



بسم الله الرحمن الرحيم

دانشگاه صنعتی اصفهان
دانشکده برق و کامپیوتر

تجزیه و تحلیل سیگنال‌ها و سیستم‌ها

مدرس: مسعود عمومی

جلسه بیست و هفتم – بخش‌های 7.1 تا 7.3 کتاب

با سلام خدمت دانشجویان محترم

THE SAMPLING THEOREM

قضیه نمونه برداری

نمونه برداری پُلی بین مباحث سیگنال‌ها و سیستم‌های زمان‌پیوسته با مباحث سیگنال‌ها و سیستم‌های زمان‌گسسته است.

مقدمه

Under certain conditions, a continuous-time signal can be completely represented by and
recoverable from knowledge of its values, or *samples*, at points equally spaced in time.

This somewhat surprising property follows from a basic result that is referred to as the
sampling theorem.

It is exploited, for example,

in moving pictures, which consist of a sequence of individual frames, each of which represents an instantaneous view (i.e., a sample in time) of a continuously changing scene. When these samples are viewed in sequence at a sufficiently fast rate, we perceive an accurate representation of the original continuously moving scene.

As another example,

printed pictures typically consist of a very fine grid of points, each corresponding to a sample of the spatially continuous scene represented in the picture.

As we will see in this chapter,

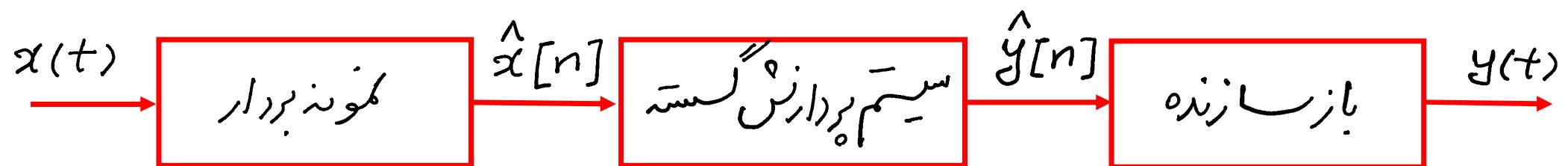
the fact that under certain conditions a continuous-time signal can be completely recovered
from a sequence of its samples provides a mechanism for representing a continuous-time
signal by a discrete-time signal.

نمیت پردازش سینال های زمان گستره

In many contexts, processing discrete-time signals is more flexible and is often preferable
to processing continuous-time signals.

The concept of sampling, then, suggests an extremely attractive and widely employed
method for using discrete-time system technology to implement continuous-time systems
and process continuous-time signals:

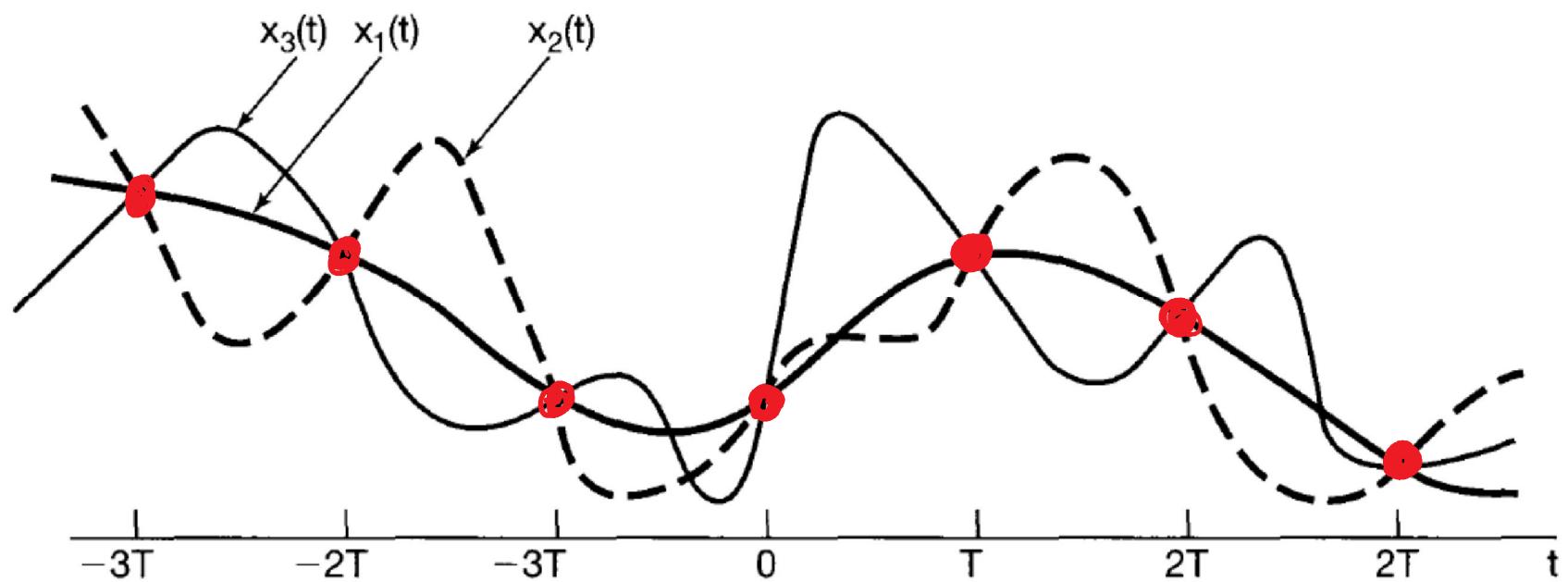
We exploit sampling to convert a continuous-time signal to a discrete-time signal,
process the discrete-time signal using a discrete-time system, and then convert
back to continuous time.



نمایش یک سیگنال زمان پیوسته توسط نمونه های متوالی آن (قضیه نمونه برداری)

three different continuous-time signals, all of which have identical values at integer multiples of T ; that is, $x_1(kT) = x_2(kT) = x_3(kT)$.

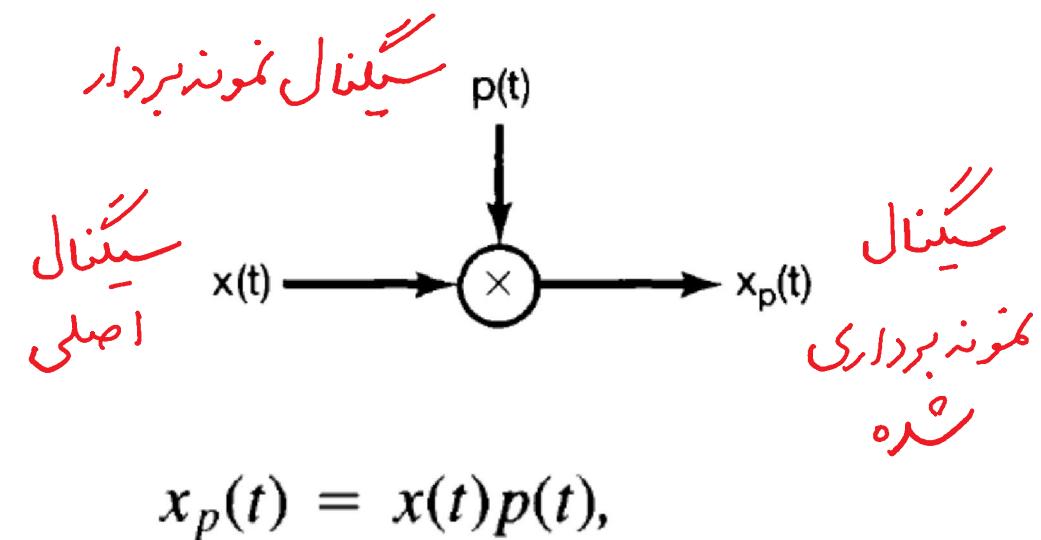
Clearly, an infinite number of signals can generate a given set of samples.



As we will see, however, if a signal is band limited—i.e., if its Fourier transform is zero outside a finite band of frequencies—and if the samples are taken sufficiently close together in relation to the highest frequency present in the signal, then the samples *uniquely* specify the signal, and we can reconstruct it perfectly. This result, known as the *sampling theorem*, is of profound importance in the practical application of the methods of signal and system analysis.

طبق قضیه نونبرداری اگر سینال زمان پوسته ای، طیف فرکانسی محدودی داشته باشد (تبیل فوریئن خارج از بیان حدهای صفر باشد)، می توان آن سینال را توسط نمونه های متواال آن با خواص مساوی کاوش (ادعه طور کامل بازنگشی کرد، بدست طبق آن که نمونه ها با ووجه به حداقل فرکانس موجود در سینال، به اندازه کافی هم نزدیک باشند.

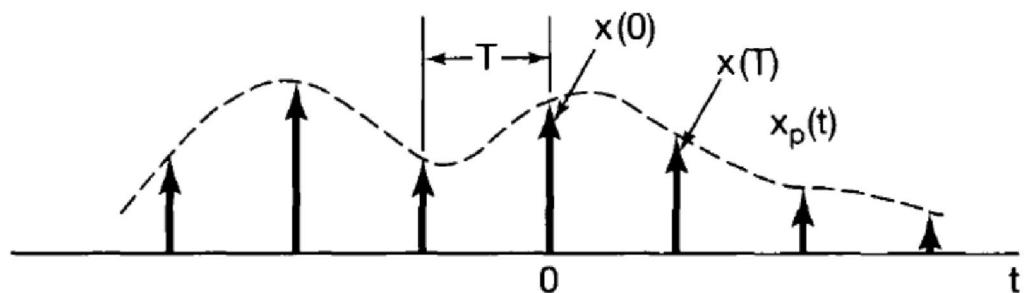
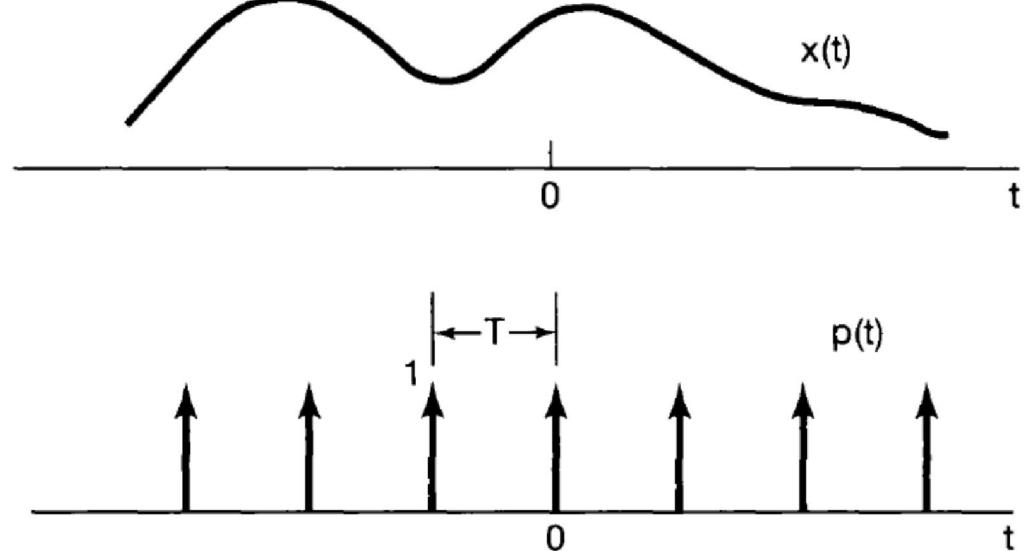
Impulse-Train Sampling



$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT).$$

دوره ثابت
نمونه برداری

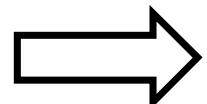
۱) نمونه برداری با قطار ضربه ایده آل



The periodic impulse train $p(t)$ is referred to as the *sampling function*, the period T as the *sampling period*, and the fundamental frequency of $p(t)$, $\omega_s = 2\pi/T$, as the *sampling frequency*.

$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$$

خاصیت غربال سیگنال ضرب



$$x_p(t) = x(t)p(t) = x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT) = \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT).$$

From the multiplication property, we know that

خاصیت ضرب در تبدیل فوریه

$$X_p(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta)P(j(\omega - \theta))d\theta.$$

we know that

$$P(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta(\omega - k\omega_s).$$

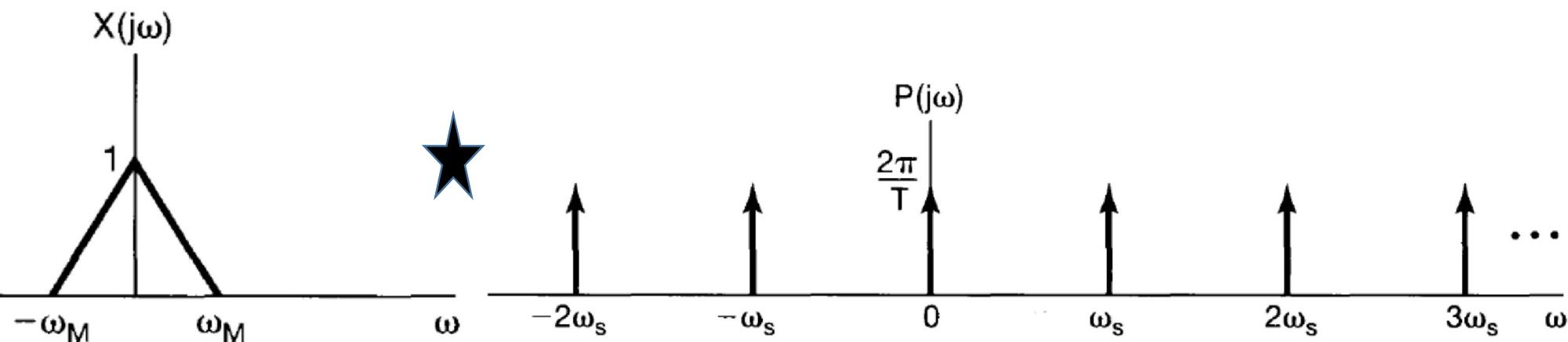
طیف فرکانسی سیگنال مخونه بردار

Since convolution with an impulse simply shifts a signal [i.e., $X(j\omega) * \delta(\omega - \omega_0) = X(j(\omega - \omega_0))$],

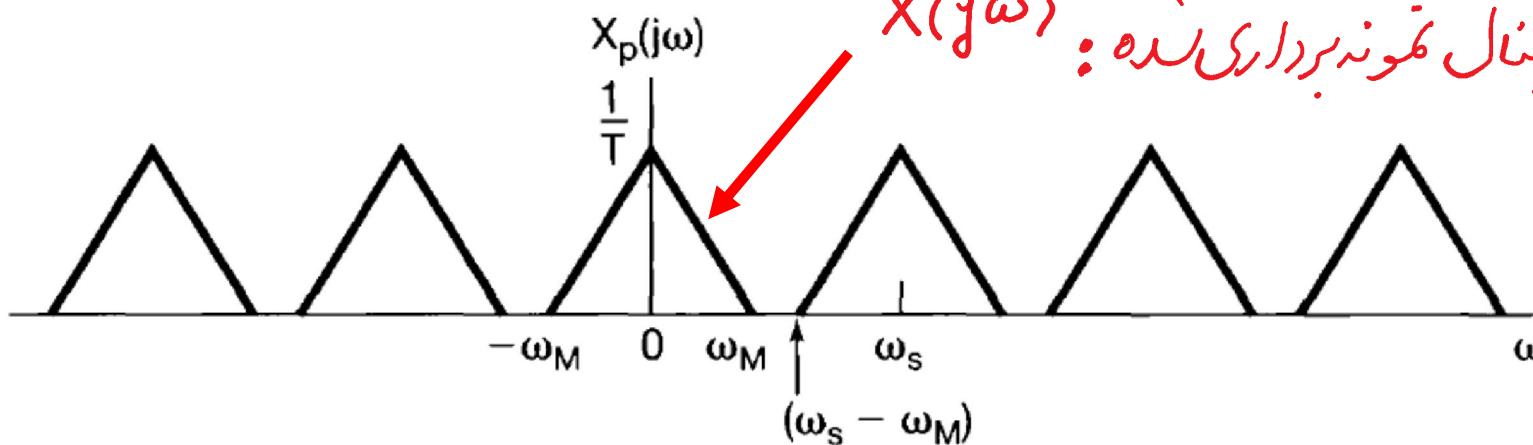
it follows that

$$X_p(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X(j(\omega - k\omega_s)).$$

طیف فرکانسی سیگنال مخونه برداری سده



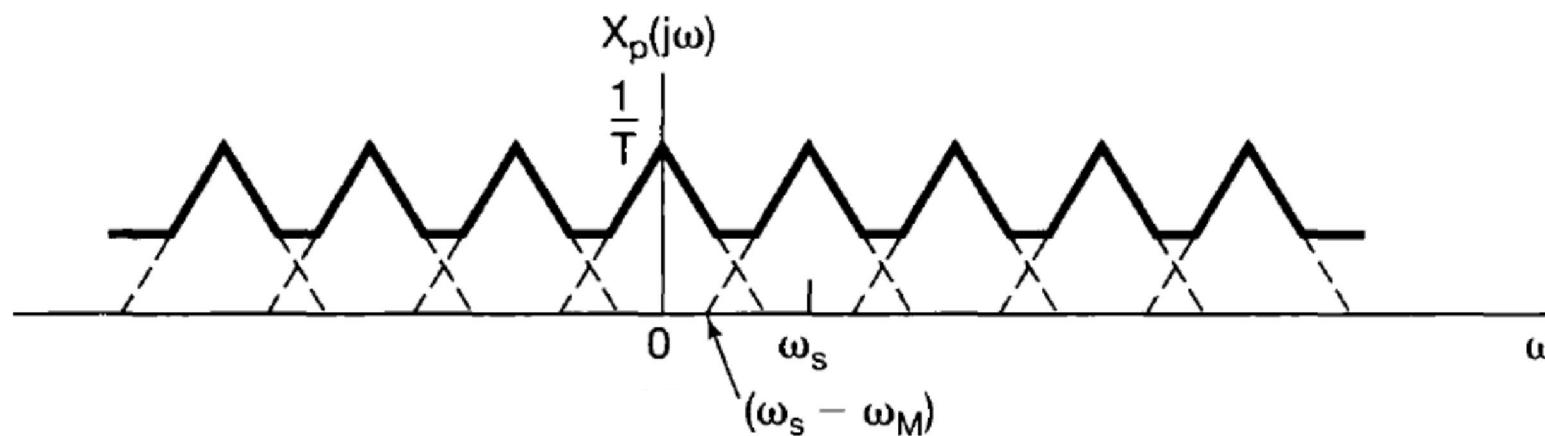
حالات های حملن در طیف سینال نمونه برداری شده: $X(j\omega)$



$$\omega_M < (\omega_s - \omega_M) \quad (1)$$

بازیابی سینال اصلی
حملن است.

spectrum of sampled signal with $\omega_s > 2\omega_M$:



$$\omega_M > (\omega_s - \omega_M) \quad (2)$$

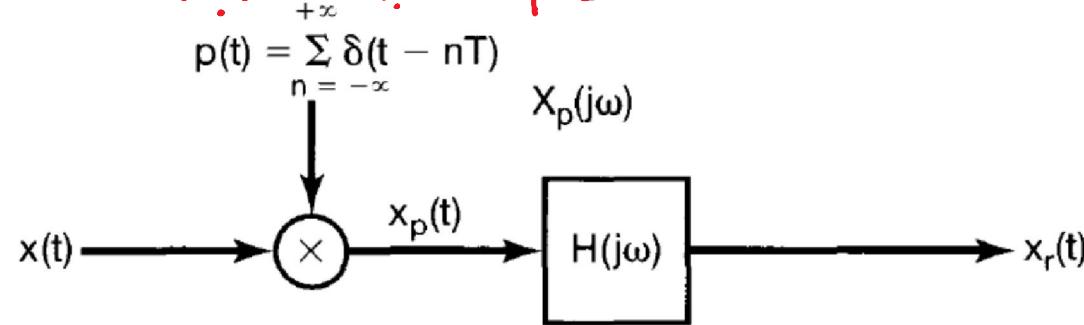
بازیابی سینال اصلی
حملن نیست.

spectrum of sampled signal with $\omega_s < 2\omega_M$.

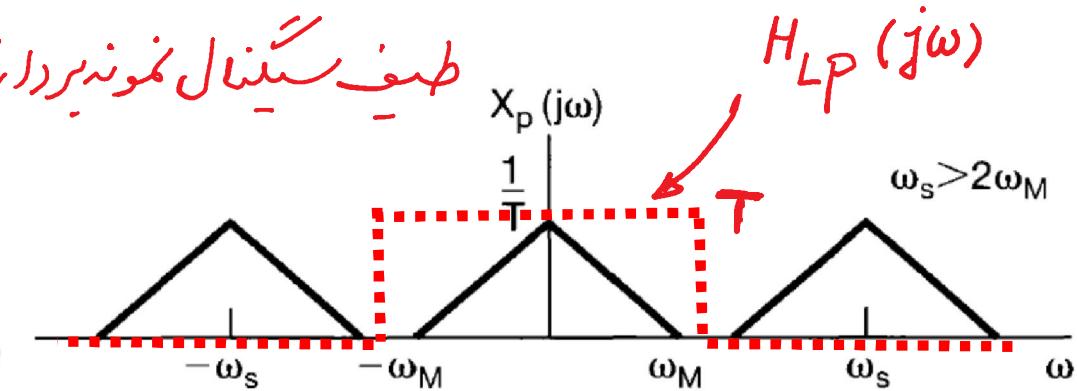
بازسازی ایدهآل سیگنال نمونهبرداری شده

Consequently, if $\omega_s > 2\omega_M$, $x(t)$ can be recovered exactly from $x_p(t)$ by means of a lowpass filter with gain T and a cutoff frequency greater than ω_M and less than $\omega_s - \omega_M$, as indicated in :

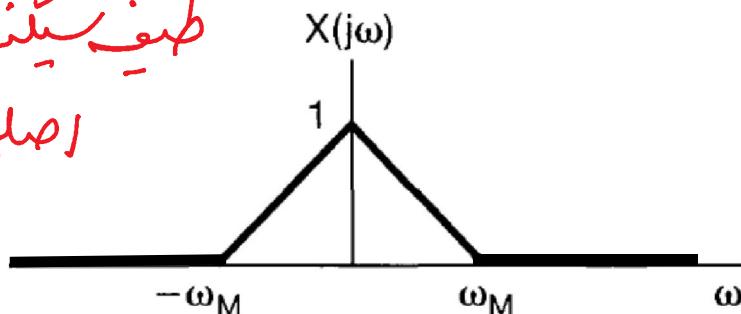
سیستم نمونهبرداری و بازسازی



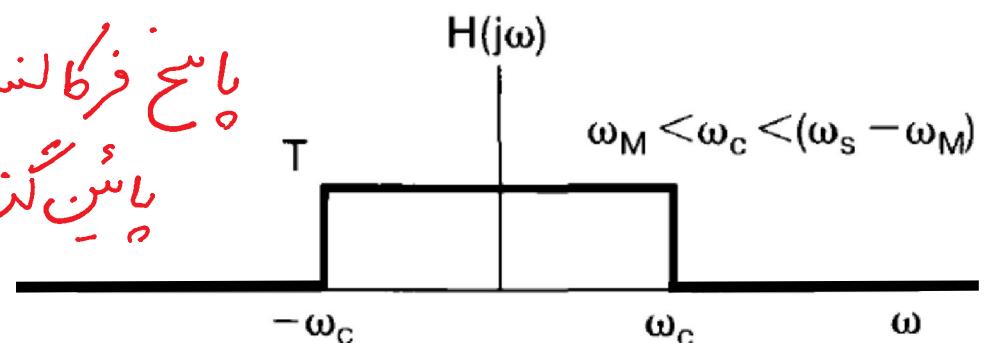
طیف سیگنال نمونهبرداری شده



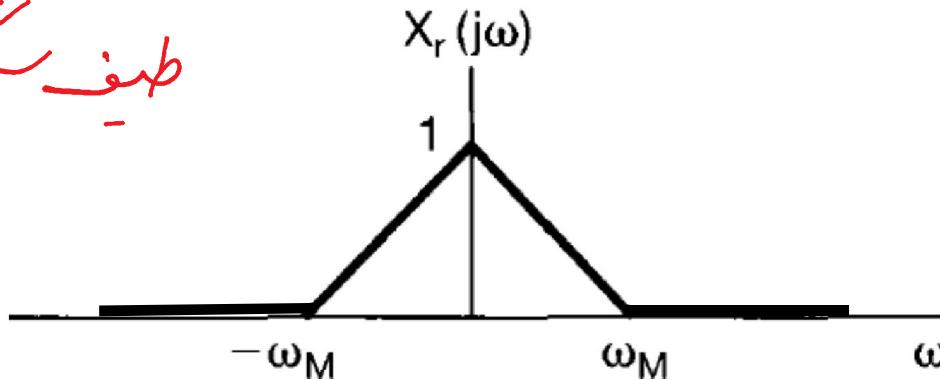
طیف سیگنال
اصلی



پاسخ فرکانسی فیلتر
پاسینگ لذت ایدهآل



طیف سینال بازسازی شده



$$X_r(j\omega) = X_p(j\omega) \cdot H(j\omega) = X(j\omega) \Rightarrow x_r(t) = x(t)$$

نتیجه عمومی: در صورتی که $x(t)$ پهناگی باند محدود ω_M داشته باشد و فرکانس اصلی قطاع رمزنده برداری یعنی $\omega_s = \frac{2\pi}{T}$ از $2\omega_M$ بیشتر باشد، بازسازی سینال $x(t)$ از روی مکونه های متوالی آن، یعنی $x(nT)$ ، $n \in \mathbb{Z}$ ممکن است.

قضیة نمونه برداری

Sampling Theorem:

Let $x(t)$ be a band-limited signal with $X(j\omega) = 0$ for $|\omega| > \omega_M$. Then $x(t)$ is uniquely determined by its samples $x(nT)$, $n = 0, \pm 1, \pm 2, \dots$, if

$$\omega_s > 2\omega_M,$$

where

$$\omega_s = \frac{2\pi}{T}.$$

Given these samples, we can reconstruct $x(t)$ by generating a periodic impulse train in which successive impulses have amplitudes that are successive sample values. This impulse train is then processed through an ideal lowpass filter with gain T and cutoff frequency greater than ω_M and less than $\omega_s - \omega_M$. The resulting output signal will exactly equal $x(t)$.

تعريف نرخ نایکوئیست

The frequency $2\omega_M$, which, under the sampling theorem, must be exceeded by the sampling frequency, is commonly referred to as the *Nyquist rate*.

نکته : در مدل به دو دلیل : ۱ - عدم امکان تکمیل قطعه ضربه ای به آن

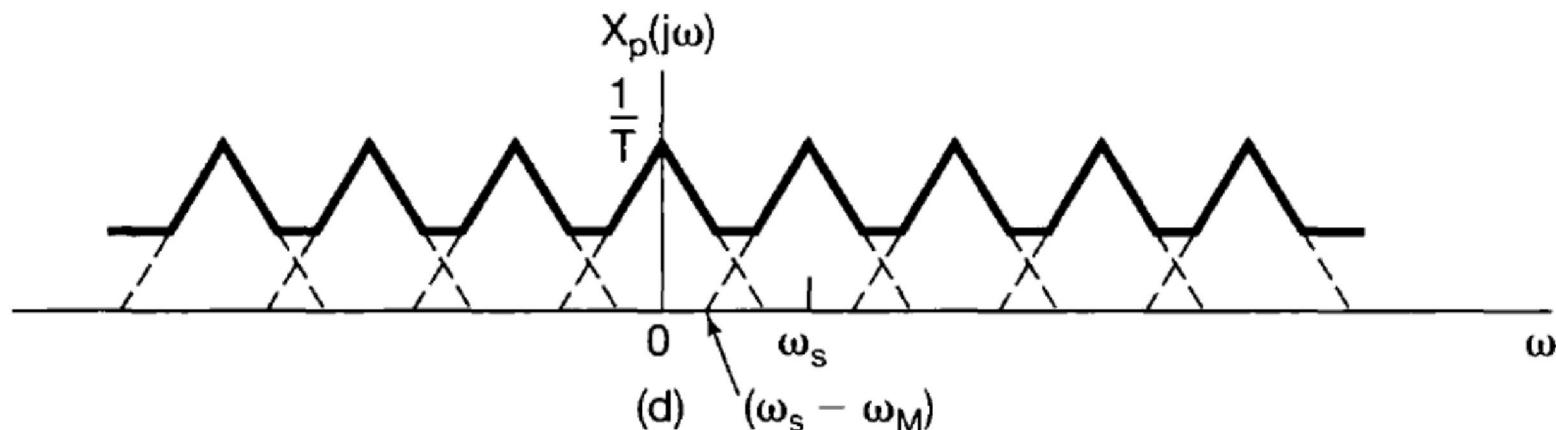
۲ - در اختیار نبودن فیلتر ایده‌آل پاسیور لذرا و این که $H(j\omega)$ یعنی پاسخ فرکانسی یک

فیلتر واقعی پاسیور لذرا در ناحیه $\omega > \omega_s - \omega_M$ تغیری دارد (ونه (صیغه) صفر است،

$$\Rightarrow X_r(j\omega) \approx X(j\omega) \Rightarrow x_r(t) \approx x(t)$$

اثر نمونه برداری با نرخ کمتر از نرخ نایکوئیست (پدیده Aliasing)

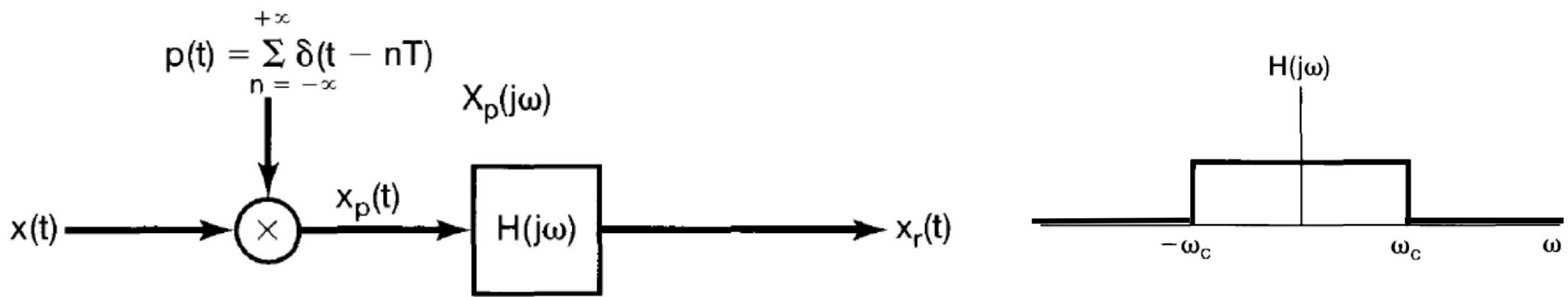
When $\omega_s < 2\omega_M$, the spectrum of $x(t)$, is no longer replicated in $X_p(j\omega)$ and thus is no longer recoverable by lowpass filtering.



spectrum of sampled signal with $\omega_s < 2\omega_M$.

This effect, in which the individual terms overlap, is referred to as *aliasing*.

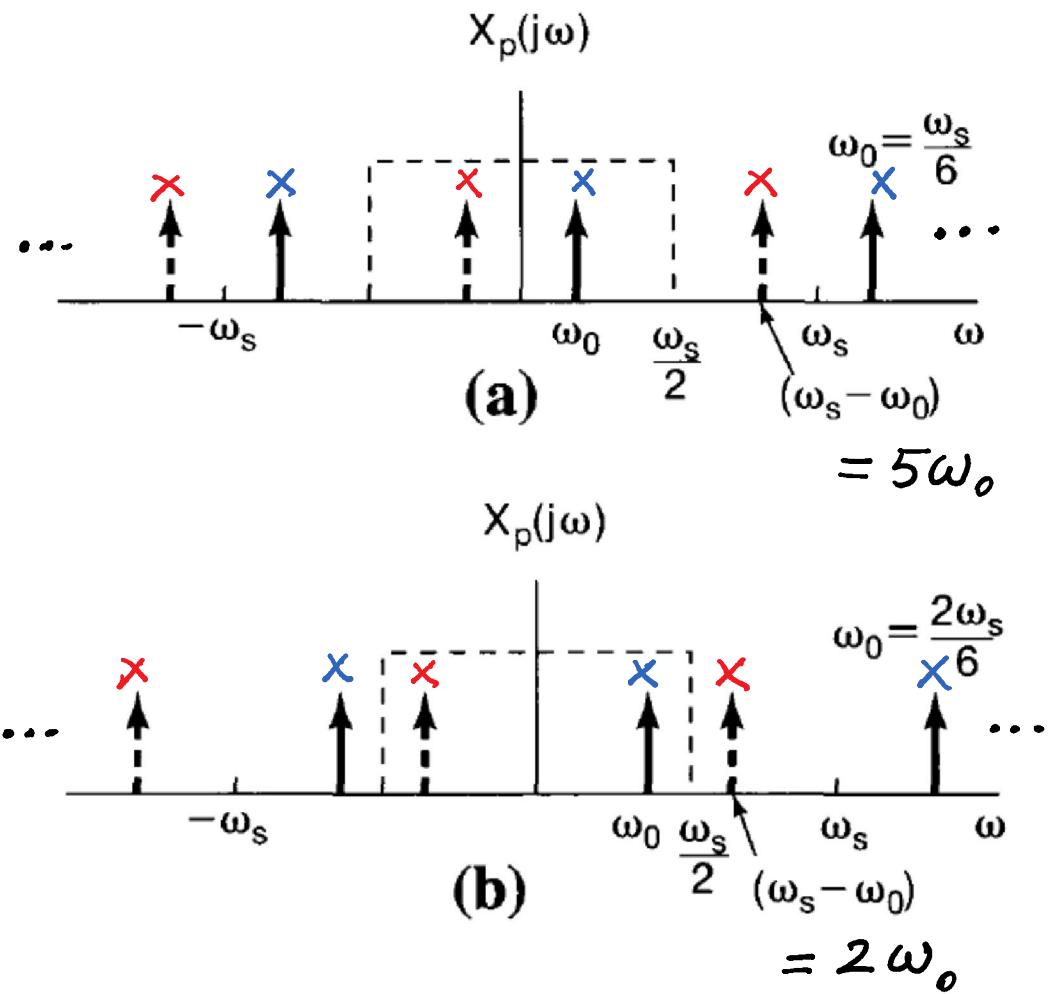
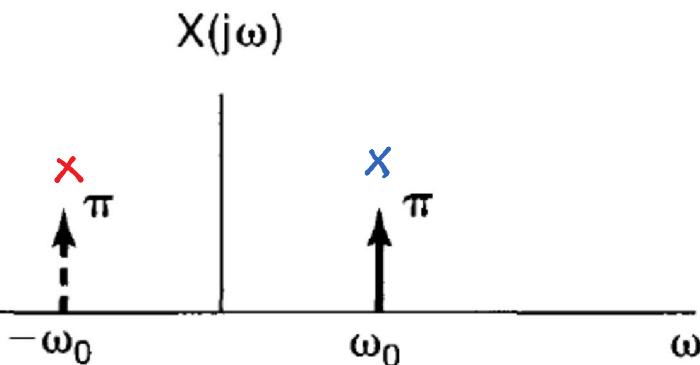
سال) در سیستم مخونه برداری با وظای اضربه و بازگشایی با فیلتر پاسی لذرسکل زیر، فرض کنیم
 $\omega_c = \frac{1}{T}$ (فرطانس مخونه برداری) و $\omega_s = \frac{2\pi}{T}$. $x(t) = \cos \omega_0 t$
 قطع (LPF) را بابت فرض حی کنیم.



برای معادله مختلف ω ، طبعی $X_r(j\omega)$ و $X_p(j\omega)$ و $x_r(t)$ چگونه است؟

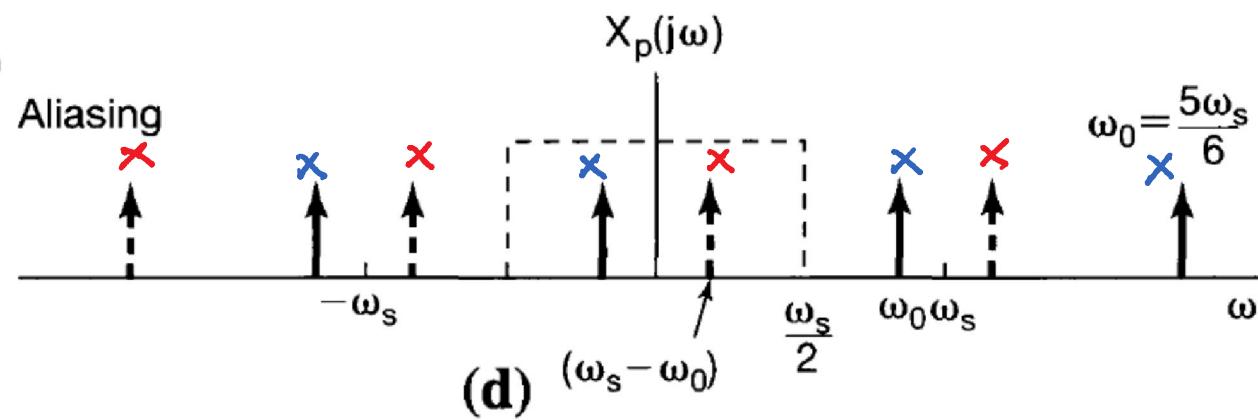
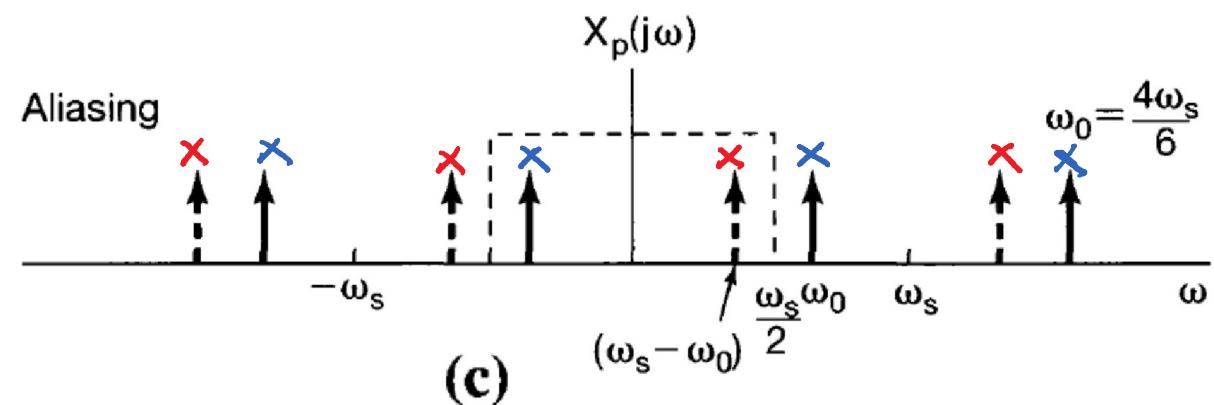
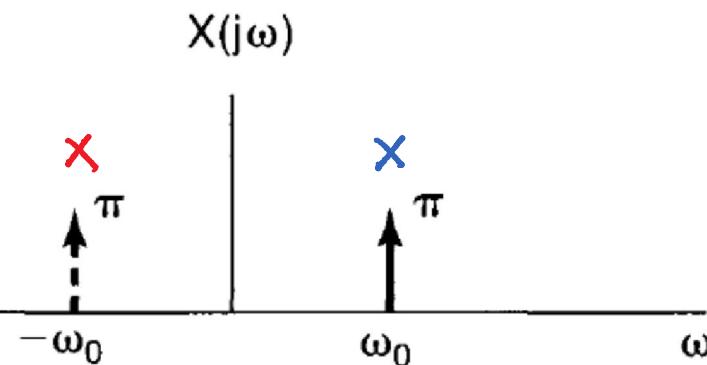
(a) $\omega_0 = \frac{\omega_s}{6}; \quad x_r(t) = \cos \omega_0 t = x(t)$

(b) $\omega_0 = \frac{2\omega_s}{6}; \quad x_r(t) = \cos \omega_0 t = x(t)$



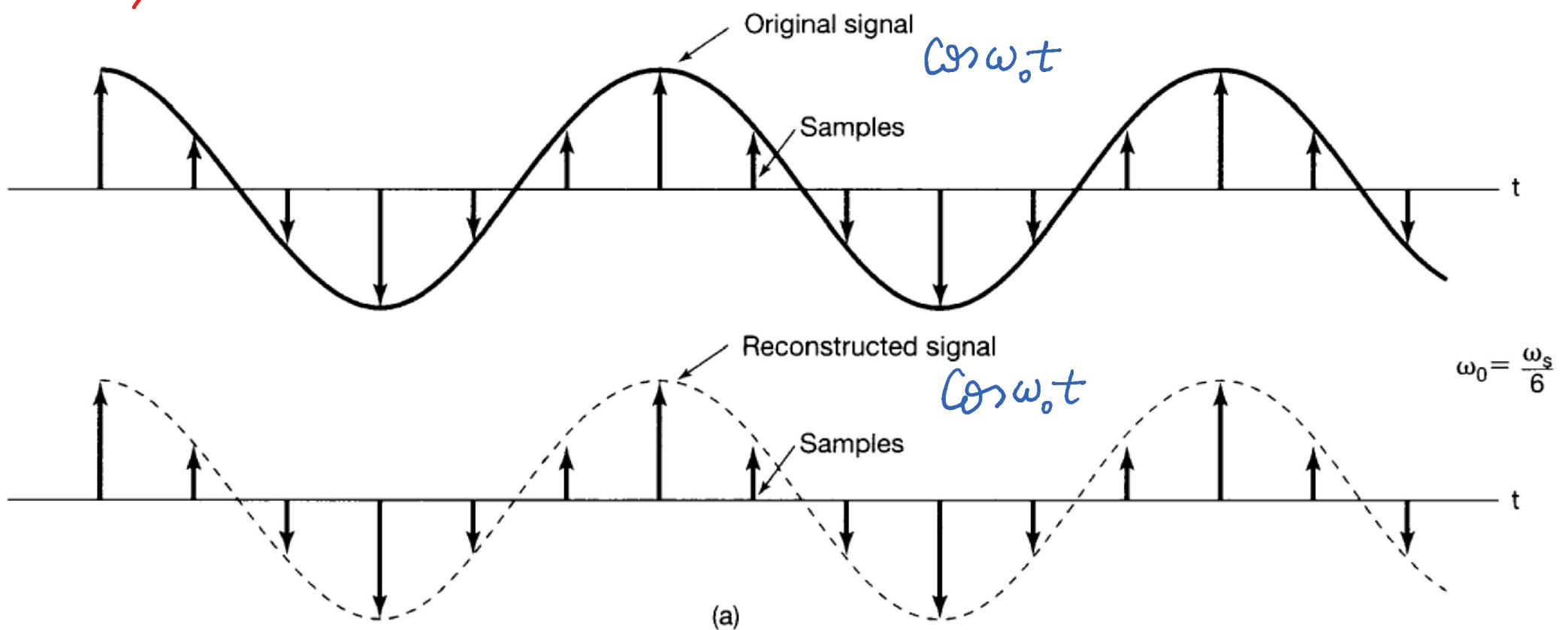
(c) $\omega_0 = \frac{4\omega_s}{6}; \quad x_r(t) = \cos(\omega_s - \omega_0)t \neq x(t)$

(d) $\omega_0 = \frac{5\omega_s}{6}; \quad x_r(t) = \cos(\omega_s - \omega_0)t \neq x(t).$

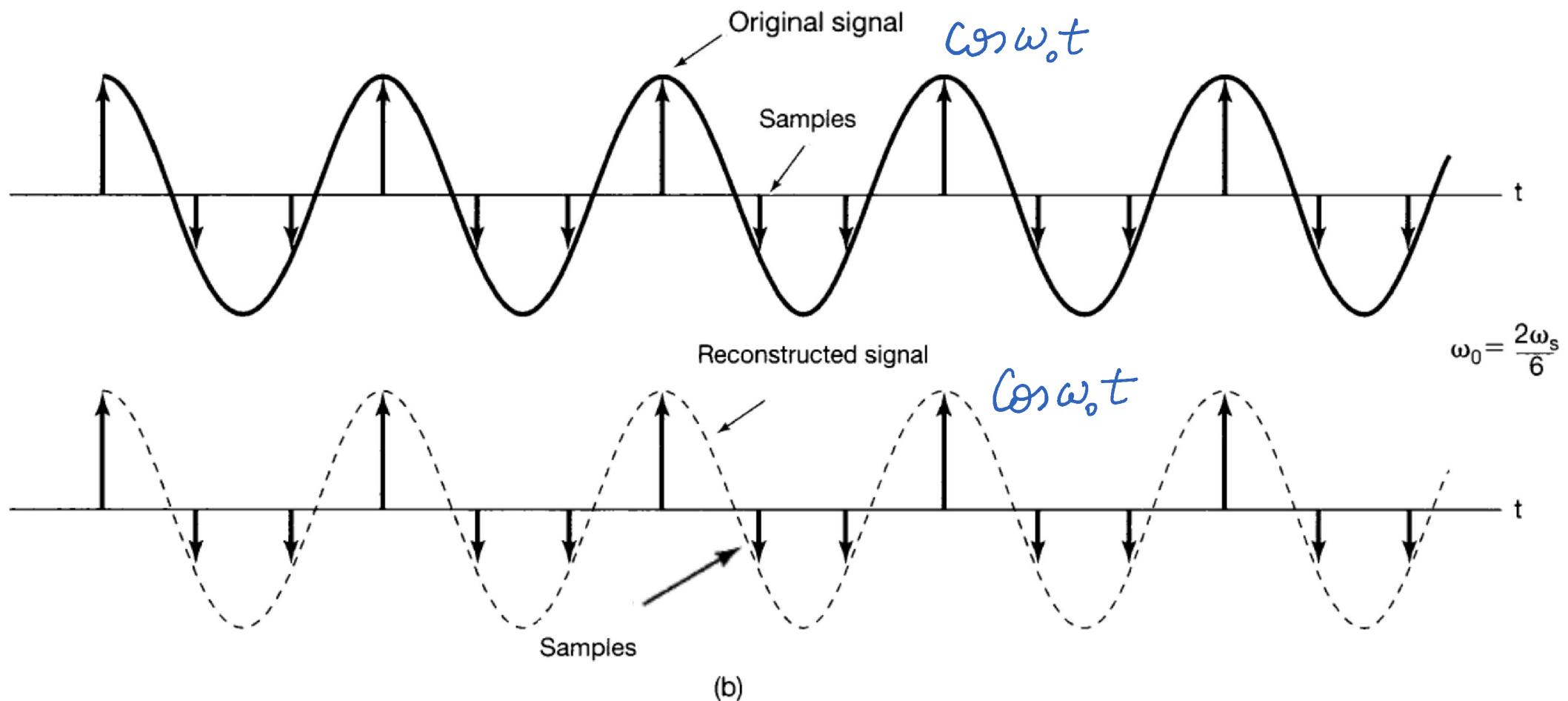


For each of the four cases , the signal $x(t)$, its samples, and the reconstructed signal $x_r(t)$ have been depicted.

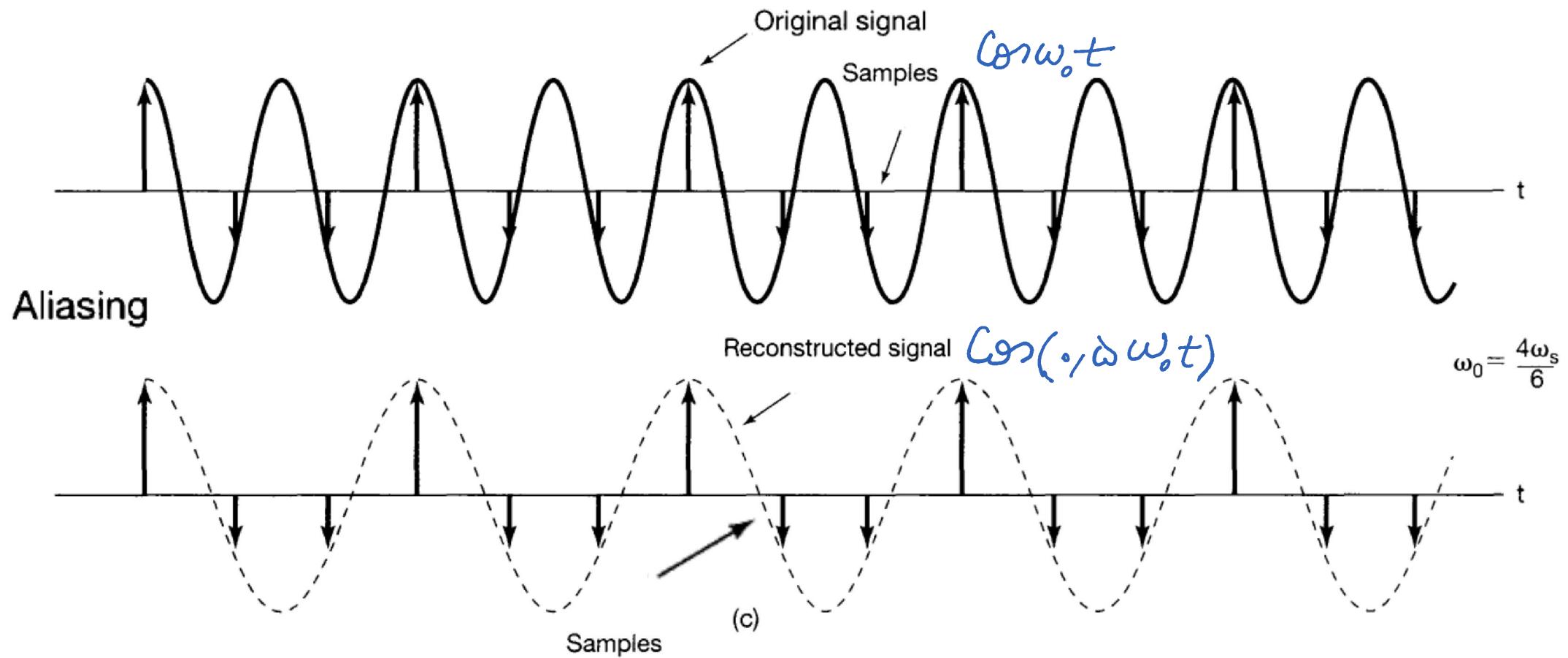
$$\omega_0 = \frac{1}{\gamma} \omega_s \Rightarrow \omega_s = \gamma \omega_0 > r \omega_m \quad , \quad \omega_c = \frac{1}{r} \omega_s = r \omega_0 > \omega_0$$



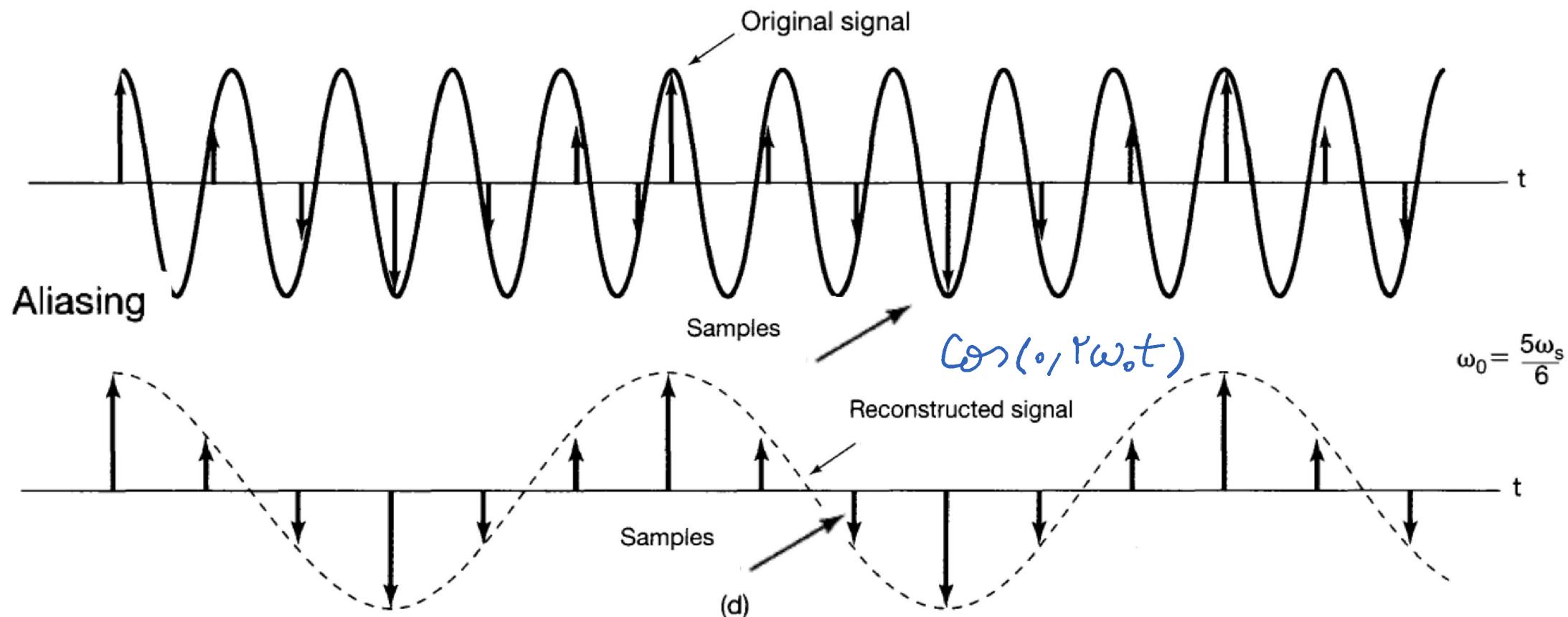
$$\omega_0 = \frac{1}{T} \omega_s \Rightarrow \omega_s = T\omega_0 > T\omega_m \quad , \quad \omega_c = \frac{1}{T} \omega_s = \frac{T}{T} \omega_0 > \omega_0$$



$$\omega_0 = \frac{r}{\gamma} \omega_s \Rightarrow \omega_s = \frac{\gamma}{r} \omega_0 < r \omega_M \quad , \quad \omega_c = \frac{1}{r} \omega_s = \frac{\gamma}{\kappa} \omega_0 < \omega_0$$



$$\omega_0 = \frac{\delta}{\gamma} \omega_s \Rightarrow \omega_s = \frac{\gamma}{\delta} \omega_0 < \gamma \omega_M \quad , \quad \omega_c = \frac{1}{r} \omega_s = \frac{\gamma}{\delta} \omega_0 < \omega_0$$



سوال: آیا مونه برداری با فرکانس دقیقاً برابر بازخ ناکلیوئیت یعنی $\omega = 2\omega_M$ ،

نکلی را در بازسازی دقیق سینال اصلی ایجاد می کند؟

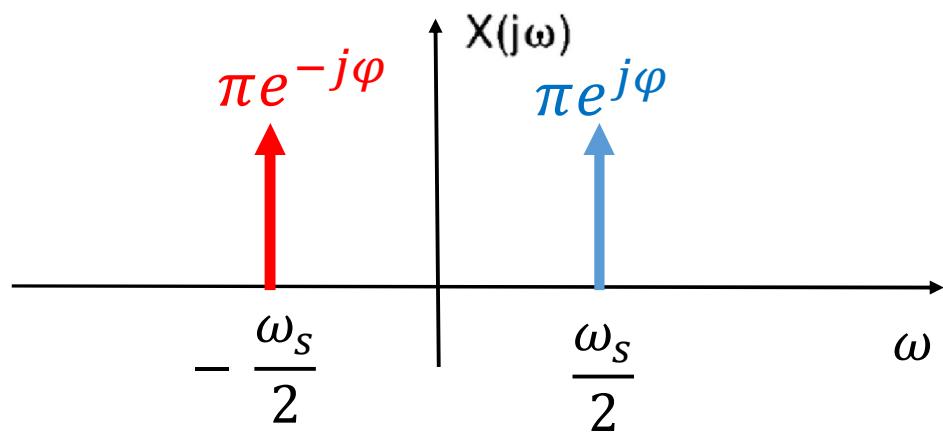
پاسخ: اگر $\omega = 2\omega_M$ ، حملن است سینال بازسازی شده $x_r(t)$ با سینال اصلی

$x(t)$ برابر نباشد.

مثال) فرض کنیم سینال اصلی $x(t) = \cos(\omega_0 t + \varphi)$ و فرکانس مونه برداری با خطا ضربه ،

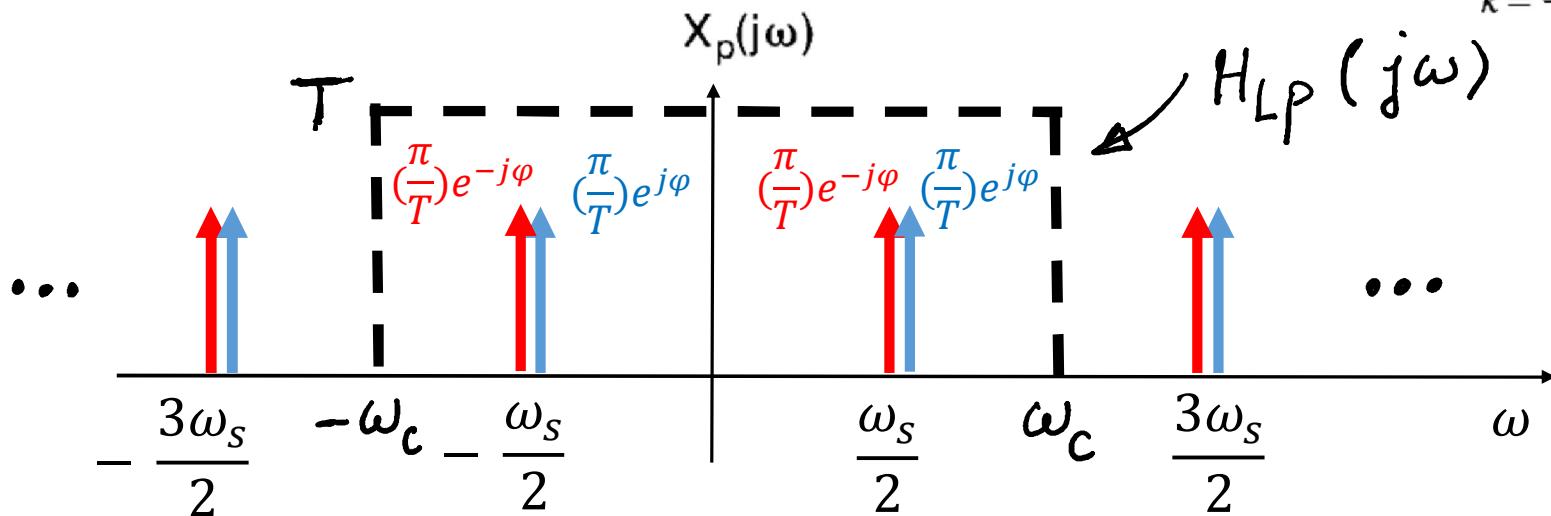
$$\omega_s = \frac{2\pi}{T} = 2\omega_0 \text{ باشد.}$$

$$X(j\omega) = \pi e^{j\varphi} \delta(\omega - \frac{\omega_s}{2}) + \pi e^{-j\varphi} \delta(\omega + \frac{\omega_s}{2})$$



طیف سیگنال مخزن برداری سده^۰

$$X_p(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X(j(\omega - k\omega_s)).$$



$$X_r(j\omega) = X_p(j\omega) \cdot H_{LP}(j\omega)$$

$$= \pi (e^{-j\varphi} + e^{j\varphi}) [\delta(\omega - \frac{1}{T}\omega_s) + \delta(\omega + \frac{1}{T}\omega_s)]$$

$$\Rightarrow \underline{x_r(t) = (\cos \varphi) \cos(\frac{1}{T}\omega_s t)}$$

نتیجه: تنها در صورتی $x_r(t) = x(t)$ که $\varphi = 0$. در حالت کلی برای $\varphi \neq 2k\pi$

$x_r(t) \neq x(t)$ بازگشایی بدروستی انجام نمی‌شود. در دوین حالت با

فرض $x(t) = \sin(\frac{1}{T}\omega_s t)$ در حالت $x_r(t) = 0$ ، $\varphi = -\pi/2$

بنابراین برای مونه برداری و بازگشایی درست سیگنال لازم است $\omega_s > 2\omega_M$.

Sampling with a Zero-Order Hold

۲) نمونه برداری با نگهدارنده مرتبه صفر

The sampling theorem, which is most easily explained in terms of impulse-train sampling, establishes the fact that a band-limited signal is uniquely represented by its samples.

طبق قضیه نمونه برداری، در صورت محقق شرایط ایده‌آل برای قطعه ضربه و فیلتر پاسنگ لذر،

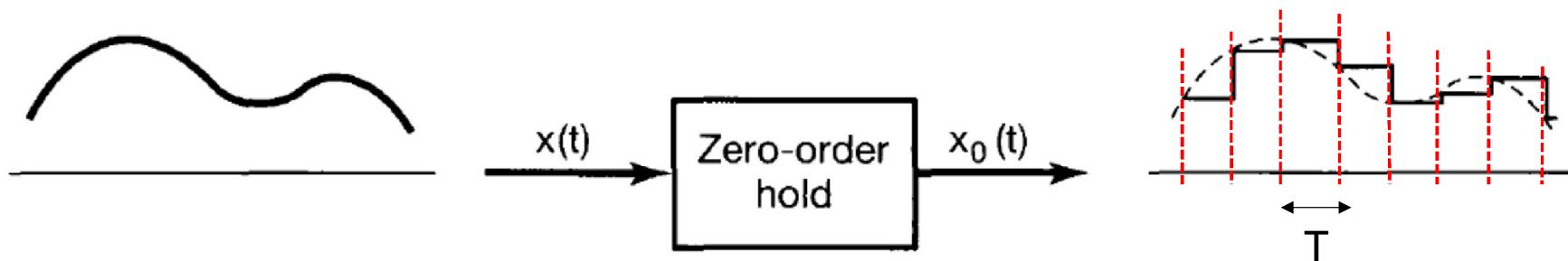
و محدود بودن پهنای باند نیک سینال زمان پیوسته، حی تو ان آن سینال را توسط نمونه‌های زمان گسته آن، به صورت یکتا نالس دارد و بازسازی کرد.

سؤال: محقق شرایط ایده‌آل فوق تا چه اندازه امکان پذیر است؟

راه حل علی کدام است؟

In practice, however, narrow, large-amplitude pulses, which approximate impulses, are also relatively difficult to generate and transmit, and it is often more convenient to generate the sampled signal in a form referred to as a *zero-order hold*.

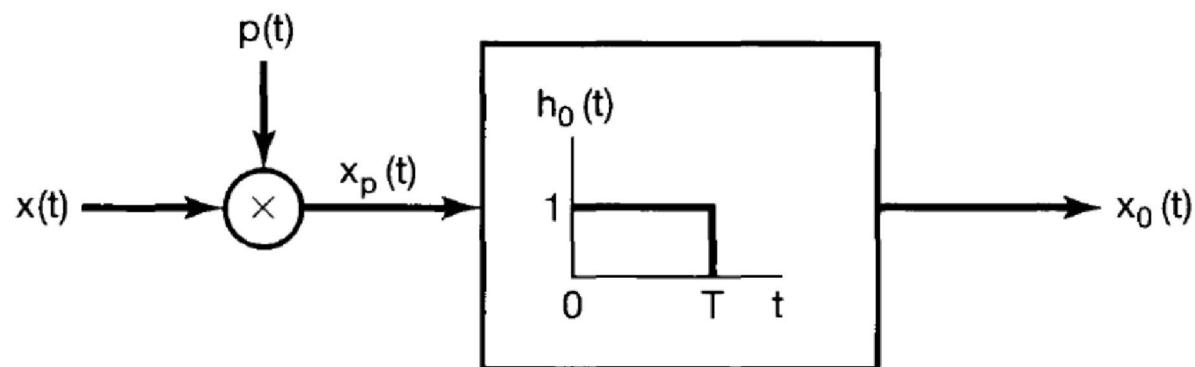
Such a system samples $x(t)$ at a given instant and holds that value until the next instant at which a sample is taken, as illustrated in



Sampling utilizing a zero-order hold.

The reconstruction of $x(t)$ from the output of a zero-order hold can again be carried out by lowpass filtering. However, in this case, the required filter no longer has constant gain in the passband.

سیستم معادل نمونهبردار با نگهدارنده مرتبه صفر



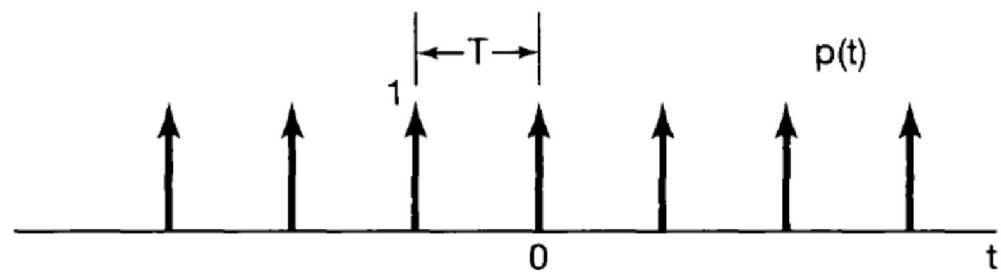
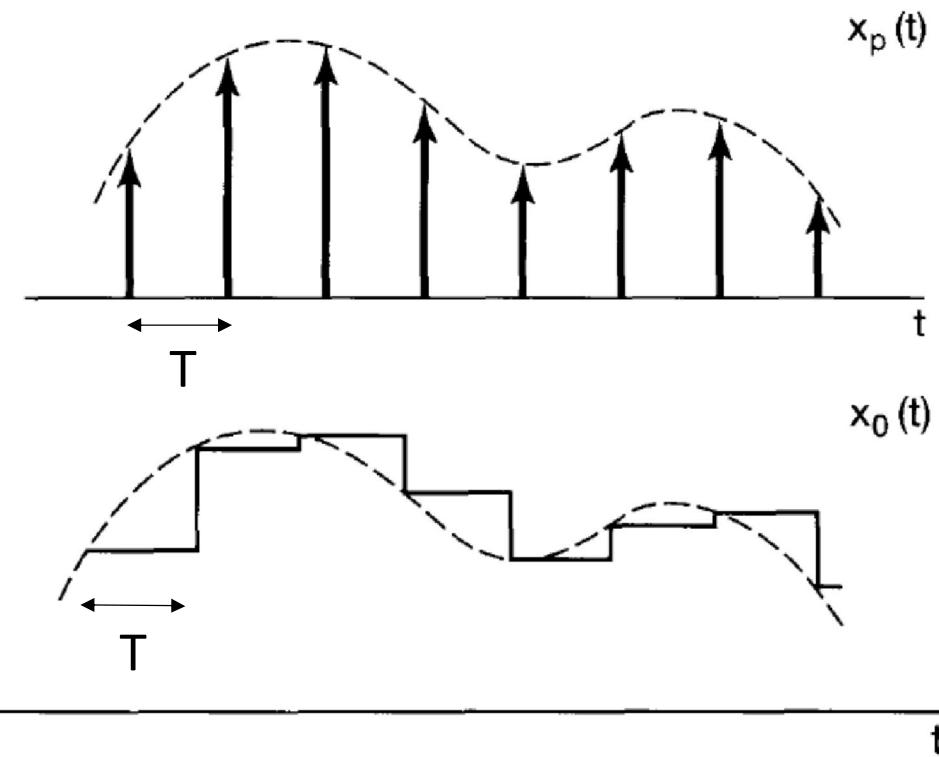
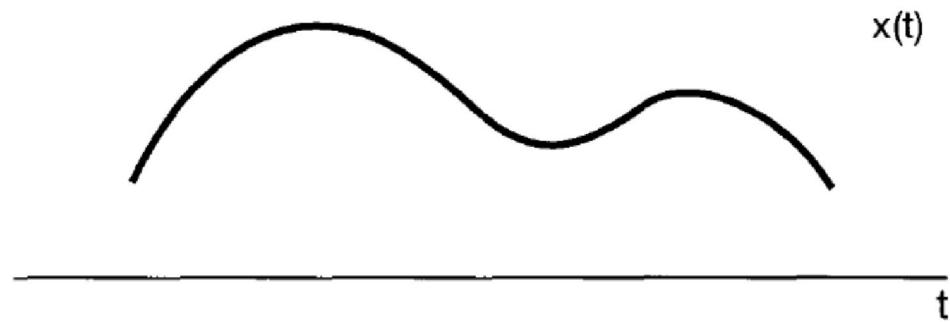
$$\text{rect}\left(\frac{t}{T}\right) = \Pi\left(\frac{t}{T}\right) = \begin{cases} 1, & |t| < \frac{T}{2} \\ 0, & \text{others.} \end{cases}$$

Rectangular Pulse

Zero-order hold as impulse-train sampling followed by an LTI system with a rectangular impulse response.

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT).$$

$$h_0(t) = \Pi\left(\frac{t - T/2}{T}\right)$$

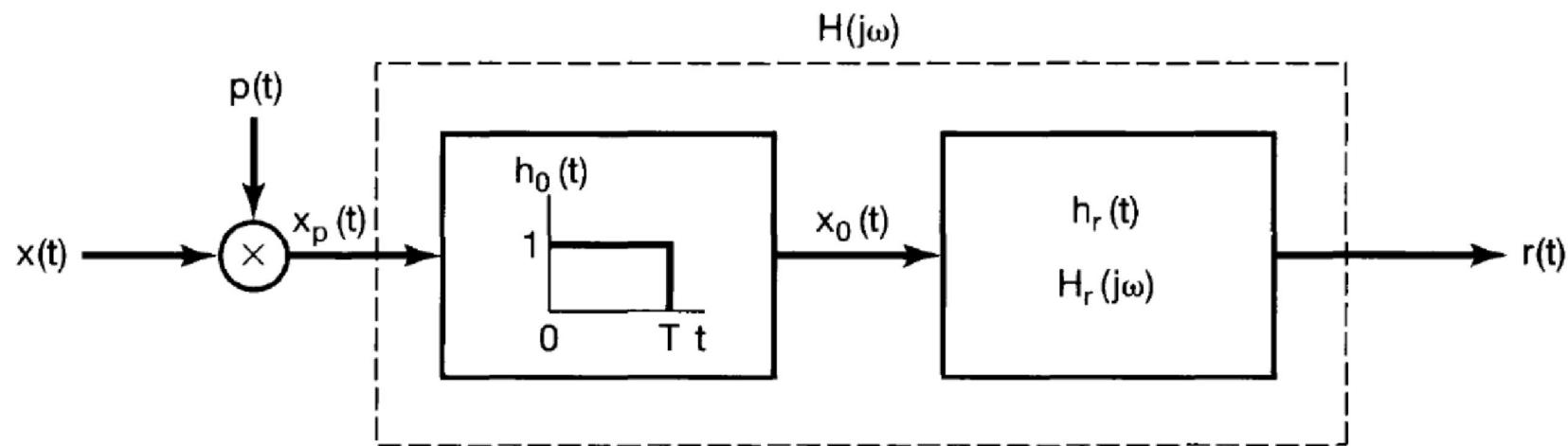


$$\begin{aligned}
 x_p(t) &= x(t)p(t) \\
 &= \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT).
 \end{aligned}$$

$$\begin{aligned}
 x_0(t) &= x_p(t) * h_0(t) \\
 &= x_p(t) * \Pi\left(\frac{t - T/2}{T}\right)
 \end{aligned}$$

بازسازی کامل سیگنال نمونهبرداری شده در سیستم نمونهبردار با نگهدارنده صفر

To reconstruct $x(t)$ from $x_0(t)$, we consider processing $x_0(t)$ with an LTI system with impulse response $h_r(t)$ and frequency response $H_r(j\omega)$.



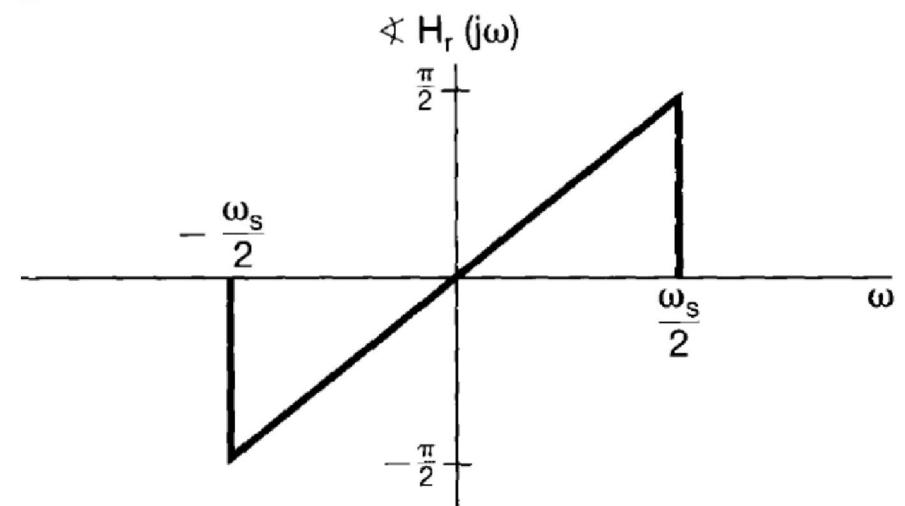
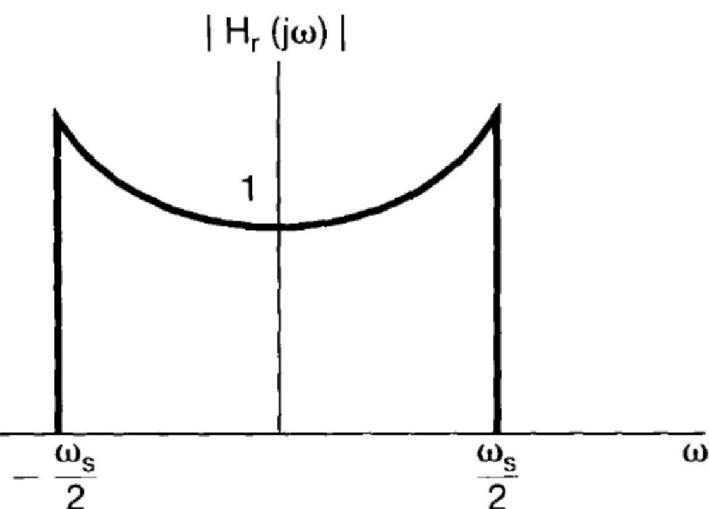
Cascade of the representation of a zero-order hold with a reconstruction filter.

We see that $r(t) = x(t)$ if the cascade combination of $h_0(t)$ and $h_r(t)$ is the ideal lowpass.

$$h_0(t) = \Pi\left(\frac{t - \frac{T}{2}}{T}\right) \quad \Rightarrow \quad H_0(j\omega) = e^{-j\omega T/2} \left[\frac{2 \sin(\omega T/2)}{\omega} \right],$$

$$\Rightarrow H_r(j\omega) = \frac{e^{j\omega T/2} H(j\omega)}{2 \sin(\omega T/2)}.$$

جایزی LPF



سؤال: آیا تَحْقِيقِ کامل فیلتری با پاسخ فرکانسی $H_r(j\omega)$ ممکن است؟ خیر.

Once again, in practice the frequency response of $H_r(j\omega)$ cannot be exactly realized, and thus an adequate approximation to it must be designed.

نکته: در بیان از سوارد، فرودی مخونه بردار با نگاهارنده سرتیه صفر، خودش تقریب کافی و مناسب از سینگنال اصلی $x(t)$ است.

In fact, in many situations, the output of the zero-order hold is considered an adequate approximation to the original signal by itself, without any additional lowpass filtering.



نکته: در برخی از طریق‌های تقریب دیگر از روش درون‌یابی نرم‌تری می‌توانند نمونه‌ها را سعاده‌تر نمود.

Alternatively, in some applications, we may wish to perform some smoother interpolation between sample values.

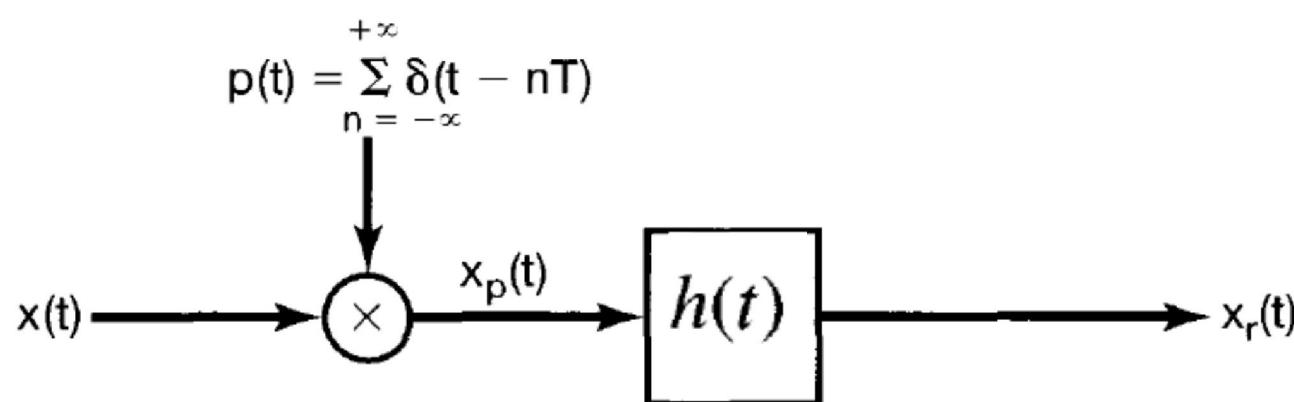
درون‌یابی یا بازسازی سیگнал پیوسته از روی نمونه‌های زمانی گسترش آن

RECONSTRUCTION OF A SIGNAL FROM ITS SAMPLES USING INTERPOLATION

Interpolation, that is, the fitting of a continuous signal to a set of sample values, is a commonly used procedure for reconstructing a function, either approximately or exactly, from samples.

روُس اول (قِيق و کامل) : برای سیگنالهای با پهنای باند محدود، مخونه برداری با قطار ضربه و بازسازی با فیلتر پاس لزر ایده‌آل، در واقع یک رُس درون یا بی قِيق است که به آن درون یا بی باند محدود کفته‌است.

درون یا بی باند محدود (Band-Limited Interpolation)



$h(t)$ پاسخ ضربه فیلتر پاس لزر ایده‌آل است.

The interpretation of the reconstruction of $x(t)$ as a process of interpolation becomes evident when we consider the effect in the time domain of the lowpass filter .

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT). \quad \rightarrow \quad x_p(t) = x(t)p(t) = \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT).$$

$$x_r(t) = x_p(t) * h(t) \quad \rightarrow \quad x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT)h(t - nT).$$

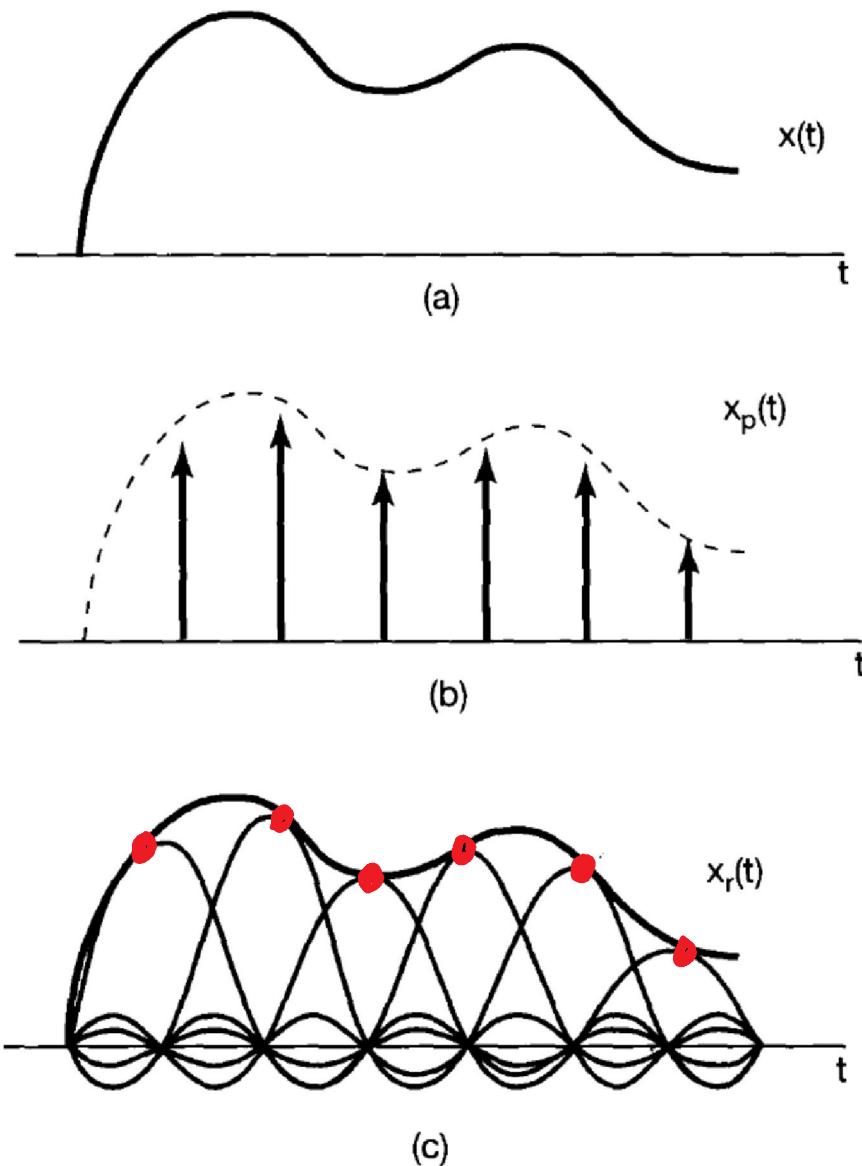
با سخ فرکانس
LPF

$$H(j\omega) = \begin{cases} T & |\omega| \leq \omega_c \\ 0 & |\omega| > \omega_c \end{cases}. \quad \rightarrow \quad h(t) = \frac{\omega_c T \sin(\omega_c t)}{\pi \omega_c t},$$

با سخ ضرب
LPF

$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT) \frac{\omega_c T}{\pi} \frac{\sin(\omega_c(t - nT))}{\omega_c(t - nT)}.$$

interpolation formula.

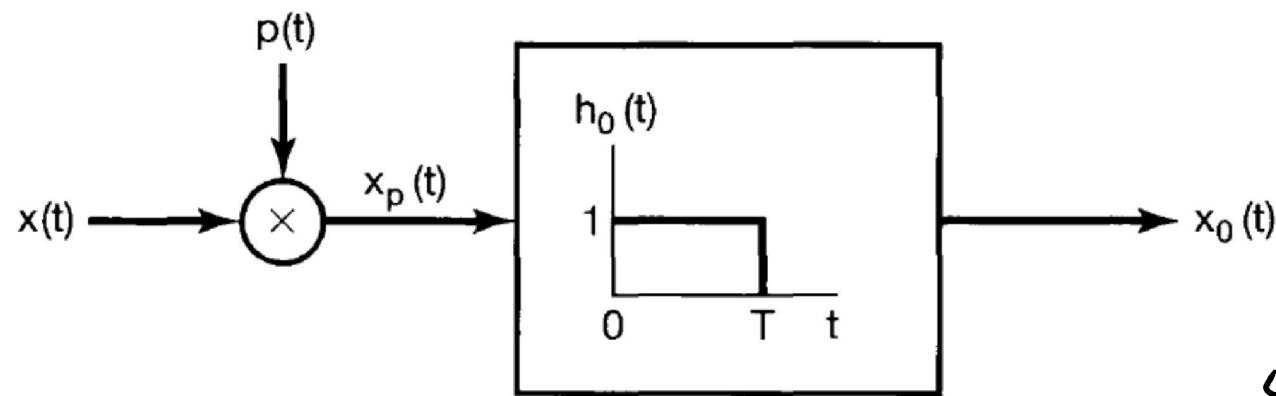


درون یابی باز محدود ایدهآل

توسط ررسی خطی توابع sinc

Ideal band-limited interpolation using the sinc function:
 (a) band-limited signal $x(t)$; (b) impulse train of samples of $x(t)$; (c) ideal band-limited interpolation in which the impulse train is replaced by a superposition of sinc functions

روُس دوم (ساده و تعریبی): استفاده سقّم از خروجی مونتاژ بردار با نگهدارنده ضربه صفر



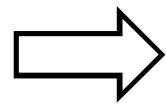
در این روش تعریبی، $h_0(t)$ را

محلوان پاسخ ضربه سیستم

درین یابی داشت که از روی سیگنال

مونتاژ برداری شده باقطران ضربه، یعنی $x_p(t)$ سیگنال $x_0(t)$ را که تقریباً خوبی از سیگنال اصلی $x(t)$ است، درین یابی و بازسازی می‌کند.

$$h_0(t) = \text{rect}\left(\frac{t - \frac{T}{2}}{T}\right)$$



$$H_0(j\omega) = e^{-j\omega T/2} \left[\frac{2 \sin(\omega T/2)}{\omega} \right],$$

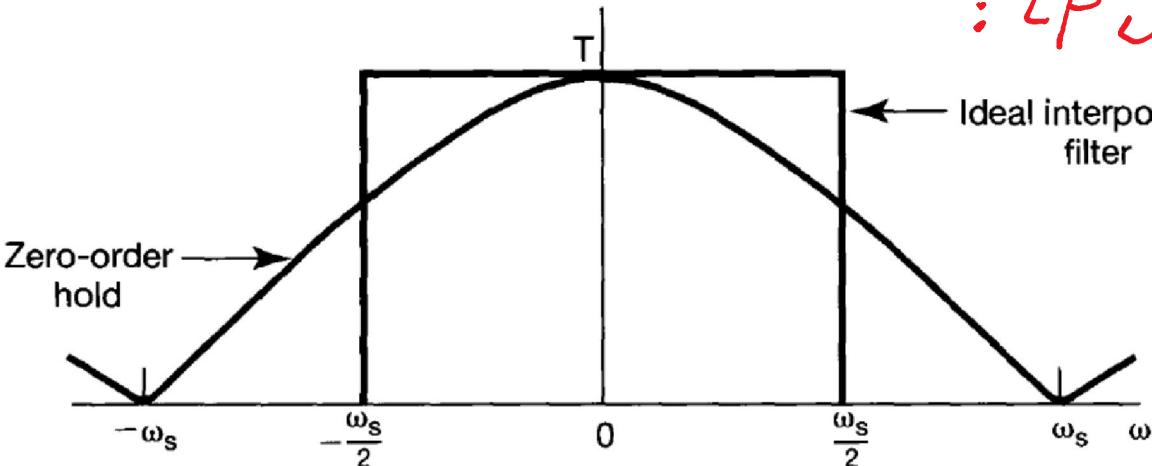
پاسخ ضربهٔ فیلتر درون‌بابی ZOH

پاسخ فرکانسی فیلتر درون‌بابی ZOH

تعابیر با فیلتر ایدهآل LPF:

$$|H(j\omega)|_{LPF}$$

$$|H_0(j\omega)|$$

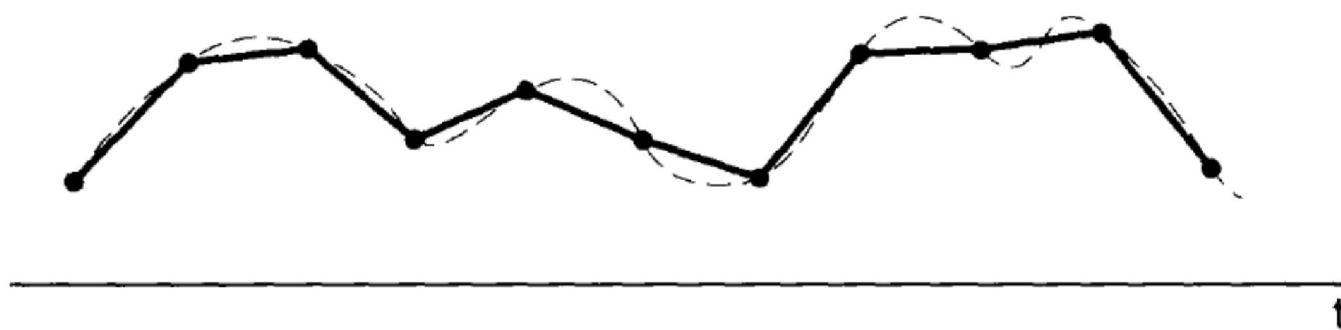


$$\omega_s = \frac{r\pi}{T}$$

$$H(j\omega)_{LPF} = \begin{cases} T & |\omega| \leq \omega_c \\ 0 & |\omega| > \omega_c \end{cases}, \quad \omega_c = \frac{\omega_s}{r}$$

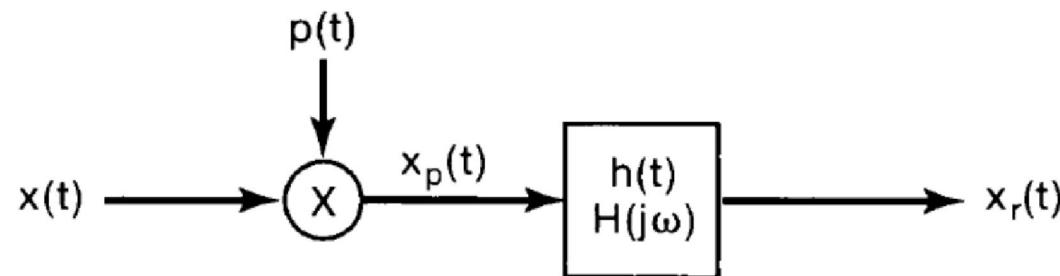
روز سوم (بالغایب کردن) : درون یابی خطی یا نکلارند مرتبه یک

Another useful form of interpolation is *linear interpolation*, whereby adjacent sample points are connected by a straight line, as illustrated in

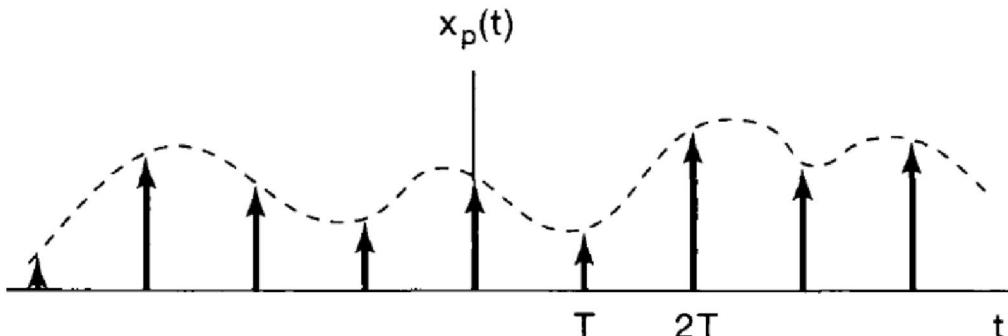


The dashed curve represents the original signal and the solid curve the linear interpolation.

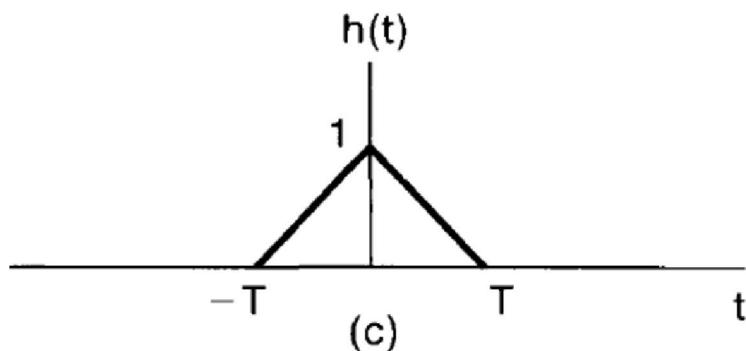
سیستم مخونه برداری با نکلارند مرتبه یک (FOH) که در آن پاسخ ضربه فیلتر درون یابی یک پالس تسلی ا است، در عمل درون یابی خطی بین مخونه های متوالی را انجام می دهد.



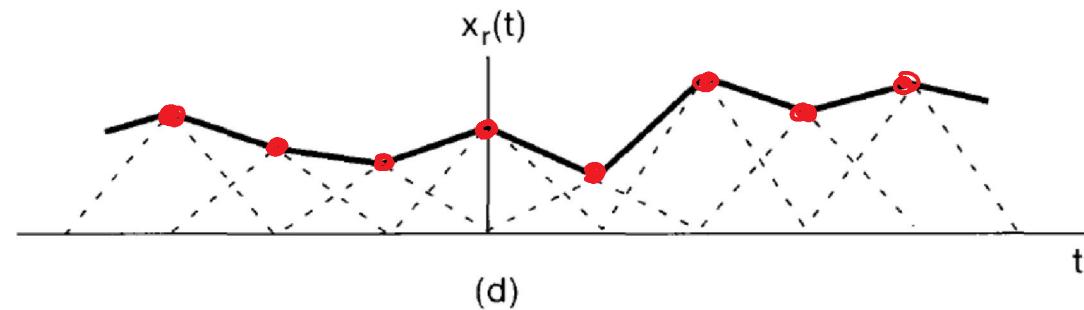
(a)



(b)



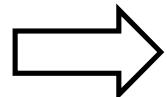
(c)



(d)

Linear interpolation
 (first-order hold) as impulse-train sampling followed by convolution with a triangular impulse response: (a) system for sampling and reconstruction; (b) impulse train of samples; (c) impulse response representing a first-order hold; (d) first-order hold applied to the sampled signal

$$h(t) = \frac{1}{T} \text{rect}\left(\frac{t}{T}\right)^* \text{rect}\left(\frac{t}{T}\right)$$

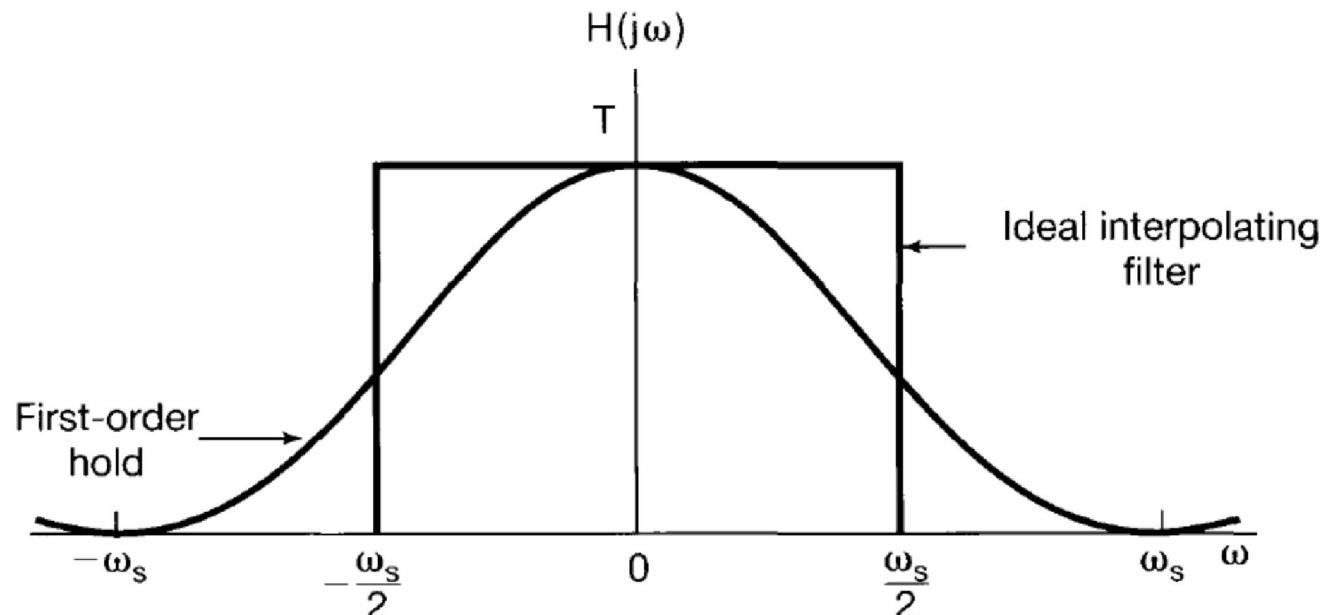


$$H(j\omega) = \frac{1}{T} \left[\frac{\sin(\omega T/2)}{\omega/2} \right]^2.$$

پاسخ ضربه فیلتر درون یابی FOH

پاسخ فرکانسی فیلتر درون یابی FOH

The transfer function of the first-order hold is shown superimposed on the transfer function for the ideal interpolating filter.



نکته: رون های درون یا بی رقیق تر (غیر خطی) مانند اندوه رتیه دوم و بالاتر

In an analogous fashion, we can define second- and higher order holds that produce reconstructions with a higher degree of smoothness.

For example, the output of a second-order hold provides an interpolation of the sample values that is continuous and has a continuous first derivative and discontinuous second derivative.



دانشگاه صنعتی اصفهان
دانشکده برق و کامپیوتر

بسم الله الرحمن الرحيم

تجزیه و تحلیل سیگنال‌ها و سیستم‌ها

مدرس: مسعود عمومی

جلسه بیست و هشتم – بخش 7.4 کتاب

با سلام خدمت دانشجویان محترم

پردازش زمان‌گسسته سیگنال‌های زمان‌پیوسته

DISCRETE-TIME PROCESSING OF CONTINUOUS-TIME SIGNALS

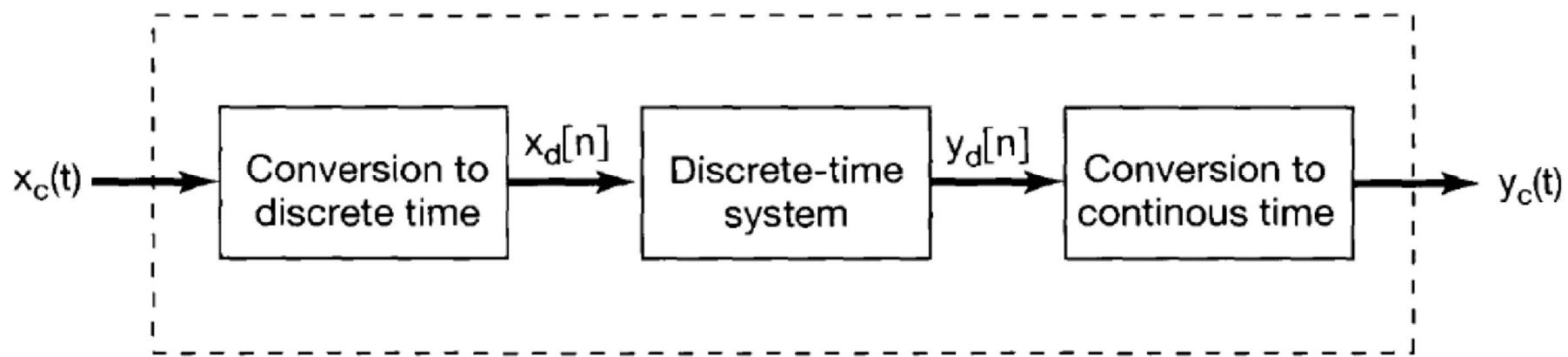
(Discrete Time Signal Processing = DSP) اهمیت پردازش زمان‌گسسته سیگنال‌ها

In many applications, there is a significant advantage offered in processing a continuous-time signal by first converting it to a discrete-time signal and, after discrete-time processing, converting back to a continuous-time signal.

The discrete-time signal processing can be implemented with any of the variety of devices that are specifically oriented toward discrete-time signal processing.

ساختار کلی روش‌های پردازش زمان‌گستره سیگنال‌های زمان‌پیوسته

In broad terms, this approach to continuous-time signal processing can be viewed as the cascade of three operations, as indicated in



Discrete-time processing of continuous-time signals.

where $x_c(t)$ and $y_c(t)$ are continuous-time signals and $x_d[n]$ and $y_d[n]$ are the discrete-time signals corresponding to $x_c(t)$ and $y_c(t)$.

پایه نظری روش‌های پردازش زمان‌گسته سیگنال‌های زمان‌پیوسته

The theoretical basis for converting a continuous-time signal to a discrete-time signal and reconstructing a continuous-time signal from its discrete-time representation lies in the sampling theorem.

Through the process of periodic sampling with the sampling frequency consistent with the conditions of the sampling theorem, the continuous-time signal $x_c(t)$ is exactly represented by a sequence of instantaneous sample values $x_c(nT)$; that is, the discrete-time sequence $x_d[n]$ is related to $x_c(t)$ by

$$x_d[n] = x_c(nT).$$

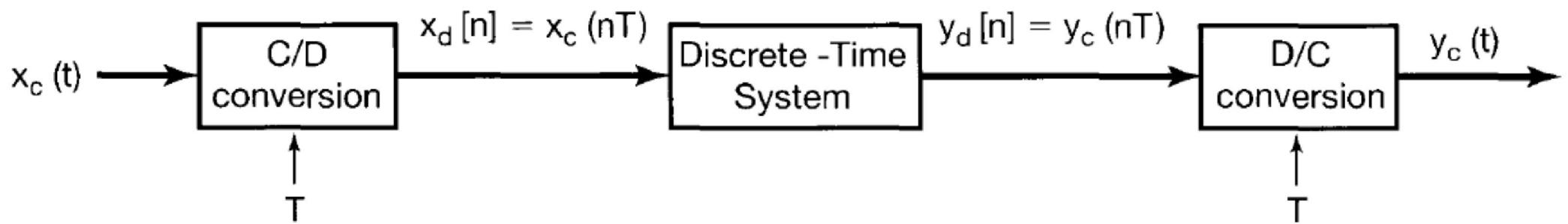
C/D مبدل

The transformation of $x_c(t)$ to $x_d[n]$ corresponding to the first system will be referred to as *continuous-to-discrete-time conversion* and will be abbreviated C/D.

D/C مبدل

The reverse operation corresponding to the third system will be abbreviated D/C, representing *discrete-time to continuous-time conversion*.

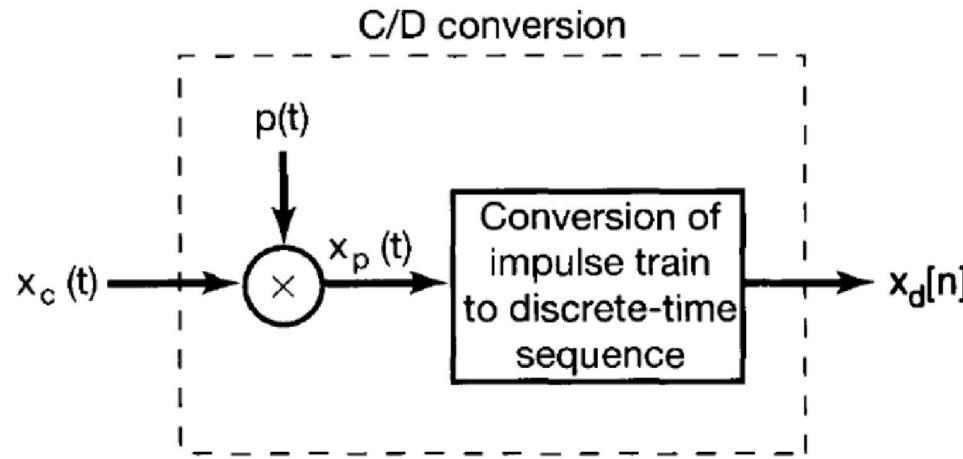
The D/C operation performs an interpolation between the sample values provided to it as input. That is, the D/C operation produces a continuous-time signal $y_c(t)$ which is related to the discrete-time signal $y_d[n]$ by $y_d[n] = y_c(nT)$.



مبدل‌های D/A و A/D

In systems such as digital computers and digital systems for which the discrete-time signal is represented in digital form, the device commonly used to implement the C/D conversion is referred to as an *analog-to-digital* (A- to-D) converter, and the device used to implement the D/C conversion is referred to as a *digital-to-analog* (D-to-A) converter.

ساختار دقیق مبدل C/D



$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT).$$

$$\xrightarrow{\hspace{1cm}} x_p(t) = \sum_{n=-\infty}^{+\infty} x_c(nT) \delta(t - nT),$$

and since the transform of $\delta(t - nT)$ is $e^{-j\omega nT}$, it follows that

$$\underline{X_p(j\omega) = \sum_{n=-\infty}^{+\infty} x_c(nT) e^{-j\omega nT}.}$$

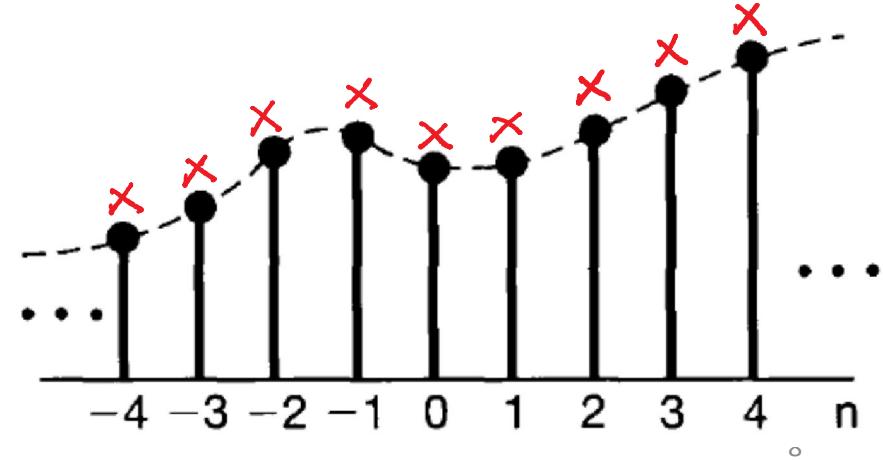
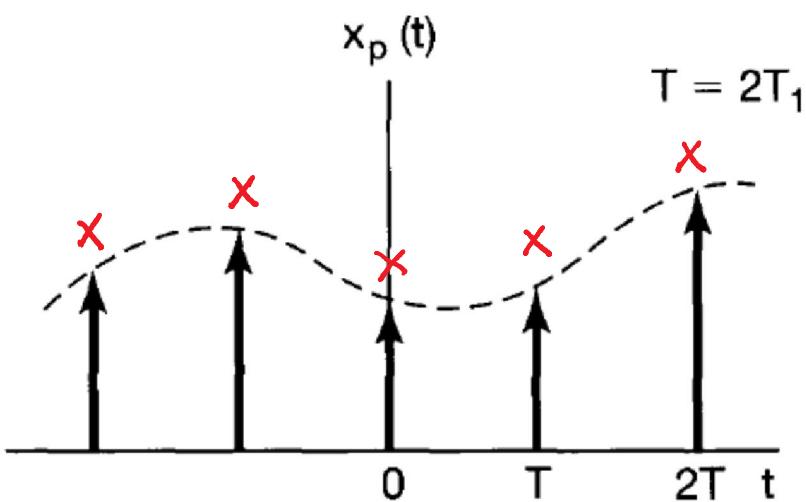
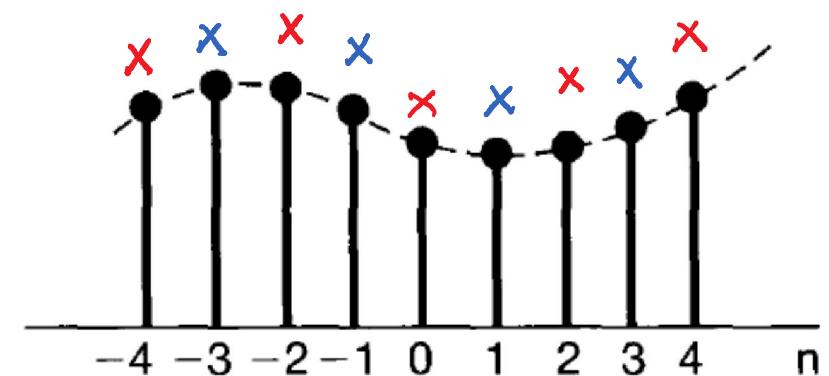
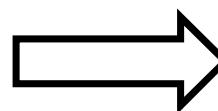
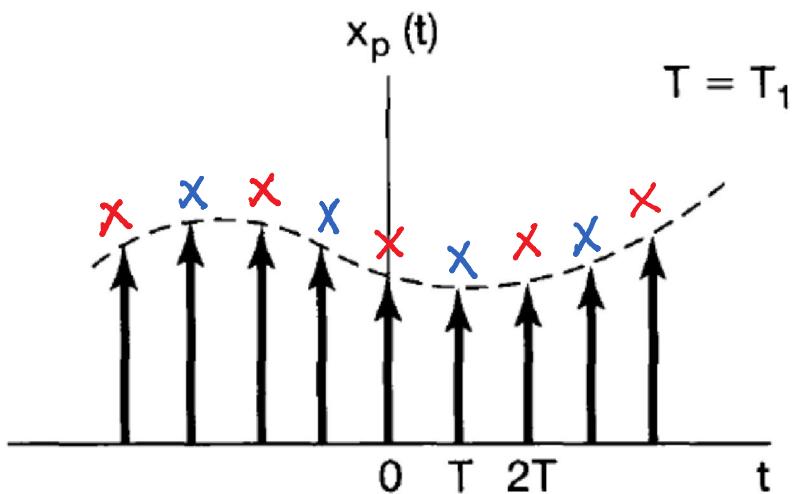
$$\underline{x_d[n] = x_c(nT)}.$$

we distinguish the continuous-time and discrete-time frequency variables by using ω in continuous time and Ω in discrete time.

$$\underline{X_d(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} x_d[n] e^{-j\Omega n}},$$

$$\begin{array}{c} \text{rad/sec} \quad \downarrow \quad \text{sec} \quad \downarrow \quad \text{rad} \\ \omega T = \Omega \Leftrightarrow \omega = \frac{\Omega}{T} \end{array}$$

نمونه برداری با روند مسافت و دنباله مساحت آن



The continuous-time Fourier transforms of $x_c(t)$ and $y_c(t)$ are $X_c(j\omega)$ and $Y_c(j\omega)$, respectively,

while the discrete-time Fourier transforms of $x_d[n]$ and $y_d[n]$ are $X_d(e^{j\Omega})$ and $Y_d(e^{j\Omega})$,

$$X_p(j\omega) = \sum_{n=-\infty}^{+\infty} x_c(nT)e^{-j\omega nT}. \quad \text{and} \quad X_d(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} x_d[n]e^{-j\Omega n},$$

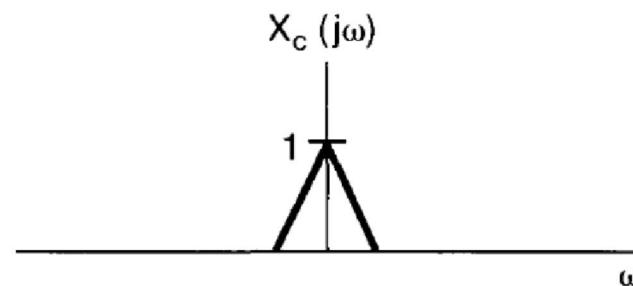
$$\longrightarrow X_d(e^{j\Omega}) = X_p(j\Omega/T).$$

$$x_p(t) = x_c(t) \cdot p(t) \quad \longrightarrow \quad X_p(j\omega) = X_c(j\omega) \cdot P(j\omega)$$

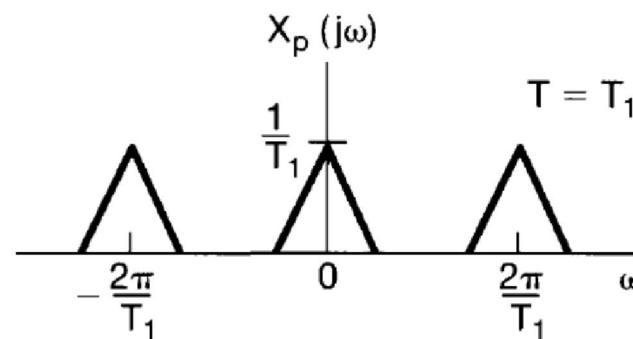
$$\longrightarrow X_p(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c(j(\omega - k\omega_s)). \quad \longrightarrow \quad X_d(e^{j\Omega}) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c(j(\Omega - 2\pi k)/T).$$

The relationship among $X_c(j\omega)$, $X_p(j\omega)$, and $X_d(e^{j\Omega})$ is illustrated for two different sampling rates.

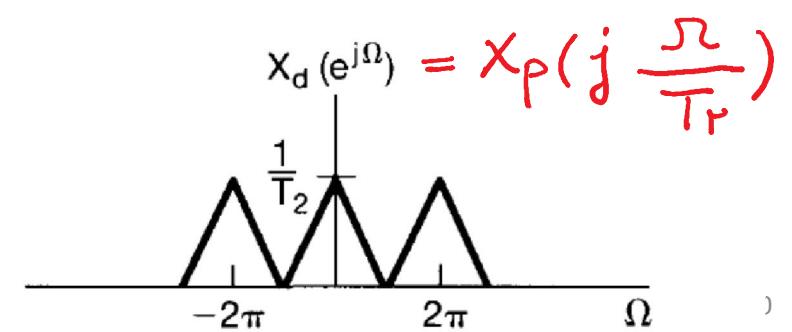
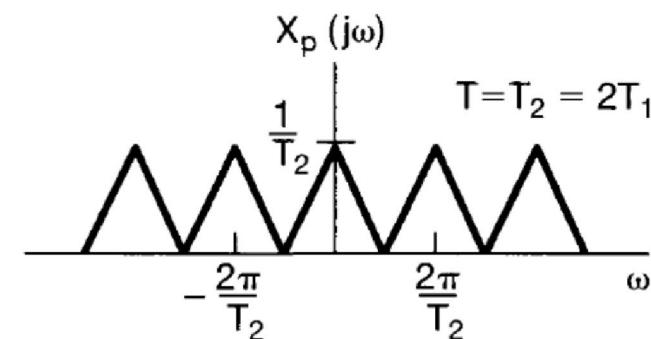
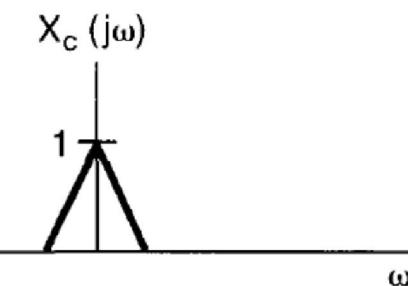
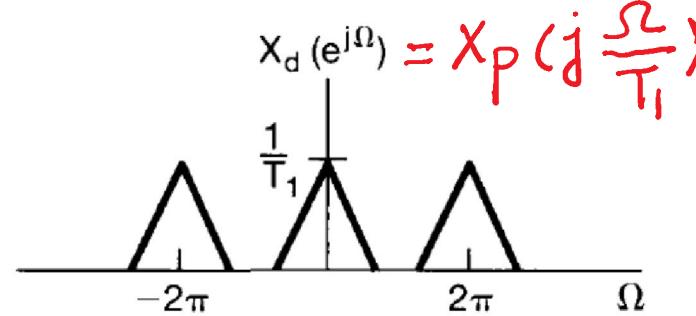
طیف فرکانسی سیگنال
زمان پیوسته اصلی



طیف فرکانسی سیگنال
نمونه برداری شده

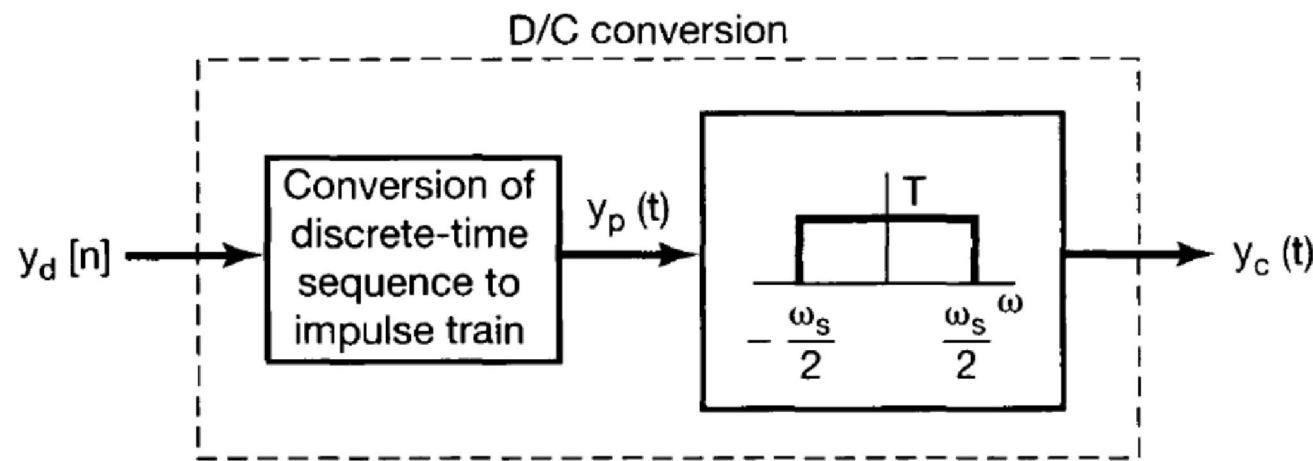


طیف فرکانسی دنباله
متناظر با سیگنال
پیوسته اصلی

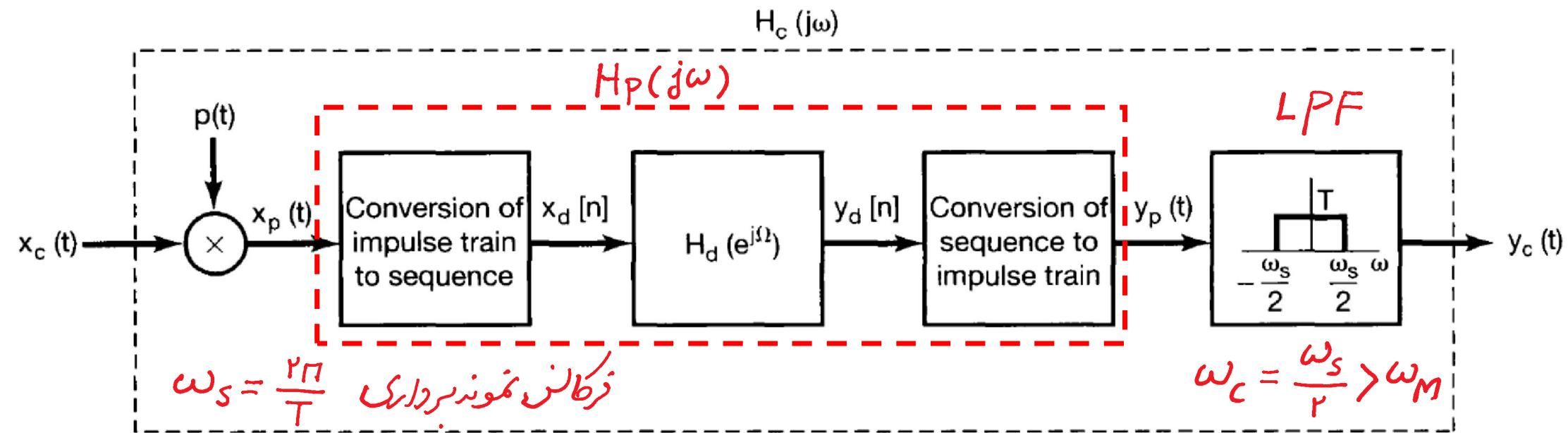


ساختار دقیق مبدل D/C

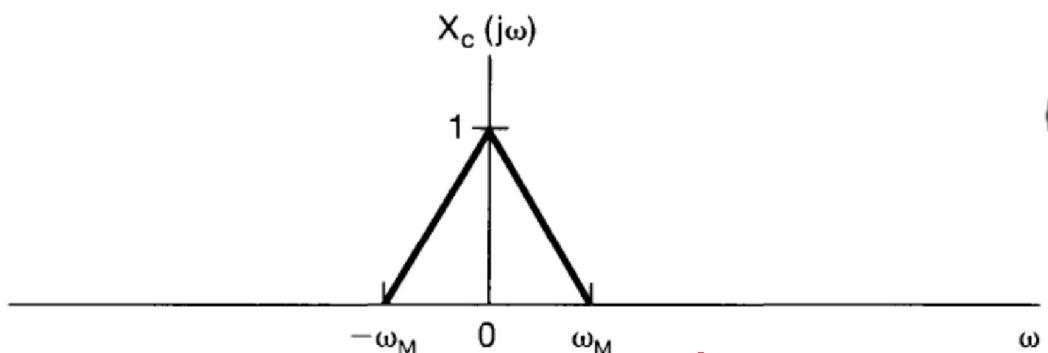
After processing with a discrete-time system, the resulting sequence is converted back to a continuous-time signal. Specifically, from the sequence $y_d[n]$, a continuous-time impulse train $y_p(t)$ can be generated. Recovery of the continuous-time signal $y_c(t)$ from this impulse train is then accomplished by means of lowpass filtering.



ساختار دقیق روش فیلترینگ یک سیگنال زمان پیوسته با استفاده از فیلتر زمان گستته



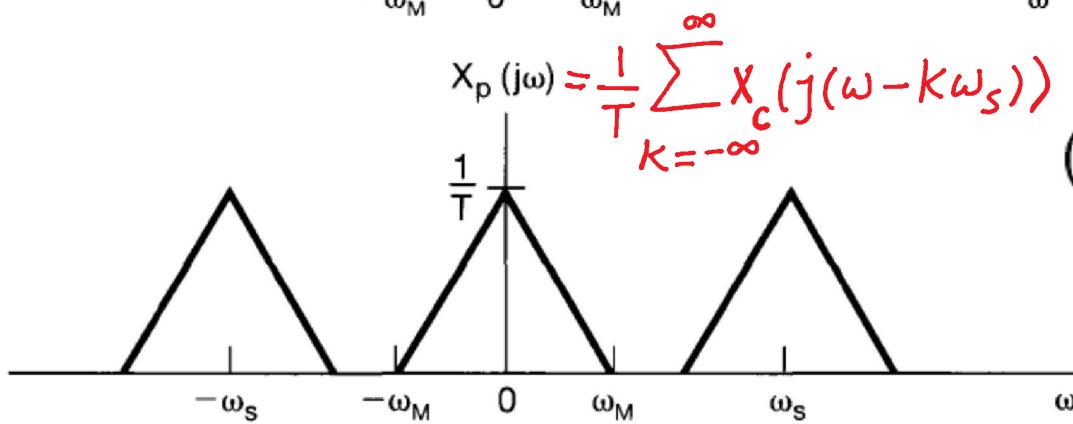
The characteristics of the overall system with a frequency response $H_d(e^{j\Omega})$ are perhaps best understood by examining the representative example depicted in the next figures.



(a) continuous-time spectrum $X_c(j\omega)$;

طیف فرکانسی سیگنال پوسته اصلی

ω_M بازه‌ای باند محدود $x_c(t)$

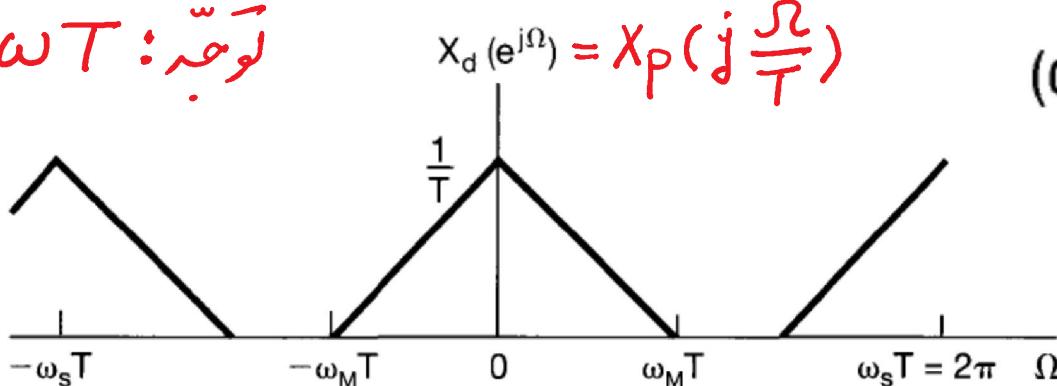


(b) spectrum after impulse-train sampling;

طیف فرکانسی سیگنال مخونه برداری شده

$\omega_s = \frac{2\pi}{T} > 2\omega_M$ بارهای $x_p(t)$

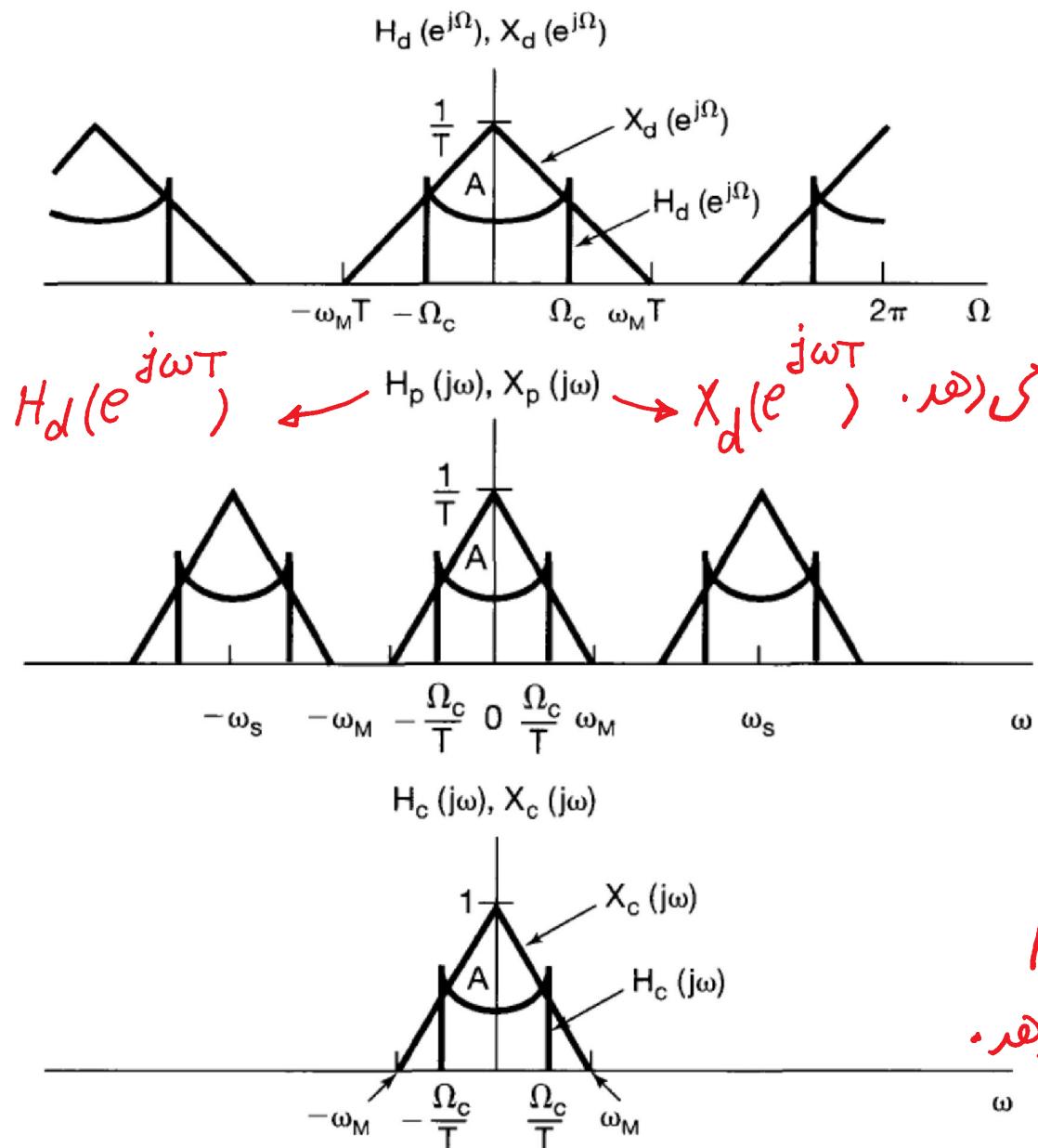
$\Omega = \omega T$: توجه:



(c) spectrum of discrete-time sequence $x_d[n]$;

طیف فرکانسی سیگنال زمان گستاخ

$X_d(e^{j\Omega}) = X_p(j\frac{\Omega}{T})$



(d) $H_d(e^{j\Omega})$ and $X_d(e^{j\Omega})$ that are multiplied to form $Y_d(e^{j\Omega})$;

پاسخ فرکانسی فیلتر گستاخ

$H_d(e^{j\omega T})$ که در (ر) $X_d(e^{j\omega T})$ ضرب شده و

(e) spectra that are multiplied to form $Y_p(j\omega)$;

پاسخ فرکانسی فیلتر پوسته معاوی

$H_p(j\omega)$ که در (ر) $X_p(j\omega)$ ضرب شده و

حیره.

(f) spectra that are multiplied to form $Y_c(j\omega)$.

پاسخ فرکانسی فیلتر پوسته معاوی

$H_c(j\omega)$ که در (ر) $X_c(j\omega)$ ضرب شده و

$$y_c(j\omega) = y_p(j\omega) \cdot H_{LP}(j\omega) \quad : \text{حلصه روابط}$$

$$= \begin{cases} y_p(j\omega) \cdot T & , |\omega| < \frac{\omega_s}{r} \\ 0 & , \text{oth} \end{cases}$$

$$\Rightarrow \text{for } |\omega| < \frac{\omega_s}{r} : y_c(j\omega) = y_p(j\omega) \cdot T = y_d(e^{j\omega T}) \cdot T$$

$$= X_d(e^{j\omega T}) \cdot H_d(e^{j\omega T}) \cdot T = X_p(j\frac{\omega T}{T}) \cdot H_d(e^{j\omega T}) \cdot T$$

$$= \frac{1}{T} X_c(j\omega) \cdot H_d(e^{j\omega T}) \cdot T \Rightarrow \underline{Y_c(j\omega) = X_c(j\omega)H_d(e^{j\omega T})}$$

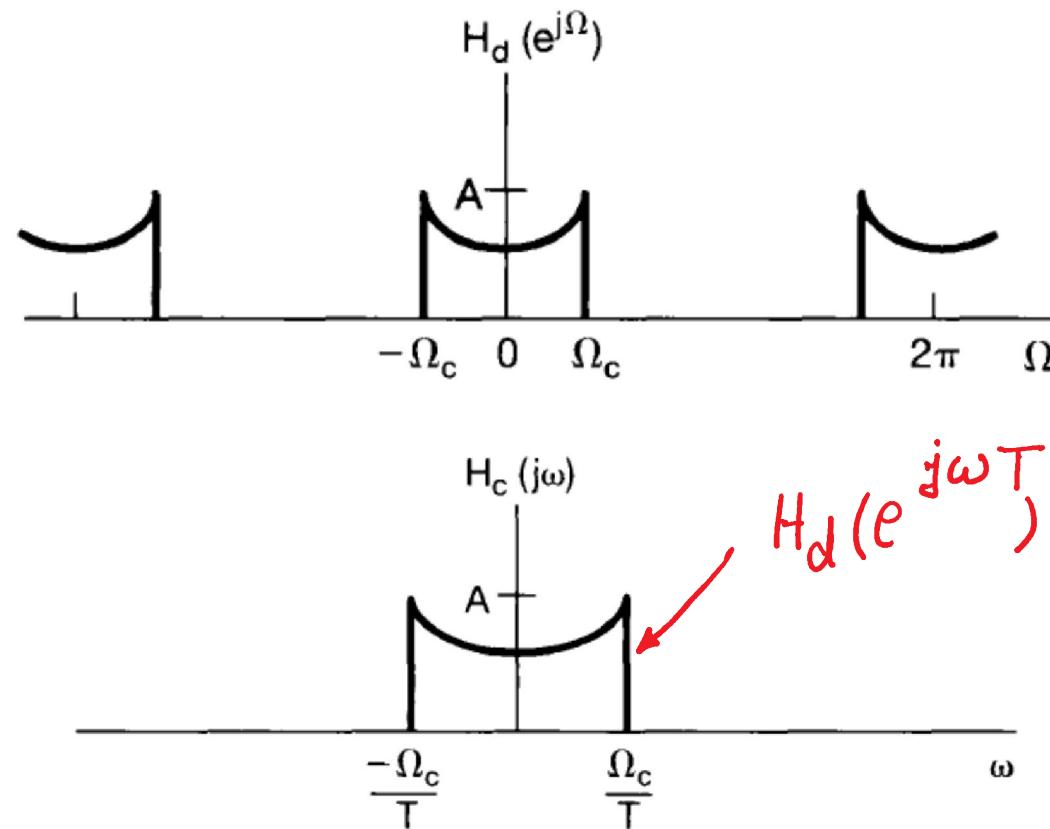
Consequently, for inputs that are sufficiently band limited, so that the sampling theorem is satisfied, the overall system is equivalent to a continuous-time LTI system with frequency response $H_c(j\omega)$ which is related to the discrete-time frequency response $H_d(e^{j\Omega})$ through

$$H_c(j\omega) = \begin{cases} H_d(e^{j\omega T}), & |\omega| < \omega_s/2 \\ 0, & |\omega| > \omega_s/2 \end{cases}$$

The equivalent frequency response for this continuous-time filter is one period of the frequency response of the discrete-time filter with a linear scale change applied to the frequency axis. This relationship between the discrete-time frequency response and the equivalent continuous-time frequency response is illustrated in the next figures.

پاسخ فرط لنسی برای فیلتر زمان پویه متعارف، یک دوره ناکر تغیر معکوس یافته از پاسخ فرط لنسی

فیلتر زمان گسته است.



Discrete-time frequency response and the equivalent continuous-time frequency response

نکته: درین حالت خاص، اگر $-\pi < \Omega < \pi$ ، فیلتر زمان‌کسته کاملاً لذراست و در نتیجه: $y_d[n] = x_d[n]$ (سیم‌هایانی کسته) در این صورت ($y_c(t) = x_c(t)$ و $y_p(t) = x_p(t)$) سیم‌هایانی پویتہ) به محابارت داشتی:

$$H_C(j\omega) = \begin{cases} 1 & , |\omega| < \frac{\omega_s}{\tau} \\ 0 & , \text{oth} \end{cases}, \quad \frac{\omega_s}{\tau} > \omega_M$$

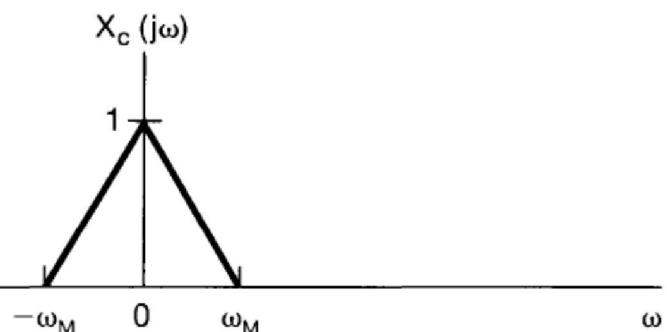
$$\Rightarrow y_c(j\omega) = X_c(j\omega) \cdot H_C(j\omega) = X_c(j\omega)$$

که پناکی باند محدود (ω_M)

Digital Differentiator

مثال ۱: فیلتر مشتق‌گیر دیجیتال با پهنازی باند محدود

Consider the discrete-time implementation of a continuous-time band-limited differentiating filter.



$$\begin{aligned} y_c(t) &= \frac{d}{dt} x_c(t) \\ \Rightarrow y_c(j\omega) &= j\omega \cdot X_c(j\omega) \\ \Rightarrow H_c(j\omega) &= j\omega \end{aligned}$$

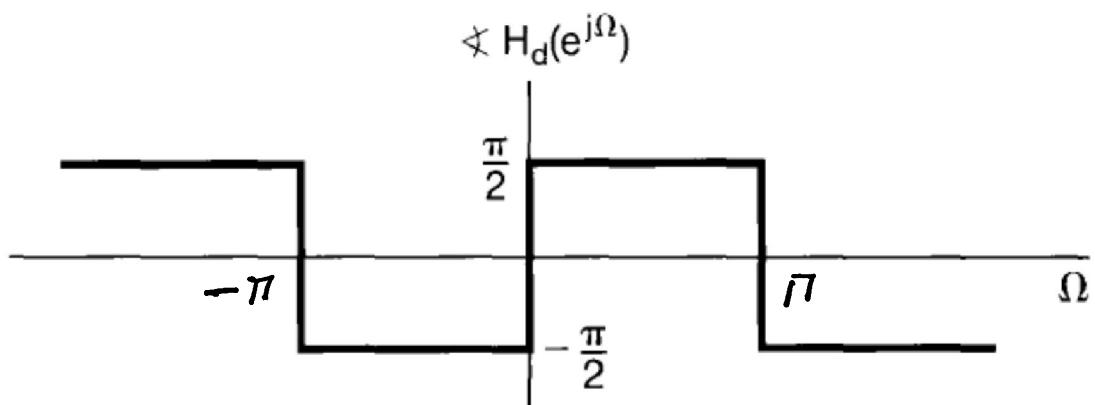
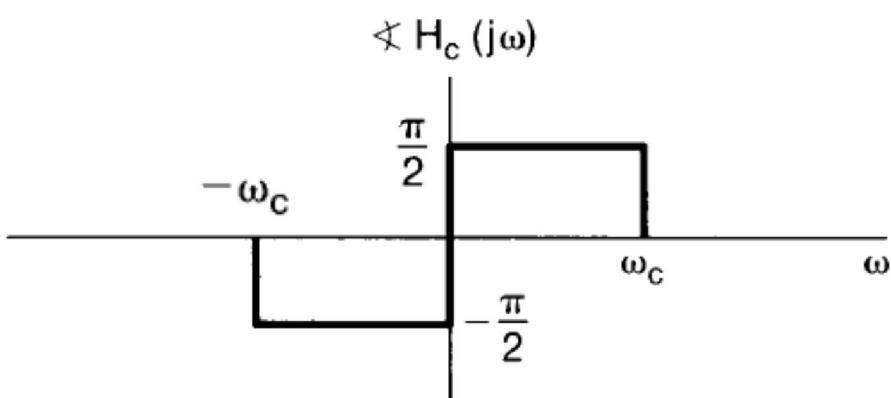
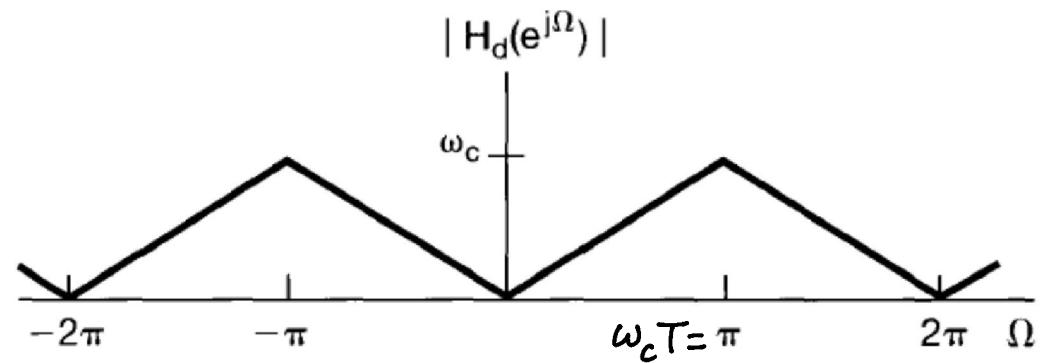
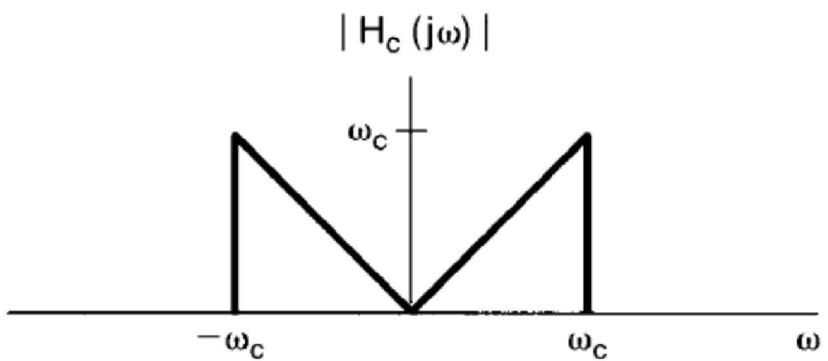
and that of a band-limited differentiator with cutoff frequency ω_c is

$$H_c(j\omega) = \begin{cases} j\omega, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases}, \quad \omega_s = 2\omega_c > 2\omega_M$$

with a sampling frequency $\omega_s = 2\omega_c$, we see that the corresponding discrete-time transfer function is

$$H_d(e^{j\Omega}) = j \left(\frac{\Omega}{T} \right), \quad |\Omega| < \pi,$$

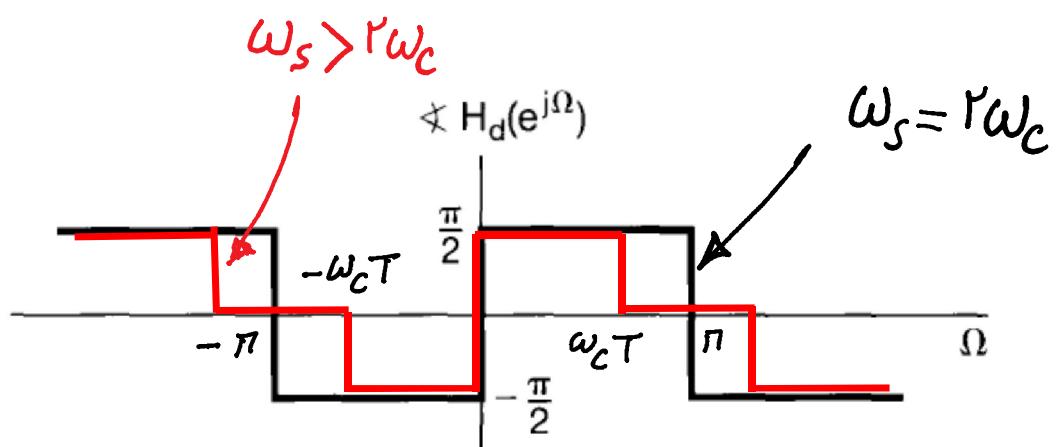
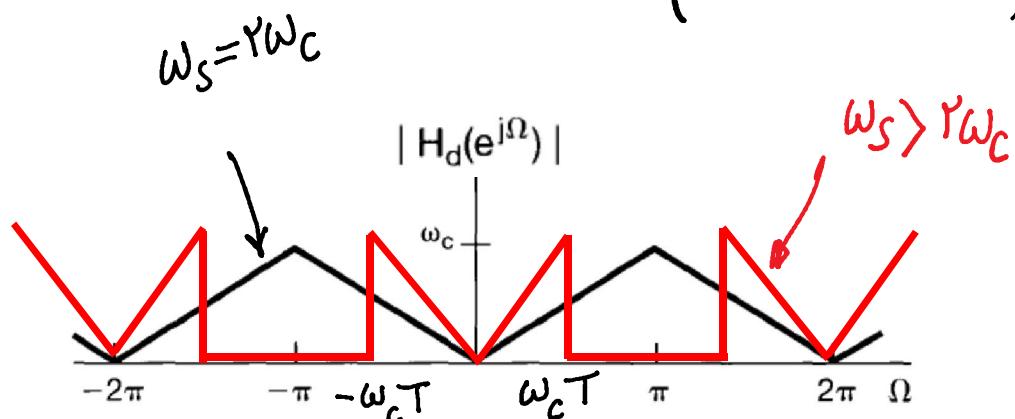
$$\omega_c = \frac{1}{r} \omega_s \Rightarrow \omega_c T = \frac{1}{r} \omega_s \times T = \frac{\pi}{T} \times T = \pi$$



نتیجه: اگر فرکانس نمونه برداری بینه از دو برابر فرکانس قطع فیلتر مستقیم (بجای میله) باشد،

$$\omega_s > 2\omega_c \Rightarrow \omega_c < \frac{\omega_s}{2} \Rightarrow \omega_c T < \frac{\omega_s}{2} \times T = \frac{\pi}{T} \times T = \pi$$

$$\Rightarrow H_d(e^{j\Omega}) = \begin{cases} j\frac{\Omega}{T}, & |\Omega| < \omega_c T \\ 0, & \omega_c T < |\Omega| < \pi \end{cases}$$



مثال ۲: تأخیردهنده دیجیتال با پهنای باند محدود

we consider the implementation of a time shift (delay) of a continuous-time signal through the use of a discrete-time processing system.

Thus, we require that the input and output of the overall system be related by

$$\underline{y_c(t) = x_c(t + \Delta)} \quad \rightarrow \omega_s = \frac{2\pi}{T} > 2\omega_M \text{ و در } \omega_M \text{ پهنای باند محدود دارد} \quad \underline{x_c(t)}$$

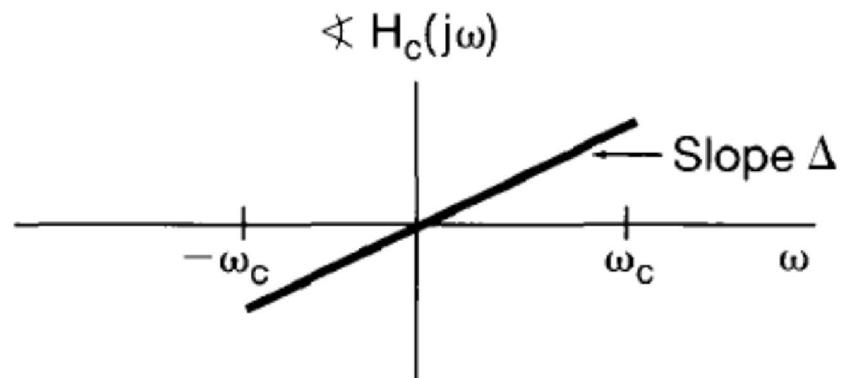
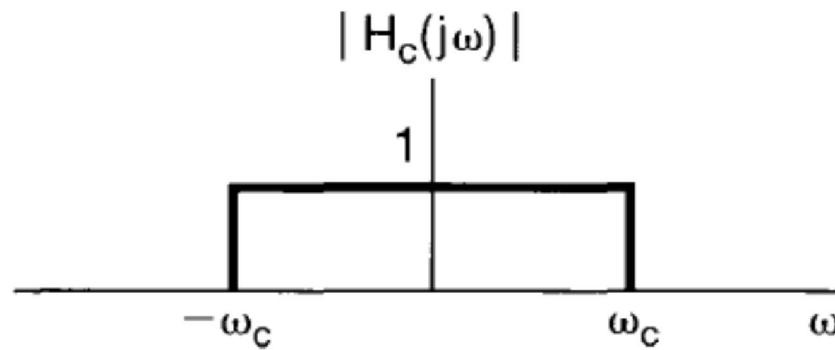
$\Rightarrow \underline{Y_c(j\omega) = e^{+j\omega\Delta}X_c(j\omega)}.$

the equivalent continuous-time system to be implemented must be band limited. Therefore,

we take

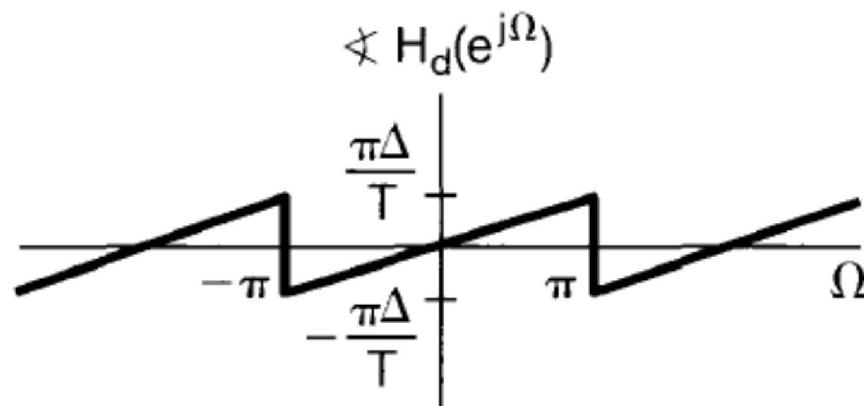
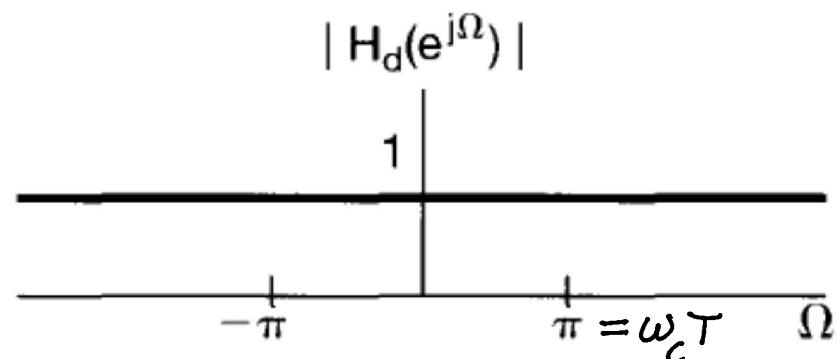
$$H_c(j\omega) = \begin{cases} e^{+j\omega\Delta}, & |\omega| < \omega_c \\ 0, & \text{otherwise} \end{cases}$$

ω_c فرکانس قطع سیم زمان پوسته



Magnitude and phase of the frequency response for a continuous-time delay;

With the sampling frequency ω_s taken as $\omega_s = 2\omega_c$, the corresponding discrete-time frequency response is $H_d(e^{j\Omega}) = e^{-j\Omega\Delta/T}$, $|\Omega| < \pi$,



magnitude and phase of the frequency response for the corresponding discrete-time delay.

For appropriately band-limited inputs, the output of the system with

$$\underline{H_d(e^{j\Omega}) = e^{-j\Omega\Delta/T}, \quad |\Omega| < \pi},$$

is a delayed replica of the input.

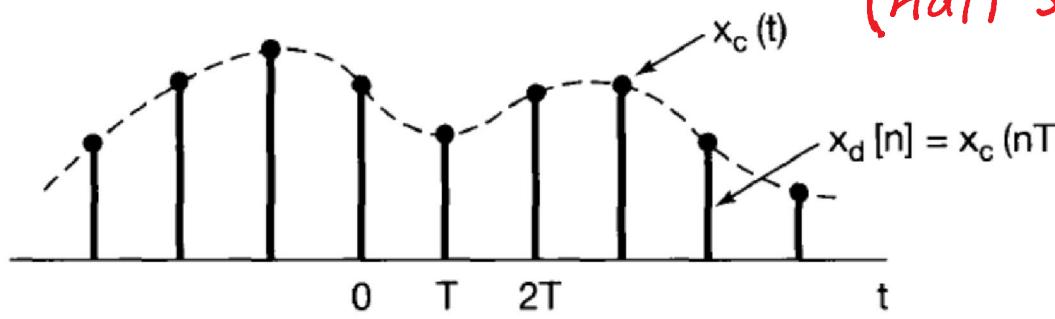
بررسی دو حالت : $\frac{\Delta}{T} \notin \mathbb{Z}$ ، $\frac{\Delta}{T} \in \mathbb{Z}$

I) For Δ/T an integer, the sequence $y_d[n]$ is a delayed replica of $x_d[n]$; that is,

$$\underline{y_d[n] = x_d\left[n + \frac{\Delta}{T}\right]}.$$

II) For Δ/T not an integer, the above equation, has no meaning, since sequences are defined only at integer values of the index. However, we can interpret the relationship between $x_d[n]$ and $y_d[n]$ in these cases in terms of band-limited interpolation.

The signals $x_c(t)$ and $x_d[n]$ are related through sampling and band-limited interpolation, as are $y_c(t)$ and $y_d[n]$. $y_d[n]$ is equal to samples of a shifted version of the band-limited interpolation of the sequence $x_d[n]$.

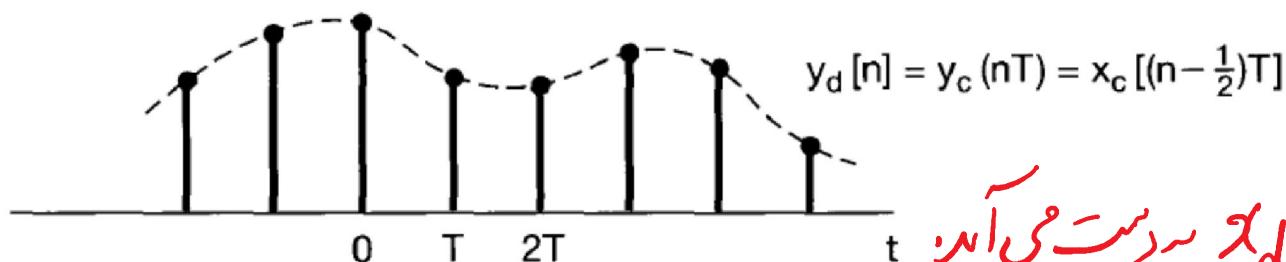


فرض نسخه: $\Delta/T = -\frac{1}{2}$

در اینجا $y_d[n]$ عبارت از مونهاي

یک سیگنال پوششی $x_c(t)$ که سمعت یافته است

است و خود $x_c(t)$ از طریق



در واقع یک پهنای باند محدود از روی $x_d[n]$ بدست می‌آید

با ساس از توجه و صوری شما

در ناه از د منان، موفق و سر بلند باشد