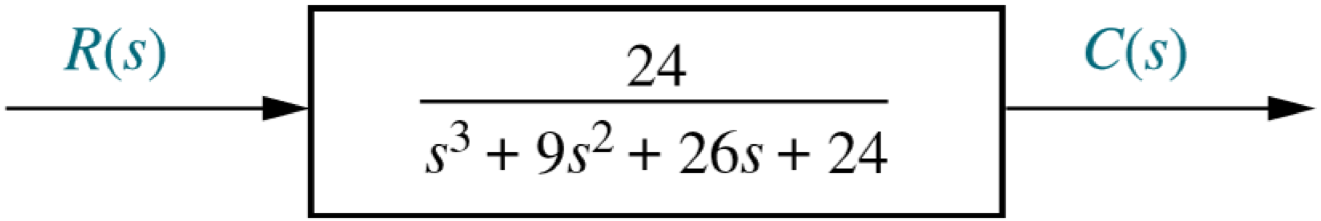
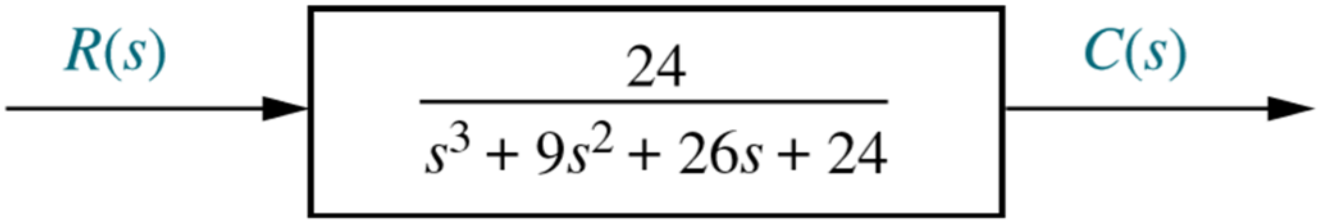
Control System I Sessional

EEE 704

Find the state-space representation of the transfer function shown in the figure using MATLAB



```
num=[24];
den=[1 9 26 24];
[A,B,C,D]=tf2ss(num,den);
P=[0 0 1;0 1 0;1 0 0];
A=inv(P)*A*P
B=inv(P)*B
C=C*P
```



PROBLEM: Given the system defined by Eq. (3.74), find the transfer function, $T(s) = Y(s)/U(s)$, where $U(s)$ is the input and $Y(s)$ is the output.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}u \tag{3.74a}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x} \tag{3.74b}$$

Do it yourself:

Hint:

```
[num,den]=ss2tf(A,B,C,D)
```

```
A=[0 1 0;0 0 1;-1 -2 -3];
B=[10 0 0]';
C=[1 0 0];
D=0;
[num,den]=ss2tf(A,B,C,D)
printsys(num,den,'s')
```

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0]\mathbf{x}$$

```
num =

     0    10    30    20


den =

    1.0000    3.0000    2.0000    1.0000


num/den =

     10 s^2 + 30 s + 20
   ---------------------
   s^3 + 3 s^2 + 2 s + 1
```

Determine pole zero location and the values of natural frequency (ω_n) and damping ratio (ξ) of a given transfer function

a. $G(s) = \dfrac{400}{s^2 + 12s + 400}$

b. $G(s) = \dfrac{900}{s^2 + 90s + 900}$

c. $G(s) = \dfrac{225}{s^2 + 30s + 225}$

d. $G(s) = \dfrac{625}{s^2 + 625}$

Solution (a)

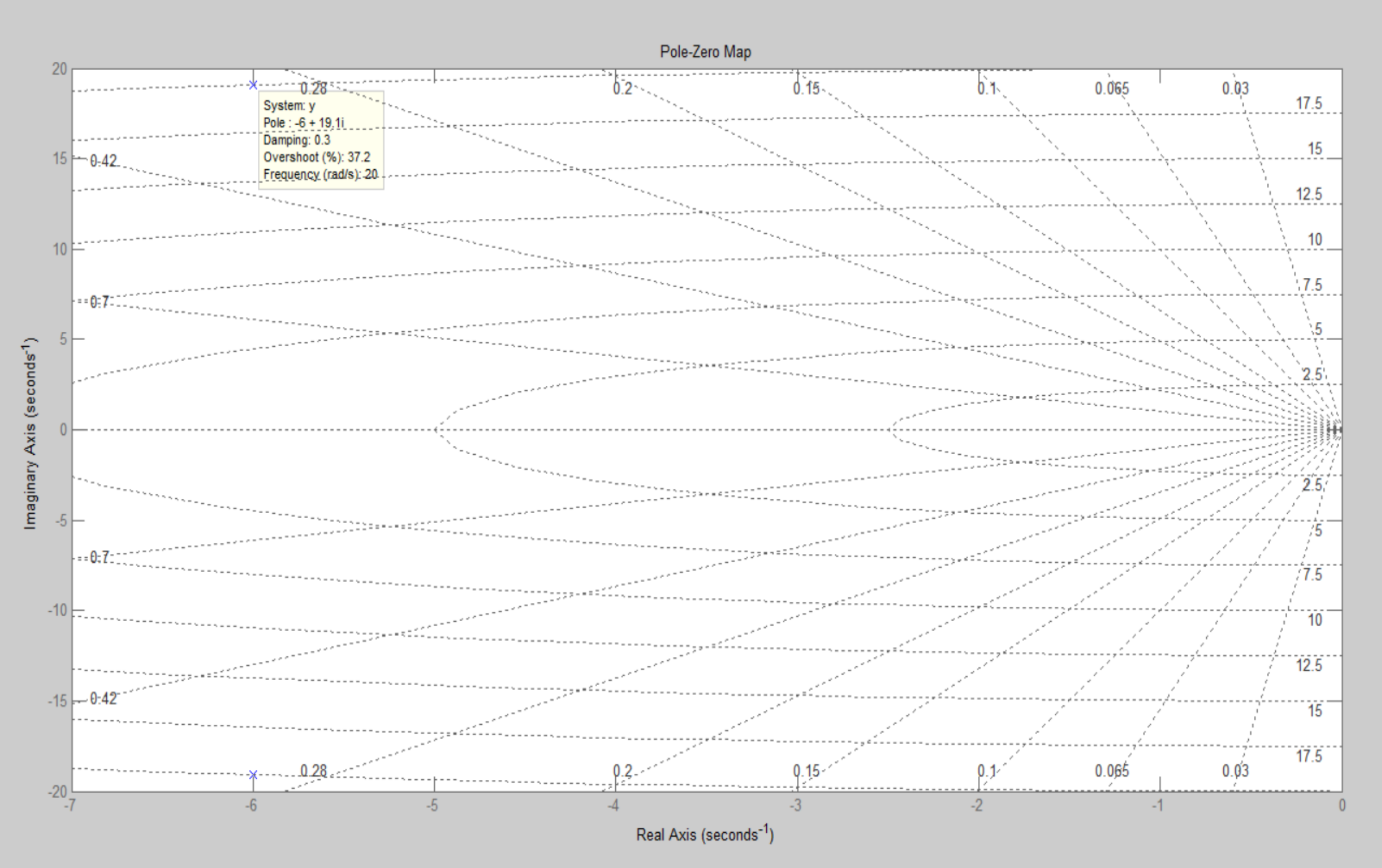
a. $G(s) = \dfrac{400}{s^2 + 12s + 400}$

```
clear all
clc
y=tf([400],[1 12 400])
pzmap(y);%shows the positions of poles and zeros
sgrid %Generate s-plane grid lines for a root locus or pole-zero map.
[wn,z]=damp(y)%shows natural frequency and damping
ratio
```

```
y =

        400
  ----------------
  s^2 + 12 s + 400

Continuous-time transfer function.


wn =

    20
    20


z =

    0.3000
    0.3000
```

Pole zero locations

Click on either of the poles which will show you the values of damping ratio and others required to characterize the nature
Of the response



Print a system when natural frequency (ω_n) and damping ratio (ξ) are given

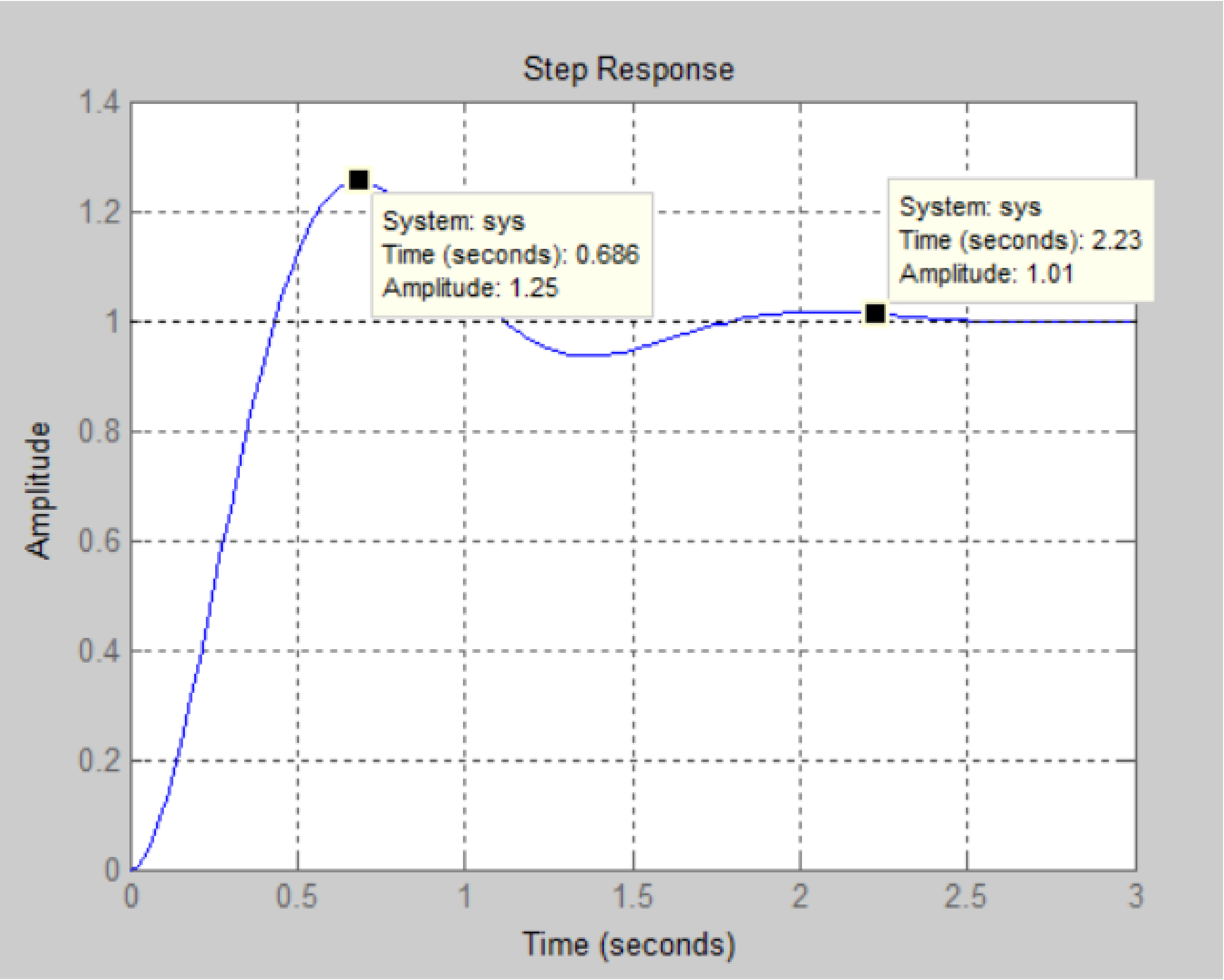
```
clc
wn=20;%given natural frequency
damping_ratio=0.3;
[num0,den]=ord2(wn,damping_ratio)% Generate continuous second order system at the pole
num=wn^2;
printsys(num,den,'s')
```

```
num0 =

     1


den =

     1     12     400


num/den =

          400
    ----------------
    s^2 + 12 s + 400
```

Determine the Unit Step Response of a given transfer function

$$G(s) = \frac{25}{s^2 + 4s + 25}$$

```
num=25;
den=[1 4 25];
step(num,den)
grid on
```



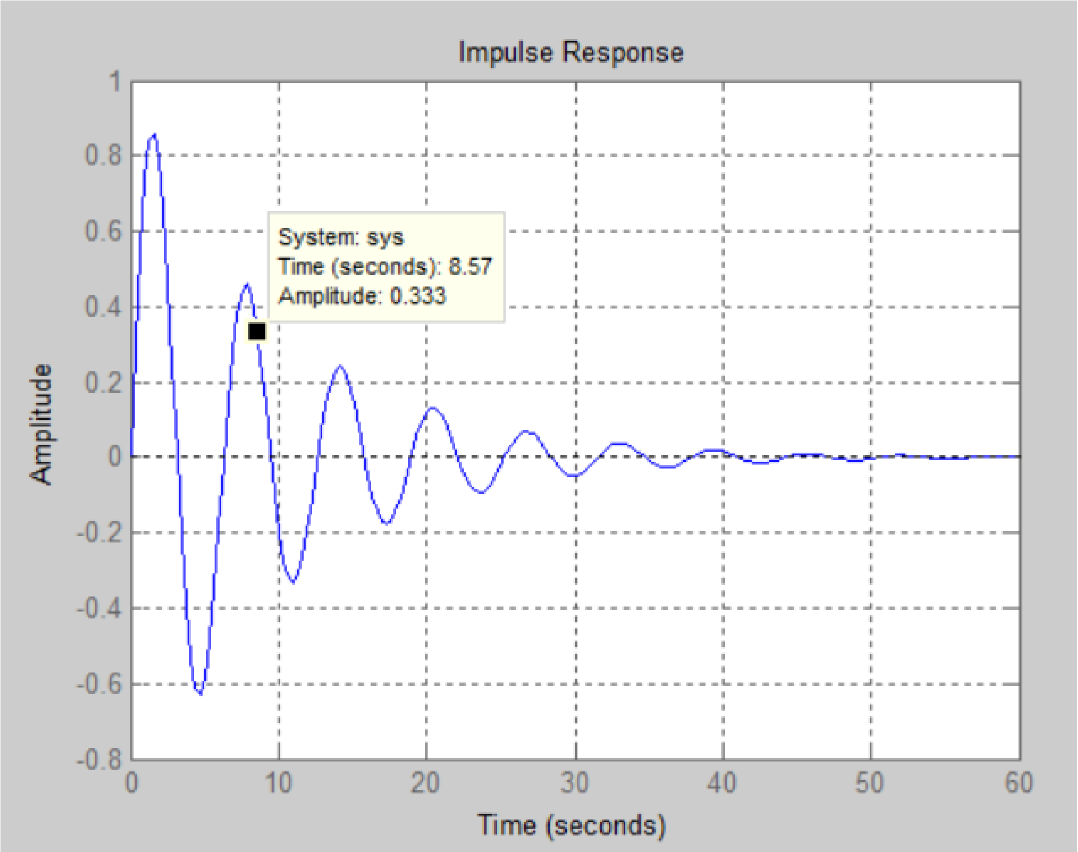
$$G(s) = \frac{25}{s^2 + 4s + 25}$$

Obtain the unit-impulse response of a transfer function

$$\frac{C(s)}{R(s)} = G(s) = \frac{1}{s^2 + 0.2s + 1}$$

```
num=1;
den=[1 0.2 1];
impulse(num,den)
grid on
```

$$\frac{C(s)}{R(s)} = G(s) = \frac{1}{s^2 + 0.2s + 1}$$



Obtain the unit-ramp response of a transfer function

$$\frac{C(s)}{R(s)} = \frac{2s + 1}{s^2 + s + 1}$$

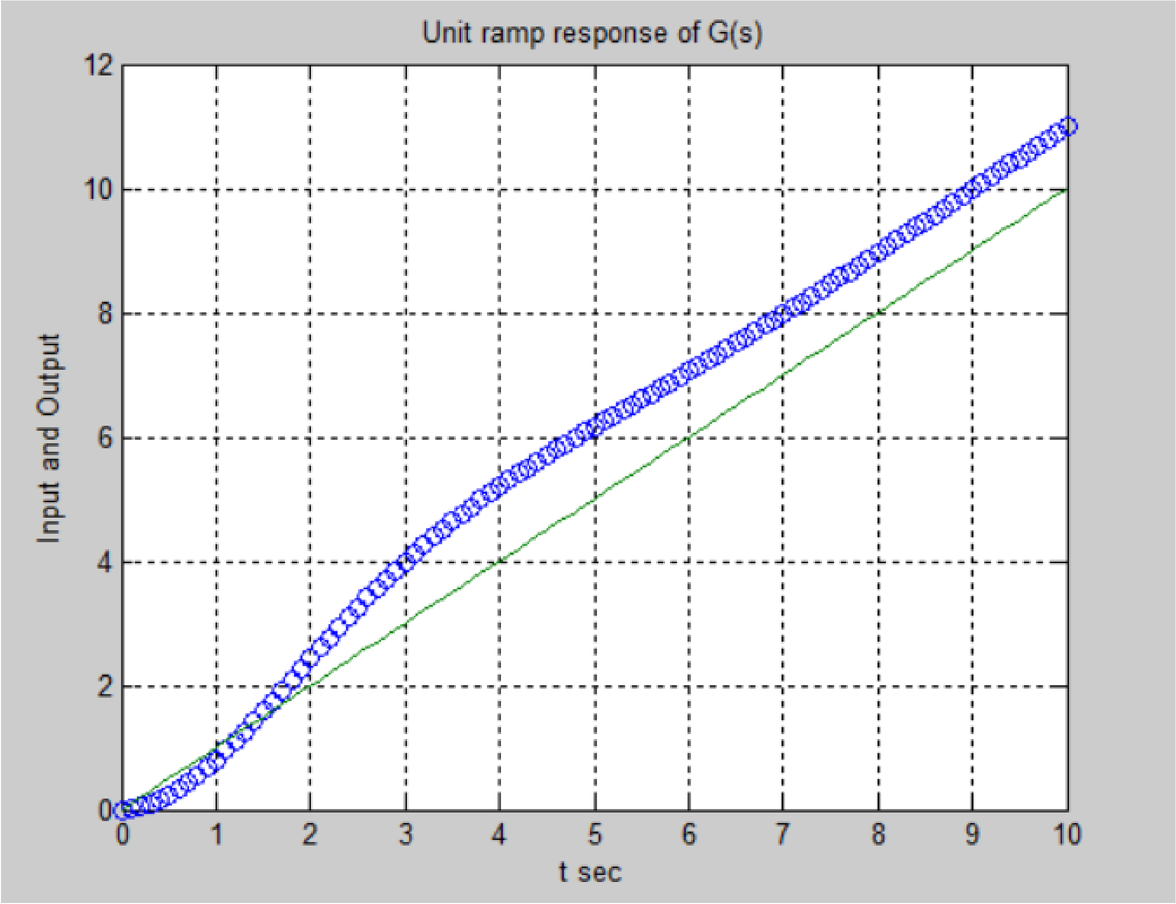
For a unit-ramp input, $R(s) = 1/s^2$. Hence

$$C(s) = \frac{2s + 1}{s^2 + s + 1}\frac{1}{s^2} = \frac{2s + 1}{(s^2 + s + 1)s}\frac{1}{s}$$

MATLAB do not have built in ramp response command. That's why we have to transfer in step response

```
num=[2 1];
den=[1 1 1 0];
t=0:0.1:10;
c=step(num,den,t);
plot(t,c,'o',t,t,'-')
grid
title('Unit ramp response of G(s)')
xlabel('t sec')
ylabel('Input and Output')
```

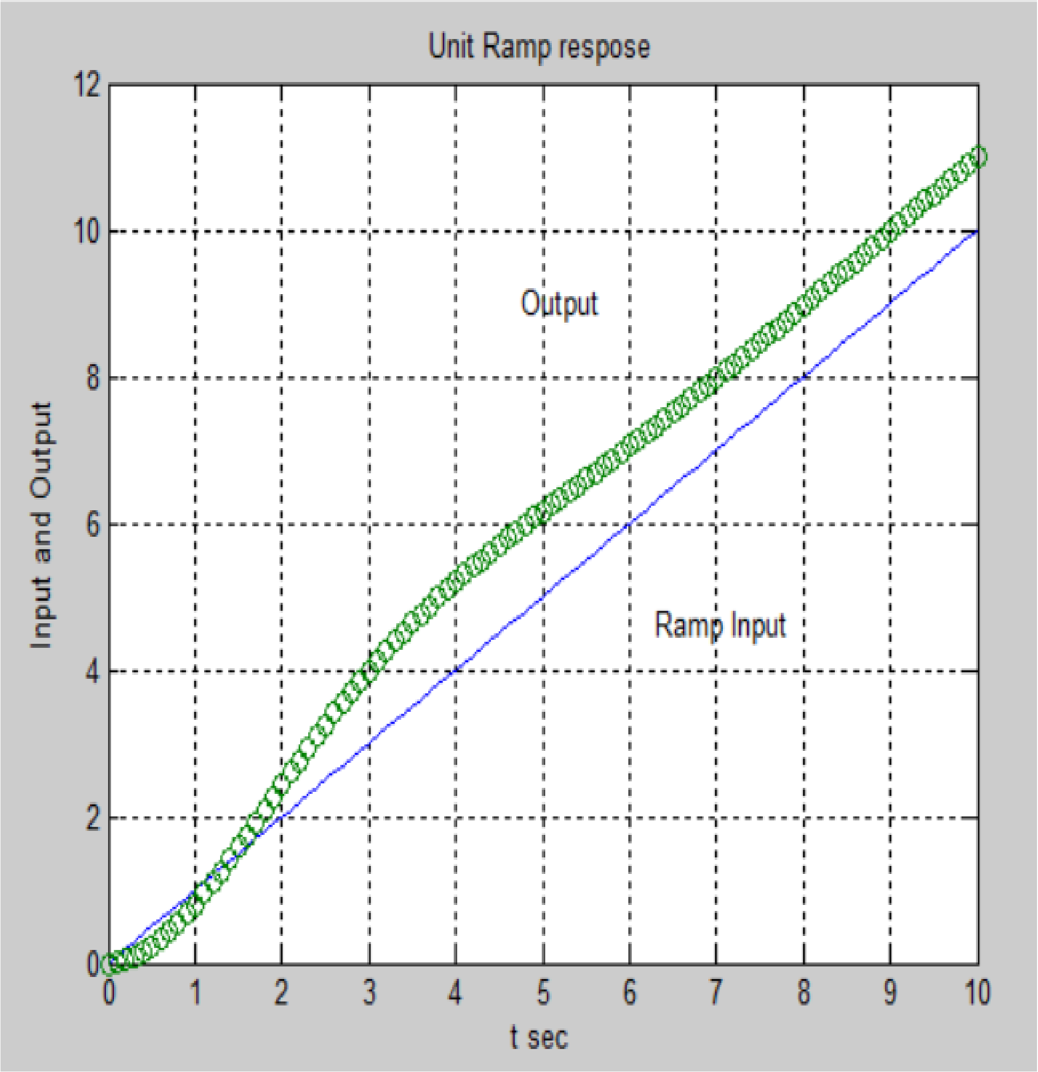
$$\frac{C(s)}{R(s)} = \frac{2s + 1}{s^2 + s + 1}$$



Obtain the unit-ramp response of a transfer function using "lsim" command

$$\frac{C(s)}{R(s)} = \frac{2s + 1}{s^2 + s + 1}$$

```
num=[2 1];
den=[1 1 1];
t=0:0.1:10;
r=t;
y=lsim(num,den,r,t);%r=ramp---t=time
plot(t,r,'-',t,y,'o')
grid
title('Unit Ramp respose')
xlabel('t sec')
ylabel('Input and Output')
text(6.3,4.6,'Ramp Input')
text(4.75,9.0,'Output')
```



Step Response of Different Damping

- Find step response of the transfer function given below

a. $G(s) = \dfrac{400}{s^2 + 12s + 400}$

b. $G(s) = \dfrac{900}{s^2 + 90s + 900}$

c. $G(s) = \dfrac{225}{s^2 + 30s + 225}$

d. $G(s) = \dfrac{625}{s^2 + 625}$

```
%Underdamped
num=[400];
den=[1 12 400];
t1=tf(num,den)
subplot(221)
step(t1)
title('Underdamped')
%Overdamped
num2=[900];
den2=[1 90 900];
t2=tf(num2,den2)
subplot(222)
step(t2)
title('Overdamped')
```

```
%Critically Damped
num3=[225];
den3=[1 30 225];
t3=tf(num3,den3)
subplot(223)
step(t3)
title('Critically Damped')
%Undamped
num4=[625];
den4=[1 0 625];
t4=tf(num4,den4)
subplot(224)
step(t4)
title('Undamped')
```

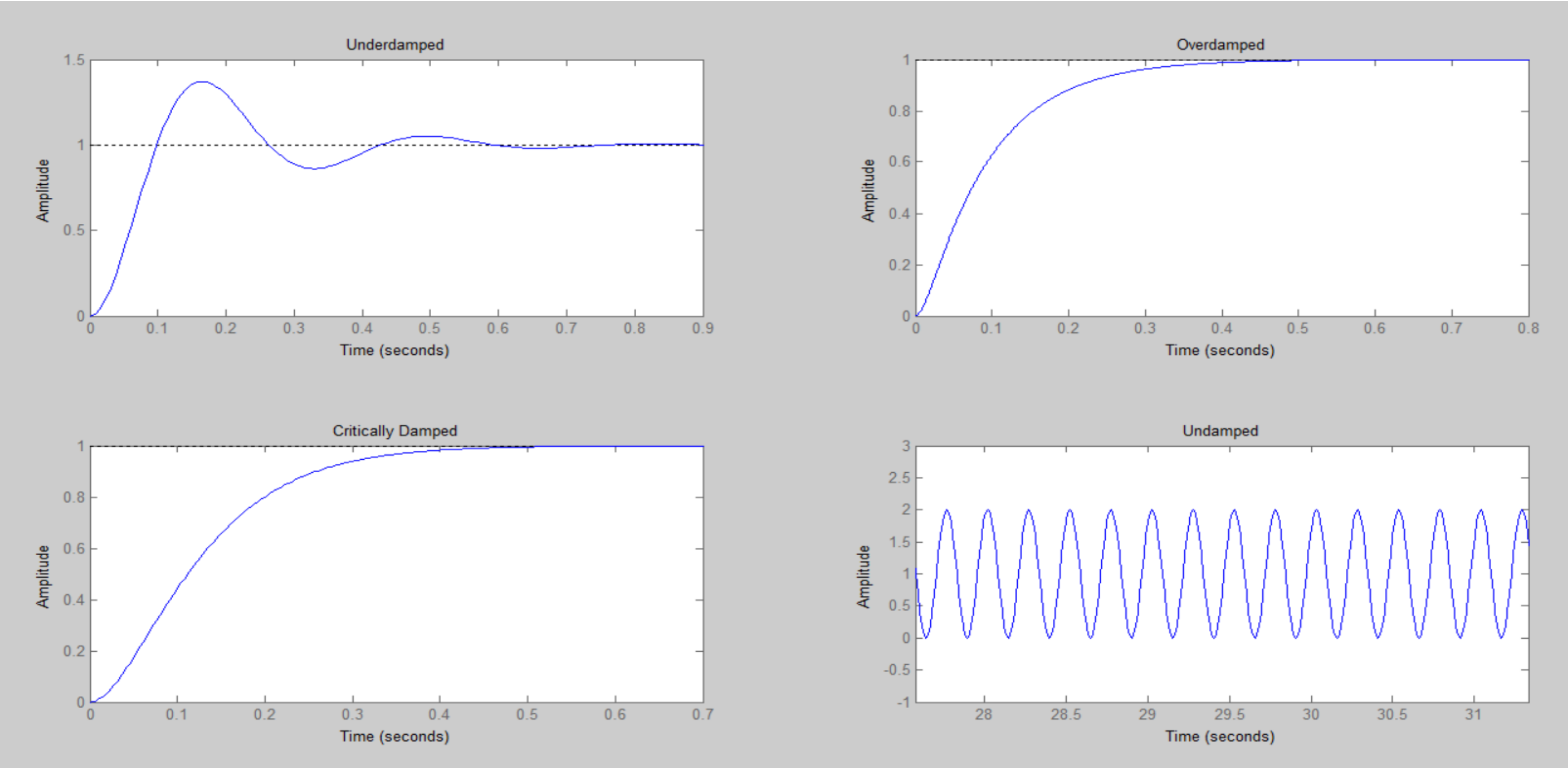
a. $G(s) = \dfrac{400}{s^2 + 12s + 400}$

b. $G(s) = \dfrac{900}{s^2 + 90s + 900}$

c. $G(s) = \dfrac{225}{s^2 + 30s + 225}$

d. $G(s) = \dfrac{625}{s^2 + 625}$

All Responses



EXP 5: State Space Representation

- Find the transfer function from the following state equations

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & -1 \\ -3 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}u(t)$$

$$y = \begin{bmatrix} 3 & 2 \end{bmatrix}\mathbf{x}$$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 6 & 5 \\ 1 & 4 & 2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}u(t)$$

$$y = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}\mathbf{x}$$

- Find the State Equation from the given Transfer Function

$$R(s) \longrightarrow \boxed{\dfrac{100}{s^4 + 20s^3 + 10s^2 + 7s + 100}} \longrightarrow C(s)$$

(*a*)

$$R(s) \longrightarrow \boxed{\dfrac{30}{s^5 + 8s^4 + 9s^3 + 6s^2 + s + 30}} \longrightarrow C(s)$$

(*b*)

EXP 6: Determine pole zero location and the values of natural frequency (ω_n) and damping ratio (ξ) of a given transfer function

a. $G(s) = \dfrac{400}{s^2 + 12s + 400}$

b. $G(s) = \dfrac{900}{s^2 + 90s + 900}$

c. $G(s) = \dfrac{225}{s^2 + 30s + 225}$

d. $G(s) = \dfrac{625}{s^2 + 625}$

Print a system when natural frequency (ω_n) and damping ratio (ξ) are given

- For $\omega_n = 10$ rad/sec and $\xi = 0.3$
- For $\omega_n = 20$ rad/sec and $\xi = 0.0$
- For $\omega_n = 25$ rad/sec and $\xi = 0.8$
- For $\omega_n = 30$ rad/sec and $\xi = 1.0$

EXP 7: Transient and Steady State Responses

- Using MATLAB, obtain the unit-step response, unit-ramp response, and unit-impulse response of the following system

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + 2s + 10}$$

EXP 8: Obtain the unit-ramp response of a transfer function using “lsim” command

- Obtain the unit-ramp response of three transfer functions using “lsim” command

EXP 9: Step Response of Different Damping

- Include step responses of overdamped, underdamped, critically damped and undamped system in a single figure using subplot command.

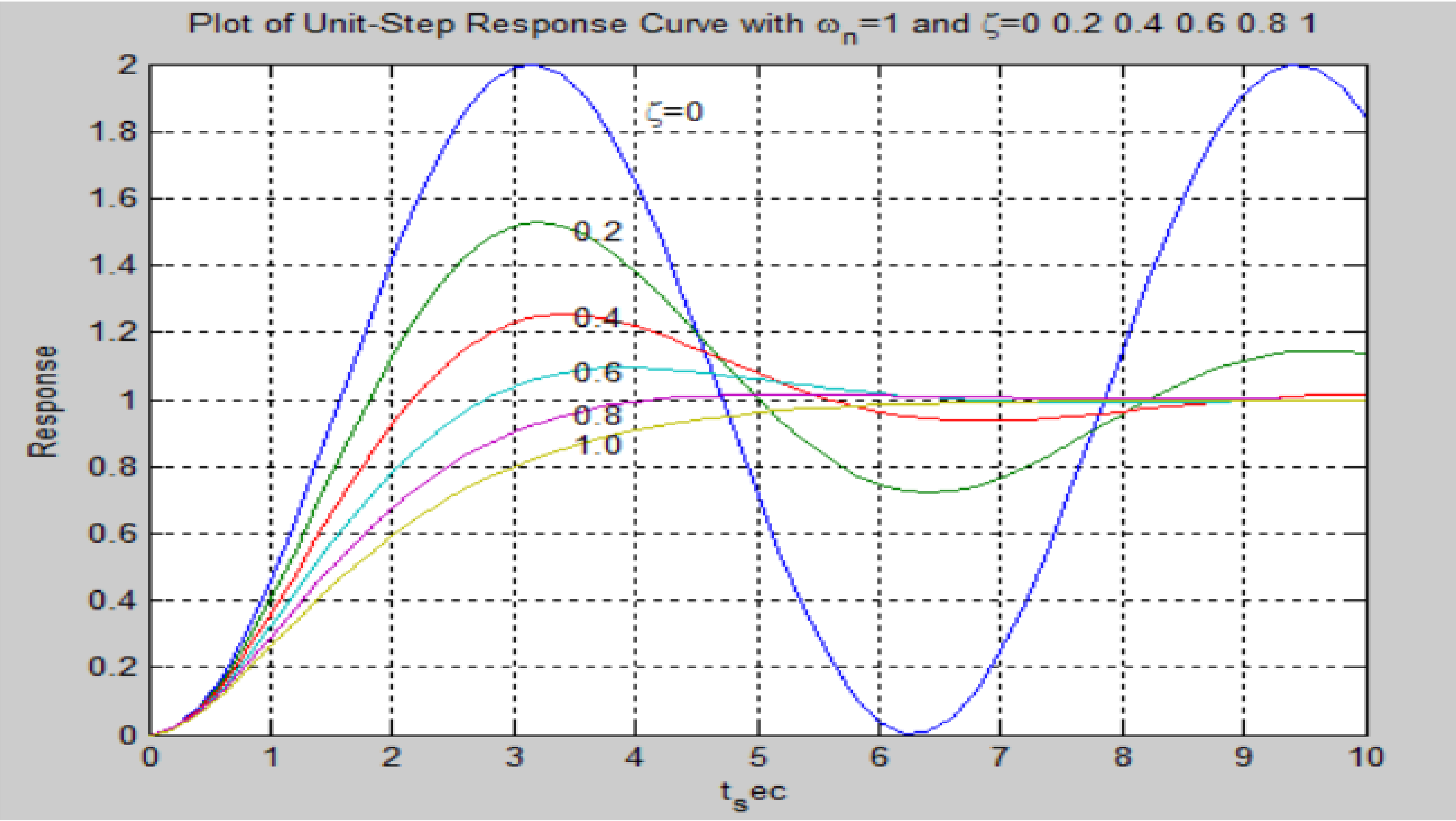
- Show graphically how the change of damping ratio changes the step response for single transfer function of constant natural frequency of 1 rad/sec which is given by

$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + 2\zeta s + 1}$$

(The undamped natural frequency ω_n is normalized to 1.) Plot unit-step response curves $c(t)$ when ζ assumes the following values:

$$\zeta = 0,\ 0.2,\ 0.4,\ 0.6.\ 0.8,\ 1.0$$

```
clc
clear all
t=0:0.2:10;
zeta=[0 0.2 0.4 0.6 0.8 1];
for n=1:6;
    num= [1];
    den= [1 2*zeta(n)*1 1]; % s2+2ewns+wn2 format
    [y(1:51,n),x,t]=step(num,den,t); %as t has 51 values so y(1:51)
end
plot(t,y)
grid
title('Plot of Unit-Step Response Curve with \omega_n=1 and \zeta=0 0.2 0.4 0.6
0.8 1')
xlabel('t_sec')
ylabel('Response')
text(4.1,1.86,'\zeta=0')
text(3.5,1.5,'0.2')
text(3.5,1.24,'0.4')
text(3.5,1.08,'0.6')
text(3.5,0.95,'0.8')
text(3.5,0.86,'1.0')
```



Plot of Unit-Step Response Curve with ωn=1 and ζ=0 0.2 0.4 0.6 0.8 1
ζ=0
0.2
0.4
0.6
0.8
1.0
Response
tsec
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
1
2
3
4
5
6
7
8
9
10

Step Responses of Second order systems according to pole movement

- Prove using MATLAB plot that

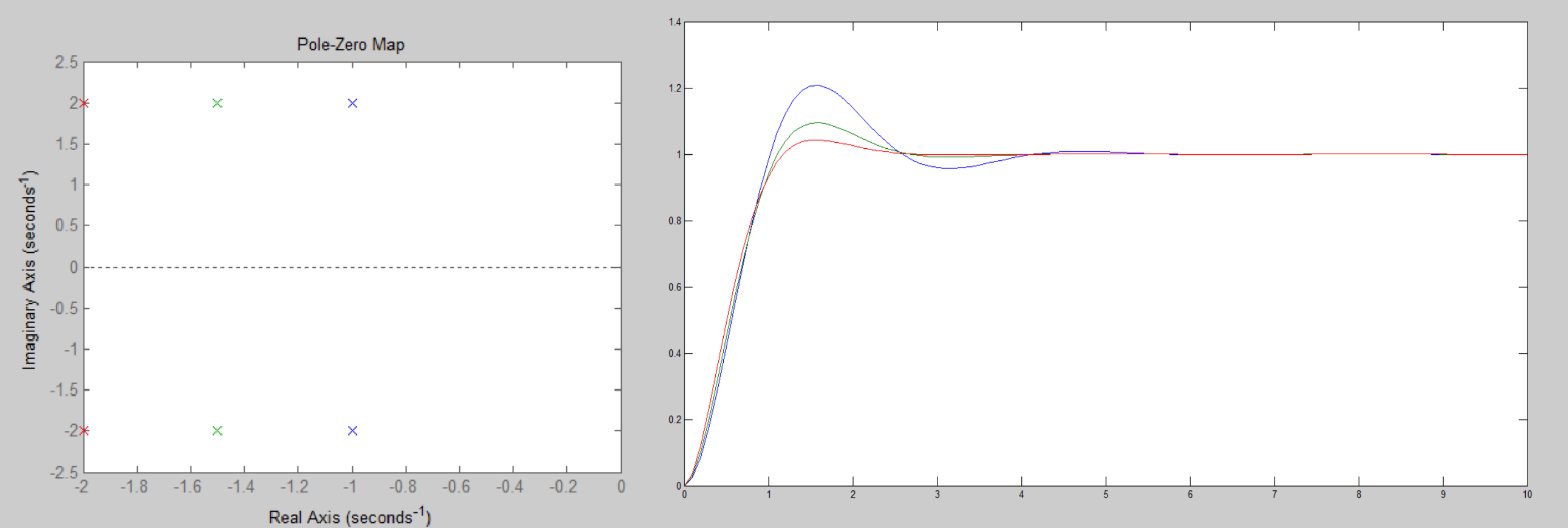
1. Frequency of oscillation remains the same for constant imaginary part
2. Envelope remains the same for constant real part
3. Overshoot remains the same for same damping ratio.

Frequency of oscillation remains the same for constant imaginary part

```
clc
den1=poly([-1+2i -1-2i]);
den2=poly([-1.5+2i -1.5-2i]);
den3=poly([-2+2i -2-2i]);
num1=den1(3);
num2=den2(3);
num3=den3(3);
```

```
f1=tf(num1,den1)
f2=tf(num2,den2)
f3=tf(num3,den3)
pzmap(f1,f2,f3)
figure
t=0:0.1:10;
c1=step(num1,den1,t);
c2=step(num2,den2,t);
c3=step(num3,den3,t);
plot(t,c1,t,c2,t,c3)
```

Frequency of oscillation remains the same for constant imaginary part



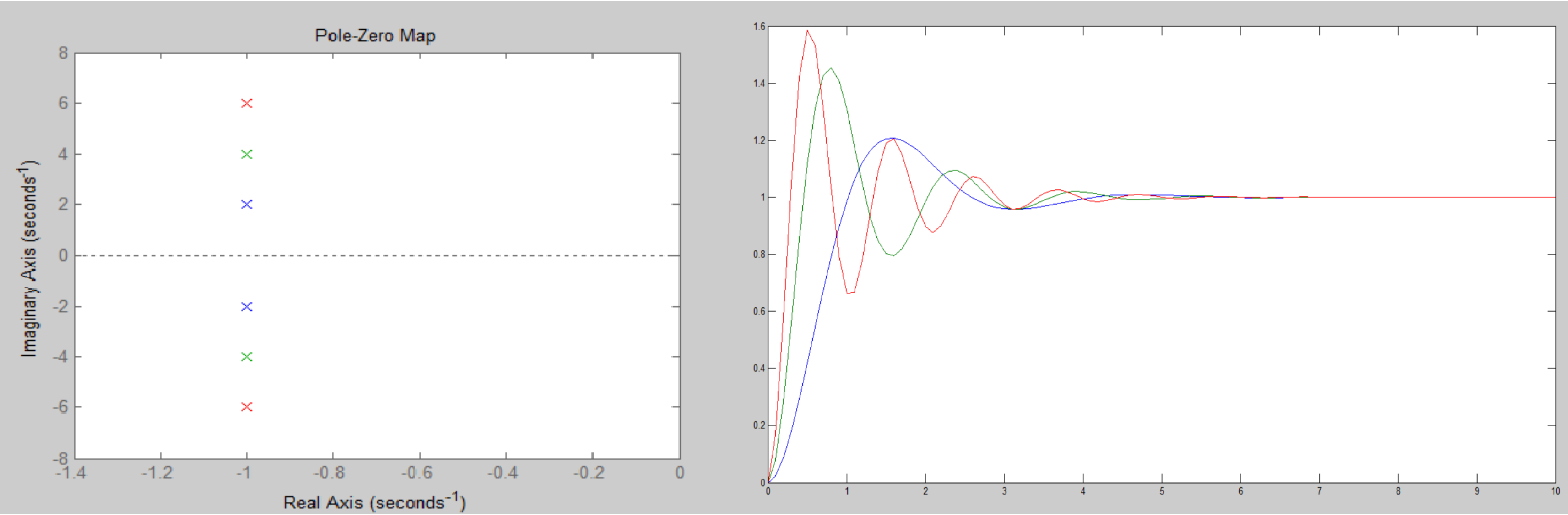
Don't get confused with natural frequency w_n

Envelope remains the same for constant real part

```
clc

den1=poly([-1+2i -1-2i]);
den2=poly([-1+4i -1-4i]);
den3=poly([-1+6i -1-6i]);
num1=den1(3);
num2=den2(3);
num3=den3(3);
```

```
f1=tf(num1,den1)
f2=tf(num2,den2)
f3=tf(num3,den3)
pzmap(f1,f2,f3)
figure
t=0:0.1:10;
c1=step(num1,den1,t);
c2=step(num2,den2,t);
c3=step(num3,den3,t);
plot(t,c1,t,c2,t,c3)
```

Envelope remains the same for constant real part

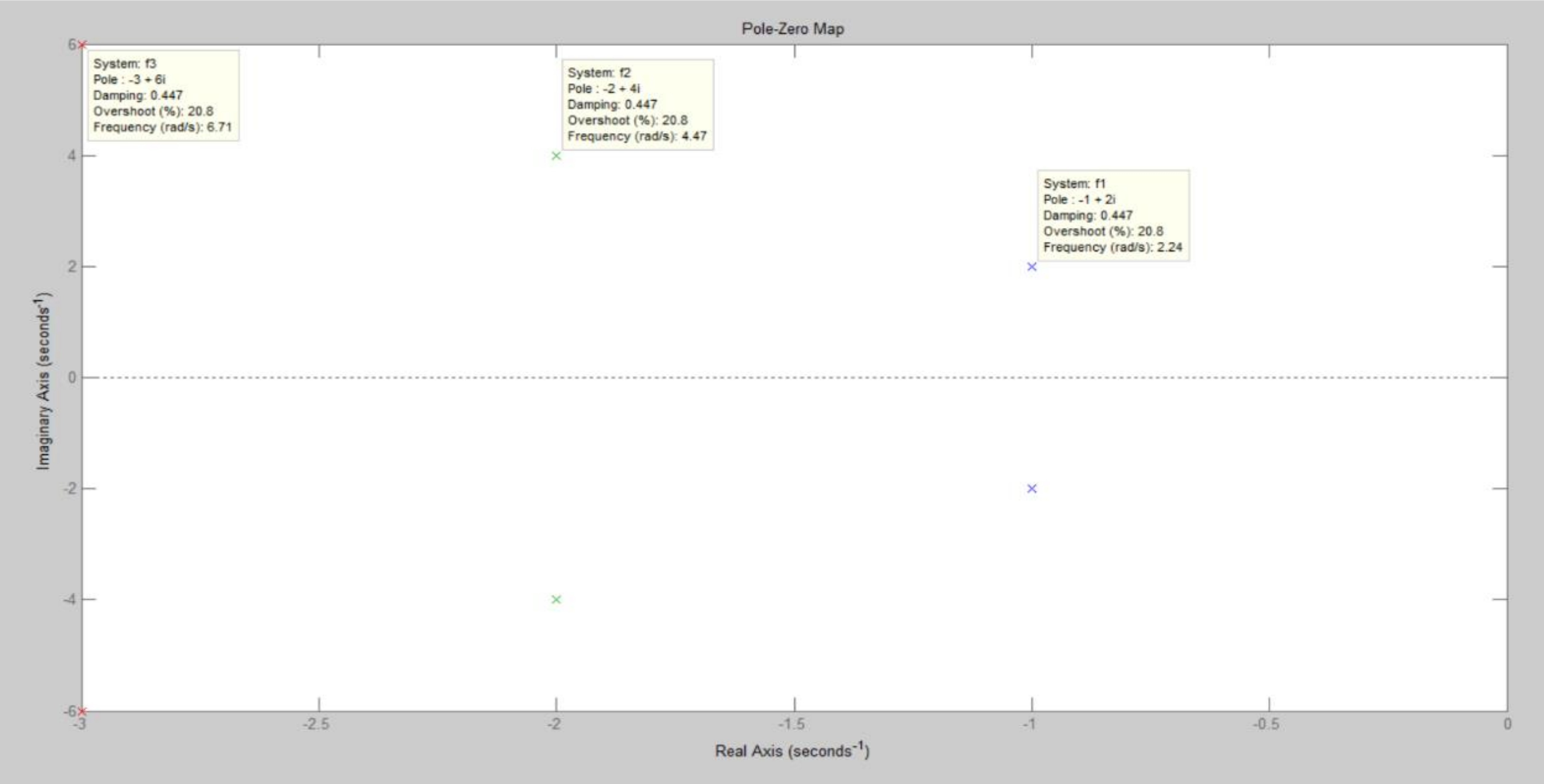


Overshoot remains the same for same damping ratio.

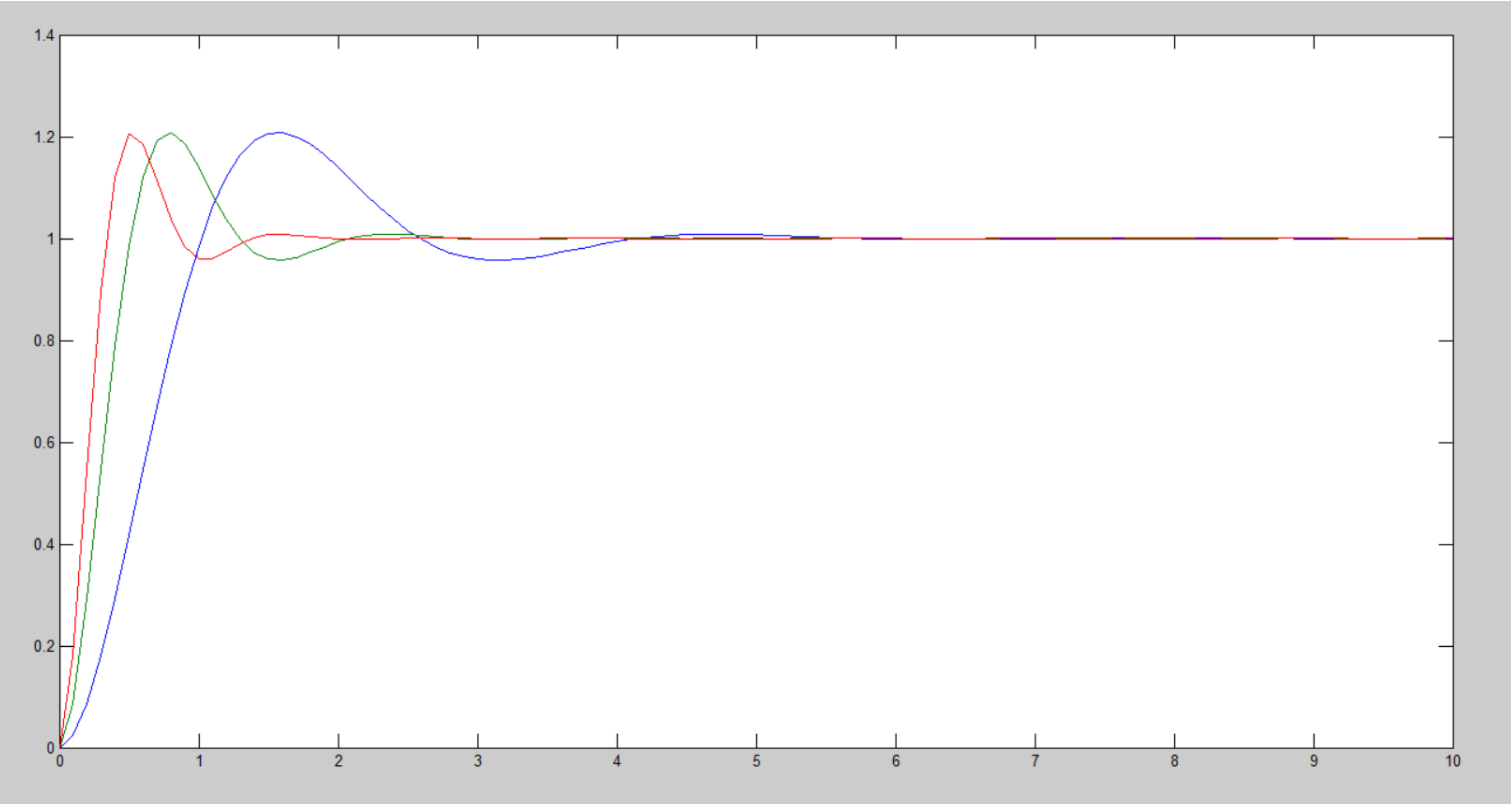
```
clc

den1=poly([-1+2i -1-2i]);
den2=poly([-2+4i -2-4i]);
den3=poly([-3+6i -3-6i]);
num1=den1(3);
num2=den2(3);
num3=den3(3);
```

```
f1=tf(num1,den1)
f2=tf(num2,den2)
f3=tf(num3,den3)
pzmap(f1,f2,f3)
figure
t=0:0.1:10;
c1=step(num1,den1,t);
c2=step(num2,den2,t);
c3=step(num3,den3,t);
plot(t,c1,t,c2,t,c3)
```

Overshoot remains the same for same damping ratio.



Same Overshoot for constant ξ



- For the given Transfer Function T(S) find out

1. Rise Time Tr
2. Peak Time Tp
3. Percentage Overshoot %OS
4. Settling Time Ts
5. Step Response

$$T(s) = \frac{16}{s^2 + 3s + 16}$$

$$T(s) = \frac{16}{s^2 + 3s + 16}$$

```
w=16;
a=3;
num=w;
den=[1 a w];
t=tf(num,den)
wn=sqrt(w)%natural frequency
zeta=a/(2*wn)%Damping Ratio
theta=acos(zeta)
Ts=4/(zeta*wn)%Settling Time
Tp=pi/(wn*sqrt(1-zeta^2))%Peak Time
Tr=(pi-theta)/(wn*sqrt(1-zeta^2))%Rise Time
OS=exp(-zeta*pi/sqrt(1-zeta^2))*100 %Percentage Overshoot
step(t)
```

Answers

Wn=4

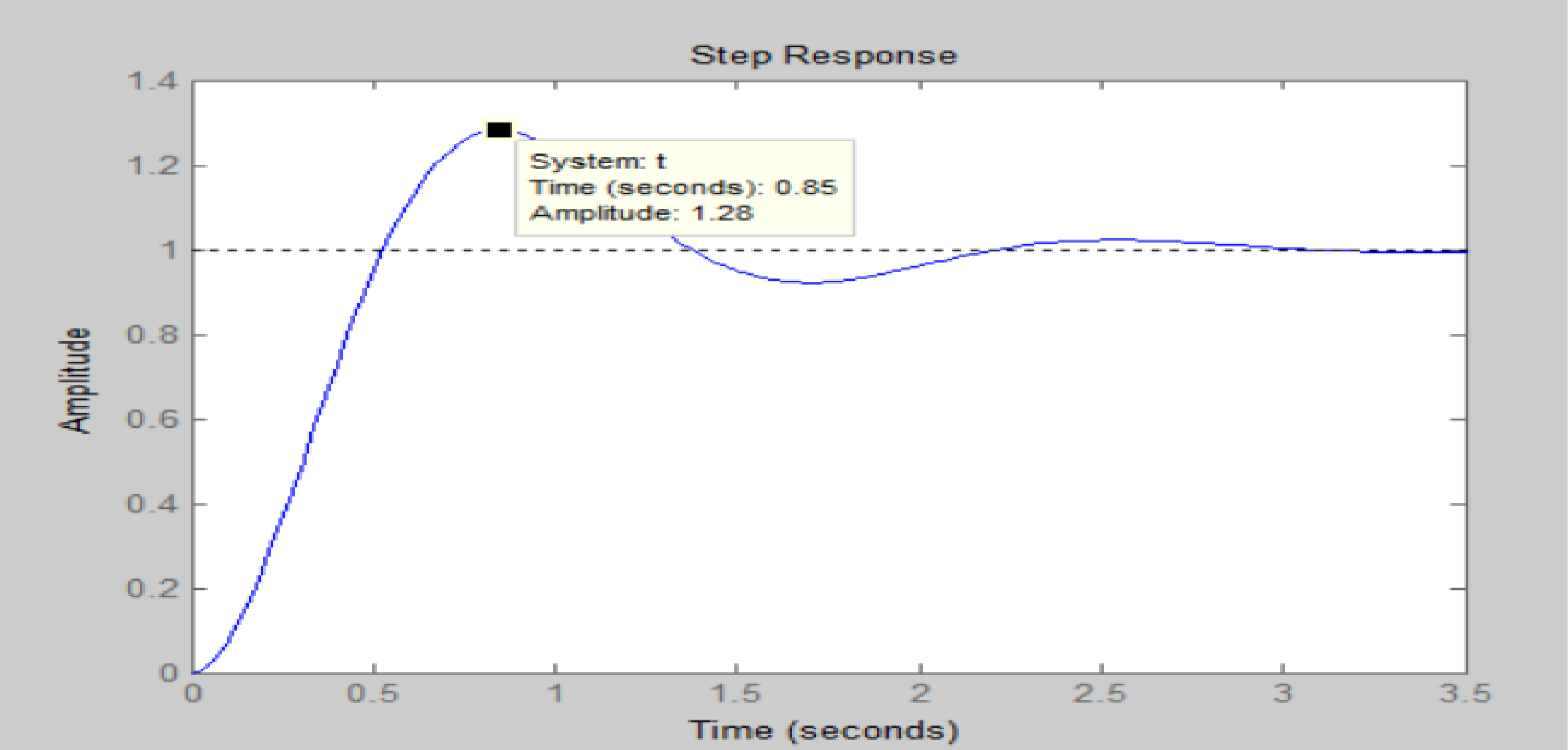
Zeta=0.375

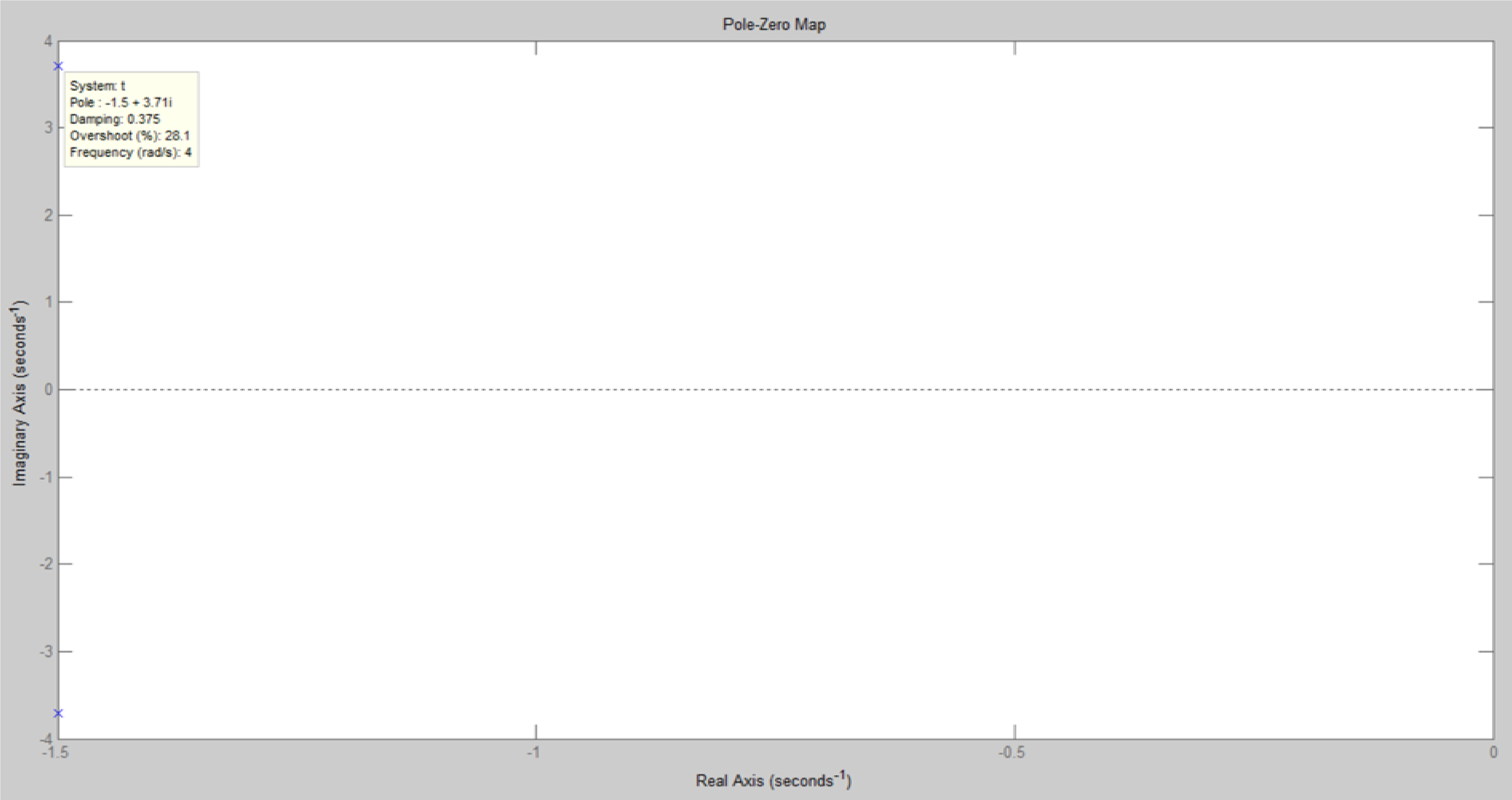
Ts=2.667

Tp=0.8472

OS=28.06%

Tr=0.5273




Pole-Zero Map
System: t
Pole : -1.5 + 3.71i
Damping: 0.375
Overshoot (%): 28.1
Frequency (rad/s): 4
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
4
3
2
1
0
-1
-2
-3
-4
-1.5
-1
-0.5
0

EXP 10: Show graphically how the change of damping ratio changes the step response for single transfer function of constant natural frequency of 1 rad/sec.

$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + 2\zeta s + 1}$$

(The undamped natural frequency ω_n is normalized to 1.) Plot unit-step response curves $c(t)$ when ζ assumes the following values:

$$\zeta = 0,\ 0.2,\ 0.4,\ 0.6.\ 0.8,\ 1.0$$

Comment on the step responses according to the change of damping

EXP 11: Step Responses of Second order systems according to pole movement

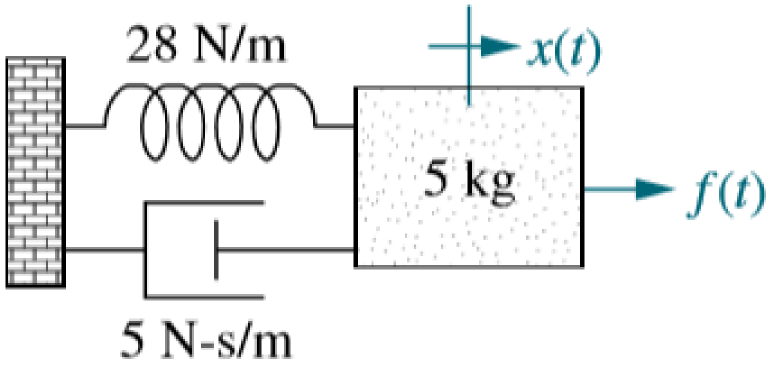
- Prove and comment using MATLAB plot that

1. Frequency of oscillation remains the same for constant imaginary part.
2. Envelope remains the same for constant real part.
3. Overshoot remains the same for same damping ratio.

EXP 12: Underdamped System Parameters

- For the figure given find using MATLAB

1. Rise Time Tr
2. Peak Time Tp
3. Percentage Overshoot %OS
4. Settling Time Ts
5. Step Response



The Transfer function should be determined by hand and included in the lab report. While measuring system parameters consider the denominator of the transfer function only.

Ref. Nise Exercise Problem no. 25 Page. 217

EXP 13: Second Order approximation

- Prove that additional poles should be as far as possible from origin for better system response.
- Prove that additional zeros should be as far as possible from origin for better system response.
- Add a zero at right half plane to observe *non minimum phase system* and comment on the step response on the system.

Ref. Nise Ch. 4, Art 4.7-4.8