

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

ساختمان‌های داده

جلسه ۱۶

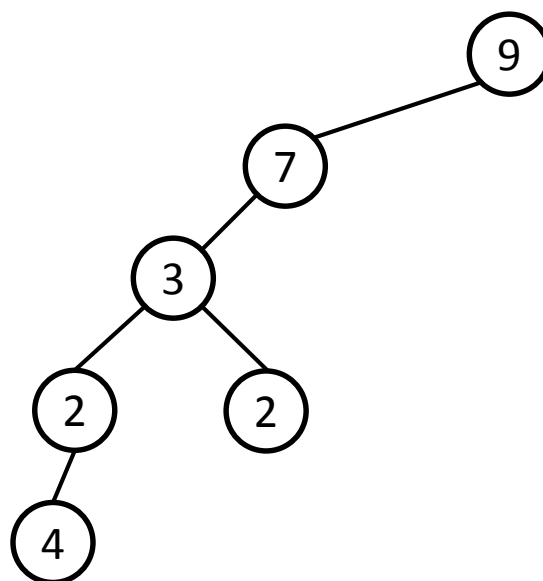
مجتبی خلیلی
دانشکده برق و کامپیوتر
دانشگاه صنعتی اصفهان

مثال

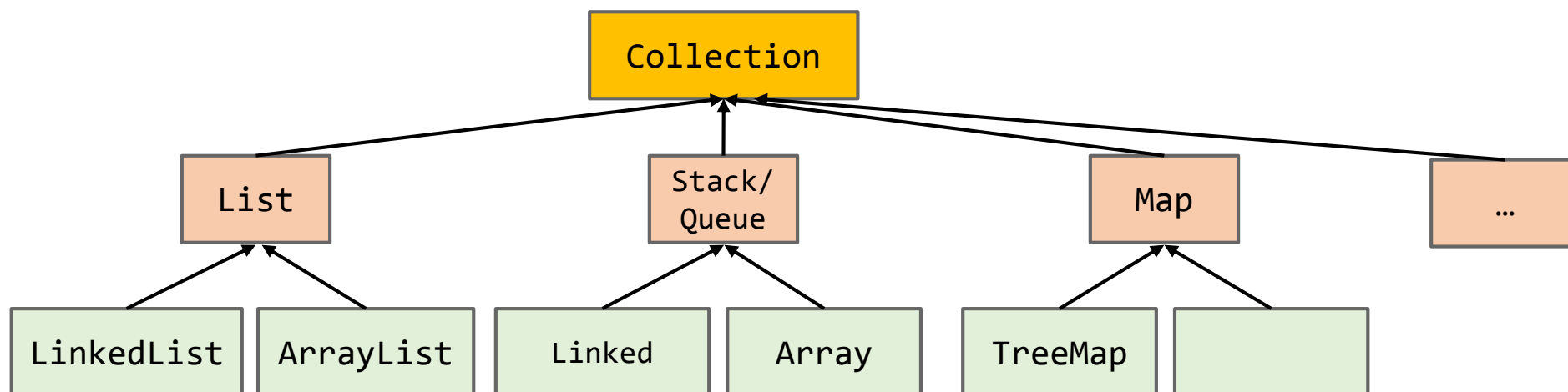
○ کمترین ارتفاع ممکن برای درخت باینری با n گره از چه مرتبه ای است؟

مثال

○ حافظه؟



ساختارهای داده



Binary Search Trees

Maps

◆ Map

Entry ADT

◆ An entry stores a key-value pair (k,v)

◆ Methods:

- **key()**: return the associated key
- **value()**: return the associated value
- **setKey(k)**: set the key to k
- **setValue(v)**: set the value to v

◆ We call this “item” or “element” or “record” exchangeably.

◆ Then, MAP stores multiple a collection of Entries

The Map ADT

- ◆ **find(k)**: if the map M has an entry with key k, return an iterator to it; else, return special iterator **end**
- ◆ **put(k, v)**: if there is no entry with key k, insert entry (k, v), and otherwise set its value to v. Return an iterator to the new/modified entry
- ◆ **erase(k)**: if the map M has an entry with key k, remove it from M
- ◆ **size()**, **empty()**
- ◆ **begin()**, **end()**: return iterators to beginning and end of M

Ordered Maps

◆ Keys come from a total order

◆ New operations:

- Each returns an **iterator** to an entry:
- **firstEntry()**: smallest key in the map
- **lastEntry()**: largest key in the map
- **floorEntry(k)**: largest key $\leq k$
- **ceilingEntry(k)**: smallest key $\geq k$
- All return **end** if the map is empty

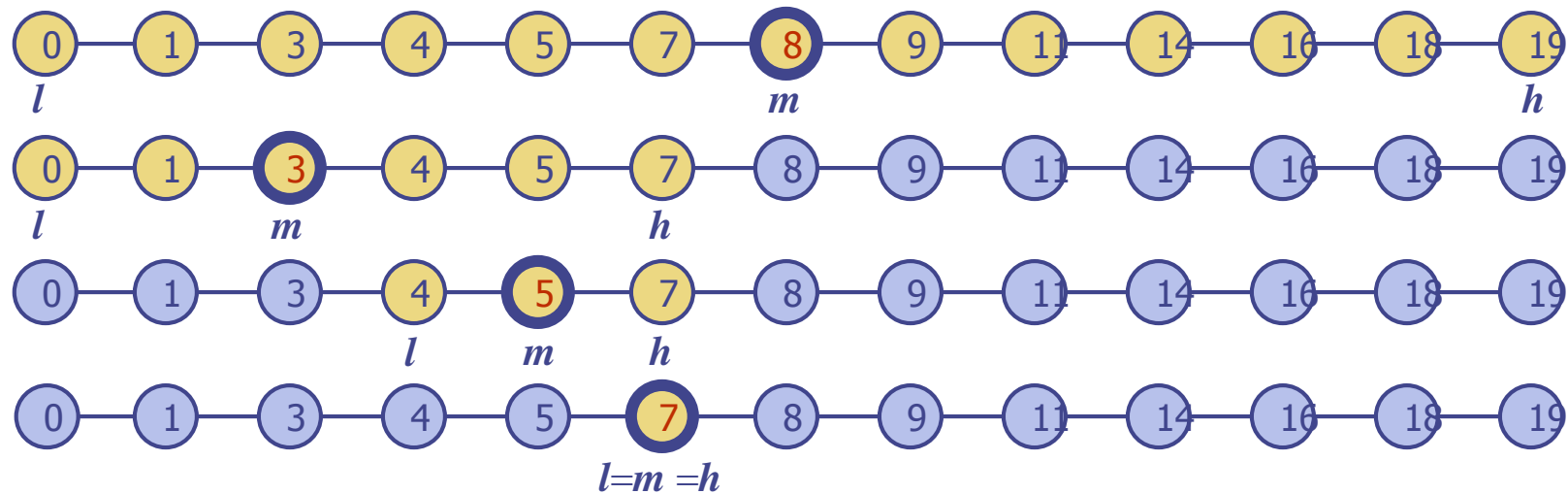
◆ **Important Issue?**

Using what data structure and algorithm, we implement Ordered Map?

Binary Search

- ◆ Binary search can perform operations **get**, **floorEntry** and **ceilingEntry** on an ordered map implemented by means of an array-based sequence, sorted by key
 - similar to the high-low game
 - at each step, the number of candidate items is halved
 - terminates after $O(\log n)$ steps

◆ Example: **find**(7)



Search Tables

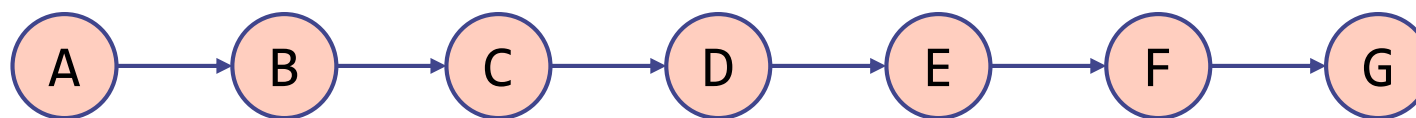
- ◆ A search table is an ordered map implemented by means of a sorted sequence
 - We store the items in an array-based sequence, sorted by key
 - We use an external comparator for the keys (for any arbitrary comparison)
- ◆ Performance:
 - **find**, **floorEntry** and **ceilingEntry** take $O(\log n)$ time, using binary search
 - **insert (put)** takes $O(n)$ time since in the worst case we have to shift $n/2$ items to make room for the new item
 - **erase (delete)** take $O(n)$ time since in the worst case we have to shift $n/2$ items to compact the items after the removal

Binary Search Trees

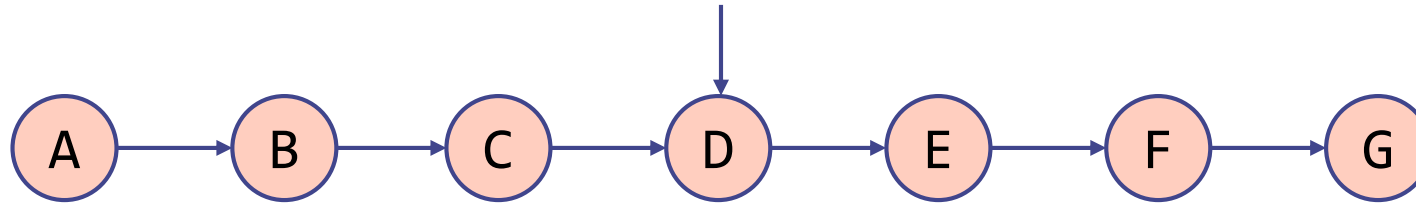
◆ Insert, find, delete?

Binary Search Tree

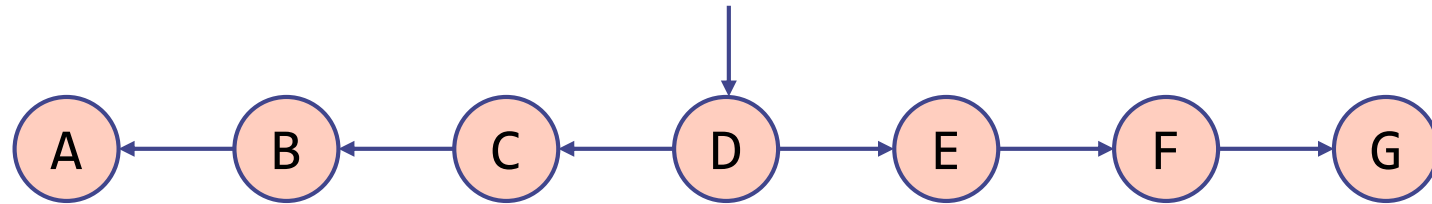
○ مرتب شده



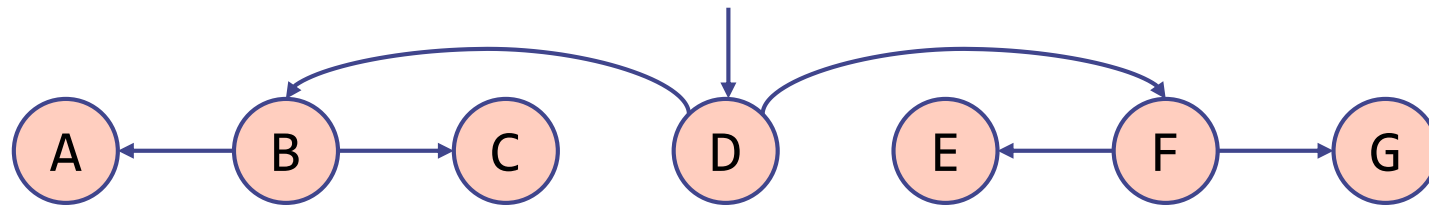
Binary Search Tree



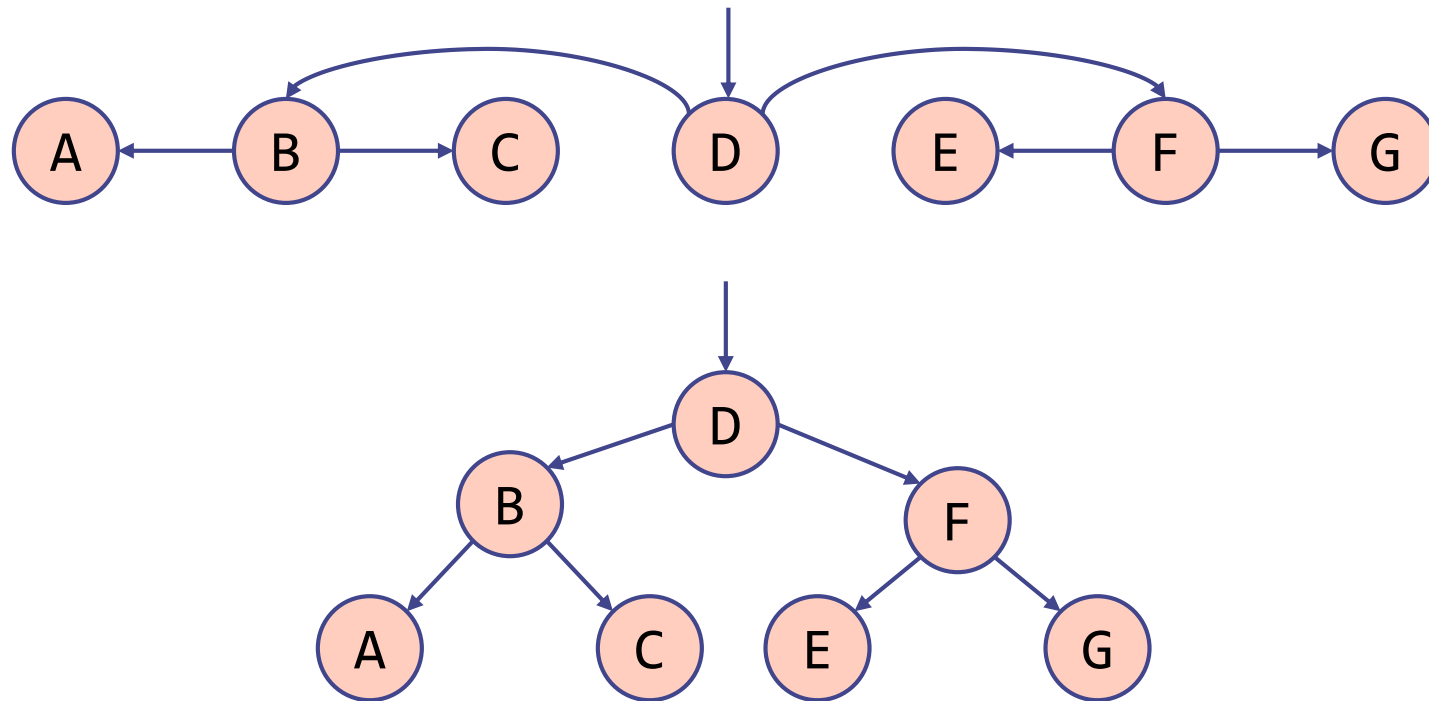
Binary Search Tree



Binary Search Tree

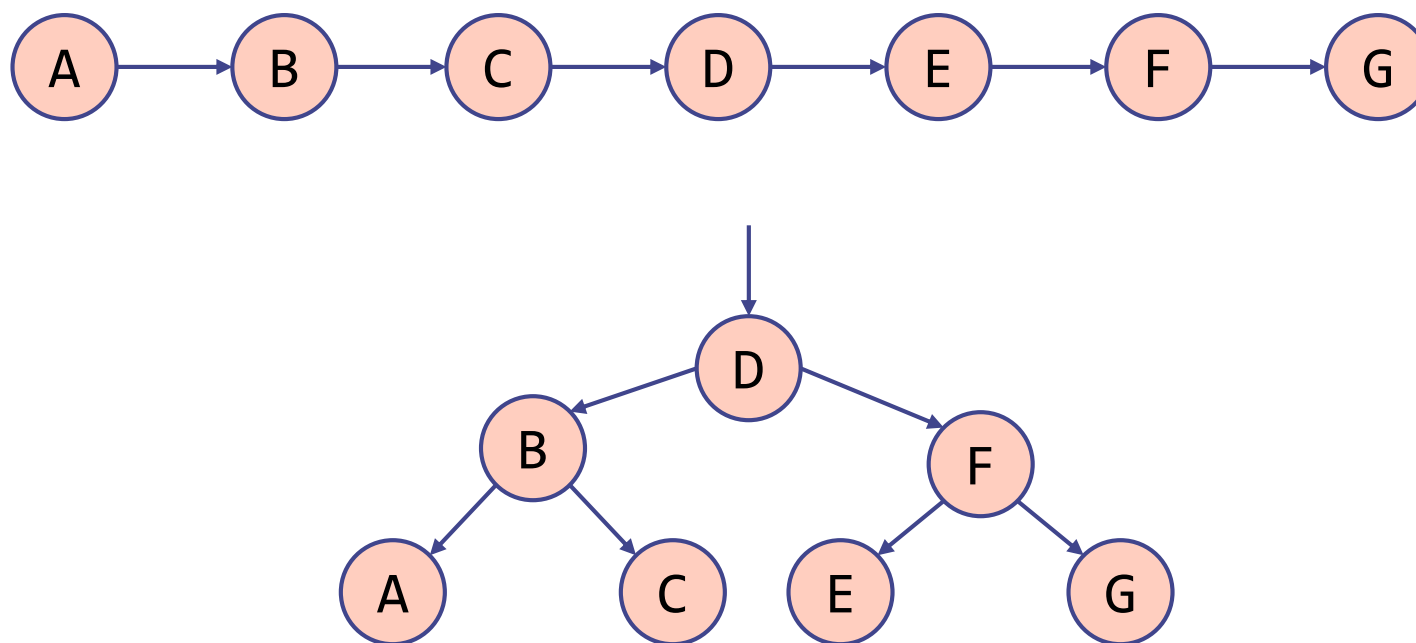


Binary Search Tree



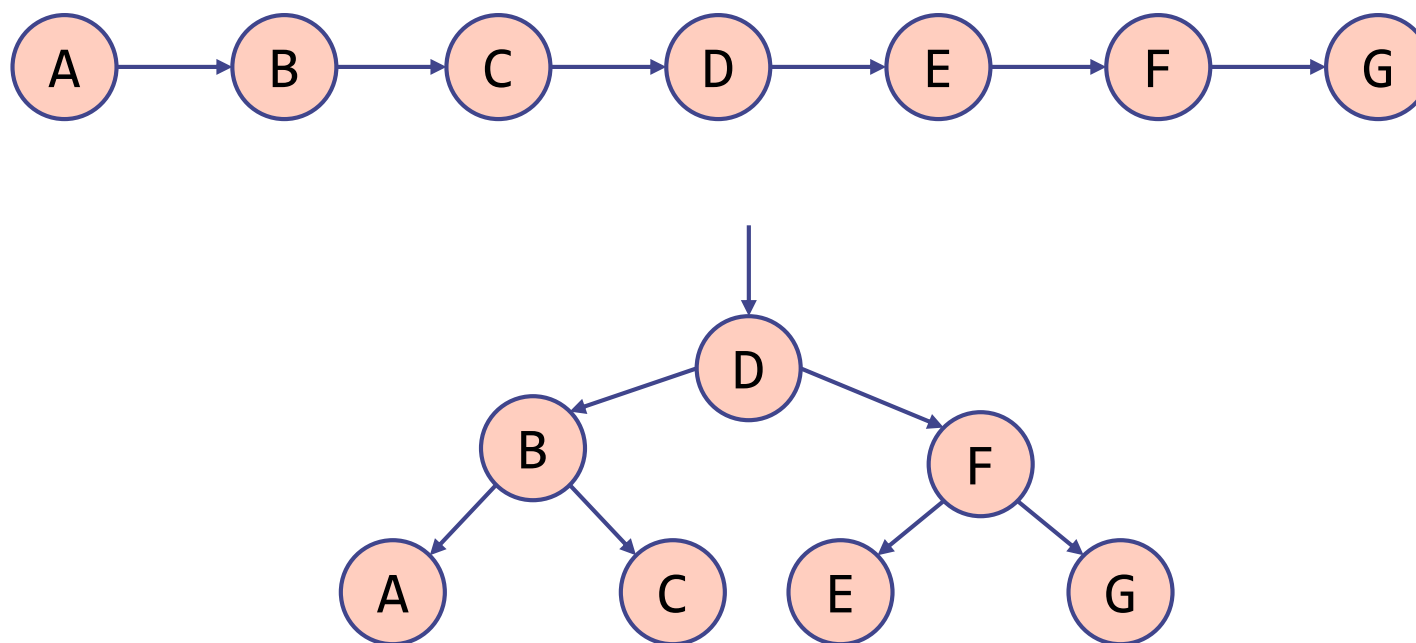
Binary Search Tree

○ دسترسی در مقایسه با لیست و آرایه؟



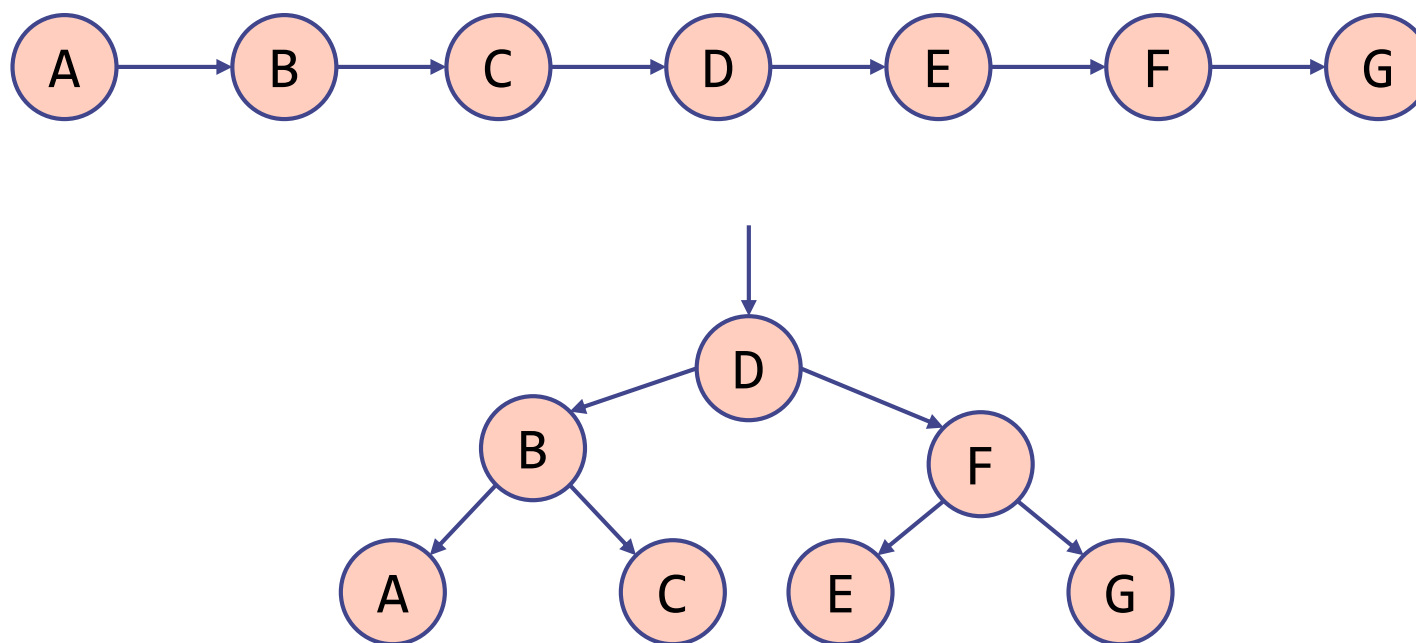
Binary Search Tree

○ جستجو در مقایسه با لیست و آرایه؟



Binary Search Tree

○ حذف در مقایسه با لیست و آرایه؟

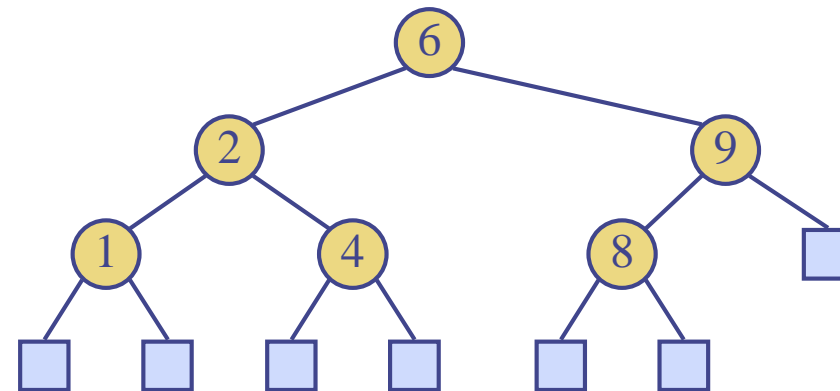


Binary Search Trees

◆ A binary search tree is a binary tree storing keys (or key-value entries) at its internal nodes and satisfying the following property:

- Let u , v , and w be three nodes such that u is in the left subtree of v and w is in the right subtree of v . We have $key(u) \leq key(v) \leq key(w)$

◆ External nodes do **not** store items (GTM version)



Binary Search Trees

○ چگونه از BST یک لیست sort شده بسازیم؟

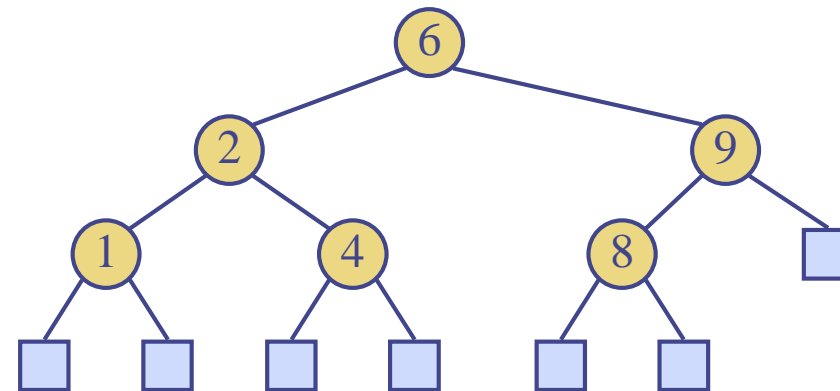
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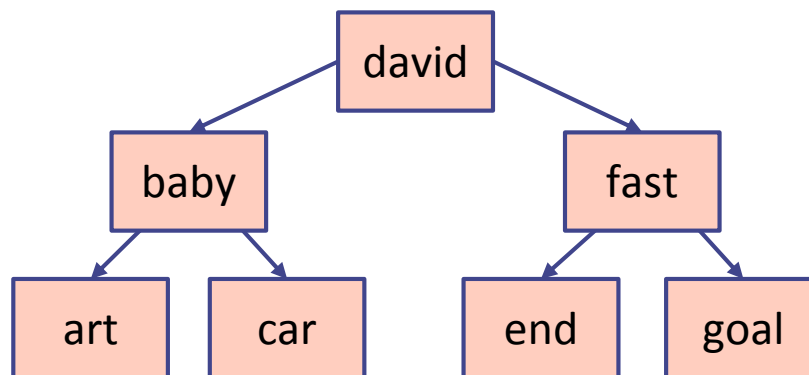
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◆ An inorder traversal of a binary search tree visits the keys in increasing order



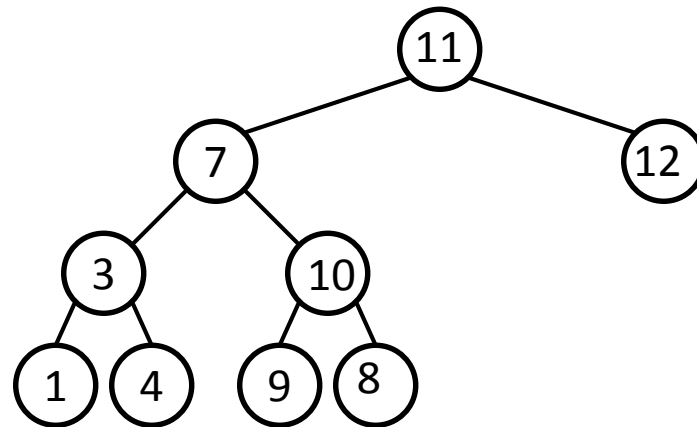
Binary Search Trees

○ کلید تکراری نداریم (ترتیب به طور شفاف و گذرا):



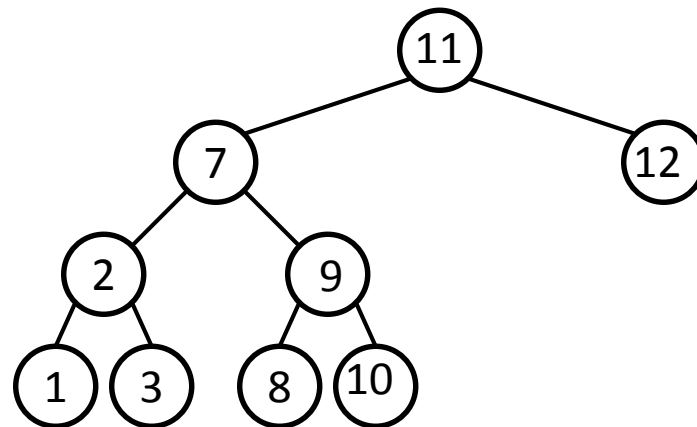
مثال

؟BST ○



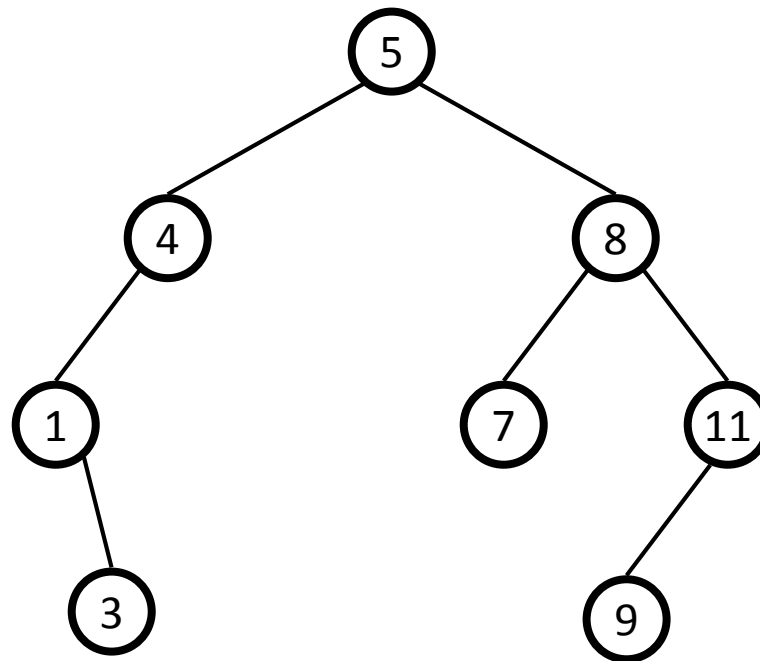
مثال

؟BST ○



مثال

؟BST ○



مثال

○ چه مقداری؟

