- The maximum friction force that can be developed at the contact surface, when body is just on the point of moving Is called limiting force of friction .



CO-EFFICIENT OF FRICTION (μ)

It is the constant ratio which the limiting friction F bears to the normal reaction N called as co-efficient of friction.

It is defined by the relationship, $\mu = \frac{F}{N}$

Where,

μ – Represents co-efficient of friction

F – Represents frictional resistance

N – Represents normal reaction.

LAWS OF STATIC FRICTION [LAWS OF DRY FRICTION (COLOUMB'S LAWS)]

The frictional resistance developed between bodies having dry surfaces of contact obey certain laws called laws of static/dry friction.

They are as follows:

1) The frictional resistance depends upon the roughness or smoothness of the surface.

2) Frictional resistance acts in a direction opposite to the motion of the body.

3) The frictional resistance is independent of the area of contact between the two bodies.

4) The ratio of the limiting friction value (F) to the normal reaction (N) is a constant is called co-efficient of friction (μ).

$$\mu = \frac{F}{N}$$

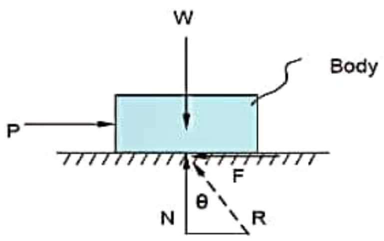
5) The magnitude of the frictional resistance developed is exactly equal to the applied force till limiting friction value is reached or where the bodies is about to move.

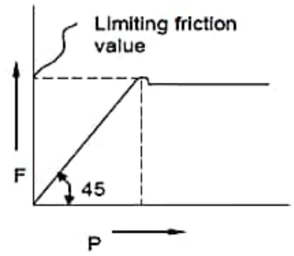
Explain the following terms

1. Angle of friction.
2. Angle of repose.
3. Cone of friction

1) ANGLE OF FRICTION (ϕ)

Definition: It is the angle made by the resultant of friction force (F) and normal reaction (N) force.





Consider a body weighing W placed on a horizontal plane. Let P be an applied force required to just move the body such that, frictional resistance reaches limiting friction value. Let R be resultant of F & N. Let θ be the angle made by the resultant with the direction of N. Such an angle θ is called the Angle of friction.

As P increases, F also increases and correspondingly θ increases. However, F cannot increase beyond the limiting friction value and as such θ can attain a maximum value only.

Let $\theta_{max} = \alpha$

Where α represents angle of limiting friction

$$\tan \theta_{max} = \tan\alpha = \frac{F}{N}$$

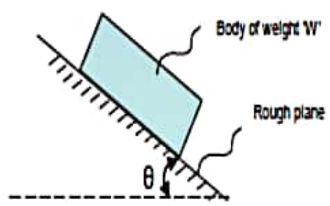
$$\text{But } \frac{F}{N} = \mu$$

Therefore $\mu = \tan\alpha$

i.e., co-efficient of friction is equal to the tangent of the angle of limiting friction.

2) ANGLE OF REPOSE (θ)

Definition: The maximum inclination of the plane with the horizontal, on which a body free from external forces can rest without sliding is called angle of repose.



Let us consider a body of weight W which is placed on an inclined plane as shown in below figure. The object is just at the point of sliding down the plane when the angle of inclination is θ. The various forces acting on the object are self-weight, normal reaction and frictional force.

Let $\theta_{max} = \Phi$

Where Φ = angle of repose

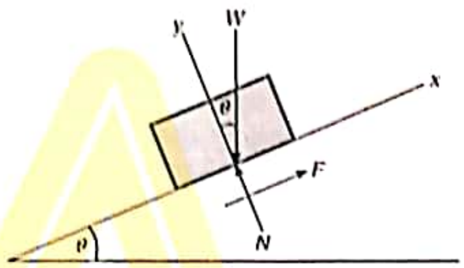


Figure 8.4 Angle of repose (θ).

Applying the conditions of equilibrium,

$$\Sigma F_x = 0;\ \Sigma F_y = 0$$

Resolving forces along the x-axis,

$$-F + W \sin \theta = 0$$

or
$$F = W \sin \theta \tag{8.2}$$

Resolving forces along the y-axis,

$$N - W \cos \theta = 0$$

or
$$N = W \cos \theta \tag{8.3}$$

We know that

$$\mu = \frac{F}{N}$$

⇒
$$\mu = \frac{W \sin \theta}{W \cos \theta} = \tan \theta \tag{8.4}$$

or
$$\tan \phi = \tan \theta$$

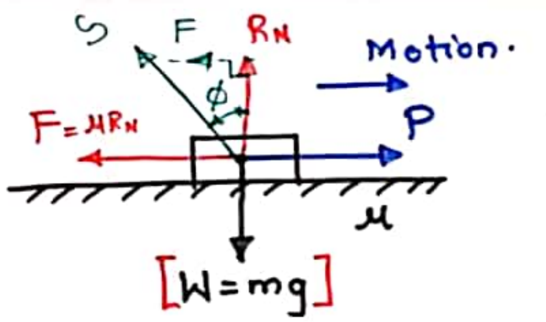
or
$$\phi = \theta$$

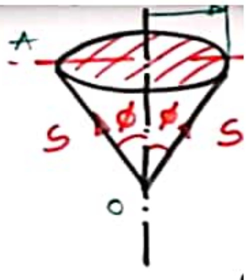
It is evident from Eqs. (8.1) and (8.4) that

Angle of friction = Angle of repose

3) CONE OF FRICTION

It is an imaginary cone of revolution formed by the resultant reaction at the contact point. If the resultant reaction is rotated about normal reaction force, it will form a right circular cone of angle 2Φ known as cone of friction.





Consider a body weighting W resting on a rough horizontal surface. Let P be a force required to just move the body such that frictional resistance reaches limiting value. Let R be the resultant of F & N making an angel with the direction of N.

If the direction of P is changed the direction of F changes and accordingly R also changes its direction. If P is rotated through 360°, R also rotates through 360° and generates an imaginary cone called cone of friction.

Note: In this discussion, all the surface that been consider are rough surfaces, such that, when the body tends to move frictional resistance opposing the motion comes into picture tangentially at the surface of contact in all the examples, the body considered is at the verge of moving such that frictional resistance reaches limiting value. We can consider the body to be at rest or in equilibrium & we can still apply conditions of equilibrium on the body to calculate unknown force.

(Numerical Problems on single and two blocks on inclined planes/friction)-Refer class notes

Tips to solve the problems

I. Draw a free body diagram
II. Draw the reference axis. We have to choose the reference axis in such a way that one of the axis must be long the direction of motion.

The following forces should be considered while drawing the free body diagram

i) Self-
yweight always acts **vertically downwards**
ii) External or internal forces
iii) In a rough surface –**a)Frictional force** is always opposite to the direction of motion at the contact surface and is parallel to the contact surface
b) Normal reaction is always perpendicular to the contact surface

iv) Write the algebraic sum of the forces along the X-axis i.e. $\Sigma F_x=0$

iv) Write the algebraic sum of the forces along the Y-axis i.e. $\Sigma F_y=0$

UNIT – 4

CENTROID AND MOMENT OF INERTIA

Centroid :

> Introduction to the Concept :

- Centre of Gravity :- It is the point where the 'whole weight' of the body is assumed to be concentrated.

(or)

It is the point on which the body can be balanced

(or)

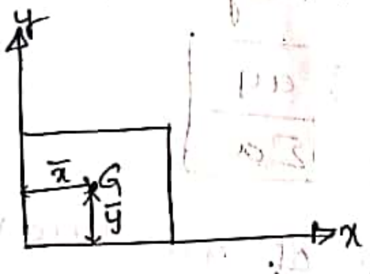
It is the point through which the weight of a body is assumed to act.

Note : This point is usually denoted by C.G (or) G

- Centroid (C.G (or) G) :

Centroid is the point where the 'whole area' of the plane figure is assumed to be concentrated

The calculation of Centroid means the determination of $\bar{x}$ and $\bar{y}$ as shown in figure.



Centroid of plane figure.

Centroid of plane Figures :

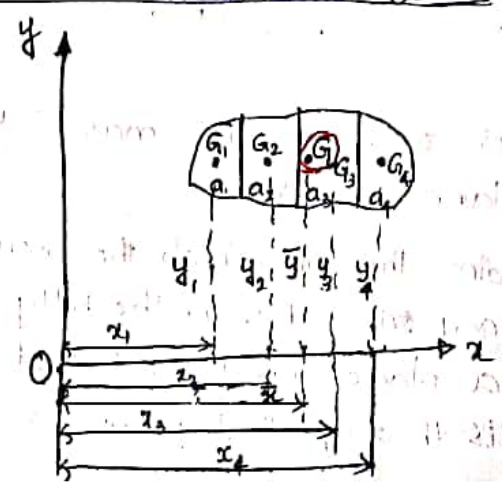


Figure shows a plane figure of total Area 'A' and unit thickness.

Let 'G' be the C.G of whole area with coordinate $(\bar{x}, \bar{y})$

Divide area into small area with centroid, then

$$A = \Sigma a = a_1 + a_2 + a_3 + a_4 \longrightarrow (1)$$

Let $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) be the coordinates of the areas a_1, a_2, a_3 and a_4 with centroid G_1, G_2, G_3 and G_4 respectively

- Moment of total area 'A' about OY axis $= A \cdot \bar{x}$
$$= \Sigma a \cdot \bar{x} \longrightarrow (2)$$

- Sum of moments of all areas about OY axis
$$= a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 = \Sigma a x \longrightarrow (3)$$

$$(2) = (3)$$

$$\Sigma . a . \bar{x} = \Sigma (a x)$$

$$\boxed{\bar{x} = \frac{\Sigma (a x)}{\Sigma a}}$$

- Similarly, if we equate moment about OX axis, we get

$$\Sigma . a . \bar{y} = \Sigma . a y$$

$$\boxed{\bar{y} = \frac{\Sigma a y}{\Sigma a}}$$

- Hence, the centre of gravity (or) centroid of any plane figure is given by

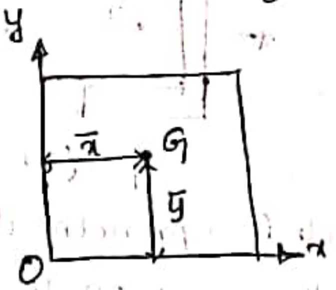
$$\bar{x} = \frac{\Sigma a x}{\Sigma a} \; ; \; \bar{y} = \frac{\Sigma a y}{\Sigma a} .$$

Note :

1) The term C.G applies to bodies with mass & weight and centroid applies to plane areas.

2) C.G of a body is a point through which the resultant gravitation force (weight) acts for any orientation of the body whereas centroid is a point in a plane area such that the moment of area about any axis through that point is zero.

> **Axes of Reference:** These are the axes with reference to which the centroid of a given figure is determined.

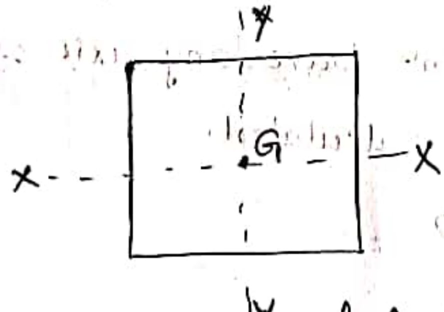


Axes of Reference.

Generally the left hand bottom corner of the plane figure is considered as the origin so that the left extreme edge and the bottom line are considered as reference axes w.r.t which the centroid of the given figure is measured.

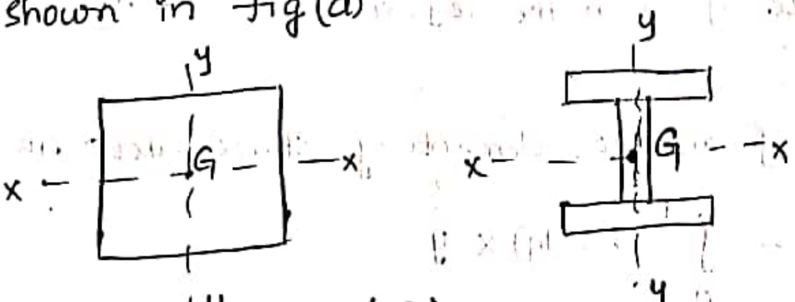
> **Centroidal Axis**

The axis which passes through the centroid of the given figure is known as centroidal axis, such as the axis X-X and the axis Y-Y as shown in figure.



Centroidal Axes.

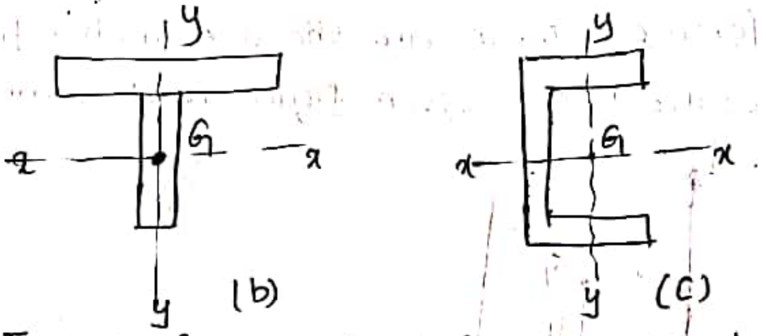
> **Symmetrical Axis:** It is the axis which divides the whole figure into equal parts, such as the axis X-X and the axis Y-Y as shown in fig (a)



(a)

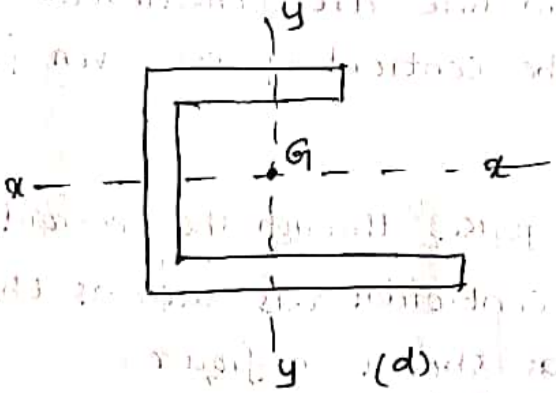
- For a figure which is symmetrical about both the axes,

  $\bar{x} = 0$ and $\bar{y} = 0$



For a figure(b) which is symmetrical about the y-y axis, $\bar{x} = 0$. For such a figure, the area on the left side of y-y axis is equal to the area on the right side of y-y axis

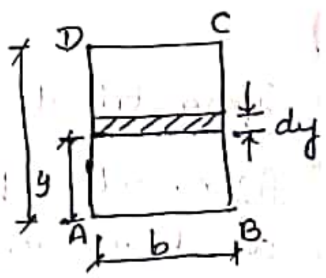
For figure (c) which is symmetrical about the x-x axis, $\bar{y} = 0$.



~~For~~ fig (d), doesn't have any axis of symmetry, so $\bar{x}$ & $\bar{y}$ can be calculated.

LOCATION OF CENTROID

1. Rectangle : Let Area be $b \times d$.



- Now consider a horizantal elementary strip of area $(b \times dy)$ which is at a distance 'y' from the reference axis AB.
- Moment of area of elementary strips about AB is given by

$$= \int_0^d (b \times dy) \times y$$

$$= b \int_0^d y \cdot dy$$

$$= b \times \left[\frac{y^2}{2}\right]_0^d = \frac{bd^2}{2}$$

Scanned with CamScanner

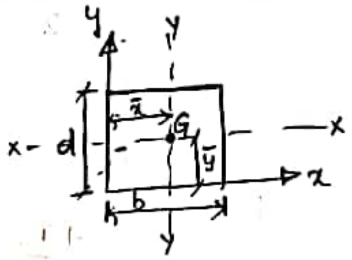
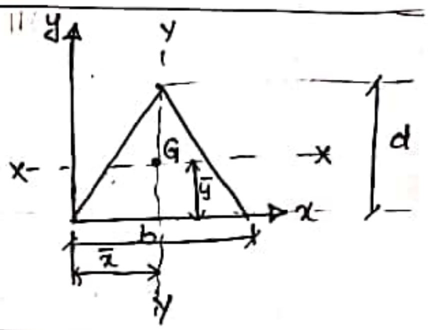
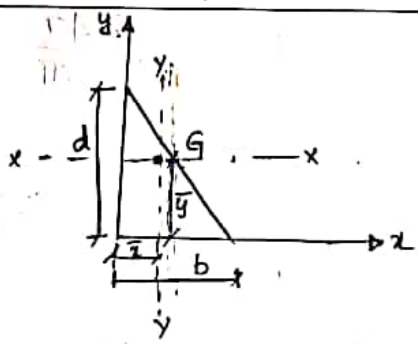
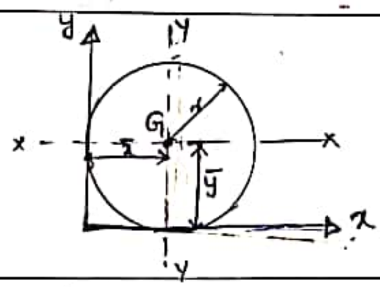
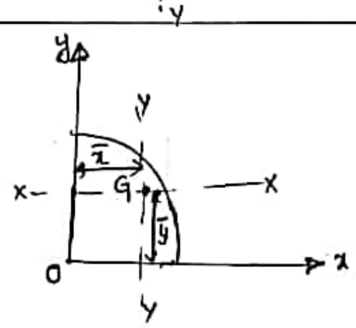
- Moment of total area about y-axis $= \frac{R^2\alpha}{2} \times \bar{y}$

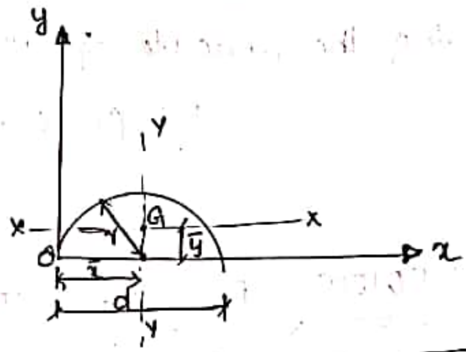
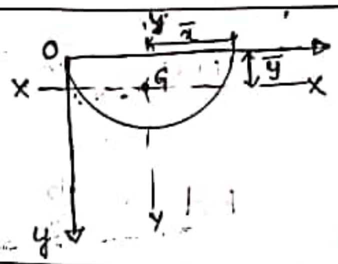
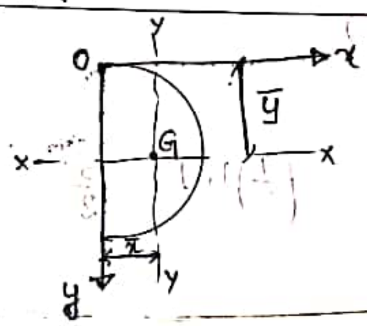
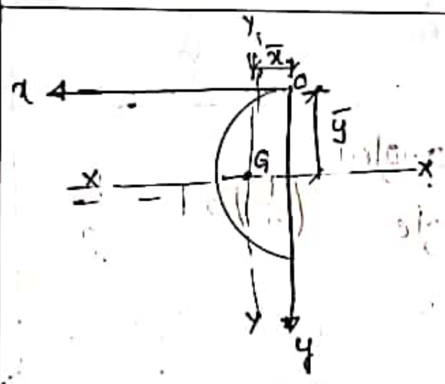
- Using the principle of moments

$$\frac{R^3}{3}(1-\cos\alpha) = \frac{R^2\alpha}{2} \times \bar{y}$$

$$\bar{y} = \frac{2R}{3\alpha}(1-\cos\alpha)$$

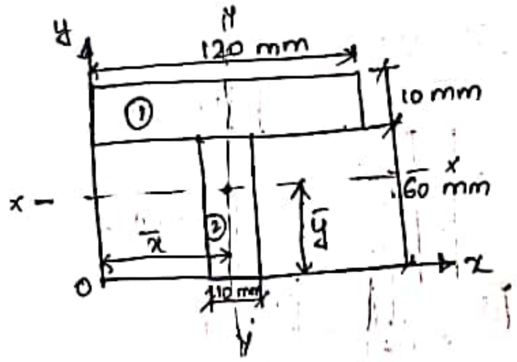
→ CENTROIDS OF SOME IMPORTANT GEOMETRICAL FIGURES

Shape	Area	$\bar{x}$	$\bar{y}$	Figure
Rectangle (Same for Square)	bd	$\frac{b}{2}$	$\frac{d}{2}$	
Triangle	$\left(\frac{1}{2}\right)bd$	$\frac{b}{2}$	$\frac{d}{3}$	
Right-angled triangle	$\left(\frac{1}{2}\right)bd$	$\frac{b}{3}$	$\frac{d}{3}$	
Circle	πr^2	$\bar{x} = r$	$\bar{y} = r$	
Quarter Circle	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	

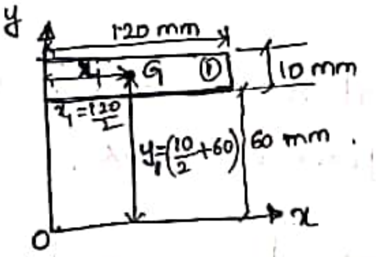
Shape	Area	$\bar{x}$	$\bar{y}$	Figure
Semi circle	$\frac{\pi r^2}{2}$	$\frac{d}{2}$	$\frac{4r}{3\pi}$	
		$\frac{d}{2}$	$-\frac{4r}{3\pi}$	
		$\frac{4r}{3\pi}$	$-\frac{d}{2}$	
		$-\frac{4r}{3\pi}$	$\frac{d}{2}$	

Problems on Centroid of Composite Sections

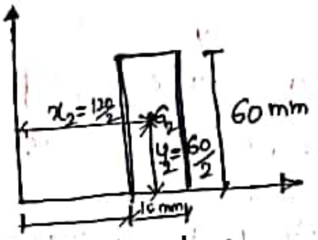
(P) Find the centroid of figure shown



Sol:



Rectangle 1



Rectangle 2

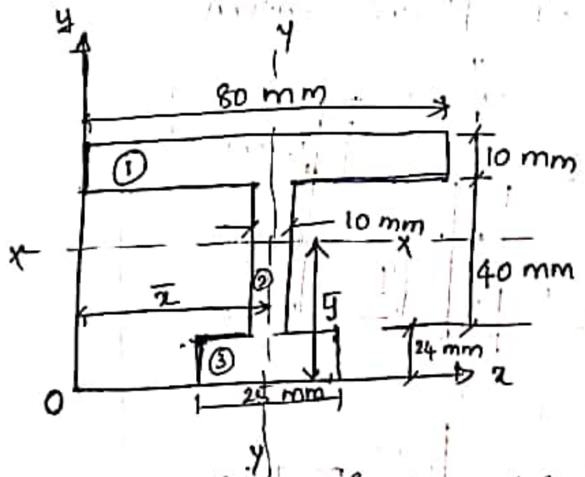
Component	Area (mm²)	Centroidal distance from y-axis (x)	Centroidal distance from x-axis(y)	ax	ay
Rectangle 1	$120 \times 10 = 1200$	$\frac{120}{2} = 60$	$60 + \frac{10}{2} = 65$	72000	78000
Rectangle 2	$10 \times 60 = 600$	$\frac{120}{2} = 60$	$\frac{60}{2} = 30$	36000	18000
Sum	$\Sigma a = 1800$			1,08,000	96,000

$$\bar{x} = \frac{\Sigma a x}{\Sigma a} \quad ; \quad \bar{y} = \frac{\Sigma a y}{\Sigma a}$$

$$\bar{x} = \frac{1,08,000}{1800} \quad ; \quad \bar{y} = \frac{96,000}{1800}$$

$$\bar{x} = 60 \text{ mm} \quad ; \quad \bar{y} = 53.33 \text{ mm}$$

(P) Find the centroid of figure shown



<u>Note</u>: The given figure is symmetrical about the y-y axis and hence we consider it as the reference y-axis.

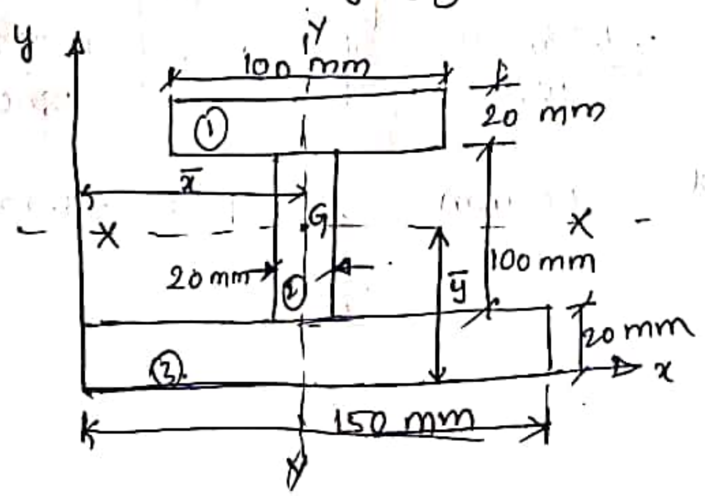
<u>Sol</u>:

Component	Area, a (mm^2)	Centroidal distance from the x-axis (y) (mm)	a × y (mm^3)
Rectangle 1	$10 \times 80 = 800$	$24 + 40 + \frac{10}{2} = 69$	55,200
Rectangle 2	$10 \times 40 = 400$	$24 + \frac{40}{2} = 44$	17,600
Rectangle 3	$25 \times 24 = 600$	$\frac{24}{2} = 12$	7200.
Sum	$\Sigma a = 1800$		$\Sigma ay = 80{,}000$

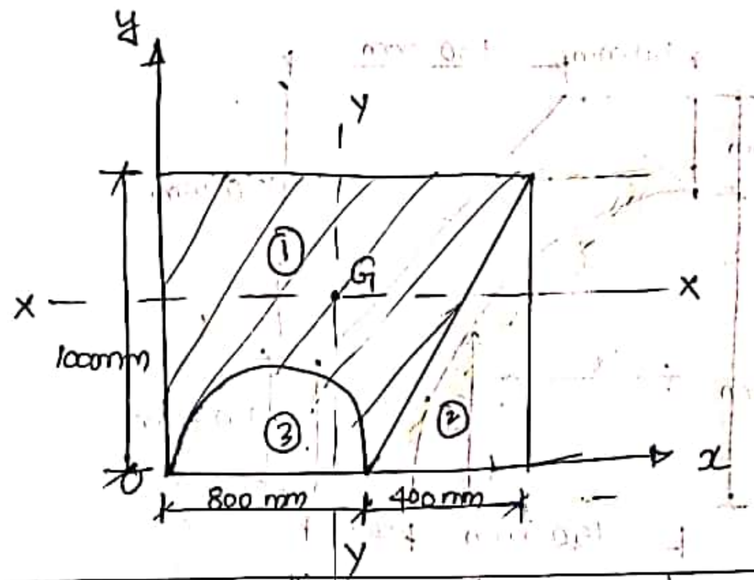
$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{80{,}000}{1800} = 44.44 \text{ mm}$$

$$\bar{x} = \frac{80}{2} = 40 \text{ mm}$$

(P) Determine the centroid of figure shown.



- (P) Locate the centre of the shaded area shown in figure ⑦



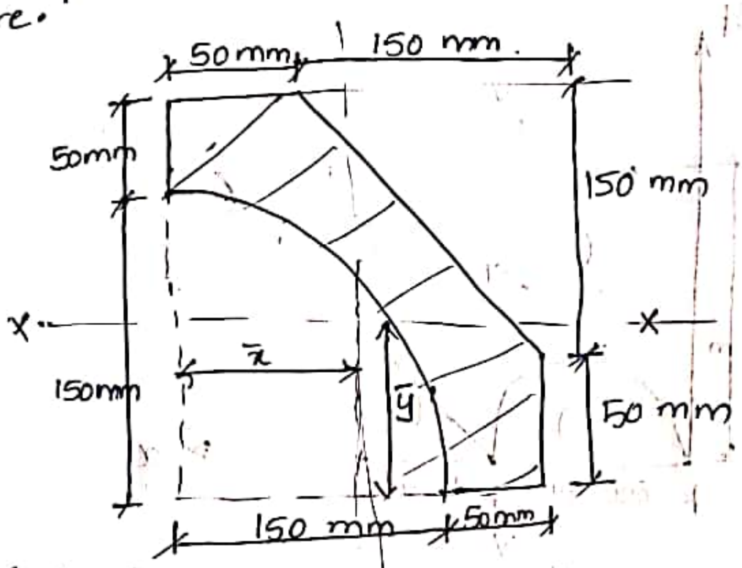
Sol:

Component	Area, a (mm^2)	x (mm)	y (mm)	ax (mm^3)	ay (mm^3)
Rectangle	1000×1200 $= 1,200,000$	$\frac{1200}{2} = 600$	$\frac{100}{2} = 500$	720000,000	600000000
Triangle	$-\frac{1}{2} \times 400 \times 1000$ $= -200000$	$800 + \frac{2}{3} \times 400$ $= 1066.67$	$\frac{1}{3} \times 1000$ $= 333.33$	-100,530,948	-42,666,58
Semi-Circle	$-\frac{\pi r^2}{2}$ $= -251,327.41$	$\frac{800}{2} = 400$	$\frac{4r}{3\pi} = 169.76$	-213,333,333.3	-66,666,666.67
Sum	$\Sigma a = 748672.59$			406135701.75	490666666.67

$$\bar{x} = \frac{\Sigma a x}{\Sigma a} = \frac{406135701.75}{748672.59} = 542.474 \text{ mm}$$

$$\bar{y} = \frac{\Sigma a y}{\Sigma a} = \frac{490666666.67}{748672.59} = 655.382 \text{ mm}$$

(P) Determine the centroid of the shaded area shown in figure.



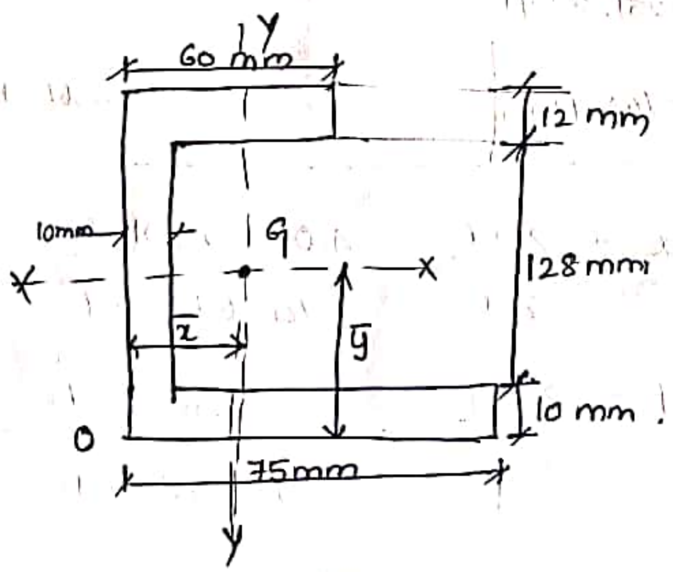
Ans: $\Sigma a = 18,578.54$; $\Sigma ax = 2,312,500.00$

$\Sigma ay = 3,954,614.22$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = 124.47 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = 212.86 \text{ mm.}$$

(P) Determine the centroid of figure



Ans: $\Sigma a = 2750$, $\Sigma ax = 56125$, $\Sigma ay = 202150$

$\bar{x} = \frac{\Sigma ax}{\Sigma a} = 20.41$ mm; $\bar{y} = \frac{\Sigma ay}{\Sigma a} = 73.51$ mm