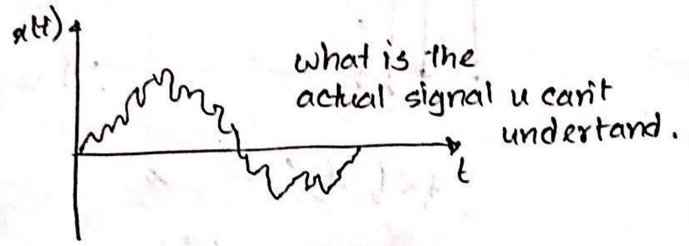
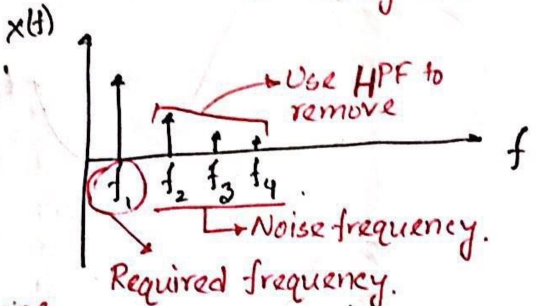
Communication

* 1. History of Communication (Slide) - Communication fundamentals.

2. Why frequency domain over time domain? → Show (slide: AM) (P-101), Fig - 3.8





<u>AM</u> → Amplitude of carrier changes with message.

Let, the message signal / modulating signal / information signal $v_m = V_m \sin 2\pi f_m t$.

- v_m → Instantaneous value of mes.
- V_m → Peak Amplitude
- f_m → frequency.

and carrier wave $v_c = V_c \sin 2\pi f_c t$.

For modulation $V_m < V_c$, otherwise will create distortion.

As it is amplitude modulation we will take only amplitude of carrier V_c, which will be added to message signal.

∴ The peak amplitude of AM wave $v_1 = V_c + v_m = V_c + V_m \sin 2\pi f_m t$.

Finally the AM (modulated) wave equ^n $v_{AM} = v_1 \sin 2\pi f_c t$

$$= (V_c + V_m \sin 2\pi f_m t) \sin 2\pi f_c t.$$

$$= V_c \left(1 + \frac{V_m}{V_c} \sin 2\pi f_m t\right) \sin 2\pi f_c t.$$

$$\boxed{= V_c (1 + m \sin 2\pi f_m t) \sin 2\pi f_c t}$$

$m = \frac{V_m}{V_c}$ = modulating index.

$$= V_c \sin \omega_c t + m V_c . \sin \omega_m t . \sin \omega_c t$$

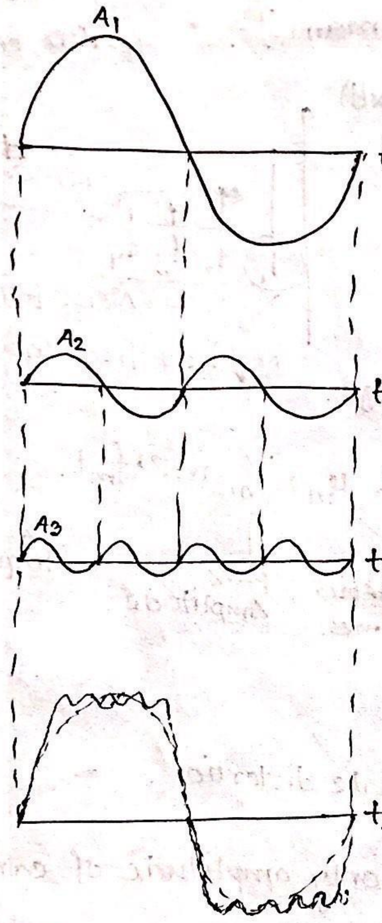
$$= V_c \sin \omega_c t + \frac{m V_c}{2} . 2 \sin \omega_m t . \sin \omega_c t$$

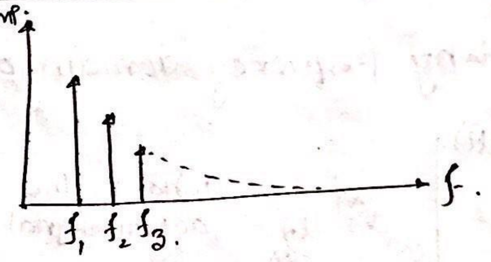
$$= V_c \sin \omega_c t + \frac{m V_c}{2} \left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

$$= V_c \sin \underbrace{\omega_c}_{2\pi f_c} t + \frac{m V_c}{2} \left[\cos 2\pi (f_c - f_m)t - \cos (f_c + f_m)t\right].$$

Slide: 9

Frequency Domain Representation of AM (Slide name: AM, P-101).





$f_1 < f_2 < f_3 < \cdots \infty$.

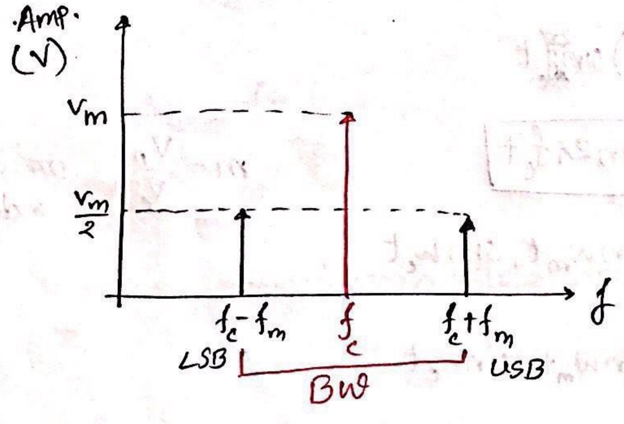
Amp of f_1 > Amp of f_2 > - -

$A_1 > A_2 > \cdots \infty$

* Principles of Electronic Communication Systems.
Louis E. Frenzel Jr.

$$v_{AM} = \underbrace{V_c \sin 2\pi f_c t}_{\text{carrier}} + \underbrace{\frac{mV_c}{2} \cos 2\pi (f_c - f_m) t}_{\text{lower side band}} - \underbrace{\frac{mV_c}{2} \cos 2\pi (f_c + f_m) t}_{\text{upper side band}}.$$

$mV_c = V_m$



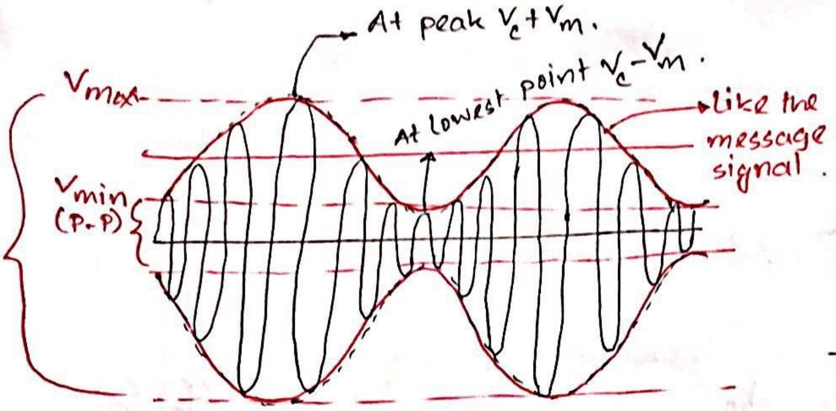
See/show:
Fig 3-6, Slide: AM, S/N-8.
P-100.

* lower SB (+ve) আর USB (-ve) উল্টা

* Carrier freq. ধ্রুবক.

* Composite AM wave এর সর্বোচ্চ বিন্দু = Σ বাকী সবগুলোর সর্বোচ্চ বিন্দুর যোগফল।

$BW = 2f_m$.
$= f_c + f_m - (f_c - f_m)$
$= 2f_m$.



modulated carrier

Slide-AM
Fig 3-1.(P-94)

V_c = Carrier Amplitude
V_m = Message "

Here,

$V_{max} = V_c + V_m$.

$V_{min} = V_c - V_m$.

Slide-AM.
P-110
Fig: 3-6.

$V_{max} + V_{min} = 2V_c$

$\therefore V_c = \dfrac{V_{max} + V_{min}}{2}$.

Similarly,

$$V_m = \frac{V_{max} - V_{min}}{2}.$$

We know, modulating index m (or μ.) $= \dfrac{V_m}{V_c}$

$$= \frac{V_{max} - V_{min}}{V_{max} + V_{min}}.$$

* Example 3-1 (P-99)

P-99 (Slide: AM)

3-3 Sidebands & the frequency domain

We know, ins. amplitude value of AM

$$v_{AM} = V_c \sin 2\pi f_c t + \frac{V_m}{2}\cos 2\pi (f_c - f_m)t - \frac{V_m}{2}\cos 2\pi (f_c + f_m)t$$

$\therefore$ USB $= f_c + f_m$

LSB $= f_c - f_m$.

* Show it in frequency domain (Slide: AM, P-101) fig 3-8

3-4

P(105) AM-POWER

instanteneous

$$v_{AM} = V_c \sin 2\pi f_c t + \frac{V_m}{2}\cos 2\pi t(f_c - f_m) - \frac{V_m}{2}\cos 2\pi t(f_c + f_m)$$

r.m.s,

$$v_{AM(r.m.s)} = \frac{V_c}{\sqrt{2}}\sin 2\pi f_c t + \frac{V_m}{2\sqrt{2}}\cos 2\pi t(f_c - f_m) - \frac{V_m}{2\sqrt{2}}\cos 2\pi t(f_c + f_m)$$

$$\therefore \text{Total Power } (P_T) = \frac{(V_c/\sqrt{2})^2}{R} + \frac{(V_m/2\sqrt{2})^2}{R} + \frac{(V_m/2\sqrt{2})^2}{R}$$

$$= \frac{V_c^2}{2R} + \frac{V_m^2}{8R} + \frac{V_m^2}{8R}.$$

$$= \frac{V_c^2}{2R} + \frac{m^2 V_c^2}{8R} + \frac{m^2 V_c^2}{8R} \quad \left[m = \frac{V_m}{V_c}\right].$$

$$= \frac{V_c^2}{2R}\left(1 + \frac{m^2}{4} + \frac{m^2}{4}\right)$$

$$= \frac{V_c^2}{2R}\left(\frac{m^2 + m^2}{4} + 1\right)$$

$$= \frac{V_c^2}{2R}\left(1 + \frac{m^2}{2}\right)$$

$$\therefore P_T = P_c\left(1 + \frac{m^2}{2}\right).$$

Example 3-3 (P-106)

Given, $P_c = 30W$

$m = 85\% = 0.85$

$\boxed{a)\ P_T = ?}$

$$P_T = P_c\left(1 + \frac{m^2}{2}\right)$$

$$= 30\left(1 + \frac{0.85^2}{2}\right)$$

$$= 40.8375W$$

$\boxed{b)\ P_{SB}\ (one) = ?}$

$$P_T = P_c + P_{SB(both)}$$

$$\Rightarrow P_{SB} = P_T - P_c$$

$$= 40.8375 - 30\ W$$

$$= 10.8375\ W.$$

*Math ref to current Ex-3-4 (P-108) (Self).

* Current Related Math.

$\boxed{\text{Efficiency of AM wave}}$

We know,

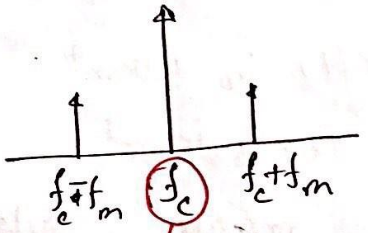
$$P_T = P_c + P_{SB(both)}$$

$$\Rightarrow P_c\left(1+\frac{m^2}{2}\right) = P_c + P_{SB(both)}$$

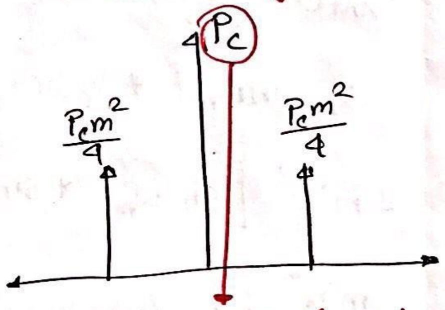
$$\Rightarrow P_c + \frac{P_c m^2}{2} = P_c + P_{SB(both)}$$

$$\Rightarrow P_{SB(both)} = \frac{P_c m^2}{2}.$$

$$\Rightarrow P_{SB(one)} = \frac{P_c m^2}{4}.$$



no-message here.



Huge power is lost in carrier propagation but here there is no message.

Now,

$$\eta = \frac{\text{useful power}}{\text{total power}}.$$

$$= \frac{P_{SB(both)}}{P_c + P_{SB(both)}} = \frac{\frac{\cancel{P_c} m^2}{2}}{\cancel{P_c}\left(\cancel{1+\frac{m^2}{2}}\right) + \frac{P_c m^2}{2}} = \frac{\frac{m^2}{\cancel{2}}}{\cancel{1/4}}$$

$$= \frac{\frac{P_c m^2}{2}}{P_c + \frac{P_c m^2}{2}}.$$

$$= \frac{\frac{m^2}{2}}{1+\frac{m^2}{2}}$$

$$= \frac{\frac{m^2}{2}}{\frac{2+m^2}{2}}$$

$$= \frac{m^2}{2+m^2}.$$

$$\therefore \eta = \frac{m^2}{m^2+2} \times 100\%.$$

$$\eta_{max}(\text{at } m=1) = \frac{1}{3} \times 100\%$$

$$= 33.33\%.$$

($2/3$rd power lost in carrier) (Very poor η).

[So, to propagate we need to omit carrier frequency f_c]

DSB-SC : Double side band Suppresed carrier.

$m(t) = V_m \sin w_m t$

$c(t) = V_c \sin w_c t.$

Use product modulator,

$e_{AM}(t) = m(t) * c(t)$

$= V_m \sin w_m t * V_c \sin w_c t$

$= m V_c^2 [\sin w_m t * \sin w_c t].$

$= \frac{m V_c^2}{2} [\underbrace{\cos(w_c - w_m)t}_{LSB} - \underbrace{\cos(w_c + w_m)t}_{USB}]$

* Both Side-bands contain message, for better accuracy we can use filters to omit one side band.

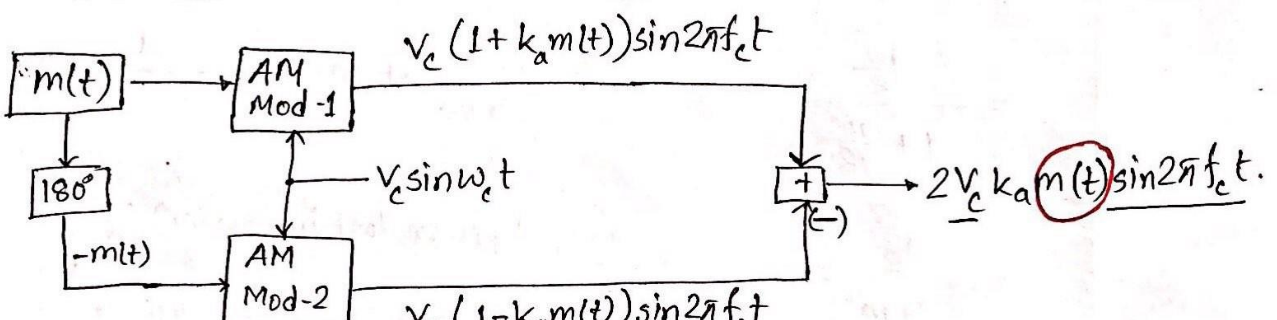
DSB-SC implementation:

we know,

$v_{AM} = v_1 \sin 2\pi f_c t = (V_c + V_m \sin 2\pi f_m t) \sin 2\pi f_c t.$

$= V_c (1 + \frac{V_m}{V_c} \sin 2\pi f_m t) \sin 2\pi f_c t$

$= V_c (1 + k_a m(t)) \sin 2\pi f_c t.$

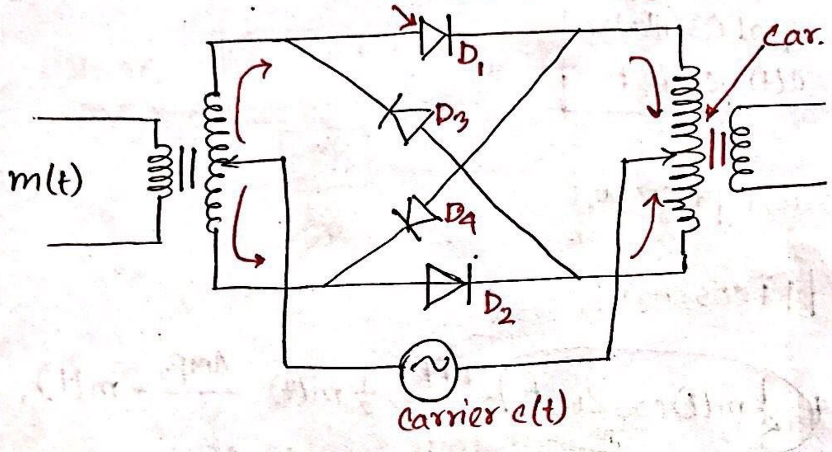


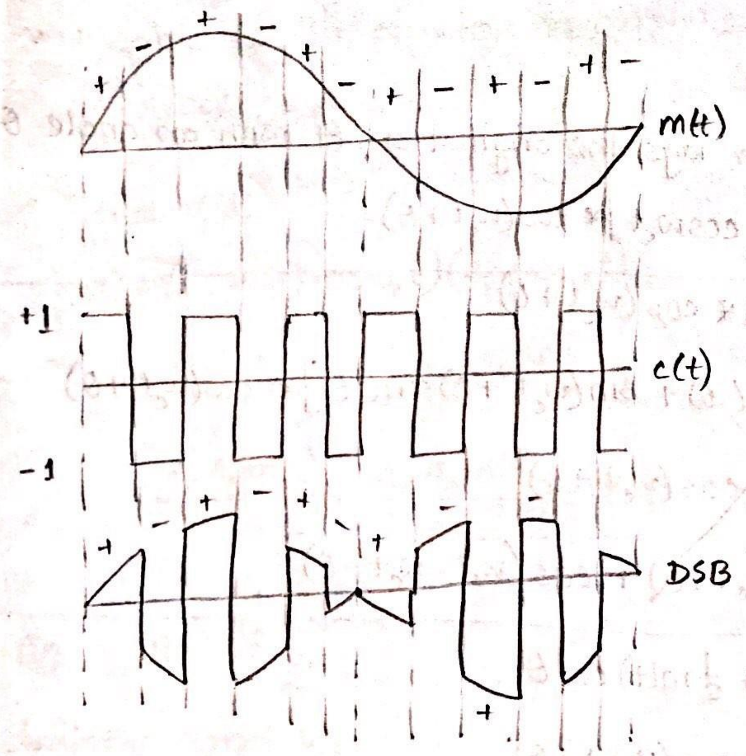
(Balanced Modulator)

Balanced Ring Modulator:

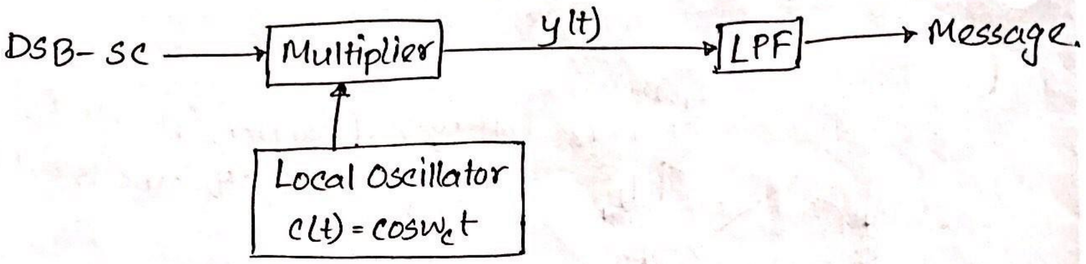
D_1, D_2 ON is +ve half cycle of carrier, Opp. for D_3 & D_4.

car. c(t) cancels out in the loop.





DSB-SC Detection:



$$y(t) = [m(t)\cos w_c t] * \cos w_c t.$$

$$= m(t) . \frac{1}{2}[1+\cos 2w_c t].$$

$$= \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2w_c t \xrightarrow{LPF} \frac{1}{2}m(t) \xrightarrow{Amp.} m(t).$$

Case 1 : Phase Error.

Let, the local oscillator lags the original signal with an angle θ, .

Now, O/p $y(t) = [m(t)\cos w_c t] * \cos(w_c t+\theta)$

$$= m(t)\cos(\underbrace{w_c t+\theta}_{A} - \underbrace{\theta}_{B}) * \cos(w_c t+\theta)$$

$$= m(t)[\cos(w_c t+\theta)\cos(-\theta) + \sin(w_c t+\theta)\sin(-\theta)] * \cos(w_c t+\theta)$$

$$= m(t) . \frac{1}{2} . 2\cos w_c t . \cos(w_c t+\theta)$$

$$= \frac{1}{2}m(t)[\cos(w_c t + w_c t + \theta) + \cos(w_c t - w_c t - \theta)].$$

$$= \frac{1}{2}m(t)\cos(2w_c t+\theta) + \frac{1}{2}m(t)\cos\theta.$$

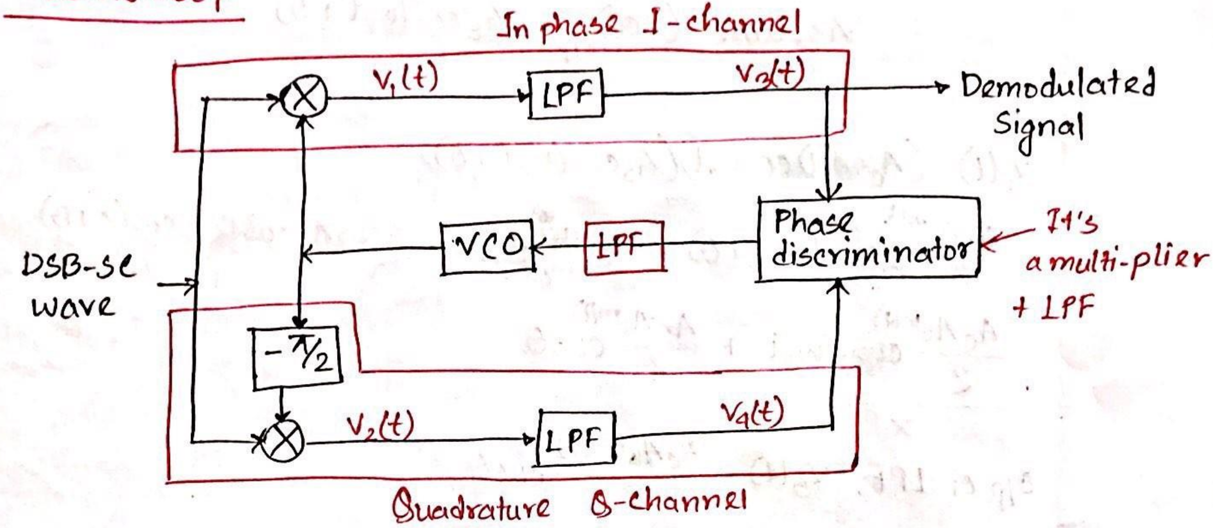
$$= \frac{1}{2}m(t)\cos\theta + \frac{1}{2}m(t)\cos(2w_c t+\theta).$$

If $\cos\theta = 0$ then the message will disappear.

$\theta = 90°$

* Balanced Ring mod. এর slide দেখতে হবে - (Lecture-02 DSB, SC, S/N-09).

Costas Loop



Case-1 Lets assume that frequency and phase of VCO output are same as that of incoming carrier, then.

$$(VCO)_{O/P} = A_0 \cos w_c t$$

- Signal $v_1(t)$

$$v_1(t) = (A_c m(t) \cos w_c t)(A_0 \cos w_c t)$$

$$= \frac{A_c A_0 m(t)}{2}(1+\cos 2w_c t)$$

$$= \frac{A_c A_0 m(t)}{2} + \underbrace{\frac{A_c A_0 m(t)}{2}\cos 2w_c t}_{\times (LPF)}$$

$\therefore$ At multiplier,
$= V_3(t) * V_4(t)$
$= 0$.

$\therefore$ VCO is free-running.

$$O/P \ \ v_3(t) = \frac{A_c A_0 m(t)}{2}$$

Similarly signal $v_2(t)$

$$v_2(t) = (A_c m(t) \cos w_c t)(A_0 \sin w_c t.)$$

$$= \frac{A_c A_0 m(t)}{2} . 2\cos w_c t . \sin w_c t .$$

$$= \frac{A_c A_0 m(t)}{2} . \sin 2w_c t \quad [2\sin A \cos A = \sin 2A]$$

O/P at $V_4(t) = 0$

Case-II Assume $(VCO)_{o/p} = A_0 \cos(w_c t + \theta)$

$$v_1(t) = (A_c m(t) \cos w_c t)(A_0 \cos(w_c t + \theta))$$

$$= \frac{A_c A_0 m(t)}{2} \{2\cos(w_c t + \theta) * \cos w_c t\}$$

$2\cos A . \cos B = \cos(A+B) + \cos(A-B)$

$$= \underbrace{\frac{A_c A_0 m(t)}{2} \cos 2w_c t}_{\times} + \frac{A_c A_0 m(t)}{2} \cos\theta$$

O/p of LPF; $v_3(t) = \dfrac{A_c A_0 m(t)}{2} \cos\theta$

$$v_2(t) = (A_c m(t) \cos w_c t)(A_0 \sin(w_c t + \theta))$$

$$= \frac{A_c A_0 m(t)}{2} \sin(2w_c t) + \frac{A_c A_0 m(t)}{2} \sin\theta$$

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$.

$\therefore$ O/p at LPF, $v_4(t) = \dfrac{A_c A_0 m(t)}{2} \sin\theta$

At Multiplier, the error signal

$$e(t) = v_3(t) * v_4(t)$$

$$= \left(\frac{A_c A_0 m(t)}{2} \cos\theta\right) * \left(\frac{A_c A_0 m(t)}{2} \sin\theta\right)$$

$$= \frac{1}{8} . (A_c A_0 m(t)) 2\sin\theta . \cos\theta$$

$$= \boxed{\frac{1}{8} . A_c A_0 m(t)} \sin 2\theta.$$

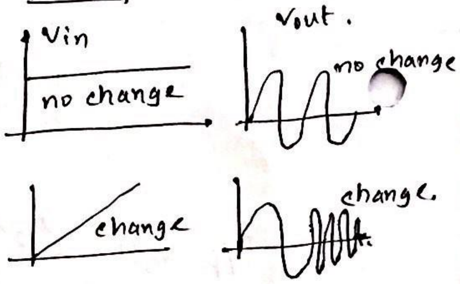
$$= C \sin 2\theta$$

constant value

O/p of LPF also $\boxed{C\sin 2\theta}$, thus VCO will give constant output which is the carrier output.

* In VCO → Frequency change with change of input.

$DC \rightarrow \boxed{VCO} \rightarrow$ AC / frequency.



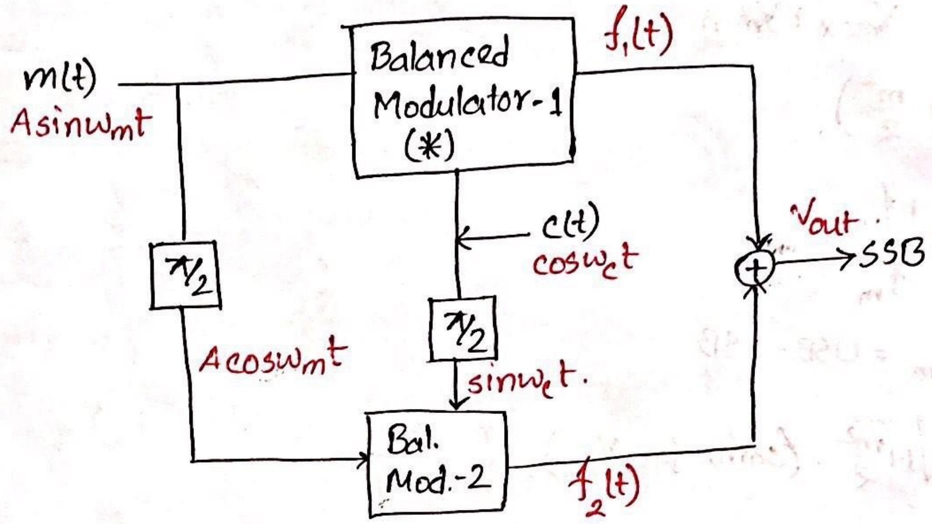
* DSB-SC Square Detection (self).

SSB-SC

Methods of generating SSB.

i) Filtering Method (self)

ii) Phasing Method :



$f_1(t) = A\sin w_m t \cdot \cos w_c t$

$f_2(t) = A\cos w_m t \cdot \sin w_c t$.

$V_{out} = f_1(t) + f_2(t)$

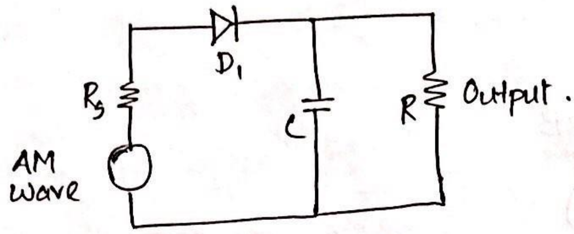
$= A\left[\sin w_m t \cdot \cos w_c t + \cos w_m t \cdot \sin w_c t\right]$

$= A\sin(w_m + w_c)t$. (only USB)

If you need LSB just use $\boxed{f_1(t) - f_2(t)}$

* VSB
* Adv. + Disadv. of SSB, DSB, AM (self).

Detection of AM wave: (Slide, Lecture 01 AM, Slide No-20)



Formulas rel^d to AM

$$m = \frac{V_m}{V_c} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}.$$

$$P_{tot} = P_c\left(1 + \frac{m^2}{2}\right).$$

$$USB = f_c + f_m$$

$$LSB = f_c - f_m$$

$$BW = 2f_m = USB - LSB$$

$$I_{tot} = I_c\sqrt{1+\frac{m^2}{2}}$$. (same for V_{tot}).

AM Rel^d Math.

~~1. A carrier wave of freq. 10MHz~~

1. An audio frequency signal $10\sin 2\pi 500t$ is used to amplitude modulate a carrier of $50\sin 2\pi 10^5 t$. Calculate:

i) Modulation index
ii) Sideband frequencies
iii) Amplitude of each side-band frequencies
iv) Band width required
v) Total Power delivered to the load of 600Ω.
vi) Transmission efficiency.

Sol^n:-

i) $m = \frac{V_m}{V_c} = \frac{10}{50} = 0.2$

ii) $f_m = 500Hz$ $\quad f_{USB} = f_c + f_m = 100.5kHz$.

$f_c = 10^5 Hz$. $\quad f_{LSB} = f_c - f_m = 99.5kHz$.

iii) $Amp_{(SB)} = \frac{mV_c}{2} = \frac{0.2\times 50}{2} = 5V$

iv) $BW = 2\times 500Hz = 1kHz$.

v) $P_{tot} = P_c\left(1+\frac{m^2}{2}\right) = \frac{V_c^2}{2R}\left(1+\frac{m^2}{2}\right) = \frac{50^2}{2\times 600}\left(1+\frac{0.2^2}{2}\right) = 2.125W$

vi) $\eta = \frac{m^2}{m^2+2}\times 100\%$

$= 1.96\%$.

The O/p from AM modulator is:

$s(t) = 5\cos(1800\pi t) + 20\cos(2000\pi t) + 5\cos(2200\pi t)$

Find a) $m = ?$ b) $P_c = ?$

Soln:-

$s(t) = 5\cos(1800\pi t) + 20\cos(2000\pi t) + 5\cos(2200\pi t)$

$= 5\left[\cos(1800\pi t) + \cos(2200\pi t)\right] + 20\cos(2000\pi t)$

$= 5\left[\cos(\underbrace{2000\pi t}_{A} + \underbrace{200\pi t}_{B}) + \cos(2000\pi t - 200\pi t)\right] + 20\cos(2000\pi t)$

$= 5 . 2\cos(2000\pi t) . \cos(200\pi t) + 20\cos(2000\pi t)$

$= 20\cos(2000\pi t)\left[1 + \frac{1}{2}\cos(200\pi t)\right]$.

$= \underbrace{20}_{V_c}\left[1 + \underbrace{\boxed{0.5}}_{m}\cos(\underbrace{\boxed{200\pi}}_{2\pi f_m} t)\right]\cos(\underbrace{\boxed{2000\pi}}_{2\pi f_c} t)$ $\left[v_{AM}(t) = V_c(1 + m\sin 2\pi f_m t)\sin 2\pi f_c t\right]$.

a) $m = 0.5$

b) $P_c = \frac{V_c^2}{2} = \frac{20^2}{2} = 200W$.

Frequency Modulation

* Noise can be easily added to amplitude modulated signal.

$$f(t) = A\cos(2\pi f t + \phi)$$

A → Amp.

f → freq.

ϕ → phase

$m(t) = A_m \cos w_m t = A_m \cos 2\pi f_m t$ (message) $\quad f_c >> f_m$

$c(t) = A_c \cos w_c t = A_c \cos 2\pi f_c t$ (carrier)

Modulated Signal. (i)

$s(t) = A_c \cos\theta \quad \theta$ = changed frequency

Instantaneous angular frequency $w_i t = 2\pi f_i t = \theta$.

Now,

$$\frac{d\theta}{dt} = 2\pi f_i$$

$$\Rightarrow d\theta = 2\pi f_i dt$$

$$\therefore \theta = 2\pi \int_0^t f_i dt \quad \cdots (ii)$$

Here

$$f_i = f_c + k_f m(t) \quad \cdots (iii).$$

k_f → frequency sensitivity.

Now from (iii),

$$\theta = 2\pi \int_0^t [f_c + k_f m(t)] dt$$

Putting this in (i)

$$s(t) = A_c \cos\left[2\pi \int_0^t f_c dt + 2\pi k_f \int_0^t m(t) dt\right] \quad \cdots\cdots (A)$$

<u>Frequency deviation</u>: Difference betn maxm inst. frequency and normal carrier frequency.

$$f_i = f_c + k_f m(t)$$

$$\Delta f = f_{i_{max}} - f_c \quad \cdots\cdots (iv)$$

$$f_i = f_c + A_m k_f \cos w_m t.$$

$$\therefore f_{i_{max}} = f_c + A_m k_f \quad \left[\text{when } w_m t = 0 \text{ or } \cos w_m t = 1\right] \cdots (v).$$

Putting (v) in (iv).

$$\Delta f = f_c + A_m k_f - f_c.$$

$$\Rightarrow \Delta f = k_f A_m \quad \cdots\cdots (vi).$$

Putting (vi) in (A).

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi . \frac{\Delta f}{A_m} \int_0^t A_m \cos 2\pi f_m t . dt\right].$$

$$= A_c \cos\left[2\pi f_c t + 2\pi . \frac{\Delta f}{A_m} . \frac{A_m \sin 2\pi f_m t}{2\pi f_m}\right].$$

$$= A_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t\right].$$

Modulation Index (β).

Depending of β, two types:

a) Narrowband frequency modulation
b) Wideband " "

$$= A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right].$$

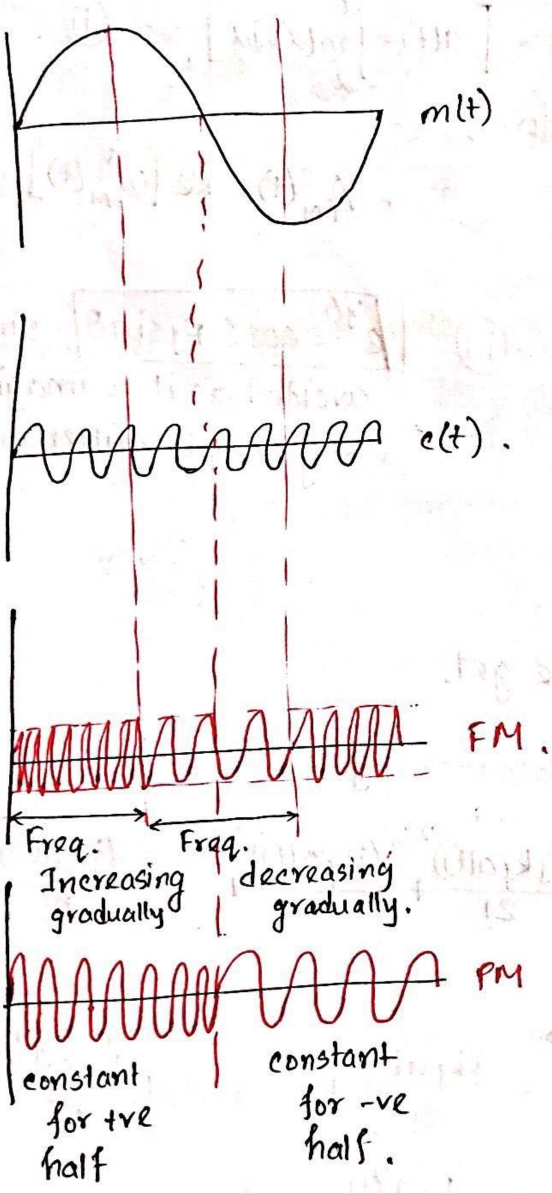
Angle Modulation

~~AM~~ FM + PM

$\phi(t) = A\cos(w_c t + km(t))$

$A\cos(w_c t + \theta)$

to phase modulate just differentiate $\frac{d}{dt}(w_c t + \theta)$
$= w_c + \frac{d\theta}{dt}$.



* $e^{j\theta} = \cos\theta + j\sin\theta$

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$

fourier of rect. pulse $\Rightarrow$ Sinc func

$\lim_{n\to\infty} \frac{x^n}{n!} \approx 0$

$$\phi_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\alpha}^{t} m(t)\,dt\right)$$

$$\phi_{PM}(t) = A\cos\left(\omega_c t + k_p m(t)\right).$$

see, ch-5, (P-203-205) (B.P. Lathi).

Bandwidth Analysis of Angle Modulated Waves: (B.P. Lathi P-209)

$$\phi_{FM} = A\cos\left(\omega_c t + k_f \int_{-\alpha}^{t} m(\alpha)\,d\alpha\right) \quad \cdots \text{(i)}$$

$$= A\cos\left(\omega_c t + k_f\, a(t)\right) \quad \left[a(t) = \int_{-\alpha}^{t} m(\alpha)\,d\alpha\right]. \quad \cdots \text{(ii)}.$$

Expanding (ii) in polar form,

$$\hat{\phi}_{FM} = A e^{j(\omega_c t + k_f a(t))} \longrightarrow \phi_{FM}(t) = Re\left[\hat{\phi}_{FM}(t)\right]$$

$$= A e^{j k_f a(t)} \cdot e^{j\omega_c t} \quad \cdots \text{(iii)}$$

[$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$, sineterm is avoided as it is imaginary, that's why ϕ_{FM} is written as $\hat{\phi}_{FM}$].

we need,

$$\therefore \phi_{FM} = Re(\hat{\phi}_{FM})$$

Expanding equ^n (iii) we get,

$$\left[e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots\right].$$

$$\hat{\phi}_{FM} = A\left[1 + jk_f a(t) + \frac{(jk_f a(t))^2}{2!} + \frac{(jk_f a(t))^3}{3!} + \cdots + \frac{(jk_f a(t))^n}{n!}\right](\cos\omega_c t + j\sin\omega_c t)$$

$$= A\left[1 + jk_f a(t) - \frac{k_f^2 a^2(t)}{2!} - \frac{jk_f^3 a^3(t)}{3!} + \cdots\cdots + \frac{j^n k_f^n a^n(t)}{n!}\right](\cos\omega_c t + j\sin\omega_c t)$$

$$= A\left[\cos\omega_c t - k_f a(t)\sin\omega_c t - \frac{k_f^2 a^2(t)}{2!}\cos\omega_c t \cdots\cdots\right]$$ (omitting all imaginary terms).

Finally.

$$\phi_{FM} = A\left[\boxed{\cos w_c t - k_f a(t)\sin w_c t} - \frac{k_f^2 a^2(t)}{2!}\cos w_c t + \cdots\right]$$

(we don't need it)

This term looks like a traditional DSB-SC AM wave

If the (~~Bandwidth~~ or freq) of ~~red box~~ a(t) is B then the total BW will be 2B.

So, the red box is called narrowband FM (NBFM).

The BW of others will be high.

If let, $a(t) = \cos\theta \Rightarrow a^2(t) = \cos^2\theta = \frac{(1+\cos 2\theta)}{2}$. [Angle increased two times so BW will also increase].

Similarly $a^3(t) = \cos^3\theta = \frac{\cos 3A + 3\cos A}{4}$ [BW tripled]

Thus the others is called wide-band FM (WBFM).

[Q] Does that mean that the BW of FM is infinity?

No, as $\frac{k_f^n a^n(t)}{n!} \approx 0$ → very small compared to n!

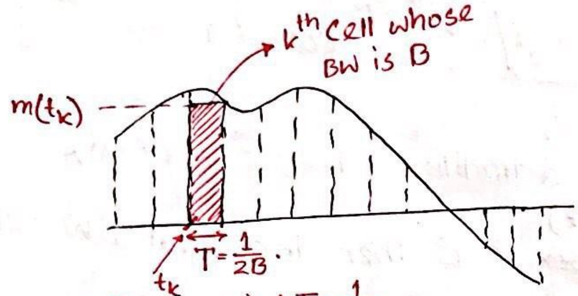
$$\frac{2^{100}}{100!} = 1.35\times 10^{-128} \text{ (Matlab)}$$

It show that the last term will tend to zero and the BW will become finite eventually.

(*) PM equ^n also similar.

$$\phi_{PM} = \cos(w_c t + k_p m(t)).$$

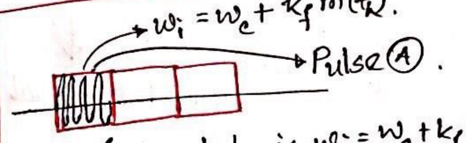
(16).20 (extra two pages)



* Time period $T = \frac{1}{2B}$ (Nyquist Theorem)

* Divide into ~~no. of~~ cells (Pulses).

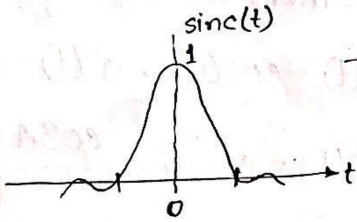
*

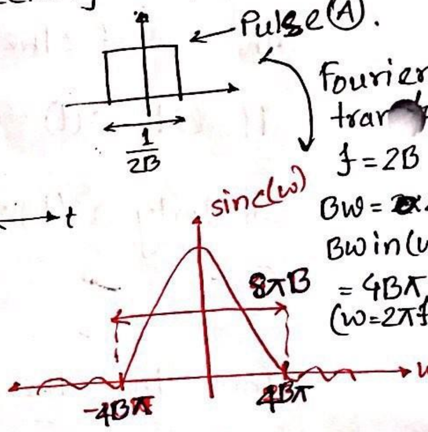


Ins. freq. at t_k is $w_i = w_c + k_f m(t)$.

* The frequency domain representation of a rectangular pulse is a sinc function.

$$\text{sinc}(t) = \begin{cases} 1 & t = 0 \\ \frac{\sin t}{t} & t \neq 0 \end{cases}$$





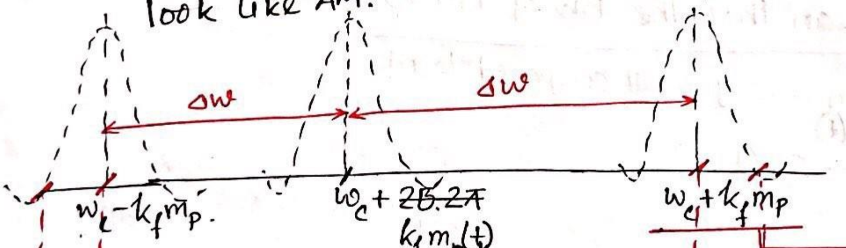
$f = 2B$

BW = 2B

BW in (w) = $4B\pi$ ($w = 2\pi f$).

Thus, the pulse (A) is Amp. modulated with freq. $w_i = w_c + k_f m(t_k)$. The freq. domain will look like AM.

$\pm m_p$ = maximum and minimum peak to peak value at $m(t_k)$ / Deviation.



see page-16 equⁿ (v) $f_{imax} = A_m k_f + f_c$.

Now,

$2\Delta w = w_c + k_f m_p - w_c + k_f m_p$

$\therefore 2\Delta w = 2 k_f m_p$.

$\therefore \Delta w = k_f m_p$.

$2B.2\pi$.

$2B.2\pi$.

2B in normal freq. To transform into ang. freq(w) multiply 2π.

Total BW_{FM} (in ang. freq) $= 2\Delta w + 4B\pi + 4B\pi$
$= 2\Delta w + 8B\pi = 2k_f m_p + 8B\pi$

In normal freq total BW_{FM} $= \frac{BW_{FM}(ang.)}{2\pi}$

$= \frac{2k_f m_p + 8B\pi}{2\pi}$

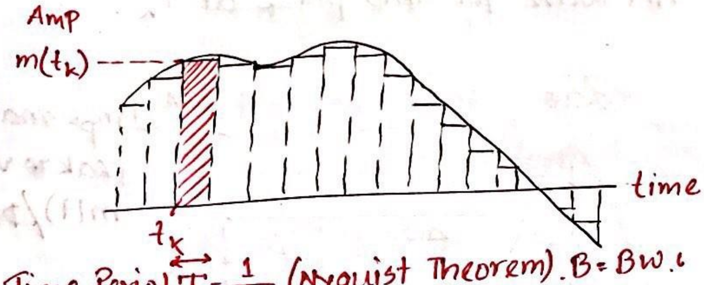
$= 2\left[\frac{k_f m_p}{2\pi} + \frac{\cancel{4}^{2}B\cancel{\pi}}{\cancel{2\pi}}\right]$

$= 2\left[\frac{k_f m_p}{2\pi} + 2B\right]$

$= 2\left[\frac{\Delta w}{2\pi} + 2B\right]$ $[\because \Delta w = k_f m_p]$

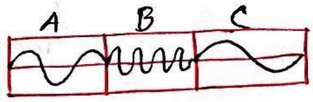
$= 2\left[\Delta f + 2B\right]$ $[\because w = 2\pi f \Rightarrow f = \frac{w}{2\pi}]$

Consider the Shape of the message Signal $m(t)$

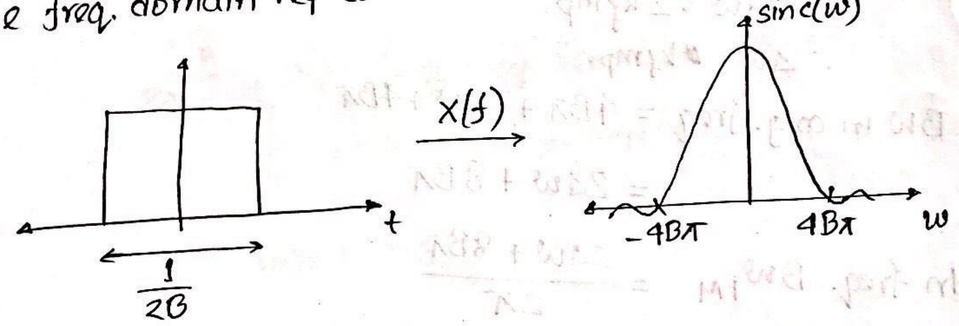


Time Period $T = \frac{1}{2B}$ (Nyquist Theorem). $B = BW$.

Sample into Pulses/cells. Thus each cell has a certain freq. or instantaneous freq. $w_i = w_c + k_f m(t_k)$



The freq. domain representation of a rectangular pulse is Sine fun^c.

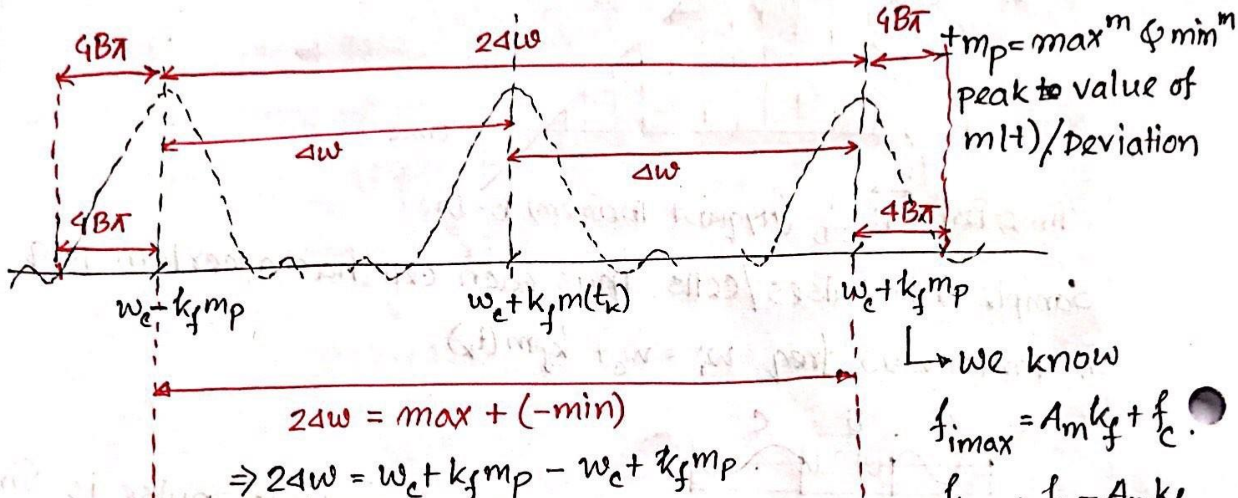


Here, time period $T = \frac{1}{2B}$.

$\therefore$ frequency $f = \frac{1}{T} = 2B$.

$\therefore$ In angular freq. $w = 2\pi f$
$= 4\pi B$.

The pulses are multiplied with sinosoids. Thus the pulse will behave like an AM wave for this pulse at t_K.



↳ we know

$f_{i\,max} = A_m k_f + f_c$.

$f_{i\,min} = f_c - A_m k_f$.

$\Rightarrow 2\Delta\omega = \omega_c + k_f m_p - \omega_c + k_f m_p$.

$\Rightarrow 2\Delta\omega = 2k_f m_p$.

$\therefore \Delta\omega = k_f m_p$.

Total BW in ang. freq $= 4B\pi + 2\Delta\omega + 4B\pi$.

$= 2\Delta\omega + 8B\pi$

$\therefore$ In freq. $BW_{FM} = \dfrac{2\Delta\omega + 8B\pi}{2\pi}$

$= \dfrac{2k_f m_p + 8B\pi}{2\pi}$

$= 2\left[\dfrac{k_f m_p}{2\pi} + \dfrac{4B\pi}{2\pi}\right]$.

$= 2\left[\dfrac{\Delta\omega}{2\pi} + 2B\right]$.

$= 2\left[\Delta f + 2B\right]$. $\left[\because \omega = 2\pi f\right]$.

Now we get;

$$BW_{FM} = 2[\Delta f + 2B]$$

This is the ideal case, but practically the required BW is much smaller.

So, Carson came up with a formula,

$$BW_{FM} = 2[\Delta f + B]$$

Also can be written as $\longrightarrow = 2B\left[\frac{\Delta f}{B} + 1\right]$.

$= 2B(\beta + 1)$ [recall $\beta = \frac{\Delta f}{f_m}$].

β = modulation index $= \frac{\Delta f}{B} = \frac{\Delta f}{f_m}$, Δf = frequency deviation, f_m/B = Message frequency/BW

$\Delta f = \frac{k_f m_p}{2\pi}$, k_f = frequency sensitivity (Hz/V)

m_p = max. peak voltage (V).

(Read B.P. Lathi P-213)

—x—

Spectral Analysis of Tone frequency Modulation (B.P Lathi P-214)
(Verification of Carson's Formula).

Let, message $m(t) = \alpha \cos w_m t$ [sinusoids are called tone modulated signals]

Now,

$$a(t) = \int_0^t m(t) = \frac{\alpha}{w_m} \cdot \sin w_m t \quad [\text{assume } a(-\infty) = 0]$$

We know,

$$\phi_{FM}(t) = A\cos\left(w_c t + k_f \int_{-\infty}^{t} m(t)\,dt\right) \cdots (i)$$

From (i) we get, the estimated value of $\phi_{FM}(t)$

$$\hat{\phi}_{FM}(t) = A e^{j(w_c t + k_f a(t))}$$

$$= A e^{j(w_c t + k_f \int_{-\infty}^{t} \alpha \cos w_m t\,dt)}$$

$$= A e^{j(w_c t + k_f \cdot \frac{\alpha}{w_m} \cdot \sin w_m t)}$$

The deviation from carrier frequency $\Delta w = k_f m_p = k_f \alpha$ [In this case].

$$\Rightarrow \Delta f = \frac{\alpha k_f}{2\pi} \cdots (ii).$$

Now, modulation index/deviation ratio $\beta = \frac{\Delta f}{B}$ [B = Bandwidth of message signal or also written as message frequency f_m] ---- (iii).

Equating (ii) and (iii) we get,

$$\beta = \frac{\Delta f}{B} = \frac{\alpha k_f}{2\pi B} = \frac{\alpha k_f}{w_m} \quad [\because 2\pi B = 2\pi f_m = w_m] \cdots (iv)$$

Recall,

$$\hat{\phi}_{FM}(t) = A e^{j(w_c t + k_f a(t))}$$

$$= A e^{j(w_c t + k_f \cdot \frac{\alpha}{w_m} \cdot \sin w_m t)}$$

Putting the value of β from (iv) in here we get,

$$\hat{\phi}_{FM}(t) = A e^{j(w_c t + \beta \sin w_m t)} \quad \text{--- (v)}$$

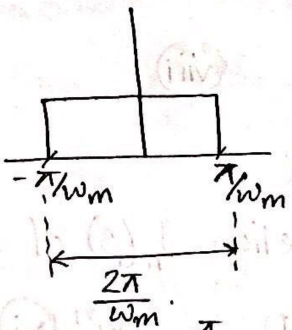
$$= A e^{j w_c t} \cdot e^{j\beta \sin w_m t}$$

Here $e^{j\beta \sin w_m t}$ looks like a periodic signal with period $2\pi/w_m$ and can be expanded by the expotential Fourier series.

$$e^{j\beta \sin w_m t} = \sum_{n=-\alpha}^{\infty} D_n e^{j w_m t} \quad \text{---- (vi)}$$

where $D_n = \frac{1}{\frac{2\pi}{w_m}} \int_{\text{initial Value}}^{\text{Final Value}} e^{j\beta \sin w_m t} \cdot e^{-jn w_m t} \, dt$

Time period



If the bandwidth $\frac{1}{T} = \frac{2\pi}{w_m}$.

So, the initial value $= -\frac{\pi}{w_m}$

& final value $= \frac{\pi}{w_m}$.

$$\therefore D_n = \frac{w_m}{2\pi} \int_{-\frac{\pi}{w_m}}^{\frac{\pi}{w_m}} e^{j\beta \sin w_m t} \cdot e^{-jn w_m t} \, dt \quad \text{-------- (vii)}$$

The equ^n ~~vi~~ vii was,

$$D_n = \frac{\omega_m}{2\pi}\int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j\beta \sin \omega_m t} \cdot e^{-jn\omega_m t}\, dt$$

Let, $\omega_m t = x$

$\Rightarrow \omega_m dt = dx$

$\Rightarrow dt = \frac{1}{\omega_m} dx.$

If, (for limits) $t \to -\frac{\pi}{\omega_m}$.

then, $\boxed{x \to -\pi}$

and $t \to \frac{\pi}{\omega_m}$.

then, $\boxed{x \to \pi}$

$dt = \frac{1}{\omega_m} dx.$

$\Rightarrow \frac{\pi}{\omega_m} = \frac{1}{\omega_m} \cdot x$

$\therefore x = \pi.$

Putting limits, x, dx in equ^n vii we get,

$$D_n = \frac{\omega_m}{2\pi}\int_{-\pi}^{\pi} e^{j\beta \sin x} \cdot e^{-jnx} \cdot \frac{dx}{\omega_m}.$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\beta \sin x} \cdot e^{-jnx} \cdot dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \cdot dx \quad \text{------ (viii)}$$

The equ^n (viii) looks like Bessel function $J_n(\beta)$ of first kind and n^{th} order. Putting the value of D_n in equ^n (vi) we get,

$$e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn\omega_m t} \quad [D_n = J_n(\beta)] \text{ --- (ix)}$$

we know from equ^n (v):

$$\hat{\phi}_{FM}(t) = A e^{j\omega_c t} . e^{j\beta \sin n\omega_m t}$$

Putting the value of (ix) in (v) we get,

$$\hat{\phi}_{FM}(t) = A . e^{j\omega_c t} . \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j\omega_m tn}.$$

$$= A \sum_{n=-\infty}^{\infty} J_n(\beta) . e^{j(\omega_c t + n\omega_m t)}$$

Finally, $\phi_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$ [Return to angular form from polar form Like equ^n (1)].

We can see that the tone modulated FM signal has a carrier component and infinite number of side-band frequencies. Like $\omega_c \pm \omega_m$, $\omega_c \pm 2\omega_m$, ..., $\omega_c \pm n\omega_m$ as $J_{-n}(\beta) = (-1)^n J_n(\beta)$.

To make the BW finite we will cancel where $J_n(\beta)$ is negligible.

Generally $J_n(\beta)$ is negligible for $n > \beta+1$

∴ The number of significant side-band impulses $= \beta+1$.

∴ The Bandwidth of FM is given by $B_{FM} = 2(\beta+1) f_m$

$$= 2(\beta f_m + f_m)$$

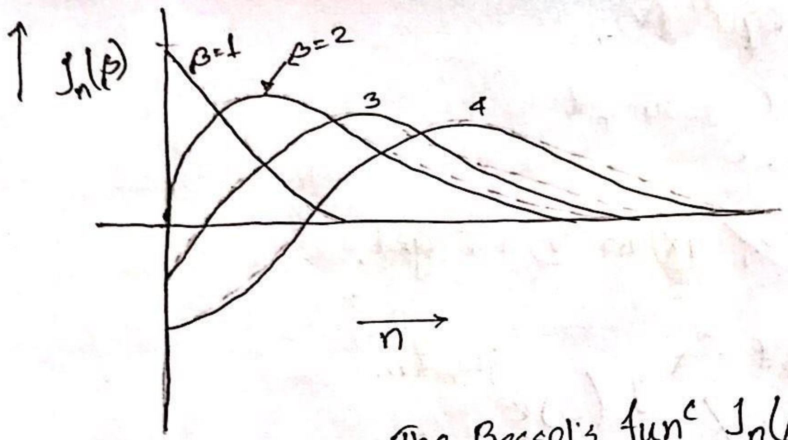
Here, $f_m = B =$ message freq./BW.

$$= 2\left(\frac{\Delta f}{f_m} . f_m + f_m\right)$$

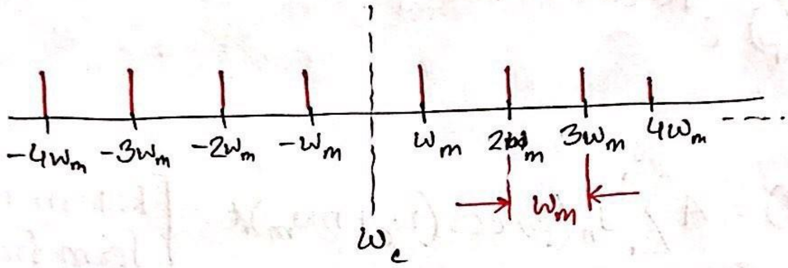
$$= 2(\Delta f + f_m)$$

$$= 2(\Delta f + B).$$

which verifies Carson's formula.



The Bessel's fun^c $J_n(\beta)$.



Notes for $B_{FM} = 2(\beta+1) f_m$.

* when $\beta \ll 1$, $B_{FM} \cong 2 f_m$, only one significant side band
Narrowband FM (NBFB).

* But $B_{FM} = 2\beta\left(1+\frac{1}{\beta}\right) f_m$

for $\beta \gg$ high, $\frac{1}{\beta} \approx 0$, and $B_{FM} \cong 2\beta . f_m$

$$= 2 . \frac{\Delta f}{f_m} . f_m$$

$$= 2\Delta f.$$ (Called wide-band FM

Problems Rel^d to FM

1. A sinusoidal wave of amplitude 10V and frequency 10kHz is applied to an FM generator that has a frequency sensitivity constant of 40Hz/Volt. Determine the frequency deviation & modulation index (also called deviation ratio)

Sol^n:- Given, $A_m = 10V$

$f_m = 10kHz.$

$k_f = 40Hz/Volt.$

$\therefore \Delta f = k_f A_m = 40 \times 10\ Hz.$

$= 400Hz.$ (Ans.)

$$\beta = \frac{\Delta f}{f_m}$$

$$= \frac{400}{10k} = 0.04$$ (Ans.).

2. Find the transmission bandwidth of single tone modulated FM signal described by $s(t) = 10\cos[2\pi 10^8 t + 6\sin(2\pi 10^3 t)]$.

Sol^n:-

Recall, $s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$.

$\therefore f_m = 1kHz$

$f_c = 10^8\ Hz$

$\beta = 6.$

$\therefore$ From Carson's rule $BW = 2(1+\beta)f_m$

$= 2(1+6)1000$

$= 14kHz.$ (Ans.)

3. Consider an FM signal with $\Delta f = 10kHz$, $f_m = 10kHz$, $A_c = 10V$, $f_c = 500kHz$. Draw the spectrum of FM signal.

<u>Sol^n</u>:- Here, $\beta = \frac{\Delta f}{f_m} = \frac{10k}{10k} = 1$.

We know, $\phi_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(w_c + n w_m)t$.

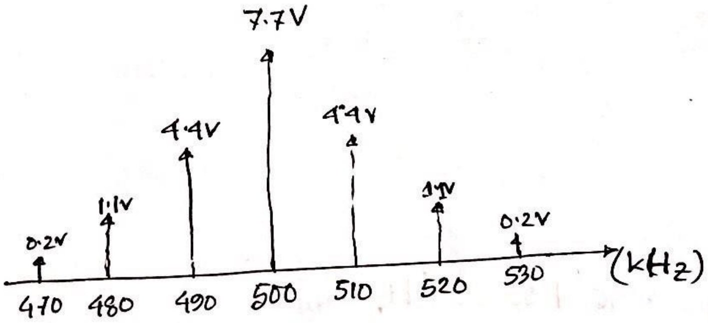
So the side-bands will be found like $w_c \pm w_m$, $w_c \pm 2w_m$, ... $w_c \pm n w_m$.

For $\beta = 1$ from Bessel chart we get, $(J_n(\beta))$

$J_0 = 0.77$ $J_1 = 0.44$ $J_2 = 0.11$ $J_3 = 0.02$, $A = 10V$, $f_m = 10kHz$, $f_c = 500kHz$.

The frequencies will be

$n \neq 0$	$A J_n(\beta)$ [Volts]	$f_c \pm n f_m$ (kHz)
0	$10 \times 0.77 = 7.7$	500kHz.
1	$10 \times 0.44 = 4.4$	490kHz, 510kHz.
2	$10 \times 0.11 = 1.1$	480kHz, 520kHz.
3	$10 \times 0.02 = 0.2$	470kHz, 530kHz.



4. An angle modulated signal with carrier frequency $w_c = 2\pi \times 10^5$ is described by the equation

$$\phi_{EM}(t) = 10\cos(w_c t + 5\sin 3000t + 10\sin 2000\pi t)$$

a) Find the power of the modulated signal
b) Find the frequency deviation Δf
c) Find the deviation ratio β.
d) Find the phase deviation $\Delta\phi$
e) Estimate the bandwidth.

Soln:-

a) $P = \frac{A^2}{2} = \frac{10^2}{2} = 50W \quad \left[\frac{\left(\frac{V}{\sqrt{2}}\right)^2}{R} = \frac{V^2}{2} \quad [R=1] \right]$. ($\frac{V}{\sqrt{2}} = V_{(rms)}$)

b) Ins. frequency $w_i = \frac{d\theta}{dt} = \frac{d}{dt}(w_c t + 5\sin 3000t + 10\sin 2000\pi t)$

$= w_c + 15000\cos 3000t + 20{,}000\pi\cos 2000\pi t$

Max^m deviation occurs at $\cos\theta = 1$.

$\therefore \Delta w = 15000 + 20{,}000\pi$

$\therefore \Delta f = \frac{\Delta w}{2\pi} = 12{,}387 \cdot 32\,Hz$.

c) $\beta = \frac{\Delta f}{f_m}$. There are two f_m here, $\frac{3000}{2\pi}$ & $\frac{2000\pi}{2\pi}$.

To find f_m take the max^m frequency which is 1000Hz.

$\therefore \beta = \frac{12387 \cdot 32}{1000} = 12 \cdot 387$

d) Phase deviation $\Delta\theta$.

Here $\theta(t) = w_c t + 5\sin 3000t + 10\sin 2000\pi t$

Max^m phase deviation occurs when $\sin\theta = 1$.

$\Delta\theta = 5 + 10 = 15$ radian.

e) Bandwidth $= 2(\Delta f + f_m)$

$= 2(12387.32 + 1000)$

$= 26774.64$ Hz.

B.P. Lathi (P-222)

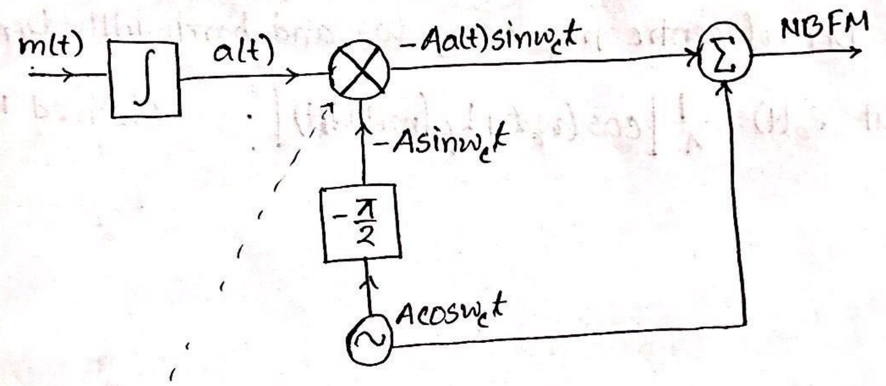
5.3 Generating FM waves

Two ways: 1. Indirect 2. Direct.

NBFM generation by indirect method.

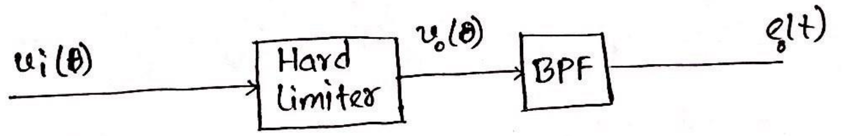
We know, $\phi_{NBFM}(t) \cong A\left[\cos\omega_c t - k_f a(t)\sin\omega_c t\right]$.

where, $a(t) = \int m(t)\,dt$. and $|k_f a(t)| \ll 1$



The multiplication is like DSB-SC of AM. So the output amplitude will vary like AM which is undesirable.

To make the amp. constant we use band-pass limiter.

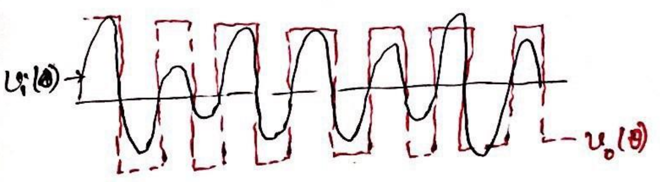


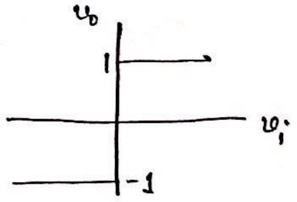
$\theta = \omega_c t + k_f \int m(t)\,dt.$

Band Pass limiter

In hard limiter

$$v_o(\theta) = \begin{cases} +1 & \cos\theta > 0 \\ -1 & \cos\theta < 0 \end{cases}$$





Hence, v_o as a function of θ is a periodic square wave fun^c with period 2π, It can be expanded in Fourier Series,

$$v_o(\theta) = \frac{4}{\pi}\left(\cos\theta - \frac{1}{3}\cos 3\theta + \frac{1}{5}\cos 5\theta + \cdots\right) \cdots (i)$$

Now, recall $s(t) = A\cos\theta$, $\theta = w_c t + k_f \int m(t)dt$.

Putting value of θ in (i)

$$v_o(\theta) = \frac{4}{\pi}\left[\cos\left(w_c t + k_f \int m(t) d(t)\right) - \frac{1}{3}\cos 3\left(w_c t + k_f \int m(t)dt\right) + \cdots\right]$$

Now, use BPF of centre frequency w_c and bandwidth B_{FM} the output $e_o(t) = \frac{4}{\pi}\left[\cos\left(w_c t + k_f \int m(t) dt\right)\right]$. $\cdots$ (desired FM wave)

FM Demodulation Basic

In transmitter, $\phi_{FM} = A\cos(w_c t + k_f\int m(t)dt)$ which will come to receiver.

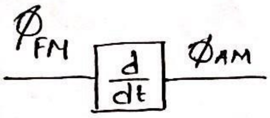
As m(t) is in integrated form, we start with differentiation.

$-\sin\theta = \sin(\theta - \pi)$

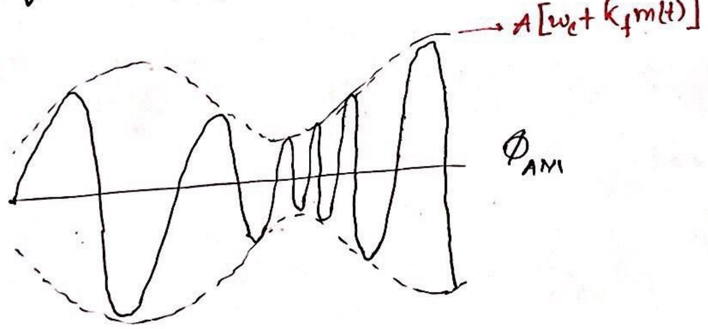
$$\phi_{FM} = A\cos(w_c t + k_f\int m(t)dt) \quad \text{----- (i)}$$

$$\frac{d\phi_{FM}}{dt} = A\left[w_c + k_f m(t)\right]\sin(w_c t + k_f\int m(t)dt - \pi) \quad \text{-- (ii)}$$

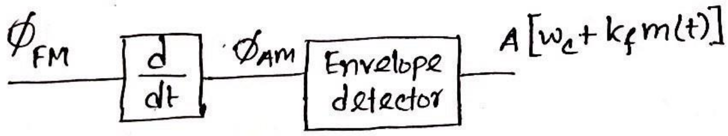
$$= \phi_{AM}$$



But ϕ_{AM} is not an actual amp. modulated signal. Both Amp. & freq. is varying here.



To get the message we can use envelope detector.



$\therefore$ to remove w_c use LPF.

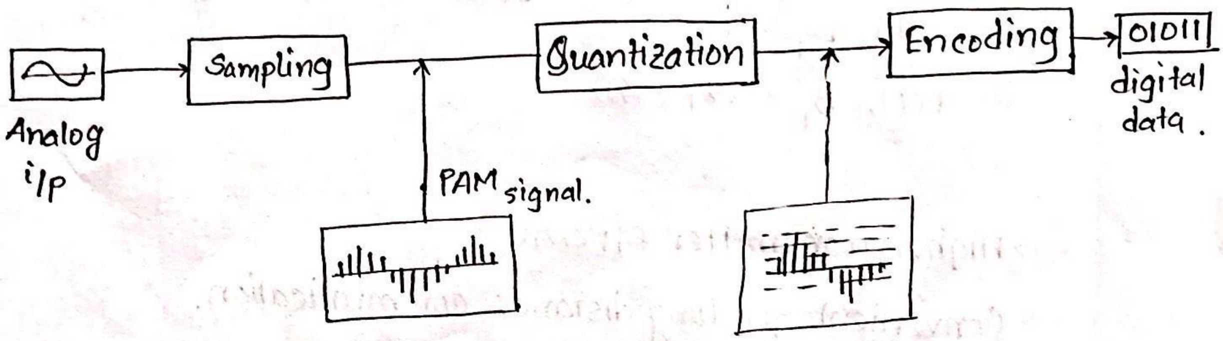
But still there is high freq. w_c. So use ~~High~~ Low Pass Filter.

Classification of FM receivers

- Freq. discriminator
  - Simple Slope detector
  - Balanced Slope detector
- Phase discriminator
  - Foster-Sealay
  - Ration detector
- PLL
- Zero Crossing Detector

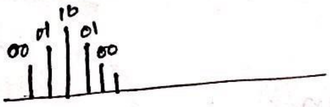
Pulse Code Modulation

* PCM is used to convert analog into digital data.

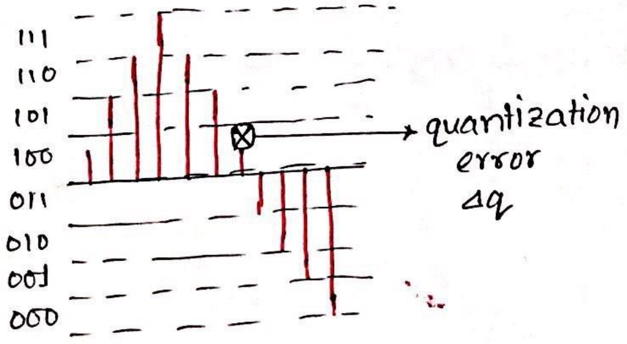


* Sampling ⇒ Analog i/p * Uniform Pulse train.

After PAM,
it is difficult to denote the binary/digital data of every amplitude.



So, quantization is done,



Level $L = 2^3 = 8$

for $L\uparrow$, $\Delta q \downarrow$ but with the price of Bandwidth.

The digital values are taken to encoder and transmitted as PCM.

Advantages:

→ Higher noise immunity.

AM, $B_T \propto \left(\frac{S}{N}\right)$.

FM, $B_T \propto \left(\frac{S}{N}\right)^2$

PCM, $B_T \propto \exp(S/N)$.

Bandwidth Efficiency = B_T.

→ Higher transmitter efficiency

→ Convenient for long distance communication.

Drawbacks:

→ Large Bandwidth

→ Encoding, decoding, quantizing circuit is very complex.

Application:

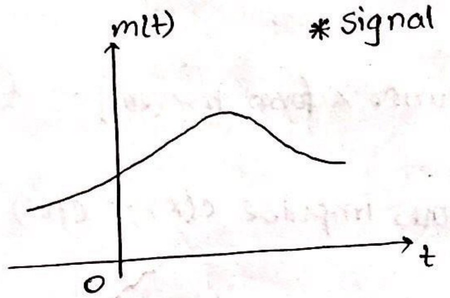
→ Used in satellite transmission

→ telephony

→ CD.

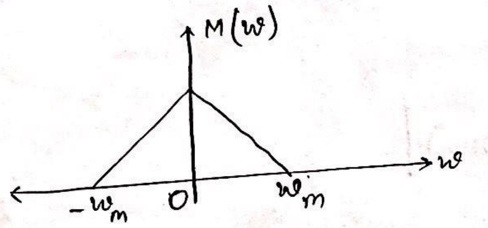
Sampling Theorem

Sampling: Continuous time Signal to discrete time signal.



* signal must band-limited.

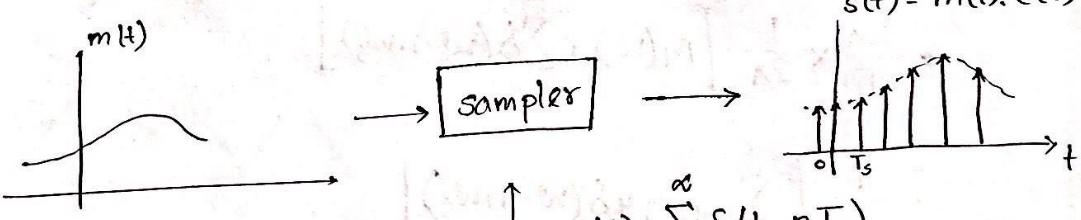
Let, the fourier transform of this signal is:



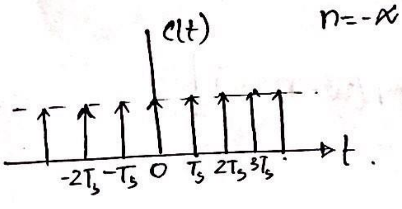
w_m = max. freq. component of message m(t)

when m(t) is multiplied in a sampler, with c(t) a continuous pulse train.
(impulse train)

$s(t) = m(t).c(t).$



$$c(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s).$$



T_s = fundamental time period / Sampling Period ~~or rate~~.

$$w_s = \frac{2\pi}{T_s} = \text{Sampling frequency.}$$

Now we have

$$s(t) = m(t) . c(t)$$

Fourier transform.

$$s(w) = \frac{1}{2\pi}\left[M(w) * c(w)\right]. \quad \{\text{fourier x-form theorem}\} \text{ --- (i)}$$

The fourier transform of continuous impulse $c(t)$ is $c(w)$

$$\therefore c(w) = w_s \sum_{n=-\infty}^{\infty} \delta(w - nw_s) \text{ ---- (ii)}$$

Now; Putting (ii) in (I)

$$s(w) = \frac{1}{2\pi}\left[M(w) * w_s \sum_{n=-\infty}^{\infty} \delta(w - nw_s)\right]$$

$$= \frac{w_s}{2\pi}\left[M(w) * \sum_{n=-\infty}^{\infty} \delta(w - nw_s)\right].$$

$$= \frac{2\pi}{T_s} \times \frac{1}{2\pi}\left[M(w) * \sum_{n=-\infty}^{\infty} \delta(w - nw_s)\right]$$

$$= \frac{1}{T_s}\left[\sum_{n=-\infty}^{\infty} M(w) * \delta(w - nw_s)\right]$$

$$= \frac{1}{T_s}\left[\sum_{n=-\infty}^{\infty} M(w - nw_s)\right]. \qquad \left[\because x(t) * \delta(t - t_1) = x(t - t_1)\right].$$

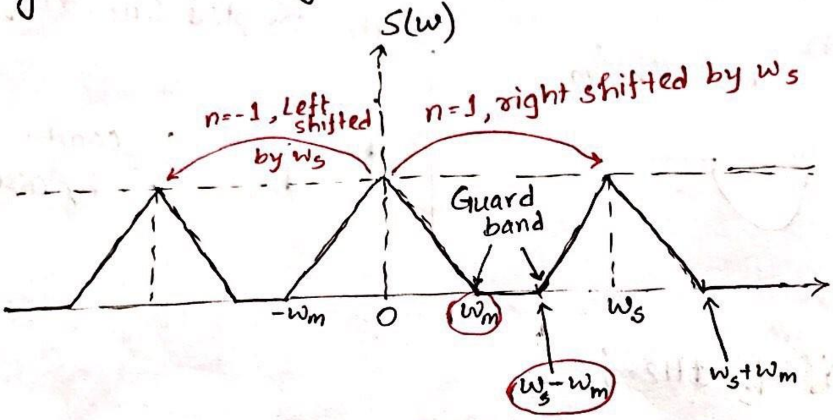
Right shifted by w_s

$$s(w) = \frac{1}{T_s}\left[\text{------} + M(w + w_s) + M(w) + M(w - w_s) + M(w - 2w_s) + \text{----}\right]$$

(Putting $n = -\infty, \text{----} -1, 0, 1 \text{--} \infty$).

Now see the fourier diagram $M(w)$

Plotting the $S(\omega)$ we get,



Here we can see,

$\omega_s - \omega_m > \omega_m$

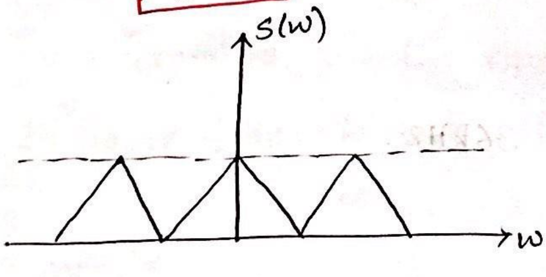
ω_s = Sampling freq.

ω_m = Max. freq. component of message $m(t)$.

$\therefore \omega_s > 2\omega_m$

when $\omega_s \geq 2\omega_m$ there is no overlapping of spectrum.

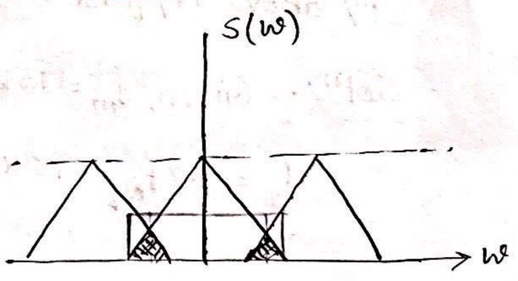
when $\boxed{\omega_s = 2\omega_m}$:



$\omega_s - \omega_m = \omega_m$

$\therefore \omega_s = 2\omega_m$.

when $\boxed{\omega_s < 2\omega_m}$. (Aliasing).



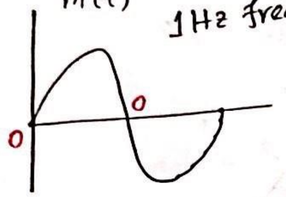
Signal can't be reconstructed.

Sampling Theorem:

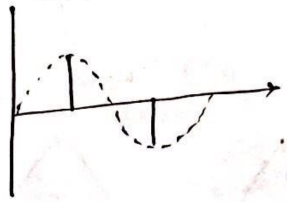
A signal can be represented in its samples and can be recovered back when sampling frequency is greater or equal to twice of maximum frequency component present in the signal.

Physical Example:

$m(t)$ 1Hz freq $= f_m$

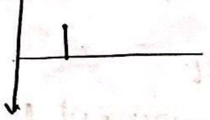


$s(t)$ sampled 2Hz. freq $= f_s$



can be constructed by assumption.

But if $f_s = 1Hz = f_m$



$m(t)$ can't be constructed.

Don't sample at zero's.

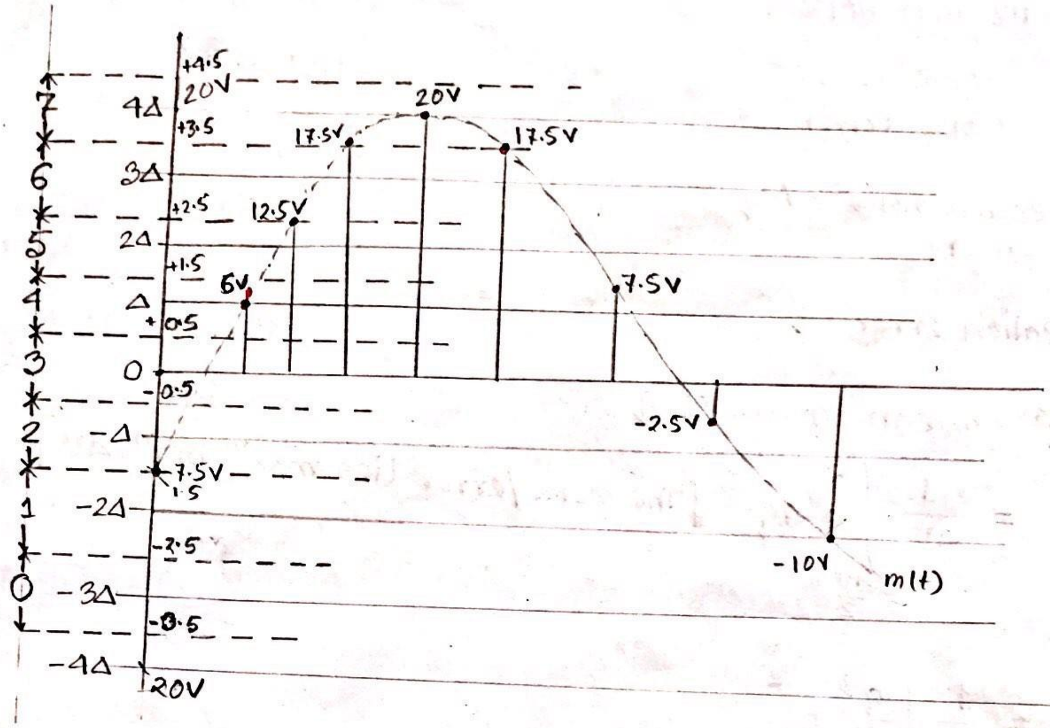
Q/ A CD records audio signals digitally by PCM. Assume audio signal's bandwidth to be 15kHz. If signal are sampled at a rate 20% above Nyquist rate, determine sampling frequency.

Sol^n:- Given, $f_m = 15kHz$.

$\therefore f_s = 1.2 f_{Nq} = 1.2 \times 2 \times 15kHz = 36kHz$.

* $m(t)$, $c(t)$ are FM modulated; Δf is ~~double~~ equal to BW if they are AM. let mess. freq $= f_m$. Find β.

Quantization



$L = 2^n = 8$

$\therefore n = 3$.

no. of bits = 3

Max^m amplitude of m(t) is ~~m(t)~~ $m_p = +20V$

Quantization level/steps $\Delta v = \frac{m_p - (-m_p)}{Level (L)} = \frac{2m_p}{L} = \frac{2 \times 20}{8} = 5$ Volts.

Then normalize. Take L normalized division from 0, 1, ... n. For encoding.

In figure if there is value bet^n +0.5 to -0.5 we will encode it as 3 (011).

$\Delta v = 5V$

Quantization noise :

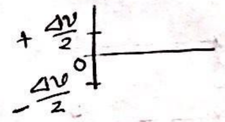
$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}}$$

Quantization noise power $= N_q$

Quantization error $= q$

$$N_q = \tilde{q^2} = \text{mean square of } q$$

$$= \frac{1}{\Delta v}\int_{-\frac{\Delta v}{2}}^{\frac{\Delta v}{2}} q^2 dq$$ [The noise/error lies max^m bet^n Δv].



$$= \frac{1}{\Delta v}\left[\frac{q^3}{3}\right]_{-\frac{\Delta v}{2}}^{\frac{\Delta v}{2}}$$

$$= \frac{1}{3\Delta v}\left[\frac{\Delta v^3}{8} + \frac{\Delta v^3}{8}\right]$$

$$= \frac{1}{3\Delta v} \times \frac{2\Delta v^3}{8_4}$$

$$= \frac{\Delta v^2}{12}$$ [Δv = q. level/step size].

We know, $\Delta v = \frac{2mp}{L}$

$$\therefore N_q = \frac{4mp^2}{L^2} \times \frac{1}{12}$$

$$= \frac{m_p^2}{3L^2}.$$

$$\therefore N_q \propto \frac{1}{L^2}.$$

Now the Signal power of message $m(t) = \widetilde{m^2(t)}$ [mean square of $m^2(t)$]

$$\therefore SNR = \frac{\widetilde{m^2(t)}}{N_q}.$$

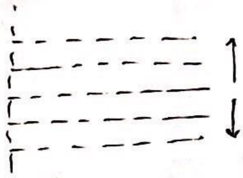
$$= 3L^2 \frac{\widetilde{m^2(t)}}{m_p^2}.$$

$$\therefore SNR \propto L^2$$

Transmission BW & O/p SNR of PCM

We know, $L = 2^n$ or $n = \log_2 L$. L = number of levels
n = number of bits.

Each quantized sample is encoded into n bits. Since, a signal m(t) band limited to B Hz requires a minimum of 2B samples per second.



$\therefore$ Total samples $= n.2B$ = Signaling Rate R_b.

Minimum channel BW $= n.B$
B_T

But $B_T > nB$ is better.

O/p SNR:

Transmitted BW $B_T = nB$ bit/sec - - (i)

$L = 2^n$ - - (ii)

$$\frac{S}{N} = 3L^2 \frac{\widetilde{m^2(t)}}{m_p^2} = L^2 \left(\frac{3\widetilde{m^2(t)}}{m_p^2}\right) \longrightarrow \text{Constant } C.$$

$$\Rightarrow \frac{S}{N} = CL^2 = C(2^n)^2$$

$$\therefore \frac{S}{N} = C.2^{2n} \text{ - - - (iii)}$$

$$\Rightarrow \frac{S}{N} = C.2^{2\left(\frac{B_T}{B}\right)} \quad [B_T = nB] \text{ - - - (iv)}$$

We can see SNR is related to exponent of transmitted BW B_T.
Thats why SNR of PCM is better than AM/FM/PM.

$$\frac{S}{N} = C.2^{2\left(\frac{B_T}{B}\right)}$$

$$\Rightarrow \left(\frac{S}{N}\right)_{dB} = 10 \log_{10} \left[C.2^{2\left(\frac{B_T}{B}\right)}\right].$$

$$C = \frac{3\widetilde{m^2(t)}}{m_p^2}.$$

$$B_T = nB.$$

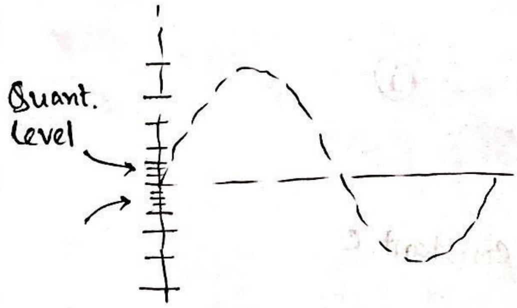
$$\Rightarrow \left(\frac{S}{N}\right)_{dB} = 10 \log_{10} \left[C.2^{2n}\right].$$

$$= 10 \log_{10} C + 20n \log_{10} 2$$

$$= \alpha + 6.02n \quad \left[\alpha = 10 \log_{10} C\right] \text{ ---- (V)}.$$

Non-uniform Quantization (Lathi P-267)

* Signals near to zero are dominant/frequent.



The quantization levels are compressed by compression. The formula of the compressors are

[μ-law] $y = \frac{1}{\ln(1+\mu)} \ln\left(1 + \mu . \frac{m}{m_p}\right) \quad 0 \le \frac{m}{m_p} \le 1.$ For N. America & Japan.

[A-law]

$$y = \begin{cases} \frac{A}{1+\ln A}\left(\frac{m}{m_p}\right) & 0 \le \frac{m}{m_p} \le 1/A \\ \frac{1}{1+\ln A}\left(1 + \ln \frac{Am}{m_p}\right) & 1/A \le \frac{m}{m_p} \le 1. \end{cases}$$

For Europe & rest of the world.

see example 6.3 (P-280) B.P. Lathi

Differential Pulse Code Modulation (DPCM): (Lathi P-290, 6.5)

Drawbacks of PCM
→ Large BW
→ Complex circuit.

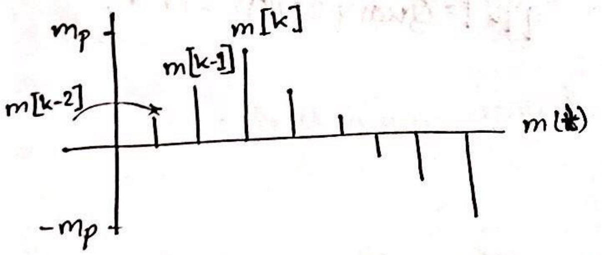
We know,

$$SNR = 3L^2 \frac{\widetilde{m^2(t)}}{m_p^2} = 3 \cdot 2^{2n} \cdot \frac{\widetilde{m^2(t)}}{m_p^2}$$

If $m_p \downarrow$ then SNR $\uparrow$.

But if we want to keep SNR constant then we can reduce $L = 2^n$.

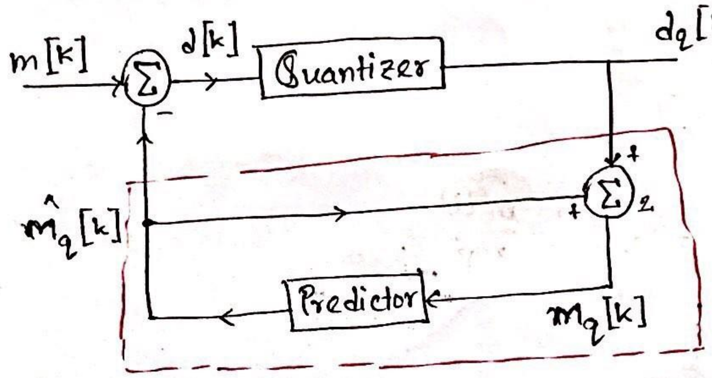
Thus if no. of bits are reduced then minimum channel BW $B_T = nB$ will also decrease.



Now difference $d[k] = m[k] - m[k-1] - m[k-2] \cdots\cdots$

$= m[k] - \hat{m}[k]$ $\left\{ \begin{array}{l} \text{estimate of } m[k] \\ \text{is } \hat{m}[k] = m[k-1] - m[k-2] - \cdots \end{array} \right\}$

DPCM transmitter :



$d_q[k]$, q is there because of quantization (noise) by quantizer.

Predictor is a kind of integrator which sums previous values.

$$d[k] = m[k] - \hat{m}_q[k] \quad \text{--- (i)}$$

After quantization,

$$d_q[k] = d[k] + q[k]$$

$$\therefore q[k] = d_q[k] - d[k] \quad \text{--- (ii)}$$

$q[k]$ = Quantization error.

At summer-2 :

$$m_q[k] = \hat{m}_q[k] + d_q[k]$$

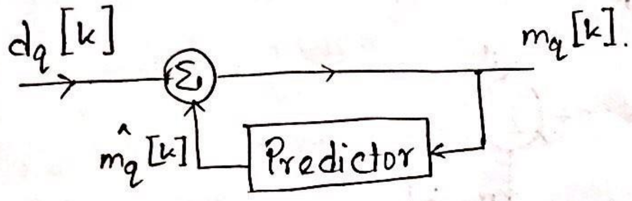
$$= m[k] - d[k] + d[k] + q[k] \quad [\text{from equ}^n \text{ (i) \& (ii)}]$$

$$= m[k] + q[k]$$

which proves $m_q[k]$ is a quantized version of $m[k]$.

DPCM receiver:

It's exactly the box portion of transmitter.



SNR improvement:

we know, $\Delta v = \frac{2m_p}{L}$ and $N_q = \frac{\Delta v^2}{12}$.

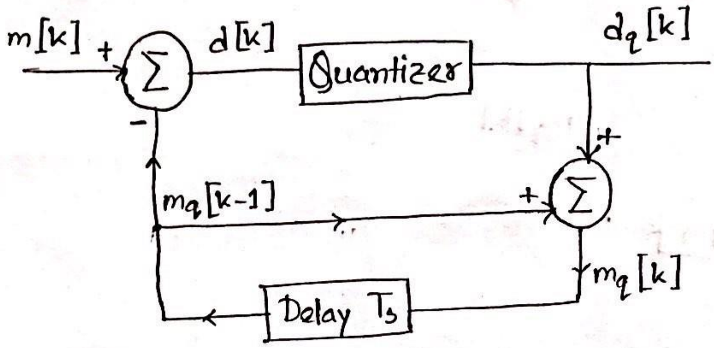
In DPCM, instead of transmitting m_p we transmit difference of m_p i.e. d_p

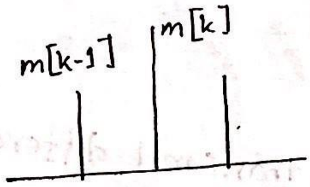
Thus, when $\overset{(d_p)}{m_p} \downarrow \Rightarrow \overset{\text{No of steps}}{\Delta v \downarrow} \Rightarrow N_q \downarrow$

$\therefore$ SNR improvement.

$$\uparrow S/N = 3L^2 \cdot \frac{\widetilde{m^2(t)}}{m_p^2 \downarrow}$$

Delta Modulation (DM): (Lathi, P-295, 6·7) Reduces Complexity of circuit in PCM, DPCM.





In DM, we just transmit the previous pulse.

$$d[k] = m[k] - m[k-1].$$

In fig: $m_q[k] = d_q[k] + m_q[k-1]$

$$m_q[k-1] = d_q[k-1] + m_q[k-2].$$

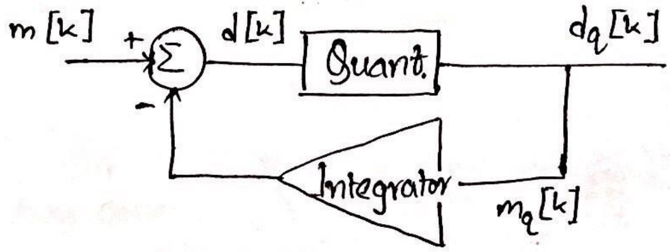
$$m_q[k-2] = d_q[k-2] + m_q[k-3].$$

$$\vdots$$

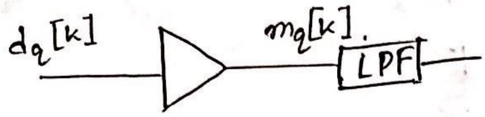
$$\therefore m_q[k] = d_q[k] + d_q[k-1] + d_q[k-2] + \cdots$$

$$= \sum_{m=0}^{k} d_q[m].$$ {Behave like an adder/accumulator/Integrator}

Hence the block diagram can be re-drawn,



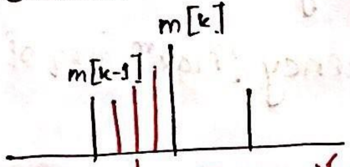
The receiver;



***Oversampling.**

$d[k] = m[k] - m[k-1]$.

In $d[k]$ increases the BW increases.



Better if we oversample $d[k]$ reduces.

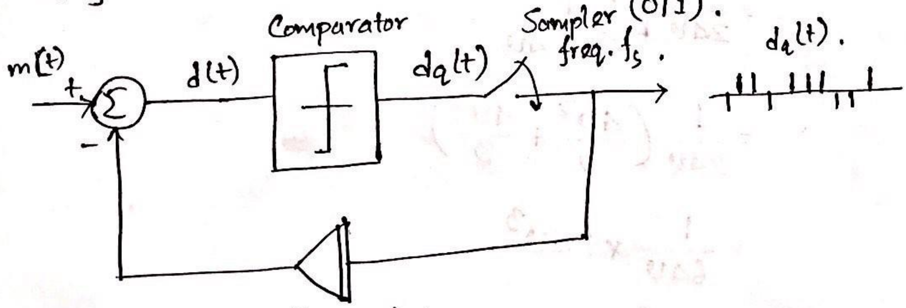
But increasing samples also increases transmission BW B_T.

To overcome it we reduce quantization level $L = 2^n$.

In DM this $n=1$ is taken, so $L=2$.

Hence the quantizer will now act like a comparator as it has only two levels, High/Low.

Finally



$\boxed{\text{Slope Overload (P-297, Lathi) (Self)}}$

To reduce slope overload, noise;

$$A_m \leq \frac{\Delta U f_s}{2\pi f_m} \quad \text{--- (i)}$$

Here, A_m = Maximum Amp. of message signal

ΔU = Step size

f_s = Sampling frequency

f_m = Message frequency (Max^m freq of message)

SNR of DM: ~~(XXX)~~

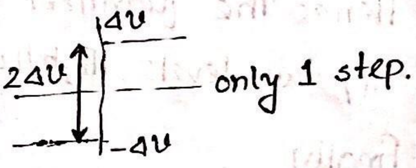
Signal Power $S = \overline{m^2(t)} = \frac{A_m^2}{2}$.

We know, $A_m = \frac{\Delta U f_s}{2\pi f_m}$.

$$\therefore S = \frac{\Delta U^2 f_s^2}{8\pi^2 f_m^2} \quad \text{---- (ii)}$$

Noise power $N = \frac{1}{2\Delta U} \int_{-\Delta U}^{\Delta U} q^2 \, dq$

We know in DM, L=2 (Level).



$$= \frac{1}{2\Delta U} \left[\frac{q^3}{3}\right]_{-\Delta U}^{\Delta U}$$

$$= \frac{1}{2\Delta U} \left(\frac{\Delta U^3}{3} + \frac{\Delta U^3}{3}\right)$$

$$= \frac{1}{6\Delta U} \times 2\Delta U^3$$

$$= \frac{\Delta U^2}{3} \quad \text{--- (iii)}$$

Full derivation, communication hand-note, pdf, (P-46).

This noise $\frac{\Delta v^2}{3}$ is transmitted and received by receiver after filtering. Let the BW of the filter is W.

For sample frequency of f_s the noise power $= \frac{\Delta v^2}{3}$.

$\therefore$ " " " " 1 " " " $= \frac{1}{f_s} \times \frac{\Delta v^2}{3}$.

$\therefore$ " " " " W " " " $= \frac{W}{f_s} \times \frac{\Delta v^2}{3}$.

$\therefore$ The noise power $N = \frac{W}{f_s} \times \frac{\Delta v^2}{3}$.

Now,

$$S = \frac{\Delta v^2 f_s^2}{8\pi^2 f_m^2}$$

$$N = \frac{W}{f_s} \times \frac{\Delta v^2}{3}.$$

$$\therefore S/N = SNR = \frac{\Delta v^2 f_s^2}{8\pi^2 f_m^2} \times \frac{f_s}{W} \times \frac{3}{\Delta v^2}$$

$$= \frac{3 f_s^3}{8\pi^2 f_m^2 W}.$$

SNR of DM;

$$P_{sig} = \frac{\Delta v^2 f_s^2}{8\pi^2 f_m^2} = \frac{A_m^2}{2} \qquad A_m \leq \frac{\Delta v f_s}{2\pi f_m}.$$

$$P_{noise} = \frac{W}{f_s} \times \frac{\Delta v^2}{3}.$$

$$SNR = \frac{P_{sig}}{P_{noise}}.$$

* A DM system is designed to operate at 3 times the nyquist rate for a signal with a 3kHz bandwidth. The quantizing step size is 250mV.

1. Determine max^m Amp. of a 1kHz i/p sinusoidal for which the delta modulation does not show slope overload.
2. Determine Post filtered SNR for the signal of part (1)

Sol^n:-

Given, $W = 3kHz$.

$\Delta v = 250mV = 0.250V$

$f_m = 1kHz$.

$\therefore f_s = 3 \times f_{NQ} = 3 \times 2 f_m = 6 f_m$

$= 6kHz$.

$$\therefore A_m \leq \frac{\Delta v f_s}{2\pi f_m}$$

$$\leq \frac{0.250 \times 6k}{2\pi \times 1k}$$

$$\leq 0.2387V$$

$\therefore$ Max^m Amp $= 0.2387V$ (Ans.)

$$P_{sig} = \frac{\Delta v^2 f_s^2}{8\pi^2 f_m^2} = \frac{(0.250)^2 \times (6k)^2}{8\pi^2 \times (1k)^2}$$

$$= \frac{A_m^2}{2} = 0.0285W$$

$$P_{noise} = \frac{W}{f_s} \times \frac{\Delta v^2}{3} = \frac{3k}{6k} \times \frac{(0.25)^2}{3}$$

$$= 0.01042W.$$

$$\therefore SNR = \frac{P_{sig}}{P_{noise}} = \frac{0.0285}{0.01042} = 2.74 \text{ (Ans.)}$$

$$\therefore (SNR)_{dB} = 10\log_{10}(SNR) = 4.37dB \text{ (Ans.)}$$

PCM math:

$\Delta v = \frac{2mp}{L}$ $\quad P_{noise} = \frac{\Delta v^2}{12} = \frac{mp^2}{3L^2}$. $\quad (SNR)_{dB} = \alpha + 6n$

$L = 2^n$. $\quad SNR = 3L^2 \frac{m^2(t)}{mp^2}$. $\quad B_T = nB$ $\quad R_b = n.2B$.

$= \cancel{nf_m}$ $\quad = \cancel{n\,2f_m}$.

$= R_b/2$. $\quad = n \times$ sampling freq.

Q. BPLathi Exercise 6·2-5.

It is desired to set up a central station for simultaneous monitoring of ECG of 10 hospital patients. The data from the rooms of the 10 patients are brought to a processing center over wires and are sampled, quantized, binary coded and time-division multiplexed. The multiplexed data is now transmitted to monitoring station. The ECG signal Bandwidth is 100Hz. The max^m acceptable error in sampled amplitude is 0·25% of the peak signal amplitude. The sampling rate must be at least twice the Nyquist rate. Determine the minimum cable bandwidth to transmit this data.

Sol^n:- Given, $f_m = 100Hz$.

$f_s = 2f_{Nq} = 2\times2\times100 = 400Hz$.

we know, in PCM max^m quantization error can occur $= \frac{\Delta v}{2}$.

$\Delta v = \frac{2mp}{L}$.

$\Rightarrow \frac{\Delta v}{2} = \frac{mp}{L}$ ---(i)

According to condition,

error $\leq 0·25\%$ of mp

$\Rightarrow \frac{\Delta v}{2} = \frac{0·25mp}{100} = \frac{mp}{400}$ ---(ii)

Equating (i) & (ii) we get

$L = 400$

$\Rightarrow 2^8 = 256$,

$2^9 = 512$.

$\therefore$ no. of bit $n = 9$.

$\therefore \cancel{B_T = nf_m = 9\times400}$

$\therefore$ Bit rate $R_b = n \times f_s$

$= 9\times400Hz$

$= 3600Hz$.

$\therefore$ Min. cable BW for single patient $= \frac{R_b}{2}$

$= 1800Hz$.

$\therefore$ For 10 patients $B_T = 18kHz$ (Ans).

Information Theory. (B.P.Lathi Ch-13)

We know, data/information transmits from transmitter to receiver

$$Tx \longrightarrow \text{channel} \longrightarrow Rx$$

The information from Tx may get disturbed in the channel due to noise. So, it becomes unpredictable. So there is a link betⁿ information & probability.

	Prob^y	Info.
i) Sun rises in the east	1	0
ii) Dog bites a man	0.8	+ve but very low
iii) Man bites a dog	0.01	Highly informative

$$\therefore \text{Information } (I_i) \propto \frac{1}{\text{Prob}^y (P_i)}$$

$$\Rightarrow I_i \propto 1/P_i$$

$$\therefore I_i = \log_2 \frac{1}{P_i}$$

(2) → for digital signal (01)

Entropy (H): Average information per message.

$$H = \sum_{i=1}^{N} I_i . P_i \qquad i = 1, 2, 3, \ldots\ldots N.$$

$$= \sum_{i=1}^{N} P_i \log_2 \frac{1}{P_i} \quad \text{bits/message}$$

$$= \sum_{i=1}^{N} P_i \log_2 \frac{1}{P_i} \quad \text{bits/message.}$$

Rate of information :(R) bits/sec.

$R = H.r$ H = Entropy

r = Baud rate or Signalling rate.

Amount of message generated per second.

Q. An event has six possible outcomes with probabilities $p_1 = 1/2$, $p_2 = 1/4$, $p_3 = 1/8$, $p_4 = 1/16$, $p_5 = 1/36$ and $p_6 = 1/32$. Find the entropy of the system. Also the rate of information if there are 16 outcomes per second. or outcome or message.

Soln :- Given Baud rate r = 16 signal/sec.

$$H = \sum_{i=1}^{6} P_i \log_2 \frac{1}{P_i}$$

$$= \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \frac{1}{6}\log_2 6 + \frac{1}{36}\log_2 36 + \frac{1}{32}\log_2 32$$

$$= \frac{31}{16} \text{ bits/message}$$

$\therefore$ we know,

$$R = \frac{31}{16} \times 16$$

$$= 31 \text{ bits/sec.}$$

Error Free Communication Over a Noisy channel:

Shannon Capacity Theorem or Channel Capacity Theorem:

For error free transmission;
The channel capacity of a white bandlimited Gaussian channel is

white → All frequency

Gaussian → Prob^y Distribution

$$C = B\log_2\left(1+\frac{S}{N}\right) \text{ bits/sec (bps)}$$

B = channel Bandwidth (Hz) . S/N = Signal to Noise Power.

Q A voice grade telephone channel has a bandwidth of 3400 Hz. If the signal to Noise ratio (SNR) on the channel is 30dB, determine the capacity of the channel.
If the above channel is to be used to transmit 4.8kbps of data determine the minimum SNR required on the channel.

$$\boxed{\log_2 x = \frac{\log_{10} x}{\log_{10} 2}}$$

Soln:-

Given, BW B = 3400Hz

SNR = 30dB = $10\log_{10}(S/N)$

$\Rightarrow \frac{S}{N} = \text{antilog}\left(\frac{30}{10}\right)$

$\Rightarrow \frac{S}{N} = 10^{\frac{30}{10}} = 10^3$

$\therefore \frac{S}{N} = 1000$

$\therefore$ Channel Capacity

$C = B\log_2\left(1+\frac{S}{N}\right)$

$= 3400\log_2(1+1000)$

$= 33.888$ kbps.

(Ans.)

For $C = 4.8$kbps, $\frac{S}{N} = ?$

$C = B\log_2\left(1+\frac{S}{N}\right)$

$\Rightarrow 4.8k = 3400\log_2\left(1+\frac{S}{N}\right)$

$\Rightarrow \log_2\left(1+\frac{S}{N}\right) = \frac{4.8k}{3400} \approx 1.412$

$\Rightarrow 1+\frac{S}{N} = 2^{1.412}$

$\Rightarrow \frac{S}{N} = 2.66 - 1 = 1.66$

Hence the minimum SNR must be 1.66 or 2.2dB to transmit the data.

(Ans.)