Filter

Example: 7.3

Obtain the co-efficients of an FIR lowpass filter to meet the specifications given below using the window method.

Passband edge frequency	1.5kHz
Transition band	0.5kHz
Stopband attenuation	$\rangle$50dB
Sampling frequency	8kHz

Solution

For lowpass filter we select the ideal impulse response $h_D(n)$.

$$h_D(n) = 2f_c \frac{\sin(n\omega_c)}{n\omega_c}, n \neq 0$$

$$h_D(n) = 2f_c[L'sHospitalrule's], n = 0$$

Here, given that stopband attenuation is greater than 50dB.

So, we can use Hamming, Blackman or Kaiser window, will satisfy the stopband attenuation requirements.

But we will use the Hamming window for simplicity.

Now, the normalzed transition width,

$$\triangle f = \frac{0.5}{8} = 0.0625$$

For Hamming window, we know,

$$\begin{aligned}\triangle f &= \frac{3.3}{N}\\ \Rightarrow N &= \frac{3.3}{\triangle f}\\ &= 52.8\end{aligned}$$

Let, N = 53.

Now, the filter co-efficients are obtained from,

$$h(n) = h_D(n)w(n); -26 \leq n \leq 26$$

Where

$$h_D(n) = 2f_c \frac{\sin(n\omega_c)}{n\omega_c}, n \neq 0$$

$$h_D(n) = 2f_c, n = 0$$

$$w(n) = 0.54 + 0.46\cos\left(\frac{2\pi n}{53}\right), -26 \leq n \leq 26$$

For the effect of window, cut-off frequency of resulting filter will be different from that given in the specification.
So,

$$f_c' = f_c + \left(\frac{\triangle f}{2}\right) = (1.5 + 0.25) = 1.75kHz \text{ or, } \frac{1.75}{8} = 0.21875$$

[Here, $\triangle$f = 0.5 not 0.0625]

Nothing that h(n) is symmetrical, we need only compute values for $h(0), h(1)......h(26)$ and then use the symmetry property to obtain the other co-efficients.

n=0:

$$h_D(0) = 2f_c = 2 \times 0.21875 = 0.4375$$
$$w(0) = 0.54 + 0.46\cos(0) = 1$$
$$h(0) = h_D(0)w(0) = 0.4375$$

n=1:

$$
\begin{aligned}
h_D(1) &= \frac{2f_c \sin(nw_c)}{nw_c} \\
&= \frac{2 \times 0.21875}{2\pi \times 0.21875} \sin(2\pi \times 0.21875) \\
&= \frac{\sin(360 \times 0.21875)}{3.1416} \\
&= 0.31219 \\
w(1) &= 0.54 + 0.46 \cos(2\pi/53) \\
&= 0.54 + 0.46 \cos(360/53) \\
&= 0.99677 \\
h(1) &= h(-1) \\
&= h_D(1) w(1) \\
&= 0.31118
\end{aligned}
$$

n=2:

$$
\begin{aligned}
h_D(2) &= \frac{\sin(2 \times 2\pi \times 0.21875)}{2\pi} \\
&= 0.06013 \\
w(2) &= 0.54 + 0.46\cos(2 \times 2\pi/53) \\
&= 0.98713 \\
h(2) &= h(-2) \\
&= h_D(2)w(2) \\
&= 0.06012
\end{aligned}
$$

n=26:

$$h_D(26) = \frac{\sin(26 \times 2\pi \times 0.21875)}{26\pi}$$

$$= -0.01131$$

$$w(26) = 0.54 + 0.46\cos(2\pi \times 26/53)$$

$$= 0.08081$$

$$h(26) = h(-26)$$

$$= h_D(26)w(26)$$

$$= -0.000914$$

We note that the indices of filter co-efficients run from -26 to 26.

To make the filter caused (necessary for implementation) we add 26 to each index so that the indices start at zero.

Example: 7.4(i)

Obtain the co-efficients of an FIR lowpass digital filter to meet the following specifications given below using the window method.

Passband edge frequency	3.4kHz
Transition width	0.6kHz
Stopband attenuation	50dB
Sampling frequency	8kHz

Include in your answer the type of window used and the reason of your choice.

Solution

For lowpass filter we select the ideal impulse response $h_D(n)$.

$$h_D(n) = 2f_c \frac{\sin(n\omega_c)}{n\omega_c}, n \neq 0$$
$$h_D(n) = 2f_c, n = 0$$

Here, given that stopband attenuation is greater than 50dB.

So, we can use Hamming, Blackman or Kaiser window, will satisfy the stopband attenuation requirements.

But we will use the Hamming window for simplicity.

Now, the normalized transition width,

$$\triangle f = \frac{0.6}{8} = 0.075$$

For Hamming window, we know,

$$\begin{aligned}\triangle f &= \frac{3.3}{N} \\ \Rightarrow N &= \frac{3.3}{\triangle f} \\ &= 44\end{aligned}$$

Let, filter length, L = N+1 = 45.

Now, the filter co-efficients are obtained from,

$$h(n) = h_D(n)w(n); -22 \leq n \leq 22$$

Where

$$h_D(n) = 2f_c\frac{\sin(n\omega_c)}{n\omega_c}, n \neq 0$$

$$h_D(n) = 2f_c, n = 0$$

$$w(n) = 0.54 + 0.46\cos\left(\frac{2\pi n}{45}\right), -22 \leq n \leq 22$$

For the effect of window, cut-off frequency of resulting filter will be different from that given in the specifications.
So,

$$f_c' = f_c + \left(\frac{\triangle f}{2}\right) = (3.4 + 0.3) = 3.7kHz \text{ or, } \frac{3.7}{8} = 0.4625$$

Nothing that h(n) is symmetrical, we need only compute values for $h(0), h(1)......h(22)$ and then use the symmetry property to obtain the other co-efficients.

n=0:

$$
\begin{aligned}
h_D(0) &= 2f_c = 2 \times 0.4625 = 0.925 \\
w(0) &= 0.54 + 0.46\cos(0) = 1 \\
h(0) &= h_D(0)w(0) = 0.925
\end{aligned}
$$

n=1:

$$
\begin{aligned}
h_D(1) &= \frac{2f_c \sin(nw_c)}{nw_c} \\
&= \frac{\sin(360 \times 0.4625)}{3.1416} \\
&= 0.0743 \\
w(1) &= 0.54 + 0.46\cos(2\pi/45) \\
&= 0.9955 \\
h(1) &= h(-1) \\
&= 0.0743 \times 0.9955 \\
&= 0.0739
\end{aligned}
$$

n=2:

$$h_D(2) = \frac{\sin(2 \times 2\pi \times 0.4625)}{2\pi}$$

$$= -0.0722$$

$$w(2) = 0.54 + 0.46\cos(2 \times 2\pi/45)$$

$$= 0.9821$$

$$h(2) = h(-2)$$

$$= (-0.0722)(0.9821)$$

$$= -0.0709$$

n=22;

$$h_D(22) = \frac{\sin(22 \times 2\pi \times 0.4625)}{22\pi}$$

$$= 0.0128$$

$$w(22) = 0.54 + 0.46\cos(2\pi \times 22/45)$$

$$= 0.0811$$

$$h(22) = h(-22)$$

$$= 0.0010$$

We note that the indices of filter co-efficients run from -22 to 22.

To make the filter caused (necessary for implementation) we add 22 to each index so that the indices start at zero.