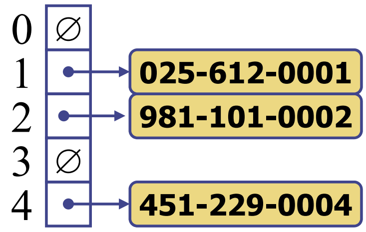
Lecture 7-2. Hash Tables

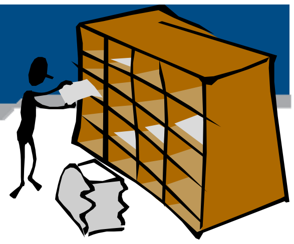


Sunghyun Cho
Professor
Division of Computer Science
chopro@hanyang.ac.kr
HANYANG UNIVERSITY

Recall the Map ADT

- get(k): if the map M has an entry with key k, return its associated value; else, return null
- put(k, v): insert entry (k, v) into the map M; if key k is not already in M, then return null; else, return old value associated with k
- remove(k): if the map M has an entry with key k, remove it from M and return its associated value; else, return null
- size(), isEmpty()
- entrySet(): return an iterable collection of the entries in M
- keySet(): return an iterable collection of the keys in M
- values(): return an iterator of the values in M

Hash Functions and Hash Tables

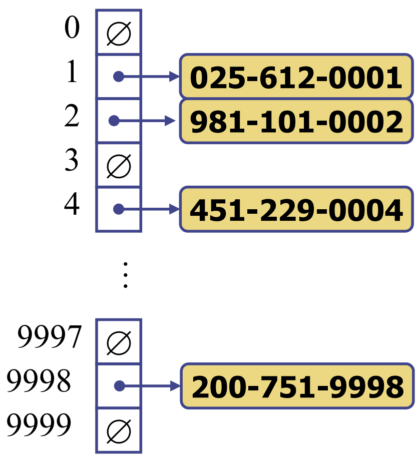


- A **hash function** ***h*** maps keys of a given type to integers in a fixed interval [0, ***N*** - 1]
- Example:
  $h(x) = x \bmod N$
  is a hash function for integer keys
- The integer $h(x)$ is called the **hash value** of key x

- A **hash table** for a given key type consists of
  - Hash function h
  - Array (called table) of size N
- When implementing a map with a hash table, the goal is to store item (k, o) at index $i = h(k)$

Example

- We design a hash table for a map storing entries as (SSN, Name), where SSN (social security number) is a nine-digit positive integer
- Our hash table uses an array of size $N = 10{,}000$ and the hash function
  $h(x) = \text{last four digits of } x$



Hash Functions

- A hash function is usually specified as the composition of two functions:

  Hash code:

  h_1: keys $\rightarrow$ integers

  Compression function:

  h_2: integers $\rightarrow [0, N-1]$

- The hash code is applied first, and the compression function is applied next on the result, i.e.,

  $$h(x) = h_2(h_1(x))$$

- The goal of the hash function is to "disperse" the keys in an apparently random way

Hash Codes

- Memory address:
  - We reinterpret the memory address of the key object as an integer (default hash code of all Java objects)
  - Good in general, except for numeric and string keys
- Integer cast:
  - We reinterpret the bits of the key as an integer
  - Suitable for keys of length less than or equal to the number of bits of the integer type (e.g., byte, short, int and float in Java)
- Component sum:
  - We partition the bits of the key into components of fixed length (e.g., 16 or 32 bits) and we sum the components (ignoring overflows)
  - Suitable for numeric keys of fixed length greater than or equal to the number of bits of the integer type (e.g., long and double in Java)

Hash Codes (cont.)

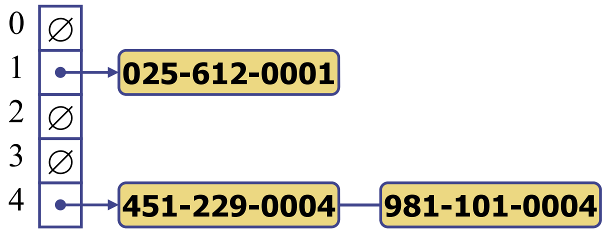
- **Polynomial accumulation:**
  - We partition the bits of the key into a sequence of components of fixed length (e.g., 8, 16 or 32 bits)
    $$a_0\, a_1 \, \ldots \, a_{n-1}$$
  - We evaluate the polynomial
    $$p(z) = a_0 + a_1 z + a_2 z^2 + \ldots$$
    $$\ldots + a_{n-1} z^{n-1}$$
    at a fixed value z, ignoring overflows
  - Especially suitable for strings (e.g., the choice $z = 33$ gives at most 6 collisions on a set of 50,000 English words)
- Polynomial $p(z)$ can be evaluated in $O(n)$ time using Horner's rule:
  - The following polynomials are successively computed, each from the previous one in $O(1)$ time
    $$p_0(z) = a_{n-1}$$
    $$p_i(z) = a_{n-i-1} + z p_{i-1}(z)$$
    $$(i = 1, 2, \ldots, n-1)$$
- We have $p(z) = p_{n-1}(z)$

Compression Functions

- Division:
  - $h_2(y) = y \bmod N$
  - The size N of the hash table is usually chosen to be a prime
  - The reason has to do with number theory and is beyond the scope of this course
- Multiply, Add and Divide (MAD):
  - $h_2(y) = (ay + b) \bmod N$
  - a and b are nonnegative integers such that
    $$a \bmod N \neq 0$$
  - Otherwise, every integer would map to the same value b

Collision Handling

- Collisions occur when different elements are mapped to the same cell
- Separate Chaining: let each cell in the table point to a linked list of entries that map there



- Separate chaining is simple, but requires additional memory outside the table

Map with Separate Chaining

Delegate operations to a list-based map at each cell:

Algorithm get(k):
return A[h(k)].get(k)

Algorithm put(k,v):
t = A[h(k)].put(k,v)
if t = **null then** {k is a new key}
    n = n + 1
return t

Algorithm remove(k):
t = A[h(k)].remove(k)
if t ≠ **null then** {k was found}
    n = n - 1
return t

Linear Probing

- **Open addressing**: the colliding item is placed in a different cell of the table
- **Linear probing**: handles collisions by placing the colliding item in the next (circularly) available table cell
- Each table cell inspected is referred to as a "probe"
- Colliding items lump together, causing future collisions to cause a longer sequence of probes

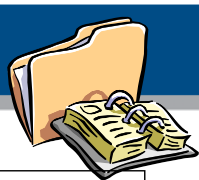
- Example:
  - $h(x) = x \bmod 13$
  - Insert keys 18, 41, 22, 44, 59, 32, 31, 73, in this order

0	1	2	3	4	5	6	7	8	9	10	11	12

⇩

		41			18	44	59	32	22	31	73	
0	1	2	3	4	5	6	7	8	9	10	11	12

Search with Linear Probing



- Consider a hash table A that uses linear probing
- get(k)
  - We start at cell $h(k)$
  - We probe consecutive locations until one of the following occurs
    - An item with key k is found, or
    - An empty cell is found, or
    - N cells have been probed unsuccessfully

```
Algorithm get(k)
    i ← h(k)
    p ← 0
    repeat
        c ← A[i]
        if c = ∅
            return null
        else if c.getKey () = k
            return c.getValue()
        else
            i ← (i + 1) mod N
            p ← p + 1
    until p = N
    return null
```

Updates with Linear Probing

- To handle insertions and deletions, we introduce a special object, called ***AVAILABLE***, which replaces deleted elements
- remove$(\boldsymbol{k})$
  - We search for an entry with key $\boldsymbol{k}$
  - If such an entry $(\boldsymbol{k}, \boldsymbol{o})$ is found, we replace it with the special item ***AVAILABLE*** and we return element $\boldsymbol{o}$
  - Else, we return ***null***
- put$(\boldsymbol{k}, \boldsymbol{o})$
  - We throw an exception if the table is full
  - We start at cell $\boldsymbol{h}(\boldsymbol{k})$
  - We probe consecutive cells until one of the following occurs
    - A cell $\boldsymbol{i}$ is found that is either empty or stores ***AVAILABLE***, or
    - $\boldsymbol{N}$ cells have been unsuccessfully probed
  - We store $(\boldsymbol{k}, \boldsymbol{o})$ in cell $\boldsymbol{i}$

Double Hashing

- Double hashing uses a secondary hash function $\boldsymbol{d}(\boldsymbol{k})$ and handles collisions by placing an item in the first available cell of the series
  $$(\boldsymbol{i} + \boldsymbol{jd}(\boldsymbol{k})) \bmod \boldsymbol{N}$$
  for $\boldsymbol{j} = 0,\ 1, \dots, \boldsymbol{N} - 1$
- The secondary hash function $\boldsymbol{d}(\boldsymbol{k})$ cannot have zero values
- The table size $\boldsymbol{N}$ must be a prime to allow probing of all the cells
- Common choice of compression function for the secondary hash function:
  $$\boldsymbol{d}_2(\boldsymbol{k}) = \boldsymbol{q} - \boldsymbol{k} \bmod \boldsymbol{q}$$
  where
  - $\boldsymbol{q} < \boldsymbol{N}$
  - $\boldsymbol{q}$ is a prime
- The possible values for $\boldsymbol{d}_2(\boldsymbol{k})$ are
  $$1, 2, \dots, \boldsymbol{q}$$

Example of Double Hashing

- Consider a hash table storing integer keys that handles collision with double hashing
  - $N = 13$
  - $h(k) = k \bmod 13$
  - $d(k) = 7 - k \bmod 7$
- Insert keys 18, 41, 22, 44, 59, 32, 31, 73, in this order

k	$h(k)$	$d(k)$	Probes		
18	5	3	5		
41	2	1	2		
22	9	6	9		
44	5	5	5	10	
59	7	4	7		
32	6	3	6		
31	5	4	5	9	0
73	8	4	8		

0	1	2	3	4	5	6	7	8	9	10	11	12

⇩

31		41			18	32	59	73	22	44		
0	1	2	3	4	5	6	7	8	9	10	11	12

Performance of Hashing

- In the worst case, searches, insertions and removals on a hash table take $O(n)$ time
- The worst case occurs when all the keys inserted into the map collide
- The load factor $\alpha = n/N$ affects the performance of a hash table
- Assuming that the hash values are like random numbers, it can be shown that the expected number of probes for an insertion with open addressing is
  $1 / (1 - \alpha)$
- The expected running time of all the dictionary ADT operations in a hash table is $O(1)$
- In practice, hashing is very fast provided the load factor is not close to 100%
- **Applications of hash tables:**
  - **small databases**
  - **compilers**
  - **browser caches**