Math Problems

Example – 01:

The following resistance values of a platinum resistance thermometer were measured at a range of temperatures. Determine the measurement sensitivity of the instrument in ohms/°C.

Resistance (Ω)	*Temperature* (°C)
307	200
314	230
321	260
328	290

Solution

If these values are plotted on a graph, the straight-line relationship between resistance change and temperature change is obvious.

For a change in temperature of 30°C, the change in resistance is 7 Ω. Hence the measurement sensitivity $= 7/30 = 0.233\ \Omega/°C$.

Example – 02:

A spring balance is calibrated in an environment at a temperature of 20°C and has the following deflection/load characteristic.

Load (kg)	0	1	2	3
Deflection (mm)	0	20	40	60

It is then used in an environment at a temperature of 30°C and the following deflection/load characteristic is measured.

Load (kg):	0	1	2	3
Deflection (mm)	5	27	49	71

Determine the zero drift and sensitivity drift per °C change in ambient temperature.

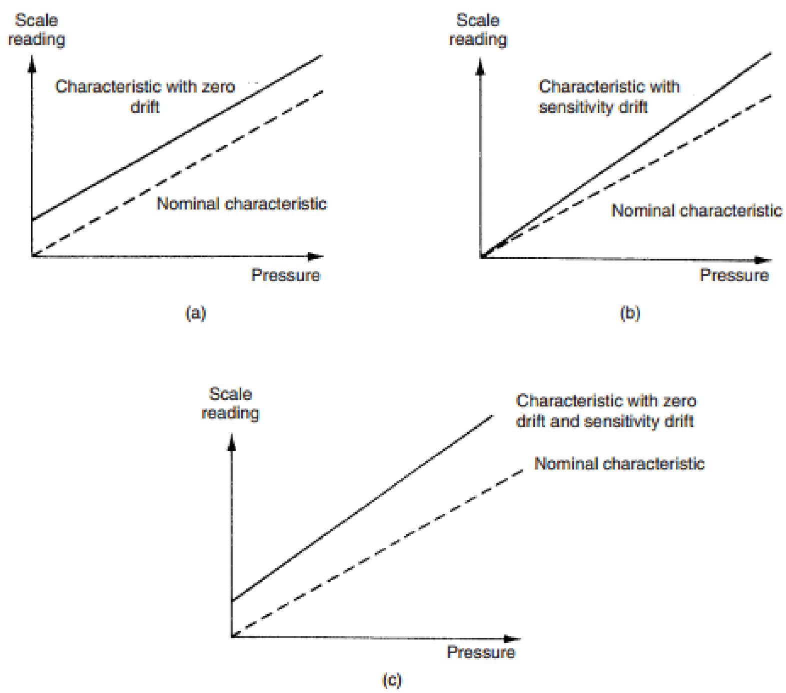


Fig. 2.7 Effects of disturbance: (a) zero drift; (b) sensitivity drift; (c) zero drift plus sensitivity drift.

Solution
At 20°C, deflection/load characteristic is a straight line. Sensitivity = 20 mm/kg.
At 30°C, deflection/load characteristic is still a straight line. Sensitivity = 22 mm/kg.
Bias (zero drift) = 5 mm (the no-load deflection)
Sensitivity drift = 2 mm/kg
Zero drift/°C = 5/10 = 0.5 mm/°C
Sensitivity drift/°C = 2/10 = 0.2 (mm per kg)/°C

Example – 03:

A balloon is equipped with temperature and altitude measuring instruments and has radio equipment that can transmit the output readings of these instruments back to ground. The balloon is initially anchored to the ground with the instrument output readings in steady state. The altitude-measuring instrument is approximately zero order and the temperature transducer first order with a time constant of 15 seconds. The

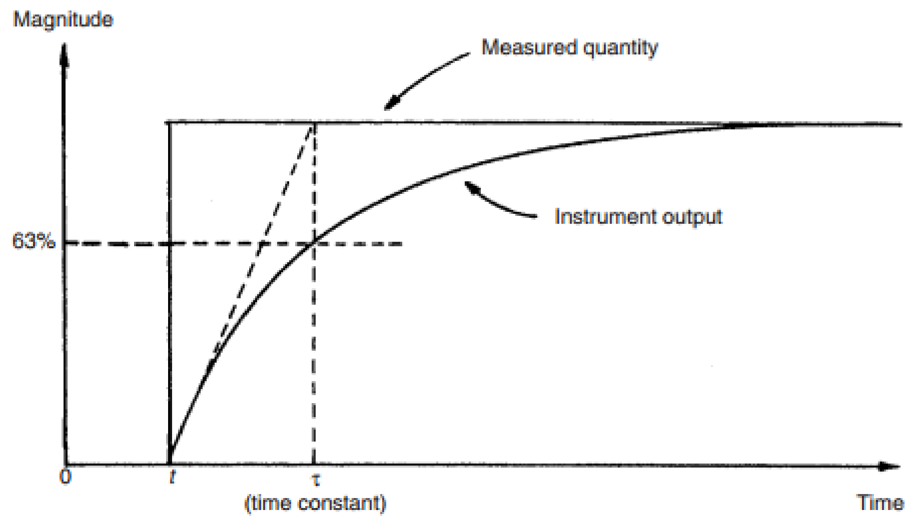


Fig. 2.11 First order instrument characteristic.

temperature on the ground, T_0, is 10°C and the temperature T_x at an altitude of x metres is given by the relation: $T_x = T_0 - 0.01x$

(a) If the balloon is released at time zero, and thereafter rises upwards at a velocity of 5 metres/second, draw a table showing the temperature and altitude measurements reported at intervals of 10 seconds over the first 50 seconds of travel. Show also in the table the error in each temperature reading.
(b) What temperature does the balloon report at an altitude of 5000 metres?

Solution
In order to answer this question, it is assumed that the solution of a first order differential equation has been presented to the reader in a mathematics course. If the reader is not so equipped, the following solution will be difficult to follow.

Let the temperature reported by the balloon at some general time t be T_r. Then T_x is related to T_r by the relation:

$$T_r = \frac{T_x}{1+\tau D} = \frac{T_0 - 0.01x}{1+\tau D} = \frac{10 - 0.01x}{1+15D}$$

It is given that $x = 5t$, thus: $T_r = \dfrac{10 - 0.05t}{1+15D}$

The transient or complementary function part of the solution ($T_x = 0$) is given by: $T_{r_{cf}} = Ce^{-t/15}$
The particular integral part of the solution is given by: $T_{r_{pi}} = 10 - 0.05(t - 15)$
Thus, the whole solution is given by: $T_r = T_{r_{cf}} + T_{r_{pi}} = Ce^{-t/15} + 10 - 0.05(t - 15)$
Applying initial conditions: At $t = 0$, $T_r = 10$, i.e. $10 = Ce^{-0} + 10 - 0.05(-15)$
Thus $C = -0.75$ and therefore: $T_r = 10 - 0.75e^{-t/15} - 0.05(t - 15)$
Using the above expression to calculate T_r for various values of t, the following table can be constructed:

Time	*Altitude*	*Temperature reading*	*Temperature error*
0	0	10	0
10	50	9.86	0.36
20	100	9.55	0.55
30	150	9.15	0.65
40	200	8.70	0.70
50	250	8.22	0.72

(b) At 5000 m, $t = 1000$ seconds. Calculating T_r from the above expression:

$$T_r = 10 - 0.75e^{-1000/15} - 0.05(1000 - 15)$$

The exponential term approximates to zero and so T_r can be written as:

$$T_r \approx 10 - 0.05(985) = -39.25°\text{C}$$

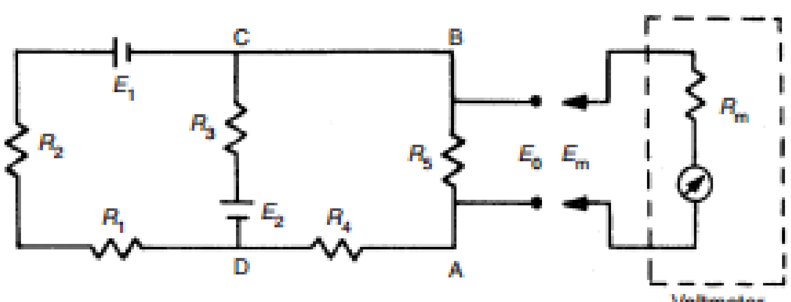
This result might have been inferred from the table above where it can be seen that the error is converging towards a value of 0.75. For large values of t, the transducer reading lags the true temperature value by a period of time equal to the time constant of 15 seconds. In this time, the balloon travels a distance of 75 metres and the temperature falls by 0.75°. Thus for large values of t, the output reading is always 0.75° less than it should be.

Example – 03:

Suppose that the components of the circuit shown in Figure 3.1(a) have the following values:

$$R_1 = 400\,\Omega; R_2 = 600\,\Omega; R_3 = 1000\,\Omega, R_4 = 500\,\Omega; R_5 = 1000\,\Omega$$

The voltage across AB is measured by a voltmeter whose internal resistance is 9500 Ω. What is the measurement error caused by the resistance of the measuring instrument?



(a)

Solution
Proceeding by applying Thévenin's theorem to find an equivalent circuit to that of Figure 3.1(a) of the form shown in Figure 3.1(b), and substituting the given component values into the equation for R_{AB} (3.1), we obtain:

$$R_{AB} = \frac{[(1000^2/2000) + 500]1000}{(1000^2/2000) + 500 + 1000} = \frac{1000^2}{2000} = 500\,\Omega$$

From equation (3.2), we have:

$$\frac{E_m}{E_0} = \frac{R_m}{R_{AB} + R_m}$$

The measurement error is given by $(E_0 - E_m)$:

$$E_0 - E_m = E_0\left(1 - \frac{R_m}{R_{AB} + R_m}\right)$$

Substituting in values:

$$E_0 - E_m = E_0\left(1 - \frac{9500}{10\,000}\right) = 0.95E_0$$

Thus, the error in the measured value is 5%.

Example – 04:

How many measurements in a data set subject to random errors lie outside deviation boundaries of $+\sigma$ and $-\sigma$, i.e. how many measurements have a deviation greater than $|\sigma|$?

Solution

The required number is represented by the sum of the two shaded areas in Figure 3.8. This can be expressed mathematically as:

$$P(E < -\sigma \text{ or } E > +\sigma) = P(E < -\sigma) + P(E > +\sigma)$$

For $E = -\sigma$, $z = -1.0$ (from equation 3.12).

Using Table 3.1:

$$P(E < -\sigma) = F(-1) = 1 - F(1) = 1 - 0.8413 = 0.1587$$

Table 3.1 Standard Gaussian table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
					$F(z)$					
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7793	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8906	0.8925	0.8943	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9648	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9926	0.9928	0.9930	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

Similarly, for $E = +\sigma$, $z = +1.0$, Table 3.1 gives:

$$P(E > +\sigma) = 1 - P(E < +\sigma) = 1 - F(1) = 1 - 0.8413 = 0.1587.$$

(This last step is valid because the frequency distribution curve is normalized such that the total area under it is unity.)

Thus

$$P[E < -\sigma] + P[E > +\sigma] = 0.1587 + 0.1587 = 0.3174 \sim 32\%$$

i.e. 32% of the measurements lie outside the $\pm\sigma$ boundaries, then 68% of the measurements lie inside.

The above analysis shows that, for Gaussian-distributed data values, 68% of the measurements have deviations that lie within the bounds of $\pm\sigma$. Similar analysis shows

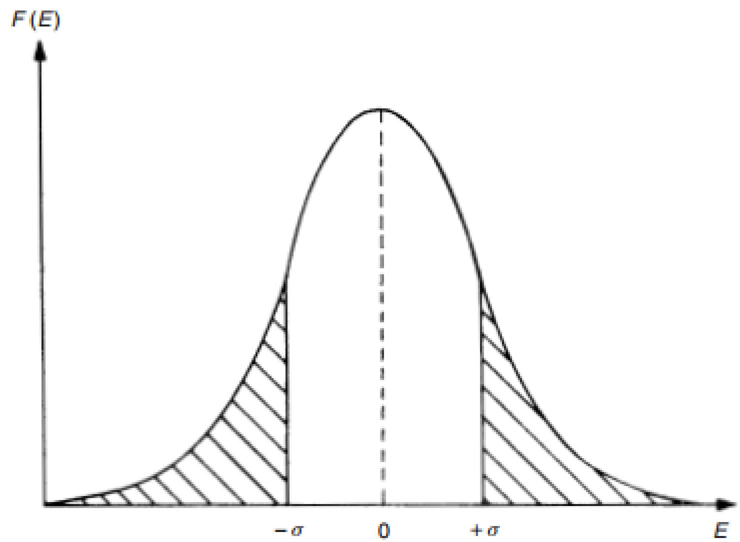


Fig. 3.8 $\pm\sigma$ boundaries.

that boundaries of $\pm 2\sigma$ contain 95.4% of data points, and extending the boundaries to $\pm 3\sigma$ encompasses 99.7% of data points. The probability of any data point lying outside particular deviation boundaries can therefore be expressed by the following table.

Deviation boundaries	*% of data points within boundary*	*Probability of any particular data point being outside boundary*
$\pm\sigma$	68.0	32.0%
$\pm 2\sigma$	95.4	4.6%
$\pm 3\sigma$	99.7	0.3%

Example – 05:

A rectangular-sided block has edges of lengths a, b and c, and its mass is m. If the values and possible errors in quantities a, b, c and m are as shown below, calculate the value of density and the possible error in this value.

$$a = 100\,\text{mm} \pm 1\%,\ b = 200\,\text{mm} \pm 1\%,\ c = 300\,\text{mm} \pm 1\%,\ m = 20\,\text{kg} \pm 0.5\%.$$

Solution

Value of $ab = 0.02\ \text{m}^2 \pm 2\%$ (possible error $= 1\% + 1\% = 2\%$)

Value of $(ab)c = 0.006\ \text{m}^3 \pm 3\%$ (possible error $= 2\% + 1\% = 3\%$)

Value of $\frac{m}{abc} = \frac{20}{0.006} = 3330\,\text{kg/m}^3 \pm 3.5\%$ (possible error $= 3\% + 0.5\% = 3.5\%$)

Example – 06:

Calculate the reading that would be observed on a moving-coil ammeter when it is measuring the current in the circuit shown in Figure 6.10.

Solution

A moving-coil meter measures mean current.

$$I_{\text{mean}} = \frac{1}{2\pi}\left(\int_0^{\pi} \frac{5\omega t}{\pi} \mathrm{d}\omega t + \int_{\pi}^{2\pi} 5 \sin(\omega t)\, \mathrm{d}\omega t\right)$$

$$= \frac{1}{2\pi}\left(\left[\frac{5(\omega t)^2}{2\pi}\right]_0^{\pi} + 5\left[-\cos(\omega t)\right]_{\pi}^{2\pi}\right)$$

$$= \frac{1}{2\pi}\left(\frac{5\pi^2}{2\pi} - 0 - 5 - 5\right) = \frac{1}{2\pi}\left(\frac{5\pi}{2} - 10\right) = \frac{5}{2\pi}\left(\frac{\pi}{2} - 2\right)$$

$$= -0.342 \text{ amps}$$

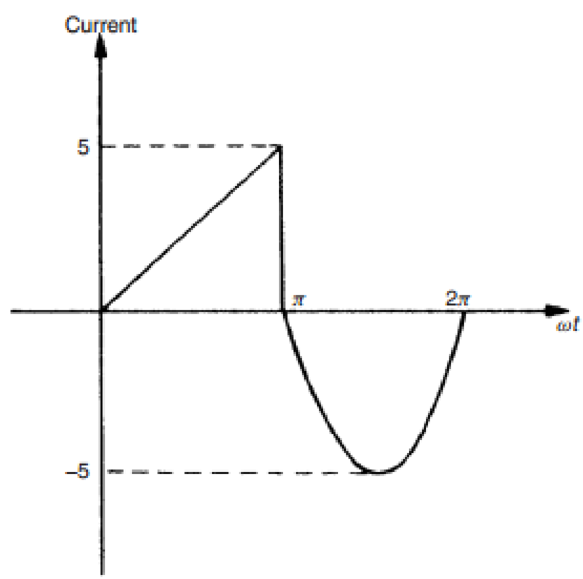


Fig. 6.10 Circuit for example 6.1.

Example – 07:

Calculate the reading that would be observed on a moving-iron ammeter when it is measuring the current in the circuit shown in Figure 6.10.

Solution

A moving-iron meter measures r.m.s. current.

$$I^2_{\text{r.m.s.}} = \frac{1}{2\pi}\left(\int_0^{\pi}\frac{25\,(\omega t)^2}{\pi^2}\mathrm{d}\omega t + \int_{\pi}^{2\pi} 25\sin^2(\omega t)\,\mathrm{d}\omega t\right)$$

$$= \frac{1}{2\pi}\left(\int_0^{\pi}\frac{25\,(\omega t)^2}{\pi^2}\mathrm{d}\omega t + \int_{\pi}^{2\pi}\frac{25\,(1-\cos 2\omega t)}{2}\mathrm{d}\omega t\right)$$

$$= \frac{25}{2\pi}\left(\left[\frac{(\omega t)^3}{3\pi^2}\right]_0^{\pi} + \left[\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right]_{\pi}^{2\pi}\right) = \frac{25}{2\pi}\left(\frac{\pi}{3}+\frac{2\pi}{2}-\frac{\pi}{2}\right)$$

$$= \frac{25}{2\pi}\left(\frac{\pi}{3}+\frac{\pi}{2}\right) = \frac{25}{2}\left(\frac{1}{3}+\frac{1}{2}\right) = 10.416$$

Thus, $I_{\text{r.m.s.}} = \sqrt{(I^2_{\text{r.m.s.}})} = 3.23\,\text{amps}$

Example – 08:

A dynamometer ammeter is connected in series with a $500\,\Omega$ resistor, a rectifying device and a $240\,\text{V}$ r.m.s. alternating sinusoidal power supply. The rectifier behaves as a resistance of $200\,\Omega$ to current flowing in one direction and as a resistance of $2\,\text{k}\Omega$ to current in the opposite direction. Calculate the reading on the meter.

Solution

$$V_{\text{peak}} = \sqrt{V_{\text{r.m.s.}}\,(2)} = 339.4\,\text{V}$$

For $0 < wt < \pi$, $R = 700\,\Omega$ and for $\pi < wt < 2\pi$, $R = 2500\,\Omega$. Thus:

$$I^2_{\text{r.m.s.}} = \frac{1}{2\pi}\left(\int_0^{\pi}\frac{(339.4\sin\omega t)^2}{700^2}\mathrm{d}\omega t + \int_{\pi}^{2\pi}\frac{(339.4\sin\omega t)^2}{2500^2}\mathrm{d}\omega t\right)$$

$$= \frac{339.4^2}{2\pi 10^4}\left(\int_0^{\pi}\frac{\sin^2\omega t}{49}\mathrm{d}\omega t + \int_{\pi}^{2\pi}\frac{\sin^2\omega t}{625}\mathrm{d}\omega t\right)$$

$$= \frac{339.4^2}{4\pi 10^4}\left(\int_0^{\pi}\frac{(1-\cos 2\omega t)}{49}\mathrm{d}\omega t + \int_{\pi}^{2\pi}\frac{(1-\cos 2\omega t)}{625}\mathrm{d}\omega t\right)$$

$$= \frac{339.4^2}{4\pi 10^4}\left(\left[\frac{\omega t}{49} - \frac{\sin 2\omega t}{98}\right]_0^{\pi} + \left[\frac{\omega t}{625} - \frac{\sin 2\omega t}{1250}\right]_{\pi}^{2\pi}\right)$$

$$= \frac{339.4^2}{4\pi 10^4}\left(\frac{\pi}{49}+\frac{\pi}{625}\right) = 0.0634$$

Hence, $I_{\text{r.m.s.}} = \sqrt{0.0634} = 0.25$ amp.

Example – 09:

A certain type of pressure transducer, designed to measure pressures in the range 0–10 bar, consists of a diaphragm with a strain gauge cemented to it to detect diaphragm

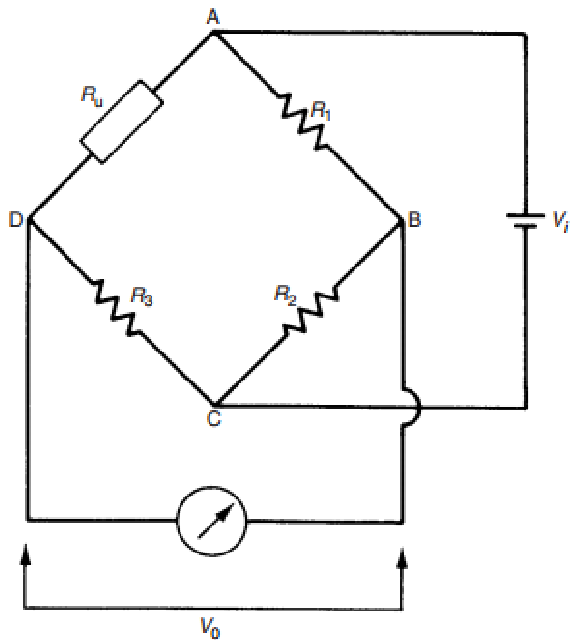


Fig. 7.2 Deflection-type d.c. bridge.

deflections. The strain gauge has a nominal resistance of 120 Ω and forms one arm of a Wheatstone bridge circuit, with the other three arms each having a resistance of 120 Ω. The bridge output is measured by an instrument whose input impedance can be assumed infinite. If, in order to limit heating effects, the maximum permissible gauge current is 30 mA, calculate the maximum permissible bridge excitation voltage. If the sensitivity of the strain gauge is 338 mΩ/bar and the maximum bridge excitation voltage is used, calculate the bridge output voltage when measuring a pressure of 10 bar.

Solution

This is the type of bridge circuit shown in Figure 7.2 in which the components have the following values:

$$R_1 = R_2 = R_3 = 120\,\Omega$$

Defining I_1 to be the current flowing in path ADC of the bridge, we can write:

$$V_i = I_1(R_u + R_3)$$

At balance, $R_u = 120$ and the maximum value allowable for I_1 is 0.03 A.

Hence:

$$V_i = 0.03(120 + 120) = 7.2\ \text{V}$$

Thus, the maximum bridge excitation voltage allowable is 7.2 volts.

For a pressure of 10 bar applied, the resistance change is 3.38 Ω, i.e. R_u is then equal to 123.38 Ω.

Applying equation (7.3), we can write:

$$V_0 = V_i\left(\frac{R_u}{R_u + R_3} - \frac{R_1}{R_1 + R_2}\right) = 7.2\left(\frac{123.38}{243.38} - \frac{120}{240}\right) = 50\,\text{mV}$$

Thus, if the maximum permissible bridge excitation voltage is used, the output voltage is 50 mV when a pressure of 10 bar is measured.

The non-linear relationship between output reading and measured quantity exhibited by equation (7.3) is inconvenient and does not conform with the normal requirement for a linear input–output relationship. The method of coping with this non-linearity varies according to the form of primary transducer involved in the measurement system.

One special case is where the change in the unknown resistance R_u is typically small compared with the nominal value of R_u. If we calculate the new voltage V_0' when the resistance R_u in equation (7.3) changes by an amount δR_u, we have:

$$V_0' = V_i \left(\frac{R_u + \delta R_u}{R_u + \delta R_u + R_3} - \frac{R_1}{R_1 + R_2} \right) \tag{7.4}$$

The change of voltage output is therefore given by:

$$\delta V_0 = V_0' - V_0 = \frac{V_i \delta R_u}{R_u + \delta R_u + R_3}$$

If $\delta R_u << R_u$, then the following linear relationship is obtained:

$$\frac{\delta V_0}{\delta R_u} = \frac{V_i}{R_u + R_3} \tag{7.5}$$

This expression describes the measurement sensitivity of the bridge. Such an approximation to make the relationship linear is valid for transducers such as strain gauges where the typical changes of resistance with strain are very small compared with the nominal gauge resistance.

However, many instruments that are inherently linear themselves at least over a limited measurement range, such as resistance thermometers, exhibit large changes in output as the input quantity changes, and the approximation of equation (7.5) cannot be applied. In such cases, specific action must be taken to improve linearity in the relationship between the bridge output voltage and the measured quantity. One common solution to this problem is to make the values of the resistances R_2 and R_3 at least ten times those of R_1 and R_u (nominal). The effect of this is best observed by looking at a numerical example.

Consider a platinum resistance thermometer with a range of 0°–50°C, whose resistance at 0°C is 500 Ω and whose resistance varies with temperature at the rate of 4 Ω/°C. Over this range of measurement, the output characteristic of the thermometer itself is nearly perfectly linear. (N.B. The subject of resistance thermometers is discussed further in Chapter 14.)

Taking first the case where $R_1 = R_2 = R_3 = 500\,\Omega$ and $V_i = 10$ V, and applying equation (7.3):

$$\text{At } 0^\circ\text{C}; \quad V_0 = 0$$

$$\text{At } 25^\circ\text{C}; \quad R_u = 600\,\Omega \quad \text{and} \quad V_0 = 10\left(\frac{600}{1100} - \frac{500}{1000}\right) = 0.455\,\text{V}$$

$$\text{At } 50^\circ\text{C}; \quad R_u = 700\,\Omega \quad \text{and} \quad V_0 = 10\left(\frac{700}{1200} - \frac{500}{1000}\right) = 0.833\,\text{V}$$

This relationship between V_0 and R_u is plotted as curve (a) in Figure 7.3 and the non-linearity is apparent. Inspection of the manner in which the output voltage V_0 above changes for equal steps of temperature change also clearly demonstrates the non-linearity.

For the temperature change from 0 to 25°C, the change in V_0 is
$(0.455 - 0) = 0.455\,\text{V}$

For the temperature change from 25 to 50°C, the change in V_0 is
$(0.833 - 0.455) = 0.378\,\text{V}$

If the relationship was linear, the change in V_0 for the 25–50°C temperature step would also be 0.455 V, giving a value for V_0 of 0.910 V at 50°C.

Now take the case where $R_1 = 500\,\Omega$ but $R_2 = R_3 = 5000\,\Omega$ and let $V_i = 26.1$ V:

$$\text{At } 0^\circ\text{C};\quad V_0 = 0$$

$$\text{At } 25^\circ\text{C};\quad R_u = 600\,\Omega \quad \text{and} \quad V_0 = 26.1\left(\frac{600}{5600} - \frac{500}{5500}\right) = 0.424\,\text{V}$$

$$\text{At } 50^\circ\text{C};\quad R_u = 700\,\Omega \quad \text{and} \quad V_0 = 26.1\left(\frac{700}{5700} - \frac{500}{5500}\right) = 0.833\,\text{V}$$

This relationship is shown as curve (b) in Figure 7.3 and a considerable improvement in linearity is achieved. This is more apparent if the differences in values for V_0 over the two temperature steps are inspected.

$$\text{From 0 to } 25^\circ\text{C, the change in } V_0 \text{ is } 0.424\,\text{V}$$

$$\text{From 25 to } 50^\circ\text{C, the change in } V_0 \text{ is } 0.409\,\text{V}$$

The changes in V_0 over the two temperature steps are much closer to being equal than before, demonstrating the improvement in linearity. However, in increasing the values of R_2 and R_3, it was also necessary to increase the excitation voltage from 10 V to 26.1 V to obtain the same output levels. In practical applications, V_i would normally be set at the maximum level consistent with the limitation of the effect of circuit heating in order to maximize the measurement sensitivity ($V_0/\delta R_u$ relationship). It would therefore not be possible to increase V_i further if R_2 and R_3 were increased, and the general effect of such an increase in R_2 and R_3 is thus a decrease in the sensitivity of the measurement system.

The importance of this inherent non-linearity in the bridge output relationship is greatly diminished if the primary transducer and bridge circuit are incorporated as elements within an intelligent instrument. In that case, digital computation is applied to produce an output in terms of the measured quantity that automatically compensates for the non-linearity in the bridge circuit.

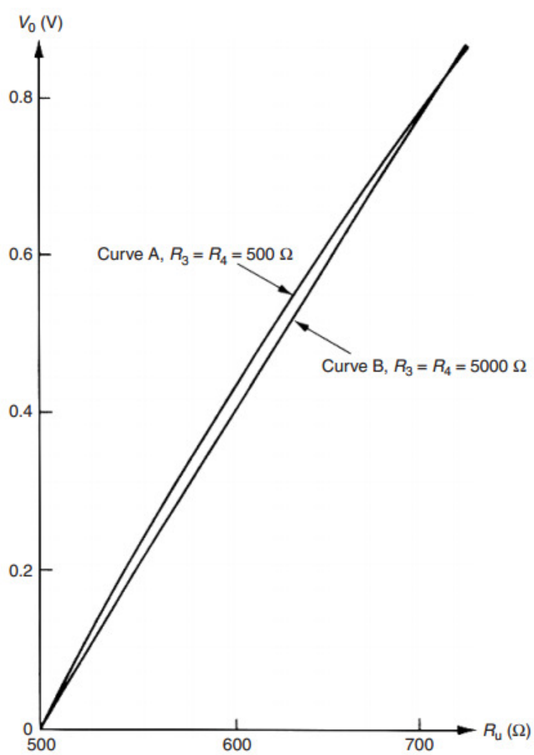


Fig. 7.3 Linearization of bridge circuit characteristic.

Example – 10:

A bridge circuit, as shown in Figure 7.5, is used to measure the value of the unknown resistance R_u of a strain gauge of nominal value 500 Ω. The output voltage measured across points DB in the bridge is measured by a voltmeter. Calculate the measurement sensitivity in volts/ohm change in R_u if

(a) the resistance R_m of the measuring instrument is neglected, and
(b) account is taken of the value of R_m.

Solution
For $R_u = 500\,\Omega$, $V_m = 0$.
To determine sensitivity, calculate V_m for $R_u = 501\,\Omega$.
(a) Applying equation (7.3): $V_m = V_i\left(\frac{R_u}{R_u + R_3} - \frac{R_1}{R_1 + R_2}\right)$

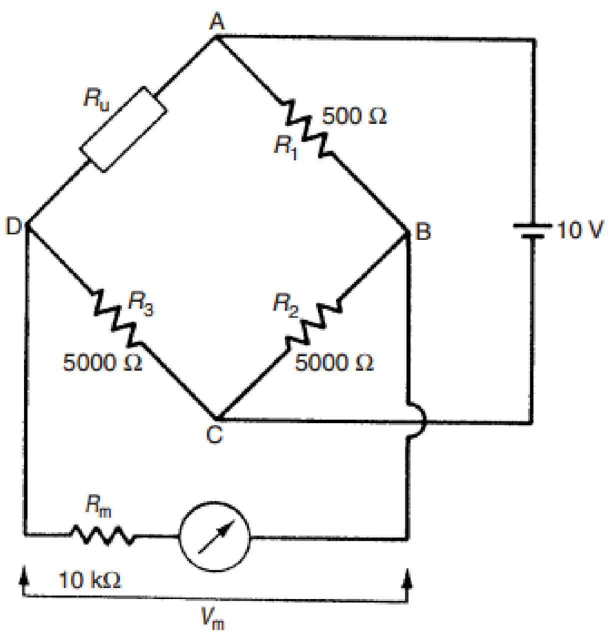


Fig. 7.5 Bridge circuit.

Substituting in values: $V_m = 10\left(\frac{501}{1001} - \frac{500}{1000}\right) = 5.00\,\text{mV}$
Thus, if the resistance of the measuring circuit is neglected, the measurement sensitivity is 5.00 mV per ohm change in R_u.
(b) Applying equation (7.10) and substituting in values:

$$V_m = \frac{10 \times 10^4 \times 500(501 - 500)}{500^2(1001) + 500 \times 501(1000) + 10^4 \times 1000 \times 1001} = 4.76\,\text{mV}$$

Thus, if proper account is taken of the 10 kΩ value of the resistance of R_m, the true measurement sensitivity is shown to be 4.76 mV per ohm change in R_u.

Example – 11:

In the Wheatstone bridge circuit of Figure 7.1, R_v is a decade resistance box with a specified inaccuracy $\pm0.2\%$ and $R_2 = R_3 = 500\,\Omega \pm 0.1\%$. If the value of R_v at the null position is $520.4\,\Omega$, determine the error band for R_u expressed as a percentage of its nominal value.

Solution

Applying equation (7.2) with $R_v = 520.4\,\Omega + 0.2\% = 521.44\,\Omega$, $R_3 = 5000\,\Omega + 0.1\% = 5005\,\Omega$, $R_2 = 5000\,\Omega - 0.1\% = 4995\,\Omega$ we get:

$$R_v = \frac{521.44 \times 5005}{4995} = 522.48\,\Omega(= +0.4\%)$$

Applying equation (7.2) with $R_v = 520.4\,\Omega - 0.2\% = 519.36\,\Omega$, $R_3 = 5000\,\Omega - 0.1\% = 4995\,\Omega$, $R_2 = 5000\,\Omega + 0.1\% = 5005\,\Omega$, we get:

$$R_v = \frac{519.36 \times 4995}{5005} = 518.32\,\Omega(= -0.4\%)$$

Thus, the error band for R_u is $\pm0.4\%$.

The cumulative effect of errors in individual bridge circuit components is clearly seen. Although the maximum error in any one component is $\pm0.2\%$, the possible error in the measured value of R_u is $\pm0.4\%$. Such a magnitude of error is often not acceptable, and special measures are taken to overcome the introduction of error by component-value tolerances. One such practical measure is the introduction of apex balancing. This is one of many methods of bridge balancing that all produce a similar result.

Example – 12:

A potentiometer R_5 is put into the apex of the bridge shown in Figure 7.6 to balance the circuit. The bridge components have the following values:

$$R_u = 500\,\Omega, R_v = 500\,\Omega, R_2 = 515\,\Omega, R_3 = 480\,\Omega, R_5 = 100\,\Omega.$$

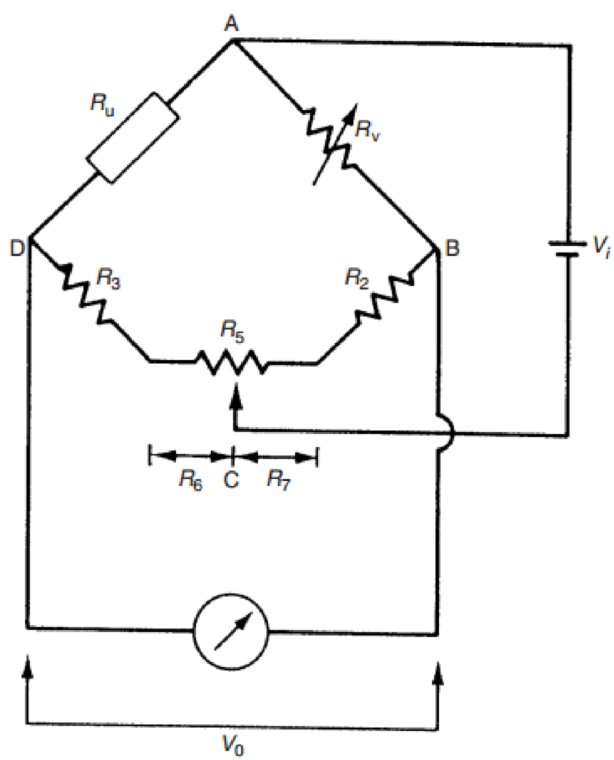


Fig. 7.6 Apex balancing.

Determine the required value of the resistances R_6 and R_7 of the parts of the potentiometer track either side of the slider in order to balance the bridge and compensate for the unequal values of R_2 and R_3.

Solution
For balance, $R_2 + R_7 = R_3 + R_6$; hence, $515 + R_7 = 480 + R_6$
Also, because R_6 and R_7 are the two parts of the potentiometer track R_5 whose resistance is $100\ \Omega$:

$R_6 + R_7 = 100$; thus $515 + R_7 = 480 + (100 - R_7)$; i.e. $2R_7 = 580 - 515 = 65$
Thus, $R_7 = 32.5$; hence, $R_6 = 100 - 32.5 = 67.5\ \Omega$.

Example – 13:

In the Maxwell bridge shown in Figure 7.8, let the fixed-value bridge components have the following values: $R_3 = 5\ \Omega$; $C = 1$ mF. Calculate the value of the unknown impedance (L_u, R_u) if $R_1 = 159\ \Omega$ and $R_2 = 10\ \Omega$ at balance. Calculate the Q factor for the unknown impedance at a supply frequency of 50 Hz.

Solution
Substituting values into the relations developed in equation (7.13) above:

$$R_u = \frac{R_2 R_3}{R_1} = \frac{10 \times 5}{159} = 0.3145\ \Omega; \quad L_u = R_2 R_3 C = \frac{10 \times 5}{1000} = 50\ \text{mH}$$

$$Q = \frac{\omega L_u}{R_u} = \frac{2\pi 50(0.05)}{0.3145} = 49.9$$

Example – 14:

A deflection bridge as shown in Figure 7.9 is used to measure an unknown capacitance, C_u. The components in the bridge have the following values:

$$V_s = 20\ \text{V}_{\text{r.m.s.}},\ C_1 = 100\ \mu\text{F},\ R_2 = 60\ \Omega,\ R_3 = 40\ \Omega$$

If $C_u = 100\ \mu$F, calculate the output voltage V_0.

Solution
From equation (7.14):

$$V_0 = V_s\left(\frac{C_1}{C_1 + C_u} - \frac{R_3}{R_2 + R_3}\right) = 20(0.5 - 0.4) = 2\text{V}_{\text{r.m.s.}}$$

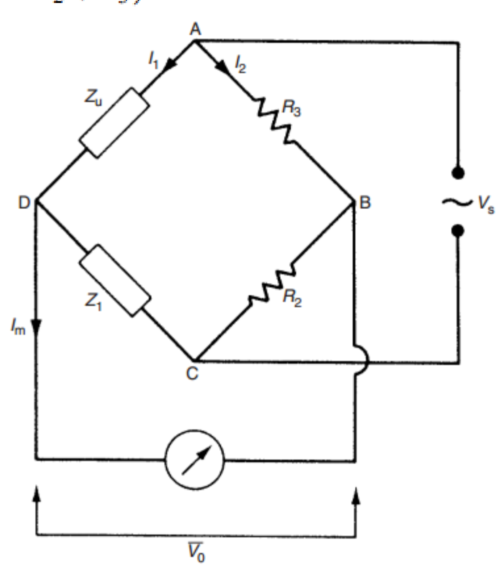


Fig. 7.9 Deflection-type a.c. bridge.

Example – 15:

An unknown inductance L_u is measured using a deflection type of bridge as shown in Figure 7.9. The components in the bridge have the following values:

$$V_s = 10\,\mathrm{V_{r.m.s.}}, L_1 = 20\,\mathrm{mH}, R_2 = 100\,\Omega, R_3 = 100\,\Omega$$

If the output voltage V_0 is 1 $\mathrm{V_{r.m.s.}}$, calculate the value of L_u.

Solution

From equation (7.15):

$$\frac{L_u}{L_1 + L_u} = \frac{V_0}{V_s} + \frac{R_3}{R_2 + R_3} = 0.1 + 0.5 = 0.6$$

Thus

$$L_u = 0.6(L_1 + L_u); \quad 0.4L_u = 0.6L_1; \quad L_u = \frac{0.6L_1}{0.4} = 30\,\mathrm{mH}$$

Example – 16:

A PMMC ammeter has the following specification

Coil dimension are $1\text{cm}\times 1\text{cm}$. Spring constant is $0.15\times10^{-6}\, N-m/rad$, Flux density is $1.5\times10^{-3}\, wb/m^2$. Determine the no. of turns required to produce a deflection of 90^0 when a current 2mA flows through the coil.

Solution:

At steady state **condition** $T_d = T_C$

$BANI = K\theta$

$\Rightarrow N = \dfrac{K\theta}{BAI}$

$A=1\times10^{-4} m^2$

$K=0.15\times10^{-6} \dfrac{N-m}{rad}$

$B=1.5\times10^{-3} wb/m^2$

$I=2\times10^{-3} A$

$\theta = 90^\circ = \dfrac{\Pi}{2} rad$

N=785 ans.

Example – 17:

The pointer of a moving coil instrument gives full scale deflection of 20mA. The potential difference across the meter when carrying 20mA is 400mV.The instrument to be used is 200A for full scale deflection. Find the shunt resistance required to achieve this, if the instrument to be used as a voltmeter for full scale reading with 1000V. Find the series resistance to be connected it?

Solution:

Case-1

V_m =400mV

$I_m = 20mA$

I=200A

$$R_m = \frac{V_m}{I_m} = \frac{400}{20} = 20\Omega$$

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$200 = 20\times10^{-3}\left[1+\frac{20}{R_{sh}}\right]$$

$$R_{sh} = 2\times10^{-3}\Omega$$

Case-II

V=1000V

$$V = V_m\left(1+\frac{R_{se}}{R_m}\right)$$

$$4000 = 400\times10^{-3}\left(1+\frac{R_{se}}{20}\right)$$

$$R_{se} = 49.98k\Omega$$

Example – 18:

A 150 v moving iron voltmeter is intended for 50HZ, has a resistance of 3kΩ. Find the series resistance required to extent the range of instrument to 300v. If the 300V instrument is used to measure a d.c. voltage of 200V. Find the voltage across the meter?

Solution:

$R_m = 3k\Omega, V_m = 150V, V = 300V$

$$V = V_m\left(1+\frac{R_{se}}{R_m}\right)$$

$$300 = 150\left(1+\frac{R_{se}}{3}\right) \Rightarrow R_{se} = 3k\Omega$$

Case-II $\quad V = V_m\left(1+\frac{R_{se}}{R_m}\right)$

$$200 = V_m\left(1+\frac{3}{3}\right)$$

$\therefore V_m = 100V$ Ans

Example – 19:

What is the value of series resistance to be used to extent '0'to 200V range of 20,000Ω/volt voltmeter to 0 to 2000 volt?

Solution:

$V_{se} = V - V = 1800$

$I_{FSD} = \frac{1}{20000} = \frac{1}{Sensitivity}$

$V_{se} = R_{se} \times i_{FSD} \Rightarrow R_{se} = 36M\Omega$ ans.

Example – 20:

A moving coil instrument whose resistance is 25Ω gives a full scale deflection with a current of 1mA. This instrument is to be used with a manganin shunt, to extent its range to 100mA. Calculate the error caused by a 10^0C rise in temperature when:

(a) Copper moving coil is connected directly across the manganin shunt.

(b) A 75 ohm manganin resistance is used in series with the instrument moving coil.

The temperature co-efficient of copper is 0.004/^{0}C and that of manganin is 0.00015^0/C.

Solution:

Case-1

$I_m = 1mA$

$R_m = 25\Omega$

I=100mA

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$100 = 1\left(1+\frac{25}{R_{sh}}\right) \Rightarrow \frac{25}{R_{sh}} = 99$$

$$\Rightarrow R_{sh} = \frac{25}{99} = 0.2525\Omega$$

Instrument resistance for 10^0C rise in temperature, $R_{mt} = 25(1+0.004\times 10)$

$R_t = R_o(1+\rho_t \times t)$

$R_{m/t=10^\circ} = 26\Omega$

Shunt resistance for 10^0C, rise in temperature

$$R_{sh/t=10^\circ} = 0.2525(1+0.00015\times 10) = 0.2529\Omega$$

Current through the meter for 100mA in the main circuit for 10^0C rise in temperature

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)\Bigg|_{t=10^\circ C}$$

$$100 = I_{mt}\left(1+\frac{26}{0.2529}\right)$$

$I_m|_{t=10} = 0.963mA$

But normal meter current=1mA

Error due to rise in temperature=(0.963-1)*100=-3.7%

Case-b As voltmeter

Total resistance in the meter circuit= $R_m + R_{sh} = 25 + 75 = 100\Omega$

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$100 = 1\left(1+\frac{100}{R_{sh}}\right)$$

$$R_{sh} = \frac{100}{100-1} = 1.01\Omega$$

Resistance of the instrument circuit for 10^0C rise in temperature

$$R_m|_{t=10} = 25(1+0.004\times10) + 75(1+0.00015\times10) = 101.11\Omega$$

Shunt resistance for 10^0C rise in temperature

$$R_{sh}|_{t=10} = 1.01(1+0.00015\times10) = 1.0115\Omega$$

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$100 = I_m\left(1+\frac{101.11}{1.0115}\right)$$

$$I_m|t = 10^{\circ} = 0.9905mA$$

Error =(0.9905-1)*100=-0.95%

Example – 21:

The coil of a 600V M.I meter has an inductance of 1 henery. It gives correct reading at 50HZ and requires 100mA. For its full scale deflection, what is % error in the meter when connected to 200V D.C. by comparing with 200V A.C?

Solution:

$V_m = 600V, I_m = 100mA$

Case-I A.C.

$$Z_m = \frac{V_m}{I_m} = \frac{600}{0.1} = 6000\Omega$$

$$X_L = 2\Pi fL = 314\Omega$$

$$R_m = \sqrt{{Z_m}^2 - X_L^2} = \sqrt{(6000)^2 - (314)^2} = 5990\Omega$$

$$I_{AC} = \frac{V_{AC}}{Z} = \frac{200}{6000} = 33.33mA$$

Case-II D.C

$$I_{DC} = \frac{V_{DC}}{R_m} = \frac{200}{5990} = 33.39mA$$

Error= $\frac{I_{DC} - I_{AC}}{I_{AC}}\times100 = \frac{33.39-33.33}{33.33}\times100 = 0.18\%$

Example – 22:

A 250V M.I. voltmeter has coil resistance of 500Ω, coil inductance 0f 1.04 H and series resistance of 2kΩ. The meter reads correctively at 250V D.C. What will be the value of capacitance to be used for shunting the series resistance to make the meter read correctly at 50HZ? What is the reading of voltmeter on A.C. without capacitance?

Solution: $C = 0.41\dfrac{L}{(R_S)^2}$

$$= 0.41 \times \frac{1.04}{(2\times10^3)^2} = 0.1\mu F$$

For A.C $Z = \sqrt{(R_m + R_{Se})^2 + X_L^2}$

$$Z = \sqrt{(500+2000)^2 + (314)^2} = 2520\Omega$$

With D.C

$R_{total} = 2500\Omega$

For $2500\Omega \rightarrow 250V$

$$1\Omega \rightarrow \frac{250}{2500}$$

$$2520\Omega \rightarrow \frac{250}{2500}\times 2520 = 248V$$

Example – 23:

The relationship between inductance of moving iron ammeter, the current and the position of pointer is as follows:

Reading (A)	1.2	1.4	1.6	1.8
Deflection (degree)	36.5	49.5	61.5	74.5
Inductance (μH)	575.2	576.5	577.8	578.8

Calculate the deflecting torque and the spring constant when the current is 1.5A?

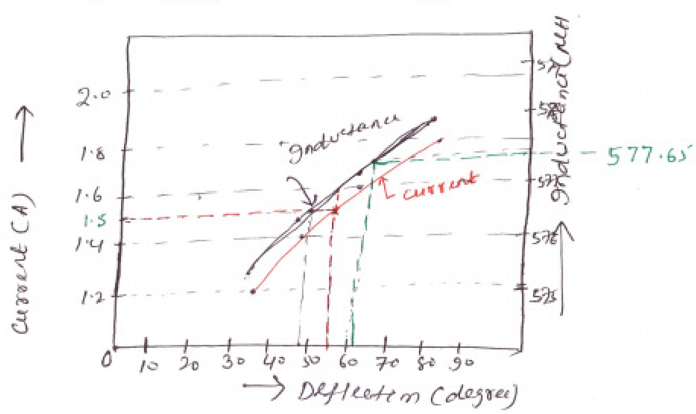
Solution:

For current I=1.5A, θ=55.5 degree=0.96865 rad

$$\frac{dL}{d\theta} = \frac{577.65 - 576.5}{60 - 49.5} = 0.11\mu H / \deg ree = 6.3\mu H / rad$$

Deflecting torque , $T_d = \dfrac{1}{2}I^2\dfrac{dL}{d\theta} = \dfrac{1}{2}(1.5)^2 \times 6.3\times10^{-6} = 7.09\times10^{-6}\, N-m$

Spring constant, $K = \dfrac{T_d}{\theta} = \dfrac{7.09\times10^{-6}}{0.968} = 7.319\times10^{-6}\,\dfrac{N-m}{rad}$



Example – 24:

For a certain dynamometer ammeter the mutual inductance 'M' varies with deflection θ as $M = -6\cos(\theta + 30^{\circ})mH$ .Find the deflecting torque produced by a direct current of 50mA corresponding to a deflection of 60^0.

Solution:

$$T_d = I_1 I_2 \frac{dM}{d\theta} = I^2 \frac{dM}{d\theta}$$

$$M = -6\cos(\theta + 30^{\circ})$$

$$\frac{dM}{d\theta} = 6\sin(\theta + 30) mH$$

$$\frac{dM}{d\theta}\Big|_{\theta=60} = 6\sin 90 = 6mH / \deg$$

$$T_d = I^2 \frac{dM}{d\theta} = (50\times10^{-3})^2 \times 6\times10^{-3} = 15\times10^{-6} N - m$$

Example – 25:

The inductance of a moving iron ammeter with a full scale deflection of 90^0 at 1.5A, is given by the expression $L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$, where θ is deflection in radian from the zero position. Estimate the angular deflection of the pointer for a current of 1.0A.

Solution:

$$L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$$

$$\frac{dL}{d\theta}\Big|_{\theta=90^{\circ}} = 40 - 8\theta - 3\theta^2 \mu H / rad$$

$$\frac{dL}{d\theta}\Big|_{\theta=90^{\circ}} = 40 - 8\times\frac{\Pi}{2} - 3(\frac{\Pi}{2})^2 \mu H / rad = 20\mu H / rad$$

$$\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta}\right)$$

$$\frac{\Pi}{2} = \frac{1}{2}\frac{(1.5)^2}{K} \times 20\times10^{-6}$$

K=Spring constant=$14.32\times10^{-6} N - m / rad$

For I=1A, $\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta}\right)$

$$\therefore \theta = \frac{1}{2} \times \frac{(1)^2}{14.32\times10^{-6}} \left(40 - 8\theta - 3\theta^2\right)$$

$$3\theta + 36.64\theta^2 - 40 = 0$$

$$\theta = 1.008 rad, 57.8^{\circ}$$

Example – 26:

The inductance of a moving iron instrument is given by $L = 10 + 5\theta - \theta^2 - \theta^3 \mu H$, where θ is the deflection in radian from zero position. The spring constant is $12\times10^{-6} N-m/rad$. Estimate the deflection for a current of 5A.

Solution:

$$\frac{dL}{d\theta} = (5-2\theta)\frac{\mu H}{rad}$$

$$\therefore \theta = \frac{1}{2K}I^2\left(\frac{dL}{d\theta}\right)$$

$$\therefore \theta = \frac{1}{2}\times\frac{(5)^2}{12\times10^{-6}}(5-2\theta)\times10^{-6}$$

$$\therefore \theta = 1.69 rad, 96.8^{\circ}$$

Example – 27:

The following figure gives the relation between deflection and inductance of a moving iron instrument.

Deflection (degree)	20	30	40	50	60	70	80	90
Inductance (μH)	335	345	355.5	366.5	376.5	385	391.2	396.5

Find the current and the torque to give a deflection of (a) 30^0 (b) 80^0. Given that control spring constant is $0.4\times10^{-6} N-m/\deg ree$

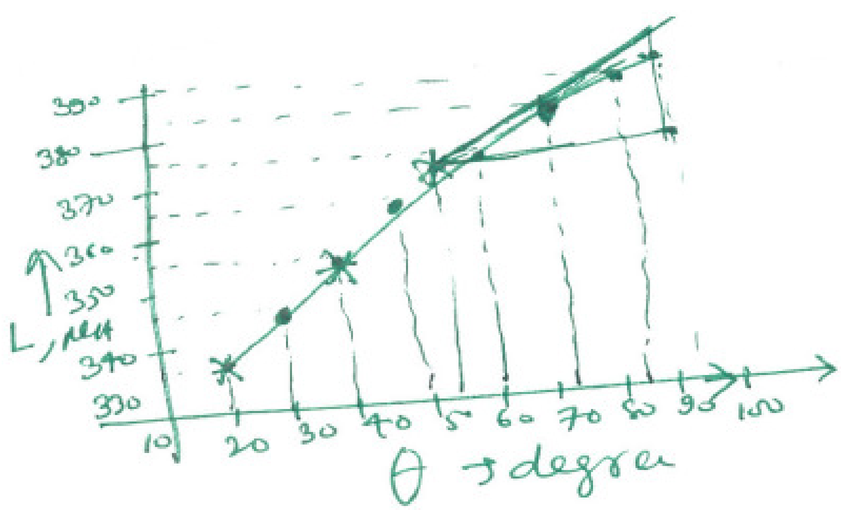
Solution:

$$\theta = \frac{1}{2K}I^2\left(\frac{dL}{d\theta}\right)$$

(a) For $\theta = 30^{\circ}$

The curve is linear

$$\therefore \left(\frac{dL}{d\theta}\right)_{\theta=30} = \frac{355.5-335}{40-20} = 1.075\mu H/\deg ree = 58.7\mu H/rad$$



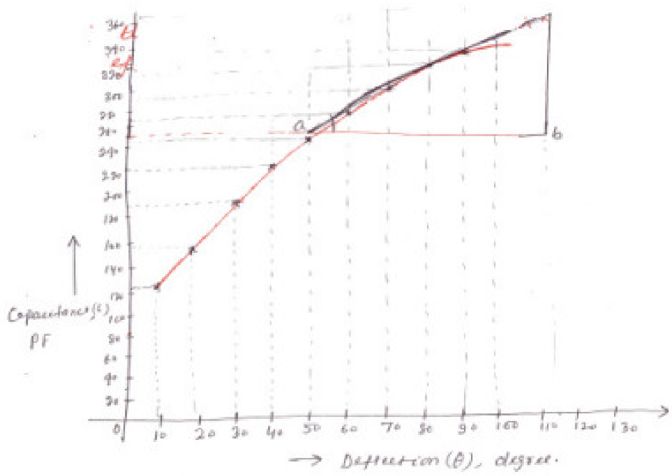
Example – 28:

In an electrostatic voltmeter the full scale deflection is obtained when the moving plate turns through 90^0. The torsional constant is $10\times10^{-6}\,N-m/rad$. The relation between the angle of deflection and capacitance between the fixed and moving plates is given by

Deflection (degree)	0	10	20	30	40	50	60	70	80	90
Capacitance (PF)	81.4	121	156	189.2	220	246	272	294	316	334

Find the voltage applied to the instrument when the deflection is 90^0?

Solution:



$$\frac{dC}{d\theta}=\tan\theta=\frac{bc}{ab}=\frac{370-250}{110-44}=1.82PF/\deg ree=104.2PF/rad$$

$$\text{Spring constant } K=10\times10^{-6}\frac{N-m}{rad}=0.1745\times10^{-6}N-m/\deg ree$$

$$\theta=\frac{1}{2K}V^2\left(\frac{dC}{d\theta}\right)\Rightarrow V=\sqrt{\frac{2K\theta}{\frac{dC}{d\theta}}}$$

$$V=\sqrt{\frac{2\times0.1745\times10^{-6}\times90}{104.2\times10^{-12}}}=549volt$$

Example – 29:

Design a multi range d.c. mille ammeter using a basic movement with an internal resistance $R_m = 50\Omega$ and a full scale deflection current $I_m = 1mA$. The ranges required are 0-10mA; 0-50mA; 0-100mA and 0-500mA.

Solution:

Case-I 0-10mA

Multiplying power $m = \frac{I}{I_m} = \frac{10}{1} = 10$

$\therefore$ Shunt resistance $R_{sh1} = \frac{R_m}{m-1} = \frac{50}{10-1} = 5.55\Omega$

Case-II 0-50mA

$m = \frac{50}{1} = 50$

$R_{sh2} = \frac{R_m}{m-1} = \frac{50}{50-1} = 1.03\Omega$

Case-III 0-100mA, $m = \frac{100}{1} = 100\Omega$

$R_{sh3} = \frac{R_m}{m-1} = \frac{50}{100-1} = 0.506\Omega$

Case-IV 0-500mA, $m = \frac{500}{1} = 500\Omega$

$R_{sh4} = \frac{R_m}{m-1} = \frac{50}{500-1} = 0.1\Omega$

Example – 30:

A moving coil voltmeter with a resistance of 20Ω gives a full scale deflection of 120^0, when a potential difference of 100mV is applied across it. The moving coil has dimension of 30mm*25mm and is wounded with 100 turns. The control spring constant is $0.375\times10^{-6} N-m/\deg ree.$ Find the flux density, in the air gap. Find also the diameter of copper wire of coil winding if 30% of instrument resistance is due to coil winding. The specific resistance for copper=$1.7\times10^{-8}\Omega m$.

Solution:

Data given

$V_m = 100mV$

$R_m = 20\Omega$

$\theta = 120^\circ$

N=100

$K = 0.375\times10^{-6} N-m/\deg ree$

$R_C = 30\% of R_m$

$\rho = 1.7\times10^{-8}\Omega m$

$$I_m = \frac{V_m}{R_m} = 5\times10^{-3} A$$

$$T_d = BANI, T_C = K\theta = 0.375\times10^{-6}\times120 = 45\times10^{-6} N-m$$

$$B = \frac{T_d}{ANI} = \frac{45\times10^{-6}}{30\times25\times10^{-6}\times100\times5\times10^{-3}} = 0.12 wb/m^2$$

$$R_C = 0.3\times20 = 6\Omega$$

Length of mean turn path =2(a+b) =2(55)=110mm

$$R_C = N\left(\frac{\rho l}{A}\right)$$

$$A = \frac{N\times\rho\times(l_t)}{R_C} = \frac{100\times1.7\times10^{-8}\times110\times10^{-3}}{6}$$

$$= 3.116\times10^{-8} m^2$$

$$= 31.16\times10^{-3} mm^2$$

$$A = \frac{\Pi}{4}d^2 \Rightarrow d = 0.2mm$$

Example – 31:

A moving coil instrument gives a full scale deflection of 10mA, when the potential difference across its terminal is 100mV. Calculate

(1) The shunt resistance for a full scale deflection corresponding to 100A

(2) The resistance for full scale reading with 1000V.

Calculate the power dissipation in each case?

Solution:

Data given

$I_m = 10mA$

$V_m = 100mV$

$I = 100A$

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$100 = 10\times10^{-3}\left(1+\frac{10}{R_{sh}}\right)$$

$$R_{sh} = 1.001\times10^{-3}\Omega$$

$$R_{se} = ??, V = 1000V$$

$$R_m = \frac{V_m}{I_m} = \frac{100}{10} = 10\Omega$$

$$V = V_m\left(1+\frac{R_{se}}{R_m}\right)$$

$$1000 = 100\times10^{-3}\left(1+\frac{R_{se}}{10}\right)$$

$$\therefore R_{se} = 99.99 K\Omega$$

Example – 32:

Design an Aryton shunt to provide an ammeter with current ranges of 1A,5A,10A and 20A. A basic meter with an internal resistance of 50w and a full scale deflection current of 1mA is to be used.

Solution: Data given

$$I_m = 1\times10^{-3}A \quad R_m = 50\Omega$$

$$I_1 = 1A \quad I_2 = 5A \quad I_3 = 10A \quad I_4 = 20A$$

$$m_1 = \frac{I_1}{I_m} = 1000A$$
$$m_2 = \frac{I_2}{I_m} = 5000A$$
$$m_3 = \frac{I_3}{I_m} = 10000A$$
$$m_4 = \frac{I_4}{I_m} = 20000A$$

$$R_{sh1} = \frac{R_m}{m_1 - 1} = \frac{50}{1000-1} = 0.05\Omega$$
$$R_{sh2} = \frac{R_m}{m_2 - 1} = \frac{50}{5000-1} = 0.01\Omega$$
$$R_{sh3} = \frac{R_m}{m_3 - 1} = \frac{50}{10000-1} = 0.005\Omega$$
$$R_{sh4} = \frac{R_m}{m_4 - 1} = \frac{50}{20000-1} = 0.0025\Omega$$

$\therefore$ The resistances of the various section of the universal shunt are

$$R_1 = R_{sh1} - R_{sh2} = 0.05 - 0.01 = 0.04\Omega$$
$$R_2 = R_{sh2} - R_{sh3} = 0.01 - 0.005 = 0.005\Omega$$
$$R_3 = R_{sh3} - R_{sh4} = 0.005 - 0.025 = 0.0025\Omega$$
$$R_4 = R_{sh4} = 0.0025\Omega$$

Example – 33:

A basic d' Arsonval meter movement with an internal resistance $R_m = 100\Omega$ and a full scale current of $I_m = 1mA$ is to be converted in to a multi range d.c. voltmeter with ranges of 0-10V, 0-50V, 0-250V, 0-500V. Find the values of various resistances using the potential divider arrangement.

Solution:

Data given

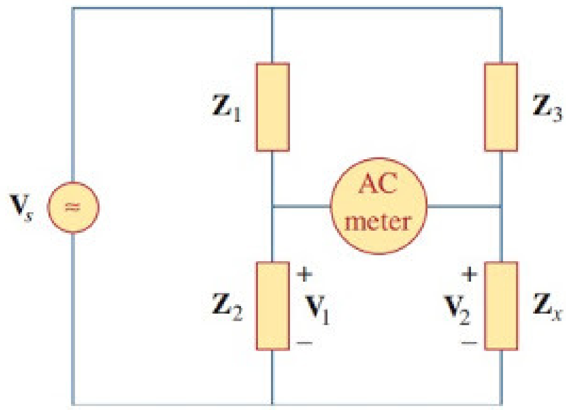
$$R_m = 100\Omega$$
$$I_m = 1mA$$
$$V_m = I_m \times R_m$$
$$V_m = 100\times1\times10^{-3}$$
$$V_m = 100mV$$

$$m_1 = \frac{V_1}{V_m} = \frac{10}{100\times10^{-3}} = 100$$
$$m_2 = \frac{V_2}{V_m} = \frac{50}{100\times10^{-3}} = 500$$
$$m_3 = \frac{V_3}{V_m} = \frac{250}{100\times10^{-3}} = 2500$$
$$m_4 = \frac{V_4}{V_m} = \frac{500}{100\times10^{-3}} = 5000$$

$$R_1 = (m_1 - 1)R_m = (100-1)\times100 = 9900\Omega$$
$$R_2 = (m_2 - m_1)R_m = (500-100)\times100 = 40K\Omega$$
$$R_3 = (m_3 - m_2)R_m = (2500-500)\times100 = 200K\Omega$$
$$R_4 = (m_4 - m_3)R_m = (5000-2500)\times100 = 250K\Omega$$

Example – 34:

The ac bridge circuit of figure balances when Z_1 is a 1 kΩ resistor, Z_2 is a 4.2 kΩ resistor, Z_3 is a parallel combination of a 1.5 MΩ resistor and a 12 pF capacitor, and f = 2kHz. Find: (a) the series components that makeup Z_x, and (b) the parallel components that makeup Z_x.



Solution:

We know,

$$\mathbf{Z}_x = \frac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2$$

Where, $Z_x = R_x + jX_x$,

Here,

$$\mathbf{Z}_1 = 1000\ \Omega, \qquad \mathbf{Z}_2 = 4200\ \Omega$$

And

$$\mathbf{Z}_3 = R_3 \parallel \frac{1}{j\omega C_3} = \frac{\dfrac{R_3}{j\omega C_3}}{R_3 + 1/j\omega C_3} = \frac{R_3}{1 + j\omega R_3 C_3}$$

Or

$$\mathbf{Z}_3 = \frac{1.5 \times 10^6}{1 + j2\pi \times 2 \times 10^3 \times 1.5 \times 10^6 \times 12 \times 10^{-12}} = \frac{1.5 \times 10^6}{1 + j0.2262}$$

$$\mathbf{Z}_3 = 1.427 - j0.3228\ \mathrm{M\Omega}$$

Equating the real and imaginary parts yields R_x = 5.993 MΩ and a capacitive reactance

$$X_x = \frac{1}{\omega C} = 1.356 \times 10^6$$

$$C = \frac{1}{\omega X_x} = \frac{1}{2\pi \times 2 \times 10^3 \times 1.356 \times 10^6} = 58.69\ \mathrm{pF}$$

(b) Z_x remains the same but R_x and X_x are in parallel. Assuming an RC parallel combination,

$$\mathbf{Z}_x = (5.993 - j1.356)\ \mathrm{M\Omega}$$
$$= R_x \parallel \frac{1}{j\omega C_x} = \frac{R_x}{1 + j\omega R_x C_x}$$

By equating the real and imaginary parts, we get

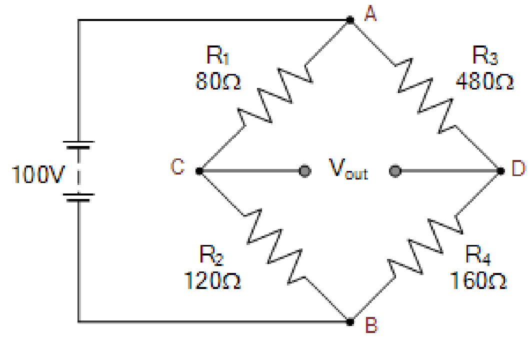
$$R_x = \frac{\mathrm{Real}(\mathbf{Z}_x)^2 + \mathrm{Imag}(\mathbf{Z}_x)^2}{\mathrm{Real}(\mathbf{Z}_x)} = \frac{5.993^2 + 1.356^2}{5.993} = \mathbf{6.3\ M\Omega}$$

$$C_x = -\frac{\mathrm{Imag}(\mathbf{Z}_x)}{\omega[\mathrm{Real}(\mathbf{Z}_x)^2 + \mathrm{Imag}(\mathbf{Z}_x)^2]}$$

$$= -\frac{-1.356}{2\pi(2000)(5.917^2 + 1.356^2)} = \mathbf{2.852\ \mu F}$$

Example – 35:

Calculate the output voltage across points C and D and the value of resistor R_4 required to balance the bridge circuit.



Solution:

For the first series arm, ACB

$$V_C = \frac{R_2}{(R_1+R_2)} \times V_S$$

$$V_C = \frac{120\Omega}{80\Omega + 120\Omega} \times 100 = 60\text{volts}$$

For the second series arm, ADB

$$V_D = \frac{R_4}{(R_3+R_4)} \times V_S$$

$$V_D = \frac{160\Omega}{480\Omega + 160\Omega} \times 100 = 25\text{volts}$$

The voltage across points C-D is given as:

$$V_{OUT} = V_C - V_D$$

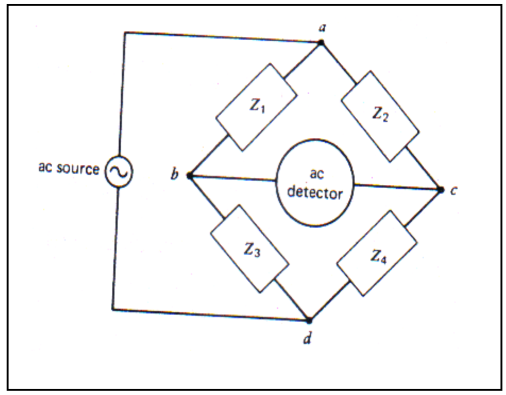
$$\therefore V_{OUT} = 60 - 25 = 35\text{volts}$$

The value of resistor, R_4 required to balance the bridge is given as:

$$R_4 = \frac{R_2 R_3}{R_1} = \frac{120\Omega \times 480\Omega}{80\Omega} = 720\Omega$$

Example – 36:

The impedances of the AC Bridge in figure are given as follows,



$$Z_1 = 200\angle 30^0\,\Omega$$

$$Z_2 = 150\angle 0^0\,\Omega$$

$$Z_3 = 250\angle -40^0\,\Omega$$

$$Z_x = Z_4 = unknown$$

Determine the constants of the unknown arm.

Solution:

The first condition for bridge balance requires that

$$Z_1Z_x = Z_2Z_3$$

$$Z_x = (Z_2Z_3/Z_1)$$

$$= [(150 * 250)/200]$$

$$= 187.5\ \Omega$$

The second condition for balance requires that the sums of the phase angles of opposite arms be equal,

$$\theta_1 + \theta_x = \theta_2 + \theta_3$$

$$\theta_x = \theta_2 + \theta_3 - \theta_1$$

$$= 0 + (-40^o) - 30^o$$

$$= -70^o$$

The unknown impedance Z_x, can be written as,

$$Z_x = 187.5\ \Omega / -70$$

$$= (64.13 - j176.19)\ \Omega$$

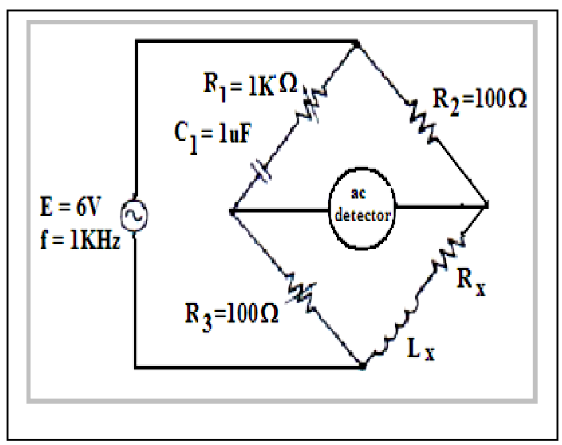
This indicates that we are dealing with a capacitive element, possibly consisting of a series resistor and a capacitor.

Example – 37:

An Opposite-Angle bridge is shown in figure. Find,

(i) The equivalent series resistance, Rx.

(ii) The inductance, L_x.



Solution:

$$Z_1Z_x = Z_2Z_3$$

$$Z_x = \frac{Z_2Z_3}{Z_1} = \frac{R_2R_3}{\left(R_1 - \frac{j}{\omega C_1}\right)}$$

$$= \frac{100\Omega * 100\Omega}{\left(1K\Omega - \frac{j}{2*\pi*1KHz*1\mu F}\right)}$$

$$Z_x = 9.75 + j1.552\Omega \quad -----(1)$$

$$R_x = 9.75\Omega$$

$$X_{Lx} = 1.552\Omega$$
$$X_{Lx} = \omega L_x$$
$$L_x = \frac{X_{Lx}}{\omega}$$
$$= \frac{1.552\Omega}{2 * \pi * 1KHz}$$
$$L_x = 247.04\mu H$$

Example – 38:

Find the equivalent series element for the unknown impedance of the Schering bridge network whose impedance measurements are to be made at null.

$R_1 = 470$ kΩ $\qquad$ $C_1 = 0.01$ mF

$R_2 = 100$ kΩ $\qquad$ $C_3 = 0.1$ mF

Find R_x and C_x ,

Solution:

$$R_x = R_2 \frac{C_1}{C_2} = \frac{(100 * 10^3) * (0.01 * 10^{-6})}{0.01 * 10^{-6}} = 10K\Omega$$

$$C_x = C_3 \frac{R_1}{R_2} = \frac{(0.01 * 10^{-6}) * (470 * 10^3)}{100 * 10^3} = 0.47 * 10^{-6} F = 0.47 \mu F$$

Example – 39:

A Maxwell-Wien bridge as shown in below operates at a supply frequency of 100Hz used to measure inductive impedance. The bridge
balanced at the following values:

$C_1 = 0.01\mu$F, $R_1 = 470\Omega$, $R_2 = 2.2k\Omega$ and $R_3 = 100\Omega$

Find the series resistance and inductance and determine its *Q-factor*.

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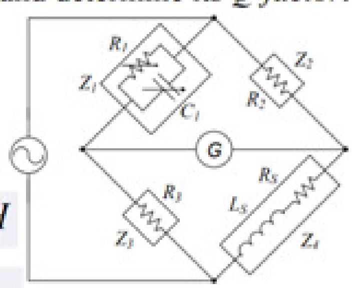
$$R_S = \frac{R_2 R_3}{R_1} = \frac{2.2k \times 100}{470} = 468.1\Omega$$

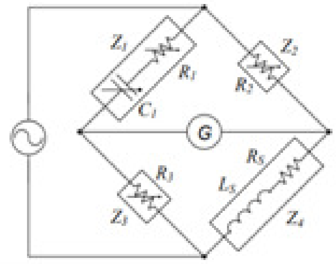
$$L_S = C_1 R_2 R_3 = 0.01\mu \times 2.2k \times 100 = 2.2mH$$

$$Q = \frac{wL_s}{R_s} = \frac{2\pi \times 100 \times 2.2 \times 10^{-3}}{468.1} = 0.00295$$



Example – 40:

Calculate the inductance and resistance of the network that causes a **Hay bridge** as shown in figure below to null with the following component values: w=3000rad/s, C_1=0.1nF, R_1=20kΩ, R_2=10kΩ and R_3=1kΩ.



SOLUTION 11

To find the series resistance and inductance, we use the above equations as:

$$L_S = \frac{C_1 R_2 R_3}{1 + w^2 R_1^2 C_1^2} = \frac{0.1n \times 10k \times 1k}{1 + (3000)^2 (20k)^2 (0.1n)^2} = 1mH$$

$$R_S = \frac{w^2 C_1^2 R_1 R_2 R_3}{1 + w^2 R_1^2 C_1^2} = \frac{(3000)^2 (0.1n)^2 \times 20k \times 10k \times 1k}{1 + (3000)^2 (20k)^2 (0.1n)^2} = 0.018\Omega$$