3.2.3.3 The Division Operation

The **division** operation, denoted by $\div$, is suited to queries that include the phrase "for all." Suppose that we wish to find all customers who have an account at *all* the branches located in Brooklyn. We can obtain all branches in Brooklyn by the expression

$$r_1 \;=\; \Pi_{branch\text{-}name}\;(\sigma_{branch\text{-}city\,=\,\text{"Brooklyn"}}\;(branch))$$

The result relation for this expression appears in Figure 3.23.

branch-name
Brighton
Downtown

Figure 3.23 Result of $\Pi_{branch\text{-}name}(\sigma_{branch\text{-}city\,=\,\text{"Brooklyn"}}\;(branch)$.

We can find all (*customer-name, branch-name*) pairs for which the customer has an account at a branch by writing

$$r_2 \;=\; \Pi_{customer\text{-}name,\; branch\text{-}name}\;(depositor \bowtie\; account)$$

Figure 3.24 shows the result relation for this expression.

Now, we need to find customers who appear in r_2 with *every* branch name in r_1. The operation that provides exactly those customers is the divide operation. We formulate the query by writing

$$\begin{aligned}&\Pi_{customer\text{-}name,\; branch\text{-}name}\;(depositor \bowtie\; account)\\ &\quad\div\, \Pi_{branch\text{-}name}\;(\sigma_{branch\text{-}city\,=\,\text{"Brooklyn"}}\;(branch))\end{aligned}$$

The result of this expression is a relation that has the schema (*customer-name*) and that contains the tuple (Johnson).

Formally, let $r(R)$ and $s(S)$ be relations, and let $S \;\subseteq\; R$; that is, every attribute of schema S is also in schema R. The relation $r \;\div\; s$ is a relation on schema $R \;-\; S$ (that is, on the schema containing all attributes of schema R that are not in schema S). A tuple *t* is in $r \;\div\; s$ if and only if both of two conditions hold:

1. t is in $\Pi_{R-S}(r)$
2. For every tuple t_s in s, there is a tuple t_r in r satisfying both of the following:
   - **a.** $t_r[S] \;=\; t_s[S]$
   - **b.** $t_r[R \;-\; S] \;=\; t$

It may surprise you to discover that, given a division operation and the schemas of the relations, we can, in fact, define the division operation in terms of the fundamental operations. Let $r(R)$ and $s(S)$ be given, with $S \;\subseteq\; R$:

$$r \;\div\; s \;=\; \Pi_{R-S}\;(r) \;-\; \Pi_{R-S}\;((\Pi_{R-S}\;(r) \;\times\; s) \;-\; \Pi_{R-S,S}(r))$$

customer-name	*branch-name*
Hayes	Perryridge
Johnson	Downtown
Johnson	Brighton
Jones	Brighton
Lindsay	Redwood
Smith	Mianus
Turner	Round Hill

Figure 3.24 Result of $\Pi_{customer\text{-}name,\ branch\text{-}name}\ (depositor \bowtie\ account)$.

To see that this expression is true, we observe that $\Pi_{R-S}\ (r)$ gives us all tuples t that satisfy the first condition of the definition of division. The expression on the right side of the set difference operator

$$\Pi_{R-S}\ ((\Pi_{R-S}\ (r)\ \times\ s)\ -\ \Pi_{R-S,S}(r))$$

serves to eliminate those tuples that fail to satisfy the second condition of the definition of division. Let us see how it does so. Consider $\Pi_{R-S}\ (r)\ \times\ s$. This relation is on schema R, and pairs every tuple in $\Pi_{R-S}\ (r)$ with every tuple in s. The expression $\Pi_{R-S,S}(r)$ merely reorders the attributes of r.

Thus, $(\Pi_{R-S}\ (r)\ \times\ s)\ -\ \Pi_{R-S,S}(r)$ gives us those pairs of tuples from $\Pi_{R-S}\ (r)$ and s that do not appear in r. If a tuple t_j is in

$$\Pi_{R-S}\ ((\Pi_{R-S}\ (r)\ \times\ s)\ -\ \Pi_{R-S,S}(r))$$

then there is some tuple t_s in s that does not combine with tuple t_j to form a tuple in r. Thus, t_j holds a value for attributes $R\ -\ S$ that does not appear in $r\ \div\ s$. It is these values that we eliminate from $\Pi_{R-S}\ (r)$.

참고

branch-name	*branch-city*	*assets*
Brighton	Brooklyn	7100000
Downtown	Brooklyn	9000000
Mianus	Horseneck	400000
North Town	Rye	3700000
Perryridge	Horseneck	1700000
Pownal	Bennington	300000
Redwood	Palo Alto	2100000
Round Hill	Horseneck	8000000

Figure 3.3 The *branch* relation.

customer-name	*account-number*
Hayes	A-102
Johnson	A-101
Johnson	A-201
Jones	A-217
Lindsay	A-222
Smith	A-215
Turner	A-305

Figure 3.5 The *depositor* relation.

account-number	*branch-name*	*balance*
A-101	Downtown	500
A-102	Perryridge	400
A-201	Brighton	900
A-215	Mianus	700
A-217	Brighton	750
A-222	Redwood	700
A-305	Round Hill	350

Figure 3.1 The *account* relation.