IMSE 440 Supplementary materials

The mean response Y and the OLS slope estimator $\hat{B}_1$ are uncorrelated.

$$\operatorname{cov}\big(\bar{Y}, \hat{B}_1\big) = 0$$

Proof

$$\begin{aligned}
\operatorname{cov}\big(\bar{Y}, \hat{B}_1\big) &= \operatorname{cov}\left(\frac{\sum_i Y_i}{n}, \hat{B}_1\right) \\
&= \frac{1}{n}\operatorname{cov}\left(\sum_i Y_i, \hat{B}_1\right) \\
&= \frac{1}{n}\sum_i \operatorname{cov}\big(Y_i, \hat{B}_1\big) \\
&= \frac{1}{n}\sum_i \operatorname{cov}\left(Y_i, \frac{\sum_j (x_j - \bar{x})Y_j}{S_{xx}}\right) \\
&= \frac{1}{nS_{xx}}\sum_i \operatorname{cov}\left(Y_i, \sum_j (x_j - \bar{x})Y_j\right) \\
&= \frac{1}{nS_{xx}}\sum_i \sum_j \operatorname{cov}\Big(Y_i, (x_j - \bar{x})Y_j\Big) \\
&= \frac{1}{nS_{xx}}\sum_i \sum_j (x_j - \bar{x})\operatorname{cov}\big(Y_i, Y_j\big)
\end{aligned}$$

When $i \neq j$, Y_i and Y_j are independent, which means $\text{cov}(Y_i, Y_j) = 0$.

$$= \frac{1}{nS_{xx}} \sum_i (x_i - \bar{x}) \text{cov}\big(Y_i, Y_i\big)$$

$$= \frac{1}{nS_{xx}} \sum_i (x_i - \bar{x}) \sigma^2$$

$$= \frac{\sigma^2}{nS_{xx}} \sum_i (x_i - \bar{x})$$

$$= \frac{\sigma^2}{nS_{xx}} \cdot 0$$

$$= 0$$