EEE 704

Control System I Sessional

Roots of a polynomial

- Find the roots the following polynomial

$$x^5 - 3x^3 + x^2 - 9 = 0,$$

Make an array using the coefficients

the coefficients are

$$\underbrace{(1)}_{C_1} x^5 + \underbrace{(0)}_{C_2} x^4 + \underbrace{(-3)}_{C_3} x^3 + \underbrace{(1)}_{C_4} x^2 + \underbrace{(0)}_{C_5} x + \underbrace{(-9)}_{C_6} = 0$$

$$x^5 - 3x^3 + x^2 - 9 = 0,$$

- Type

```
c=[1 0 -3 1 0 -9];
roots(c)
ans =

   1.9316 + 0.0000i
   0.5898 + 1.1934i
   0.5898 - 1.1934i
  -1.5556 + 0.4574i
  -1.5556 - 0.4574i
```

Zero Pole Gain Model

- `f=zpk([-2],[-1 -2 -2],2)`

```
f =


      2 (s+2)
  -------------
  (s+1) (s+2)^2

Continuous-time zero/pole/gain model.
```

Partial Fraction Expansion

- Do the partial on the following example

$$F(s) = \frac{2}{(s+1)(s+2)}$$

Partial Fraction Expansion

```
numf=2;
denf=poly([-1 -2]);
[z,p,k]=residue(numf,denf)
```

$$F(s) = \frac{2}{(s+1)(s+2)}$$

$$= \frac{2}{s+1} - \frac{2}{s+2}$$

```
z =

    -2
     2

p =

    -2
    -1

k =

     []
```

Transfer Function

- `f=tf([3],[1 2 5 0])`

```
f =

          3
  -----------------
  s^3 + 2 s^2 + 5 s

Continuous-time transfer function.
```

Example 2.37. *Generate the transfer function using MATLAB.*

$$G(s) = \frac{3(s+9)(s+21)(s+57)}{s(s+30)(s^2+5s+35)(s^2+28s+42)}$$

Using

(*a*) *the ratio of factors*

(*b*) *the ratio of polynomials.*

```
gzpk=zpk([-9 -21 -57],[0 -30 roots([1 5 35])' roots([1 28 42])'], 3 )
rtf=tf(gzpk)
```

$$G(s) = \frac{3(s+9)(s+21)(s+57)}{s(s+30)(s^2+5s+35)(s^2+28s+42)}$$

Output

```
gzpk =

                3 (s+9) (s+21) (s+57)
  ----------------------------------------------
  s (s+30) (s+26.41) (s+1.59) (s^2 + 5s + 35)

Continuous-time zero/pole/gain model.


rtf =

                 3 s^3 + 261 s^2 + 5697 s + 32319
  --------------------------------------------------------------
  s^6 + 63 s^5 + 1207 s^4 + 7700 s^3 + 37170 s^2 + 44100 s

Continuous-time transfer function.
```

Laplace Transform

- Find the Laplace transform of

$$f(t) = 7t^3 \cos(5t + 60^\circ)$$

Laplace Transform

```
syms t %Short-cut for constructing symbolic variables
f=7*t^3*cos(5*t+(pi/3));
z=laplace(f)
pretty(z)
```

```
z =

(7*3^(1/2)*((120*s)/(s^2 + 25)^3 - (240*s^3)/(s^2 + 25)^4))/2 + 21/(s^2 + 25)^2 - (168*s^2)/(s^2 + 25)^3 + (168*s^4)/(s^2 + 25)^4


           /                   3   \
      1/2 |    120 s        240 s   |
   7 3    | ---------- - ---------- |
          |   2      3     2      4 |
          \ (s  + 25)    (s  + 25)  /        21           168 s^2 ... 
```

$$\frac{7\sqrt{3}\left(\frac{120\,s}{(s^2+25)^3}-\frac{240\,s^3}{(s^2+25)^4}\right)}{2}+\frac{21}{(s^2+25)^2}-\frac{168\,s^2}{(s^2+25)^3}+\frac{168\,s^4}{(s^2+25)^4}$$

Inverse Laplace Transform

- Find the Inverse Laplace of

$$F(s) = \frac{3}{s(s^2 + 2s + 5)}$$

Inverse Laplace

```
syms s
f=ilaplace(3/(s*(s^2+2*s+5)))
pretty(f)
```

```
f =

3/5 - (3*exp(-t)*(cos(2*t) + sin(2*t)/2))/5


                  /             sin(2 t) \
      3 exp(-t) | cos(2 t) + -------- |
                  \                2     /
  3/5 - ---------------------------------
                       5
```

Exp. 1: Determination of Roots of Equations

Find the roots of the polynomials given below

$$p_1(x) = x^5 + 2x^4 - 3x^3 + 7x^2 - 8x + 7$$

$$p_2(x) = x^4 + 3x^3 - 5x^2 + 9x + 11$$

$$p_3(x) = x^3 - 2x^2 - 3x + 9$$

$$p_4(x) = x^2 - 5x + 13$$

$$p_5(x) = x + 5$$

Exp. 2: Partial Fraction Expansion

Expand the following function $F(s)$ into partial fractions with MATLAB:

$$F(s) = \frac{5s^3 + 7s^2 + 8s + 30}{s^4 + 15s^3 + 62s^2 + 85s + 25}$$

$$F(s) = \frac{2}{(s+1)(s+2)}$$

$$F(s) = \frac{32}{s(s^2 + 12s + 32)}$$

$$F(s) = \frac{2}{(s+1)(s+2)^2}$$

Exp. 3: Laplace Transform Review

Determine Laplace of the given functions

(a) $f(t) = 7t^3 \cos(5t + 60°)$

(b) $f(t) = -7te^{-5t}$

(c) $f(t) = -3\cos 5t$

(d) $f(t) = t \sin 7t$

(e) $f(t) = 5e^{-2t}\cos 5t$

(f) $f(t) = 3\sin(5t + 45^{\circ})$

Determine Inverse Laplace

$$(a)\ F(s) = \frac{s}{s(s+2)(s+6)}$$

$$(b)\ F(s) = \frac{1}{s^2(s+5)}$$

$$(c)\ F(s) = \frac{3s+1}{(s^2+2s+9)}$$

$$(d)\ F(s) = \frac{s-25}{s(s^2+3s+20)}$$

Exp. 4: Modelling in Zero Pole Gain and Transfer Function

Example 2.37. *Generate the transfer function using MATLAB.*

$$G(s) = \frac{3(s+9)(s+21)(s+57)}{s(s+30)(s^2+5s+35)(s^2+28s+42)}$$

Using

(*a*) *the ratio of factors*

(*b*) *the ratio of polynomials.*

Example 2.38. *Generate the transfer function using MATLAB.*

$$G(s) = \frac{s^4+20s^3+27s^2+17s+35}{s^5+8s^4+9s^3+20s^2+29s+32}.$$

Using

(*a*) *the ratio of factors*

(*b*) *the ratio of polynomials.*