

①

(a)

$$P_X(x) = \binom{n}{x} p^x (1-p)^{n-x} \quad \text{head } \leq 5, \text{ tail } \leq 5 \Rightarrow P_X(x) = \binom{10}{5} \cdot 0.5^5 (1-0.5)^5 = 0.246$$

(b)

$$\text{head } \leq 6 + \text{head } \leq 7 + \text{head } \leq 8 + \text{head } \leq 9 + \text{head } \leq 10 \Rightarrow P_X(6) + P_X(7) + P_X(8) + P_X(9) + P_X(10)$$

$$\binom{10}{6} \cdot 0.5^6 (1-0.5)^4 + \binom{10}{7} \cdot 0.5^7 (1-0.5)^3 + \dots = 0.377$$

②

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}$$

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(A) \cdot P(B|A) + P(A') \cdot P(B|A')}$$

A = تست مثبت

B = برای انتقال، B' = برای انتقال

$$P(A|B') = 0.999$$

$$P(B') = 0.2, P(B) = 0.98$$

$$P(A|B) = 0.995 \rightarrow P(B'|A) = ?$$

$$P(A) = P(A|B') \cdot P(B') + P(A|B) \cdot P(B) = 0.999 \times 0.2 + 0.995 \times 0.98 = 0.1998 + 0.9749 = 0.12488$$

$$P(B'|A) = \frac{P(A|B') \cdot P(B')}{P(A)} = \frac{0.999 \times 0.2}{0.12488} = \frac{0.1998}{0.12488} \approx 0.803 \rightarrow 80.3\%$$

$$P(A|B) = 0.4$$

$$P(A \cap B) = ? , P(A' \cap B) = ?$$

$$P(B) = 0.5$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \rightarrow 0.4 = \frac{P(A \cap B)}{0.5} \rightarrow P(A \cap B) = 0.2$$

$$P(A' \cap B) = P(B) - P(A \cap B) = 0.5 - 0.2 = 0.3$$

$$P(S) = 0.4, P(P) = 0.25, P(L) = 0.35$$

$$P(D|S) = 0.01, P(D|P) = 0.01, P(D|L) = 0.02$$

$$P(D) = P(D|S)P(S) + P(D|P)P(P) + P(D|L)P(L) = (0.01 \times 0.4) + (0.01 \times 0.25) + (0.02 \times 0.35) = 0.0135$$

$$P(L|D) = \frac{P(D|L) \cdot P(L)}{P(D)} = \frac{0.02 \times 0.35}{0.0135} \approx 0.5185$$

$$P_X(x) = \binom{n}{x} p^x (1-p)^{n-x} \quad (a) \quad (5)$$

$n: 5$, $p: 0.13$ و $p' = 1-p = 0.87$, x متغیر تصادفی

$$P_X(5) = (1-p)^4 \times p = 0.13 \times (0.87)^4 = 3.71 \times 10^{-5}$$

$$P_X(3) = \binom{5}{3} (0.13)^3 (0.87)^2 = 0.0166 \quad (b)$$

$$P(X \geq 1) = 1 - P(X=0) \rightarrow P(X=0) = \binom{5}{0} (0.13)^0 (0.87)^5 \quad (c)$$

$$P(X \geq 1) = 1 - P(X=0) = 0.5016$$

$$P_X(4) + P_X(5) = \binom{5}{4} (0.13)^4 (0.87)^1 + \binom{5}{5} (0.13)^5 (0.87)^0 = 0.00128 \quad (d)$$

A : انتخاب درست کردند $P(A) = 2/3 \quad (6)$

B : انتخاب تصادفی نکرد $P(B|A) = 5 P(B|A')$

$$P(B|A') = 0.1$$

$$P(A|B) = ?$$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} = \frac{0.5 \times 2/3}{0.36} \approx \boxed{0.907}$$

$$P(B) = P(B|A) \cdot P(A) + P(B|A') \cdot P(A') = (0.5 \times 2/3) + (0.1 \times 1/3) = 0.36$$