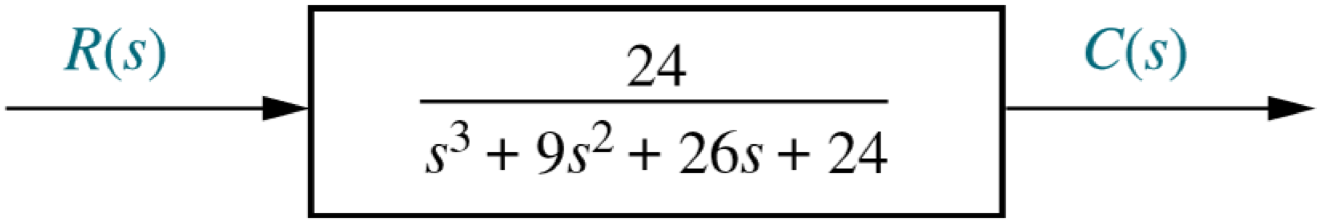
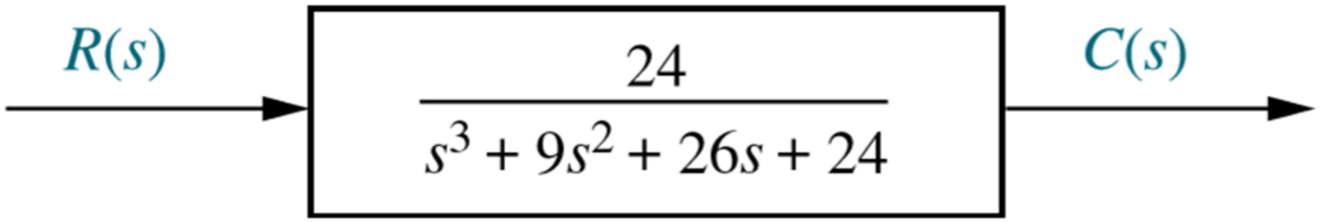
# Control System I Sessional

EEE 704

# Find the state-space representation of the transfer function shown in the figure using MATLAB



```
num=[24];
den=[1 9 26 24];
[A,B,C,D]=tf2ss(num,den);
P=[0 0 1;0 1 0;1 0 0];
A=inv(P)*A*P
B=inv(P)*B
C=C*P
```



**PROBLEM:** Given the system defined by Eq. (3.74), find the transfer function, $T(s) = Y(s)/U(s)$, where $U(s)$ is the input and $Y(s)$ is the output.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}u \tag{3.74a}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x} \tag{3.74b}$$

Do it yourself:

*Hint:*

```
[num,den]=ss2tf(A,B,C,D)
```

```
A=[0 1 0;0 0 1;-1 -2 -3];
B=[10 0 0]';
C=[1 0 0];
D=0;
[num,den]=ss2tf(A,B,C,D)
printsys(num,den,'s')
```

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}u$$

$$y = [1 \quad 0 \quad 0]\mathbf{x}$$

```
num =

     0    10    30    20


den =

    1.0000    3.0000    2.0000    1.0000


num/den =

     10 s^2 + 30 s + 20
   ---------------------
   s^3 + 3 s^2 + 2 s + 1
```

Determine pole zero location and the values of natural frequency ($\omega_n$) and damping ratio ($\xi$) of a given transfer function

**a.** $G(s) = \dfrac{400}{s^2 + 12s + 400}$

**b.** $G(s) = \dfrac{900}{s^2 + 90s + 900}$

**c.** $G(s) = \dfrac{225}{s^2 + 30s + 225}$

**d.** $G(s) = \dfrac{625}{s^2 + 625}$

# Solution (a)

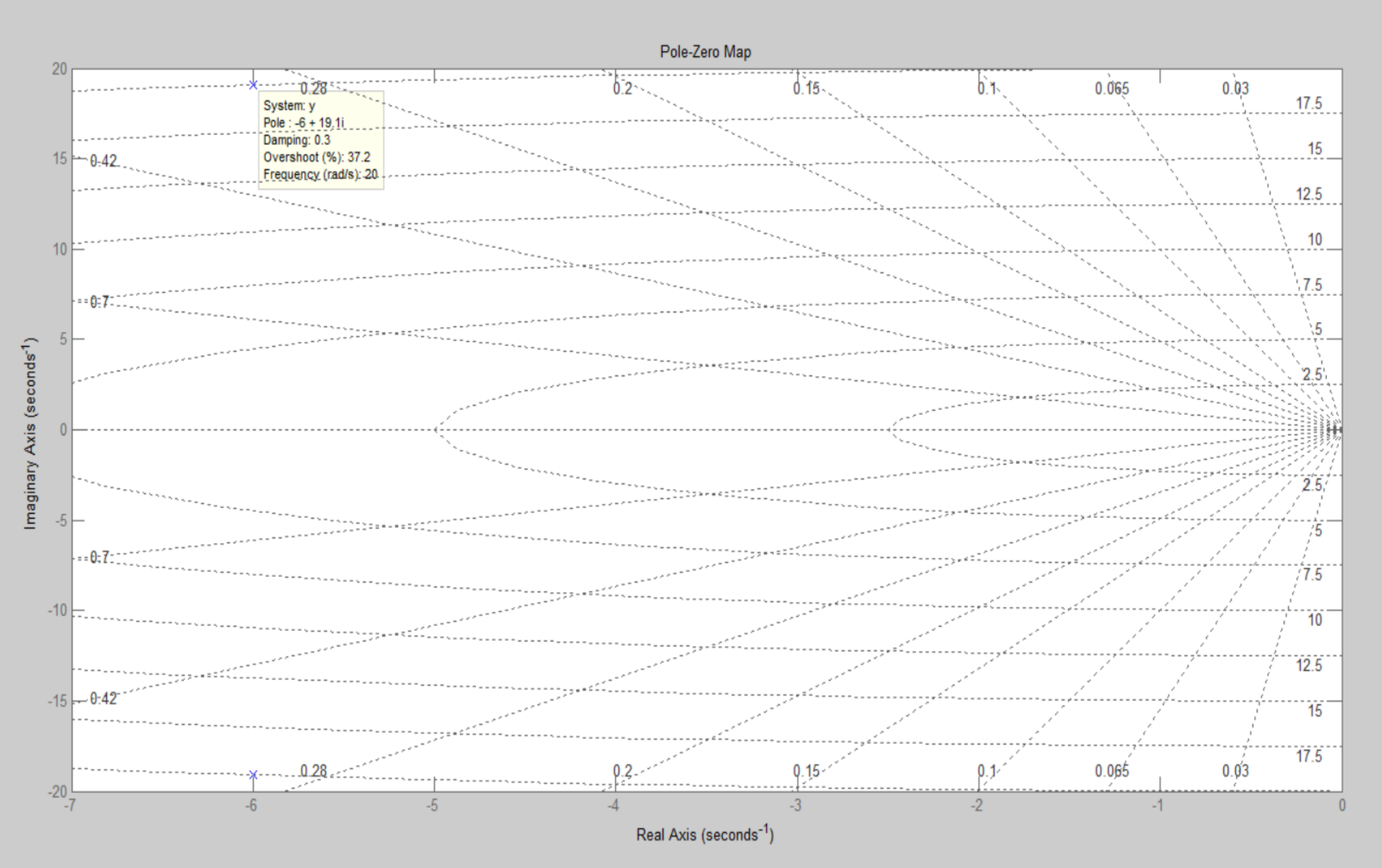
**a.** $G(s) = \dfrac{400}{s^2 + 12s + 400}$

```
clear all
clc
y=tf([400],[1 12 400])
pzmap(y);%shows the positions of poles and zeros
sgrid %Generate s-plane grid lines for a root locus or pole-zero map.
[wn,z]=damp(y)%shows natural frequency and damping
ratio
```

```
y =

        400
  ----------------
  s^2 + 12 s + 400

Continuous-time transfer function.


wn =

    20
    20


z =

    0.3000
    0.3000
```

# Pole zero locations

Click on either of the poles which will show you the values of damping ratio and others required to characterize the nature
Of the response



# Print a system when natural frequency ($\omega_n$) and damping ratio ($\xi$) are given

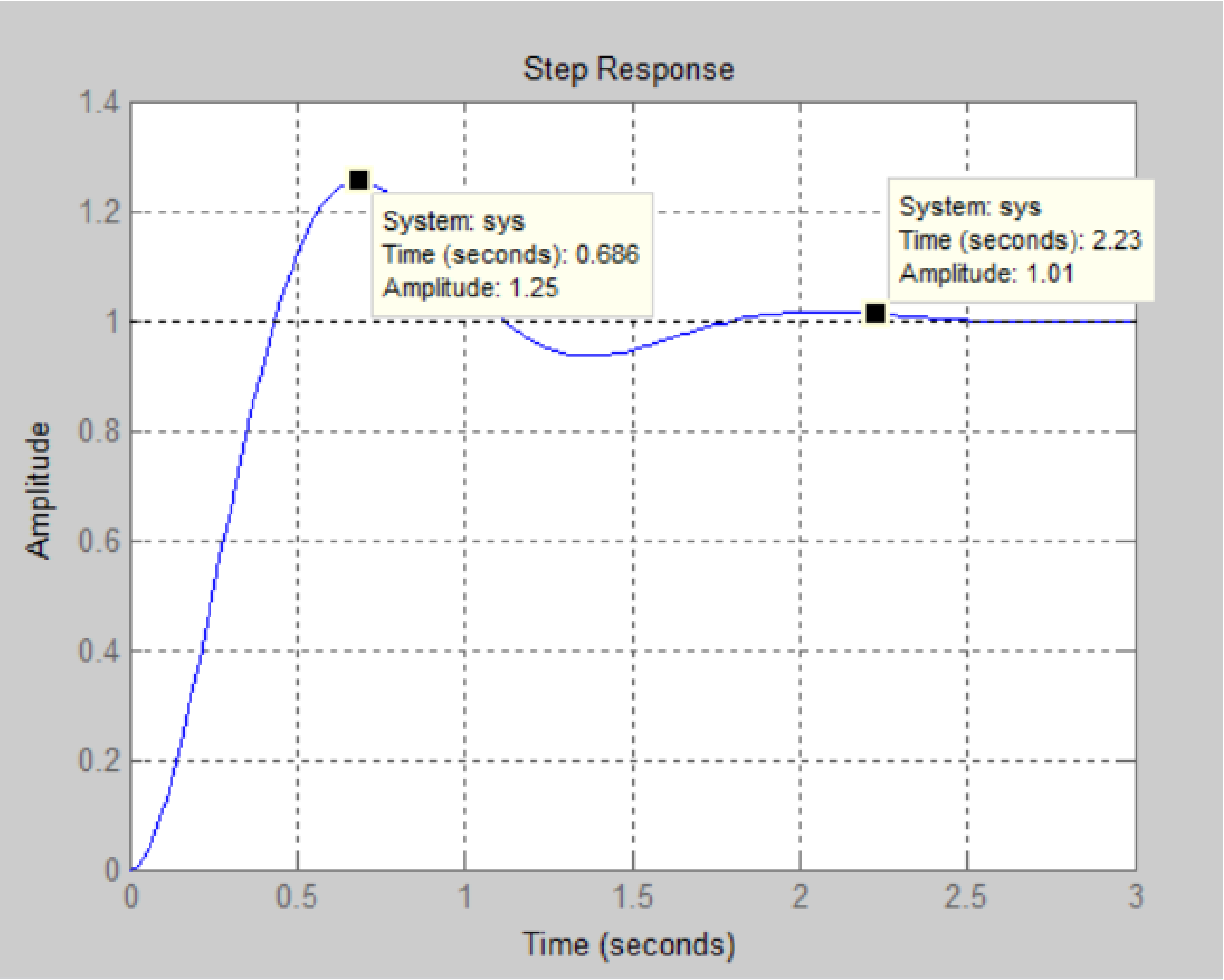
```
clc
wn=20;%given natural frequency
damping_ratio=0.3;
[num0,den]=ord2(wn,damping_ratio)% Generate continuous second order system at the pole
num=wn^2;
printsys(num,den,'s')
```

```
num0 =

     1


den =

     1    12    400


num/den =

        400
   ----------------
   s^2 + 12 s + 400
```

# Determine the Unit Step Response of a given transfer function

$$G(s) = \frac{25}{s^2 + 4s + 25}$$

```
num=25;
den=[1 4 25];
step(num,den)
grid on
```



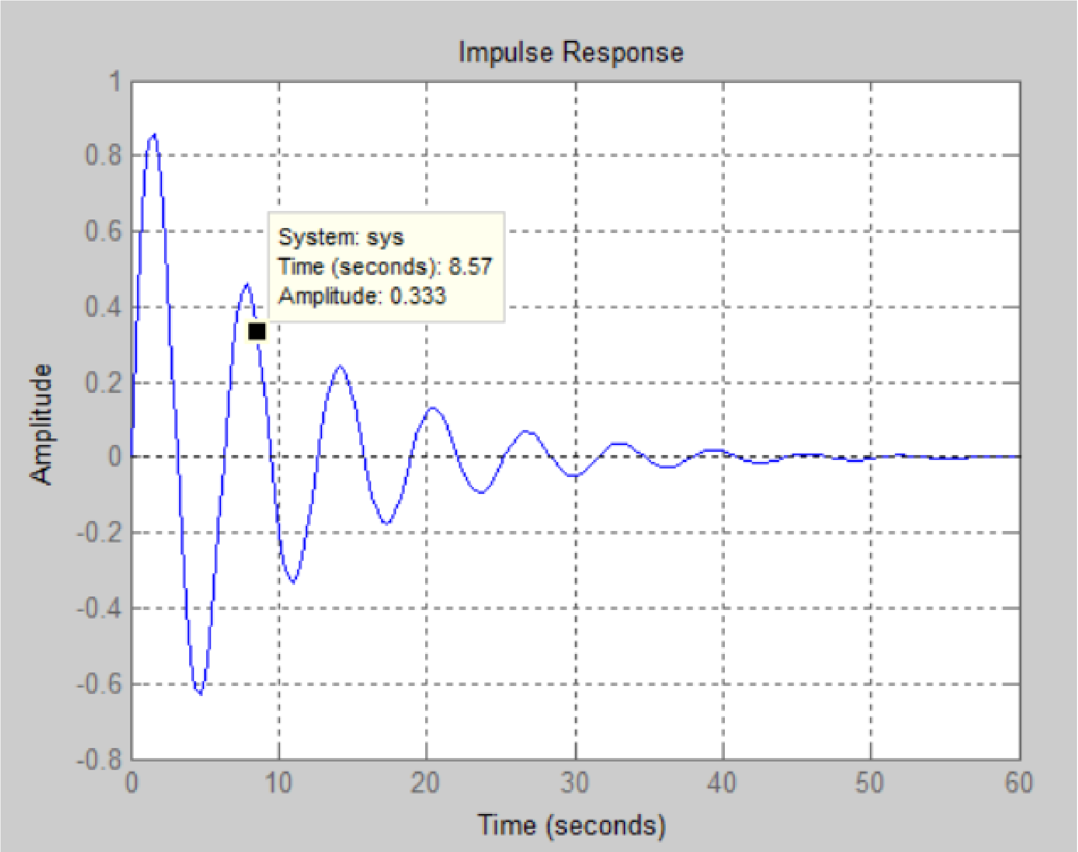

$$G(s) = \frac{25}{s^2 + 4s + 25}$$

# Obtain the unit-impulse response of a transfer function

$$\frac{C(s)}{R(s)} = G(s) = \frac{1}{s^2 + 0.2s + 1}$$

```
num=1;
den=[1 0.2 1];
impulse(num,den)
grid on
```

$$\frac{C(s)}{R(s)} = G(s) = \frac{1}{s^2 + 0.2s + 1}$$



# Obtain the unit-ramp response of a transfer function

$$\frac{C(s)}{R(s)} = \frac{2s + 1}{s^2 + s + 1}$$

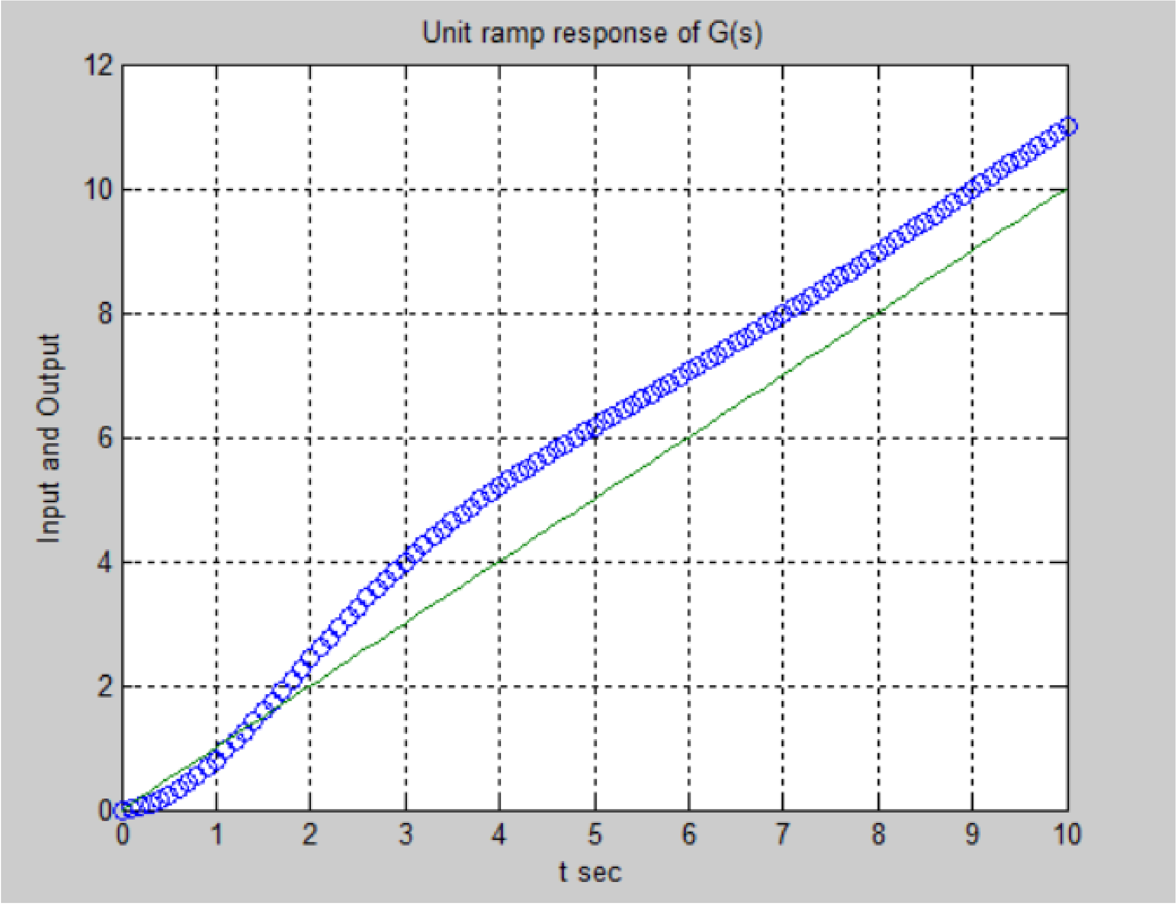
For a unit-ramp input, $R(s) = 1/s^2$. Hence

$$C(s) = \frac{2s + 1}{s^2 + s + 1}\frac{1}{s^2} = \frac{2s + 1}{(s^2 + s + 1)s}\frac{1}{s}$$

MATLAB do not have built in ramp response command. That's why we have to transfer in step response

```
num=[2 1];
den=[1 1 1 0];
t=0:0.1:10;
c=step(num,den,t);
plot(t,c,'o',t,t,'-')
grid
title('Unit ramp response of G(s)')
xlabel('t sec')
ylabel('Input and Output')
```

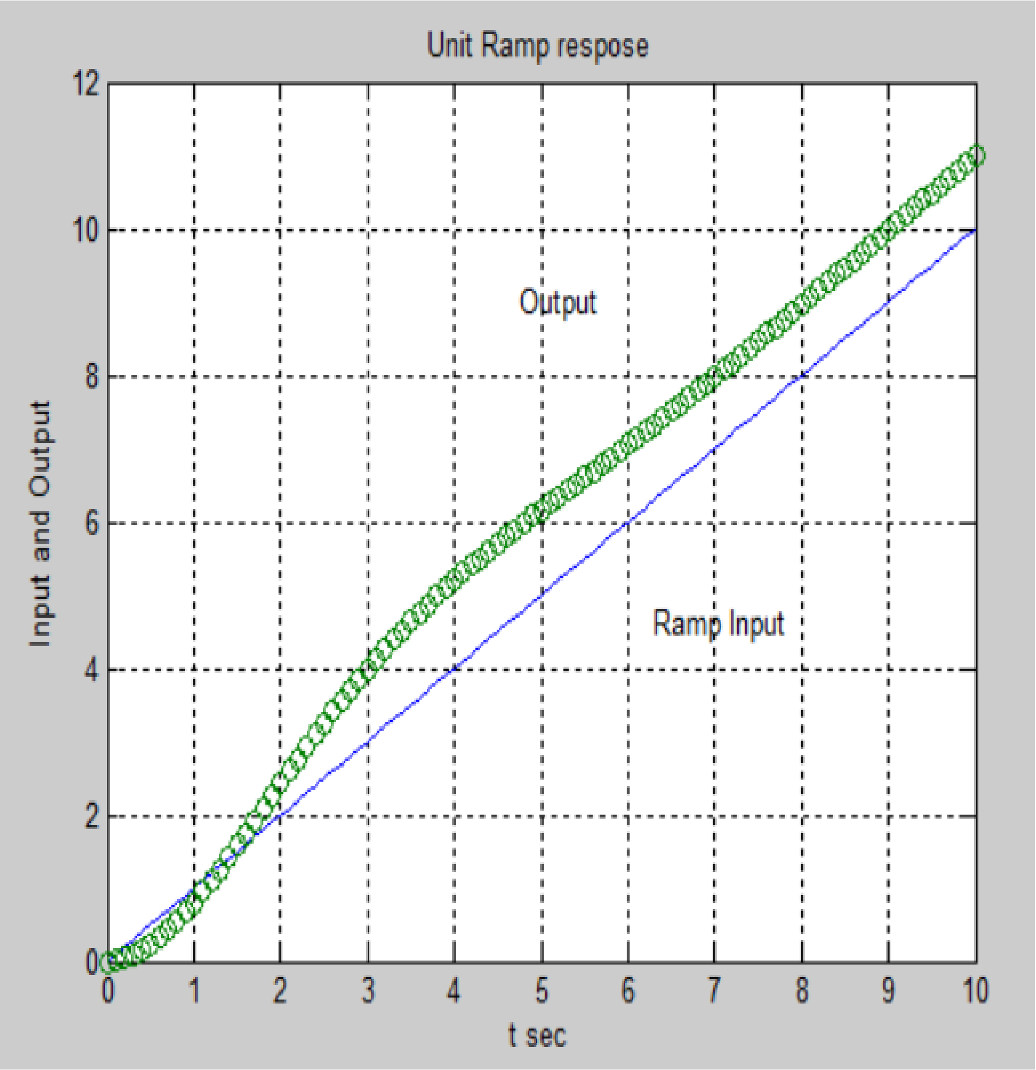
$$\frac{C(s)}{R(s)} = \frac{2s + 1}{s^2 + s + 1}$$



# Obtain the unit-ramp response of a transfer function using "lsim" command

$$\frac{C(s)}{R(s)} = \frac{2s + 1}{s^2 + s + 1}$$

```
num=[2 1];
den=[1 1 1];
t=0:0.1:10;
r=t;
y=lsim(num,den,r,t);%r=ramp---t=time
plot(t,r,'-',t,y,'o')
grid
title('Unit Ramp respose')
xlabel('t sec')
ylabel('Input and Output')
text(6.3,4.6,'Ramp Input')
text(4.75,9.0,'Output')
```



# Step Response of Different Damping

- Find step response of the transfer function given below

**a.** $G(s) = \dfrac{400}{s^2 + 12s + 400}$

**b.** $G(s) = \dfrac{900}{s^2 + 90s + 900}$

**c.** $G(s) = \dfrac{225}{s^2 + 30s + 225}$

**d.** $G(s) = \dfrac{625}{s^2 + 625}$

```matlab
%Underdamped
num=[400];
den=[1 12 400];
t1=tf(num,den)
subplot(221)
step(t1)
title('Underdamped')
%Overdamped
num2=[900];
den2=[1 90 900];
t2=tf(num2,den2)
subplot(222)
step(t2)
title('Overdamped')
```

```matlab
%Critically Damped
num3=[225];
den3=[1 30 225];
t3=tf(num3,den3)
subplot(223)
step(t3)
title('Critically Damped')
%Undamped
num4=[625];
den4=[1 0 625];
t4=tf(num4,den4)
subplot(224)
step(t4)
title('Undamped')
```

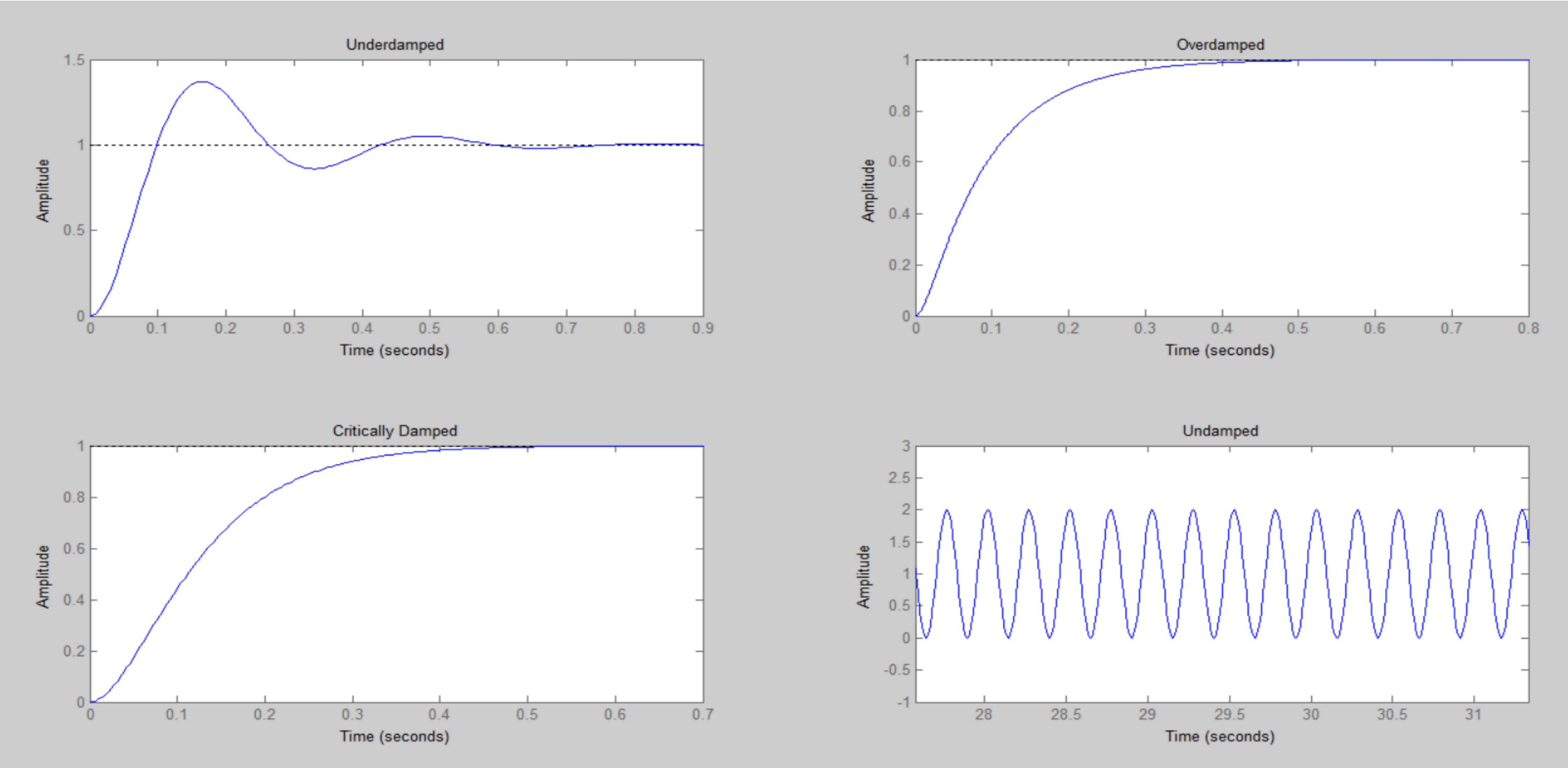
**a.** $G(s) = \dfrac{400}{s^2 + 12s + 400}$

**b.** $G(s) = \dfrac{900}{s^2 + 90s + 900}$

**c.** $G(s) = \dfrac{225}{s^2 + 30s + 225}$

**d.** $G(s) = \dfrac{625}{s^2 + 625}$

# All Responses



# EXP 5: State Space Representation

- Find the transfer function from the following state equations

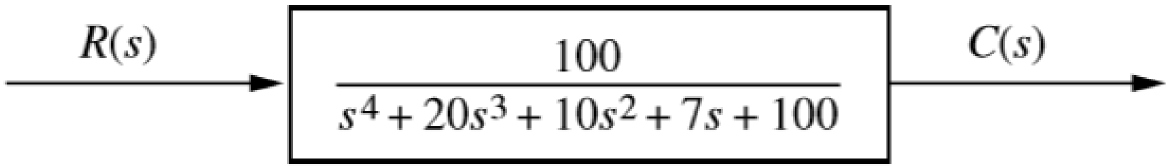
$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & -1 \\ -3 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}u(t)$$

$$y = [3 \quad 2]\mathbf{x}$$

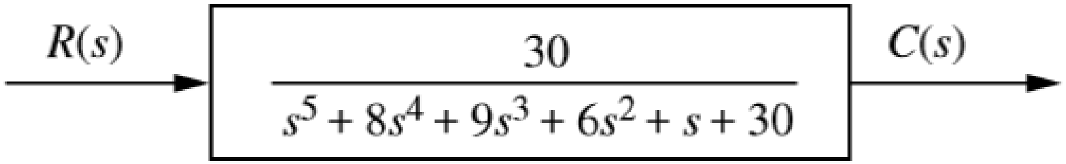
$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 6 & 5 \\ 1 & 4 & 2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}u(t)$$

$$y = [1 \quad 2 \quad 0]\mathbf{x}$$

- Find the State Equation from the given Transfer Function




(*a*)




(*b*)

# EXP 6: Determine pole zero location and the values of natural frequency ($\omega_n$) and damping ratio ($\xi$) of a given transfer function

**a.** $G(s) = \dfrac{400}{s^2 + 12s + 400}$

**b.** $G(s) = \dfrac{900}{s^2 + 90s + 900}$

**c.** $G(s) = \dfrac{225}{s^2 + 30s + 225}$

**d.** $G(s) = \dfrac{625}{s^2 + 625}$

# Print a system when natural frequency ($\omega_n$) and damping ratio ($\xi$) are given

- For $\omega_n = 10$ rad/sec and $\xi = 0.3$
- For $\omega_n = 20$ rad/sec and $\xi = 0.0$
- For $\omega_n = 25$ rad/sec and $\xi = 0.8$
- For $\omega_n = 30$ rad/sec and $\xi = 1.0$

# EXP 7: Transient and Steady State Responses

- Using MATLAB, obtain the unit-step response, unit-ramp response, and unit-impulse response of the following system

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + 2s + 10}$$

# EXP 8: Obtain the unit-ramp response of a transfer function using "lsim" command

- Obtain the unit-ramp response of three transfer functions using "lsim" command

# EXP 9: Step Response of Different Damping

- Include step responses of overdamped, underdamped, critically damped and undamped system in a single figure using subplot command.