Measurement

Definition: Measurement is the act, or the result, of a quantitative comparison between a given quantity and a quantity of the same kind chosen as a unit.
Measurement of a given quantity is essentially an act or result of comparison between the quantity (whose magnitude is unknown) and predetermined or predefined standards.

Basic requirements for a meaningful measurement:

- The standard used for comparison purposes must be accurately defined and should be commonly accepted.
- The apparatus used and the method adopted must be provable (verifiable).

Significance of Measurement

Importance of Measurement is simply and eloquently expressed in the following statement of famous physicist Lord Kelvin: "I often say that when you can measure what you are speaking about and can express it in numbers, you know something about it; when you cannot express in it numbers your knowledge is of meager and unsatisfactory kind"

A measured variable is represented in terms of measurement units, which are categorized into

1. Fundamental units
2. Derived units

Fundamental Units

The fundamental units in measurement science are measures of length, mass and time. Since length, mass and time are fundamental to most other physical quantities; they are called the primary fundamental units.
The dimensional symbols for the fundamental units of length, mass and time are L, M and T respectively.

Derived units

These are units which can be expressed in terms of the fundamental units. Derived unit originates from some physical law defining that unit. For example, the area (A) of a rectangle is proportional to its length (l) and breadth (b), or A = lb. if the metre has been chosen as the unit of length then the area of a rectangle of 5 metres by 7 metres is 35 m2 . Note that the numbers of measure are multiplied as well as the units. The derived unit for area (A) is then the metre square (m2).
A derived unit is recognized by its dimensions, which can be defined as the complete algebraic formula for the derived unit. The dimensional symbol for the derived unit of area is L 2 and that for volume is L3. The dimensional symbol for the unit of force is MLT-2, which follows from the defining equation for force F=ma. The dimensional formulas of the derived units are particularly useful for converting units from one system to another. For convenience, some derived units have been given new names. For example, the derived unit of force in the SI system is called the newton (N), instead of the dimensionally correct kg-m/s-2.

Measurement standards

A standard of measurement is a physical representation of a unit of measurement. A unit is realised by reference to an arbitrary material standard or to natural phenomena including physical and atomic constants. For example, the fundamental unit of mass in the SI system is the kilogram, defined as the mass of the cubic decimeter of water at its temperature of maximum of 4°C.

This unit of mass is represented by a material standard; the mass of the international prototype kilogram consisting of a platinum–iridium hollow cylinder. This unit is preserved at the International Bureau of Weights and Measures at Sevres, near Paris, and is the material representation of the kilogram.

The classifications of standards are

1. International standards
2. Primary standards
3. Secondary standards
4. Working standards
5. Current standards
6. Voltage standards
7. Resistance standards
8. Capacitance standards
9. Time and frequency standards

Measuring Units

Table 1.1 Definitions of standard units

Physical quantity	**Standard unit**	**Definition**
Length	meter	The length of path travelled by light in an interval of 1/299 792 458 seconds
Mass	kilogram	The mass of a platinum –iridium cylinder kept in the International Bureau of Weights and Measures Sevres, Paris
Time	second	9.192631770 × 109 cycles of radiation from vaporized caesium-133 (an accuracy of 1 in 10^{12} or 1 second in 36 000 years)
Temperature	kelvin	The temperature difference between absolute zero and the triple point of water is defined as 273.16 kelvin
Current	ampere	One ampere is the current flowing through two infinitely long parallel conductors of negligible cross-section placed 1 meter apart in a vacuum and producing a force of 2 ð 10 7 Newton's per meter length of conductor
Luminous intensity	candela	One candela is the luminous intensity in a given direction from a source emitting monochromatic radiation at a frequency of 540 terahertz (Hz ð 1012) and with a radiant density in that direction of 1.4641 mW/steradian. (1 steradian is the solid angle which, having its vertex at the centre of a sphere, cuts off an area of the sphere surface equal to that of a square with sides of length equal to the sphere radius)
Matter	mole	The number of atoms in a 0.012 kg mass of carbon-12

Table 1.2 Fundamental and derived SI units

(a) Fundamental units

Quantity	Standard unit	Symbol
Length	meter	m
Mass	kilogram	Kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Matter	mole	Mol

(b) Supplementary fundamental units

Quantity	Standard unit	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

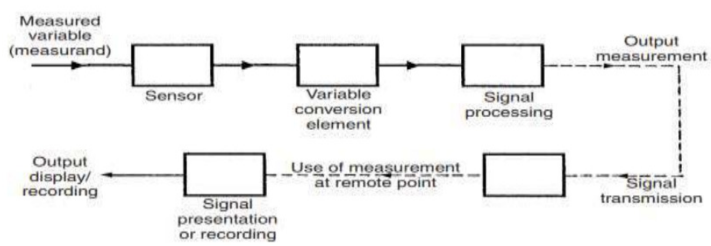
(c) Derived units

Quantity	Standard unit	Symbol	Formula
Area	square meter	m^2	
Volume	cubic meter	m^3	
Velocity	meter per second	m/s	
Acceleration	meter per second squared	m/s^2	
Angular velocity	radian per second	rad/s	
Angular acceleration	radian per second squared	rad/s^2	
Density	kilogram per cubic meter	kg/m^3	
Specific volume	cubic meter per kilogram	m^3/kg	
Mass flow rate	kilogram per second	kg/s	
Volume flow rate	cubic meter per second	m^3/s	
Force	newton	N	$kg\ m/s^2$
Pressure	newton per square meter	N/m^2	
Torque	newton meter	N m	
Momentum	kilogram meter per second	kg m/s	
Moment of inertia	kilogram meter squared	$kg\ m^2$	
Kinematic viscosity	square meter per second	m^2/s	
Dynamic viscosity	newton second per square meter	$N\ s/m^2$	
Work, energy, heat	joule	J	Nm
Specific energy	joule per cubic metre	J/m^3	
Power	watt	W	J/s
Thermal conductivity	watt per meter kelvin	W/m K	
Electric charge	coulomb	C	A s
Voltage, e.m.f., pot. diff.	volt	V	W/A
Electric field strength	volt per meter	V/m	
Electric resistance	ohm	Ω	V/A
Electric capacitance	farad	F	A s/V
Electric inductance	henry	H	V s/A
Electric conductance	siemen	S	A/V
Resistivity	ohm meter	Ω m	
Permittivity	farad per meter	F/m	
Permeability	henry per meter	H/m	

Current density	ampere per square meter	A/m^2	
Magnetic flux	weber	Wb	V s
Magnetic flux density	tesla	T	Wb/m^2
Magnetic field strength	ampere per meter	A/m	
Frequency	hertz	Hz	s^{-1}
Luminous flux	lumen	lm	cd sr
Luminance	candela per square meter	cd/m2	
Illumination	lux	Lx	lm/m^2
Molar volume	cubic meter per mole	m^3/mol	
Molarity	mole per kilogram	mol/kg	
Molar energy	joule per mole	J/mol	

Measurement system and its element

A measurement system may be defined as systematic arrangement different components for the measurement or determination of an unknown quantity and analysis of instrumentation. The generalized measurement system and its different components/elements are shown in Figure 2



i. **Sensor**

This gives an output that is a function of the measured (the input applied to it). For examples of primary sensors are a liquid-in-glass thermometer, a thermocouple and a strain gauge. In the case of the mercury-in-glass thermometer, the output reading is given in terms of the level of the mercury.

ii. **Variable conversion element**

It is used for changing the form of the measured signal into a desired form for manipulation by the instrument. For instance, the displacement-measuring strain gauge has an output in the form of a varying resistance. The resistance change cannot be easily measured and so it is converted to a change in voltage by a bridge circuit, which is a typical example of a variable conversion element. In some cases, the primary sensor and variable conversion element are combined, and the combination is known as a transducer.

iii. **Signal processing**

Signal processing elements improve the quality of the output of a measurement system. Example is the electronic amplifier, which amplifies the output of the primary transducer or variable conversion element, thus improving the sensitivity and resolution of measurement. This element of a measuring system is particularly important where the primary transducer has a low output. For example, thermocouples have a typical output of only a few millivolts. Other types of signal processing element are those that filter out induced noise and remove mean levels etc. In some devices, signal processing is incorporated into a transducer, which is then known as a transmitter.

iv. Data transmission elements

The data transmission elements are required to transmit the data containing the information of the signal from one system to another. For example, satellites are physically separated from the earth where the control stations guiding their movement are located.

Instruments and Measurement Systems

- Measurement involves the use of instruments as a physical means of determining quantities or variables.
- Because of modular nature of the elements within it, it is common to refer the measuring instrument as a MEASUREMENT SYSTEM.

Classification of instruments

The measuring instruments are firstly classified into

1. Absolute Instruments
2. Secondary Instruments

1. Absolute Instruments

The instruments of this type give the value of the measurand in terms of instrument constant and its deflection. Such instruments do not require comparison with any other standard. The example of this type of instrument is tangent galvanometer, which gives the value of the current to be measured in terms of tangent of the angle of deflection produced, the horizontal component of the earth's magnetic field, the radius and the number of turns of the wire used.

Rayleigh current balance and absolute electrometer are other examples of absolute instruments.

2. Secondary Instruments

These instruments are so constructed that the deflection of such instruments gives the magnitude of the electrical quantity to be measured directly. These instruments are required to be calibrated by comparison with either an absolute instrument or with another secondary instrument, which has already been calibrated before the use. These instruments are generally used in practice.

Secondary instruments are further classified as

- Indicating instruments
- Integrating instruments
- Recording instruments

(i) Indicating Instruments

Indicating instruments are those which indicate the magnitude of an electrical quantity at the time when it is being measured. The indications are given by a pointer moving over a calibrated (pre-graduated) scale. Ordinary ammeters, voltmeters, wattmeters, frequency meters, power factor meters, etc., fall into this category.

(ii) Integrating Instruments

Integrating instruments are those which measure the total amount of either quantity of electricity (ampere-hours) or electrical energy supplied over a period of time. The summation, given by such an instrument, is the product of time and an electrical quantity under measurement. The ampere-hour meters and energy meters fall in this class.

(iii) Recording Instruments

Recording instruments are those which keep a continuous record of the variation of the magnitude of an electrical quantity to be observed over a definite period of time. In such instruments, the moving system carries an inked pen which touches lightly a sheet of paper wrapped over a drum moving with uniform slow motion in a direction perpendicular to that of the direction of the pointer. Thus, a curve is traced which shows the variations in the magnitude of the electrical quantity under observation over a definite period of time. Such instruments are generally used in powerhouses where the current, voltage, power, etc., are to be maintained within certain acceptable limit.

Analogue and digital instruments

a) Analogue Instruments

The signals of an analogue unit vary in a continuous fashion and can take on infinite number of values in a given range. Fuel gauge, ammeter and voltmeters, wrist watch, speedometer fall in this category.

b) Digital instruments

Signals varying in discrete steps and taking on a finite number of different values in a given range are digital signals and the corresponding instruments are of digital type. Digital instruments have some advantages over analogue meters, in that they have high accuracy and high speed of operation. It eliminates the human operational errors. Digital instruments can store the result for future purposes. A digital multimeter is the example of a digital instrument.

Mechanical, Electrical and electronic instruments

a. Mechanical instruments

Mechanical instruments are very reliable for static and stable conditions. They are unable to respond rapidly to the measurement of dynamic and transient conditions due to the fact that they have moving parts that are rigid, heavy and bulky and consequently have a large mass. Mass presents inertia problems and hence these instruments cannot faithfully follow the rapid changes which are involved in dynamic instruments. Also, most of the mechanical instruments causes noise pollution.

Advantages of Mechanical Instruments

- Relatively cheaper in cost
- More durable due to rugged construction
- Simple in design and easy to use
- No external power supply required for operation
- Reliable and accurate for measurement of stable and time invariant quantity

Disadvantages of Mechanical Instruments

- Poor frequency response to transient and dynamic measurements
- Large force required to overcome mechanical friction
- Incompatible when remote indication and control needed
- Cause noise pollution

b. Electrical instruments

When the instrument pointer deflection is caused by the action of some electrical methods then it is called an electrical instrument. The time of operation of an electrical instrument is more rapid than that of a mechanical instrument. Unfortunately, an electrical system normally depends upon a mechanical measurement as an indicating device. This mechanical movement has some inertia due to which the frequency response of these instruments is poor.

c. Electronic instruments

Electronic instruments use semiconductor devices. Most of the scientific and industrial instrumentations require very fast responses. Such requirements cannot be met with by mechanical and electrical instruments. In electronic devices, since the only movement involved is that of electrons, the response time is extremely small owing to very small inertia of the electrons. With the use of electronic devices, a very weak signal can be detected by using pre-amplifiers and amplifiers.

Advantages of Electrical/Electronic Instruments

- Non-contact measurements are possible
- These instruments consume less power
- Compact in size and more reliable in operation
- Greater flexibility
- Good frequency and transient response
- Remote indication and recording possible

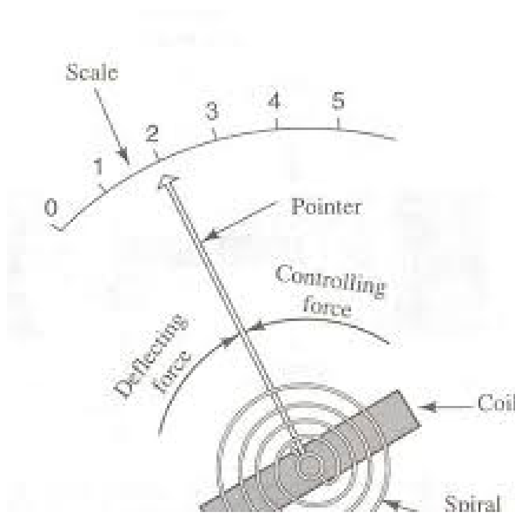
Electromechanical indicating instrument

For satisfactory operation electromechanical indicating instrument, three forces are necessary. They are

1. Deflecting force
2. Controlling force
3. Damping force

1. Deflecting force

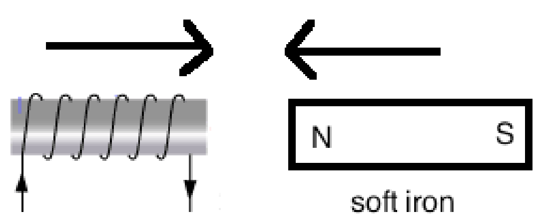
When there is no input signal to the instrument, the pointer will be at its zero position. To deflect the pointer from its zero position, a force is necessary which is known as deflecting force. A system which produces the deflecting force is known as a deflecting system. Generally a deflecting system converts an electrical signal to a mechanical force.



Some important terms in deflecting force:

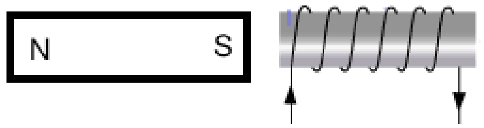
Magnitude effect

When a current passes through the coil (Fig), it produces a imaginary bar magnet. When a soft-iron piece is brought near this coil it is magnetized. Depending upon the current direction the poles are produced in such a way that there will be a force of attraction between the coil and the soft iron piece. This principle is used in moving iron attraction type instrument.



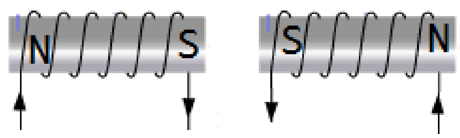
If two soft iron pieces are place near a current carrying coil there will be a force of repulsion between the two soft iron pieces. This principle is utilized in the moving iron repulsion type instrument.

Force between a permanent magnet and a current carrying coil



When a current carrying coil is placed under the influence of magnetic field produced by a permanent magnet and a force is produced between them. This principle is utilized in the moving coil type instrument.

Force between two current carrying coils



When two current carrying coils are placed closer to each other there will be a force of repulsion between them. If one coil is movable and other is fixed, the movable coil will move away from the fixed one. This principle is utilized in electrodynamometer type instrument.

2. Controlling force

To make the measurement indicated by the pointer definite (constant) a force is necessary which will be acting in the opposite direction to the deflecting force. This force is known as controlling force. A system which produces this force is known as a controlled system. When the external signal to be measured by the instrument is removed, the pointer should return back to the zero position. This is possibly due to the controlling force and the pointer will be indicating a steady value when the deflecting torque is equal to controlling torque.

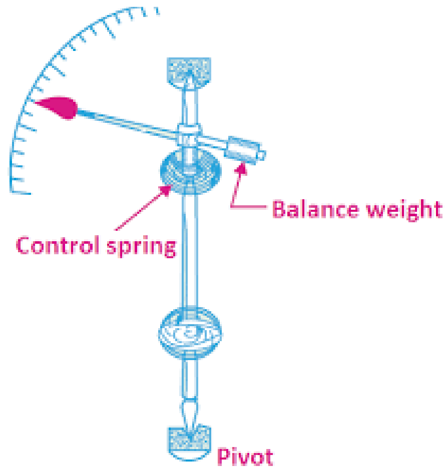
$$T_d = T_c \tag{1.1}$$

Spring control

Two springs are attached on either end of spindle (Fig).The spindle is placed in jeweled bearing, so that the frictional force between the pivot and spindle will be minimum. Two springs are provided in opposite direction to compensate the temperature error. The spring is made of phosphorous bronze.

When a current is supply, the pointer deflects due to rotation of the spindle. While spindle is rotate, the spring attached with the spindle will oppose the movements of the pointer. The torque produced by the spring is directly proportional to the pointer deflection θ .

$$T_C \propto \theta \tag{1.2}$$



The deflecting torque produced T_d proportional to 'I'. When $T_C = T_d$, the pointer will come to a steady position. Therefore

$$\theta \propto I \qquad (1.3)$$

Since, θ and I are directly proportional to the scale of such instrument which uses spring controlled is uniform.

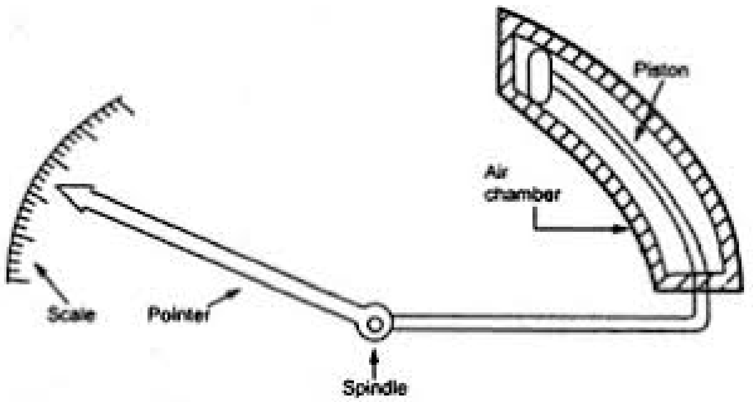
3. Damping force

The deflection torque and controlling torque produced by systems are electro mechanical. Due to inertia produced by this system, the pointer oscillates about it final steady position before coming to rest. The time required to take the measurement is more. To damp out the oscillation is quickly, a damping force is necessary. This force is produced by different systems.

(a) Air friction damping
(b) Fluid friction damping
(c) Eddy current damping

Air friction damping

The piston is mechanically connected to a spindle through the connecting rod (Fig. 1.6). The pointer is fixed to the spindle moves over a calibrated dial. When the pointer oscillates in clockwise direction, the piston goes inside and the cylinder gets compressed. The air pushes the piston upwards and the pointer tends to move in anticlockwise direction.



If the pointer oscillates in anticlockwise direction the piston moves away and the pressure of the air inside cylinder gets reduced. The external pressure is more than that of the internal pressure. Therefore the piston moves down wards. The pointer tends to move in clock wise direction.

Eddy current damping

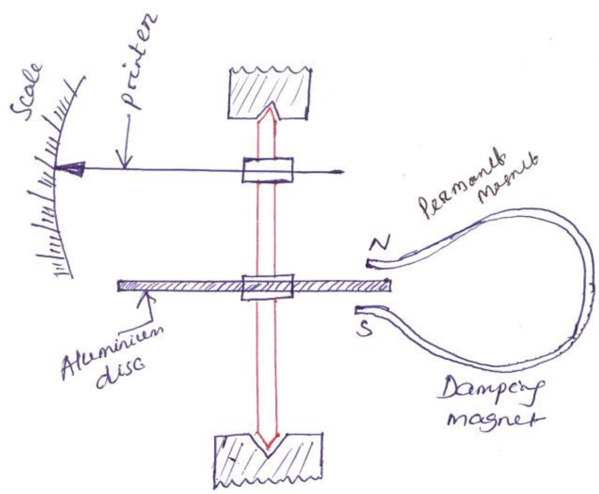


Fig. Disc type

An aluminum circular disc is fixed to the spindle (Fig). This disc is made to move in the magnetic field produced by a permanent magnet.

When the disc oscillates it cuts the magnetic flux produced by damping magnet. An emf is induced in the circular disc by faradays law. Eddy currents are established in the disc since it has several closed paths. By Lenz's law, the current carrying disc produced a force in a direction opposite to oscillating force. The damping force can be varied by varying the projection of the magnet over the circular disc.

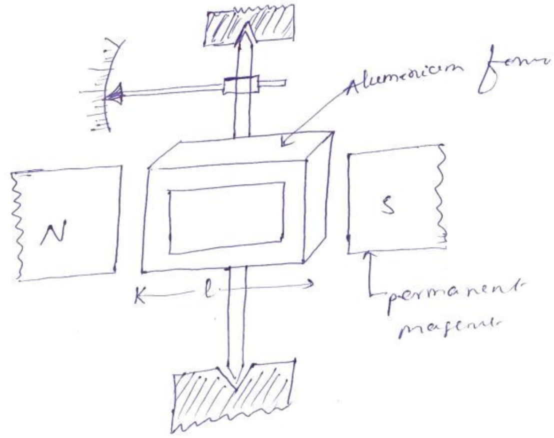


Fig. Rectangular type

Manual and automatic instruments

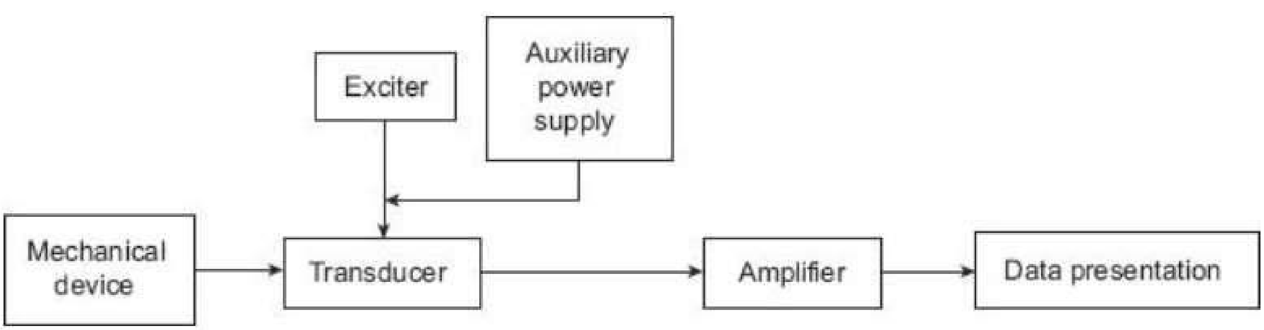
In manual instruments, the service of an operator is required. For example, measurement of temperature by a resistance thermometer incorporating a Wheatstone bridge in its circuit, an operator is required to indicate the temperature being measured.

In an automatic type of instrument, no operator is required all the time. For example, measurement of temperature by mercury-in-glass thermometer

Self-operated and power operated instruments

Self-operated instruments are those in which no outside power is required for operation. The output energy is supplied wholly or almost wholly by the input measurand. Dial indicating type instruments belong to this category.

The power-operated instruments are those in which some external power such as electricity, compressed air, hydraulic supply is required for operation. In such cases, the input signal supplies only an insignificant portion of the output power. Electromechanical instruments shown in Figure fall in this category:



Deflection and null output instruments

In a deflection-type instrument, the deflection of the instrument indicates the measurement of the unknown quantity. The measurand quantity produces some physical effect which deflects or produces a mechanical displacement in the moving system of the instrument. An opposite effect is built in the instrument which opposes the deflection or the mechanical displacement of the moving system. The balance is achieved when opposing effect equals the actuating cause producing the deflection or the mechanical displacement. The deflection or the mechanical displacement at this point gives the value of the unknown input quantity. These type of instruments are suited for measurement under dynamic condition. Permanent Magnet Moving Coil (PMMC), Moving Iron (MI), etc., type instruments are examples of this category.

In null-type instruments, a zero or null indication leads to determination of the magnitude of the measurand quantity. The null condition depends upon some other known conditions. These are more accurate and highly sensitive as compared to deflection-type instruments. A dc potentiometer is a null- type instrument.

Static characteristics

The static characteristics of measuring instruments are concerned only with the steady state reading that the instrument settles down to, such as the accuracy of the reading etc.

i. Accuracy

Accuracy is the closeness with which the instrument reading approaches the true value of the variable under measurement. Accuracy is determined as the maximum amount by which the result differs from the true value. It is almost impossible to determine experimentally the true value. The true value is not indicated by any measurement system due to the loading effect, lags and mechanical problems (e.g., wear, hysteresis, noise, etc.).

Accuracy of the measured signal depends upon the following factors:

- Intrinsic accuracy of the instrument itself;
- Accuracy of the observer;
- Variation of the signal to be measured; and
- Whether or not the quantity is being truly impressed upon the instrument.

ii. Precision

Precision is a measure of the reproducibility of the measurements, i.e., precision is a measure of the degree to which successive measurements differ from one another. Precision is indicated from the number of significant figures in which it is expressed. Significant figures actually convey the information regarding the magnitude and the measurement precision of a quantity. More significant figures imply greater precision of the measurement.

iii. Resolution

If the input is slowly increased from some arbitrary value it will be noticed that the output does not change at all until the increment exceeds a certain value called the resolution or discrimination of the instrument. Thus, the resolution or discrimination of any instrument is the smallest change in the input signal (quantity under measurement) which can be detected by the instrument. It may be expressed as an accrual value or as a fraction or percentage of the full scale value. Resolution is sometimes referred as sensitivity. The largest change of input quantity for which there is no output of the instrument is called the dead zone of that instrument.

The sensitivity gives the relation between the input signal to an instrument or a part of the instrument system and the output. Thus, the sensitivity is defined as the ratio of output signal or response of the instrument to a change of input signal or the quantity under measurement.

$$\frac{\text{scale deflection}}{\text{value of measurand producing deflection}}$$

iv. Speed of response

The quickness of an instrument to read the measurand variable is called the speed of response. Alternately, speed of response is defined as the time elapsed between the start of the measurement to the reading taken. This time depends upon the mechanical moving system, friction, etc.

v. Tolerance

Tolerance is a term that is closely related to accuracy and defines the maximum error that is to be expected in some value. Whilst it is not, strictly speaking, a static characteristic of measuring instruments, it is mentioned here because the accuracy of some instruments is sometimes quoted as a tolerance figure. When used correctly, tolerance describes the maximum deviation of a manufactured component from some specified value.

vi. Repeatability/reproducibility

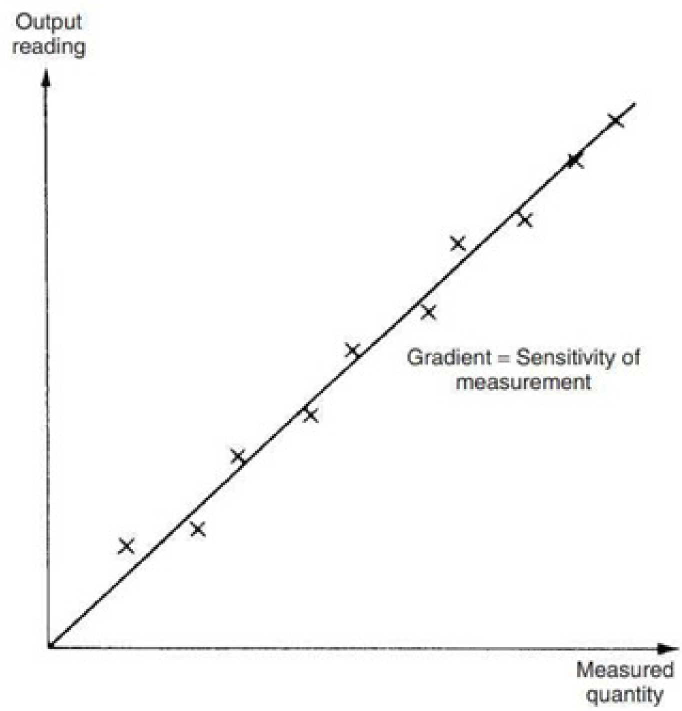
Repeatability describes the closeness of output readings when the same input is applied repetitively over a short period of time, with the same measurement conditions, same instrument and observer, same location and same conditions of use maintained throughout.

Reproducibility describes the closeness of output readings for the same input when there are changes in the method of measurement, observer, measuring instrument, location, conditions of use and time of measurement. Both terms thus describe the spread of output readings for the same input. This spread is referred to as repeatability if the measurement conditions are constant and as reproducibility if the measurement conditions vary.

vii. **Range/Span**

The range or span of an instrument defines the minimum and maximum values of a quantity that the instrument is designed to measure.

It is normally desirable that the output reading of an instrument is linearly proportional to the quantity being measured. The Xs marked on Figure below show a plot of the typical output readings of an instrument when sequences of input quantities are applied to it. Normal procedure is to draw a good fit straight line through the Xs, as shown in Figure below (Whilst this can often be done with reasonable accuracy by eye, it is always preferable to apply a mathematical least-squares line-fitting technique, as described in Chapter 11.) The non-linearity is then defined as the maximum deviation of any of the output readings marked X from this straight line. Non-linearity is usually expressed as a percentage of full-scale reading.



viii. Linearity

It is normally desirable that the output reading of an instrument is linearly proportional to the quantity being measured. The Xs marked on Figure 2.6 show a plot of the typical output readings of an instrument when sequences of input quantities are applied to it. Normal procedure is to draw a good fit straight line through the Xs, as shown in Figure 2.6. (Whilst this can often be done with reasonable accuracy by eye, it is always preferable to apply a mathematical least-squares line-fitting technique, as described in Chapter 11.) The non-linearity is then defined as the maximum deviation of any of the output readings marked X from this straight line. Non-linearity is usually expressed as a percentage of full-scale reading.

ix. Sensitivity of measurement

The sensitivity of measurement is a measure of the change in instrument output that occurs when the quantity being measured changes by a given amount. Thus, sensitivity is the ratio:

$$\frac{\text{scale deflection}}{\text{value of measurand producing deflection}}$$

The sensitivity of measurement is therefore the slope of the straight line drawn on Figure 2.6. If, for example, a pressure of 2 bar produces a deflection of 10 degrees in a pressure transducer, the sensitivity of the instrument is 5 degrees/bar (assuming that the deflection is zero with zero pressure applied).

x. Threshold

If the input to an instrument is gradually increased from zero, the input will have to reach a certain minimum level before the change in the instrument output reading is of a large enough magnitude to be detectable. This minimum level of input is known as the threshold of the instrument. Manufacturers vary in the way that they specify threshold for instruments.

xi. Sensitivity of Disturbance

All calibrations and specifications of an instrument are only valid under controlled conditions of temperature, pressure etc. These standard ambient conditions are usually defined in the instrument specification. As variations occur in the ambient temperature etc., certain static instrument characteristics change, and the sensitivity to disturbance is a measure of the magnitude of this change. Such environmental changes affect instruments in two main ways, known as zero drift and sensitivity drift. Zero drift is sometimes known by the alternative term, bias.

i. Hysteresis

If the input measured quantity to the instrument is steadily increased from a negative value, the output reading varies in the manner shown in curve (a). If the input variable is then steadily decreased, the output varies in the manner shown in curve (b). The non-coincidence between these loading and unloading curves is known as hysteresis. Two quantities are defined, maximum input hysteresis and maximum output hysteresis, as shown in Figure

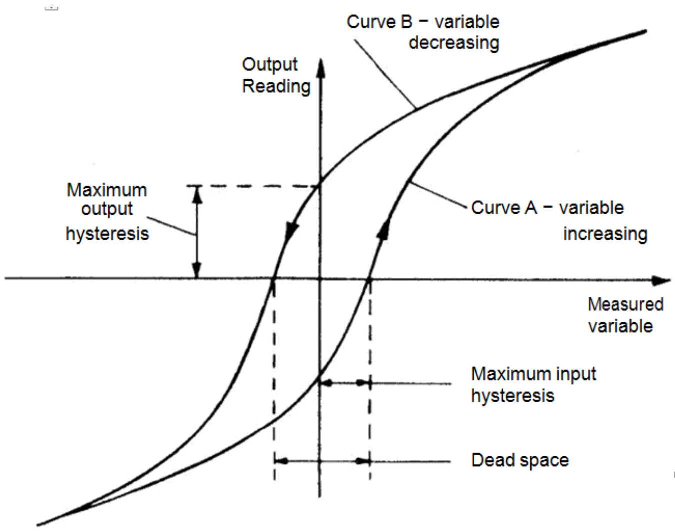


Fig. 2.8 Instrument characteristic with hysteresis.

ii. Dead space

Dead space is defined as the range of different input values over which there is no change in output value. Any instrument that exhibits hysteresis also displays dead space, as marked on Figure 2.8. Some instruments that do not suffer from any significant hysteresis can still exhibit a dead space in their output characteristics, however.

Dynamic characteristics

The dynamic characteristics of a measuring instrument describe its behavior between the time a measured quantity changes value and the time when the instrument output attains a steady value in response. As with static characteristics, any values for dynamic characteristics quoted in instrument data sheets only apply when the instrument is used under specified environmental conditions. Outside these calibration conditions, some variation in the dynamic parameters can be expected.

In any linear, time-invariant measuring system, the following general relation can be written between input and output for time (t) > 0:

$$a_n \frac{d^n q_0}{dt^n} + a_{n-1} \frac{d^{n-1} q_0}{dt^{n-1}} + \cdots + a_1 \frac{dq_0}{dt} + a_0 q_0 = b_m \frac{d^m q_i}{dt^m} + b_{m-1} \frac{d^{m-1} q_i}{dt^{m-1}} + \cdots + b_1 \frac{dq_i}{dt} + b_0 q_i \tag{2.1}$$

Where qi is the measured quantity, q0 is the output reading and a0 . . . an, b0 . . . bm are constants.

Rewriting equation 2.1 to limit of step changes we get:

$$a_n \frac{d^n q_0}{dt^n} + a_{n-1} \frac{d^{n-1} q_0}{dt^{n-1}} + \cdots + a_1 \frac{dq_0}{dt} + a_0 q_0 = b_0 q_i \tag{2.2}$$

Further simplification can be made by considering several measurement instruments in terms of:

i. Zero order instrument

ii. Second order instruments

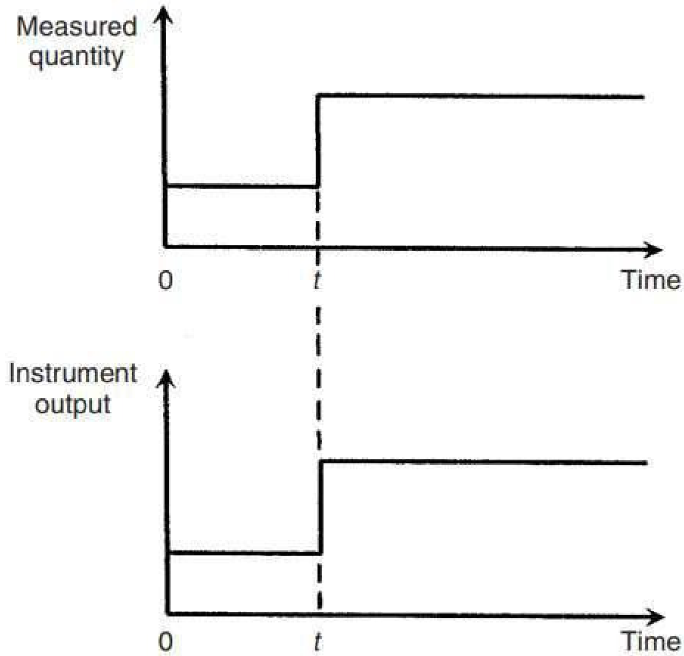
iii. Third order instruments

If all the coefficients a_1 **…** a_n other than a_0 in equation (2.2) are assumed zero, then:

$$a_0 q_0 = b_0 q_i \quad \text{or} \quad q_0 = b_0 q_i / a_0 = K q_i \tag{2.3}$$

Where K is a constant known as the instrument sensitivity as defined earlier.

Any instrument that behaves according to equation (2.3) is said to be of zero order type. A potentiometer, which measures motion, is a good example of such an instrument, where the output voltage changes instantaneously as the slider is displaced along the potentiometer track.



First order instruments

If all the coefficients a2 . . . an except for a0 and a1 are assumed zero in equation (2.2) then:

$$a_1 \frac{dq_0}{dt} + a_0 q_0 = b_0 q_i \qquad (2.4)$$

Any instrument that behaves according to equation (2.4) is known as a first order instrument. If d/dt is replaced by the D operator in equation (2.4), we get:

$$a_1 D q_0 + a_0 q_0 = b_0 q_i \quad \text{and rearranging this then gives} \quad q_0 = \frac{(b_0/a_0) q_i}{[1 + (a_1/a_0) D]} \qquad (2.5)$$

Defining K = b0/a0 as the static sensitivity and = a1/a0 as the time constant of the system, equation (2.5) becomes:

$$q_0 = \frac{K q_i}{1 + \tau D} \qquad (2.6)$$

If equation (2.6) is solved analytically, the output quantity q0 in response to a step change in qi at time t varies with time in the manner shown in Figure 2.11. The time constant of the step response is the time taken for the output quantity q0 to reach 63% of its final value. Example is the liquid-in-glass thermometer.

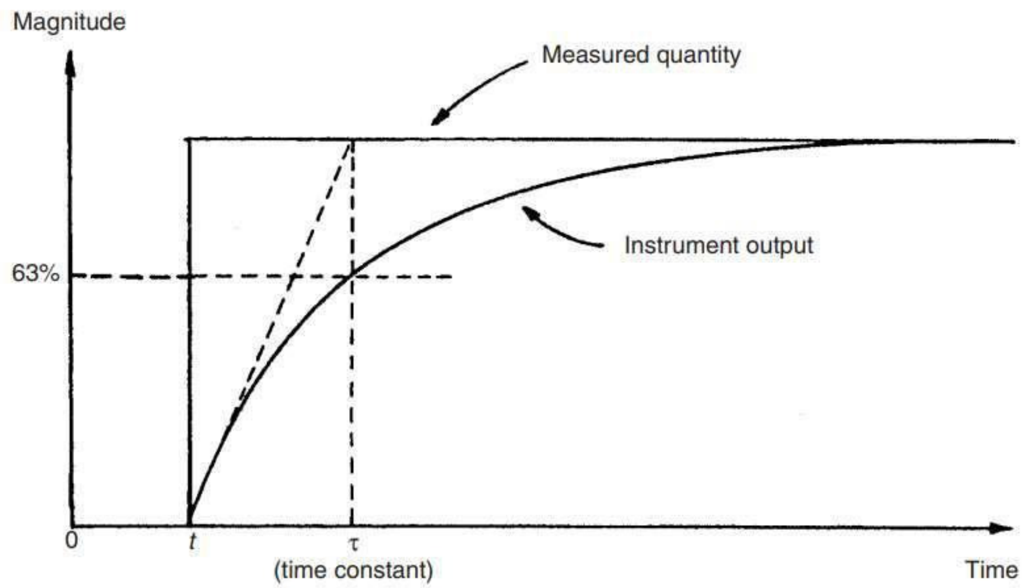


Fig. 2.11 First order instrument characteristic.

Second order Instrument

If all coefficients a3 . . . an other than a0, a1 and a2 in equation (2.2) are assumed zero, then we get:

$$a_2 \frac{d^2 q_0}{dt^2} + a_1 \frac{dq_0}{dt} + a_0 q_0 = b_0 q_i \tag{2.7}$$

Applying the D operator again: $a_2 D^2 q_0 + a_1 D q_0 + a_0 q_0 = b_0 q_i$, and rearranging:

$$q_0 = \frac{b_0 q_i}{a_0 + a_1 D + a_2 D^2} \tag{2.8}$$

It is convenient to re-express the variables a0, a1, a2 and b0 in equation (2.8) in terms of three parameters K (static sensitivity),(undamped natural frequency) and (damping ratio), where:

$$K = b_0/a_0; \quad \omega = a_0/a_2; \quad \xi = a_1/2a_0a_2$$

Re-expressing equation (2.8) in terms of K, ω and ξ we get:

$$\frac{q_0}{q_i} = \frac{K}{D^2/\omega^2 + 2\xi D/\omega + 1} \tag{2.9}$$

This is the standard equation for a second order system and any instrument whose response can be described by it is known as a second order instrument.

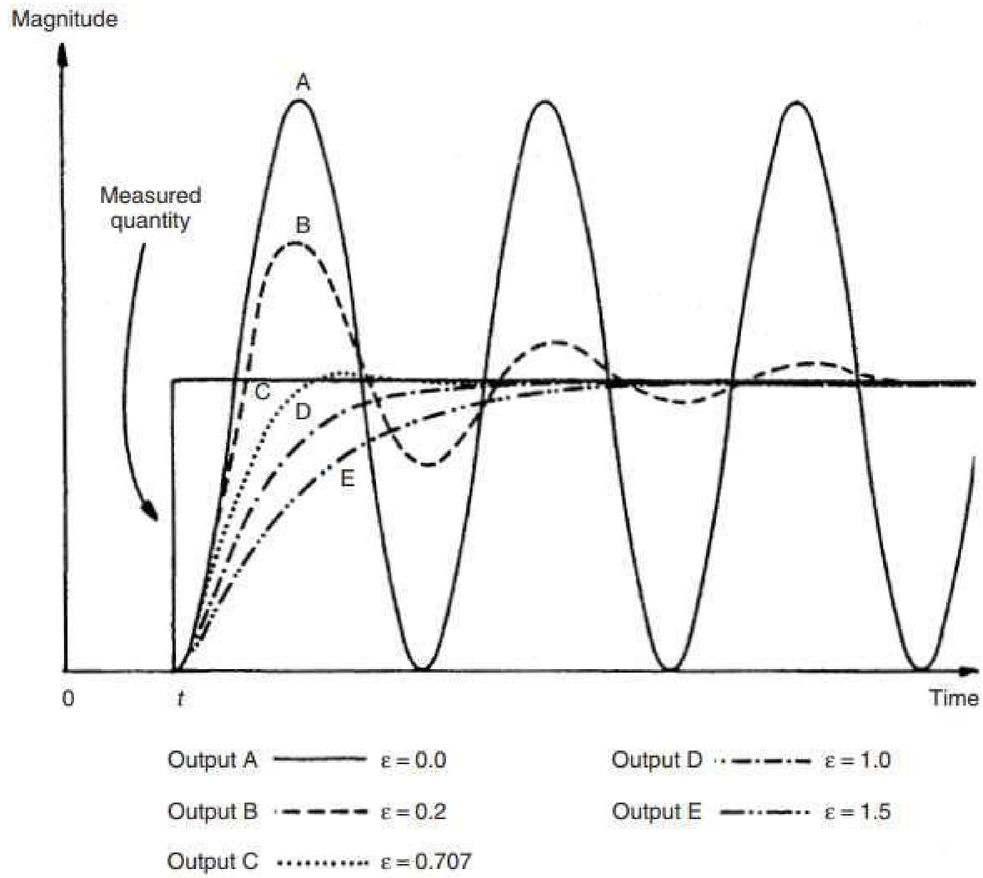


Fig. 2.12 Response characteristics of second order instruments.

The shape of the response curve depends on the damping ratio.