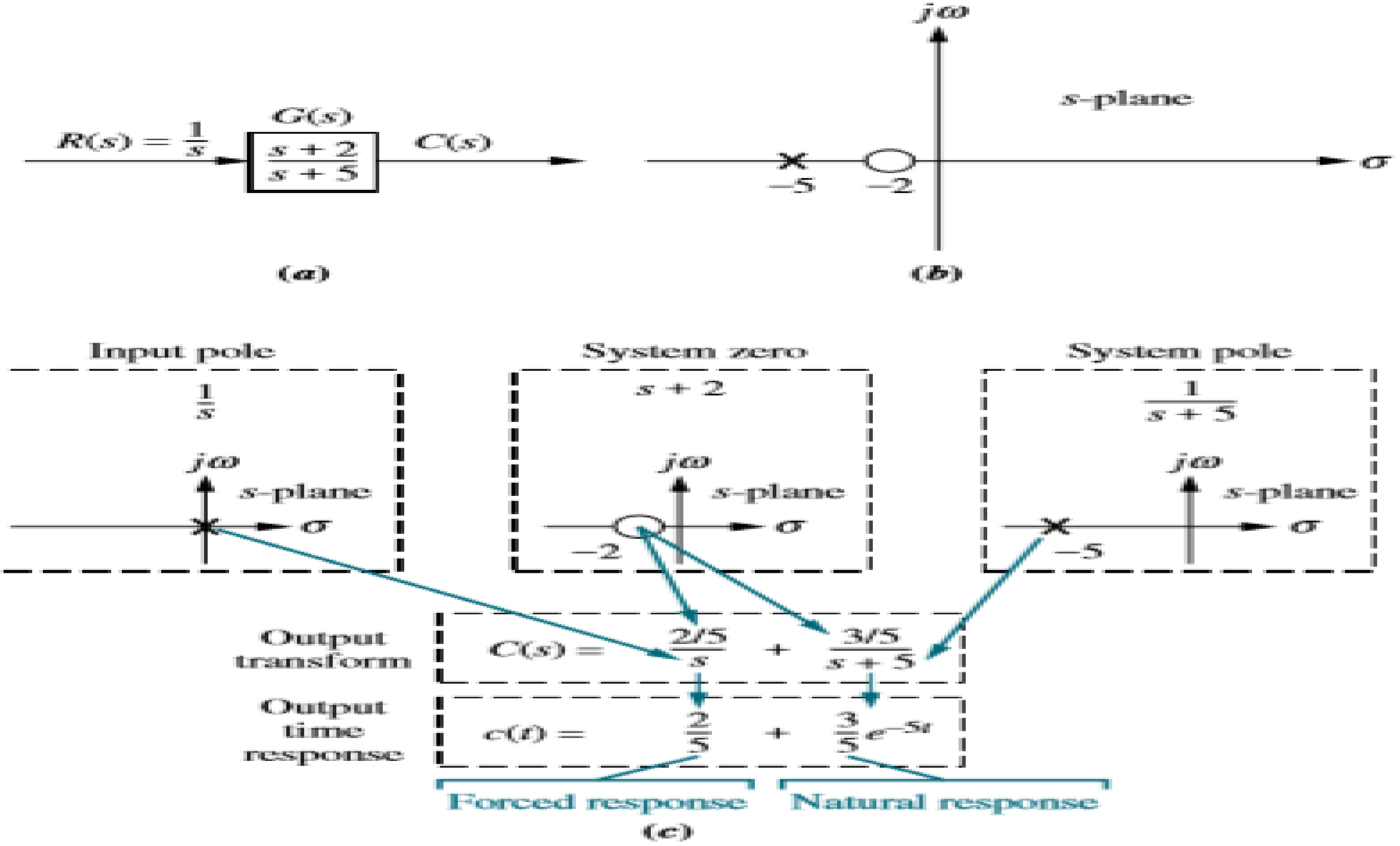
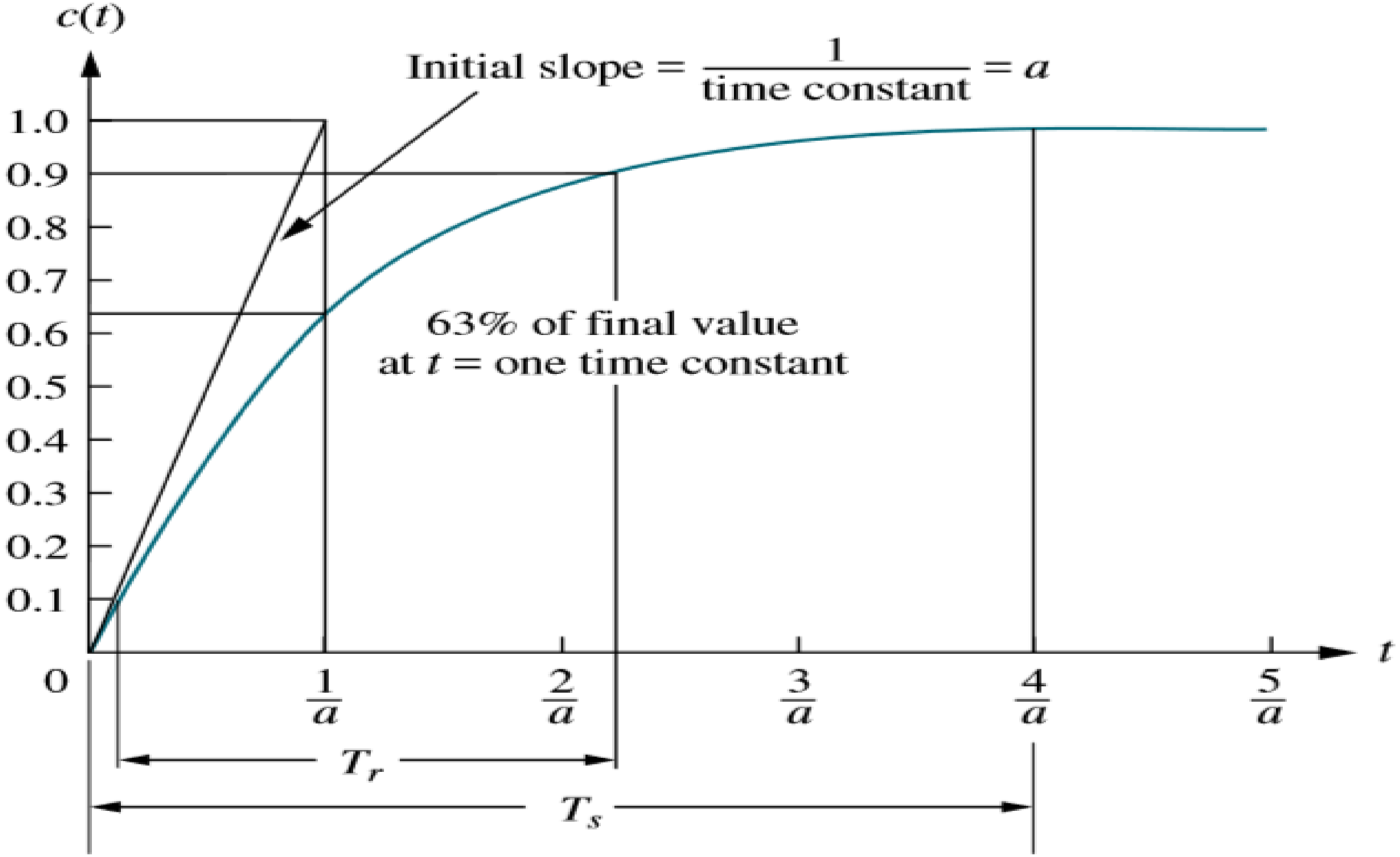
Time Response

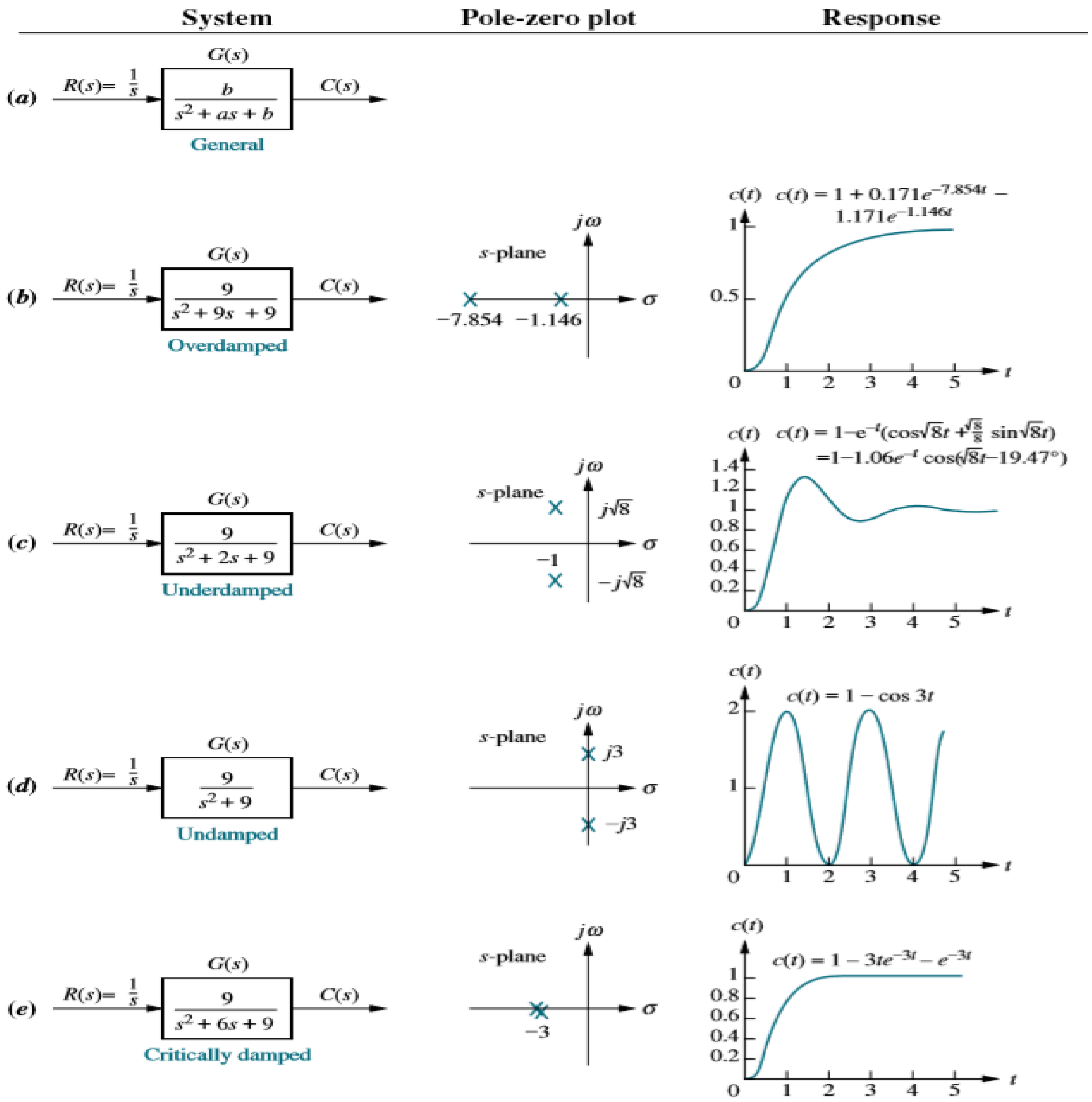
Evolution of System Response



First-order system response to a unit step



Second-order systems, pole plots, and step responses



Second Order Responses

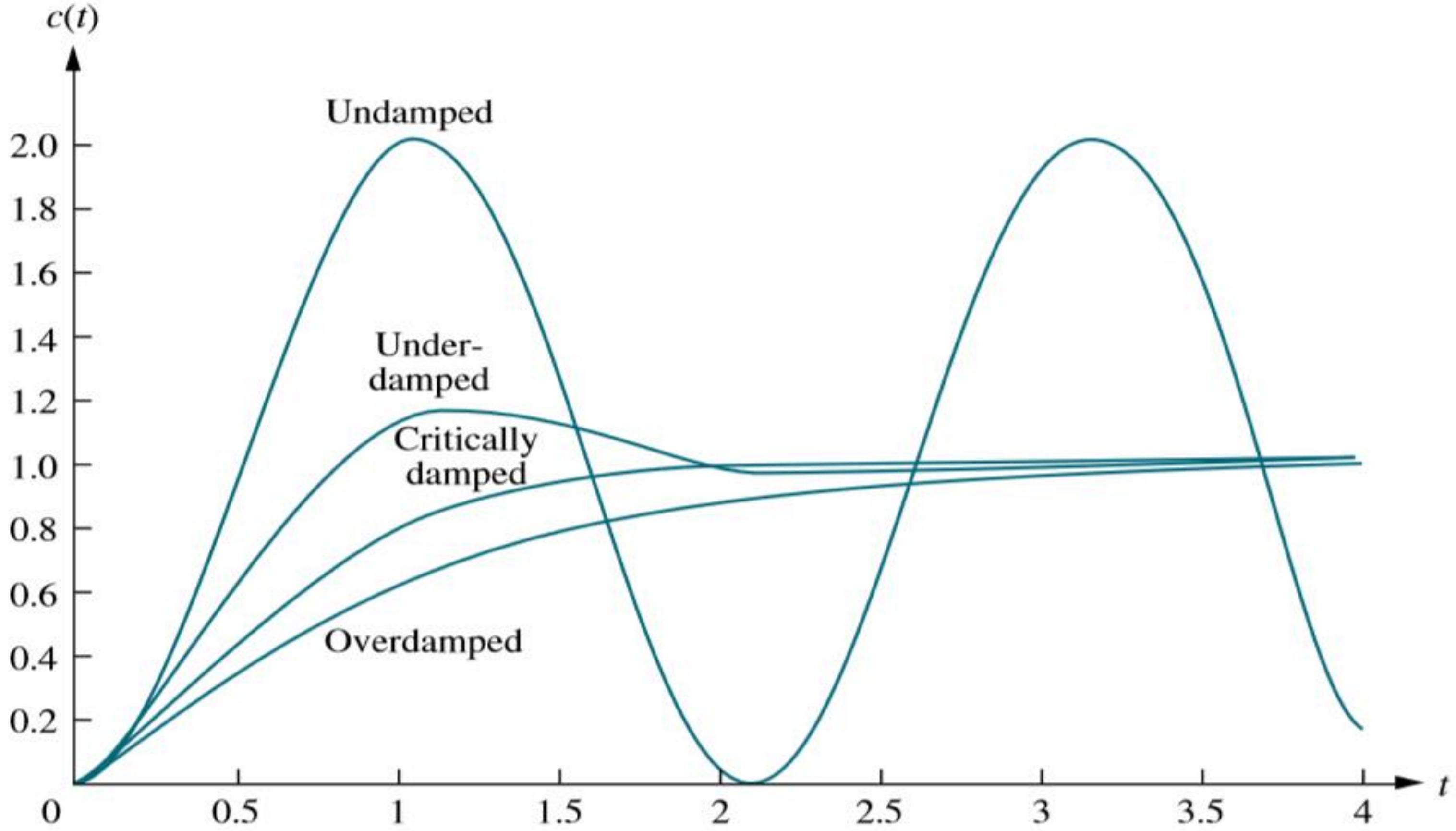
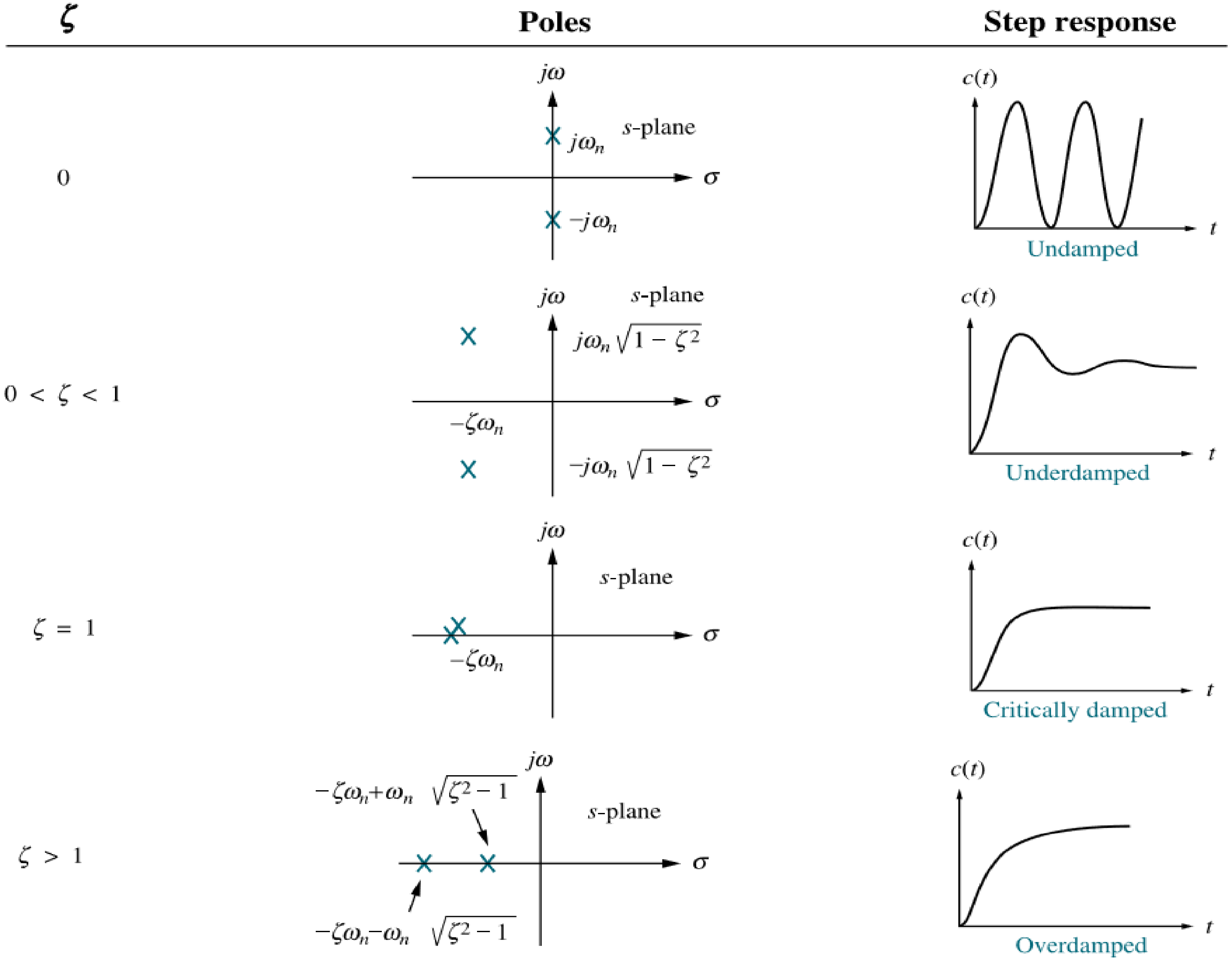
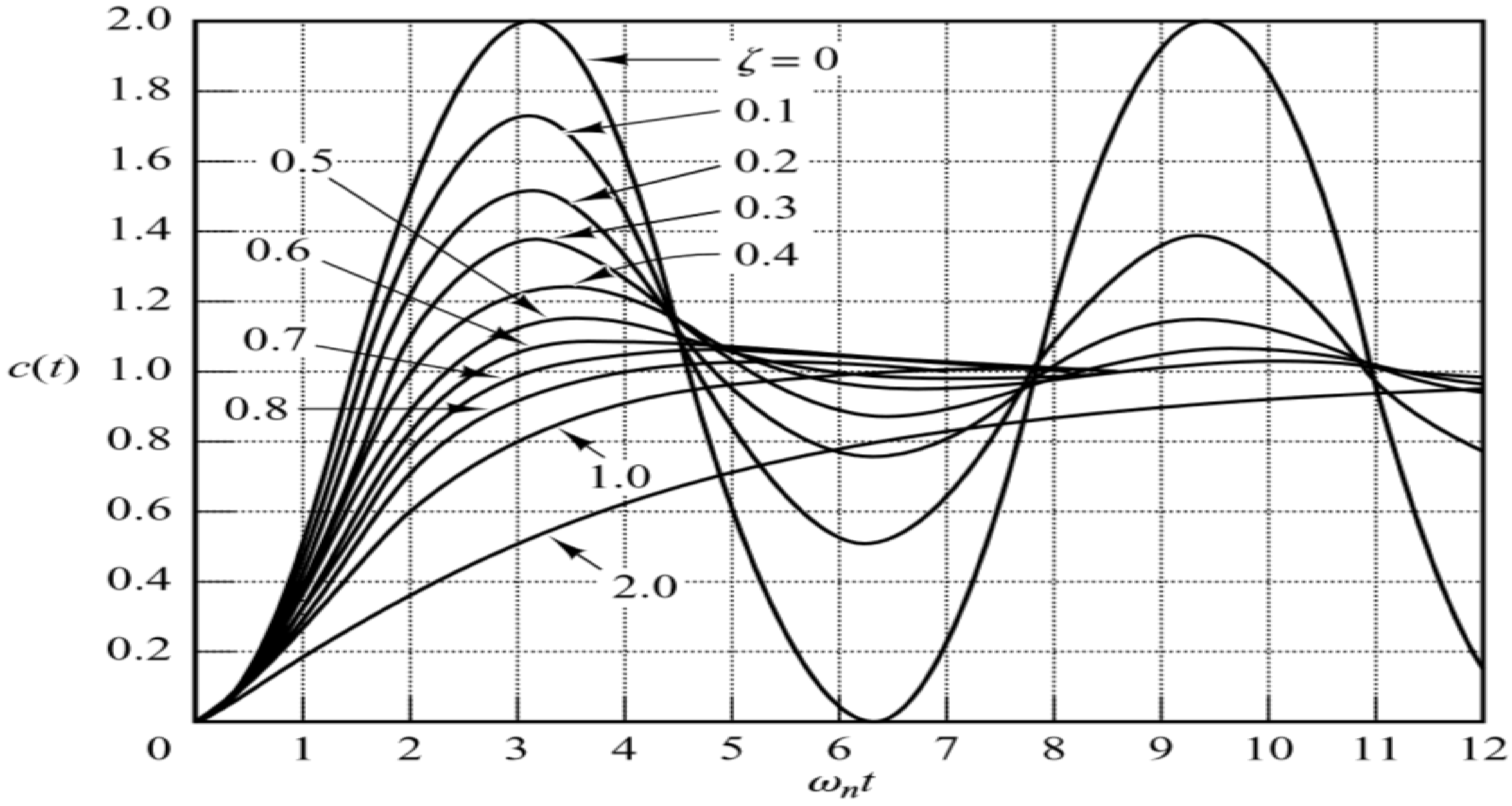


FIGURE 4.10 Step responses for second-order system damping cases

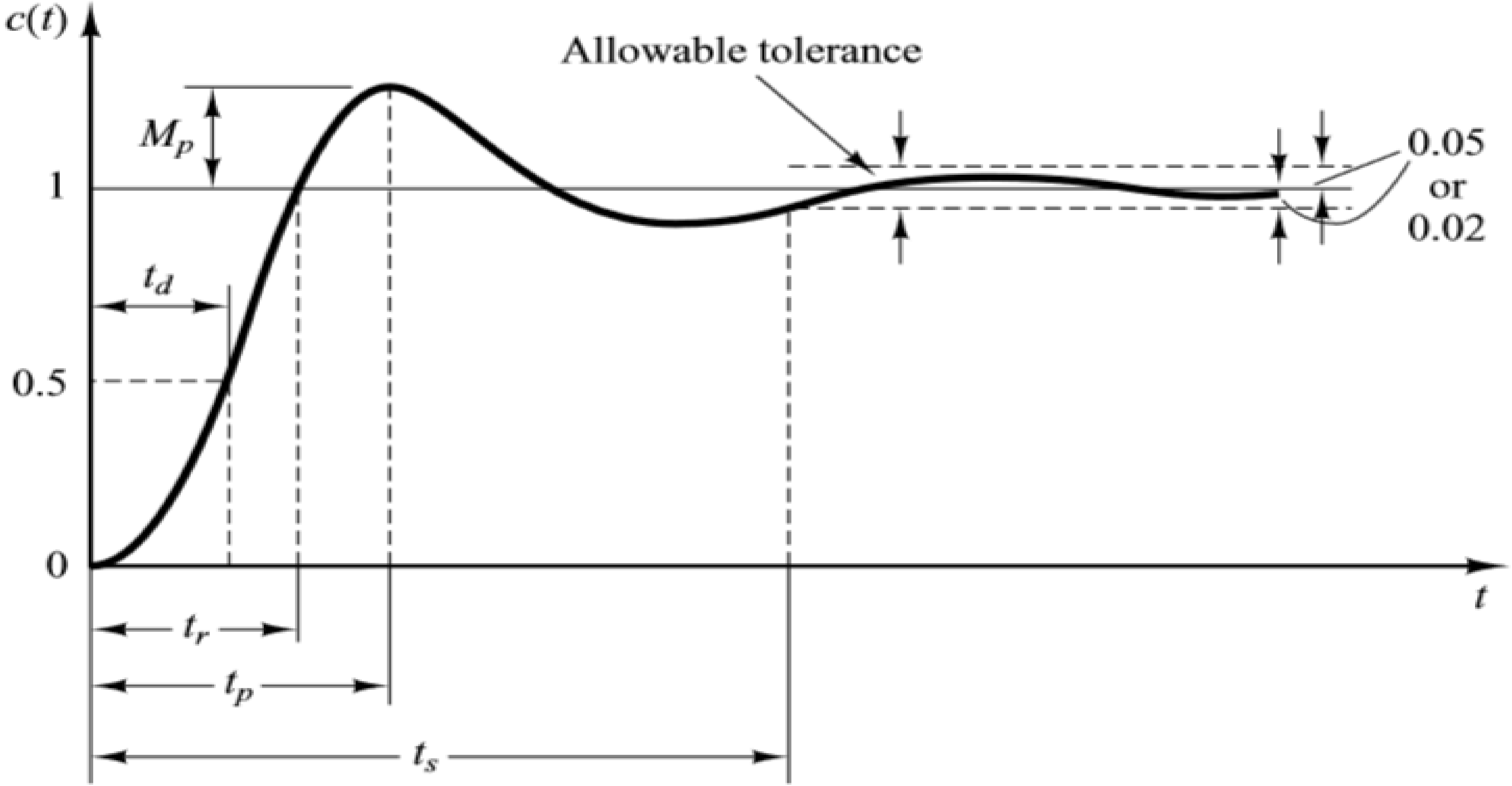
2nd Order response as a function of damping ratio ξ



A family of unit-step response curves c(t) with various values of ξ

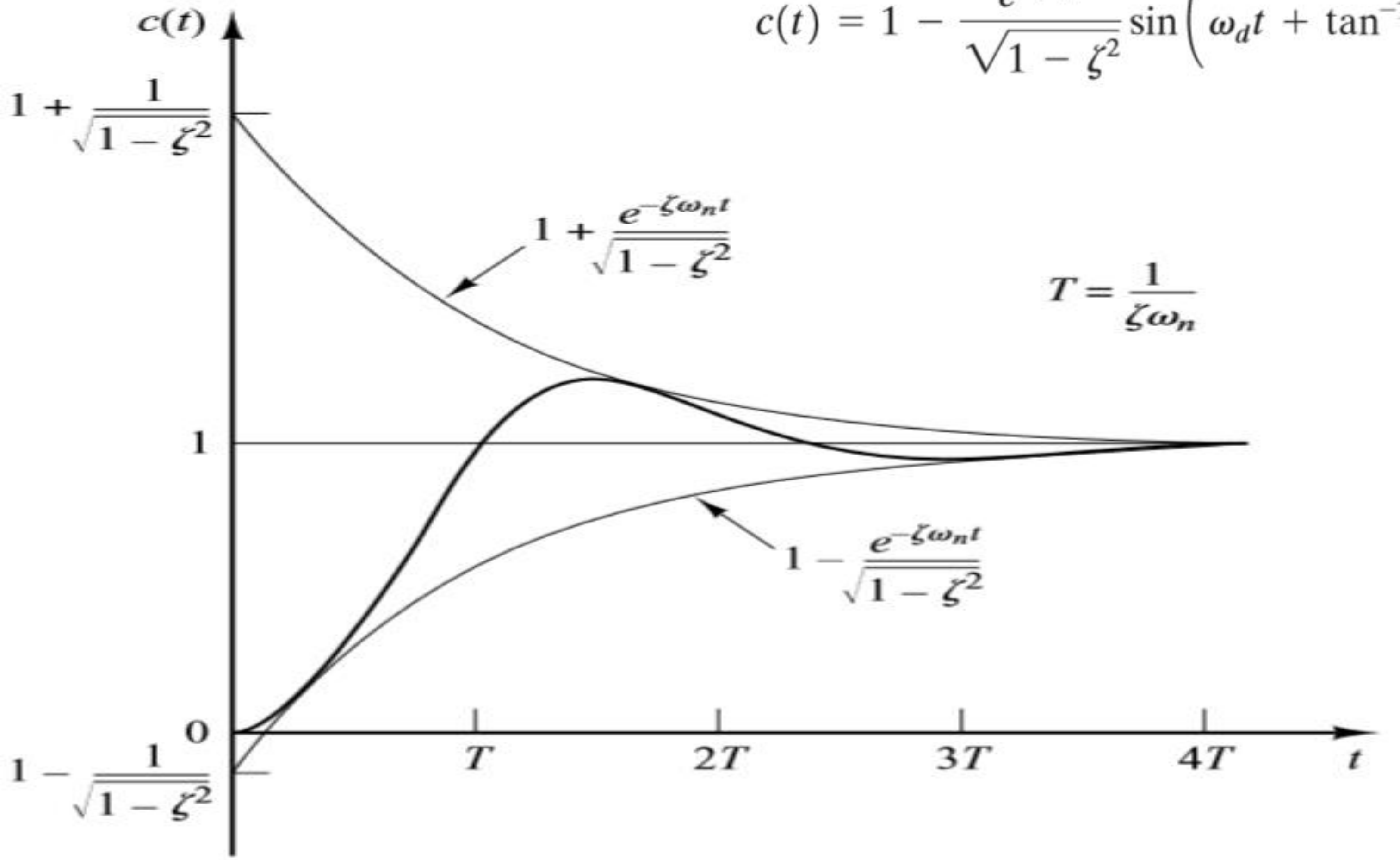


Unit-step response curve showing td, tr, tp, %OS(Mp) and ts.

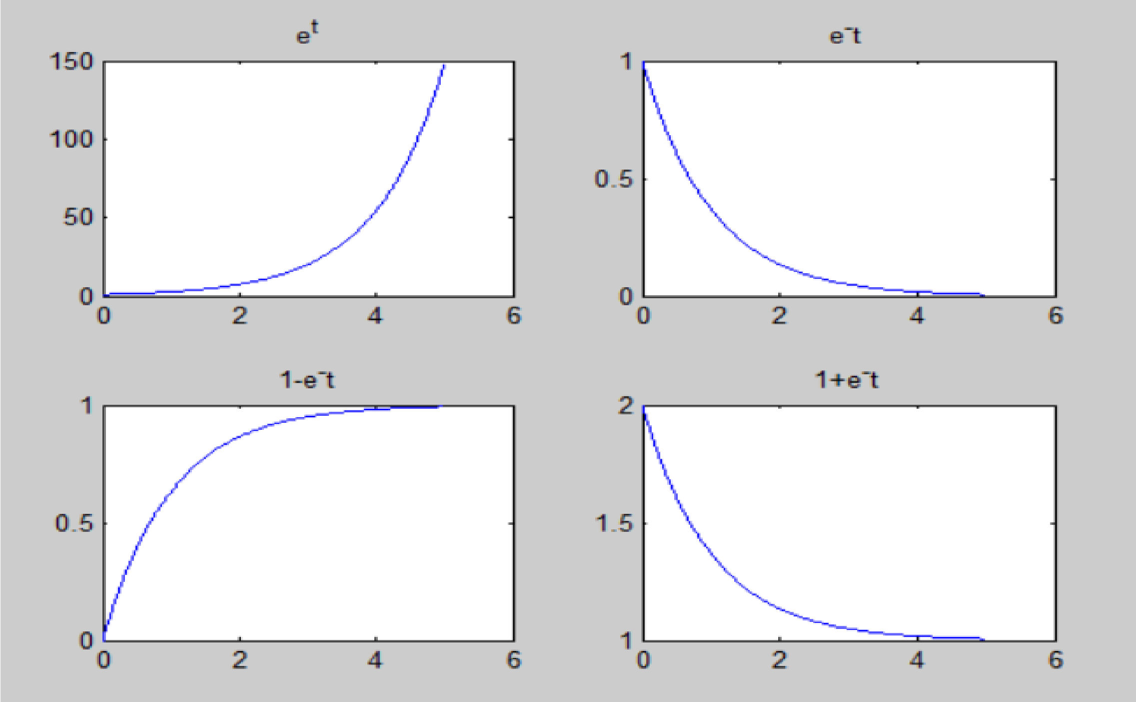


Generation of Underdamped Response Curve

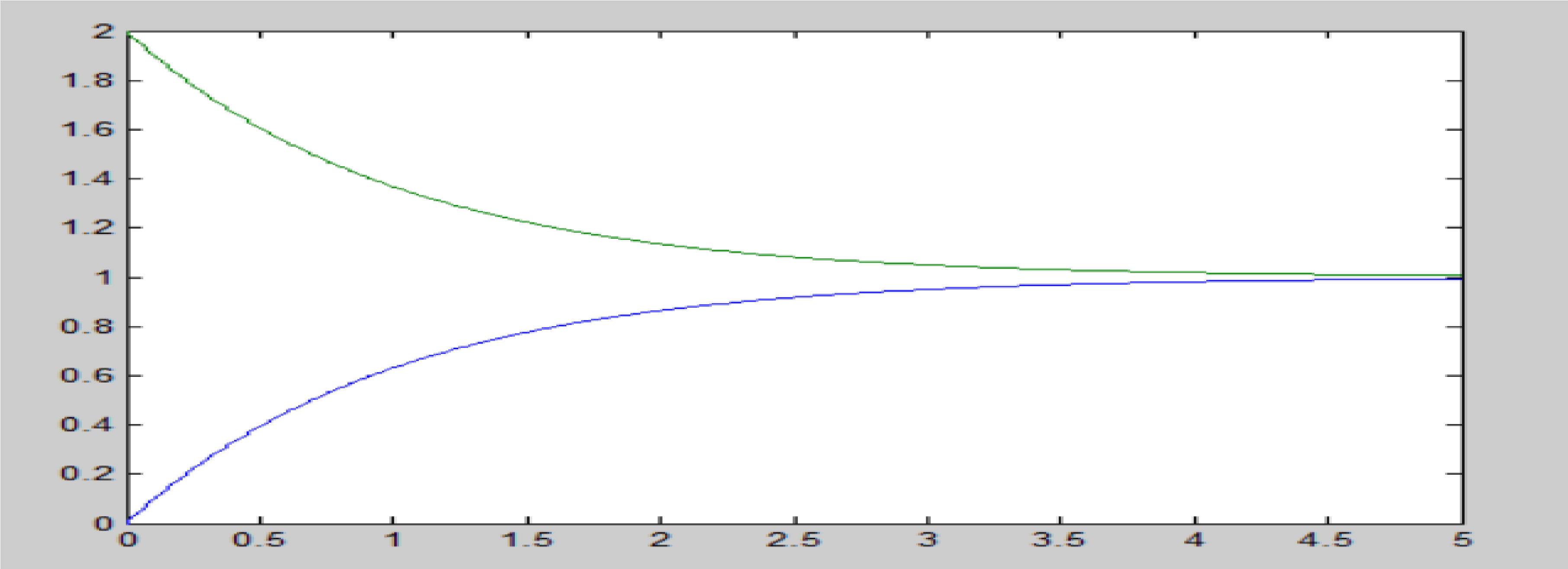
$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right), \qquad \text{for } t \geq 0$$



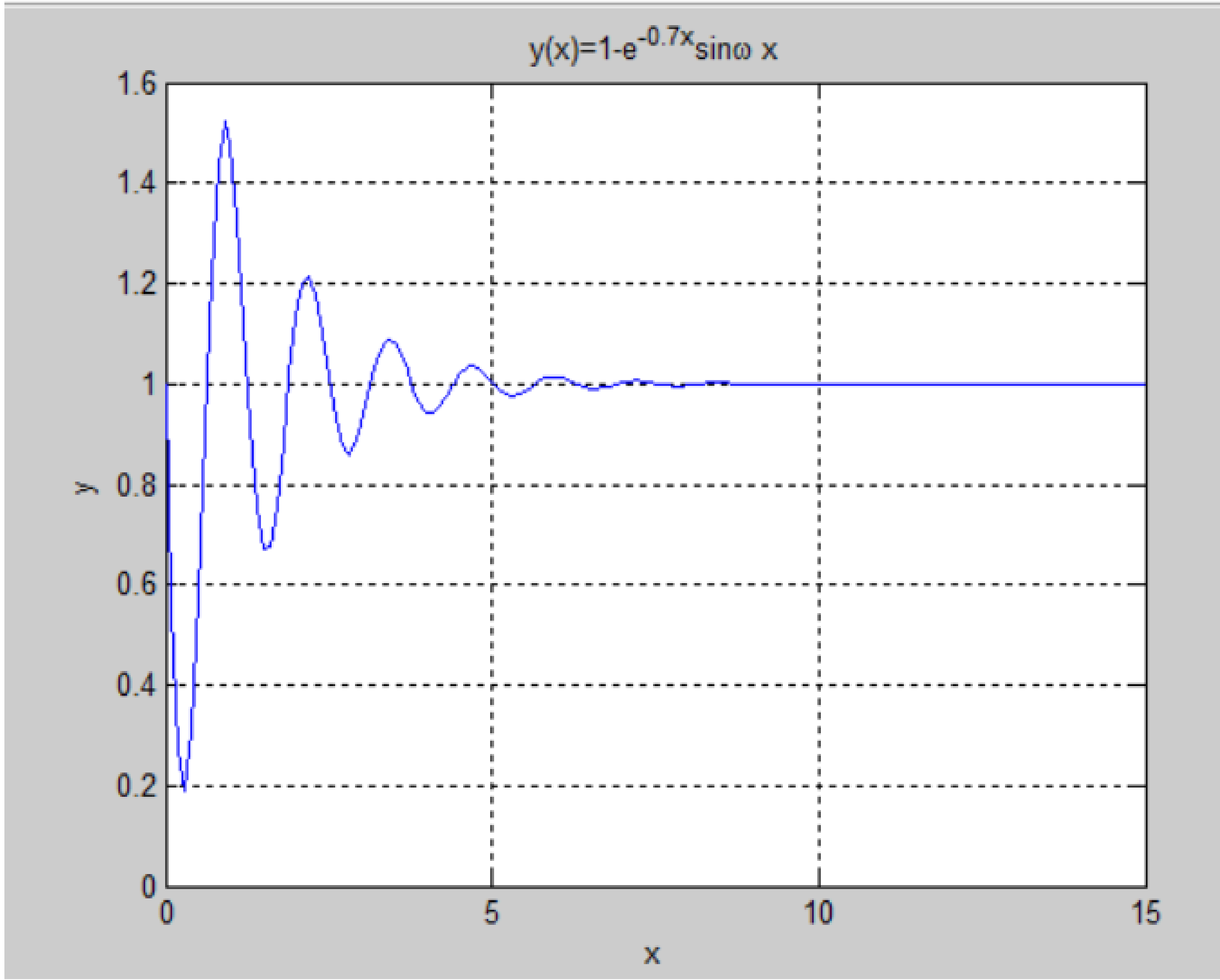
- t=0:0.01:5;
- y1=exp(t);
- y2=exp(-t);
- y3=1-exp(-t);
- y4=1+exp(-t);
- subplot(221)
- plot(t,y1)
- title('e^t')
- subplot(222)
- plot(t,y2)
- title('e^-t')
- subplot(223)
- plot(t,y3)
- title('1-e^-t')
- subplot(224)
- plot(t,y4)
- title('1+e^-t')
- figure
- plot(t,y3,t,y4)



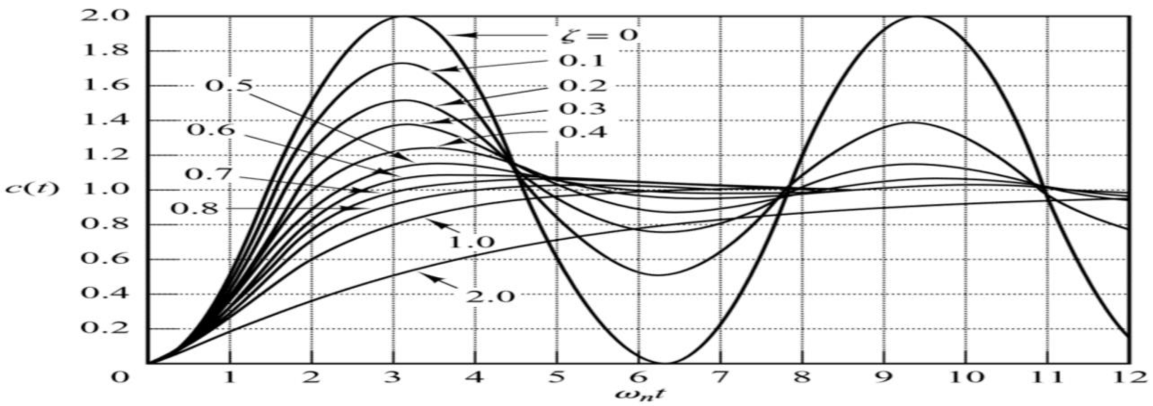
Formation of Exponential Decay Envelope due to real part of the pole



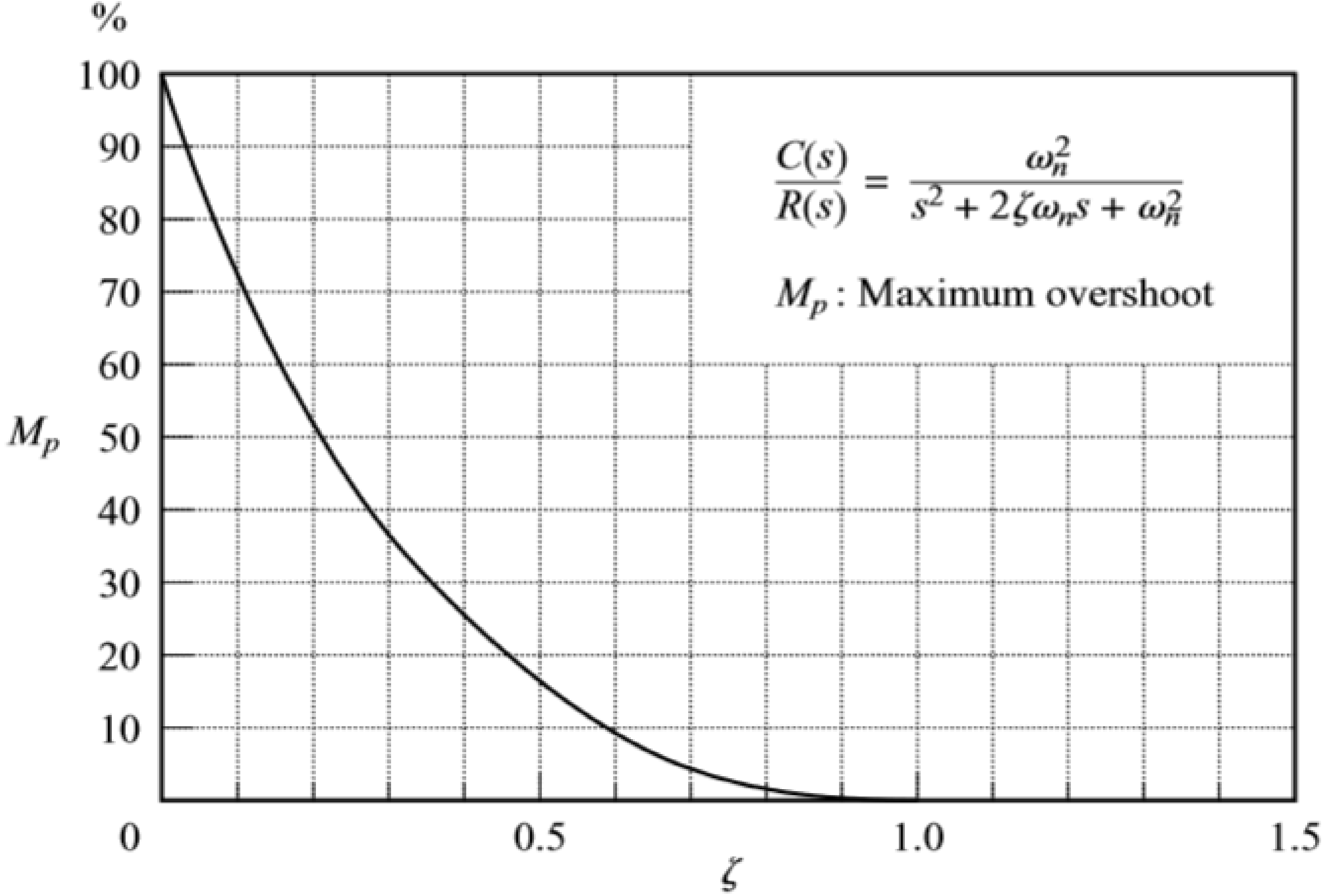
```
x=0:0.1:15;
w=5;
y=1-exp(-0.7*x).*sin(w*x);
plot(x,y)
title('y(x)=1-e^-^0^.^7^xsin\omega x')
xlabel('x')
ylabel('y')
grid on
```



Mp vs ξ curve

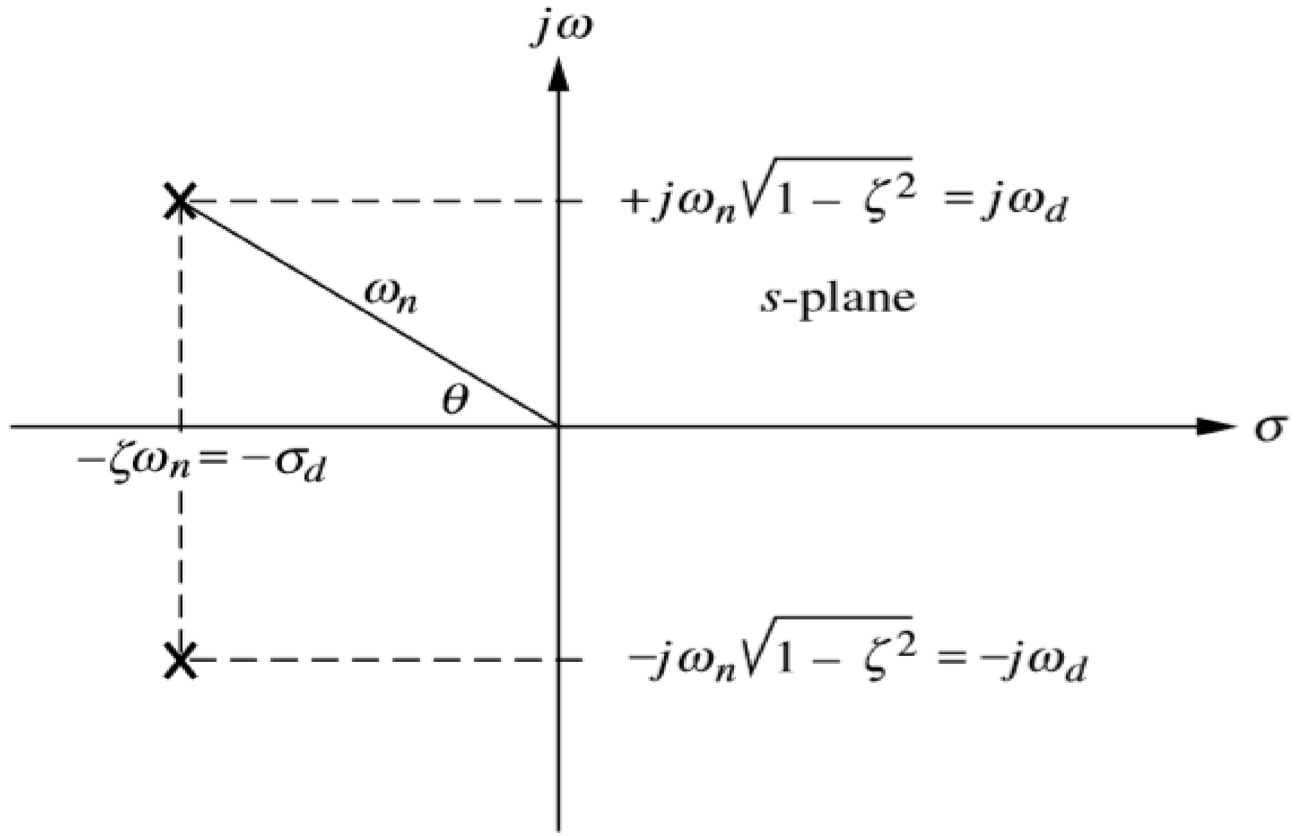


$\zeta = 0$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
1.0
2.0
$c(t)$
$\omega_n t$



$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
M_p : Maximum overshoot
%
M_p
ζ

Pole plot for an Underdamped Response



Step responses of second-order underdamped systems as poles move

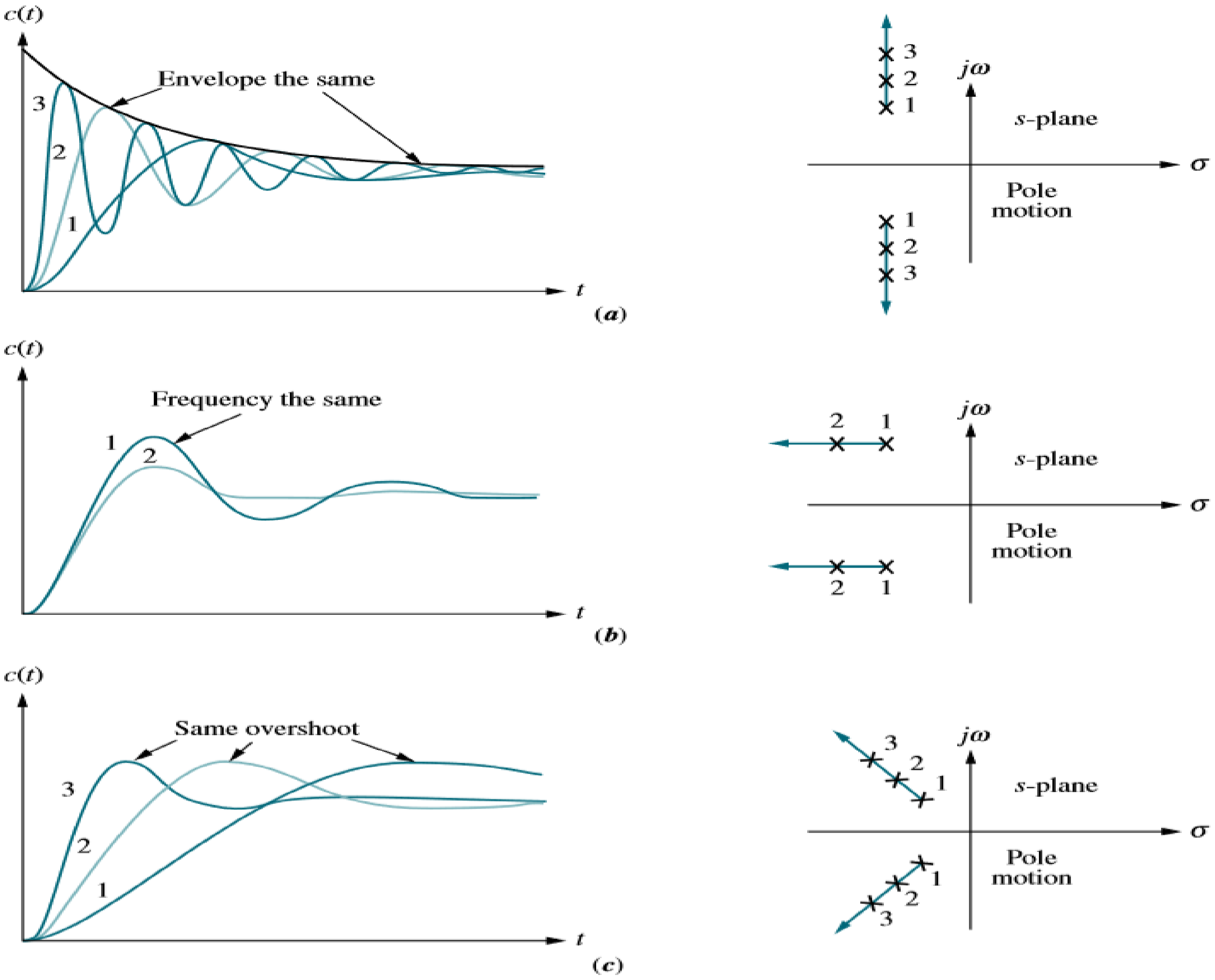


FIGURE 4.19 Step responses of second-order underdamped systems as poles move: **a.** with constant real part; **b.** with constant imaginary part; **c.** with constant damping ratio

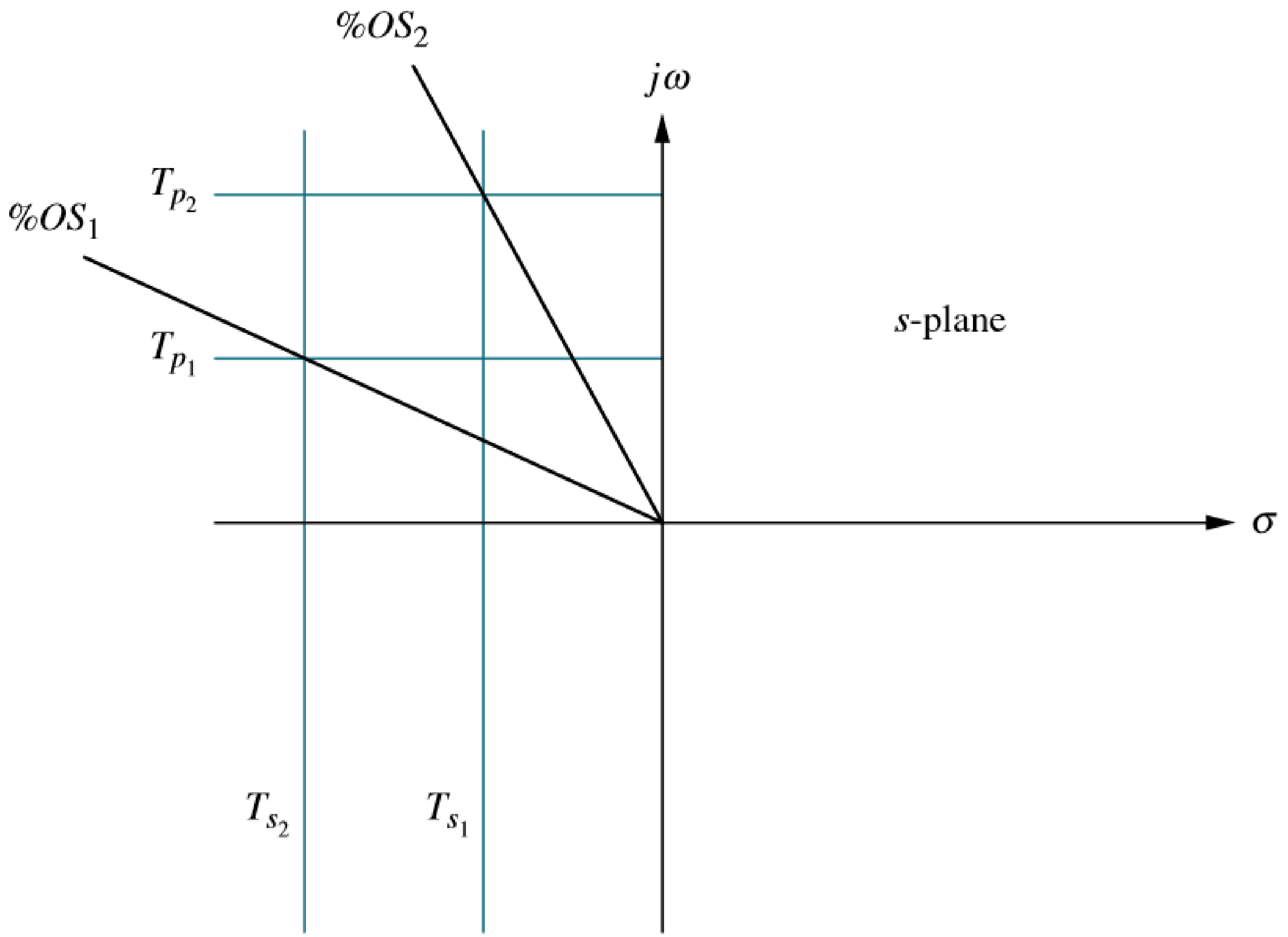


FIGURE 4.18 Lines of constant peak time, T_p, settling time, T_s, and percent overshoot, $\%OS$. Note: $T_{s_2} < T_{s_1}$; $T_{p2} < T_{p1}$; $\%OS_1 < \%OS_2$.

Mathematical Problems

For the given systems:

1. Find ξ and ω_n
2. Characterize the nature of response

a. $G(s) = \dfrac{400}{s^2 + 12s + 400}$

b. $G(s) = \dfrac{900}{s^2 + 90s + 900}$

c. $G(s) = \dfrac{225}{s^2 + 30s + 225}$

d. $G(s) = \dfrac{625}{s^2 + 625}$

Exercise 70 (Norman Nise)

70. Consider the translational mechanical system shown in Figure P4.17. A 1-pound force, $f(t)$, is applied at $t = 0$. If $f_v = 1$, find K and M such that the response is characterized by a 4-second settling time and a 1-second peak time. Also, what is the resulting percent overshoot? [Section: 4.6]

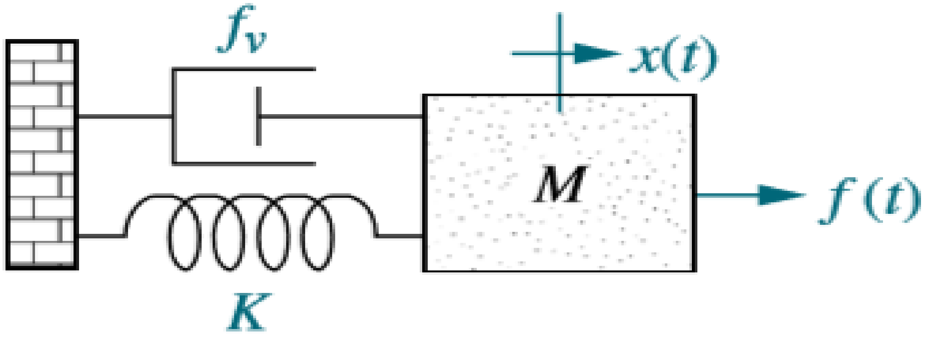


FIGURE P4.17

Example 4.7

Transient Response Through Component Design

PROBLEM: Given the system shown in Figure 4.21, find J and D to yield 20% overshoot and a settling time of 2 seconds for a step input of torque $T(t)$.

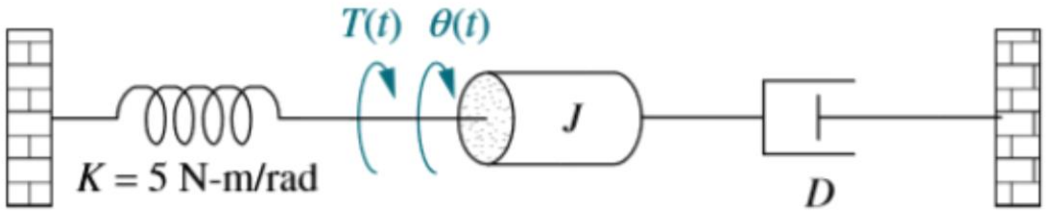


FIGURE 4.21 Rotational mechanical system for Example 4.7

Whose transfer function is given by

$$G(s) = \frac{1/J}{s^2 + \frac{D}{J}s + \frac{K}{J}}$$

56. The response of the deflection of a fluid-filled catheter to changes in pressure can be modeled using a second-order model. Knowledge of the parameters of the model is important because in cardiovascular applications the undamped natural frequency should be close to five times the heart rate. However, due to sterility and other considerations, measurement of the parameters is difficult. A method to obtain transfer functions using measurements of the amplitudes of two consecutive peaks of the response and their timing has been developed (*Glantz*, *1979*). Assume that Figure P4.13 is obtained from catheter measurements. Using the information shown and assuming a second-order model excited by a unit step input, find the corresponding transfer function.

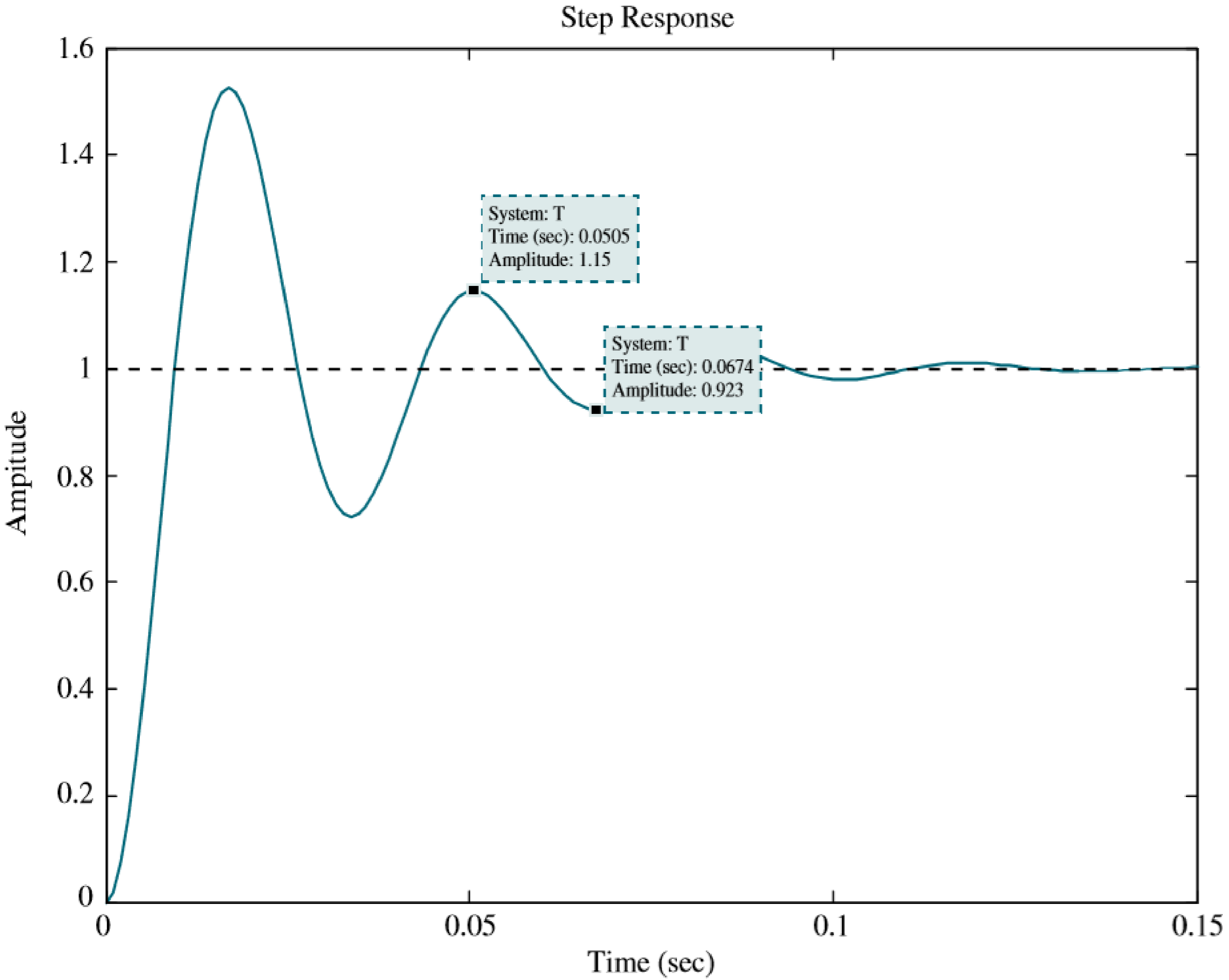


FIGURE P4.13

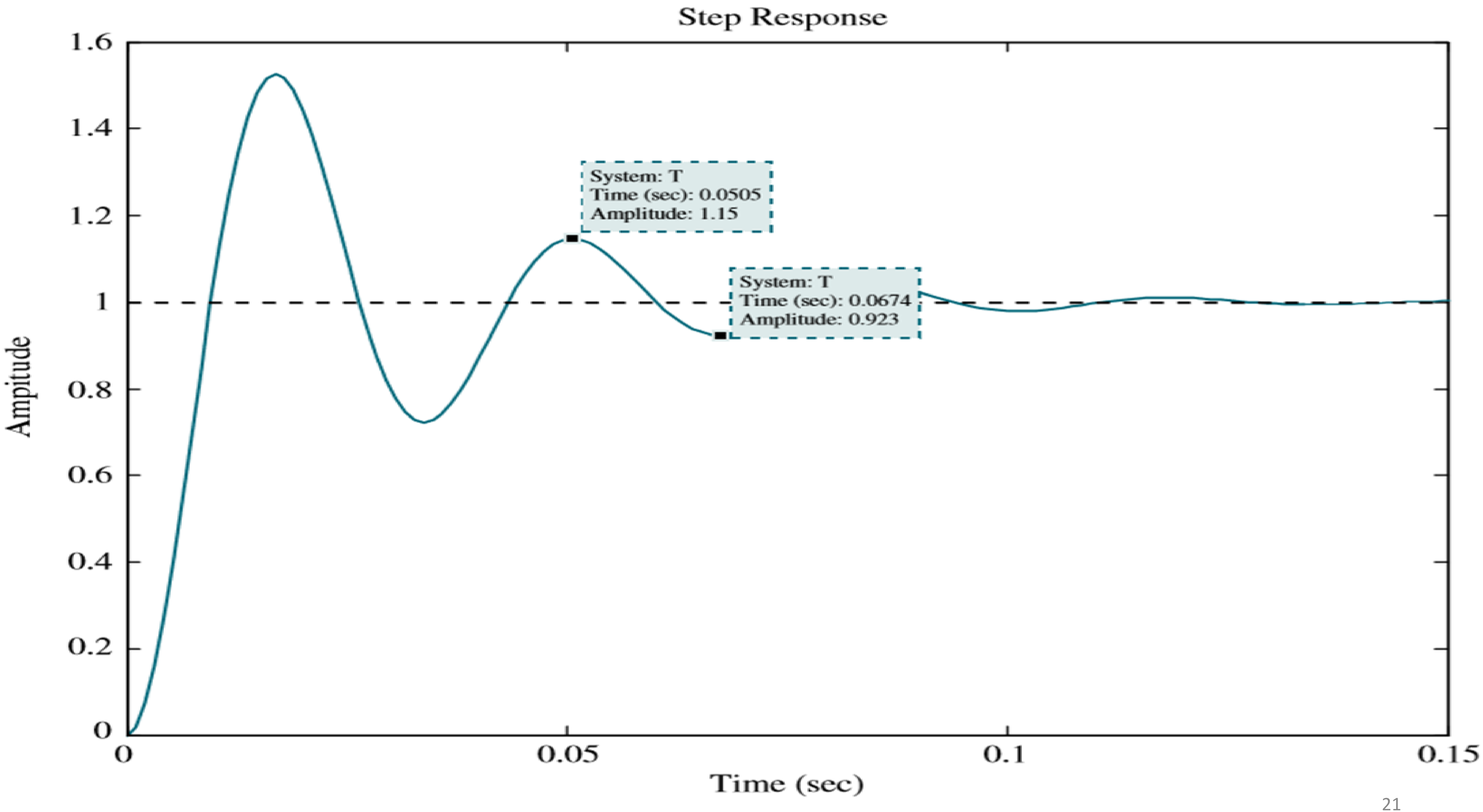
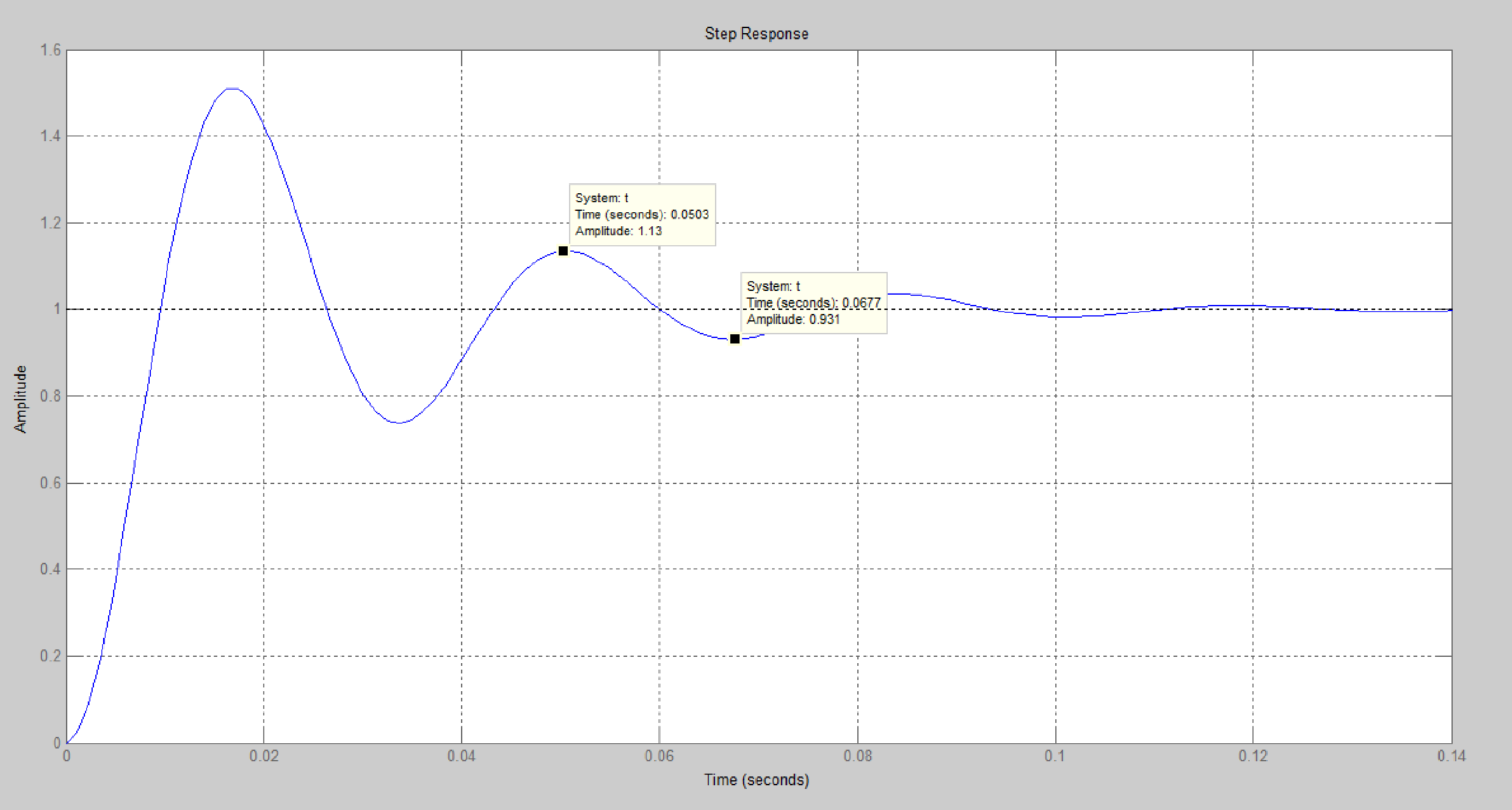

Step Response
System: T
Time (sec): 0.0505
Amplitude: 1.15
System: T
Time (sec): 0.0674
Amplitude: 0.923
Amplitude
Time (sec)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.05
0.1
0.15

FIGURE P4.13

Which is Verified in MATLAB

```
clear all
clc
wn=190.8;
z=0.208;
[n1,d]=ord2(wn,z);
n=wn^2;
t=tf(n,d)
step(t)
grid on
```


Step Response
System: t
Time (seconds): 0.0503
Amplitude: 1.13
System: t
Time (seconds): 0.0677
Amplitude: 0.931
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0.02
0.04
0.06
0.08
0.1
0.12
0.14

Addition of Pole

$$C(s) = \frac{A}{s} + \frac{B(s + \zeta\omega_n) + C\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2} + \frac{D}{s + \alpha_r}$$

or, in the time domain,

$$c(t) = Au(t) + e^{-\zeta\omega_n t}(B \cos \omega_d t + C \sin \omega_d t) + De^{-\alpha_r t}$$

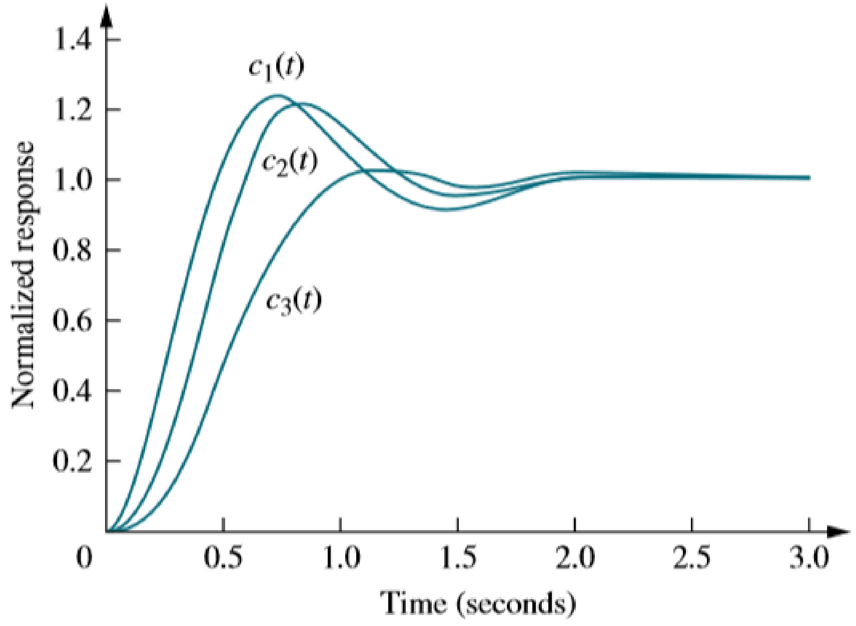
How much farther from the dominant poles does the third pole have to be for its effect on the second-order response to be negligible?

- if the real pole is five times farther to the left than the dominant poles, we assume that the system is represented by its dominant second-order pair of poles.

$$T_1(s) = \frac{24.542}{s^2 + 4s + 24.542}$$

$$T_2(s) = \frac{245.42}{(s+10)(s^2 + 4s + 24.542)}$$

$$T_3(s) = \frac{73.626}{(s+3)(s^2 + 4s + 24.542)}$$

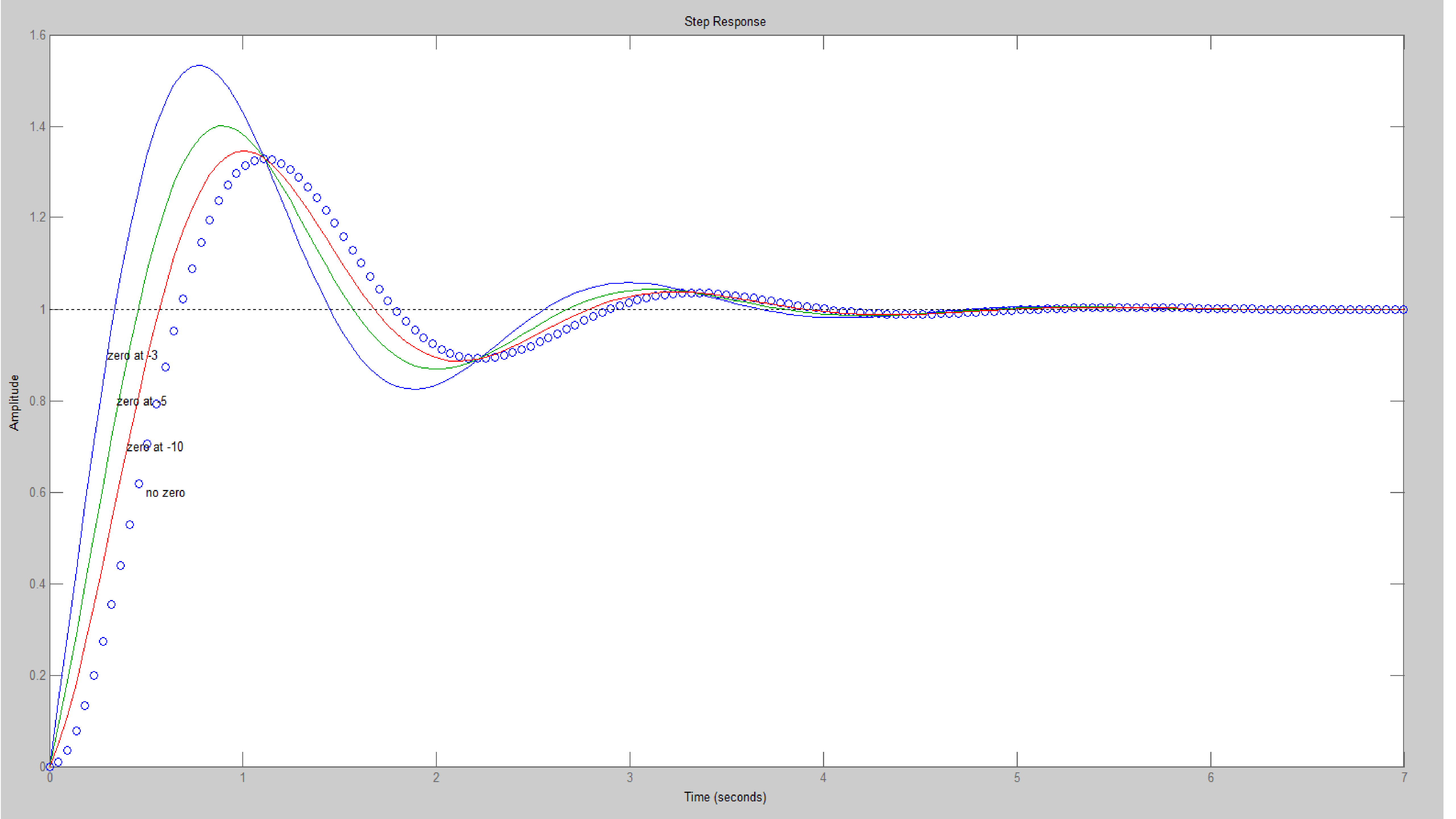


Addition of Zero

- The zero should be as far as possible from the origin.
- If the zero is not far enough the derivative component becomes dominant which changes rise time, peak time, overshoot etc.

$$(s+a)C(s) = sC(s) + aC(s)$$

```
• clc
• deng=[1 2 9];%poles at -1 +- j2.828
• Ta=tf([1 3]*9/3,deng)%zero at -3 (3s+9)
• Tb=tf([1 5]*9/5,deng)%zero at -5 (1.8s+9)
• Tc=tf([1 10]*9/10,deng)%zero at -10 (0.9+9)
• T=tf(9,deng)%no zero
• step(T,'o',Ta,Tb,Tc)
• text(0.5,0.6,'no zero')
• text(0.4,0.7,'zero at -10')
• text(0.35,0.8,'zero at -5')
• text(0.3,0.9,'zero at -3')
```



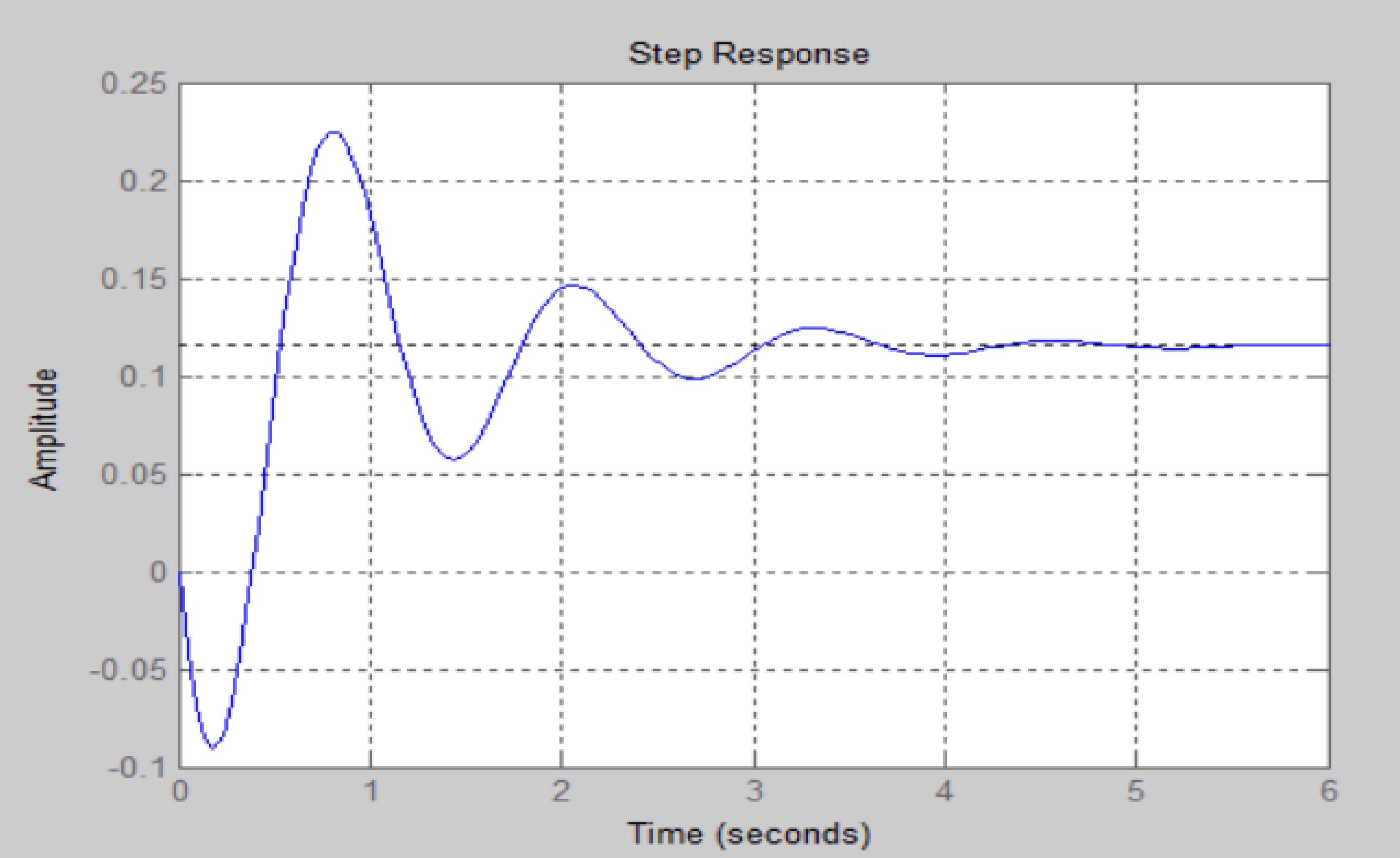
Step Response
zero at -3
zero at -5
zero at -10
no zero
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0
1
2
3
4
5
6
7

Non Minimum Phase System

- Occurs if the additional zero is at right half plane.

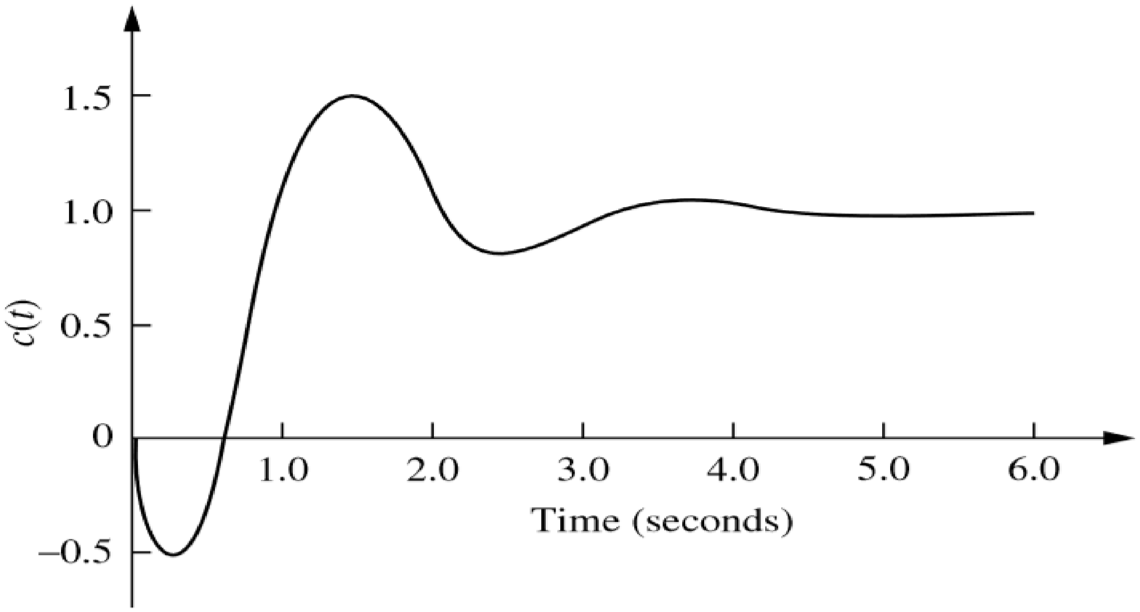
```
     -s + 3
  --------------
  s^2 + 2 s + 26
```

```
%non minimum phase system
clear all
clc
num=poly([3]);
den=poly([-1+5i -1-5i]);
t1=tf(-num,den)
step(t1)
grid on
```



Consequences of Non Minimum Phase System

- If a motorcycle or airplane was a *non minimum-phase system*, it would initially veer left when commanded to steer right.



Assignment

- Answer the following questions (ref. Nise Ch.4 Page: 214)

1. Name the performance specification for first and second-order systems.
2. In a system with an input and an output, what poles generate the steady-state and transient response?
3. The imaginary and real part of a pole generates what part of a response?
4. What is the difference between the natural frequency and the damped frequency of oscillation?
5. If a pole is moved with a constant imaginary part, what will the responses have in common?
6. If a pole is moved with a constant real part, what will the responses have in common?
7. If a pole is moved along a radial line extending from the origin, what will the responses have in common?
8. What pole locations characterize (1) the underdamped system, (2) the overdamped system and (3) the critically damped system?
9. Name two conditions under which the response generated by a pole can be neglected.
10. How can you justify pole-zero cancellation?

The End