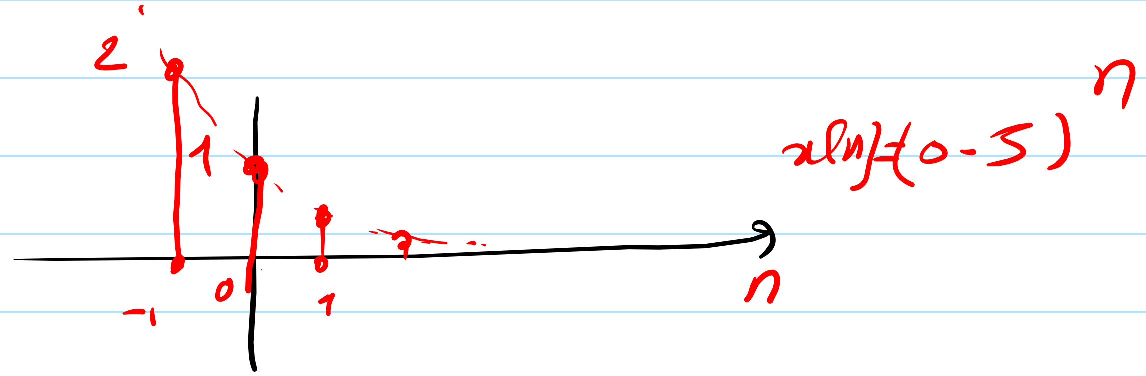
(به نام خدا)

سیگنال های زمان گسسته نمایی

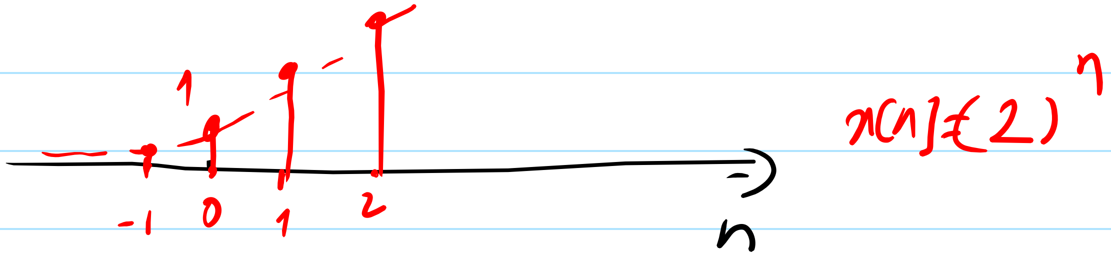
$x[n] = c\alpha^n$ مثلاً $c = 1$ → ابتداً c: دلخواه

1) α: حقیقی → $x[n] = \alpha^n$ $\alpha > 0$ $\alpha < 0$

$0 < \alpha < 1$ →



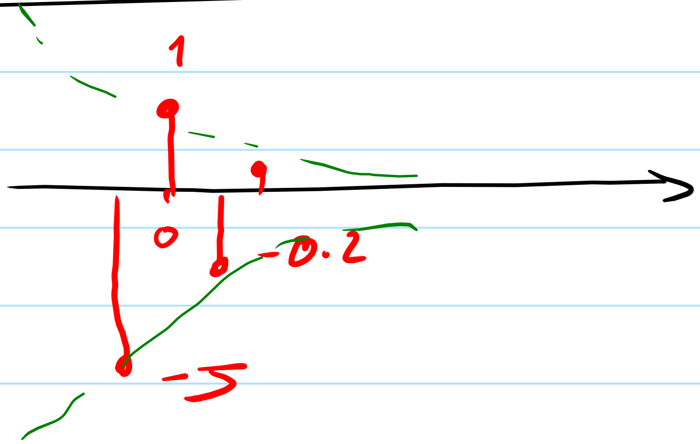
$1 < \alpha$ →



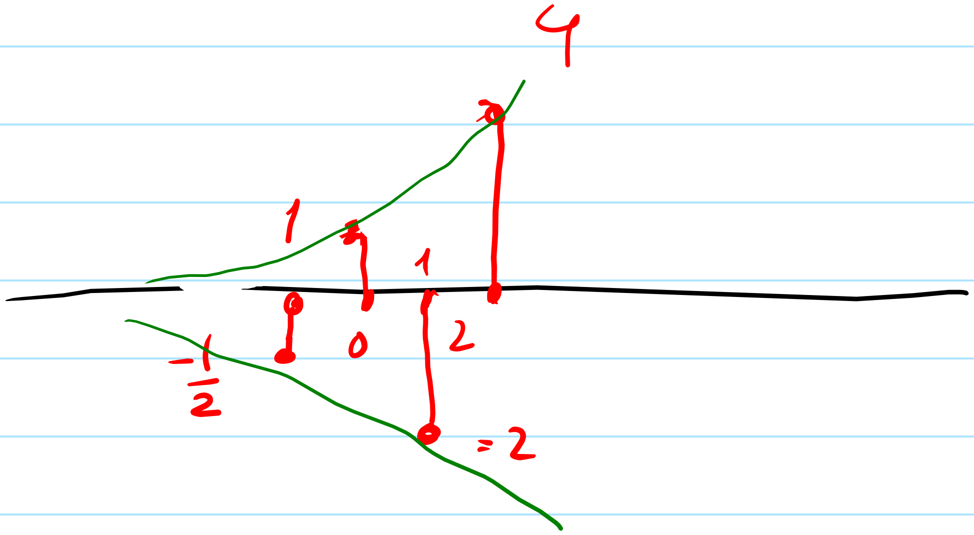
$-1 < \alpha < 0$

$x[n] = (-0.2)^n$

$(-0.2)^{-1} = \frac{1}{-0.2} = -5$

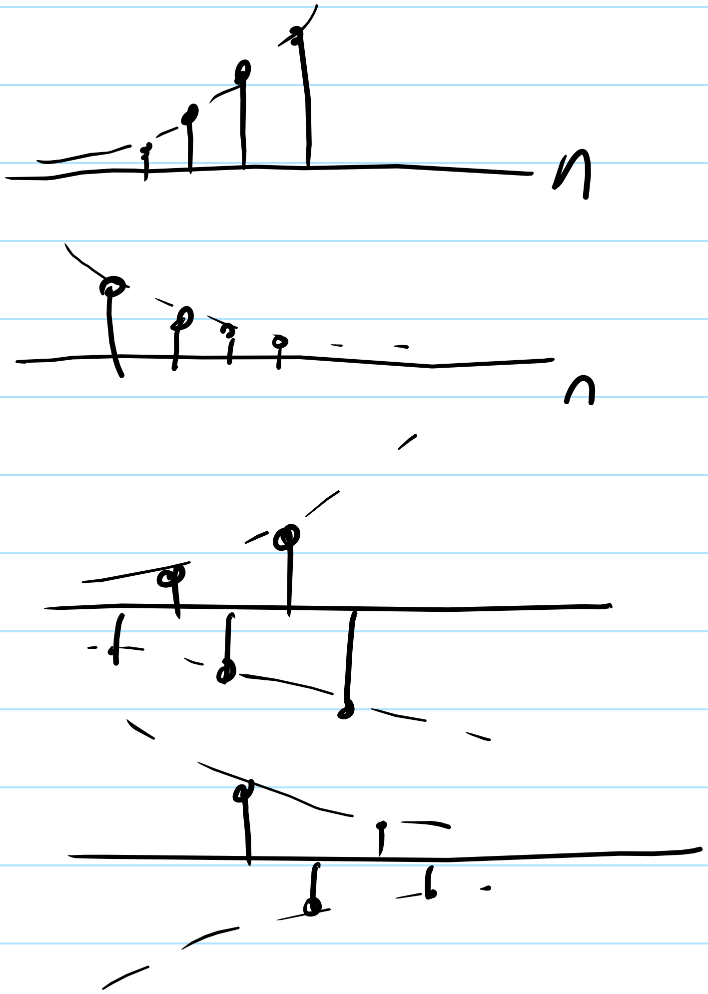


$\alpha < -1 \qquad \alpha = -2 \rightarrow x[n] = (-2)^n$



$x[n] = \alpha^n$

α : منفی



α: مختلط

$\alpha =$ موهومی خالص

$$\alpha = j\omega_0 \rightarrow x[n] = e^{jn\omega_0} = \underline{\underline{\cos n\omega_0}} + j\,\underline{\underline{\sin n\omega_0}}$$

برای تست: $e^{jn\omega_0}$

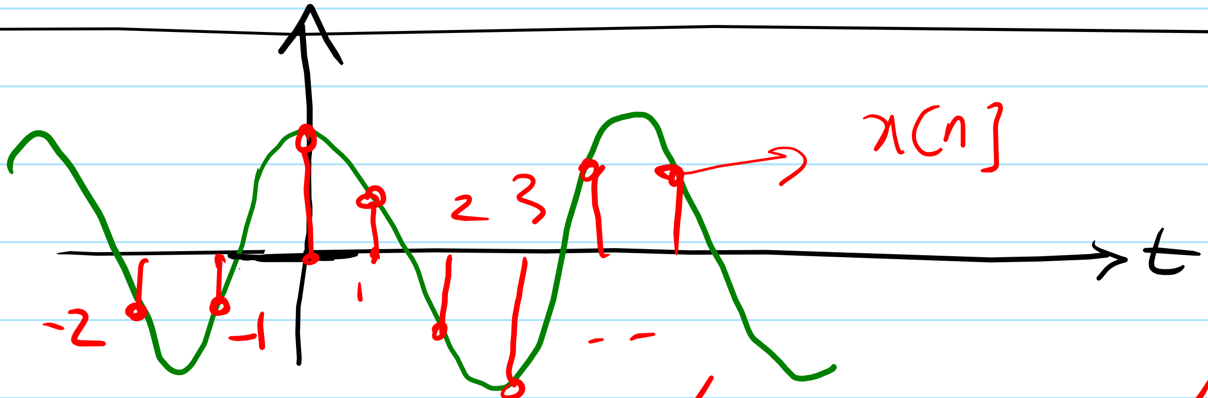
$$x(n) = e^{j\omega_0 n} \qquad \omega_0 \rightarrow \omega_0 + 2\pi$$

$$x[n] = e^{j\omega_0 n} \qquad x_1[n] = e^{j(\omega_0 + 2\pi)n}$$

$$= e^{j\omega_0 n + j2\pi n} = e^{j\omega_0 n}\,\underline{\underline{e^{j2\pi n}}}_{1}$$

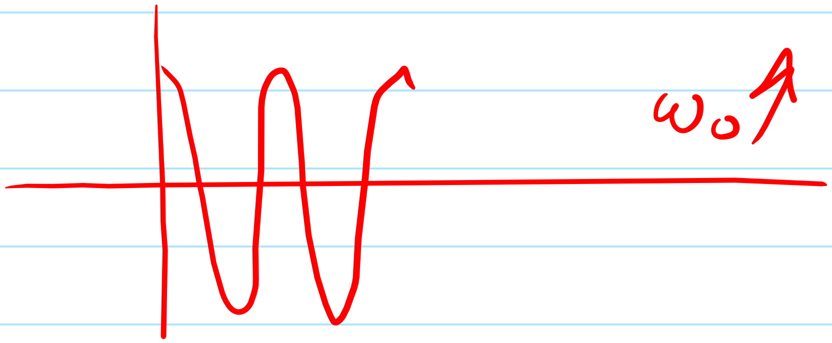
$= e^{j\omega_0 n} \rightarrow$ با $x[n]$ یکی

$$\cos\omega_0 n \rightarrow \cos\omega_0 t$$

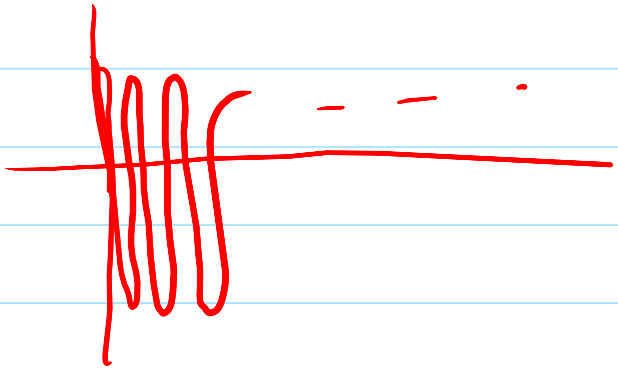


$\omega_0,\ \omega_0 \pm 2\pi,\ \omega_0 \pm 4\pi, \dots \rightarrow$ شکل $\cos n\omega_0$ تغییر نمی‌کند

$\cos \omega_0 t$ $\omega_0 \uparrow \rightarrow$ شکل فشرده‌تر می‌شود



$\omega_0 \uparrow$



$\cos \omega_0 n$ شکل در بازه‌ها به عرض 2π تکرار می‌شود

برای ω_0

$x[n] = x[n+N]$ فرض کنیم N دوره تناوب باشد

از این فرض استفاده کرده داشته باشیم:

$$e^{j\omega_0 n} = e^{j\omega_0 (n+N)} \rightarrow \underset{\text{حذف}}{\underline{\underline{e^{j\omega_0 n}}} = \underline{\underline{e^{j\omega_0 n}}}}\, e^{j\omega_0 N} \rightarrow e^{j\omega_0 N} = 1$$

$$\Rightarrow \boxed{\omega_0 N = 2\pi m}$$

شرط تناوب $e^{j\omega_0 n}$ این است که N را داشته باشیم: (N کوچکترین عدد طبیعی)

$\omega_0 N = 2\pi m \rightarrow N = \underline{\frac{2\pi}{\omega_0} \times m} \rightarrow$ باید $\underline{\underline{\frac{2\pi}{\omega_0}}}$: کسر گویا باشد

مثلاً $\frac{2\pi}{\omega_0} = \underline{\underline{\frac{p}{q}}}$

ساده شدنی نباشد

$\Rightarrow N = \frac{p}{q} \times m \rightarrow \underset{\text{قرار میدهیم}}{m = q} \rightarrow \boxed{N = p} \longrightarrow$ N کوچکترین مقدار ممکن

اگر $\frac{2\pi}{\omega_0}$ کسر گویا نباشد، $x[n]$ متناوب نیست.

مثال $x[n] = e^{j3n}$ $\omega_0 = 3$ $\frac{2\pi}{3}$: کسر گویا نیست

$\rightarrow x[n]$ متناوب نیست

$\rightarrow \cos 3n , \sin 3n$: متناوب نیستند

$x[n] = e^{j\frac{3\pi}{11}n}$ حساب

حل

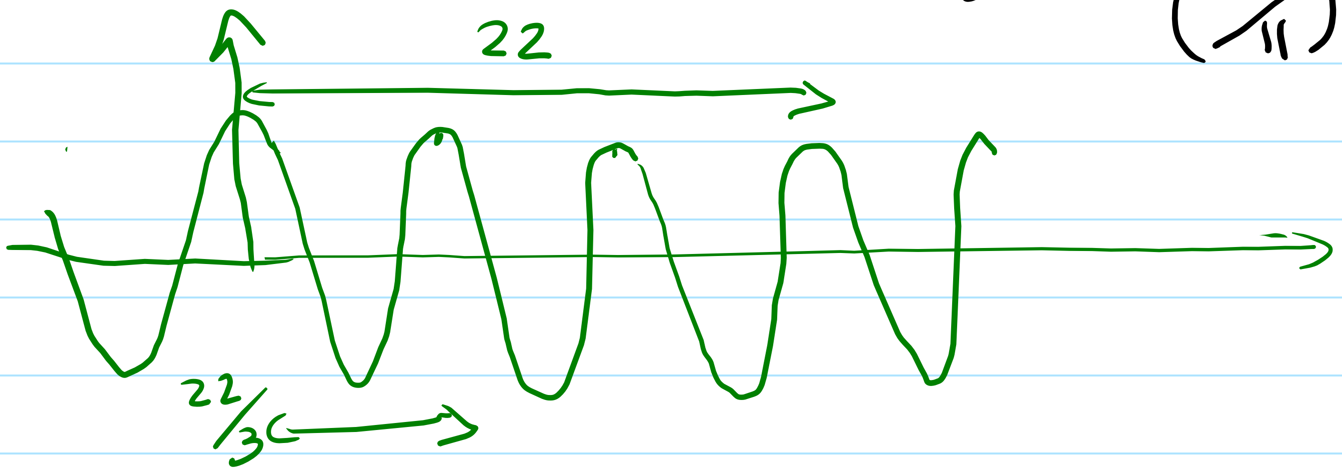
$\omega_0 = \frac{3\pi}{11} \rightarrow \frac{2\pi}{\omega_0} = \frac{2\pi}{\left(\frac{3\pi}{11}\right)} = \frac{2\pi}{3\pi} \times 11 = \frac{2\times 11}{3} = \frac{22}{3}$

$N = \underset{\uparrow}{\frac{2\pi}{\omega_0} \times m} = \frac{22}{3} \times \underset{\underset{3}{\uparrow}}{m} = \frac{22}{3} \times 3 = 22$

کوچکترین m که جواب بده

$\rightarrow \boxed{N = 22}$

$x(t) = e^{j\frac{3\pi}{11}t} \rightarrow \underline{T} = \frac{2\pi}{\omega_0} = \frac{2\pi}{\left(\frac{3\pi}{11}\right)} = \frac{22}{3}$



$$x[n]=e^{j\omega_0 n}=\underbrace{\cos\omega_0 n}+j\sin\omega_0 n$$

$$x(t)=e^{j\omega_0 t}=\underbrace{\cos\omega_0 t}+j\sin\omega_0 t$$

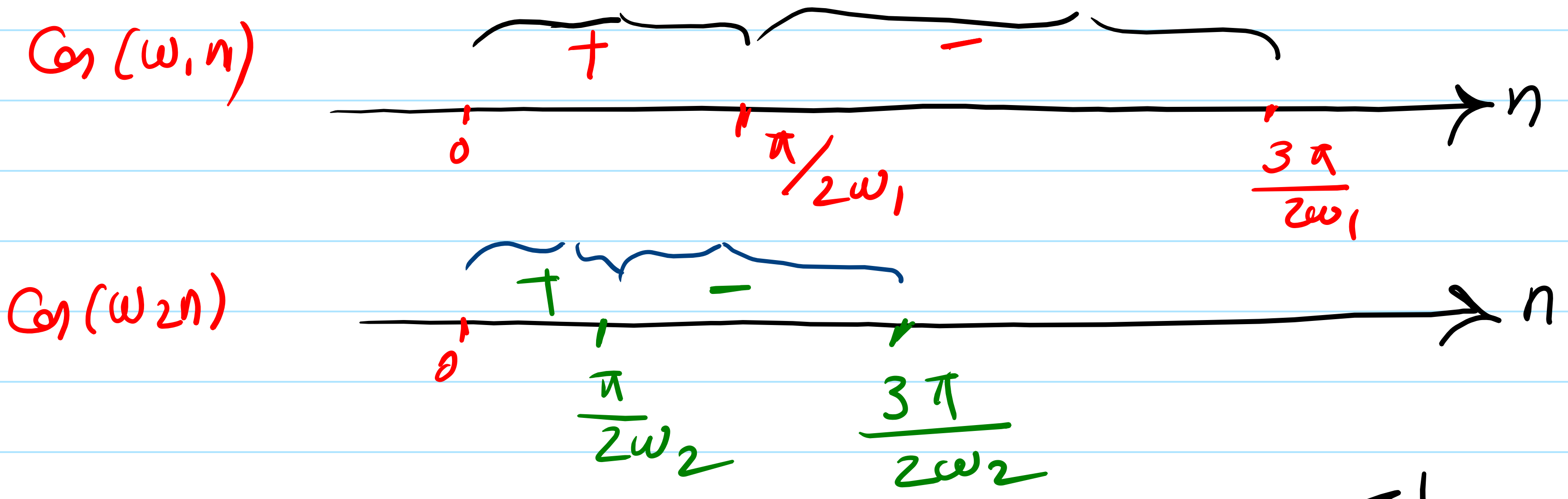
↑ نرخ نوسان   با افزایش ω_0 :   زیاد شدن

$\omega_1<\omega_2$

$\cos\theta$

$0<\theta<\frac{\pi}{2}$ $+$

$\frac{\pi}{2}<\theta<\frac{3\pi}{2}$ $-$

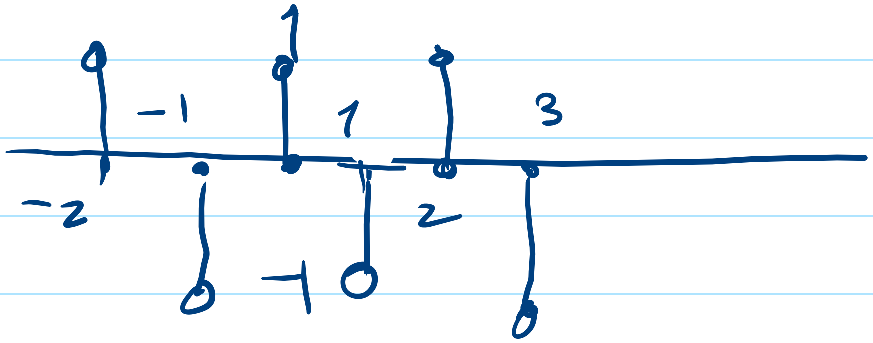


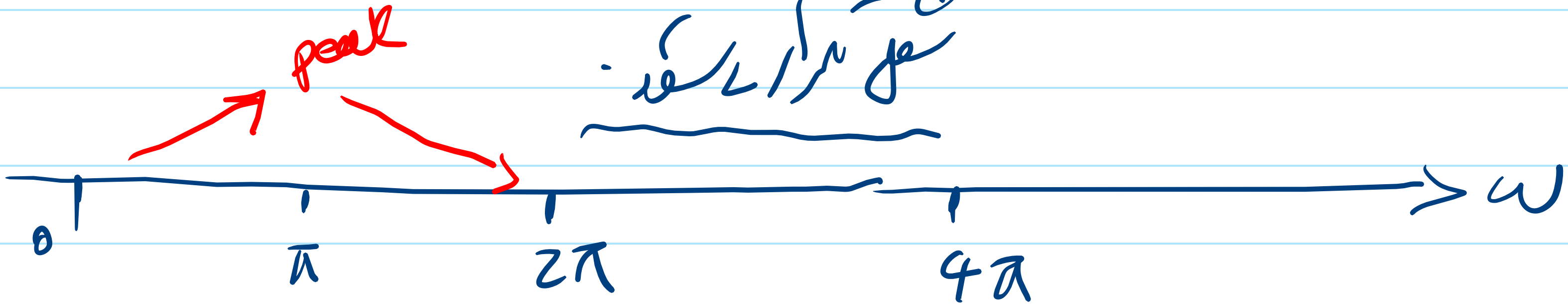
$0<\omega_1 n<\frac{\pi}{2}\rightarrow$

$0<n<\frac{\pi}{2\omega_1}$

منظور از نرخ نوسان، نرخ تغییرات دامنه $x[n]$ (حقیقی)

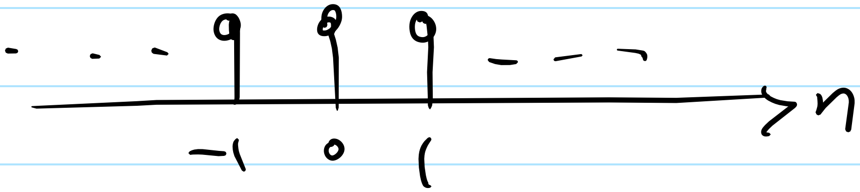
$\omega \rightarrow \pi \quad \rightarrow \quad x(n) = \cos n\pi$





ω ها تکرار می‌شوند.

$x[n] = \cos 2\pi n \rightarrow x[n] = 1$



$e^{j\omega_0 t}$: هارمونیک اصلی (پایه)

$e^{jk\omega_0 t}$ $\quad k = \pm 2, \pm 3, \ldots$

هارمونیک عام

$$\phi_1[n] = e^{j\frac{2\pi}{N}n} \qquad \omega_0 = \frac{2\pi}{N}$$

دوره تناوب : N

$\varphi_1[n]=e^{j\omega_0 n}=e^{j\frac{2\pi}{N}n}$ N : دوره تناوب $\omega_0=\frac{2\pi}{N}$

$\varphi_2[n]=e^{j2\omega_0 n}$

$\varphi_3[n]=e^{j3\omega_0 n}$

$\vdots$

$\varphi_{k+N}[n]=e^{j(k+N)\omega_0 n}$ شکل : $\varphi_k[n]=e^{jk\omega_0 n}=e^{jk(\frac{2\pi}{N})n}$ گسسته :

یک

برابر

دلیل

$$\varphi_{k+N}[n]=e^{jk\omega_0 n+jN\omega_0 n}$$

$$=\underbrace{e^{jk\omega_0 n}}\times\boxed{e^{jN\overset{\frac{2\pi}{N}}{\omega_0}n}}=1$$

$$e^{j\cancel{N}\frac{2\pi}{\cancel{N}}n}=e^{j2\pi n}=1$$

$$\rightarrow\ \varphi_{k+N}[n]=\varphi_k[n]$$

$\varphi_0[n] = 1 = e^{j\times 0\times \omega_0 n}$ $\quad e^{jk\omega_0 t}$ $\quad k=0$ $\quad \underline{\underline{DC}}$

$\varphi_0[n] \quad \varphi_1[n] \quad \cdots\cdots \quad \varphi_{N-1}[n]$

$\|$

$\varphi_N[n] \quad \varphi_{N+1}[n] \quad \varphi_{2N-1}[n]$

$\varphi_{k+N}[n] = \varphi_k[n]$

هارمونیک kام

مثال $N=3$ $\quad \varphi_0[n]=1 \quad \varphi_1[n] \quad \varphi_2[n]$

$$e^{j\times 8\times\frac{2\pi}{3}n} = e^{j\,2\times\frac{2\pi}{3}n} = \varphi_2[n]$$

$k = 8 = \underline{\underline{3}} + \underline{\underline{3}} + 2$

$x[n] = \alpha^n \qquad x[n] = e^{j\omega_0 n}$

$x[n] = C\alpha^n$ $\quad C$: ثابت $\quad \alpha$: حقیقی

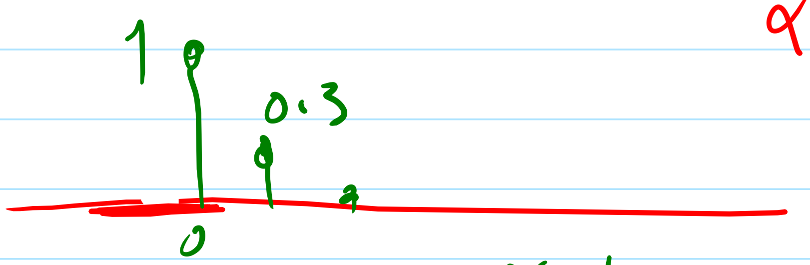
$x[n] = C\alpha^n \qquad C = |C| e^{j\varphi} = |C|\cos\varphi + j|C|\sin\varphi$

$$\rightarrow x[n] = (|C|\cos\varphi + j|C|\sin\varphi)\alpha^n = \overbrace{\underbrace{(|C|\cos\varphi)\alpha^n}}^{Re} + \overbrace{j(|C|\sin\varphi)\alpha^n}^{Im}$$

$\varphi = \frac{\pi}{3} \rightarrow \cos\varphi = \frac{1}{2} \qquad |C| = 1 \qquad C = e^{j\pi/3}$

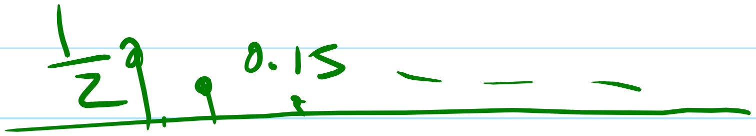
$\alpha = 0.3$

Re: $x[n] = \frac{1}{2}\alpha^n$

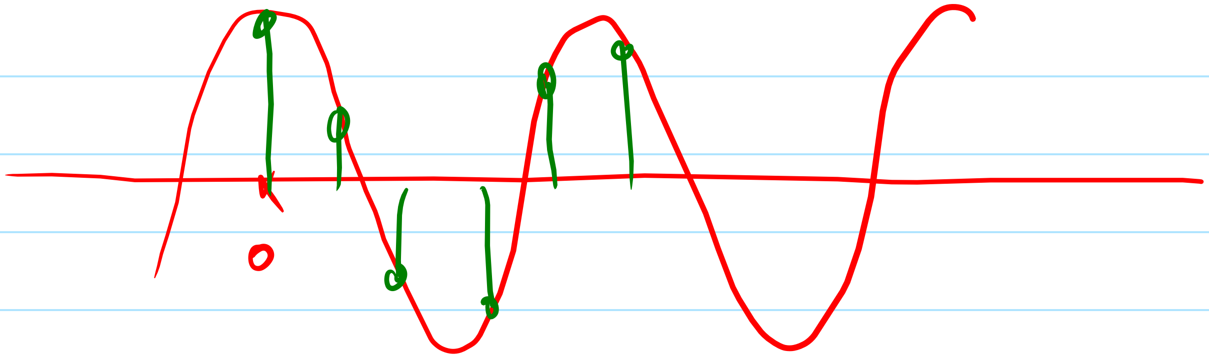


$\times \frac{1}{2}$

$\frac{1}{2}(0.3)^n$

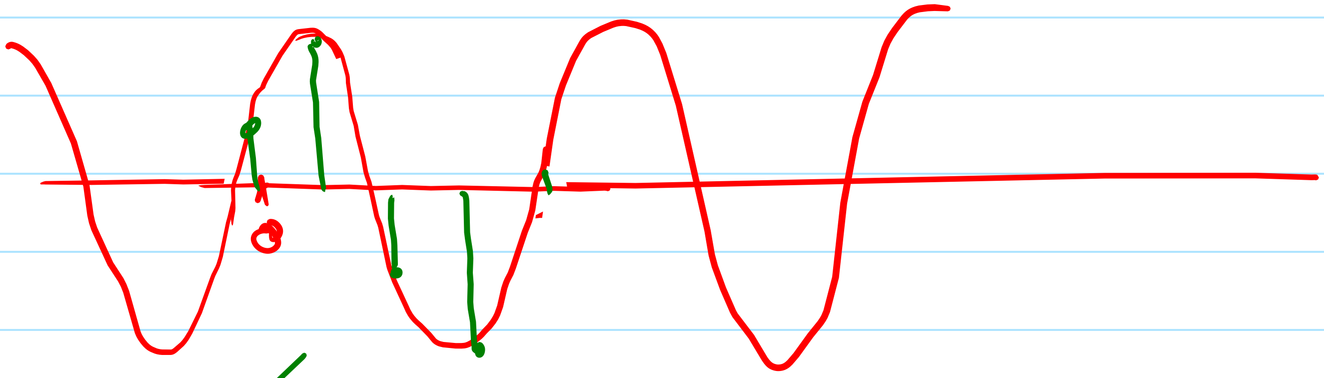


$$x[n] = Ce^{j\omega_0 n} = |C|e^{j\varphi}e^{j\omega_0 n} = |C|e^{j(\omega_0 n+\varphi)} = \overbrace{|C|\cos(n\omega_0+\varphi)}^{Re} + \overbrace{j|C|\sin(n\omega_0+\varphi)}^{Im}$$



$\cos n\omega_0$

$\cos(n\omega_0+\varphi)$



در زمان پیوسته $\cos\omega_0 t$ و $\cos(\omega_0 t+\varphi)$ شکل یکسانی دارند (فقط شیفت یافته یکدیگرند)
ولی در زمان گسسته با شیفت شکل‌ها شکل عوض می‌کنند.

$$x[n]=Ce^{j\omega_0 n}$$

$$x[n]=e^{(a+j\omega_0)n}=e^{an}e^{j\omega_0 n}$$

$$=e^{an}\cos\omega_0 n+je^{an}\sin\omega_0 n$$

$$e^{(\sigma+j\omega_0)n} = \underbrace{e^{\sigma n}\cos\omega_0 n}_{Re} + j e^{\sigma n}\sin\omega_0 n$$

خلاصه

α^n — $\alpha>0$ — $\alpha<0$ — تغییر علامت — پریودیک

$\alpha>0$: صعودی، نزولی

$e^{j\omega_0 n} \rightarrow \frac{2\pi}{\omega_0} = \frac{p}{q} \rightarrow$ متناوب $\rightarrow \boxed{N=p}$

ب.م.م $(p,q)=1$ شرط o,c

$$1 \qquad \underset{\text{هارمونیک اصلی}}{\underline{\underline{e^{j\frac{2\pi}{N}n}}}} \qquad \underset{\text{دوم}}{\underline{\underline{e^{j2\frac{2\pi}{N}n}}}} \qquad \underset{k\text{ام}}{\underline{\underline{e^{jk\frac{2\pi}{N}n}}}} \qquad \underset{(N-1)\text{ام}}{\underline{\underline{e^{j(N-1)\frac{2\pi}{N}n}}}}$$

دوره تناوب N