Problems Related to Angle Modulation

1. A sinusoidal wave of amplitude 10V and frequency 10 kHz is applied to an FM generator that has a sensitivity constant of 40Hz/Volt. Determine the frequency deviation and modulation index (deviation ratio).

Solution:

Given, A_m=10V

f_m=10 kHz

k_f=40Hz/Volt.

We know

$\Delta f = k_f A_m$= 400Hz

$\beta = \Delta f / f_m$

= 400/10k

=0.04

2. Find the transmission bandwidth of single tone modulated FM signal described by $s(t)=10\cos[2\pi 10^8 t+6\sin(2\pi 10^3 t)]$

Solution:

We know FM modulated wave

$s(t)=A_c\cos[2\pi f_c t+\beta\sin(2\pi f_m t)]$

Here,

$f_m=1kHz$

$f_{c=}10^8Hz$

$\beta=6$

From Carson's rule

$BW=2(1+\beta)f_m$

$=2(1+6)1000$

$=14kHz$

3. Consider an FM signal with Δf=10kHz, f_m=10kHz, A=10V, f_c=500kHz. Draw the spectrum of this FM signal.

Solution

Here $\beta=\Delta f/f_m=1$

We know $\Phi(t)=A\sum_{-\infty}^{\infty} Jn(\beta)\cos 2\pi(fc+nfm)t$

So the sidebands of carrier f_c will be $f_c \pm f_m$, $f_c \pm 2f_m$, $f_c \pm 3f_m$…… $f_c \pm nf_m$ for n=0 (only carrier frequency),1,2……n and the corresponding Bessels function$[J_n(\beta)]$ will be ***J<u>0</u>, J<u>1</u>, J<u>2</u>……*** and so on where β=1.

Also it is given that, f_m=10kHz, A=10V, f_c=500kHz

The frequencies and corresponding amplitude will be the taken from Bessel Table.(Take the value for modulation index β=1)

BESSEL TABLE

Modulation index	Carrier J_0	Sidebands J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}
0.0	1.00	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.06	0.02	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02
8.0	0.17	0.23	−0.11	−0.29	0.10	0.19	0.34	0.32	0.22	0.13	0.06

Tabulated value for Bessel Function for the first kind of the n^{th} order

From Bessel Table we get

J_0=0.77

J_1=0.44

J_2=0.11

J_3=0.02

BESSEL TABLE

Modulation index	Carrier J_0	Sidebands J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}
0.0	1.00	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.06	0.02	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—
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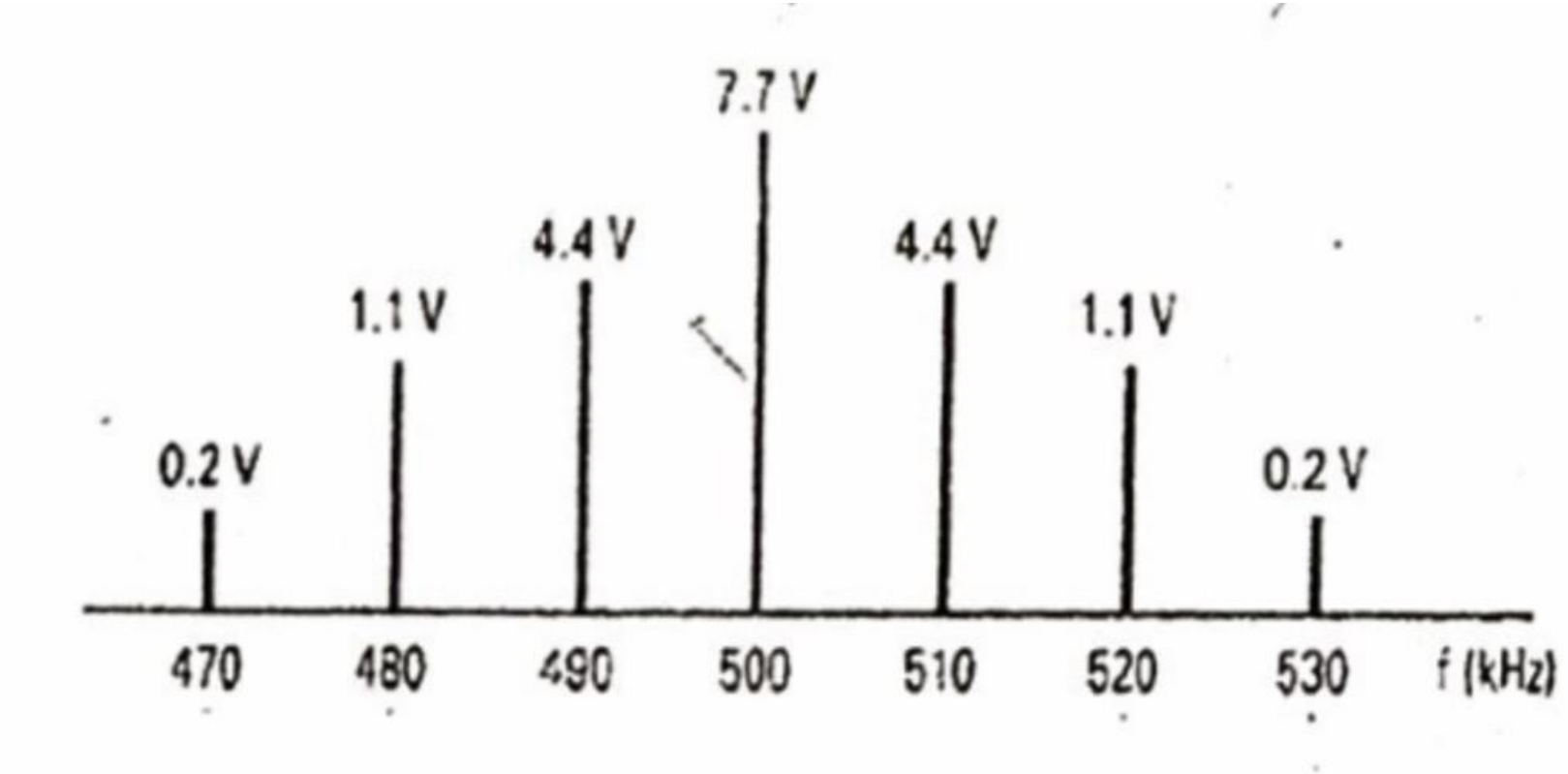
Tabulated value for Bessel Function for the first kind of the n^{th} order

Taking Values from Bessel table

n	Aj_n (β) Volts β=1	fc±nfm kHz
0	10*0.77=7.7	500
1	10*0.44=4.4	490,510
2	10*0.11=1.1	480,520
3	10*0.02=0.2	470,530

Final the spectrum for β=1



Try yourself

An angle modulated signal with carrier frequency $w_c=2\pi 10^5$ is described by the equation $\Phi(t)=10\cos(w_c t+5\sin 3000t+10\sin 2000\pi t)$.

1. Find the power of the modulated signal
2. Find the frequency deviation Δf
3. Find deviation ratio β
4. Find the phase deviation $\Delta\Phi$
5. Estimate the Bandwidth

Ref. B.P. Lathi