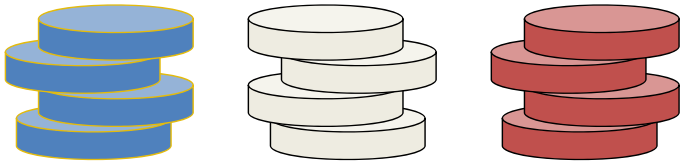
Lecture 4-1. Stacks



Sunghyun Cho
Professor
Division of Computer Science
chopro@hanyang.ac.kr

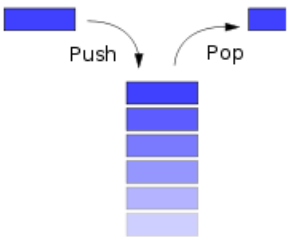
HANYANG UNIVERSITY

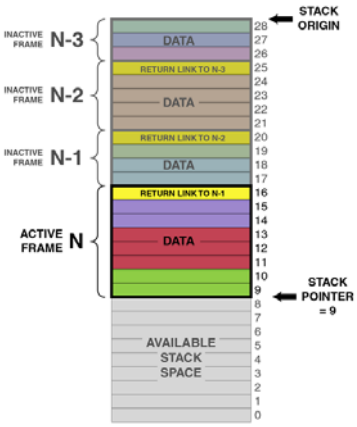
Review

- Data Structure
  - Data storing structure + access methods
- Asymptotic Analysis
  - Compare the performance of each data structures and algorithms
  - Pseudo-code → Mathematic Exp. → Big O
- List
  - The simplest and fundamental data structure in CS
  - Array based list
  - Linked list
  - O(n) for add and remove method

Keywords

- Definition of Stack
- Implementation of Stack
- Usages of Stack





Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure
- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations
- Example: ADT modeling a simple stock trading system
  - The data stored are buy/sell orders
  - The operations supported are
    - order buy(stock, shares, price)
    - order sell(stock, shares, price)
    - void cancel(order)
  - Error conditions:
    - Buy/sell a nonexistent stock
    - Cancel a nonexistent order

The Stack ADT

- The Stack ADT stores arbitrary objects
- **Insertions and deletions follow the last-in first-out scheme**
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object): inserts an element
  - object pop(): removes and returns the last inserted element
- Auxiliary stack operations:
  - object top(): returns the last inserted element without removing it
  - integer size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored

Example 6.3: Stack Operations (textbook p. 209)

Stack Interface in Java

- Java interface corresponding to our Stack ADT
- Requires the definition of class EmptyStackException
- Different from the built-in Java class java.util.Stack

```
public interface Stack<E> {

   public int size();

   public boolean isEmpty();

   public E top()
         throws EmptyStackException;

   public void push(E element);

   public E pop()
         throws EmptyStackException;
}
```

Exceptions

- Attempting the execution of an operation of ADT may sometimes cause an error condition, called an "**Exception**"
- Exceptions are said to be "thrown" by an operation that cannot be executed
- In the Stack ADT, operations pop and top cannot be performed if the stack is empty → Exception Cases
- Attempting the execution of pop or top on an empty stack throws an EmptyStackException

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the Java Virtual Machine
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

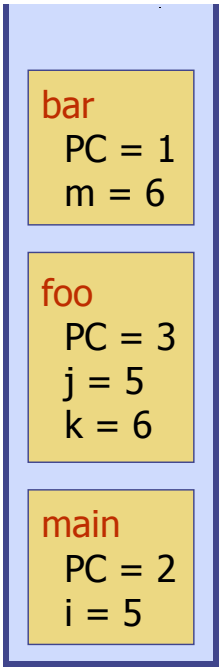
Method Stack in the JVM

- The Java Virtual Machine (JVM) keeps track of the chain of active methods with a stack
- When a method is called, the JVM pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When a method ends, its frame is popped from the stack and control is passed to the method on top of the stack
- Allows for recursion

```
main() {
   int i = 5;
   foo(i);
   }

foo(int j) {
   int k;
   k = j+1;
   bar(k);
   }

bar(int m) {
   …
   }
```



Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable **t** keeps track of the index of the top element

Algorithm *size*()
  return $t + 1$

Algorithm *pop*()
  if *isEmpty*() **then**
    throw *EmptyStackException*
  else
    $t \leftarrow t - 1$
    return $S[t + 1]$

=> S[t]

Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then throw a FullStackException
  - Limitation of the array-based implementation
  - Not intrinsic to the Stack ADT

```
Algorithm push(o)
  if t = S.length − 1 then
    throw FullStackException
  else
    t ← t + 1
    S[t] ← o
```

Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(N)$ $(N \neq n)$
  - Each operation runs in time $O(1)$

n/2까지 제거됐다가 하는 경우 스택을 사용하면 안됨

- Limitations
  - **The maximum size of the stack** must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes **an implementation-specific exception**

Array-based Stack in Java

```
public class ArrayStack<E>
      implements Stack<E> {

   // holds the stack elements
   private E S[ ];

   // index to top element
   private int top = -1;

   // constructor
   public ArrayStack(int capacity) {
       S = (E[]) new Object[capacity]);
    }
```

```
   public E pop()
         throws EmptyStackException {
    if isEmpty()
      throw new EmptyStackException
            ("Empty stack: cannot pop");
     E temp = S[top];
     // facilitate garbage collection:
     S[top] = null;
     top = top – 1;
     return temp;
   }

…  (other methods of Stack interface)
```

Example use in Java

```
public class Tester {

   // … other methods
   public intReverse(Integer a[]) {
       Stack<Integer> s;
       s = new ArrayStack<Integer>();

       … (code to reverse array a) …
    }
```

```
   public floatReverse(Float f[]) {
       Stack<Float> s;
       s = new ArrayStack<Float>();

       … (code to reverse array f) …
    }
```

Linked List based Stack

- Using linked list to implement the stack ADT
  - use singly linked list
  - need to decide if the top of the stack is at the head or at the tail of the list
    - ➢ to insert and delete elements within constant time, **the head should be the top of the stack**
  - in order to perform operation size in constant time, we keep track of the current number of elements in an instance variable
- JAVA code
  - refer to Code Fragment 6.4 (textbook p. 215)

Performance

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$

Parentheses Matching

- Each “(”, “{”, or “[” must be paired with a matching “)”, “}”, or “[”
  - correct: ( )(( )){([( )])}
  - correct: ((( )(( )){([( )])}
  - incorrect:)(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

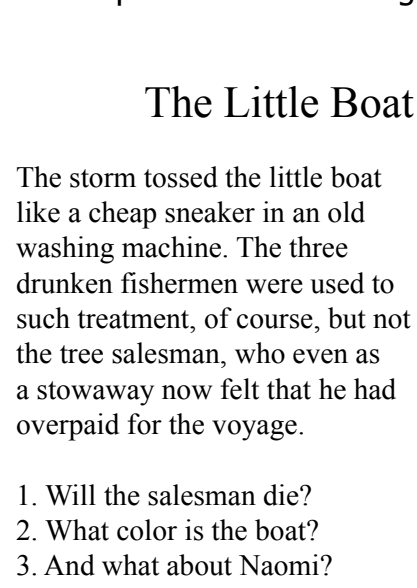
Parentheses Matching Algorithm

Algorithm ParenMatch(*X,n*):
Input: An array *X* of *n* tokens, each of which is either a grouping symbol, a variable, an arithmetic operator, or a number
Output: **true** if and only if all the grouping symbols in *X* match
Let *S* be an empty stack
for *i*=0 to *n*-1 **do**
    if *X*[*i*] is an **opening grouping symbol** **then**
        S.**push**(*X*[*i*])
    else if *X*[*i*] is a **closing grouping symbol** **then**
        if *S*.isEmpty() **then**
            return false {nothing to match with}
        if *S*.**pop**() does not match the type of *X*[*i*] **then**
            return false {wrong type}
if *S*.isEmpty() **then**
    return true {every symbol matched}
else return false {some symbols were never matched}

HTML Tag Matching

- For fully-correct HTML, each <name> should pair with a matching </name>

```
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine. The three
drunken fishermen were used to
such treatment, of course, but
not the tree salesman, who even as
a stowaway now felt that he
had overpaid for the voyage. </p>
<ol>
<li> Will the salesman die? </li>
<li> What color is the boat? </li>
<li> And what about Naomi? </li>
</ol>
</body>
```



Tag Matching Algorithm (in Java)

```
import java.io.*;
import java.util.Scanner;
import net.datastructures.*;
/** Simplified test of matching tags in an HTML document. */
public class HTML {
  /** Strip the first and last characters off a <tag> string. */
  public static String stripEnds(String t) {
    if (t.length() <= 2) return null;              // this is a degenerate tag
    return t.substring(1,t.length()-1);
  }
  /** Test if a stripped tag string is empty or a true opening tag. */
  public static boolean isOpeningTag(String tag) {
    return (tag.length() == 0) || (tag.charAt(0) != '/');
  }
```

refer to Code Fragment 6.6 (textbook p. 218)

Tag Matching Algorithm (cont.)

```
/** Test if stripped tag1 matches closing tag2 (first character is '/'). */
 public static boolean areMatchingTags(String tag1, String tag2) {
   return tag1.equals(tag2.substring(1)); // test against name after '/'
 }
 /** Test if every opening tag has a matching closing tag. */
 public static boolean isHTMLMatched(String[] tag) {
   Stack<String> S = new NodeStack<String>();     // Stack for matching tags
   for (int i = 0; (i < tag.length) && (tag[i] != null); i++) {
     if (isOpeningTag(tag[i]))
     S.push(tag[i]); // opening tag; push it on the stack
     else {
     if (S.isEmpty())
       return false;                    // nothing to match
     if (!areMatchingTags(S.pop(), tag[i]))
       return false;      // wrong match
     }
     }
   if (S.isEmpty()) return true; // we matched everything
   return false; // we have some tags that never were matched
 }
```

Tag Matching Algorithm (cont.)

```
  public final static int CAPACITY = 1000; // Tag array size
  /* Parse an HTML document into an array of html tags */
  public static String[] parseHTML(Scanner s) {
    String[] tag = new String[CAPACITY]; // our tag array (initially all null)
    int count = 0;                                  // tag counter
    String token;                                   // token returned by the scanner s
    while (s.hasNextLine()) {
      while ((token = s.findInLine("<[^>]*>")) != null)  // find the next tag
        tag[count++] = stripEnds(token); // strip the ends off this tag
      s.nextLine(); // go to the next line
    }
    return tag; // our array of (stripped) tags
  }
  public static void main(String[] args) throws IOException { // tester
    if (isHTMLMatched(parseHTML(new Scanner(System.in))))
      System.out.println("The input file is a matched HTML document.");
    else
      System.out.println("The input file is not a matched HTML document.");
  }
 }
```

Evaluating Arithmetic Expressions

14 – 3 * 2 + 7 = (14 – (3 * 2)) + 7

Operator precedence

* has precedence over +/–

Associativity

operators of the same precedence group evaluated from left to right

Example: (x – y) + z rather than x – (y + z)

Idea: push each operator on the stack, but first pop and perform higher and *equal* precedence operations.

Algorithm for Evaluating Expressions

Two stacks:

- opStk holds operators
- valStk holds values
- Use $ as special "end of input" token with lowest precedence

Algorithm doOp()

```
x ← valStk.pop();
y ← valStk.pop();
op ← opStk.pop();
valStk.push( y op x )
```

Algorithm repeatOps(refOp):

```
while ( valStk.size() > 1 ∧
        prec(refOp) ≤
        prec(opStk.top())
   doOp()
```

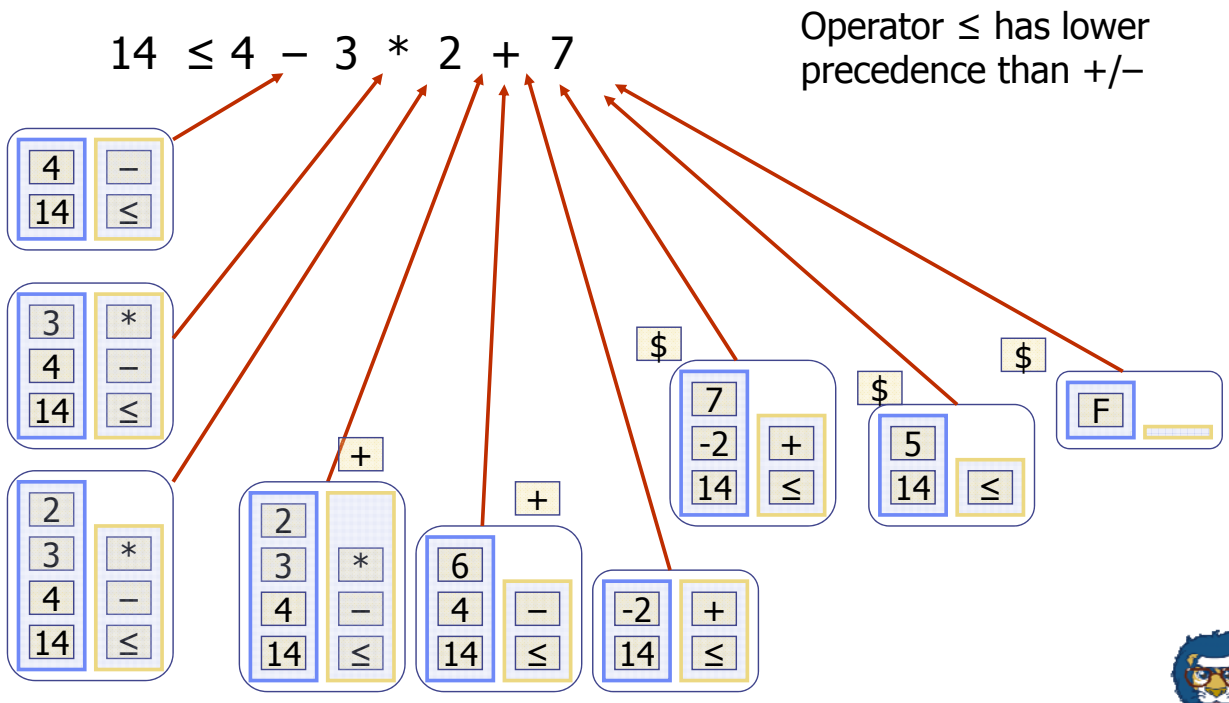
Algorithm EvalExp()

Input: a stream of tokens representing an arithmetic expression (with numbers)

Output: the value of the expression

```
while there's another token z
   if isNumber(z) then
        valStk.push(z)
   else
        repeatOps(z);
        opStk.push(z)
repeatOps($);
return valStk.top()
```

Algorithm on an Example Expression



HANYANG UNIVERSITY