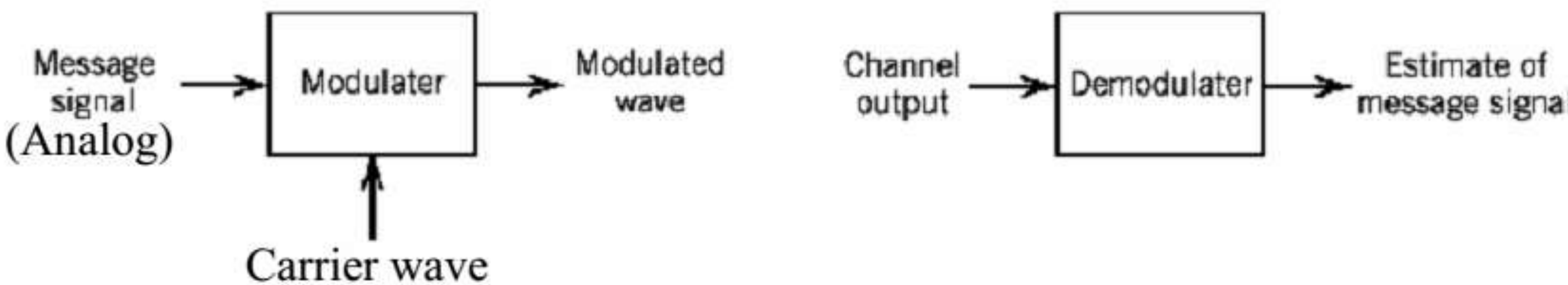
Continuous-Wave Modulation

2.1 Introduction

- Analog communication system
  - The most common carrier is the sinusoidal wave.



2.1 Introduction

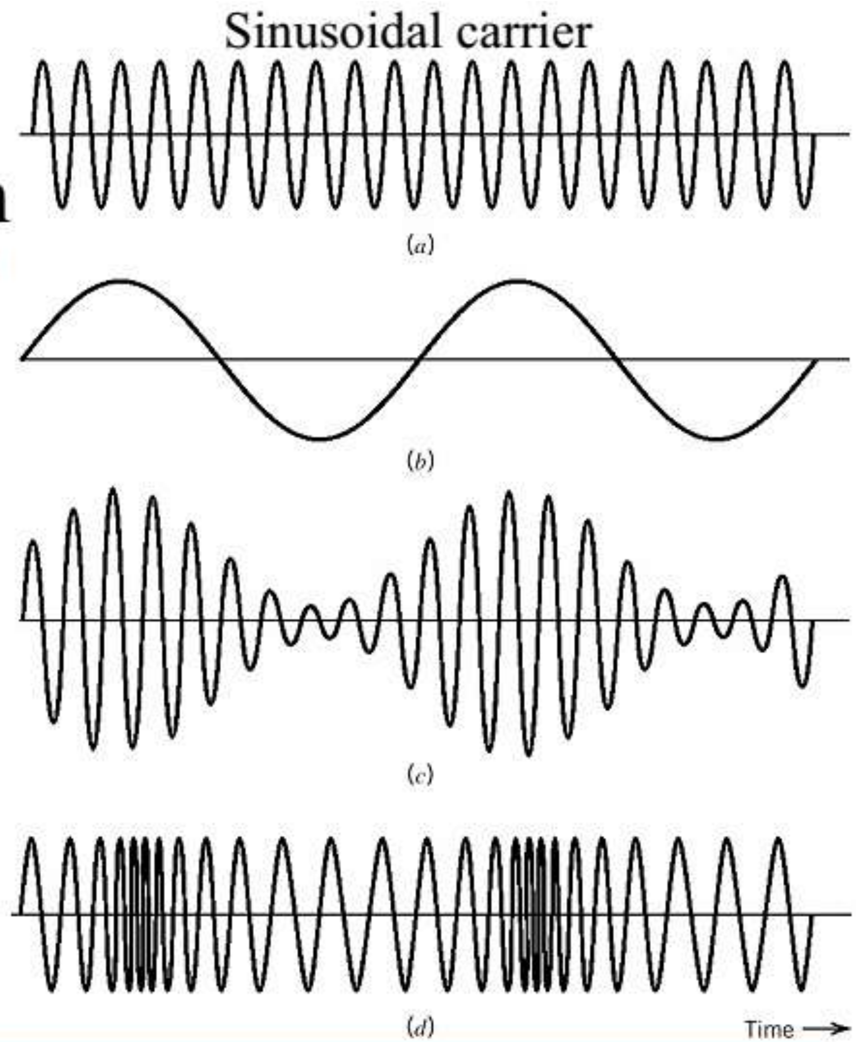
- Modulation
  - A process by which *some characteristic of a carrier* is varied in accordance with a *modulating wave* (baseband signal).
- Sinusoidal Continuous-Wave (CW) modulation
  - Amplitude modulation
  - Angle modulation

2.1 Introduction

Baseband signal

Amplitude Modulation

Frequency Modulation



2.2 Double-Sideband with Carrier or simply Amplitude Modulation

Carrier $c(t) = A_c \cos(2\pi f_c)$

Baseband $m(t)$

Modulated Signal $s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$,

where k_a is amplitude sensitivity or modulation index

- Two required conditions on amplitude sensitivity
  - $1 + k_a m(t) \geq 0$, which is ensured by $|k_a m(t)| \leqq 1$.
    - The case of $|k_a m(t)| > 1$ is called *overmodulation*.
    - The value of $|k_a m(t)|$ is sometimes represented by "percentage" (because it is limited by 1), and is named $(|k_a m(t)| \times 100)\%$ modulation.
  - $f_c >> W$, where W is the message bandwidth.
    - Violation of this condition will cause **nonvisualized envelope**.

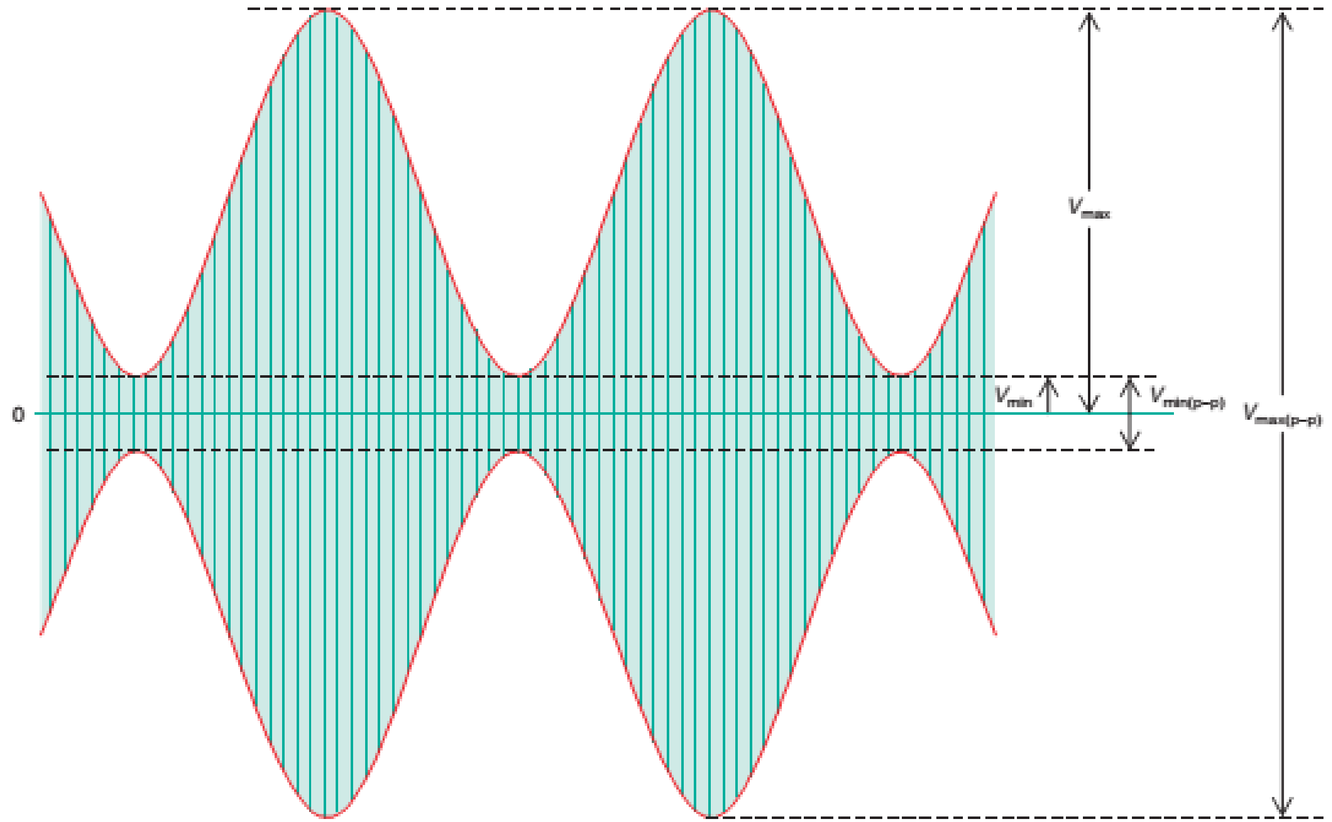
Modulation Index and Percentage of Modulation

As stated previously, for undistorted AM to occur, the modulating signal voltage V_m must be less than the carrier voltage V_c. Therefore the relationship between the amplitude of the modulating signal and the amplitude of the carrier signal is important. This relationship, known as the *modulation index m* (also called the modulating factor or coefficient, or the degree of modulation), is the ratio

$$m = \frac{V_m}{V_c}$$

These are the peak values of the signals, and the carrier voltage is the unmodulated value.

Multiplying the modulation index by 100 gives the *percentage of modulation*. For example, if the carrier voltage is 9 V and the modulating signal voltage is 7.5 V, the modulation factor is 0.8333 and the percentage of modulation is $0.833 \times 100 = 83.33$.



$$V_m = \frac{V_{max} - V_{min}}{2} \qquad V_c = \frac{V_{max} + V_{min}}{2}$$

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

Suppose that on an AM signal, the $V_{max(p-p)}$ value read from the graticule on the oscilloscope screen is 5.9 divisions and $V_{min(p-p)}$ is 1.2 divisions.

a. What is the modulation index?

$$m\frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{5.9 - 1.2}{5.9 + 1.2} = \frac{4.7}{7.1} = 0.662$$

b. Calculate V_c, V_m, and m if the vertical scale is 2 V per division. (*Hint:* Sketch the signal.)

$$V_c = \frac{V_{max} + V_{min}}{2} = \frac{5.9 + 1.2}{2} = \frac{7.1}{2} = 3.55 @ \frac{2\text{ V}}{\text{div}}$$

$$V_c = 3.55 \times 2\text{ V} = 7.1\text{ V}$$

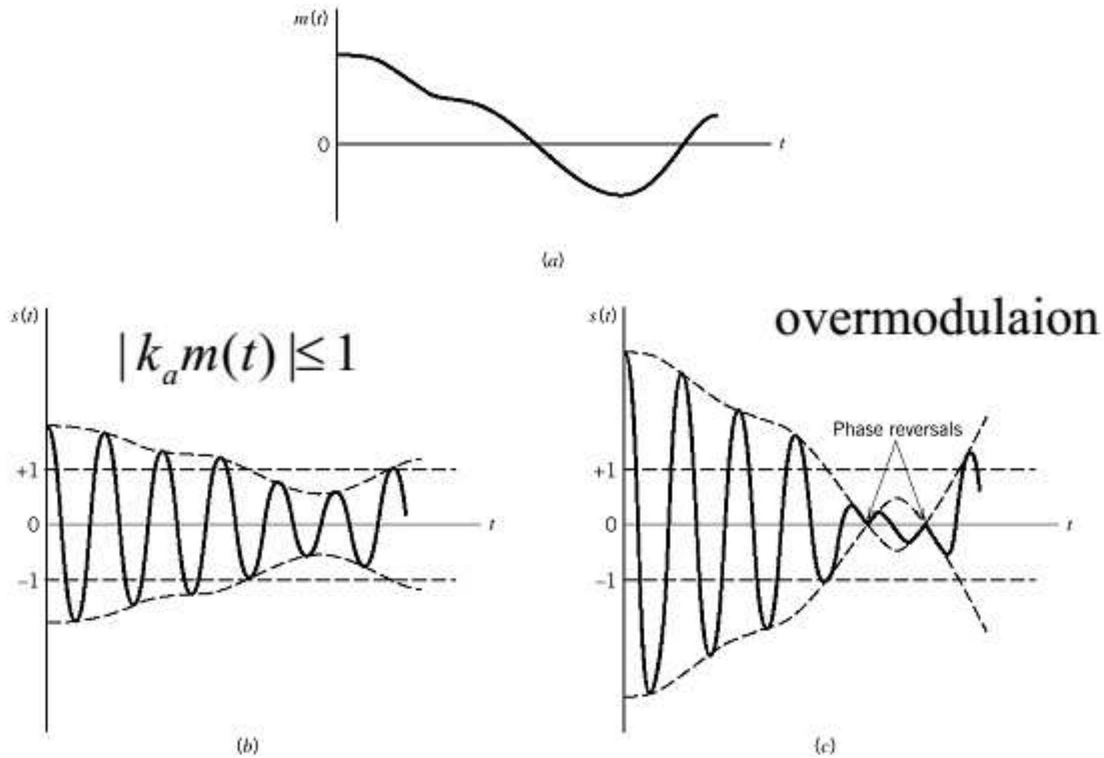
$$V_m = \frac{V_{max} - V_{min}}{2} = \frac{5.9 - 1.2}{2} = \frac{4.7}{2}$$

$$= 2.35 @ \frac{2\text{ V}}{\text{div}}$$

$$V_m = 2.35 \times 2\text{ V} = 4.7\text{ V}$$

$$m = \frac{V_m}{V_c} = \frac{4.7}{7.1} = 0.662$$

2.2 Overmodulation



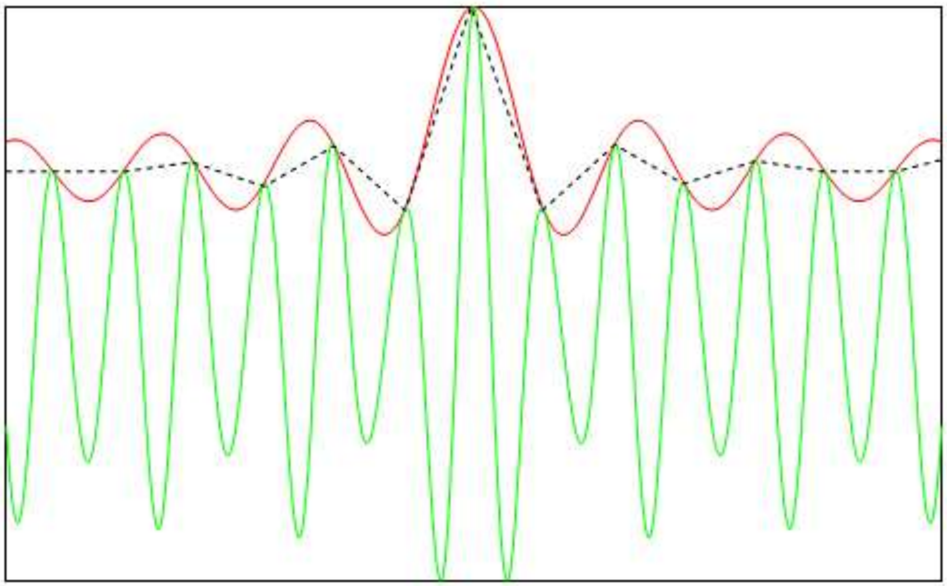
- The modulation index should be a number between 0 and 1.
- If the amplitude of the modulating voltage is higher than the carrier voltage, m will be greater than 1, causing distortion of the modulated waveform.
- If the distortion is great enough, the intelligence signal becomes unintelligible.
- Distortion of voice transmissions produces garbled, harsh, or unnatural sounds in the speaker.
- Distortion of video signals produces a scrambled and inaccurate picture on a TV screen.

- The modulating voltage is much greater than
- the carrier voltage, resulting in a condition called overmodulation.

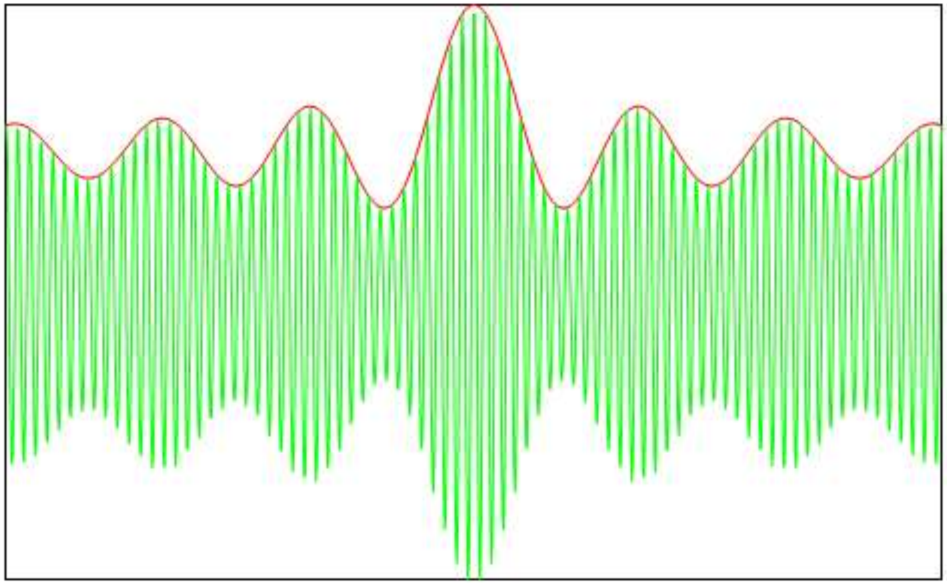
Preventing overmodulation is tricky.

- Normally, the amplitude of the modulating signal is adjusted so that only the voice peaks produce 100 percent modulation. This prevents overmodulation and distortion.
  - Automatic circuits called compression circuits solve this problem by amplifying the lower-level signals and suppressing or compressing the higher-level signals.
  - The result is a higher average power output level without overmodulation.
- Distortion caused by overmodulation also produces adjacent channel interference.
- Distortion produces a non sinusoidal information signal.
  - According to Fourier theory, any non sinusoidal signal can be treated as a fundamental sine wave at the frequency of the information signal plus harmonics.
  - Obviously, these harmonics also modulate the carrier and can cause interference with other signals on channels adjacent to the carrier.

2.2 Example of *Non-Visualized Envelope*



2.2 Example of *Visualized Envelope*

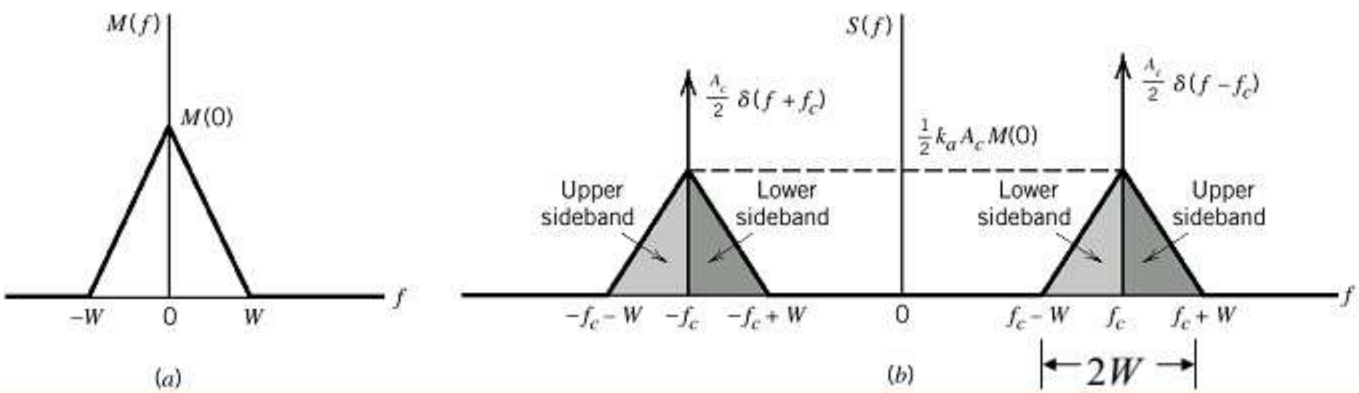


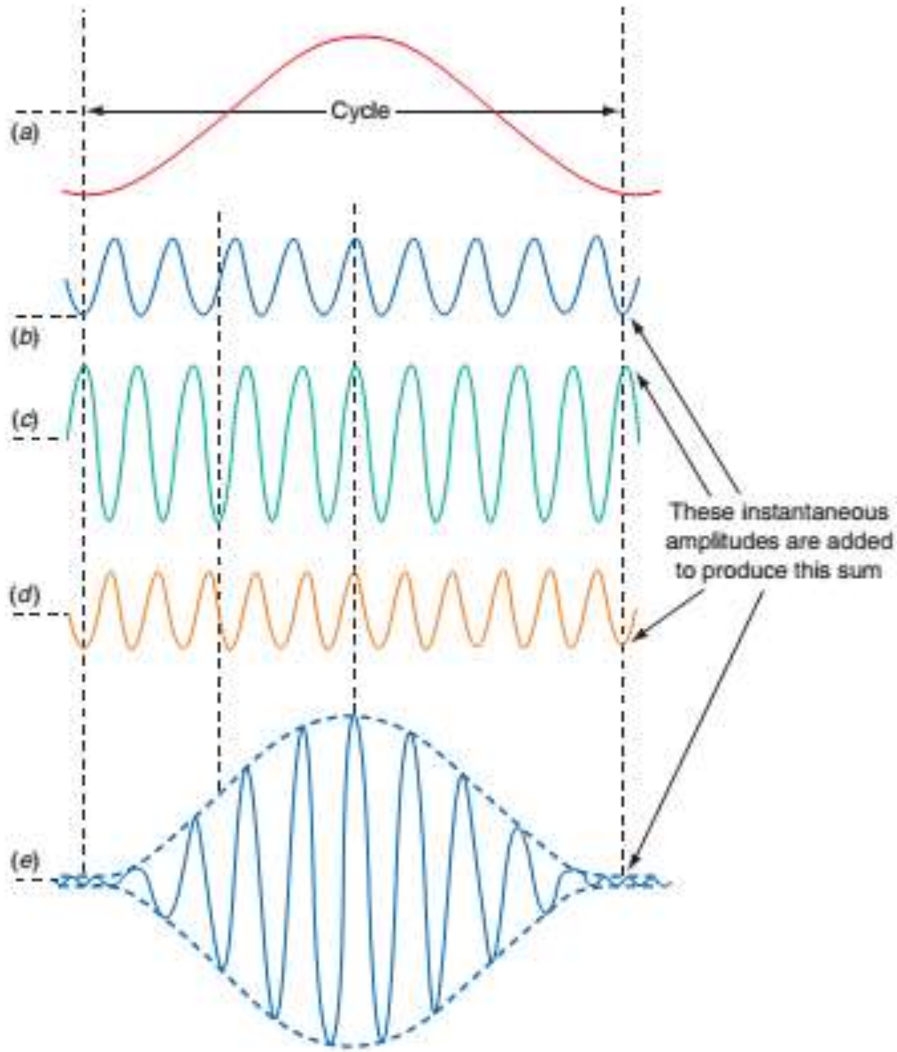
2.2 Transmission Bandwidth

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

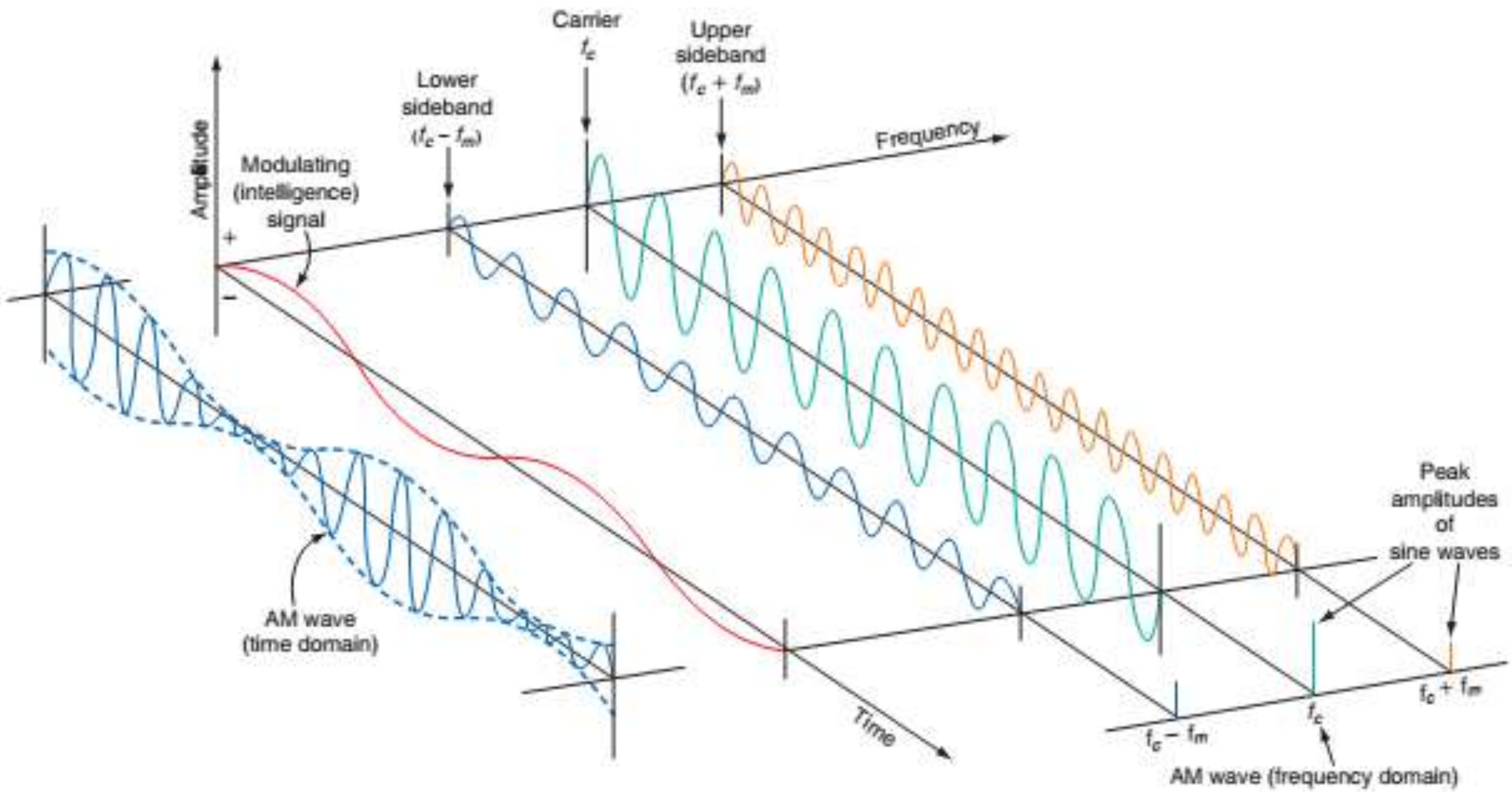
$$\Rightarrow S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

Transmission bandwidth $B_T = 2W$.





The AM wave is the algebraic sum of the carrier and upper and lower sideband sine waves. (*a*) Intelligence or modulating signal. (*b*) Lower sideband. (*c*) Carrier. (*d*) Upper sideband. (*e*) Composite AM wave.



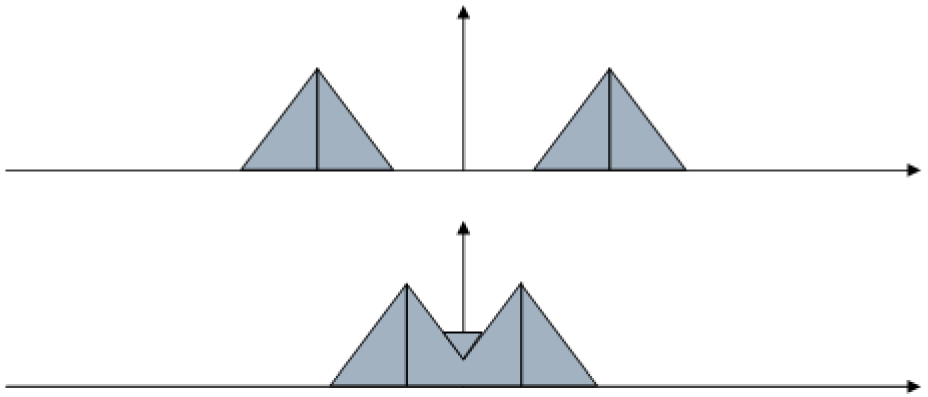
The relationship between the time and frequency domains.

2.2 Transmission Bandwidth

- Transmission bandwidth of an AM wave
  - For positive frequencies, the highest frequency component of the AM wave equals $f_c + W$, and the lowest frequency component equals $f_c - W$.
  - The difference between these two frequencies defines the transmission bandwidth B_T for an AM wave.

2.2 Transmission Bandwidth

- The condition of $f_c > W$ ensures that the sidebands do not overlap.



DEMODULATION OF AM SIGNALS

Demodulation: Restoring (or recovering) the message signal from the received modulated waveform that is generally corrupted by noise

Types of AM detectors:

Name of detector	Type of det.	Circuit used	Suitable for
Envelope detector	Noncoherent	Diode with an RC filter	DSB, VSB
Product detector	Coherent	Analog multiplier	Every AM

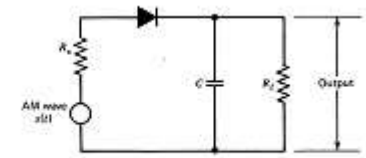
ENVELOPE DETECTOR

- On positive half-cycle of RF input signal $s(t)$ the diode is forward biased and the capacitor C charges up rapidly to the peak value of RF input signal
- When RF input falls below the output voltage then the diode becomes reverse-biased and the capacitor C discharges slowly through the load resistor R_l
- If

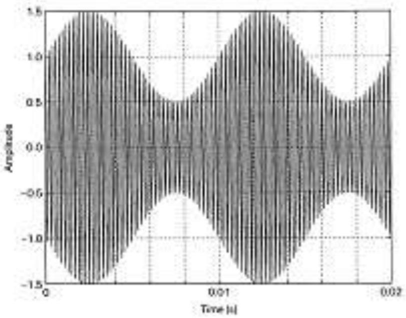
$$\frac{1}{f_c} << R_l C << \frac{1}{W}$$

then the average value of output voltage is equal to the message signal

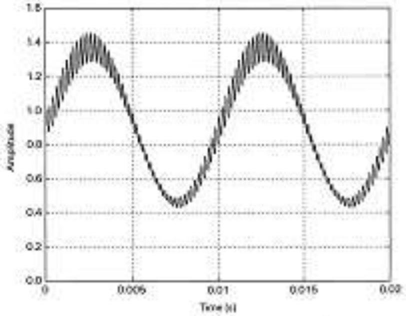
Circuit



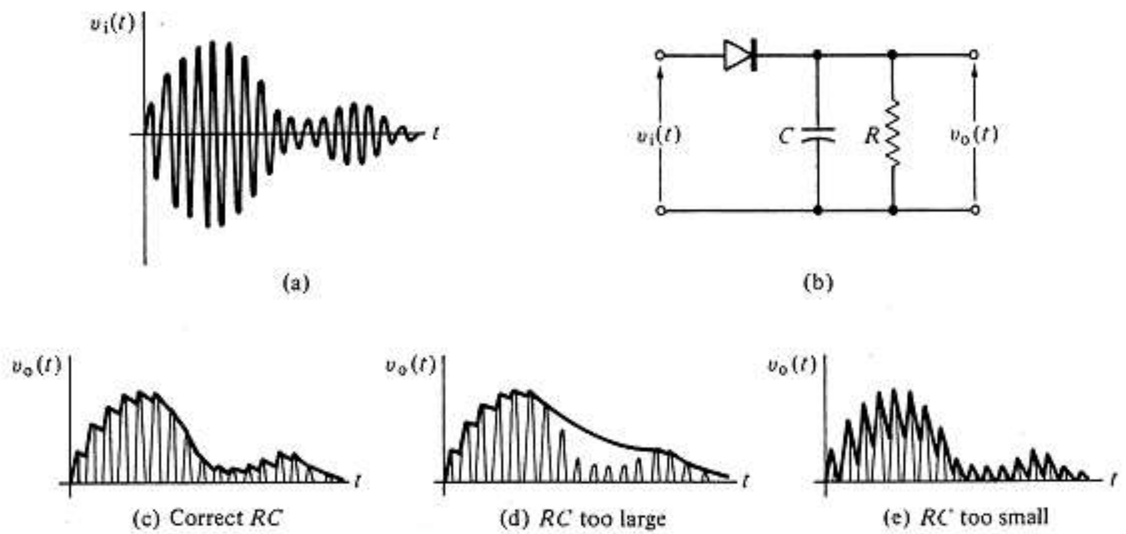
AM wave



Demo-dulated output



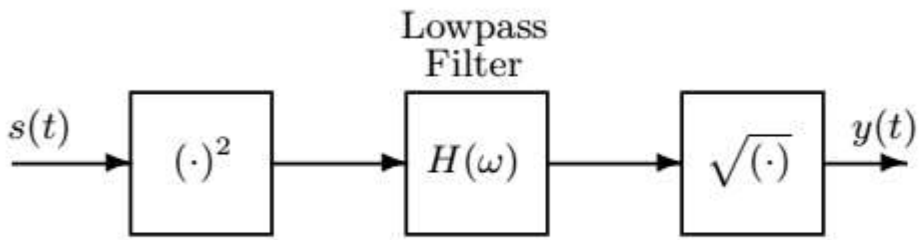
Distortion in envelope detector caused by the wrong time constant



Question: Why the frequency domain was not used in the analysis of envelope detector?

Recall: Envelope detector can be used to demodulate only DSB (no distortion) and VSB (little distortion) signals

Method 1: Square-Law Demodulation



Square-Law Envelope Detector

$$s(t) = A_c[1 + k_a m(t)] \cos \omega_c t$$

The baseband message $m(t)$ is a lowpass signal with cutoff frequency W, that is, $M(\omega) = 0$ for $|\omega| \geq W$.

The squarer output is

$$\begin{aligned} s^2(t) &= A_c^2[1 + k_a m(t)]^2 \cos^2 \omega_c t \\ &= 0.5A_c^2[1 + k_a m(t)]^2 \\ &\quad + 0.5A_c^2[1 + k_a m(t)]^2 \cos 2\omega_c t \end{aligned}$$

- The first term on the right-hand side is a lowpass signal except that the cutoff frequency has been increased to $2W$ by the squaring operation.

- The second term has a spectrum centered about $\pm 2\omega_c$. For positive frequencies, this spectrum is confined to the interval $(2\omega_c - 2W, 2\omega_c + 2W)$.

- The spectra for these two terms must not overlap. This requirement is met if

$$2W < 2\omega_c - 2W \quad \text{or} \quad \omega_c > 2W$$

- $H(\omega)$ is an ideal lowpass filter with cutoff frequency $2W$ so that its output is $0.5A_c^2[1 + k_a m(t)]^2$.

- The square-rooter output is proportional to $m(t)$ with a dc offset.

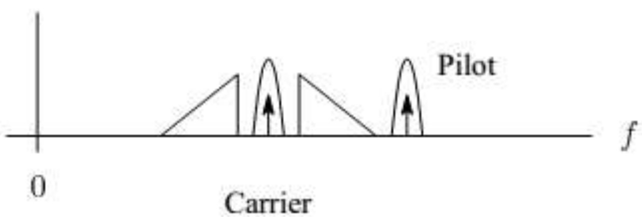
PRODUCT DETECTOR

In *coherent detection* or *synchronous demodulation* both the phase and frequency of carrier must be known at the detector. Carrier is recovered by the **carrier recovery circuit** at the receiver.

Techniques for providing the carrier signal:

1. Carrier is transmitted
2. A pilot signal is transmitted outside the pass-band of modulated signal

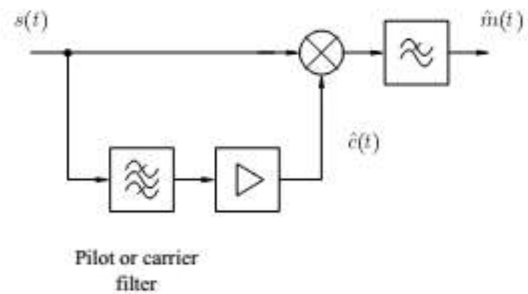
Spectrum of an AM signal (Only the positive-frequency side is shown)



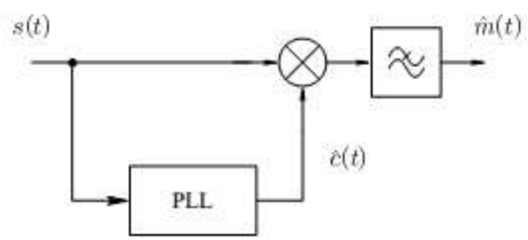
Note: Signals to be recovered are separated in the frequency domain (See frequency regions marked by curves) $\Longrightarrow$ Filtering is used

Techniques for recovering the carrier signal:

1. Recovery of carrier by a band-pass filter



2. Recovery of carrier by *phase-locked loop* (PLL)



Note: The demodulator contains a carrier recovery circuit [its output is the recovered carrier $\hat{c}(t)$] and a product detector (see the analog multiplier and low-pass filter)

Product detector

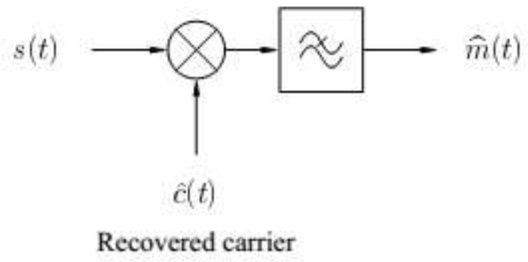
AM modulated input signal: $s(t) = A_c f[m(t)] \cos(2\pi f_c t)$

Recovered carrier: $\hat{c}(t) = A_r \cos(2\pi f_c t)$

Output of analog multiplier: $\widehat{m}(t) = s(t)\hat{c}(t) = \frac{A_c A_r}{2} f[m(t)] \left(1 + \cos[2\pi(2f_c)t]\right)$

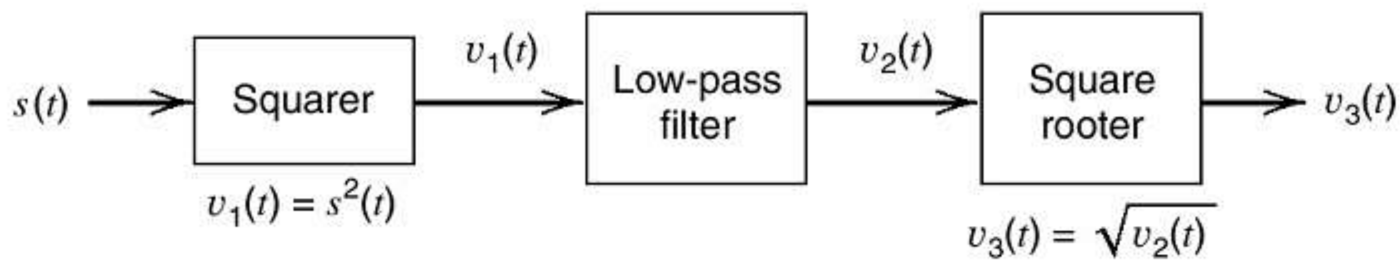
Note:
- The first term contains the message signal
- A low-pass filter is required to suppress the sum-frequency output

Block diagram of product detector



2.2 Virtues of Amplitude Modulation

- AM receiver can be implemented in terms of *simple circuit* with *inexpensive electrical components*.
  - E.g., AM receiver

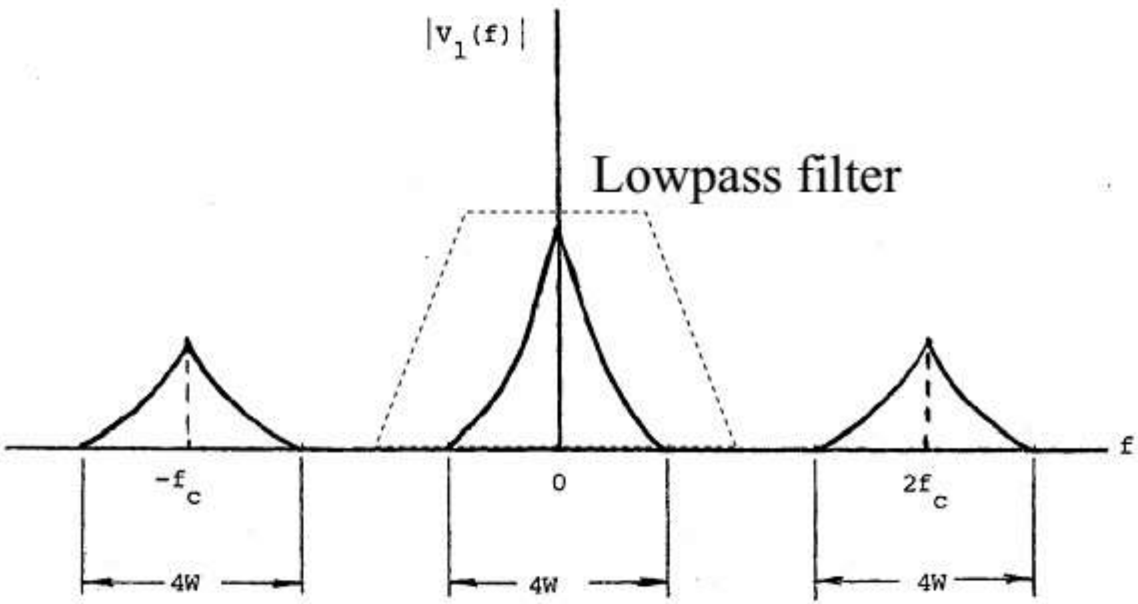


$$v_1(t) = s^2(t) = A_c^2[1 + k_a m(t)]^2 \cos^2(2\pi f_c t)$$

$$= \frac{A_c^2}{2}[1 + k_a m(t)]^2[1 + \cos(4\pi f_c t)]$$

2.2 Virtues of Amplitude Modulation

- The bandwidth of $m^2(t)$ is twice of $m(t)$. (So to speak, the bandwidth of $m(f)*m(f)$ is twice of $m(f)$.)



2.2 Virtues of Amplitude Modulation

- So if $2f_c > 4W$,

$$\Rightarrow v_2(t) = \frac{A_c^2}{2}[1 + k_a m(t)]^2$$

$$\Rightarrow v_3(t) = \frac{A_c}{\sqrt{2}}[1 + k_a m(t)]$$

$$\text{if } m(t) \text{ is zero mean} \overset{\text{block DC}}{\Rightarrow} \frac{A_c k_a}{\sqrt{2}} m(t)$$

By means of a squarer, the receiver can recover the information-bearing signal **without the need of a local carrier**.

2.2 Limitations of Amplitude Modulation (DSB-C)

- Wasteful of power and bandwidth

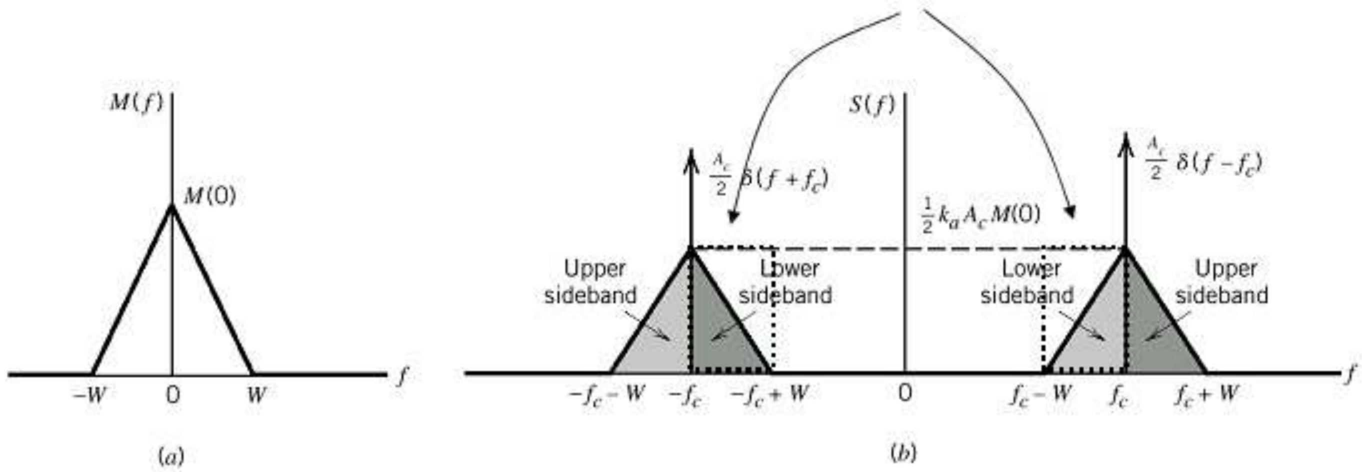
$$\begin{aligned} s(t) &= A_c[1 + k_a m(t)]\cos(2\pi f_c t) \\ &= \underbrace{A_c \cos(2\pi f_c t)}_{\text{with carrier}} + k_a m(t)\cos(2\pi f_c t) \end{aligned}$$

Waste of power in the information-less "with-carrier" part.

2.2 Limitations of Amplitude Modulation

- Wasteful of power and bandwidth

Only requires half of bandwidth after modulation



2.3 Linear Modulation

Definition

- Both $s_I(t)$ and $s_Q(t)$ in

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

are *linear* function of $m(t)$.

2.3 Linear Modulation

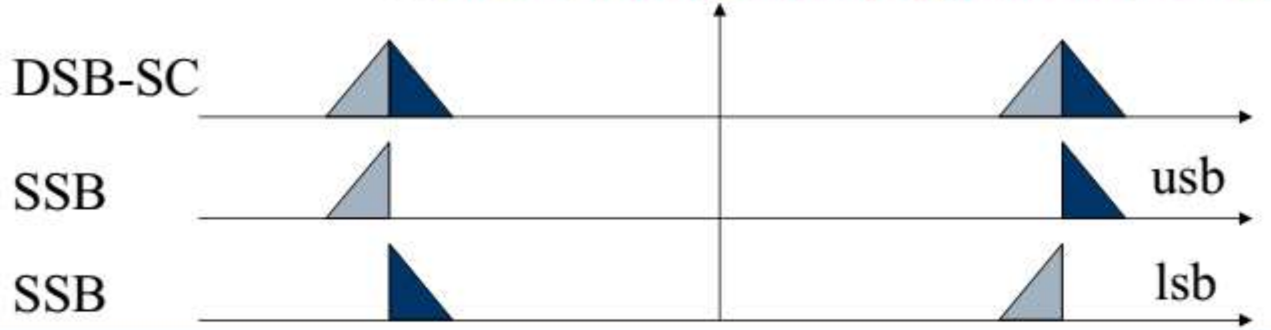
- For a single real-valued $m(t)$, three types of modulations can be identified according to how $s_Q(t)$ are *linearly* related to $m(t)$, at the case that $s_I(t)$ is exactly $m(t)$:
  - *(Some modulation may have $m_I(t)$ and $m_Q(t)$ that respectively bear independent information.)*

1. Double SideBand-Suppressed Carrier modulation (DSB-SC)
2. Single SideBand (SSB) modulation
3. Vestigial SideBand (VSB) modulation

2.3 DSB-SC and SSB

Type of modulation	$s_I(t)$	$s_Q(t)$	
DSB-SC	$m(t)$	0	
SSB	$m(t)$	$\hat{m}(t)$	Upper side band transmission
SSB	$m(t)$	$-\hat{m}(t)$	Lower side band transmission

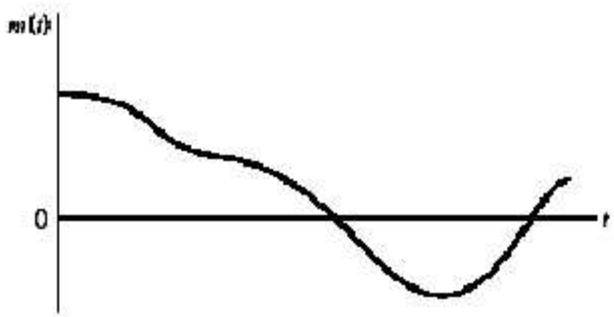
* $\hat{m}(t)$ = Hilbert transform of $m(t)$, which is used to completely "suppress" the other sideband.

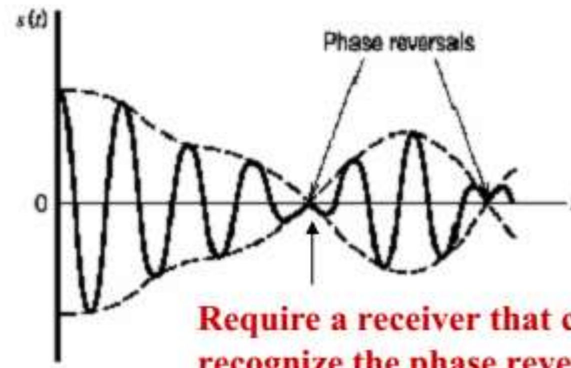


2.3 DSB-SC

- Different from DSB-C, DSB-SC $s(t)$ undergoes a *phase reversal* whenever $m(t)$ crosses zero.

$$s(t) = m(t)\cos(2\pi f_c t)$$

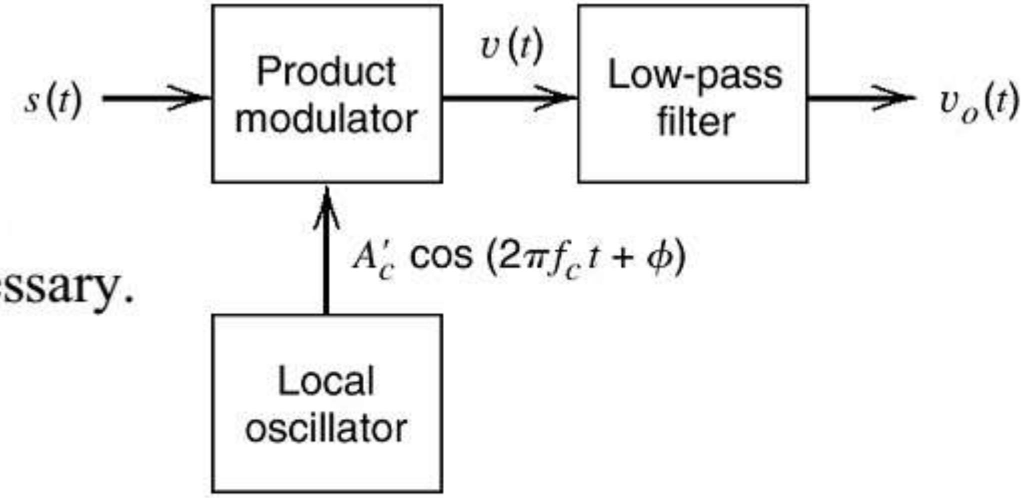




Require a receiver that can recognize the phase reversal!

2.3 Coherent Detection for DSB-SC

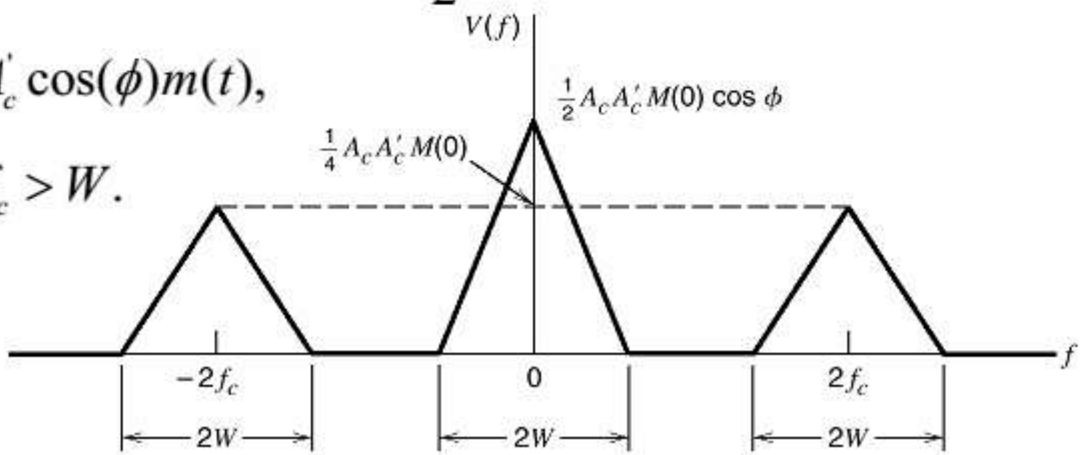
- For DSB-SC, we can no longer use the "envelope detector" (as used for DSB-C), in which no local carrier is required for the receiver.
- The *coherent detection* or *synchronous demodulation* becomes necessary.



2.3 Coherent Detection for DSB-SC

$$v(t) = A_c^{'}\cos(2\pi f_c t + \phi)s(t)$$
$$= A_c A_c^{'}\cos(2\pi f_c t)\cos(2\pi f_c t + \phi)m(t)$$
$$= \frac{1}{2}A_c A_c^{'}\cos(4\pi f_c t + \phi)m(t) + \frac{1}{2}A_c A_c^{'}\cos(\phi)m(t)$$
$$\xrightarrow{LowPass} \frac{1}{2}A_c A_c^{'}\cos(\phi)m(t),$$

provided $f_c > W$.



2.3 Coherent Detection for DSB-SC

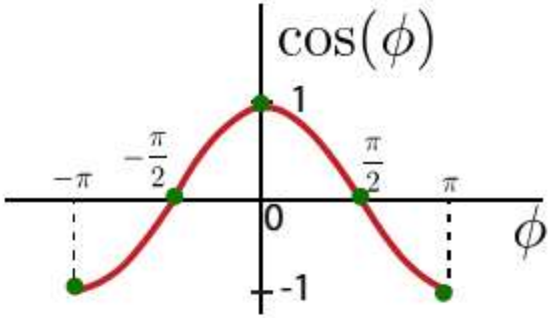
- *Quadrature null effect* of the coherent detector.
  - If $\phi = \pi/2$ or $-\pi/2$, the output of coherent detector for DSB-SC is *nullified*.
- If ϕ is not equal to either $\pi/2$ or $-\pi/2$, the output of coherent detector for DSB-SC is simply attenuated by a factor of $\cos(\phi)$, **if ϕ is a constant, independent of time.**
- However, in practice, ϕ often varies with time; therefore, it is necessary to have an additional mechanism to maintain the local carrier in the receiver in perfect synchronization with the local carrier in the transmitter.
- Such an additional mechanism adds the system complexity of the receiver.

$\boxed{\text{Costas Receiver}}$

Coherent Demodulation Output

$$v_0(t) = \frac{1}{2} A_c A_c' \cos(\phi) m(t)$$

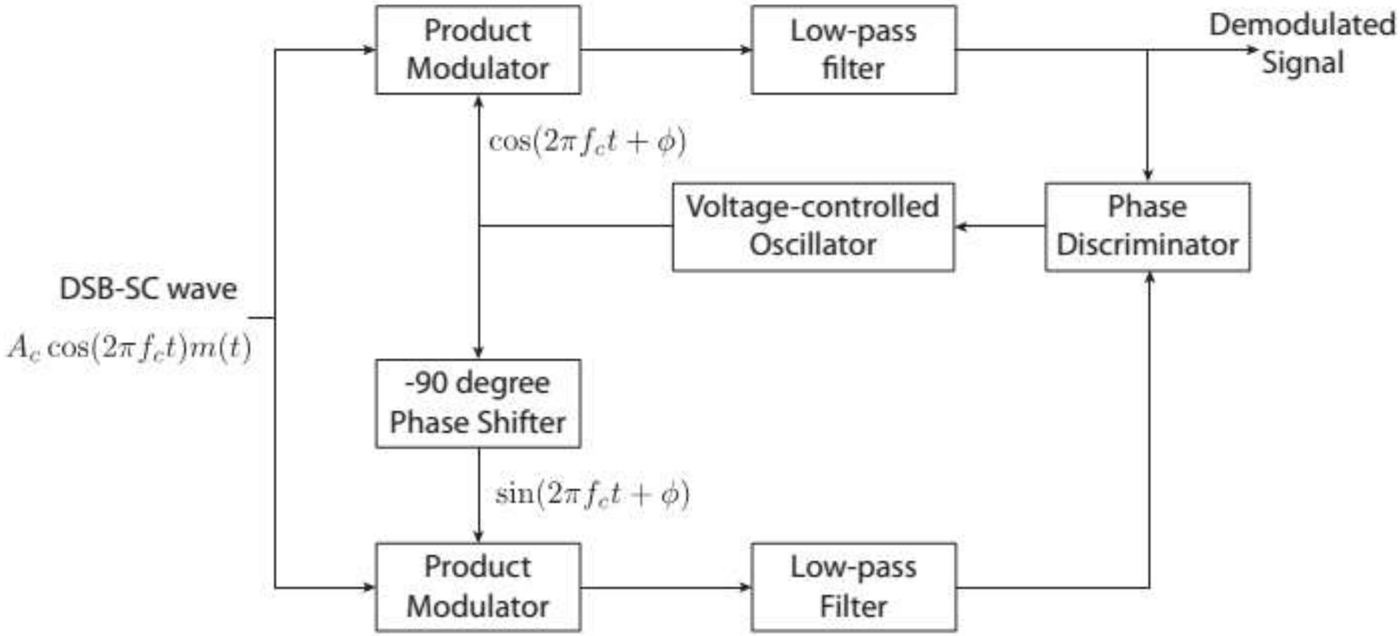
- To maximize $v_0(t)$, would like $\phi \approx 0$.



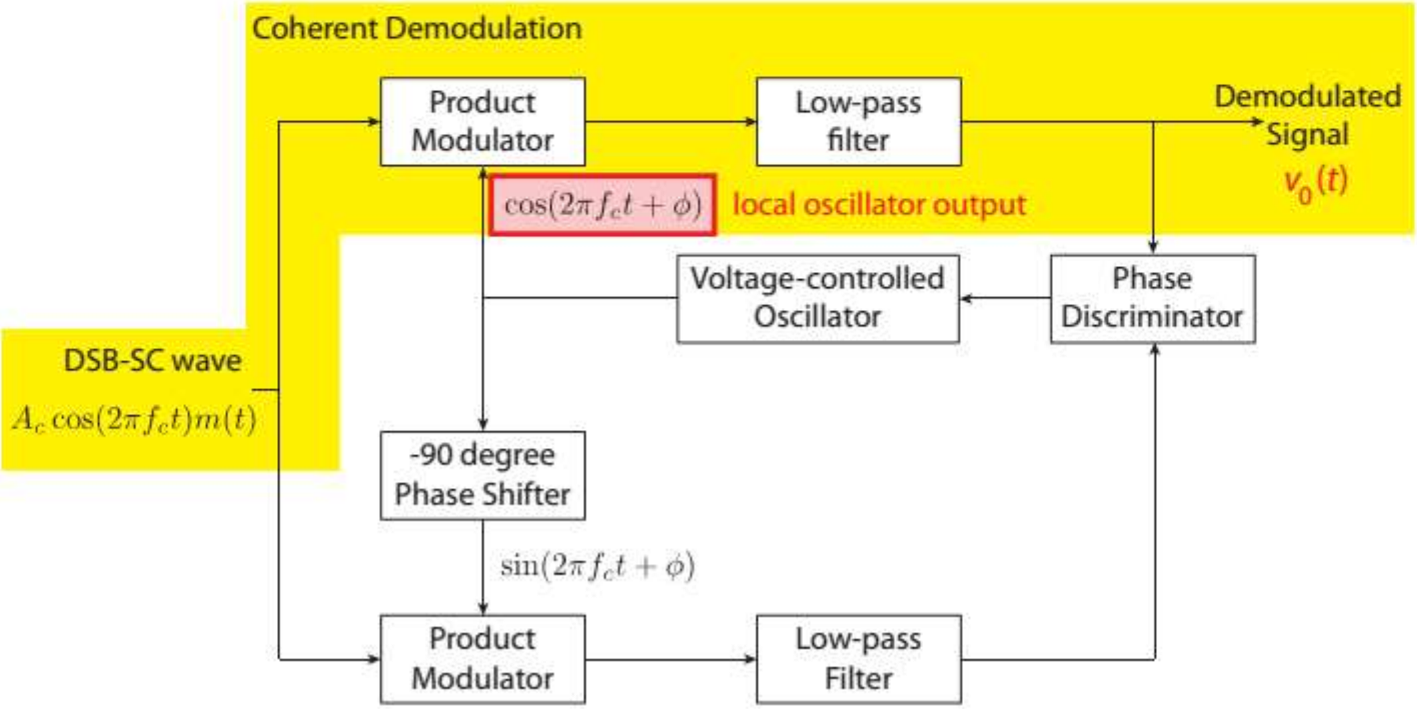
- For $v_0(t)$ to be proportional to $m(t)$, ϕ should be constant.

Thus, we wish to synchronize the local oscillator.
Let $A_c' = 1$ for simplicity.

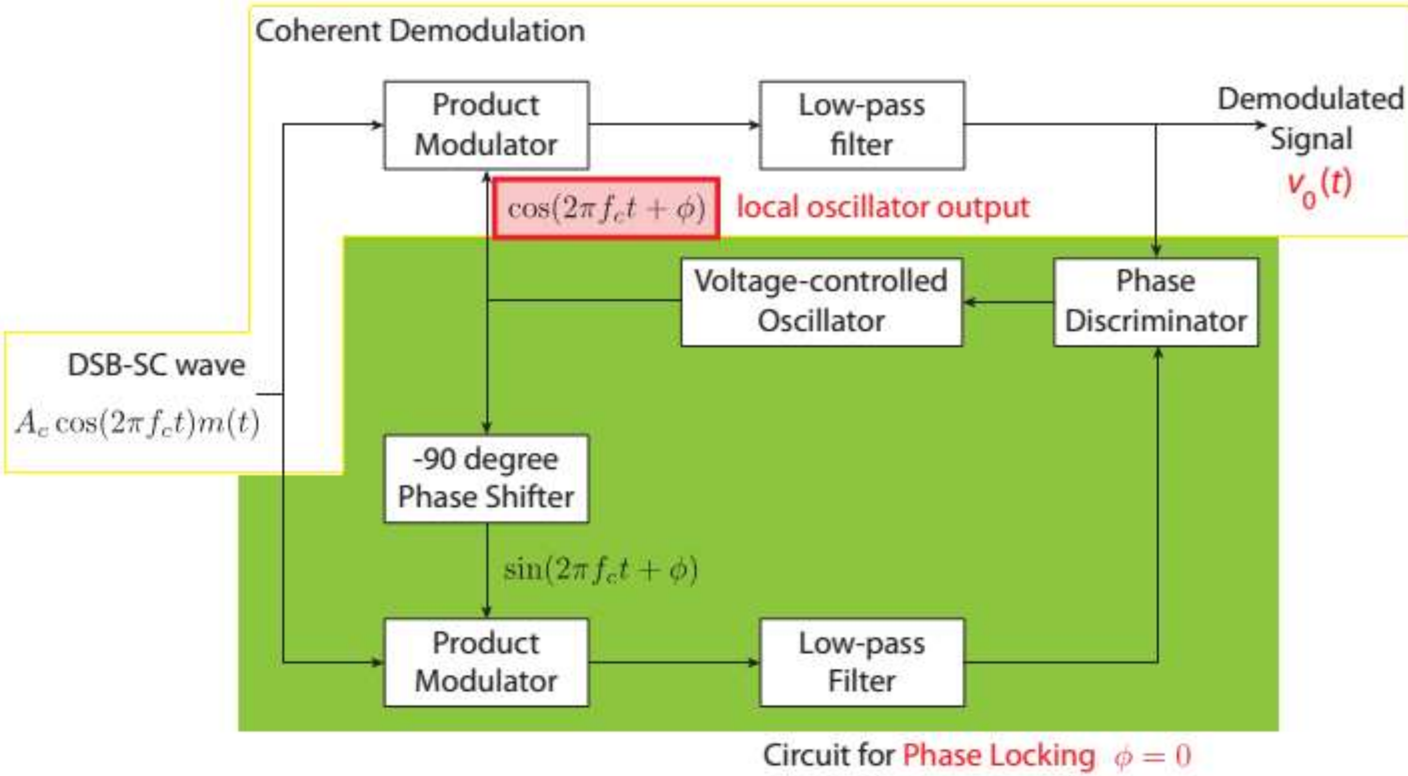
Costas Receiver



Costas Receiver: Coherent Demodulation

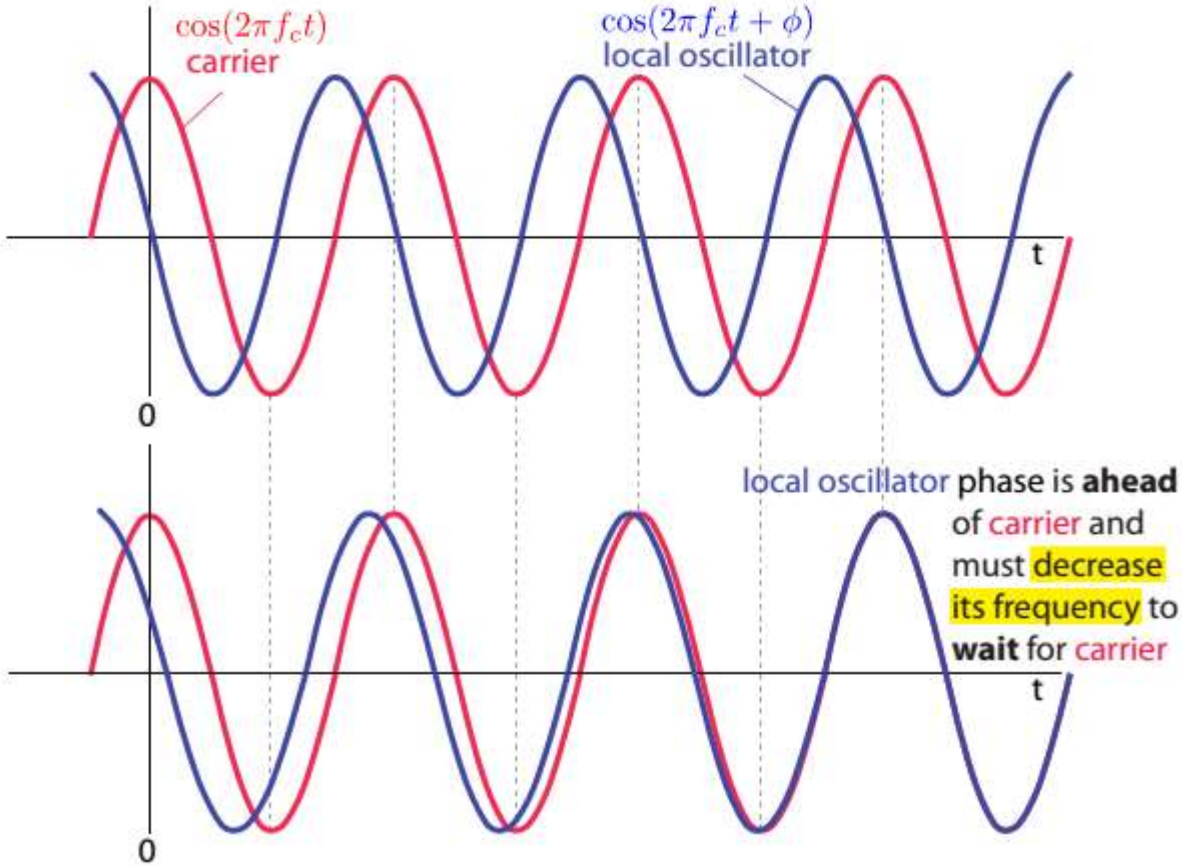


Costas Receiver: Phase Lock Circuit



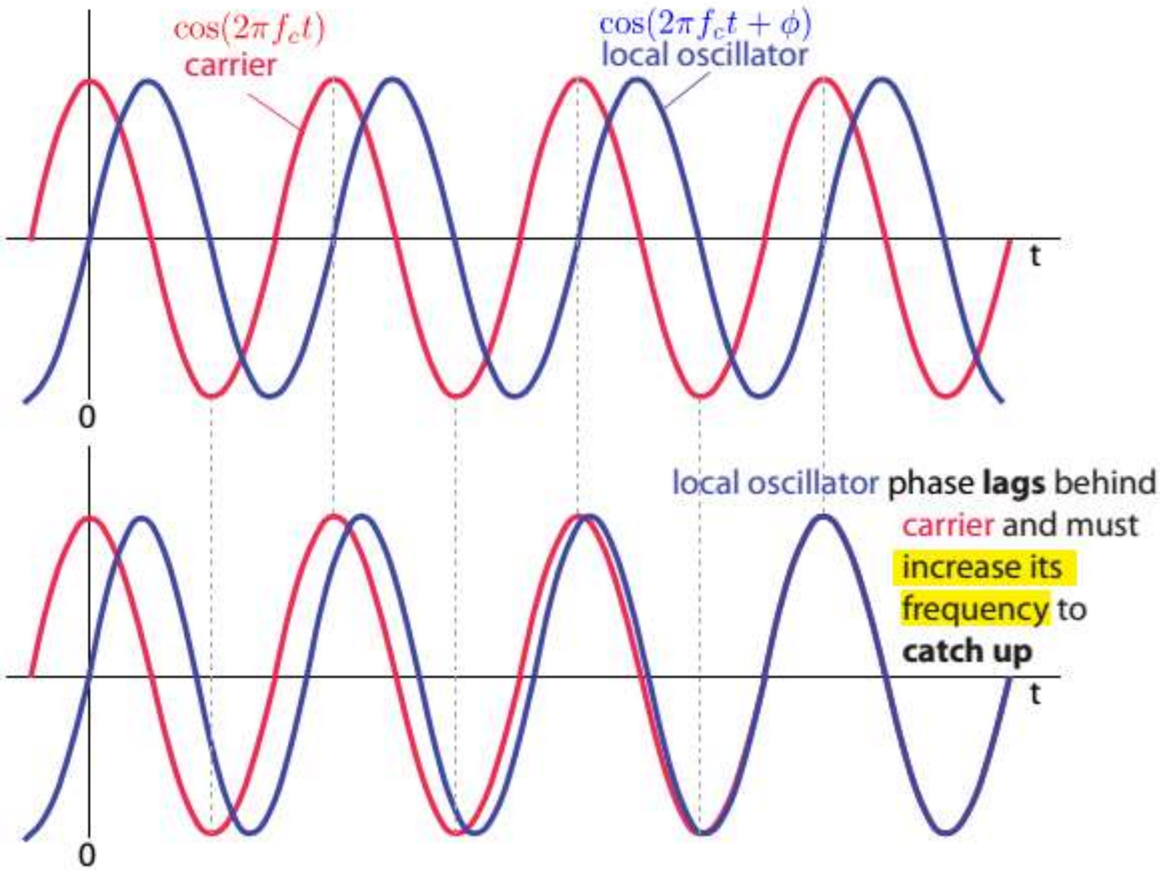
Costas Receiver: Phase Lock Circuit

$\phi > 0$: Freq of local oscillator needs to temporarily decrease

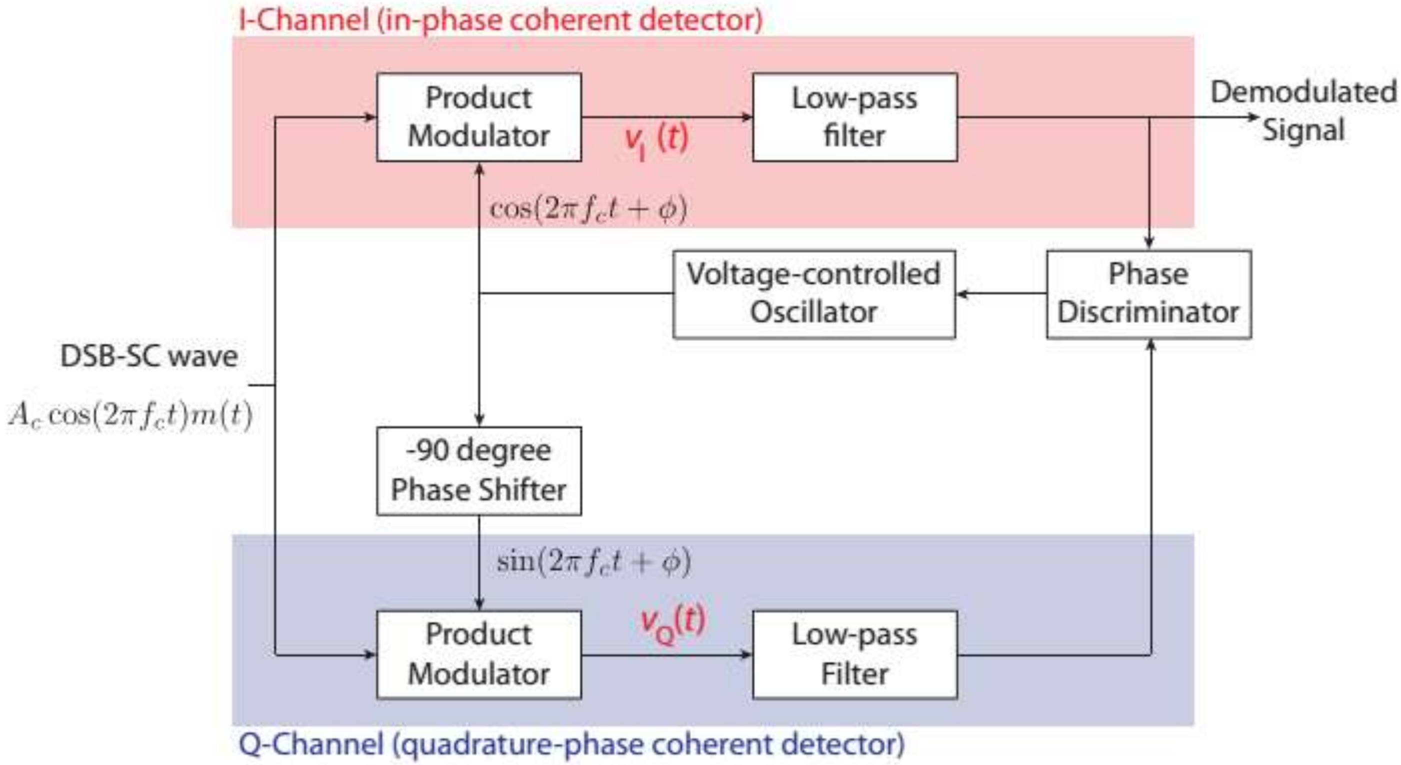


Costas Receiver: Phase Lock Circuit

$\phi < 0$: Freq of local oscillator needs to temporarily increase



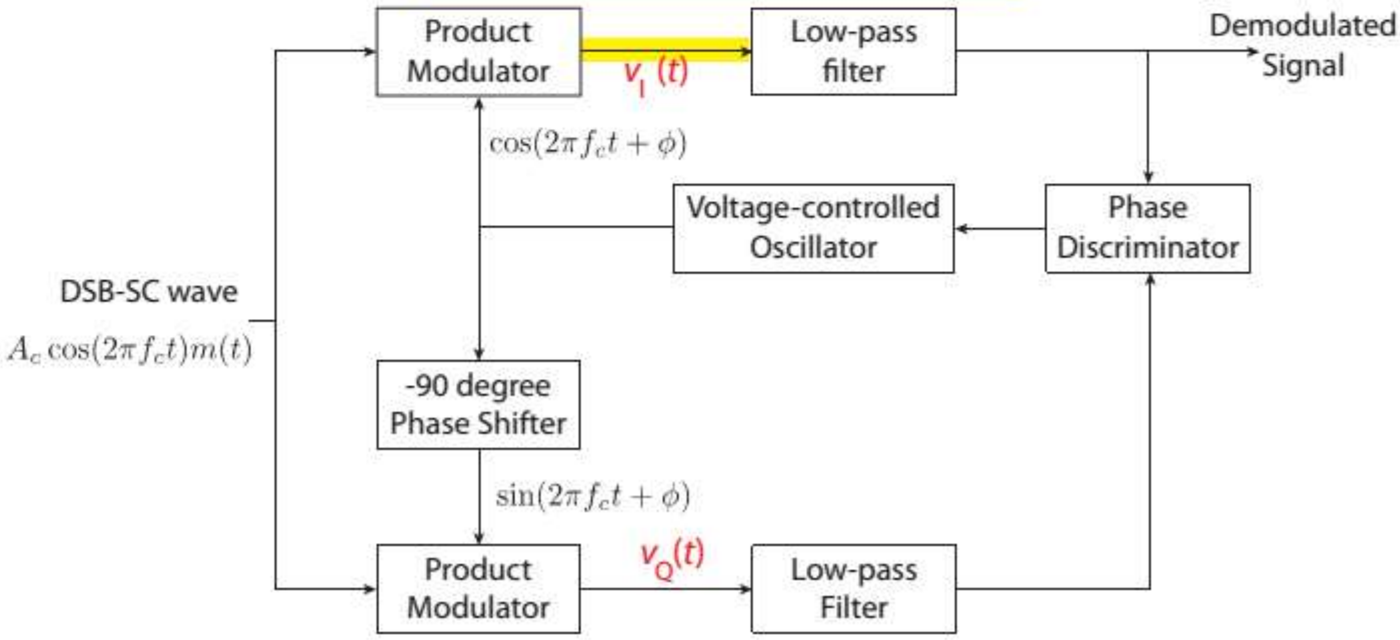
Costas Receiver: In-Phase and Quadrature-Phase



Costas Receiver: In-Phase Coherent Detector

$\cos(A)\cos(B) = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(B-A)$

$$A_c \cos(2\pi f_c t)\cos(2\pi f_c t + \phi)m(t)$$
$$= \frac{A_c}{2}\cos(4\pi f_c t + \phi)m(t) + \frac{A_c}{2}\cos(\phi)m(t)$$



Costas Receiver: In-Phase Coherent Detector

$$\begin{aligned} s(t) &= A_c \cos(2\pi f_c t) m(t) \\ v_I(t) &= s(t) \cdot \cos(2\pi f_c t + \phi) \end{aligned}$$

Recall

$$\cos(2\pi f_c t + \phi) = \frac{e^{j\phi}}{2} e^{j2\pi f_c t} + \frac{e^{-j\phi}}{2} e^{-j2\pi f_c t} \rightleftharpoons \frac{e^{j\phi}}{2}\delta(f - f_c) + \frac{e^{-j\phi}}{2}\delta(f + f_c)$$

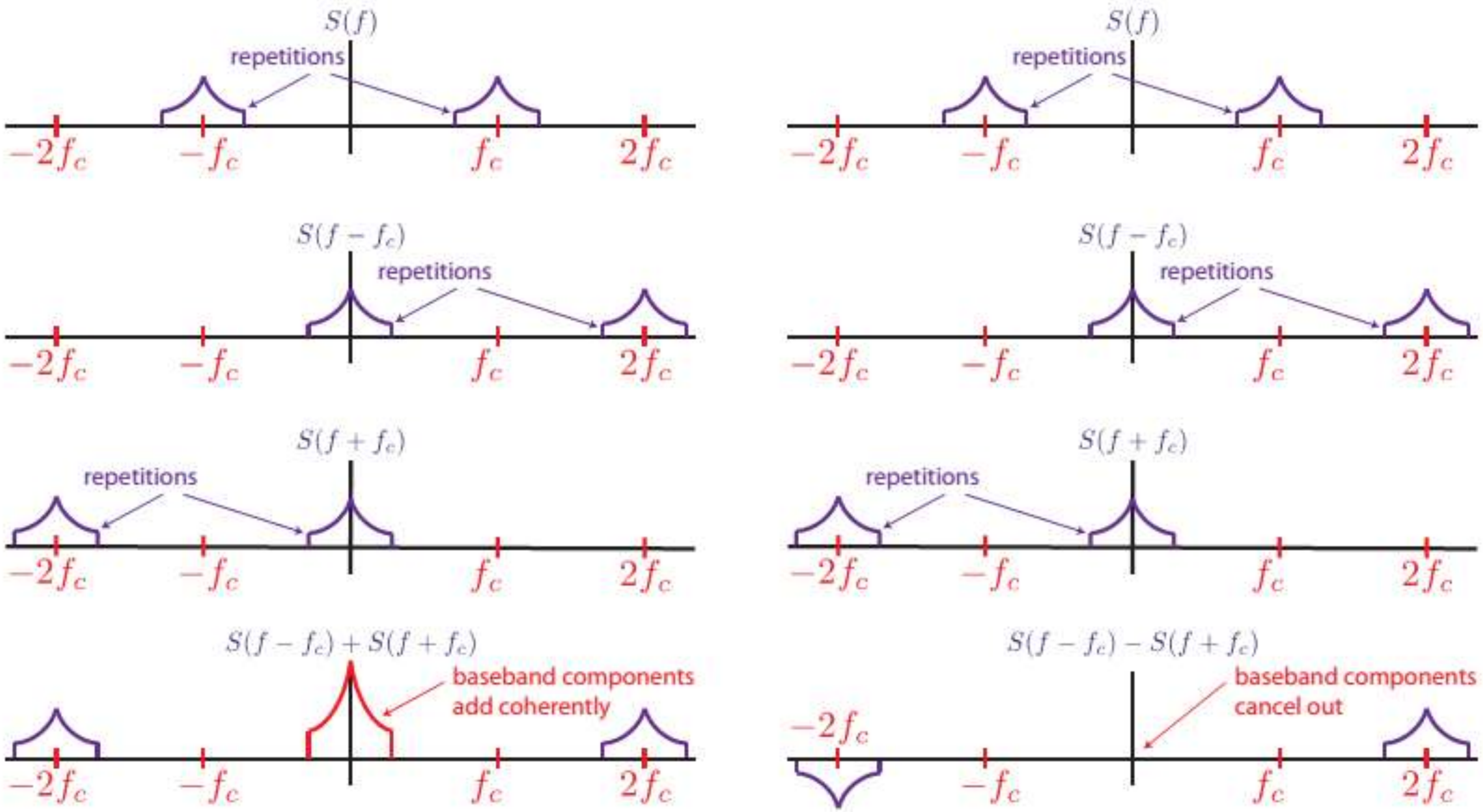
$$\begin{aligned} V_I(f) &= S(f) \star \left[\frac{e^{j\phi}}{2}\delta(f - f_c) + \frac{e^{-j\phi}}{2}\delta(f + f_c)\right] \\ &= \frac{e^{j\phi}}{2} S(f) \star \delta(f - f_c) + \frac{e^{-j\phi}}{2} S(f) \star \delta(f + f_c) \\ &= \frac{e^{j\phi}}{2} S(f - f_c) + \frac{e^{-j\phi}}{2} S(f + f_c) \end{aligned}$$

Costas Receiver: In-Phase Coherent Detector

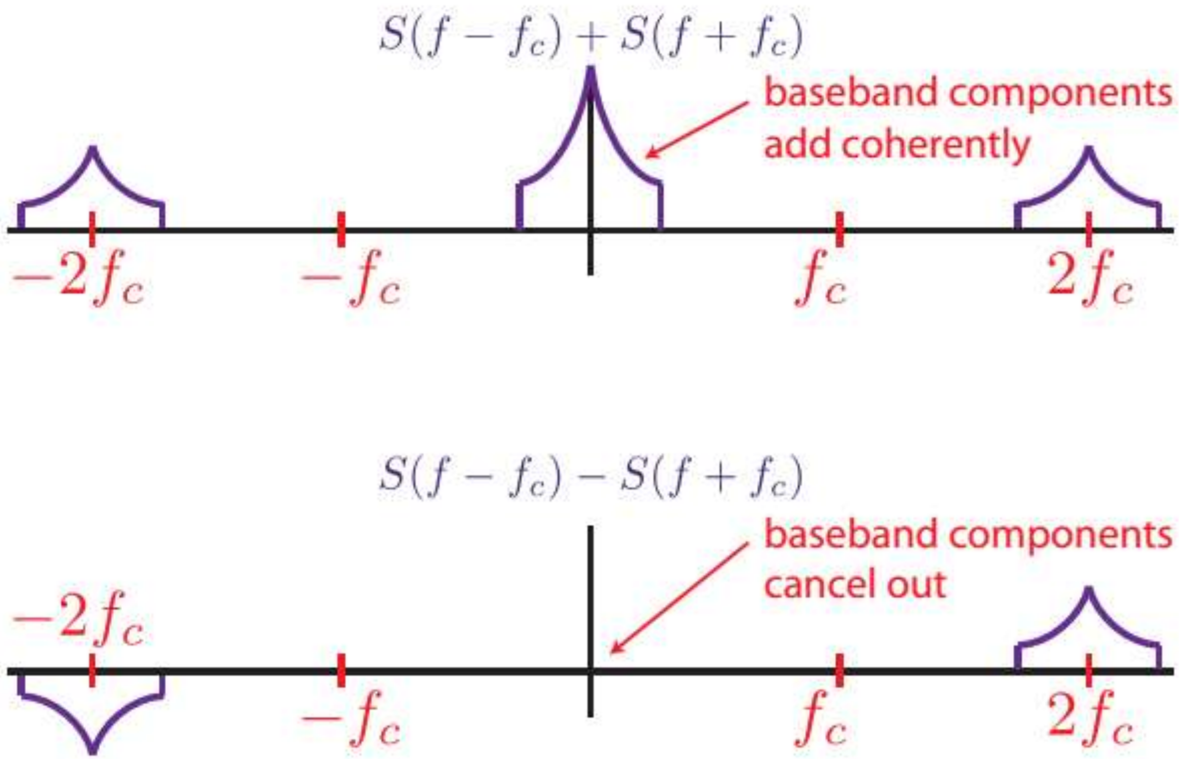
$$
\begin{aligned}
V_I(f) &= \frac{e^{j\phi}}{2}S(f-f_c)+\frac{e^{-j\phi}}{2}S(f+f_c)\\
&= \frac{\cos(\phi)+j\sin(\phi)}{2}S(f-f_c)+\frac{\cos(-\phi)+j\sin(-\phi)}{2}S(f+f_c)\\
&= \frac{\cos(\phi)+j\sin(\phi)}{2}S(f-f_c)+\frac{\cos(\phi)-j\sin(\phi)}{2}S(f+f_c)\\
&= \underbrace{\frac{\cos(\phi)}{2}}_{\approx 1/2}[S(f-f_c)+S(f+f_c)]+j\underbrace{\frac{\sin(\phi)}{2}}_{\text{small}}[S(f-f_c)-S(f+f_c)]
\end{aligned}
$$

for $\phi \ll 1$.

Costas Receiver: In-Phase Coherent Detector



Costas Receiver: In-Phase Coherent Detector



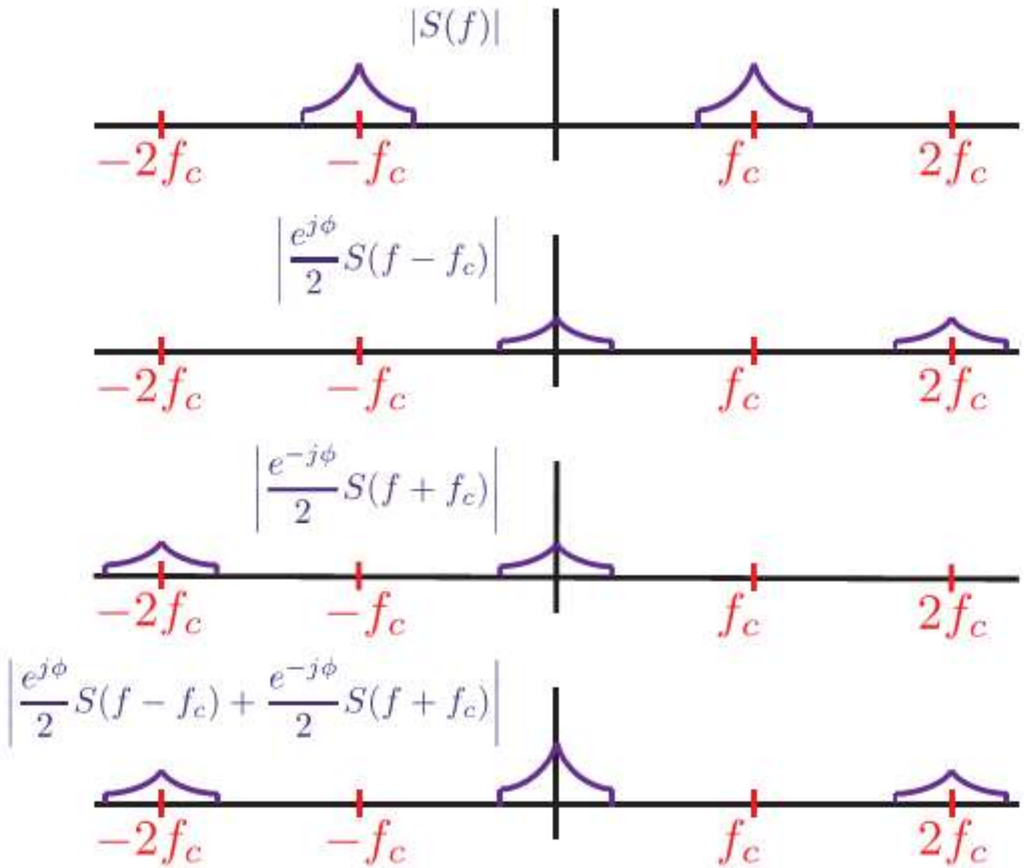
Costas Receiver: In-Phase Coherent Detector

$$
\begin{aligned}
V_I(f) &= \frac{e^{j\phi}}{2} S(f-f_c) + \frac{e^{-j\phi}}{2} S(f+f_c) \\
&= \frac{\cos(\phi) + j\sin(\phi)}{2} S(f-f_c) + \frac{\cos(-\phi) + j\sin(-\phi)}{2} S(f+f_c) \\
&= \frac{\cos(\phi) + j\sin(\phi)}{2} S(f-f_c) + \frac{\cos(\phi) - j\sin(\phi)}{2} S(f+f_c) \\
&= \underbrace{\frac{\cos(\phi)}{2}}_{\approx 1/2} \left[\underbrace{S(f-f_c) + S(f+f_c)}_{\text{significant baseband}} \right] + j \underbrace{\frac{\sin(\phi)}{2}}_{\text{small}} \left[\underbrace{S(f-f_c) - S(f+f_c)}_{\text{negligible baseband}} \right]
\end{aligned}
$$

for $\phi \ll 1$.

The closer ϕ is to zero, the more significant the baseband term of $v_I(t)$ and vice versa.

Costas Receiver: In-Phase Coherent Detector

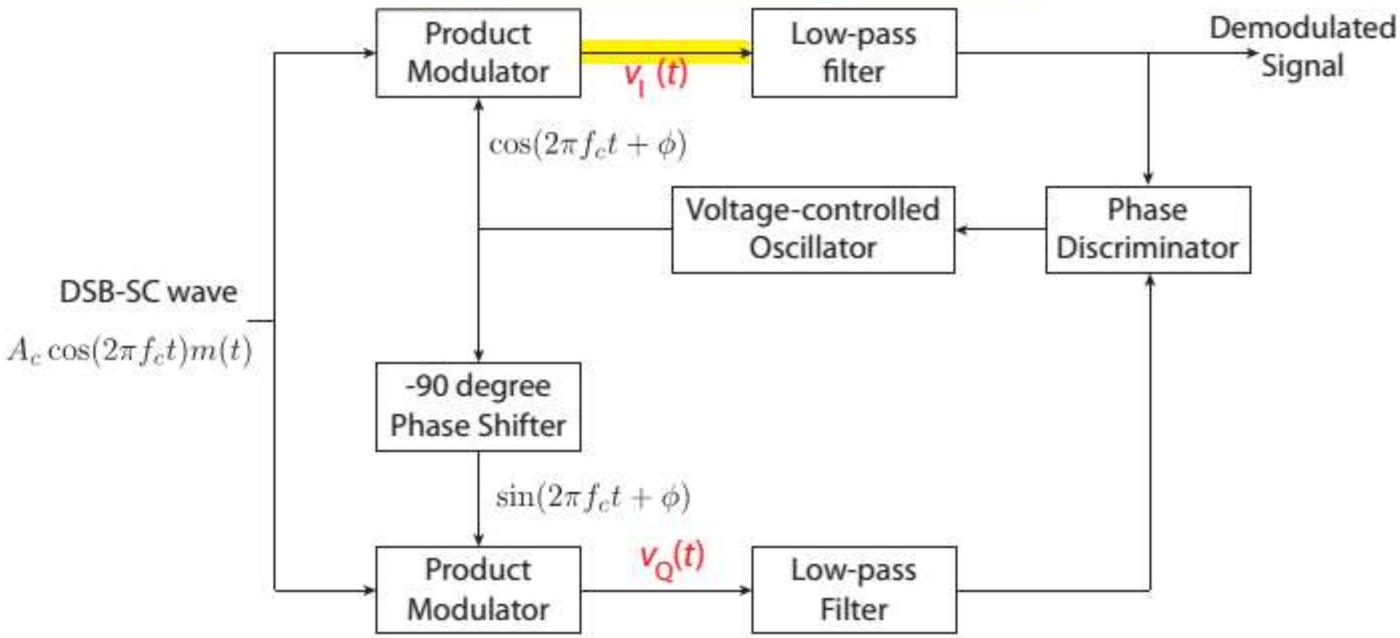


For ϕ small.

Costas Receiver: Quadrature-Phase Detector

$\cos(A)\cos(B) = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(B-A)$

$$A_c\cos(2\pi f_c t)\cos(2\pi f_c t + \phi)m(t)$$
$$= \frac{A_c}{2}\cos(4\pi f_c t + \phi)m(t) + \frac{A_c}{2}\cos(\phi)m(t)$$



$\cos(A)\sin(B) = \frac{1}{2}\sin(A+B) + \frac{1}{2}\sin(B-A)$

$$A_c\cos(2\pi f_c t)\sin(2\pi f_c t + \phi)m(t)$$
$$= \frac{A_c}{2}\sin(4\pi f_c t + \phi)m(t) + \frac{A_c}{2}\sin(\phi)m(t)$$

Costas Receiver: Quadrature-Phase Detector

$$
\begin{aligned}
s(t) &= A_c \cos(2\pi f_c t) m(t) \\
v_Q(t) &= s(t) \cdot \sin(2\pi f_c t + \phi)
\end{aligned}
$$

Recall

$$\sin(2\pi f_c t + \phi) = \frac{e^{j\phi}}{2j} e^{j2\pi f_c t} - \frac{e^{-j\phi}}{2j} e^{-j2\pi f_c t} \rightleftharpoons \frac{e^{j\phi}}{2j}\delta(f - f_c) - \frac{e^{-j\phi}}{2j}\delta(f + f_c)$$

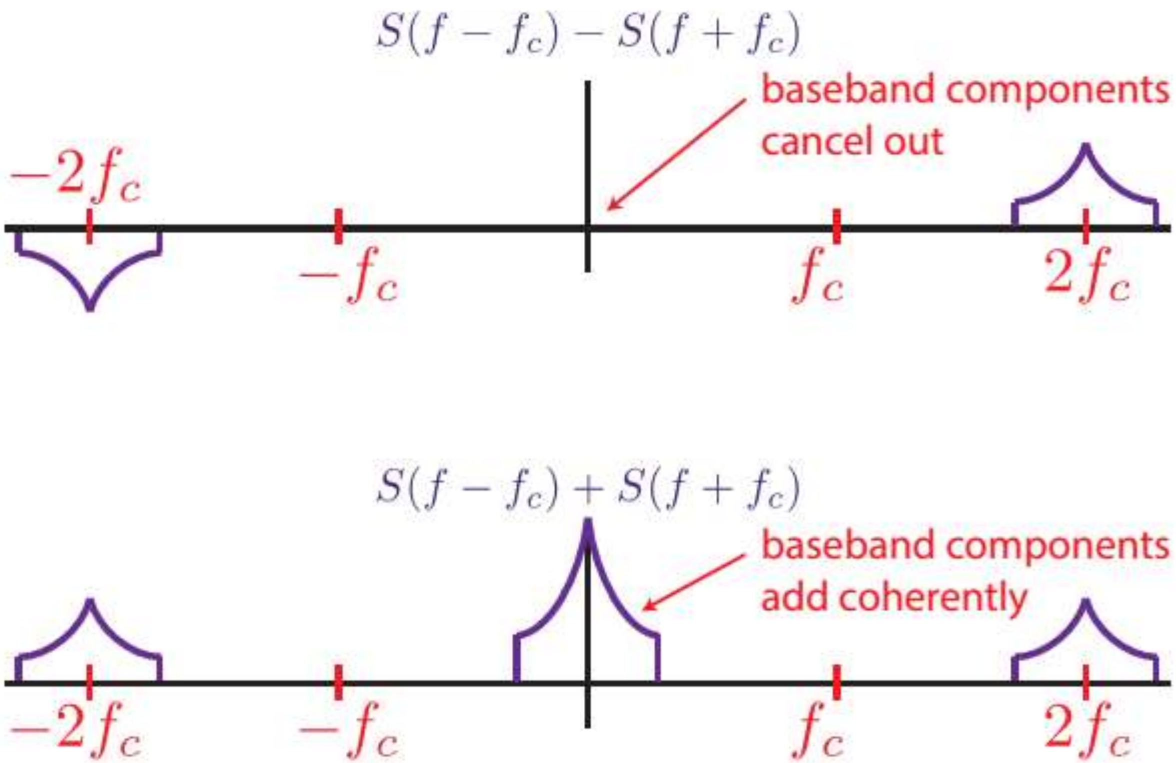
$$
\begin{aligned}
V_Q(f) &= S(f) \star \left[\frac{e^{j\phi}}{2j}\delta(f - f_c) - \frac{e^{-j\phi}}{2j}\delta(f + f_c)\right] \\
&= \frac{e^{j\phi}}{2j} S(f) \star \delta(f - f_c) - \frac{e^{-j\phi}}{2j} S(f) \star \delta(f + f_c) \\
&= \frac{e^{j\phi}}{2j} S(f - f_c) - \frac{e^{-j\phi}}{2j} S(f + f_c)
\end{aligned}
$$

Costas Receiver: Quadrature-Phase Detector

$$
\begin{aligned}
V_Q(f) &= \frac{e^{j\phi}}{2j}S(f-f_c) - \frac{e^{-j\phi}}{2j}S(f+f_c) \\
&= \frac{\cos(\phi)+j\sin(\phi)}{2j}S(f-f_c) - \frac{\cos(-\phi)+j\sin(-\phi)}{2j}S(f+f_c) \\
&= \frac{\cos(\phi)+j\sin(\phi)}{2j}S(f-f_c) - \frac{\cos(\phi)-j\sin(\phi)}{2j}S(f+f_c) \\
&= \underbrace{\frac{\cos(\phi)}{2j}}_{\approx 1/2j}[S(f-f_c)-S(f+f_c)] + j\underbrace{\frac{\sin(\phi)}{2j}}_{\text{small}}[S(f-f_c)+S(f+f_c)]
\end{aligned}
$$

for $\phi \ll 1$.

Costas Receiver: Quadrature-Phase Detector



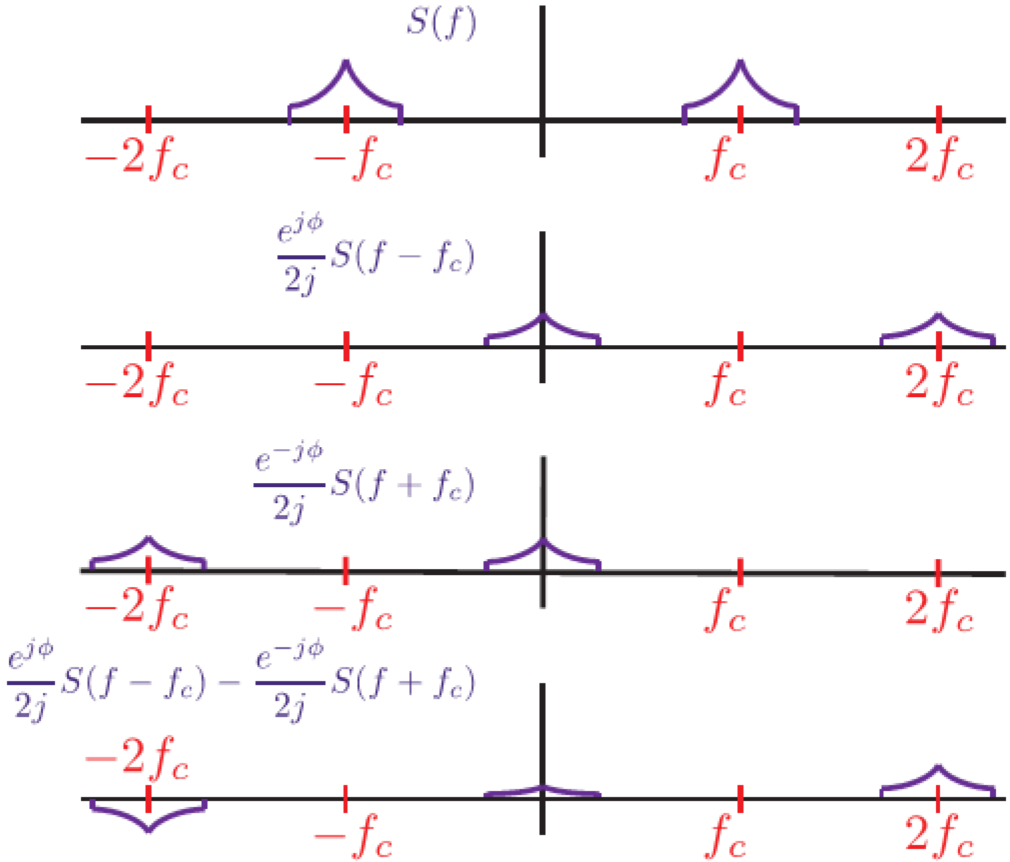
Costas Receiver: Quadrature-Phase Detector

$$
\begin{aligned}
V_Q(f) &= \frac{e^{j\phi}}{2j} S(f - f_c) - \frac{e^{-j\phi}}{2j} S(f + f_c) \\
&= \frac{\cos(\phi) + j\sin(\phi)}{2j} S(f - f_c) - \frac{\cos(-\phi) + j\sin(-\phi)}{2j} S(f + f_c) \\
&= \frac{\cos(\phi) + j\sin(\phi)}{2j} S(f - f_c) - \frac{\cos(\phi) - j\sin(\phi)}{2j} S(f + f_c) \\
&= \underbrace{\frac{\cos(\phi)}{2j}}_{\approx 1/2j} \Bigg[\underbrace{S(f - f_c) - S(f + f_c)}_{\text{negligible baseband}} \Bigg] + j \underbrace{\frac{\sin(\phi)}{2j}}_{\text{small}} \Bigg[\underbrace{S(f - f_c) + S(f + f_c)}_{\text{significant baseband}} \Bigg]
\end{aligned}
$$

for $\phi \ll 1$.

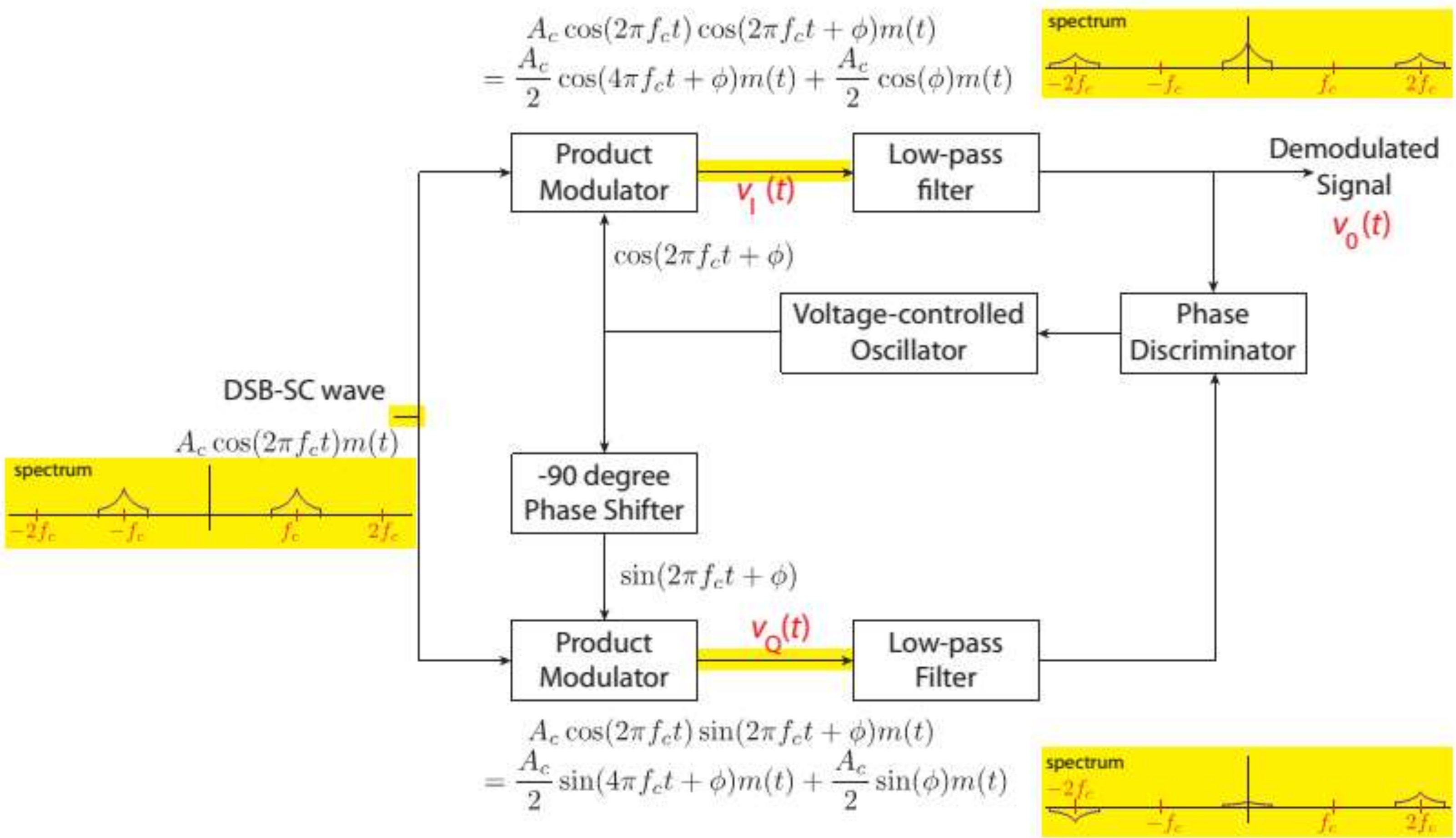
The closer ϕ is to zero, the more negligible the baseband term of $v_Q(t)$ and vice versa.

Costas Receiver: Quadrature-Phase Detector

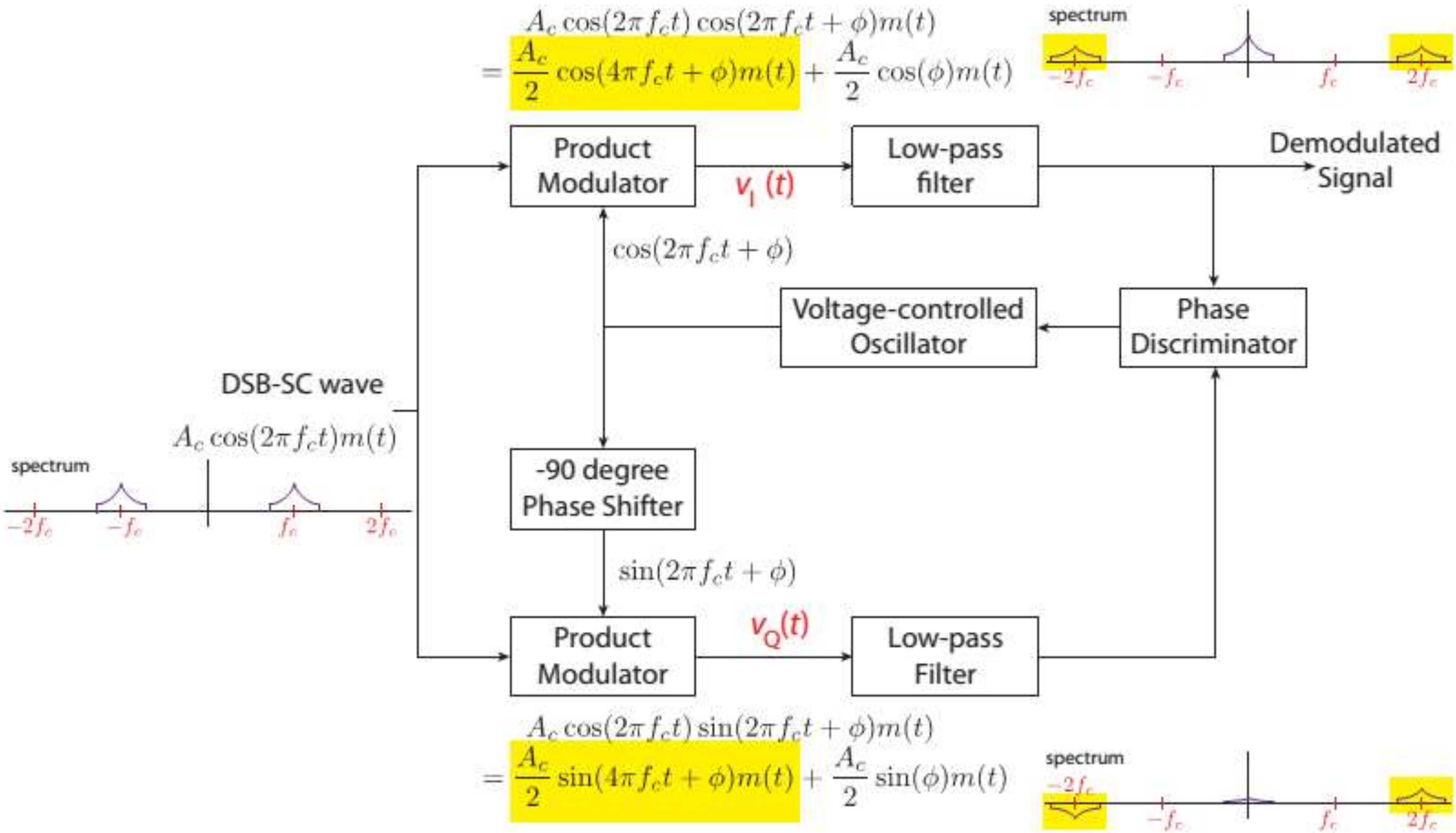


For ϕ small.

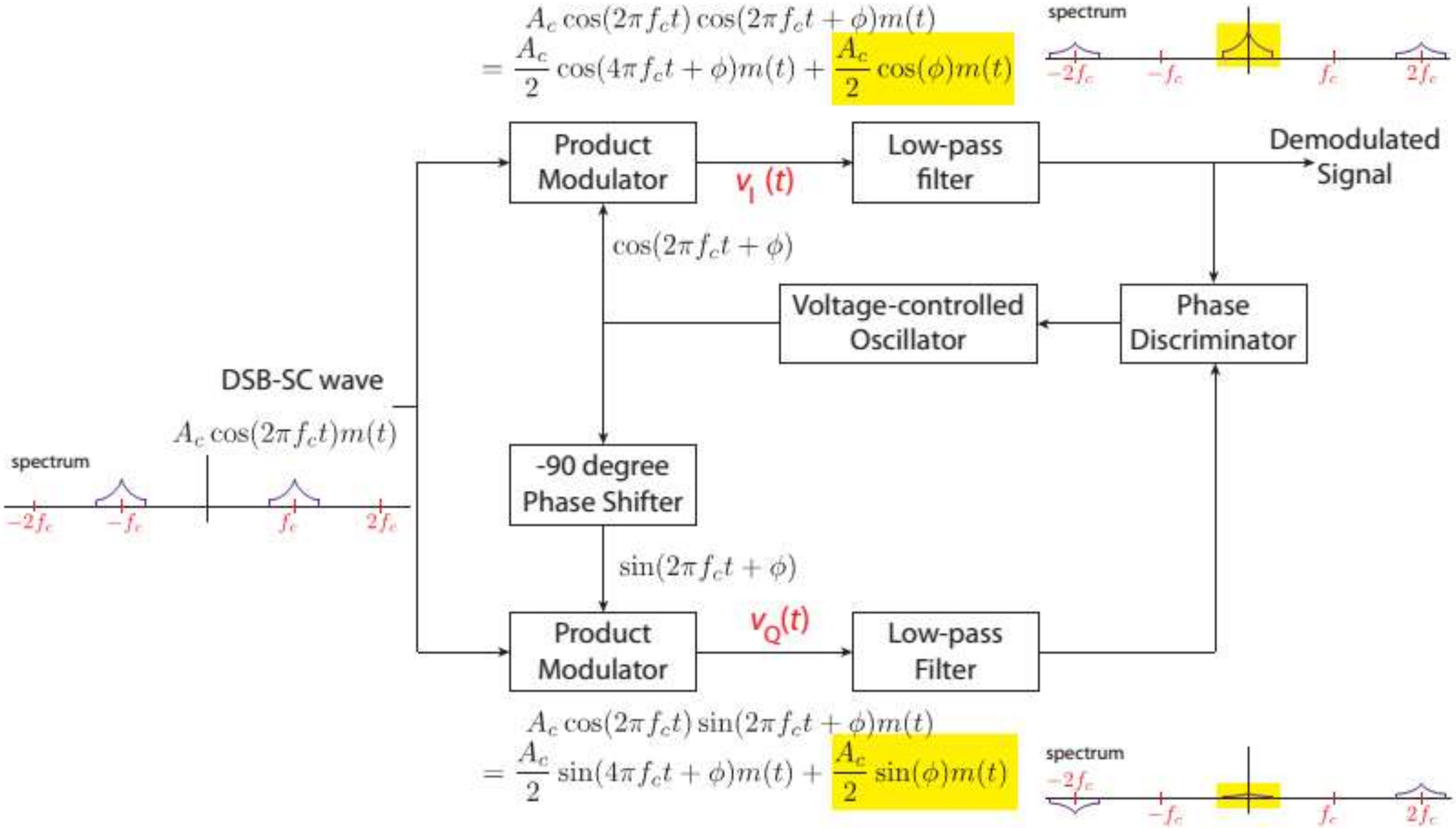
Costas Receiver: $v_I(t)$ and $v_Q(t)$



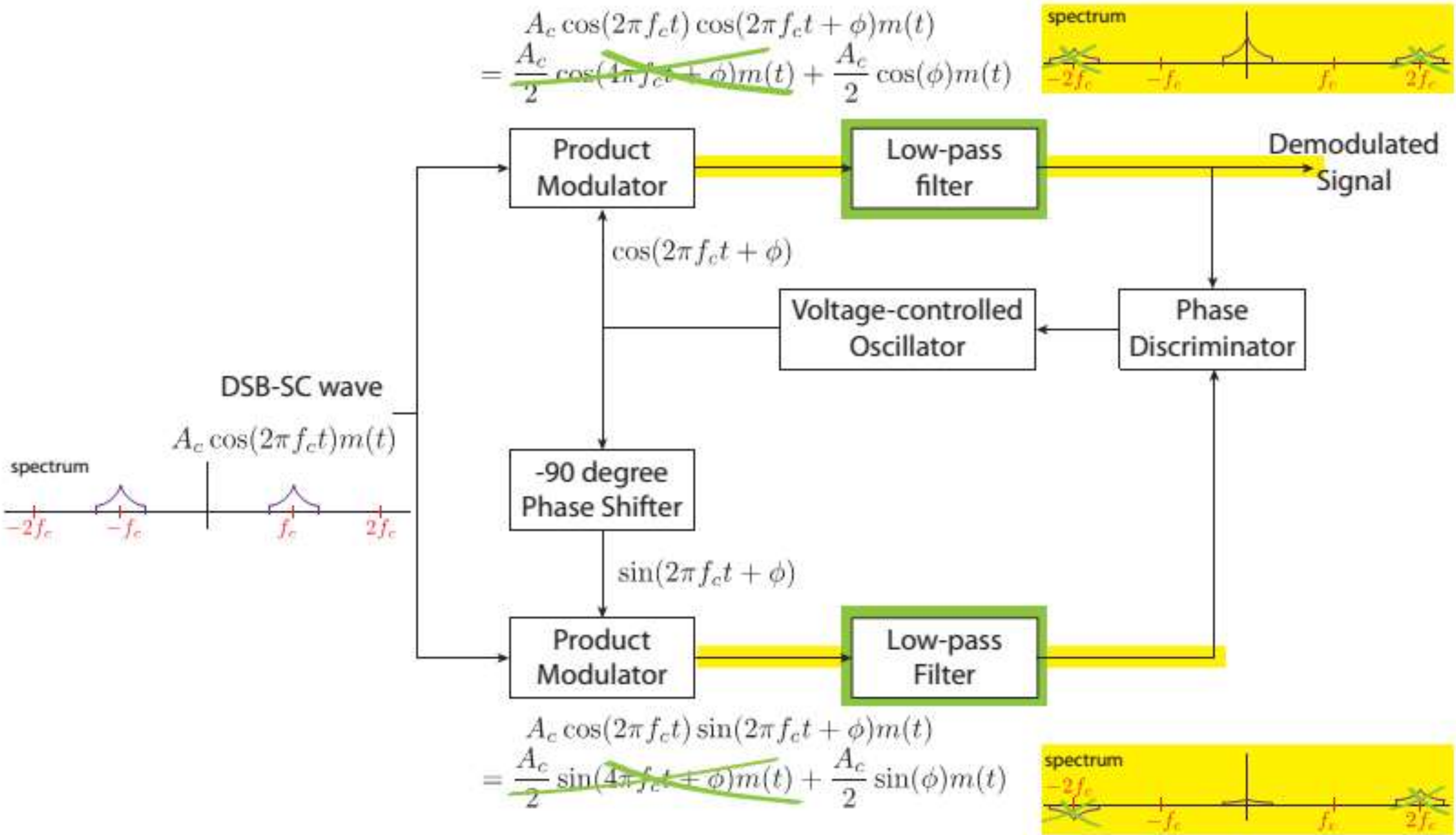
Costas Receiver: $v_I(t)$ and $v_Q(t)$



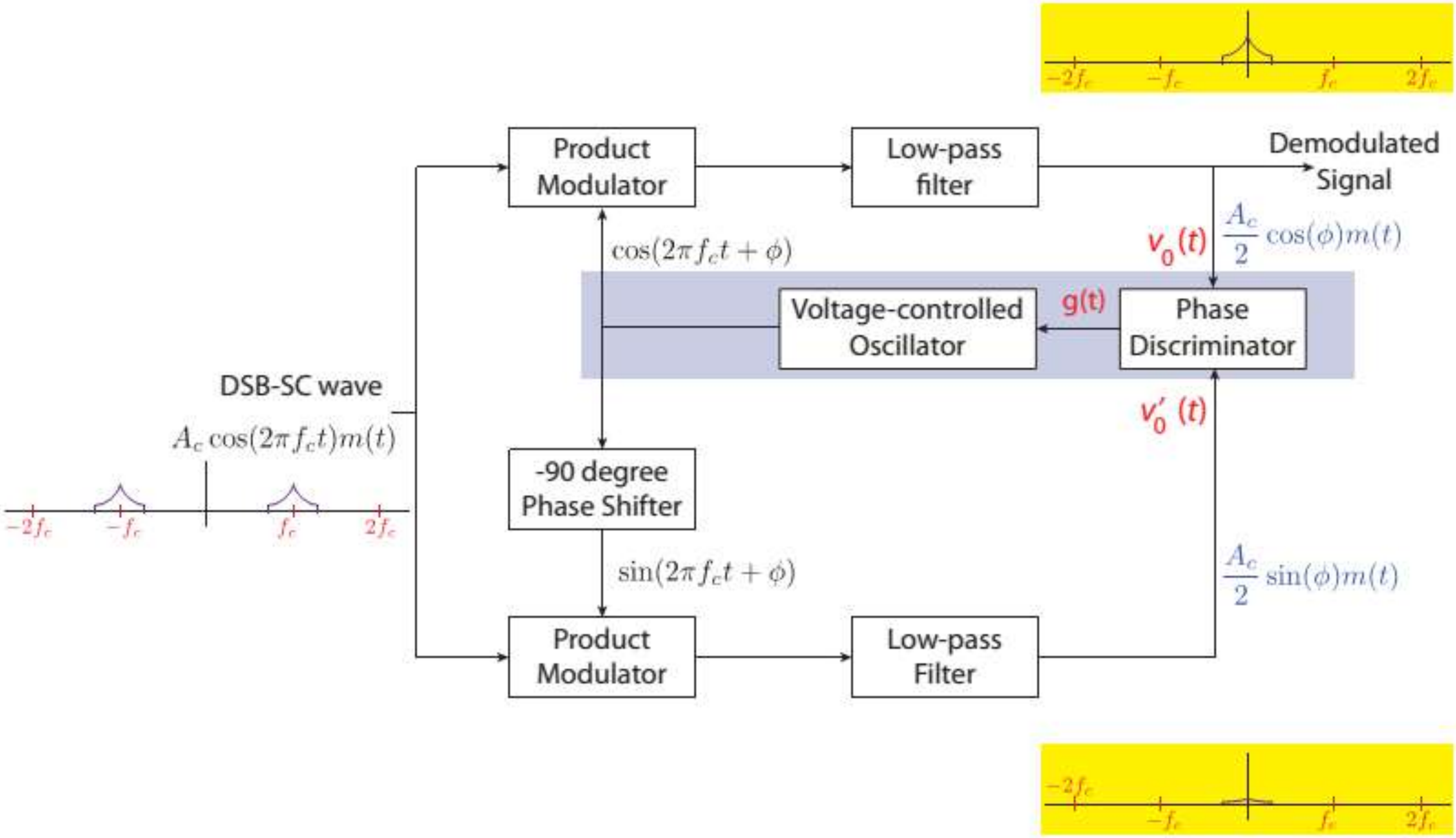
Costas Receiver: $v_I(t)$ and $v_Q(t)$



Costas Receiver: Low-Pass Filter



Costas Receiver: Local Oscillator Control



Costas Receiver: Phase Discriminator

Two components in sequence:

(1) multiplier

$$
\begin{aligned}
v_0(t) \cdot v_0'(t) &= \frac{A_c}{2}\cos(\phi)m(t) \cdot \frac{A_c}{2}\sin(\phi)m(t) \\
&= \frac{A_c^2}{4}\underbrace{\cos(\phi)}_{\approx 1}\underbrace{\sin(\phi)}_{\approx \phi} m^2(t) \\
&= \frac{A_c^2}{4}\phi m^2(t)
\end{aligned}
$$

for $\phi \ll 1$.

Costas Receiver: Phase Discriminator

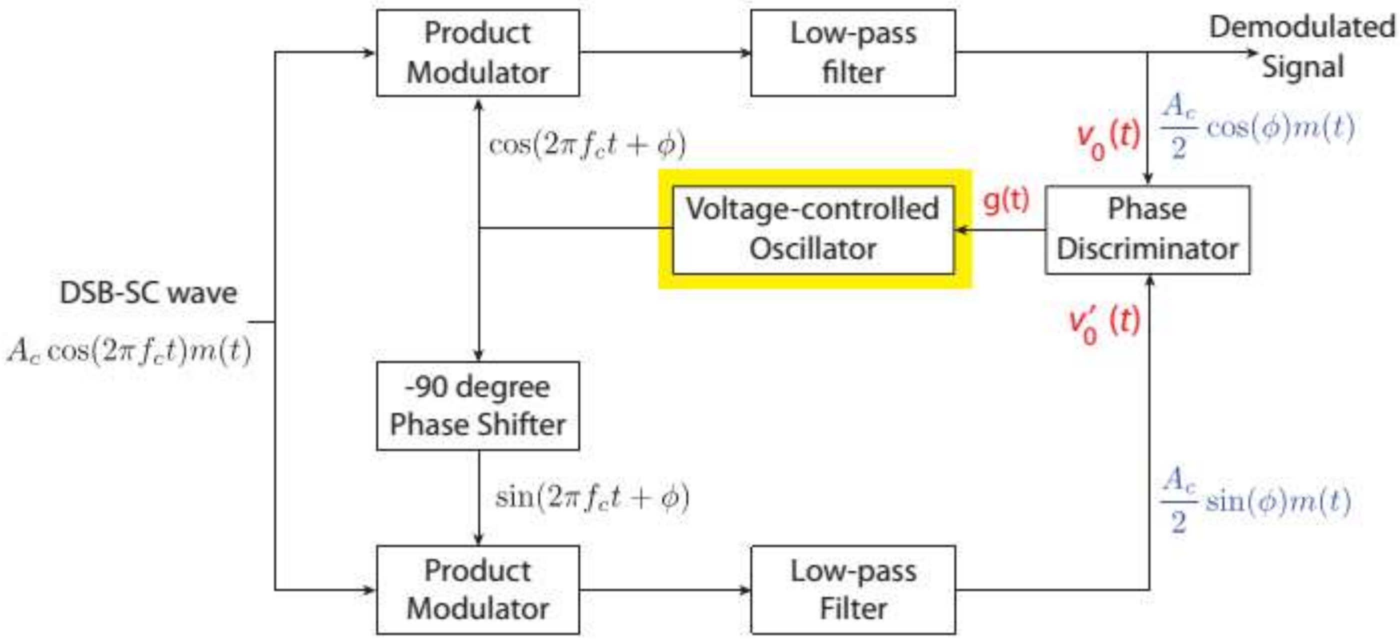
Two components in sequence:

(2) time-averaging unit

$$
\begin{aligned}
g(t) &= \frac{1}{2T}\int_{-T}^{T}\frac{A_c^2}{4}\phi m^2(t)dt \\
&= \frac{A_c^2}{4}\phi \quad \underbrace{\frac{1}{2T}\int_{-T}^{T}m^2(t)dt}_{\text{power of } m(t)\text{; for } T \text{ large is constant}}
\end{aligned}
$$

Therefore, $g(t)$ is proportional to ϕ, and is the same sign as the phase error ϕ.

Costas Receiver: Voltage-controlled Oscillator



Costas Receiver: Voltage-controlled Oscillator

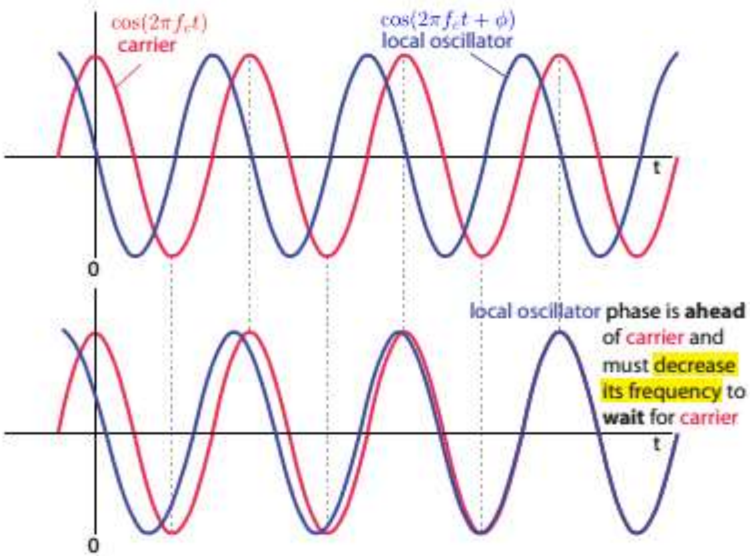
- If $g(t) > 0$ (or $\phi > 0$), then the local oscillator will <u>decrease</u> from f_c proportional to the value of $g(t)$ (or ϕ).

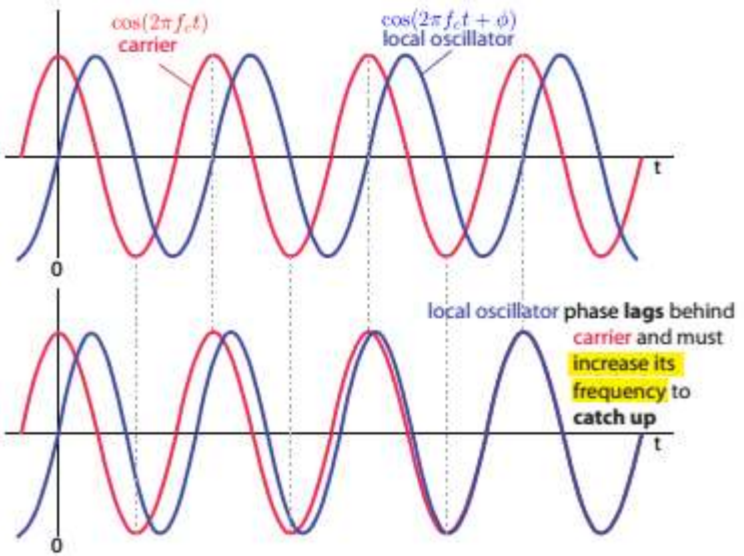
- If $g(t) < 0$ (or $\phi < 0$), then the local oscillator will <u>increase</u> from f_c proportional to the value of $g(t)$ (or ϕ).

Costas Receiver: Voltage-controlled Oscillator

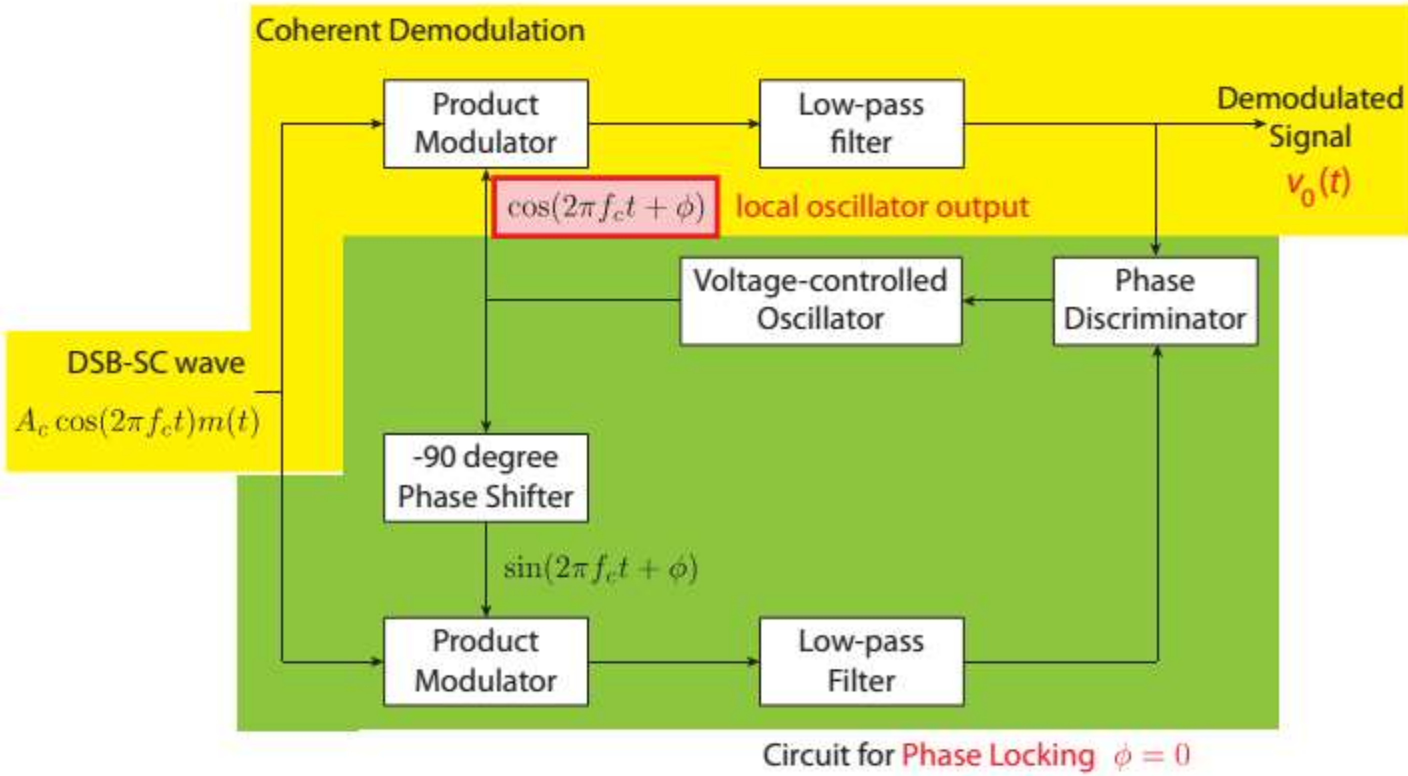
$\phi > 0$: Freq of local oscillator needs to temporarily <u>decrease</u>

$\phi < 0$: Freq of local oscillator needs to temporarily <u>increase</u>





Costas Receiver



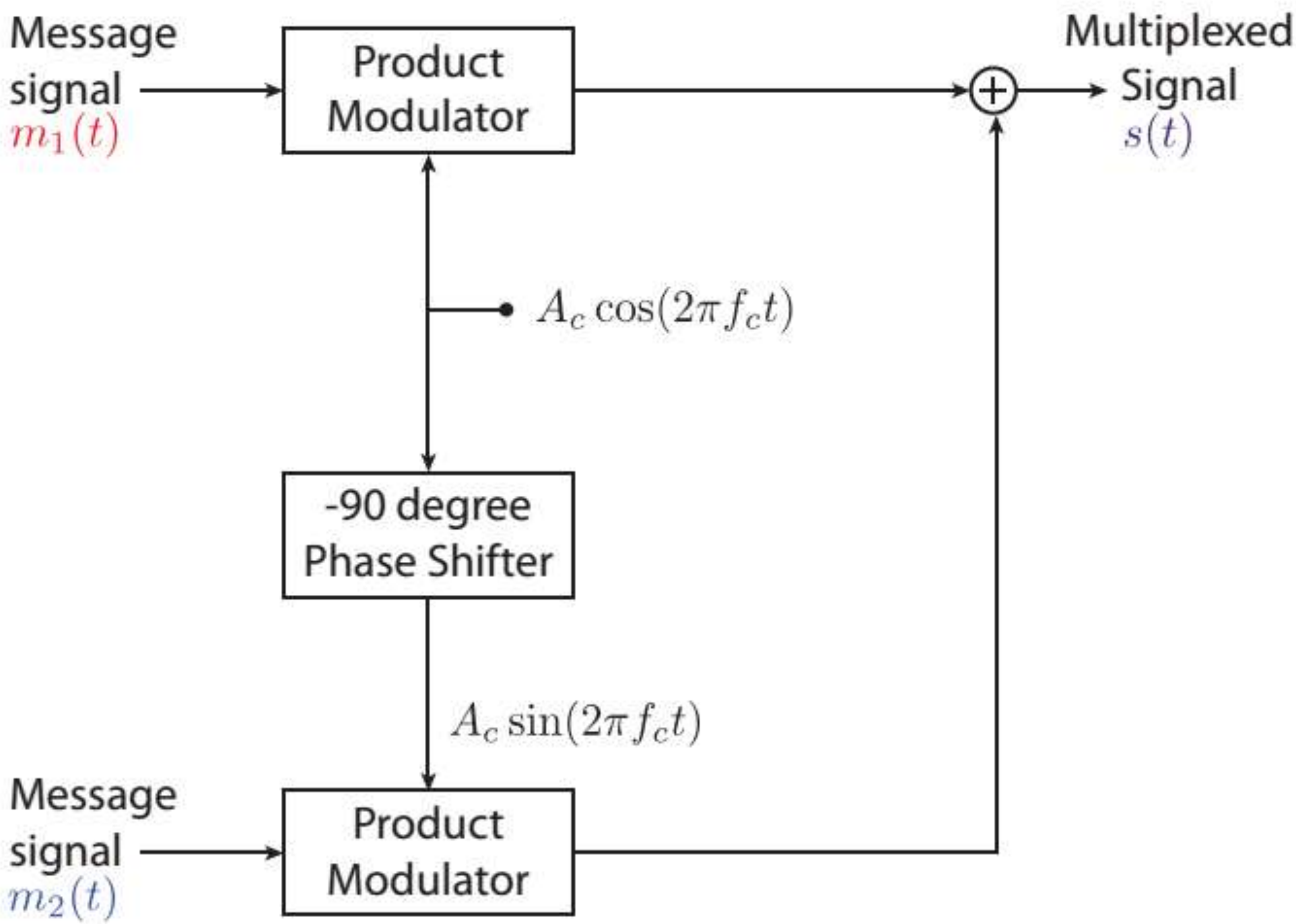
$$\boxed{\text{Quadrature Amplitude Multiplexing}}$$

Multiplexing and QAM

Multiplexing: to send multiple message simultaneously

Quadrature Amplitude Multiplexing (QAM): (a.k.a quadrature-carrier multiplexing) amplitude modulation scheme that enables two DSB-SC waves with independent message signals to occupy the same channel bandwidth (i.e., same frequency channel) yet still be separated at the receiver.

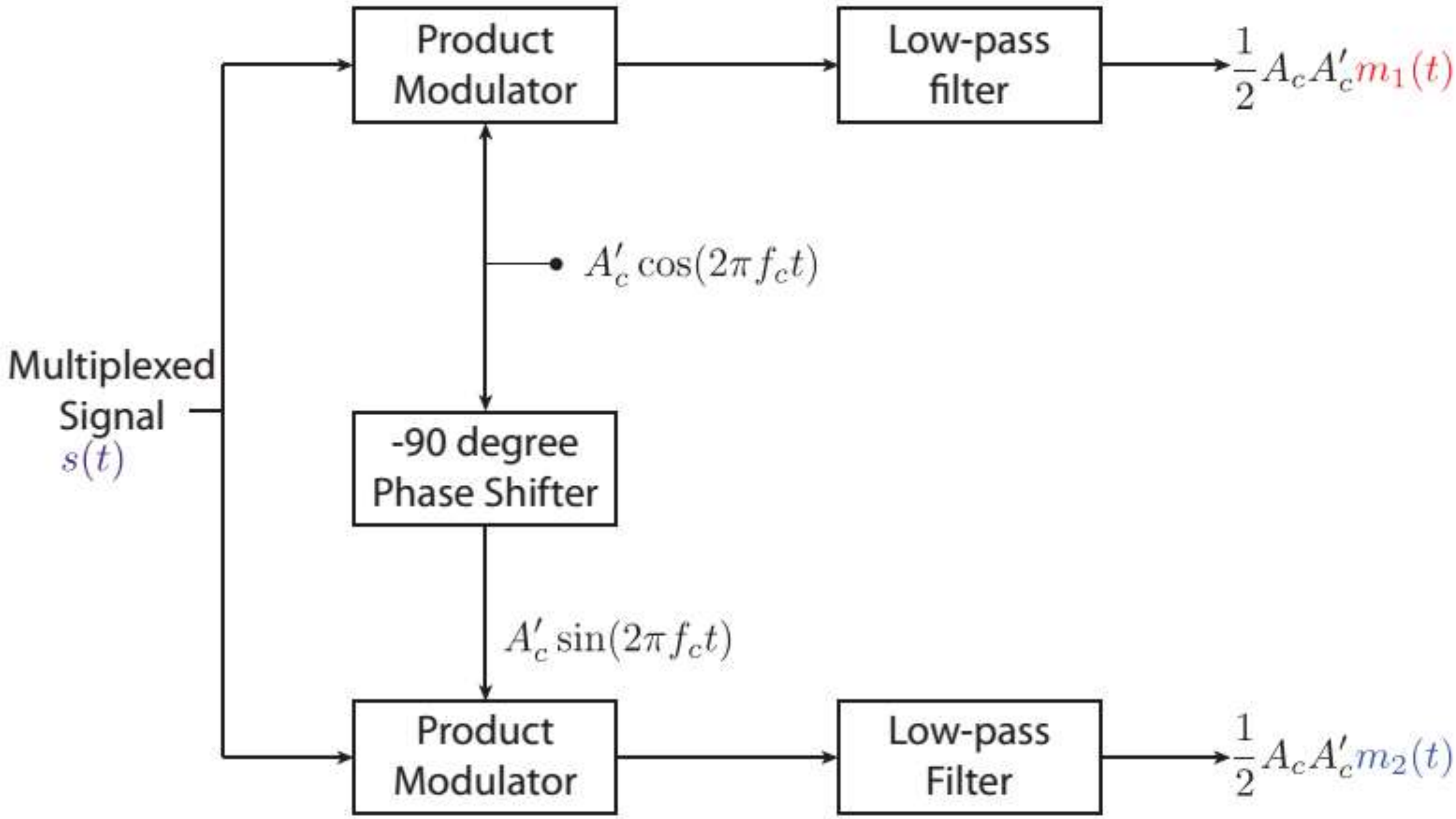
QAM: Transmitter



QAM: Transmitter

$$s(t) = A_c m_1(t) \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t)$$

QAM: Receiver



QAM: Receiver

- Costas receiver may be used to synchronize the local oscillator for demodulation.

■