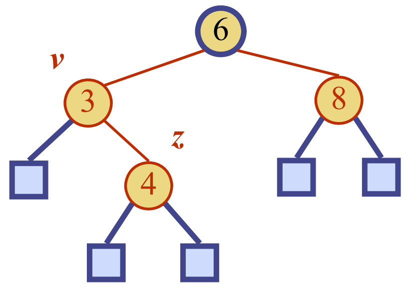
Lecture 8-2: AVL Trees



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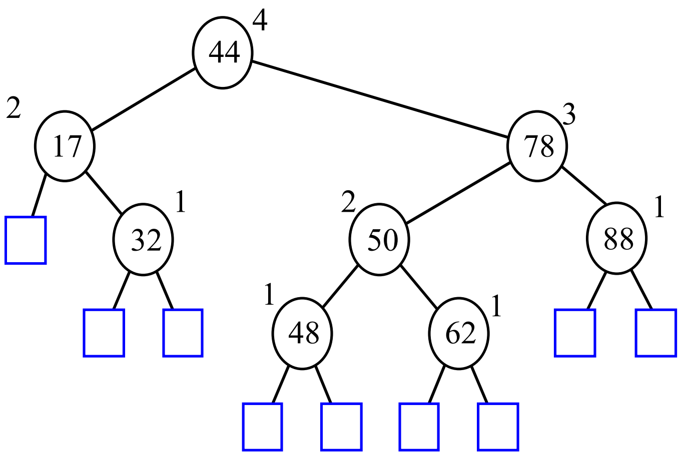
AVL Tree Definition

- AVL trees are balanced
  "Height Balance Property"
- An AVL Tree is a binary search tree such that for every internal node v of T, the heights of the children of v can differ by at most 1

바뀌어진 트리들의 높이 속성이 원래 모양이었던 모양의 높이의 차가 1이하 인것만 허용해 노드를 만듬

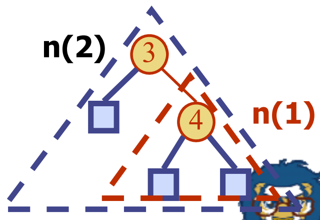


An example of an AVL tree where the heights are shown next to the nodes:

Georgy Adelson-Velsky and E. M. Landis

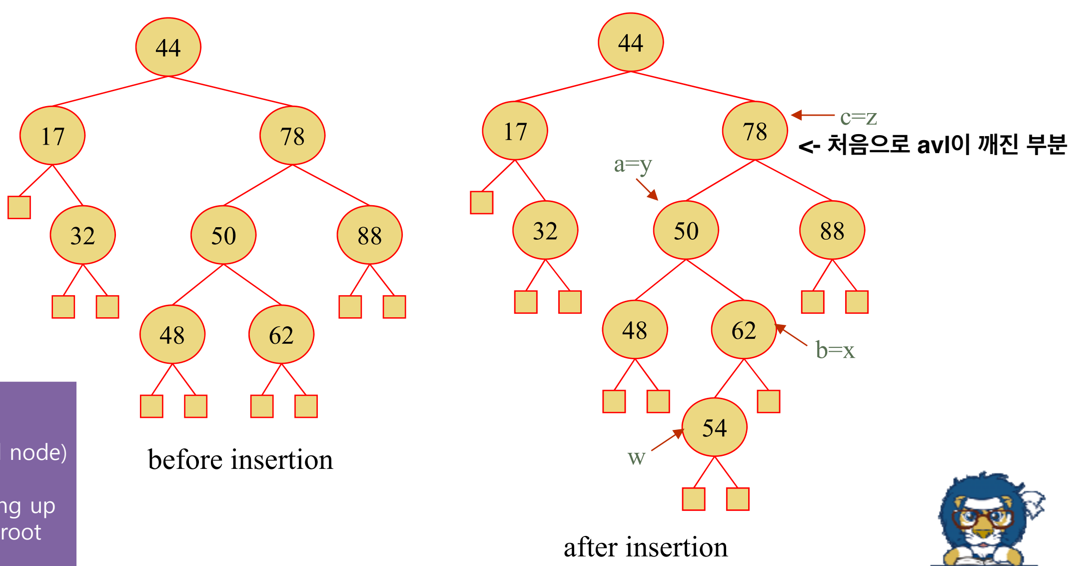
Height of an AVL Tree

- Fact: The height of an AVL tree storing n keys is O(log n).
- Proof: Let us bound n(h): the minimum number of internal nodes of an AVL tree of height h.
- We easily see that n(1) = 1 and n(2) = 2
- For n > 2, an AVL tree of height h contains the root node, one AVL subtree of height h-1 and another of height h-2.
- That is, n(h) = 1 + n(h-1) + n(h-2)
- Knowing n(h-1) > n(h-2), we get n(h) > 2n(h-2). So
  n(h) > 2n(h-2), n(h) > 4n(h-4), n(h) > 8n(n-6), … (by induction),
  n(h) > 2^in(h-2i)
- Solving the base case we get: n(h) > $2^{h/2-1}$
- Taking logarithms: h < 2log n(h) +2
- Thus the height of an AVL tree is O(log n)



Insertion

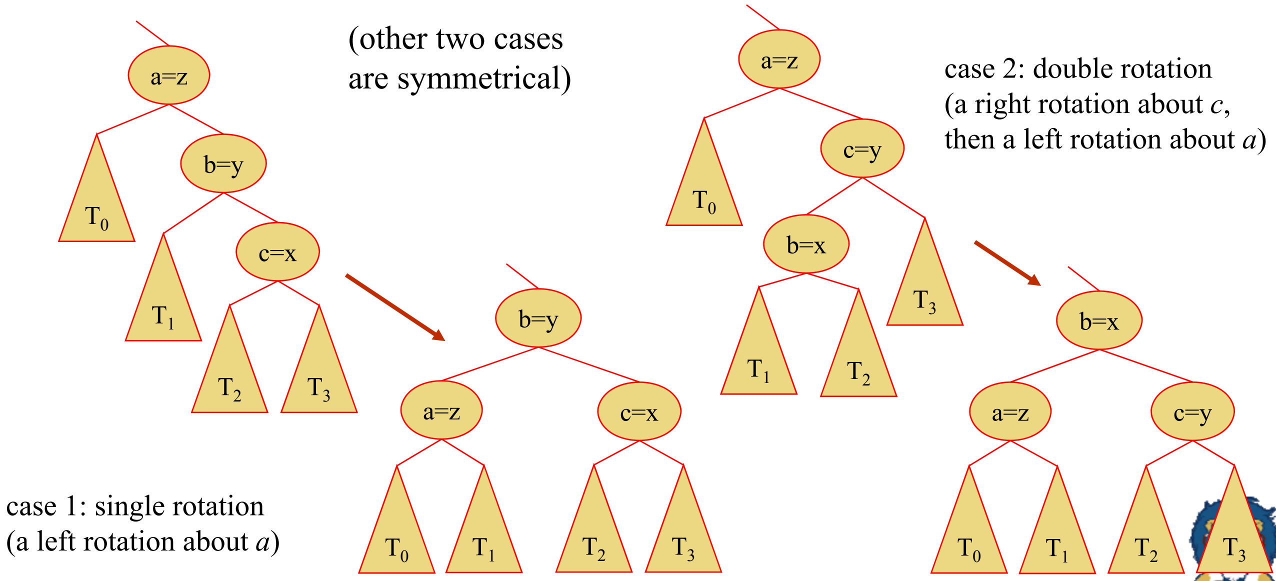
- Insertion is as in a binary search tree
- Always done by expanding an external node.
- Example:



- x, y, z selection criteria: ancestor's of w (inserted node)
  - z: 1st unbalanced node encountered in going up from w toward the root

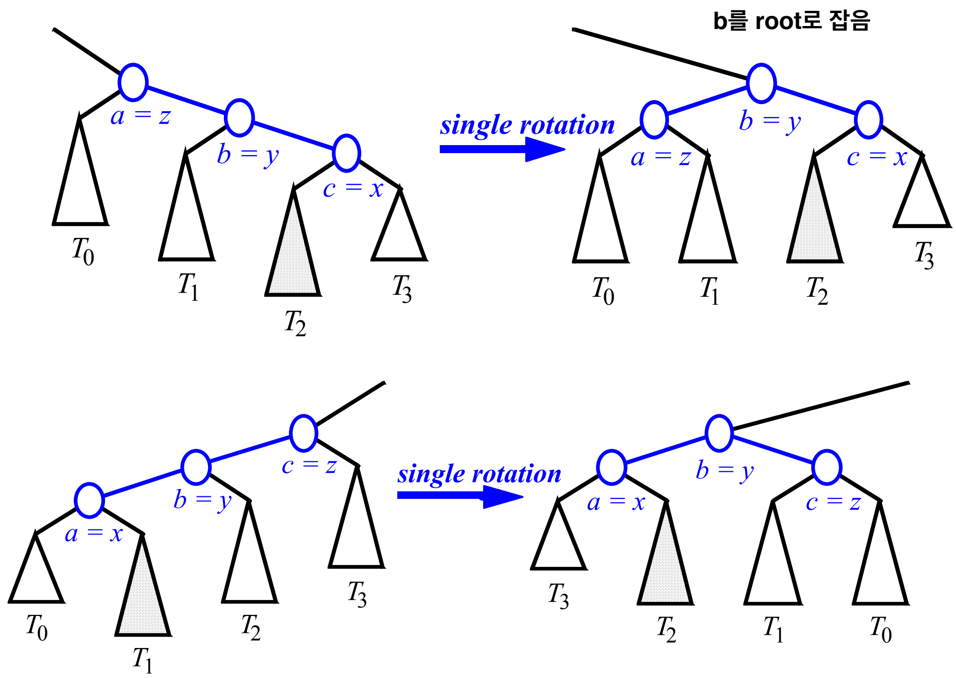
Trinode Restructuring

- let (a,b,c) be an inorder listing of x, y, z
- perform the rotations needed to **make b the topmost node of the three**



Restructuring (as Single Rotations)

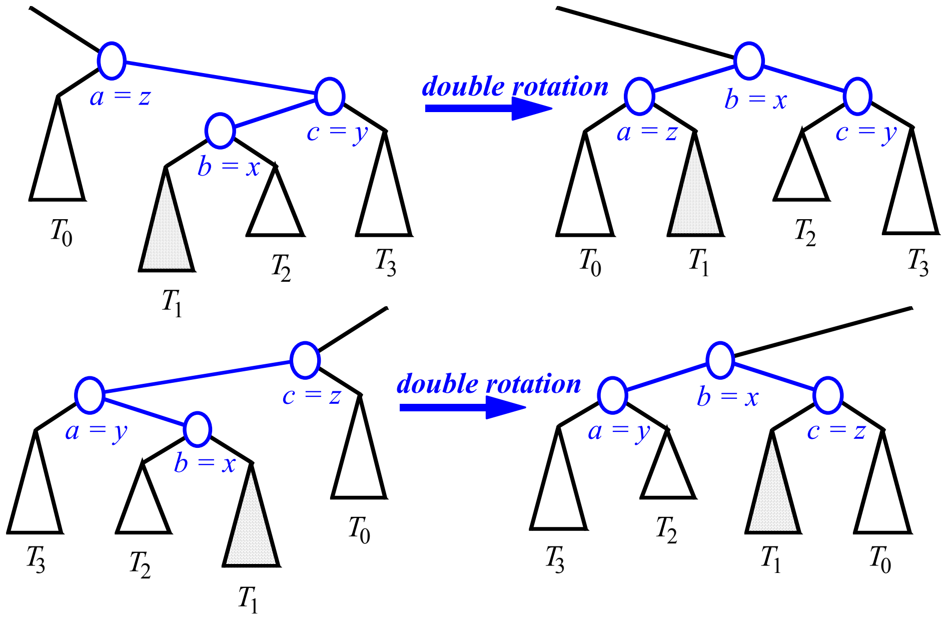
- Single Rotations:



inorder순서: a,b,c

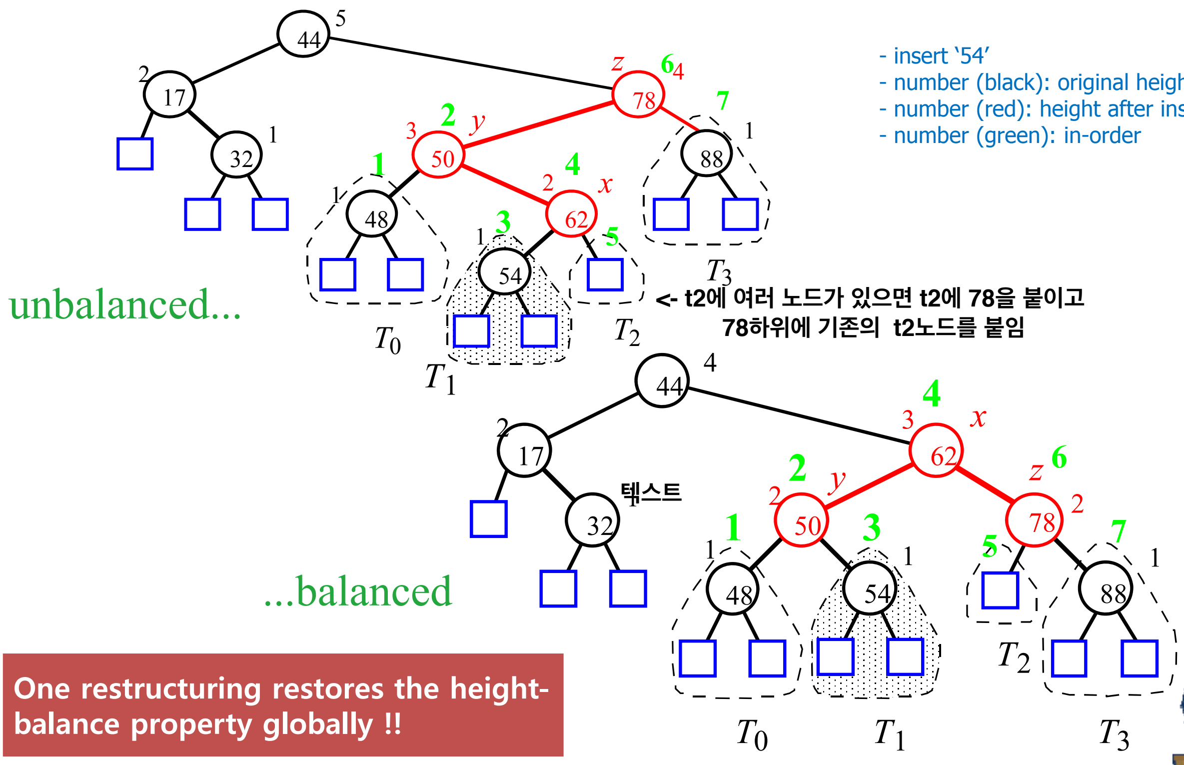
Restructuring (as Double Rotations)

- double rotations:



Insertion Example, continued

- insert '54'
- number (black): original height
- number (red): height after insertion
- number (green): in-order

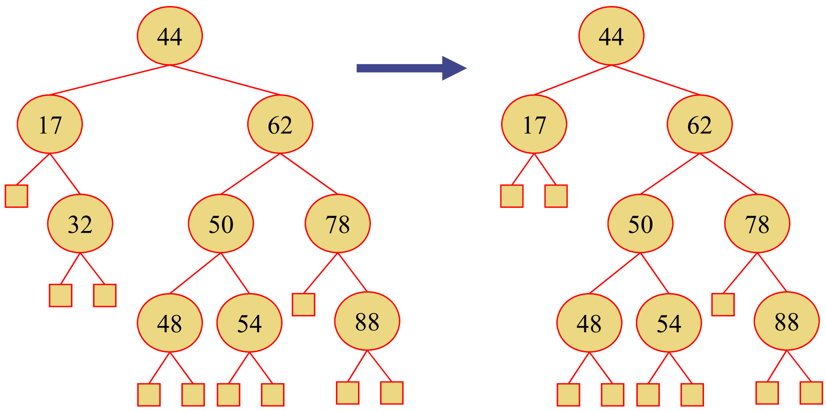


One restructuring restores the height-balance property globally !!

inorder순서: a,b,c

Removal

- Removal begins as in a binary search tree, which means the node removed will become an empty external node. Its parent, w, may cause an imbalance.
- Example:



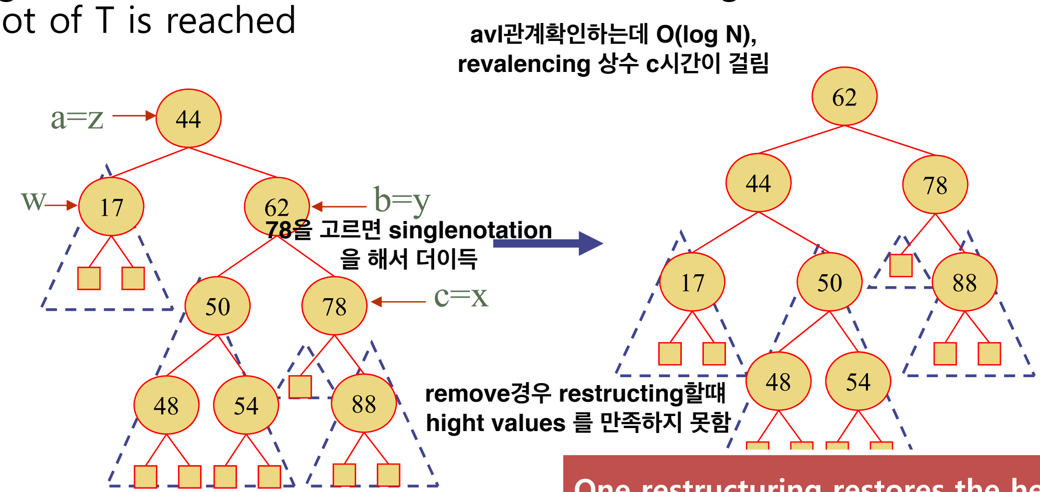
before deletion of 32

after deletion

- x, y, z selection criteria:
  z: 1st unbalanced node encountered
  y: is the child of z with larger height
  x: taller child of y
  else the same side as y

Rebalancing after a Removal

- Let z be the first unbalanced node encountered while travelling up the tree from w. Also, let y be the child of z with the larger height, and let x be the child of y with the larger height
- We perform restructure(x) to restore balance at z
- As this restructuring may upset the balance of another node higher in the tree, we must continue checking for balance until the root of T is reached



One restructuring restores the height-balance property locally !!

AVL Tree Performance

- a single restructure takes O(1) time
  - using a linked-structure binary tree
- get takes O(log n) time
  - height of tree is O(log n), no restructures needed
- put takes O(log n) time
  - initial find is O(log n)
  - Restructuring up the tree, maintaining heights is O(log n)
- remove takes O(log n) time
  - initial find is O(log n)
  - Restructuring up the tree, maintaining heights is O(log n)

Q & A