

## Power System II

### Mechanical Design of overhead Lines.

- B Main Component of Overhead Lines:** In general the main components of an overhead line are
- (i) Conductors: which carry electric power from the sending end station to the receiving end station.
  - (ii) Supports: which may be poles (etc towers) and keep the conductors at a suitable level above the ground.
  - (iii) Insulators: which are attached to supports and insulate the conductors from the ground.
  - (iv) Cross arms: which provide support to the insulators.
  - (v) Miscellaneous items: such as phase plates, danger plates, lightning arrestors, anti climbing wires etc.

#### Conductor Properties:

- 1) High electrical conductivity
- 2) High tensile strength in order to withstand mechanical stresses.
- 3) Low cost so that it can be used for long distances.

(iv) low specific gravity so that weight per unit volume is small.  
If there are  $n$  layers, the total number of individual wires is  $3n(n+1)/2$

Al conductor mostly used because  
(i) low cost (ii) low specific gravity.

### Line Support Properties:

- (i) High mechanical strength to withstand the weight of conductors and wind loads etc.
- (ii) Light in weight without the loss of mechanical strength.
- (iii) Cheap in cost and economical to maintain.
- (iv) Longer life.
- (v) Easy accessibility of conductors for maintenance.

### Types of poles:

- (i) Wooden pole
- (ii) Steel pole
- (iii) R.C.C. pole
- (iv) Steel tower.

## Q) Insulators

### B) Properties

- (i) High resistivity
- (ii) High mechanical strength.
- (iii) Dielectric strength.
- (iv) High permittivity
- (v) High ratio of puncture strength to flashover
- (vi) free from, non-porous impurities, crack.

### C) Types: (i) Pin type $\leftarrow 11\text{KV}$ upto 33 KV.

- (ii) Suspension type or Disc type  $> 33\text{KV}$
- (iii) Strain type ( $11\text{KV}$ )
- (iv) Shackle type (low voltage distribution)

D) String Efficiency: The voltage applied across the string of suspension insulators is not uniformly distributed across various units or discs. The disc nearest to the conductor has much higher potential than the other discs. This unequal potential distribution is undesirable and usually

expressed in terms of string efficiency. The ratio of voltage across the whole string to the product of number of discs and the voltage across the discs nearest to the conductor is known as string efficiency.

$$\text{String efficiency} = \frac{\text{Voltage across the string}}{n \times \text{Voltage across disc nearest to the conductor}}$$

$n = \text{no. of discs in the string.}$

String efficiency is an important consideration since it decides the potential distribution along the string. The greater the string efficiency, the more uniform is the voltage distribution.

### Mathematical expression:

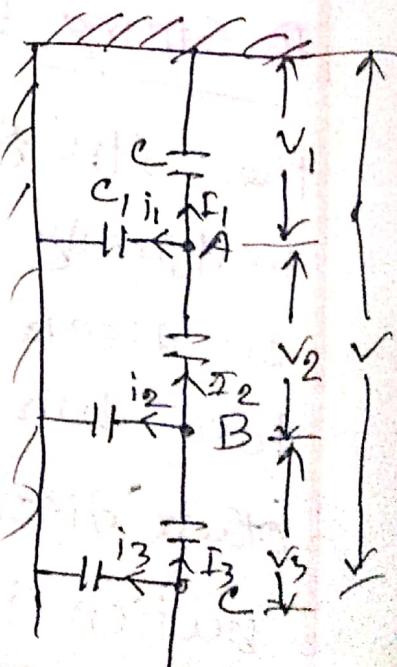
here self capacitance  $\equiv c$ ,

shunt capacitance  $\equiv c_1$

$k$  is the ratio of self and

shunt capacitance.

$$k_1 = \frac{c}{c_1} \Rightarrow c_1 = kc$$



Voltage across each unit is  $v_1, v_2, v_3, \dots$

At node A.

$$I_2 = I_1 + i_1$$

$$\Rightarrow v_2 w_c = v_1 w_c + v_1 w_c i_1$$

$$\Rightarrow v_2 w_c = v_1 w_c + v_1 w_k c$$

$$\Rightarrow v_2 = v_1 (1+k) \quad \dots \text{.} \quad (i)$$

At B node

$$I_3 = I_2 + i_2$$

$$\Rightarrow v_3 w_c = v_2 w_c + (v_1 + v_2) w_c$$

$$\Rightarrow v_3 w_c = v_2 w_c + (v_1 + v_2) w_k c$$

$$\Rightarrow v_3 = v_2 + (v_1 + v_2) k$$

$$= v_1 (1+k) + v_1 k + v_1 (1+k) k$$

$$= v_1 (1+k + k + k^2)$$

$$= v_1 (1+3k+k^2) \quad \dots \text{.} \quad (ii)$$

Now total voltage

$$V = v_1 + v_2 + v_3$$

$$= v_1 + v_1 (1+k) + v_1 (1+3k+k^2)$$

$$= v_1 (1+1+k+1+3k+k^2)$$

$$= v_1 (3+4k+k^2)$$

$$V = V_1(1+k)(3+k) \quad \text{--- (II)}$$

From (I), (II) and (III) we get -

$$\frac{V_1}{1} = \frac{V_2}{1+k} = \frac{V_3}{1+3k+k^2} = \frac{V}{(1+k)(3+k)}$$

$\therefore$  voltage across top unit  $V_1 = \frac{V}{(1+k)(3+k)}$

$$\text{n} \quad \text{n} \quad \text{2nd} \quad \text{n} \quad V_2 = V_1(1+k)$$

$$\text{n} \quad \text{n} \quad \text{3rd} \quad \text{n} \quad V_3 = V_1(1+3k+k^2)$$

voltage across string  
Phase string efficiency = nx voltage across disc nearest to conductor

$$\text{Efficiency} = \frac{\sqrt{V_1}}{3 \times V_3} \times 100$$

II. Critical disruptive voltage; it is the minimum

phase-neutral voltage at which corona occurs

$$g = \frac{V}{\pi \log_e \frac{d}{r}} \text{ V/cm}$$

In order that corona is formed, the value of  $g$  must be made equal to the breakdown

strength of air. The breakdown strength of air at 76 cm pressure and temp.

at  $25^{\circ}\text{C}$  is  $30 \text{ kV/cm}$  (max) or  $21.2 \text{ kV/cm}$  (rms) and is denoted by  $\beta_0$ .

$$\beta_0 = \frac{V_c}{\pi \log_e \frac{d}{R}}$$

$$5.5 V_c = \beta_0 \pi \log_e \frac{d}{R}$$

$$\text{or } V_c = m_0 \beta_0 \pi \log_e \frac{d}{R} \text{ kV/phase}$$

$m_0 = 1$  for polished conductors

$= 0.98$  to  $0.92$  for dirty conductors

$= 0.87$  to  $0.8$  for stranded conductors

Visual critical voltage: It is the minimum phase-neutral voltage at which corona glow appears all along the line conductors.

Power loss due to corona: When disruptive

voltage is exceeded, the power loss due to corona is given by:

$$P = 242.2 \left( \frac{f+25}{8} \right) \sqrt{\frac{\pi}{d}} (V - V_c) \times 10^5 \text{ kW/km/phase}$$

$f$  = freq. in Hz

$V$  = phase-neutral voltage (n.m.s)

$V_c$  = disruptive voltage (r.m.s) per phase.  
 $r$  = radius  
 $D$  = space.

### Advantages of corona

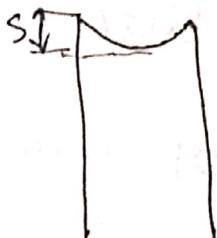
- ① Due to corona formation, the air surrounding the conductor becomes conducting and the virtual diameter of the conductor is increased. The increased diameter reduces the electrostatic stresses between conductors.
- ② Corona reduces the effects of transients produced by surges.

### Disadvantages

- ① Corona is accompanied by loss of energy. This affects the transmission efficiency of the line.
- ② Ozone is produced by corona and cause corrosion of the conductor due to chemical action.

(iii) The current drawn by the line due to corona is non-sinusoidal and hence non-sinusoidal voltage drop occurs in the line. This may cause inductive interference with neighbouring communication line.

I Sag in Overhead line: The difference in level between points of supports and the lowest point on the conductor is called sag.



II Sag And Tension: The conductor sag

should be kept to a minimum in order to reduce the conductor material required and to avoid extra pole height for sufficient clearance above ground level.

It is also desirable that tension in the conductor should be low to avoid the mechanical failure of conductor and to permit the use of less strong supports. However low conductor tension and minimum sag are not possible. It is because low sag means a tight wire and high tension, whereas a low tension means a loose wire and increased sag. Sag and tension are opposite to each other.

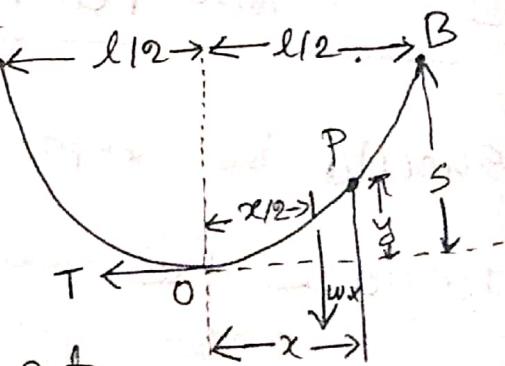
### Calculation of sag:

#### (1) When supports at equal levels:

Consider a conductor between two equivalent supports A and B with O as the lowest point.

It can be proved that

lowest point will be at the mid-span.



Let,  $\lambda$  = Length of span

$w$  = Weight per unit of conductor.

$T$  = Tension in the conductor.

Consider a point  $P$  on the conductor. Taking the lowest point  $O$  as the origin, let the co-ordinates of point  $P$  be  $x$  and  $y$ .

Assuming that the curvature is so small that curved length is equal to its horizontal projection ( $OP=x$ ), the two forces acting on the portion  $OP$  of the cond<sup>n</sup> are:

(a) The weight  $wx$  of conductor acting at a distance  $x/2$  from  $O$ .

(b) The tension  $T$  acting at  $O$

Equating the moments of above two forces

forces -

$$Ty = wx \times \frac{x}{2}$$

$$y = \frac{wx^2}{2T}$$

The maximum sag is represented by the value of  $y$  at either of supports A and B

At support A,  $x = l/2$  and  $y = S$

$$\text{Sag. } S = \frac{w(l/2)^2}{2T} = \frac{wl^2}{8T}$$

# When supports are at unequal levels: In hilly areas, conductors suspended between supports at unequal levels.

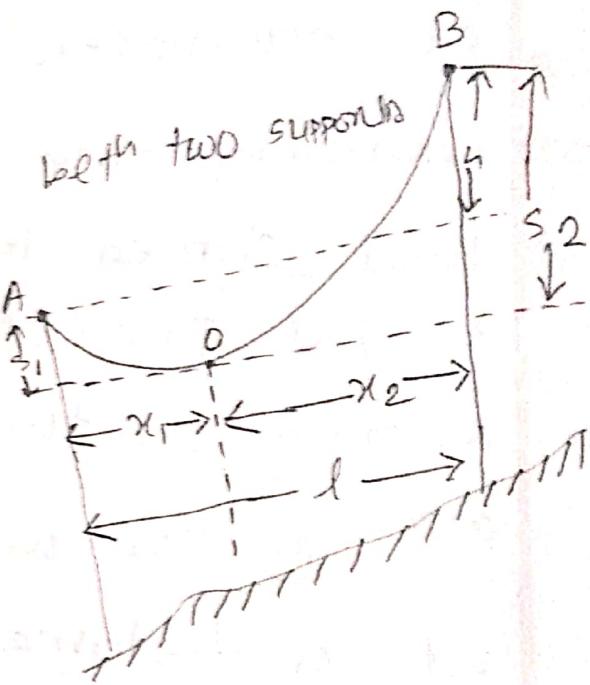
Let,  $l$  = span length

$h$  = Difference in levels b/w two supports

$x_1$  = Distance of support A at lower level

$x_2$  = Distance of support B at higher level.

$T$  = tension



If  $w$  is the weight per unit length of the conductor, then

$$wx_1^2 \quad \text{--- (I)}$$

$$\text{Sag } S_1 = \frac{wx_1^2}{2T}$$

$$wx_2^2 \quad \text{--- (II)}$$

$$\text{and, Sag } S_2 = \frac{wx_2^2}{2T} \quad \text{--- (III)}$$

$$\text{Also } x_1 + x_2 = l$$

(iii) - (i)

$$S_2 - S_1 = \frac{wx_2^L}{2T} - \frac{wx_1^L}{2T}$$

$$\Rightarrow h = \frac{w}{2T} [x_2^L - x_1^L]$$

$$= \frac{w}{2T} (x_2 + x_1)(x_2 - x_1)$$

$$= \frac{wl}{2T} (x_2 - x_1)$$

$$x_2 - x_1 = \frac{2Th}{wl} \quad \text{--- (iv)}$$

$$\therefore x_1 = \frac{l}{2} - \frac{Th}{wl}$$

$$x_2 = \frac{l}{2} + \frac{Th}{wl}$$

# Effect of wind and ice loading: In actual practice

a conductor may have ice coating and simultaneously subjected to wind pressure.

The weight of ice acts vertically downwards.

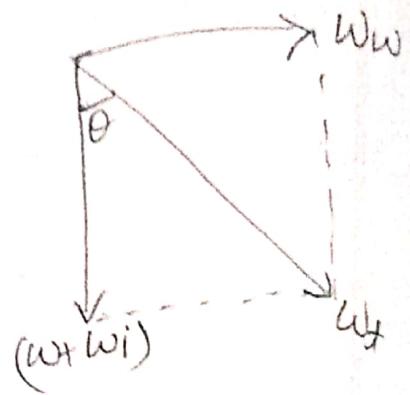
i.e. ~~at~~ in the same direction as the weight of conductor. The force due to the wind

is assumed to act horizontally i.e. at right angle to the projected surface of the

Conductors. Hence the total force on the conductors is the vector sum of horizontal and vertical forces.

Total weight of conductor per unit length is -

$$w_t = \sqrt{(w+w_i)^2 + (W_w)^2}$$



w = weight of conductor per unit length

= conductor material density  $\times$  volume per unit length

w\_i = weight of ice per unit length  
= density of ice  $\times$  volume of ice per unit length.

w\_w = wind force per unit length.

= wind pressure per unit area  $\times$  projected area per unit length.

When the conductor has wind and ice loading also, following points may be noted -

- ① Conductor sets itself in a plane at an angle  $\theta$  to the vertical where

$$\tan \theta = \frac{w_w}{w + w_i}$$

$$\theta = \tan^{-1} \frac{w_w}{w + w_i}$$

- (ii) The sag in the conductor is -

$$s = \frac{w l^2}{2 T}$$

Hence  $s$  represents the slant sag in a direction making an angle  $\theta$  to the vertical. If no specific mention is made in the problem, then slant sag is calculated by using the above formula.

- (iii) The vertical sag =  $s \cos \theta$ .

## Electrical design of overhead line

### II Inductance of composite-conductor lines<sup>18</sup>

Conductors  $X$  is composed of  $n$  identical, parallel filaments, each of which carries the current  $I/n$ . conductor  $Y$ , which is the return ckt for the current in conductor  $X$ , is composed of  $m$  identical, parallel filaments, each of which carries the current  $-I/m$ .

Now for filament  $a$  of conductor  $X$ , we

obtain flux linkage of filament  $a$

$$\Psi_a = 2 \times 10^{-7} \frac{I}{n} \left( \ln \frac{1}{r_{a1}} + \ln \frac{1}{D_{ab}} + \ln \frac{1}{D_{ae}} + \dots + \ln \frac{1}{D_{an}} \right)$$

$$= 2 \times 10^{-7} \frac{I}{m} \left( \ln \frac{1}{D_{a1}} + \ln \frac{1}{D_{ab}} + \ln \frac{1}{D_{ae}} + \dots + \ln \frac{1}{D_{am}} \right)$$

$$\Psi_a = 2 \times 10^{-7} I \ln \frac{\sqrt[m]{D_{a1} D_{ab} D_{ae} \dots D_{am}}}{\sqrt[n]{r_{a1} D_{ab} D_{ae} \dots D_{an}}}$$

$$\therefore L_a = \frac{\mu_0}{l/n} = 2n \times 10^7 \ln \frac{\sqrt[n]{D_{aa'} D_{ab} D_{ac} \dots D_{an}}}{\sqrt[n]{D_{ba'} D_{bb} D_{bc} \dots D_{bn}}} \text{ H/m}$$

Similarly for filament b

$$L_b = 2n \times 10^7 \ln \frac{\sqrt[n]{D_{bb'} D_{bb} D_{bc} \dots D_{bn}}}{\sqrt[n]{D_{ba} D_{ba} D_{bc} \dots D_{bn}}} \text{ H/m}$$

The average inductance of the filaments of conductor x is

$$L_{av} = \frac{L_a + L_b + L_c + \dots + L_n}{n}$$

conductor x is composed of n filaments electrically in parallel. If all the filaments had the same inductance, the inductance of the conductor would be  $\frac{1}{n} \times L_{av}$ . Hence all filaments have different inductances but the inductance of all of them in parallel is  $1/n$  times the average inductance. Thus the inductance of conductor x is

$$L_x = \frac{L_{av}}{n} = \frac{L_a + L_b + L_c + \dots + L_n}{n^2}$$

$$L_x = 2 \times 10^7 \ln \frac{\sqrt[n]{(D_{aa'} D_{ab} D_{ac} \dots D_{an})(D_{ba'} D_{bb} D_{bc} \dots D_{bn})(D_{ca'} D_{cb} D_{cc} \dots D_{cn})}}{\sqrt[n]{(D_{aa} D_{ab} D_{ac} \dots D_{an})(D_{ba} D_{bb} D_{bc} \dots D_{bn})(D_{ca} D_{cb} D_{cc} \dots D_{cn})}}$$

The numerator is  $m$ th root of  $\sigma a^n (D_m m)$  terms, which are the products of the distance from all  $n$  filaments of conductor  $x$  to the  $m$ th root of the product of  $n$  distances between conductor  $x$  and mean distance between conductors  $y$ . It is abbreviated as  $D_m$  or GMD and is also called the mutual GMD between two conductors.

The inductance of conductor  $y$  is determined in a similar manner and the inductance of the line is  $L = L_x + L_y$

### If Inductance of 34 lines with Equilateral Spacing

There are conductors of an equilateral triangle. If there is no neutral wire, etc. if we assume balanced

Three phase phasor currents  $I_a + I_b + I_c = 0$ .

Flux linkage of conductor a:

$$\Psi_a = 2 \times 10^{-7} \left( I_a \ln \frac{1}{D_s} + I_b \ln \frac{1}{D} + I_c \ln \frac{1}{D} \right) \text{ Wb/m}$$

$$I_a = -(I_b + I_c)$$

$$\Psi_a = 2 \times 10^{-7} \left( I_a \ln \frac{1}{D_s} - I_a \ln \frac{1}{D} \right)$$

$$= 2 \times 10^{-7} I_a \ln \frac{D}{D_s} \text{ Wb/m.}$$

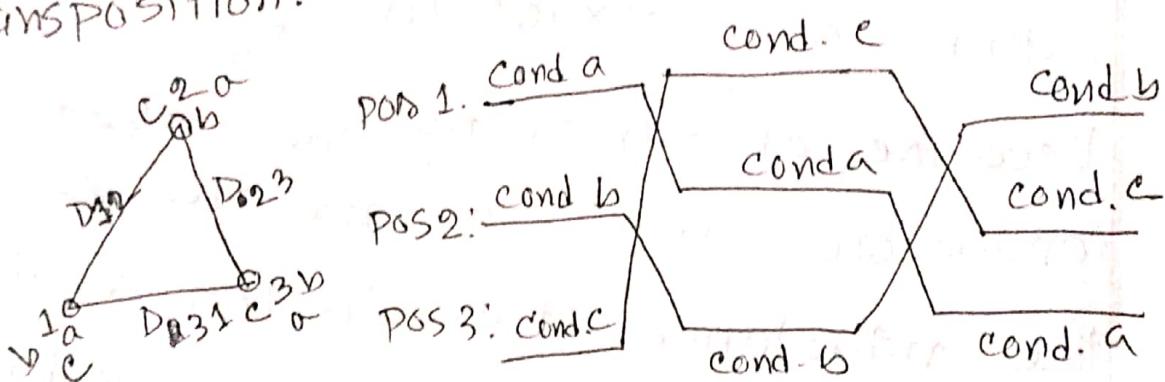
$$\text{and } L_a = 2 \times 10^{-7} \ln \frac{D}{D_s} \text{ H/m}$$

Because of symmetry, the inductance of conductors b and c are the same as the inductance of conductor a. Since each phase consists of only one conductor.

Inductance of 3-phase lines with unsymmetrical spacing:

When the conductors of a three phase line are not spaced equilaterally, the problem of finding the inductance becomes more difficult. Then the flux linkages and inductance of each phase are not the same.

A different inductance in each phase results in an unbalanced cut. Balance of each three phases can be restored by exchanging the position of the conductors at regular intervals along the line so that each cond. occupies the original position of every other conductor over an equal distance. Such an exchange of conductor positions is called transposition.



The phase conductors are designated a, b and c, and the positions occupied are numbered 1, 2 and 3. Transposition results in each conductor having the same average inductance over the whole cycle.

To find the average inductance of one cond. of a transposed line, the flux linkages of

a cond. are found for each position it occupies in the transposition cycle, and the average flux linkages are determined.

Let cond. a @ in position 1, bin 2 and e in 3

$$\Psi_{a1} = 2 \times 10^7 \left[ I_a \ln \frac{1}{D_S} + I_b \ln \frac{1}{D_2} + I_e \ln \frac{1}{D_{31}} \right] \text{ wb/lm}$$

When a in position 2, bin 3, e in 1.

$$\Psi_{a2} = 2 \times 10^7 \left[ I_a \ln \frac{1}{D_S} + I_b \ln \frac{1}{D_{23}} + I_e \ln \frac{1}{D_{12}} \right]$$

when a in position 3, bin 1, e in 2

$$\Psi_{a3} = 2 \times 10^7 \left[ I_a \ln \frac{1}{D_S} + I_b \ln \frac{1}{D_{31}} + I_e \ln \frac{1}{D_{23}} \right]$$

The average value of flux linkages of a is:

$$\begin{aligned} \Psi_a &= \frac{\Psi_{a1} + \Psi_{a2} + \Psi_{a3}}{3} \\ &= \frac{2 \times 10^7}{3} \left( 3 I_a \ln \frac{1}{D_S} + I_b \ln \frac{1}{D_{12} D_{23} D_{31}} + I_e \ln \frac{1}{D_{12} D_{23} D_{31}} \right) \end{aligned}$$

with  $I_a = -(I_b + I_e)$

$$\Psi_a = \frac{2 \times 10^7}{3} \left( 3 I_a \ln \frac{1}{D_S} - I_a \ln \frac{1}{D_{12} D_{23} D_{31}} \right)$$

$$= 2 \times 10^{-7} I_a \ln \frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{D_S}$$

and the average inductance per phase is

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_S} \text{ H/m}$$

$$D_{eq} = \sqrt[3]{D_{12} D_{23} D_{31}}$$

# Gauss's law of Electric field: The law states that the total electric charge within a closed surface equals the total electric flux emerging from the surface.

$$\vec{E} = \frac{\vec{D}}{\epsilon_0 k}$$

In other words the total charge within a closed surface equals the integral over the surface of the normal component of electric flux density.

$$\vec{D} = \epsilon_0 \vec{E}$$

All points equidistant from such a conductor are points of equipotential and have the

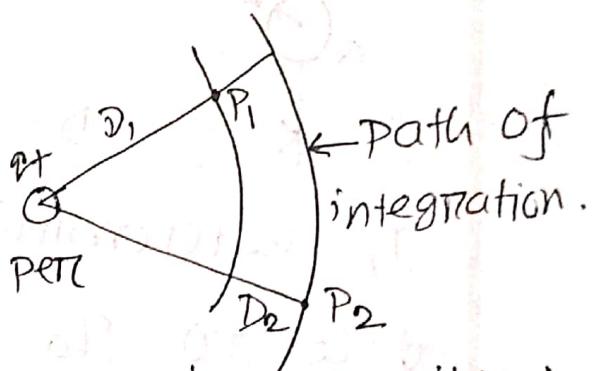
electric flux density. So, the electric flux density on the surface is equal to the flux leaving the conductor per meter of length divided by the area of the surface in an axial length of 1m. So

$$D_f = \frac{q}{2\pi x} \text{ C/m}^2$$

$$\therefore E = \frac{D_f}{K} = \frac{q}{2\pi x K} \text{ V/m}$$

Potential difference b/w

two points due to a charge work in jouls per



coulomb's. The electric field intensity is a measure of the force on a charge in the field.

$$V_2 = \int_{D_1}^{D_2} E dx = \int_{D_1}^{D_2} \frac{q}{2\pi x K} dx$$

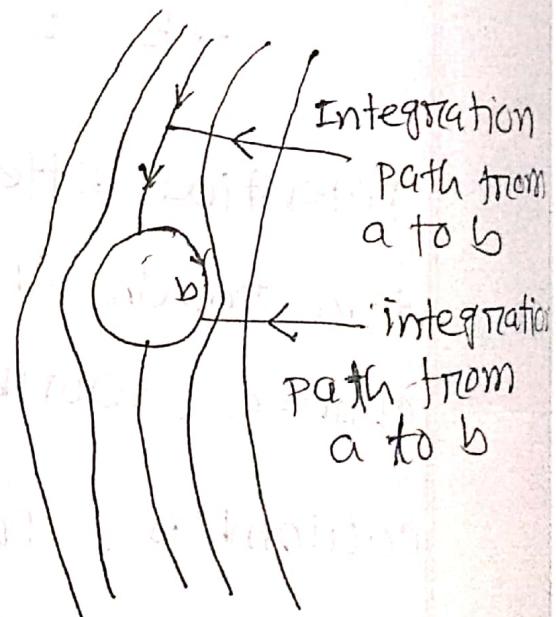
$$= \frac{q}{2\pi K} \int_{D_1}^{D_2} \frac{1}{x} dx$$

$$= \frac{q}{2\pi K} [\ln x]_{D_1}^{D_2}$$

$$= \frac{q}{2\pi k} \ln \frac{D_2}{D_1}$$

# Capacitance of a two wire line: Capacitance betw the two conductors of a two-wire line was defined as the charge on the conductor per unit of potential difference betw them.

$C_{AB} = \frac{q}{V} \text{ F/m}$



In determining  $V_{ab}$  due to  $q_a$  we follow the path through the undistorted region.

$$V_{ab} = \frac{q_a}{2\pi k} \ln \frac{D}{r_a} + \frac{q_b}{2\pi k} \ln \frac{\pi b}{D} \text{ V}$$

$$\therefore q_a = -q_b$$

$$\therefore V_{ab} = \frac{q_a}{2\pi k} \left( \ln \frac{D}{r_a} - \ln \frac{\pi b}{D} \right) \text{ V}$$

$$= \frac{q_a}{2\pi k} \ln \frac{D}{r_a \pi b} \text{ V}$$

$\therefore$  Capacitance b/w conductors is

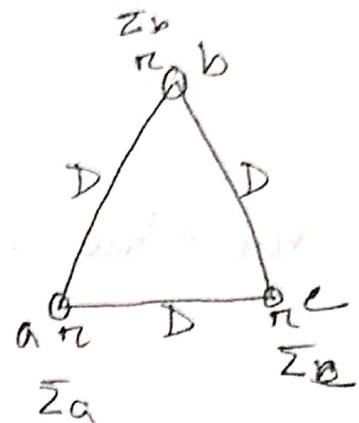
$$C_{ab} = \frac{q_a}{V_{ab}} = \frac{\frac{2\pi K}{2\pi K} \ln(D/\text{mean})}{\ln(D/\pi)} = \frac{\pi K}{\ln(D/\pi)} \quad [\text{mean} = \frac{D}{2}]$$

If Capacitance of a 3-phase line with equilateral spacing:

Hence,

$$V_{ab} = \frac{q_a}{2\pi K} \ln \frac{D}{\pi} + \frac{q_b}{2\pi K} \ln \frac{\pi}{D} +$$

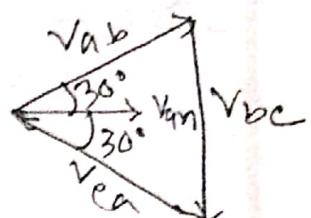
$$\frac{q_c}{2\pi K} \ln \frac{D}{D}$$



$$\text{and } V_{ac} = \frac{q_a}{2\pi K} \ln \frac{D}{\pi} + \frac{q_c}{2\pi K} \ln \frac{\pi}{D} + \frac{q_b}{2\pi K} \ln \frac{D}{D}$$

$$V_{ab} + V_{ac} = 2 \frac{q_a}{2\pi K} \ln \frac{D}{\pi} + (q_b + q_c) \frac{1}{2\pi K} \ln \frac{\pi}{D}$$

Here  $q_a + q_b + q_c = 0$  as neutral is far from cond.



$$\therefore V_{ab} + V_{ac} = 2 \frac{q_a}{2\pi K} \ln \frac{D}{\pi} - \frac{q_a}{2\pi K} \ln \frac{\pi}{D}$$

$$= 2 \frac{q_a}{2\pi K} \ln \frac{D}{\pi} + \frac{q_a}{2\pi K} \ln \frac{D}{\pi}$$

$$= \frac{3q_a}{2\pi K} \ln \frac{D}{\pi}$$

$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$$

$$= \sqrt{3} V_{an} (1.866 + j.5)$$

$$V_{ac} = -V_{ca} = \sqrt{3} V_{an} \angle -30^\circ$$

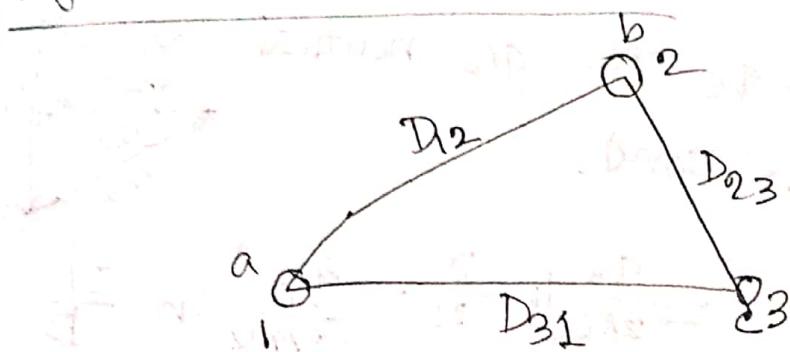
$$= \sqrt{3} V_{an} (0.866 - j.5)$$

$$\therefore V_{ab} + V_{ac} = 3 V_{an} = \frac{3 q_a}{2 \pi K} \ln \frac{D}{\pi}$$

$$\Rightarrow V_{an} = \frac{q_a}{2 \pi K} \ln \frac{D}{\pi}$$

$$\therefore C_{an} = \frac{q_a}{V_{an}} = \frac{2 \pi K}{\ln(D/\pi)}$$

# Capacitance of three phase line with unsymmetrical spacing



with phase a in position 1, b in 2, c in 3

$$V_{ab} = \frac{q_a}{2 \pi K} \ln \frac{D_{12}}{\pi} + \frac{q_b}{2 \pi K} \ln \frac{\pi}{D_{12}} + \frac{q_c}{2 \pi K} \ln \frac{D_{23}}{D_{31}}$$

Now a in position 2, b in 3 and c in 1

$$V_{ab} = -\frac{q_a}{2\pi k} \ln \frac{D_{23}}{\pi} + \frac{q_b}{2\pi k} \ln \frac{\pi}{D_{23}} + \frac{q_c}{2\pi k} \ln \frac{D_{23}}{D_{12}}$$

with a in 3, b in 1 and c in 2

$$V_{ab} = -\frac{q_a}{2\pi k} \ln \frac{D_{31}}{\pi} + \frac{q_b}{2\pi k} \ln \frac{\pi}{D_{31}} + \frac{q_c}{2\pi k} \ln \frac{D_{12}}{D_{23}}$$

$$\therefore 3V_{ab} = -\frac{q_a}{2\pi k} \left( \ln \frac{D_{12} D_{23} D_{31}}{\pi^3} \right) + \frac{q_b}{2\pi k} \left( \ln \frac{\pi^3}{D_{12} D_{23} D_{31}} \right) \\ + \frac{q_c}{2\pi k} \left( \ln \frac{D_{12} D_{23} D_{31}}{D_{12} D_{23} D_{31}} \right)$$

$$V_{ab} = -\frac{q_a}{2\pi k} \left( \ln \frac{D_{ea}}{\pi^3} \right) + \frac{q_b}{2\pi k} \left( \ln \frac{\pi}{D_{ea}} \right)$$

Similarly, the average voltage drop from a to c is

$$V_{ac} = -\frac{1}{2\pi k} \left( q_a \ln \frac{D_{ea}}{\pi} + q_c \ln \frac{\pi}{D_{ea}} \right) V$$

$$\therefore 3V_{an} = V_{ab} + V_{ac}$$

$$= -\frac{2q_a}{2\pi k} \ln \frac{D_{ea}}{\pi} + \frac{q_b}{2\pi k} \ln \frac{\pi}{D_{ea}} + \frac{q_c}{2\pi k} \ln \frac{\pi}{D_{ea}}$$

as since  $q_a + q_b + q_c = 0$  in balanced phase

$$3V_{an} = -\frac{3q_a}{2\pi k} \ln \frac{D_{ea}}{\pi} V$$

$$\therefore C_{an} = \frac{v_a}{v_{an}} = \frac{2\pi k}{\ln(Da/T)} \text{ F/m}$$

∴  $\frac{v_a}{v_{an}} = \frac{2\pi k}{\ln(Da/T)}$

∴  $v_a = v_{an} \cdot \frac{2\pi k}{\ln(Da/T)}$

∴  $v_a = v_{an} \cdot \frac{2\pi k}{\ln(Da/T)} \cdot \frac{\partial P}{\partial T}$

(differentiate w.r.t.  $T$ )

$\therefore \frac{\partial v_a}{\partial T} = v_{an} \cdot \frac{2\pi k}{\ln(Da/T)} \cdot \frac{\partial^2 P}{\partial T^2}$

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(first differentiate  $\frac{\partial P}{\partial T}$  w.r.t.  $T$ )

$\therefore \frac{\partial v_a}{\partial T} = v_{an} \cdot \frac{2\pi k}{\ln(Da/T)} \cdot \frac{\partial^2 P}{\partial T^2}$

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