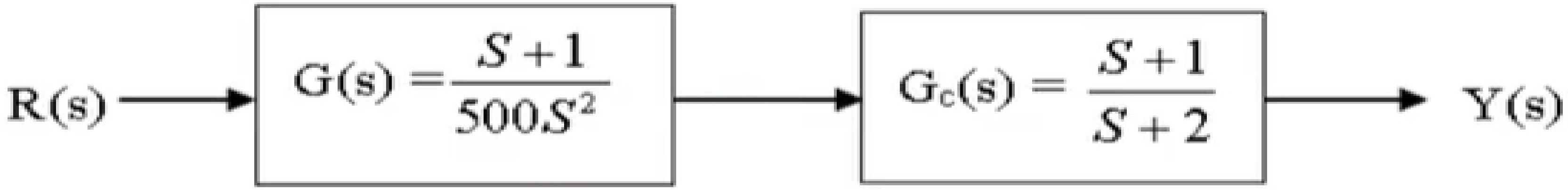
Control Systems Lab

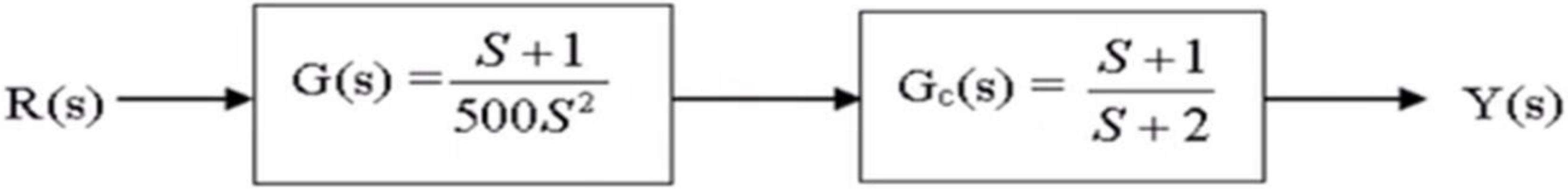
1. REDUCTION OF MULTIPLE SUBSYSTEMS
2. STEADY STATE ERROR
3. ROOT LOCUS
4. FREQUENCY RESPONSE (BODE PLOT)

Reduction Of Multiple Subsystems

Two systems connected in series (Cascaded System)

- Find Y(S)



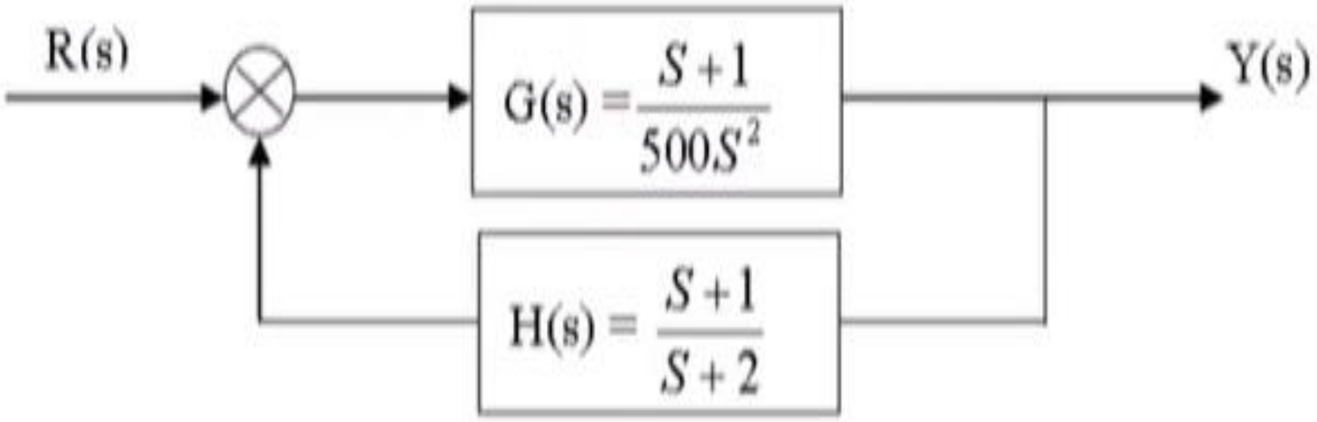



```
clc
clear all
numg=[1 1];
deng=[500 0 0];
numh=[1 1];
denh=[1 2];
[num,den]=series(numg,deng,numh,denh);
printsys(num,den)
```

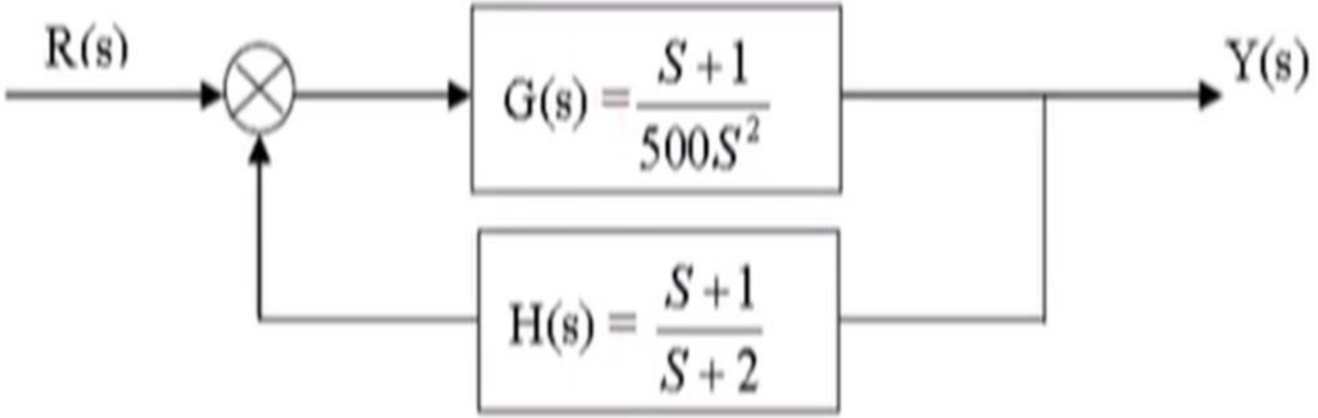
```
num/den =

      s^2 + 2 s + 1
   ------------------
   500 s^3 + 1000 s^2
```

Simple Feedback System

- Find Y(S)



```
clc
clear all
numg=[1 1];
deng=[500 0 0];
numh=[1 1];
denh=[1 2];
[num,den]=feedback(numg,deng,numh,denh,-1);
%negative feedback is built in.... if positive
feedback is used, use 1.
printsys(num,den)
```



Similarly for Parallel Connection

- Use “parallel” command

Multiple Subsystem Reduction

Example 5.2

Block Diagram Reduction by Moving Blocks

PROBLEM: Reduce the system shown in Figure 5.11 to a single transfer function.

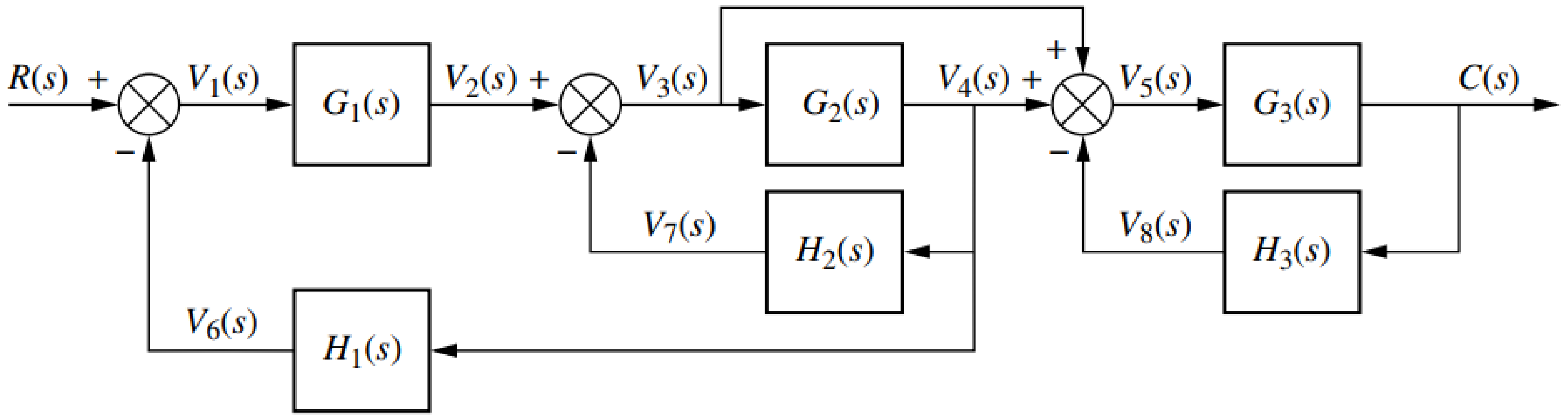
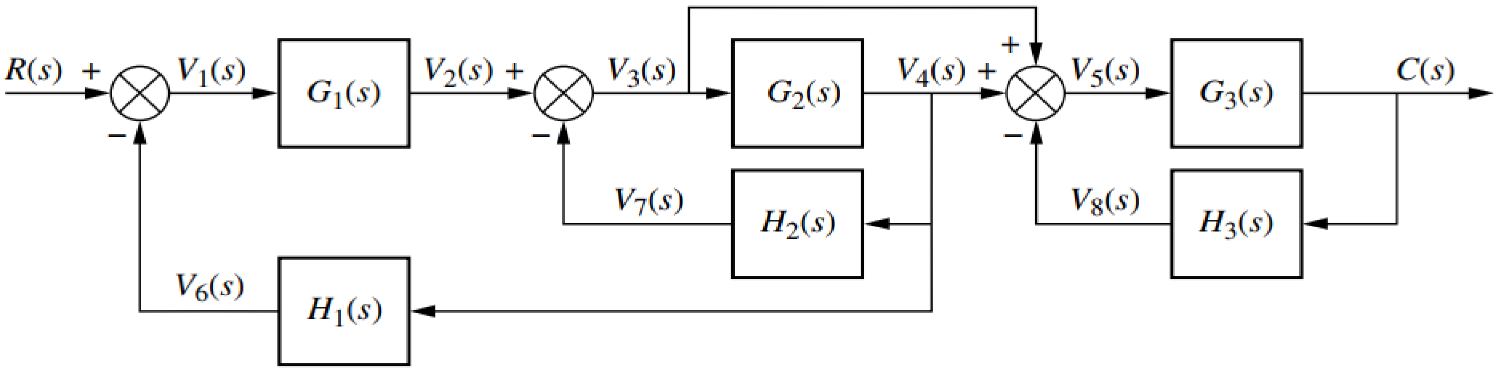


FIGURE 5.11 Block diagram for Example 5.2

Consider all $G_i(S)=1/(S+1)$ and $H_i(S)=1/S$




```
clc; clear all;
G1=tf(1,[1 1]);
G2=G1;G3=G1;
H1=tf(1,[1 0]);
H2=H1;H3=H1;
System=append(G1,G2,G3,H1,H2,H3);
input=1;output=3;
q=[1 -4 0 0 0
    2 1 -5 0 0
    3 2 1 -5 -6
    4 2 0 0 0
    5 2 0 0 0
    6 3 0 0 0];
T=connect(System,q,input,output);
T=tf(T)
T=minreal(T)
```

Output Transfer Function

```
T =

                   s^4 + 2 s^3
  -----------------------------------------------------
  s^6 + 3 s^5 + 5 s^4 + 6 s^3 + 4 s^2 + 2 s + 7.334e-17

Continuous-time transfer function.


T =

             s^3 + 2 s^2
  -------------------------------------
  s^5 + 3 s^4 + 5 s^3 + 6 s^2 + 4 s + 2

Continuous-time transfer function.
```

Steady State Error

- A unity feedback system has the following forward transfer function

$$G(s) = \frac{1000(s+8)}{(s+7)(s+9)}$$

Determine positional error constant k_p and steady state error e_{ss}

K_p can be obtained by simply putting *s=0* in G(s) according to final value theorem or using MATLAB command `dcgain`

$$G(s) = \frac{1000(s+8)}{(s+7)(s+9)}$$

```
clc
close all
clear all
numg=1000*[1 8];
deng=poly([-7 -9]);
G=tf(numg,deng);
kp=dcgain(G)
estep=1/(1+kp)
```

$$G(s) = \frac{1000(s+8)}{(s+7)(s+9)}$$

Output

```
kp =

  126.9841


estep =

    0.0078
```

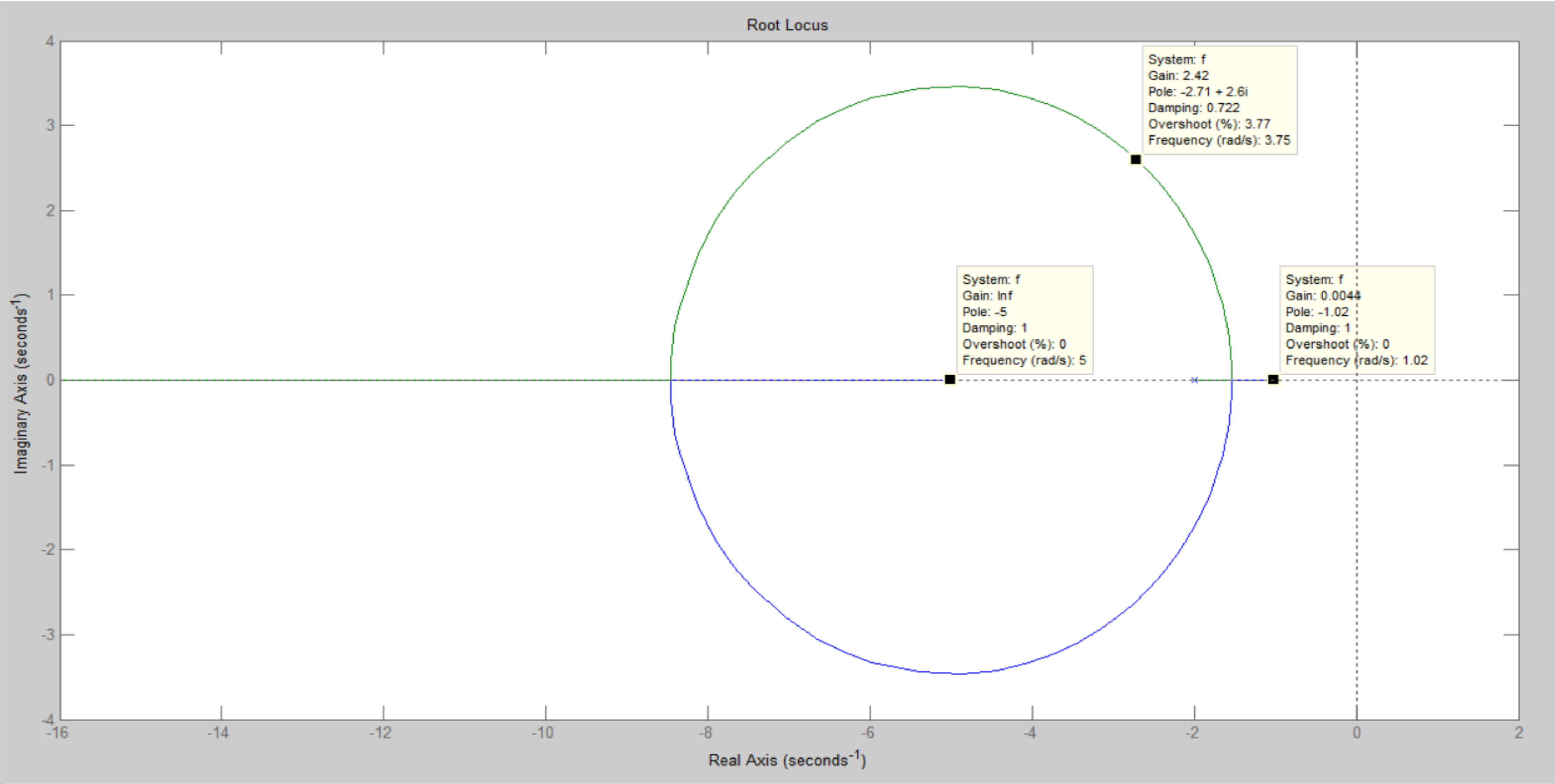
Root Locus

Plot the root locus of the given transfer function

```
f =

      (s+5)
  -----------
  (s+1) (s+2)
```

```
f=zpk([-5],[-1 -2],1)
rlocus(f)
```

Root Locus

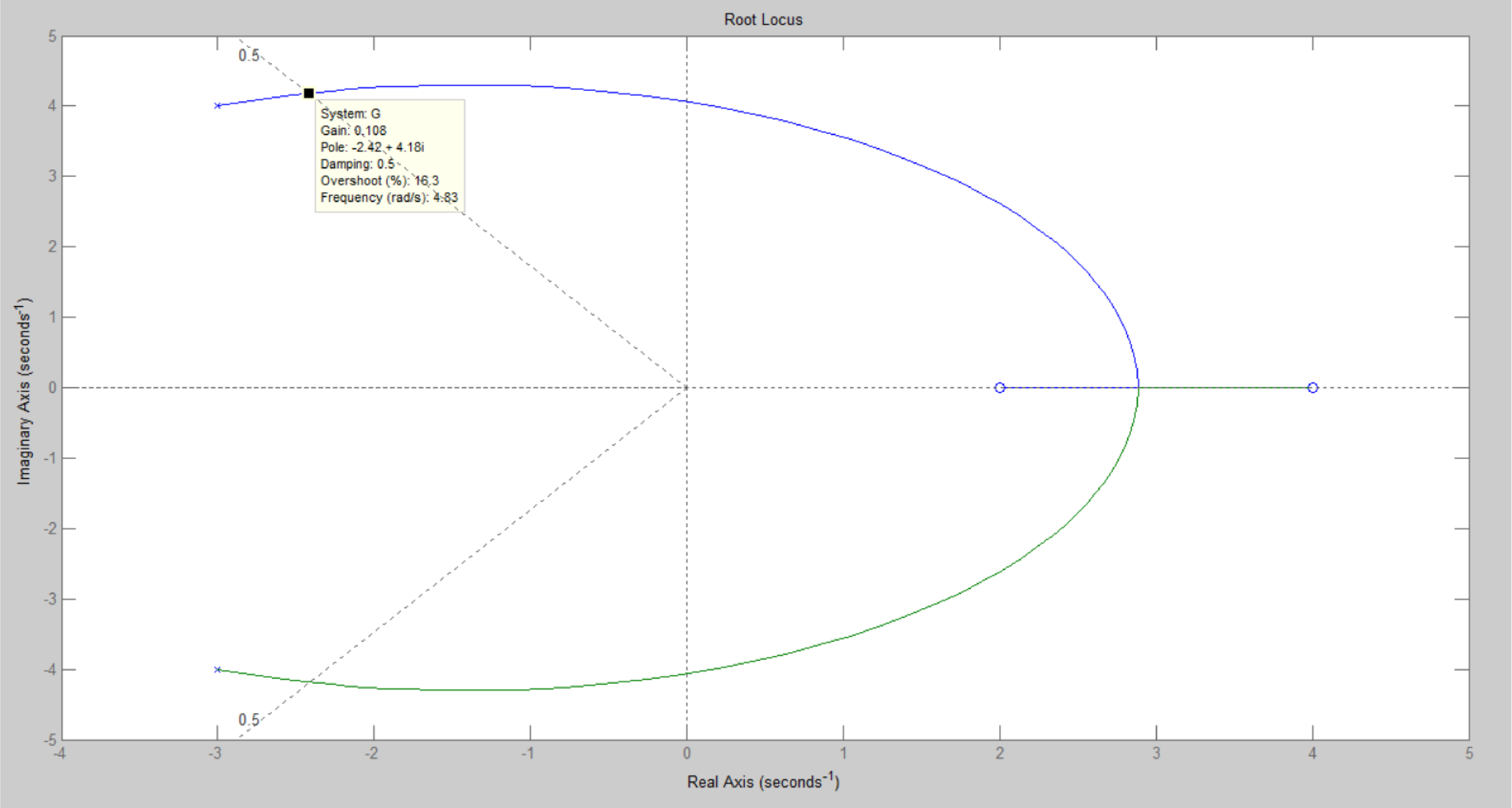


Root Locus
System: f
Gain: 2.42
Pole: -2.71 + 2.6i
Damping: 0.722
Overshoot (%): 3.77
Frequency (rad/s): 3.75
System: f
Gain: Inf
Pole: -5
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 5
System: f
Gain: 0.0044
Pole: -1.02
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 1.02
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
4
3
2
1
0
-1
-2
-3
-4
-16
-14
-12
-10
-8
-6
-4
-2
0
2

Determine the roots at a given value of damping ratio(ξ)

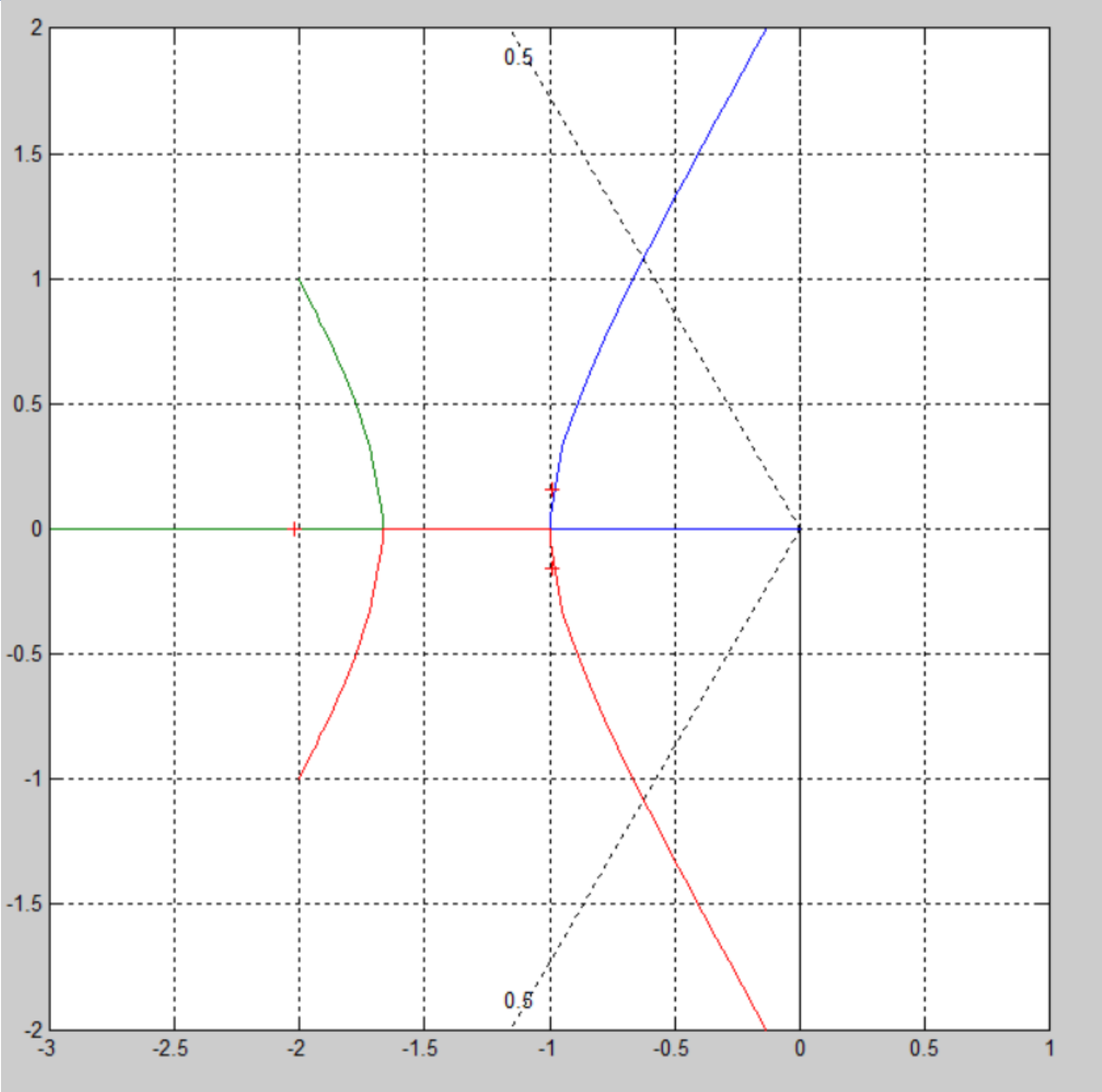
```
clc
close all
clear all
numg=poly([2 4]);
deng=[1 6 25];
G=tf(numg,deng)
rlocus(G)
z=0.5;
sgrid(z,0)
```

```
G =

  s^2 - 6 s + 8
  --------------
  s^2 + 6 s + 25

Continuous-time transfer function.
```


Root Locus
System: G
Gain: 0.108
Pole: -2.42 + 4.18i
Damping: 0.5
Overshoot (%): 16.3
Frequency (rad/s): 4.83
0.5
0.5
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
5
4
3
2
1
0
-1
-2
-3
-4
-5
-4
-3
-2
-1
0
1
2
3
4
5

Another Method

```
clc
clear all
num=[1];
den=[1 4 5 0];
r=rlocus(num,den);
plot(r,'-');
axis([-3 1 -2 2]);
axis('square')
grid
sgrid(0.5,[]);
[K,r]=rlocfind(num,den)
```



In this method the answers can obtained in command window……….

```
Select a point in the graphics window

selected_point =

  -0.6289 + 1.0772i


K =

    4.2633


r =

  -2.7441 + 0.0000i
  -0.6280 + 1.0767i
  -0.6280 - 1.0767i
```

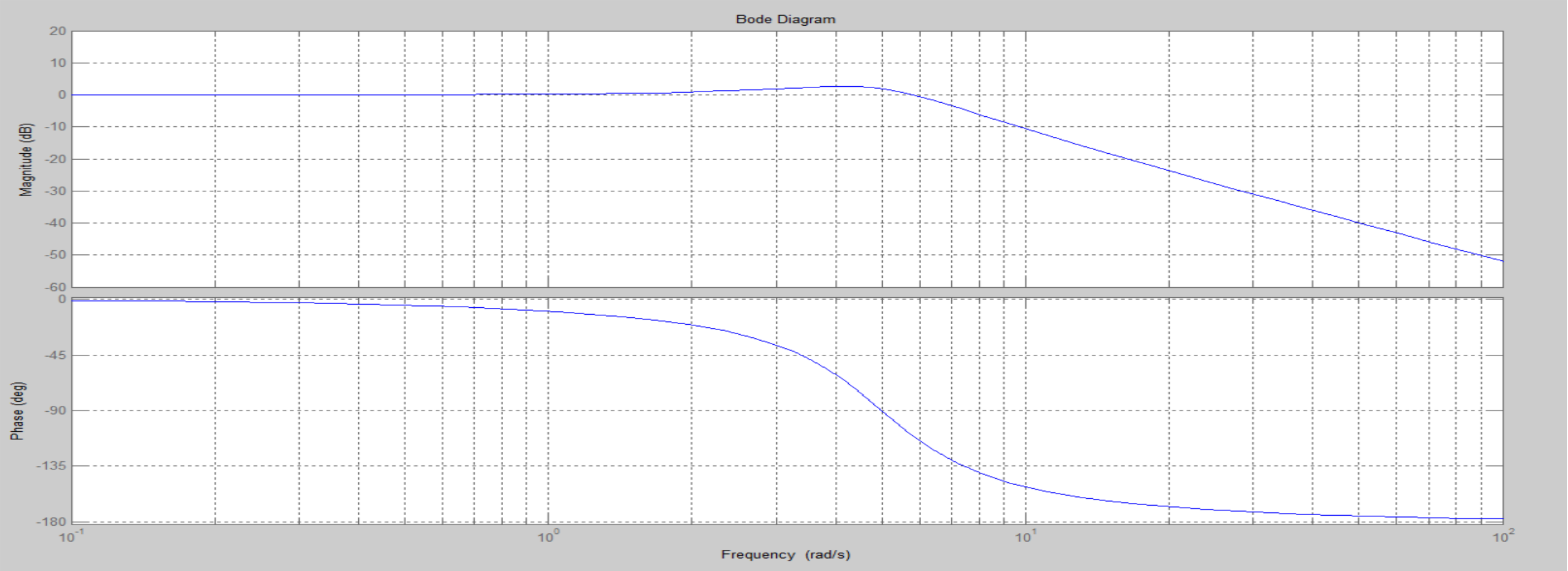
Bode Plot

Consider the following transfer function:

$$G(s) = \frac{25}{s^2 + 4s + 25}$$

Plot a Bode diagram for this transfer function.

```
clc
clear all
num = [25];
den = [1 4 25];
bode(num,den)
grid on
```



Gain Margin, Phase Margin, Gain Cross Over Frequency, Phase Cross Over Frequency

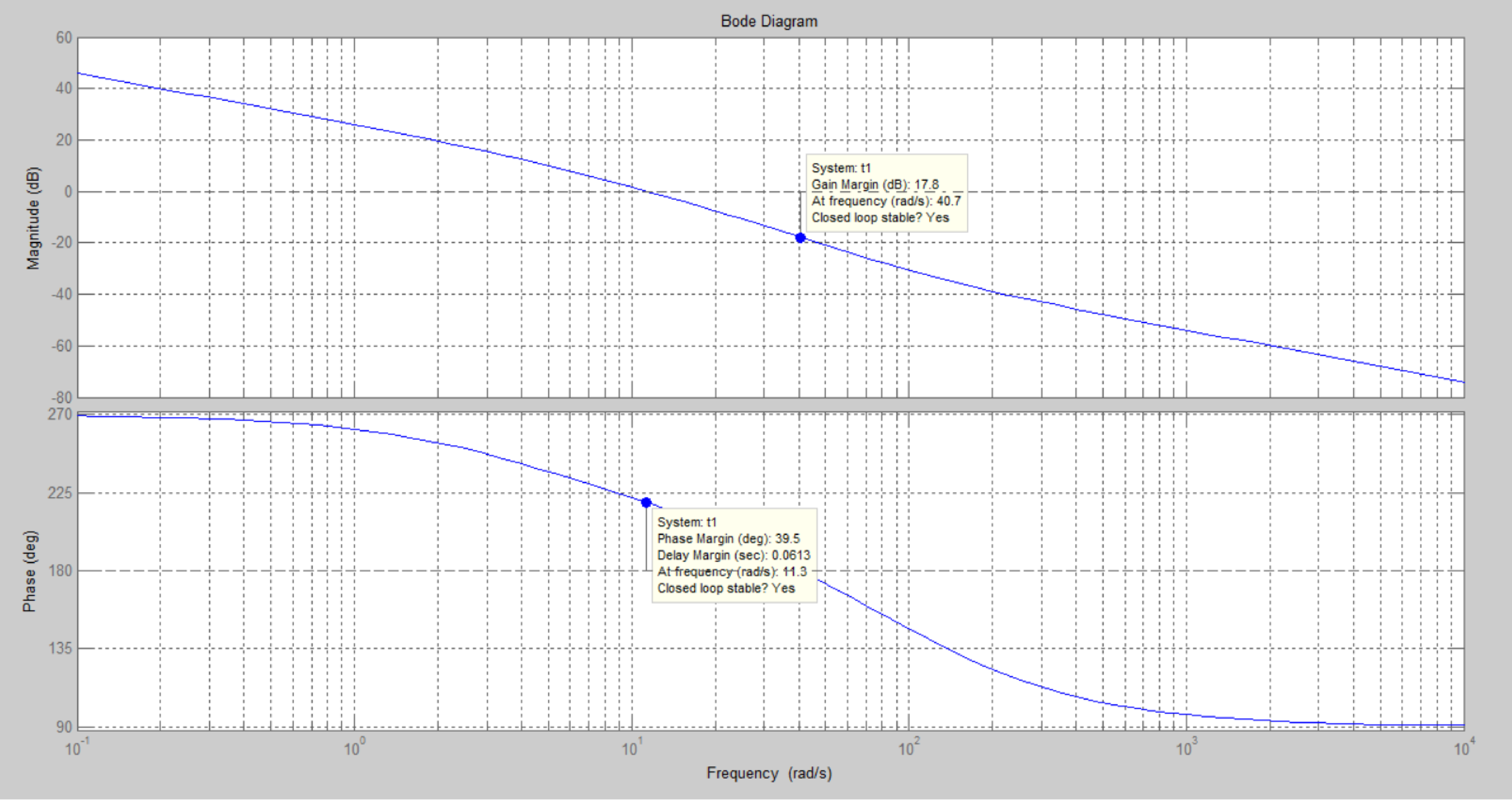
```
clear all
clc
num=[-2 200 2000];
den=poly([0 -5 -20]);
t1=tf(num,den)
bode(t1);
[Gm,Pm,Wgc,Wpc]=margin(t1)
Gm_db=20*log10(Gm)%The gain margin is to be
tranformed in dB
grid on
```

Answers are given at Command Window

```
Gm =

    7.8011


Pm =

   39.5344


Wgc =

   40.7458


Wpc =

   11.2646


Gm_db =

   17.8431
```

In the Figure

- Right Click
- Enter *Characteristics*
- Enter *Minimum Stability Margins*


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
60
40
20
0
-20
-40
-60
-80
270
225
180
135
90
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
System: t1
Gain Margin (dB): 17.8
At frequency (rad/s): 40.7
Closed loop stable? Yes
System: t1
Phase Margin (deg): 39.5
Delay Margin (sec): 0.0613
At frequency (rad/s): 11.3
Closed loop stable? Yes

Bode Diagram for State Space Representation

Consider the following system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -25 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 25 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

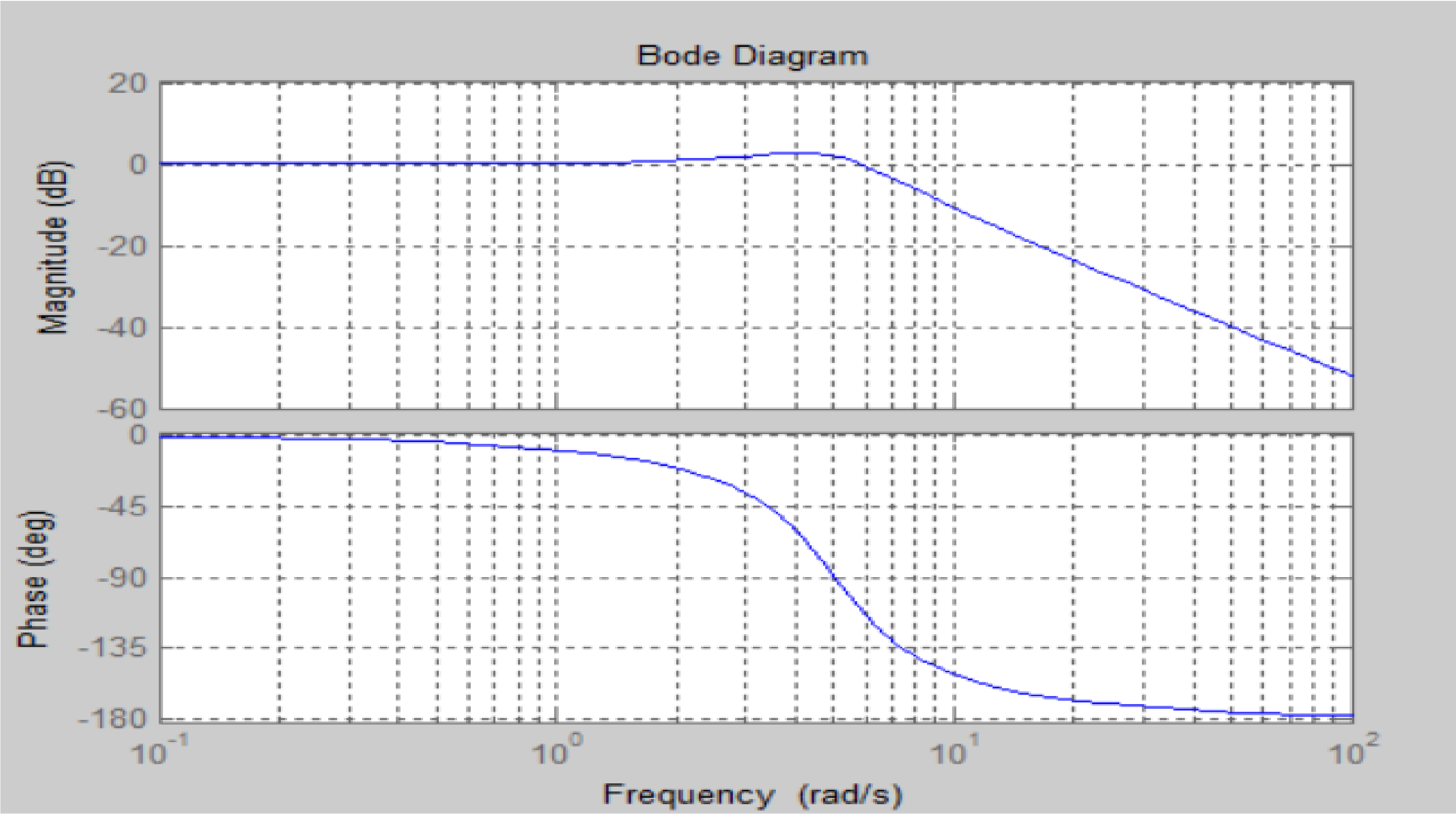
This system has one input u and one output y. By using the command

bode(A,B,C,D)

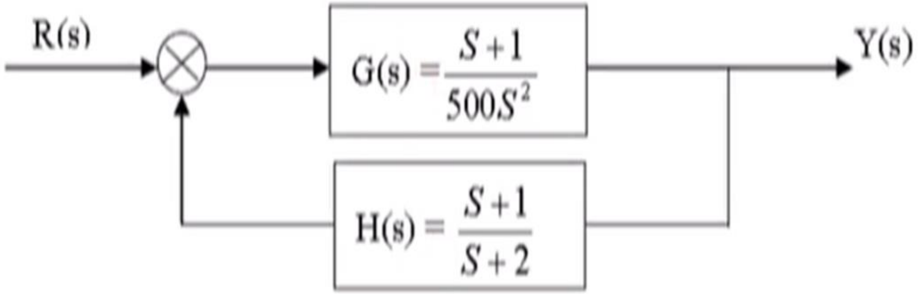
```
clear all
close all
clc
A=[0 1;-25 -4];
B=[0;25];
C=[1 0];
D=[0];
bode(A,B,C,D)
grid on
title('Bode Diagram')
```

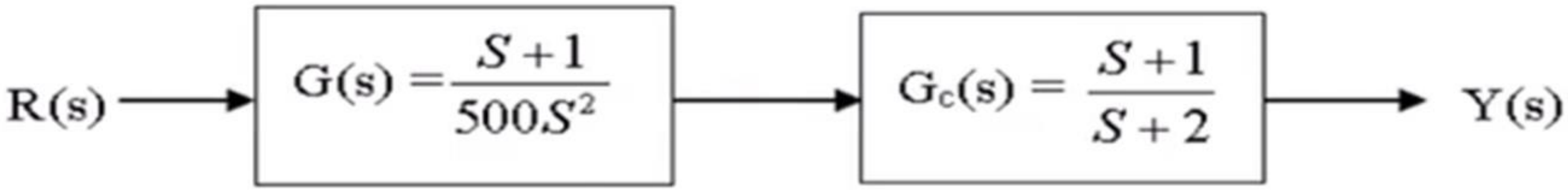
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -25 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 25 \end{bmatrix} u$$

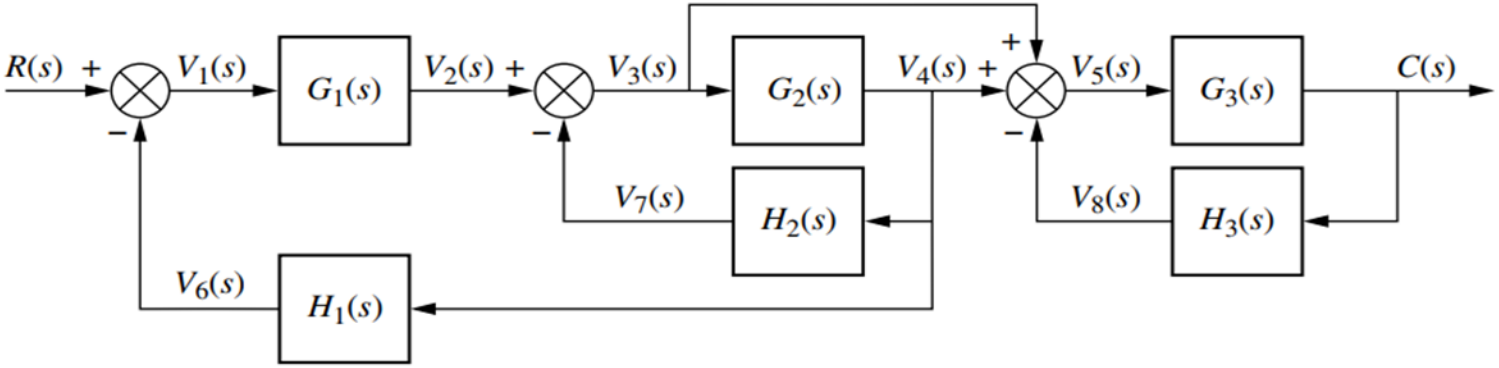
$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



Experiment 14: Reduction Of Multiple Subsystems







Experiment 15: Steady State Error

- A unity feedback system has the following forward transfer function

$$G(s) = \frac{1000(s+8)}{(s+7)(s+9)}$$

Determine positional error constant k_p and steady state error e_{ss}

Experiment 16: Root Locus Technique

- Plot the root locus of the following transfer function using both MATLAB and manually with hand the following transfer function

$$G(S)=\frac{1}{s(s+2)(s^2+2s+10)}$$

- Find the position of your roots in a given transfer function of your choice when the value of damping ratio is given.

Experiment 17: Frequency Response Analysis

- Draw the bode plot of the given transfer functions showing minimum stability margins in the figure.

a. $G(s) = \dfrac{10}{s(s+1)(s+2)}$

b. $G(s) = \dfrac{1000}{(s+3)(s+4)(s+5)(s+6)}$

c. $G(s) = \dfrac{50(s+3)}{s(s+2)(s+4)}$

- For each closed-loop system with the following performance characteristics, find the closed-loop bandwidth:

a. $\zeta = 0.2,\ T_s = 3$ seconds

b. $\zeta = 0.2,\ T_p = 3$ seconds

c. $T_s = 4$ seconds, $T_p = 2$ seconds

d. $\zeta = 0.3,\ T_r = 4$ seconds

- Determine the Gain Margin, Phase Margin, Gain Cross Over Frequency, Phase Cross Over Frequency of any transfer function using both figure and "margin" command.

- Find the bode plot of the following state models showing minimum stability margins in the figure

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -25 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 25 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Experiment 18: Routh-Hurwitz criterion for stability

- Given the unity feedback system G(S) write a program using MATLAB to determine the value of K for stability.

$$G(s) = \frac{Ks(s+2)}{(s^2 - 4s + 8)(s+3)}$$

Hint: See the solution of Ch.7 Exercise: 37 of N. Nise in Page: 329

Lab Report Top page Format

Name:
Roll:
Course Title:
Course Code:

Exp. No:	Experiment Name	Remarks
1.	Determination of Roots of Equations	
2.	Partial Fraction Expansion	
3.	Laplace Transform Review	
4.	Modelling in Zero Pole Gain and Transfer Function	

Before submitting the whole lab reports all together one has to make this Top Page (where all experiment names are written in a single table)