EQUILIBRIUM OF COPLANAR CONCURRENT AND NON CONCURRENT FORCE SYSTEMS

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

LECTURE 5

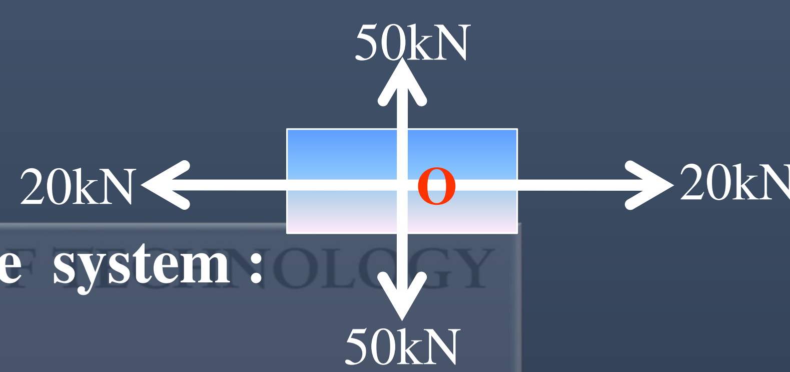
- Equilibrium of a particle
- Conditions for Equilibrium
- Space Diagram & Free Body Diagram (FBD)
- Application problems

EQUILIBRIUM OF FORCE SYSTEMS

Equilibrium of a particle :

If the net effect of the given forces is zero, the particle is said to be in equilibrium.

A body is said to be in equilibrium when it does not have any translatory or rotatory motion in any direction.



Equilibrium of concurrent coplanar force system :

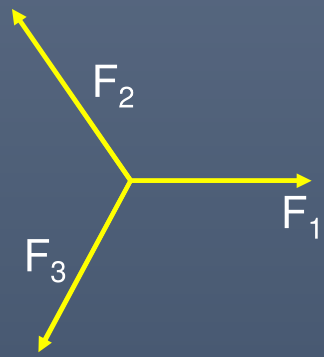
Equilibrium conditions:

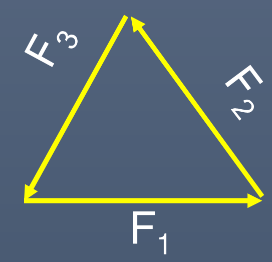
The algebraic sum of the components of the forces along each of three mutually perpendicular axes is zero.

In case of coplanar forces, $\Sigma F_X = 0$; $\Sigma F_Y = 0$

Triangle Law:

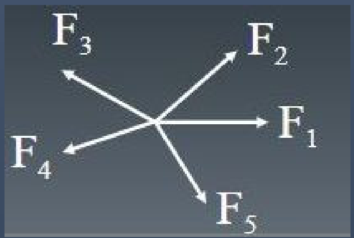
If three forces are in equilibrium, then, they form a closed triangle when represented in a Tip to Tail arrangement.

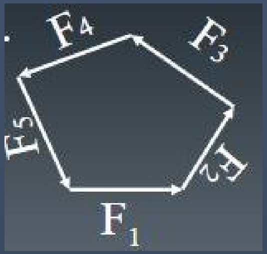




Polygonal Law:

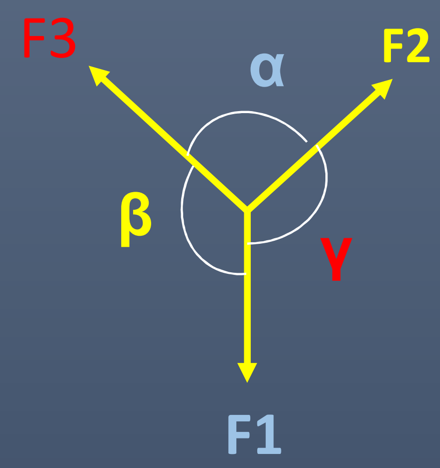
If more than three forces are in equilibrium, then, they form a closed polygon when represented in a Tip to Tail arrangement





Lami's theorem:

If a system of three "concurrent" forces is in equilibrium, then, each force of the system is proportional to sine of the angle between the other two forces (and constant of proportionality is the same for all the forces).



Note: While using Lami's theorem, all the three forces should be either directed away or all directed towards the point of concurrence.

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

Equilibrium of Non-concurrent Coplanar force system

When a body is in equilibrium, it has neither translatory nor rotatory motion in any direction.

Thus the resultant force R and the resultant couple M are both zero, and we have the **equilibrium equations** for two dimensional force system as

$$\sum F_x = 0; \qquad \sum F_y = 0 \quad \& \quad \sum M = 0$$

These requirements are both necessary and sufficient conditions for equilibrium.

Space and Free body diagrams

Space diagram :

The sketch showing the physical conditions of the problem, like, the nature of supports provided; size, shape and location of various bodies; forces applied on bodies, etc.,

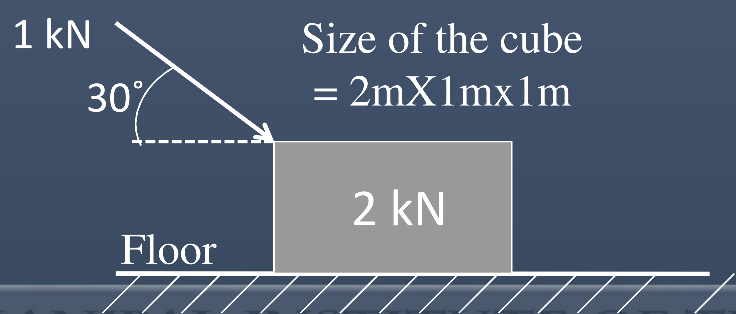
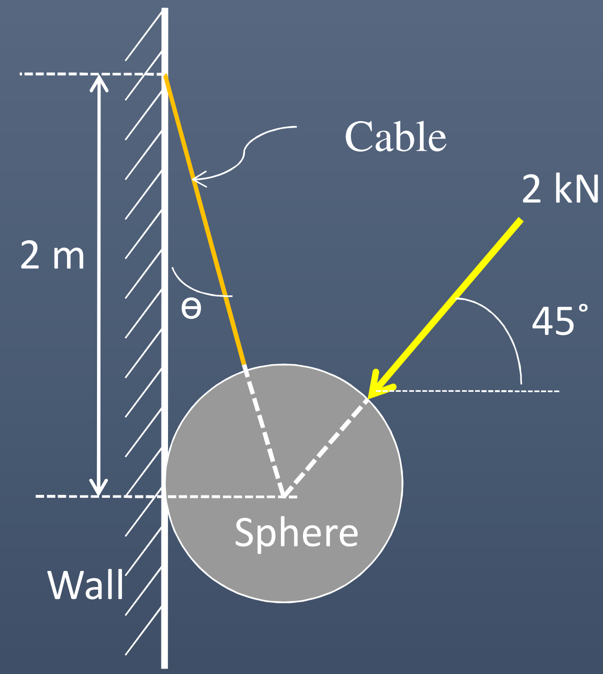


Figure 1



Wt. of the sphere = 1 kN

Radius of the sphere = 0.5 m

Figure 2

Free body diagram :

An isolated view of a body which shows only the external forces acting on the body

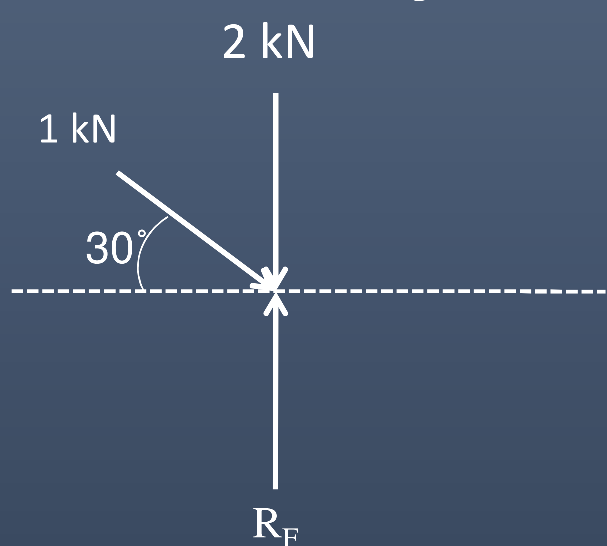


Figure 1

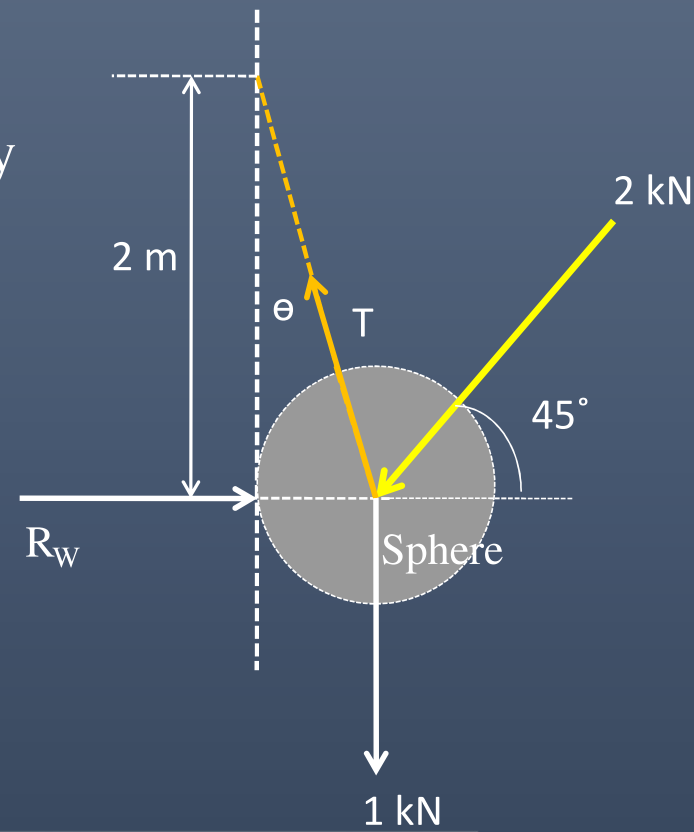


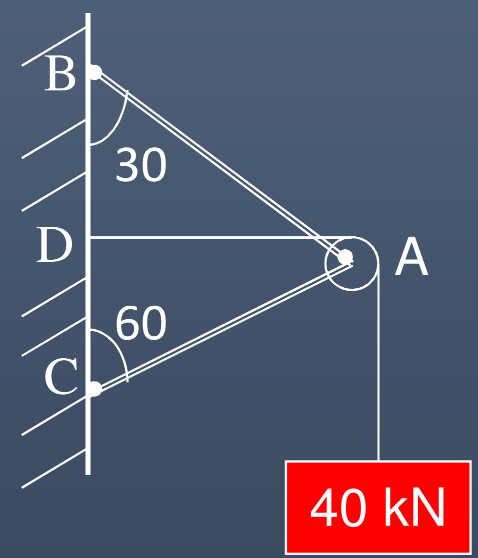
Figure 2

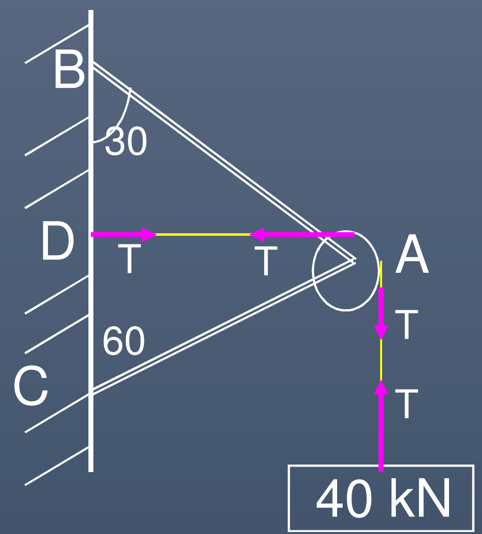
Few Guidelines for Drawing FBD:

1. Tensile Force – vector directed away from the body
2. Compressive Force - vector directed towards the body
3. Reactions at smooth surfaces - normal to the surface and pointing towards the body
4. Forces in Link rods/connecting rods – along the axis (towards or away from the body)
5. Forces in Cables (Strings or Chords) - along the cable and directed away from the body (tension)

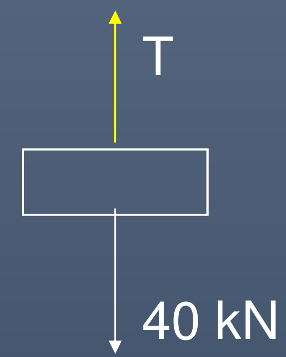
Application Problems

1. Determine the magnitude and nature of the forces in the bars AB and AC shown in figure. Neglect size and weight of the pulley.

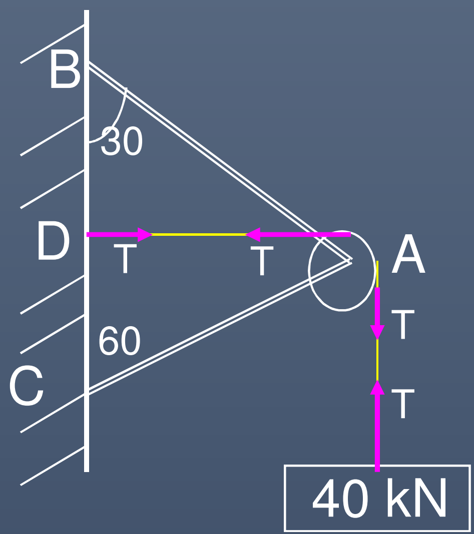


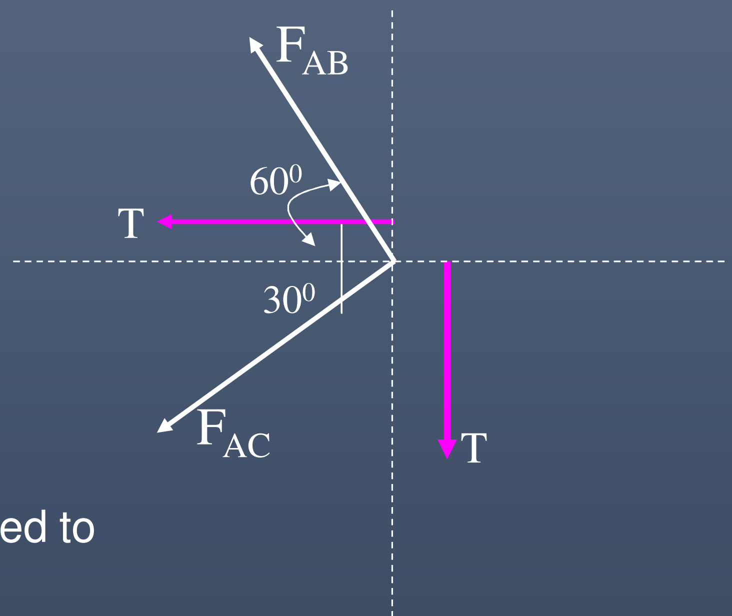


FBD of 40kN weight



If the pulley is frictionless then tension in the rope on either side of it is same





The AB and AC may be subjected to either tension or compression

Hence initially assume one direction

Angle between F_{AB} and F_{AC} = 90deg.

The joint A is under equilibrium and hence sum of all forces acting at joint A is zero.

Taking F_{AC} as X-axis and F_{AB} as Y– axis

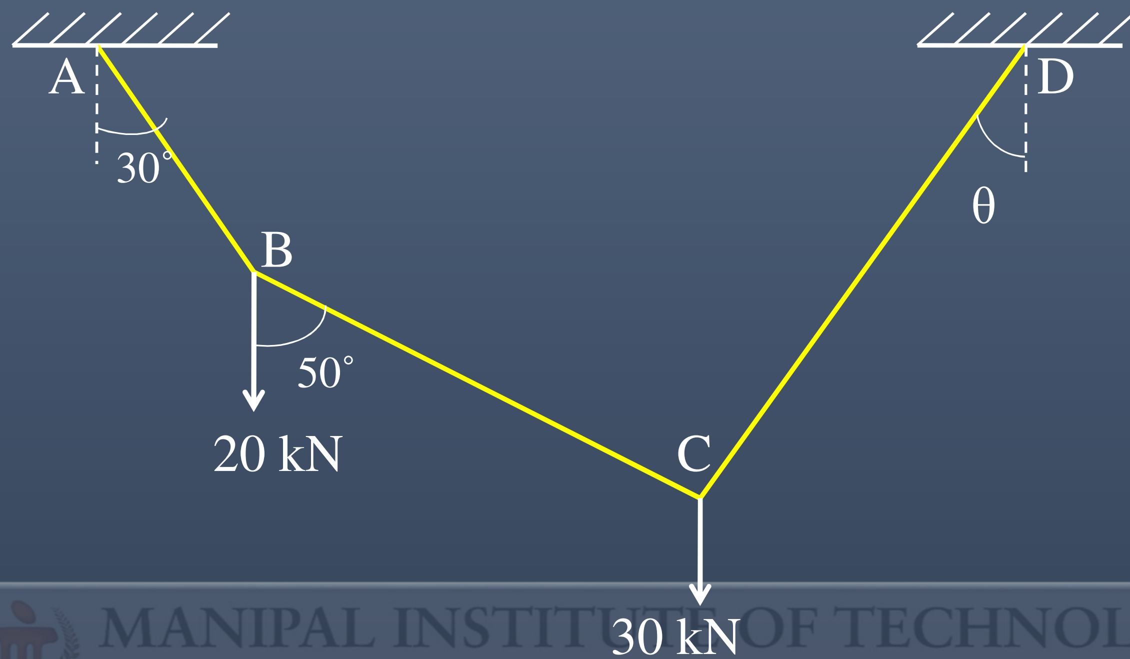
$$
\begin{aligned}
&From \overset{+}{\rightarrow} \sum F_X = 0,\\
&-F_{AC} - W_X - T_X = 0\\
&\therefore F_{AC} = -40Cos60 - 40Cos30 = -54.641kN
\end{aligned}
$$

F_{AC} is –ve , F_{AC} is towards 'A', So it is Compressive.

$$
\begin{aligned}
&From + \uparrow \sum Fy = 0,\\
&+F_{AB} + T_Y - W_Y = 0\\
&\therefore F_{AB} = 40Sin60 - 40Sin30 = +14.641kN
\end{aligned}
$$

F_{AB} is +ve. F_{AB} is away from 'A', So it is Tensile.

2. Determine the tension in the strings AB, BC, CD and inclination of the segment CD to the vertical, in the system shown.



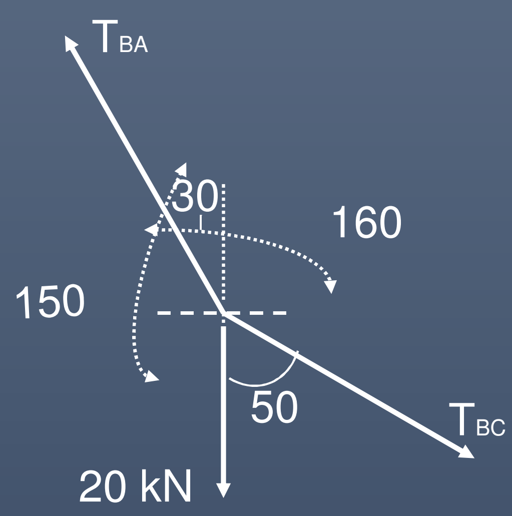


Fig Q2.4a FBD of Joint 'B'

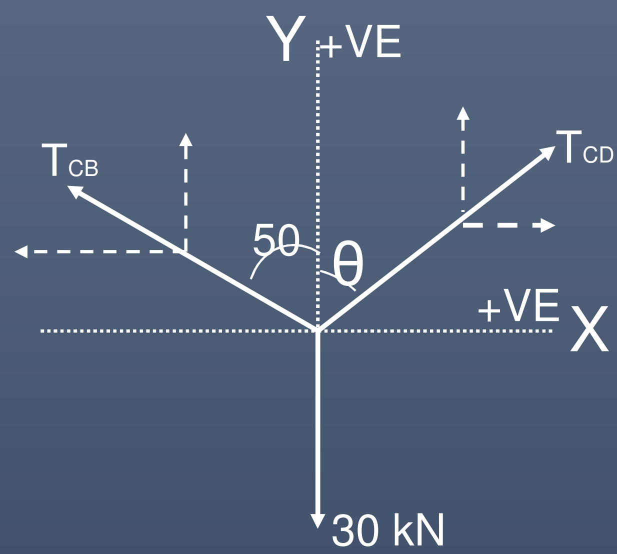


Fig Q2.4b FBD of Joint 'C'

Considering FBD of Joint 'C', We have,

$$From \sum F_X = 0, \xrightarrow{+}$$

$$T_{CD} Sin\theta = 29.24 Sin50$$ ------Eqn (1)

$$From \sum F_Y = 0, + \uparrow$$

$$T_{CD} Cos\theta = 30 - 29.24 Cos50$$ ------Eqn (2)

Dividing Eqn(1) by (2), we get,

$$\tan\theta = \frac{22.40}{11.20}$$

$$\theta = 63.42^o$$

$$\therefore T_{CD} = 25.05 kN$$

(<u>NOTE:</u> For this FBD, if we use Lami's Theorem,we have to expand Sin(50+θ) and solve for θ, which can take more time.)

LECTURE 6

- Types of Beam supports & Support reactions
- Types of Beams
- Types on Loads on beam

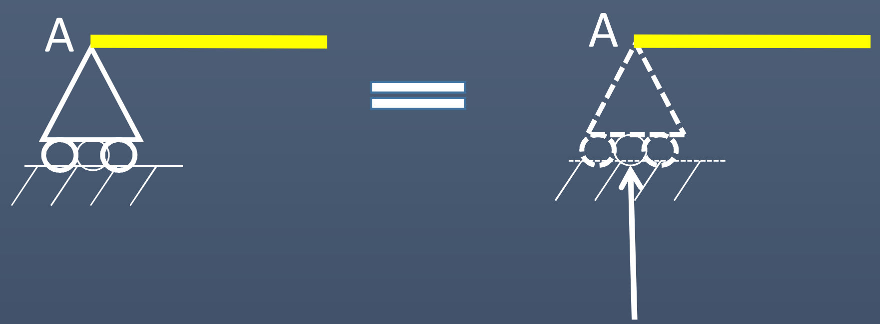
Types of Beam Supports & Support Reactions

Beam: A beam is a structural element which is capable of withstanding load primarily by resisting bending.

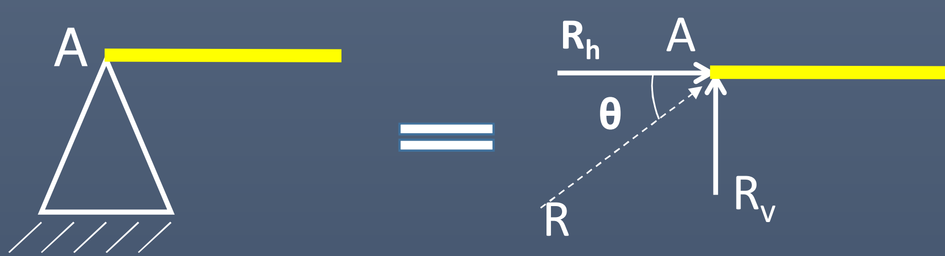
Supports: A structure is subjected to external forces and transfers these forces through the supports on to the foundation. Therefore the support reactions and the external forces together keep the structure in equilibrium.

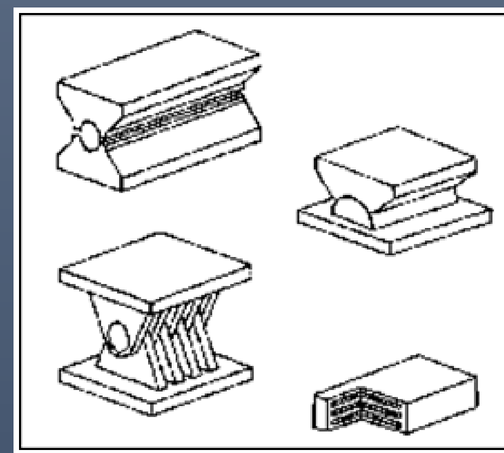
1. Roller Support
2. Hinged or pinned support
3. Fixed or built in support

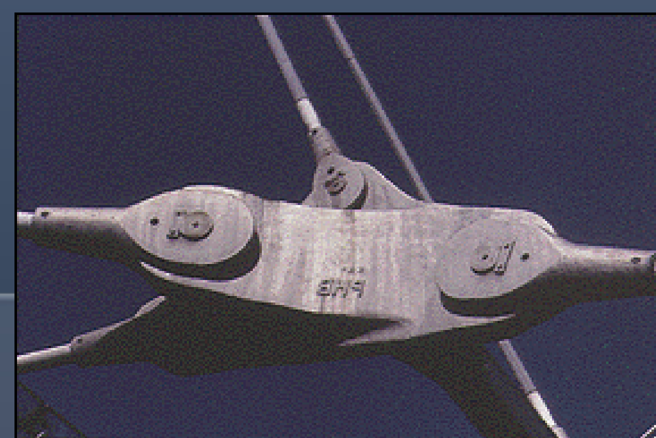
1. Roller Support

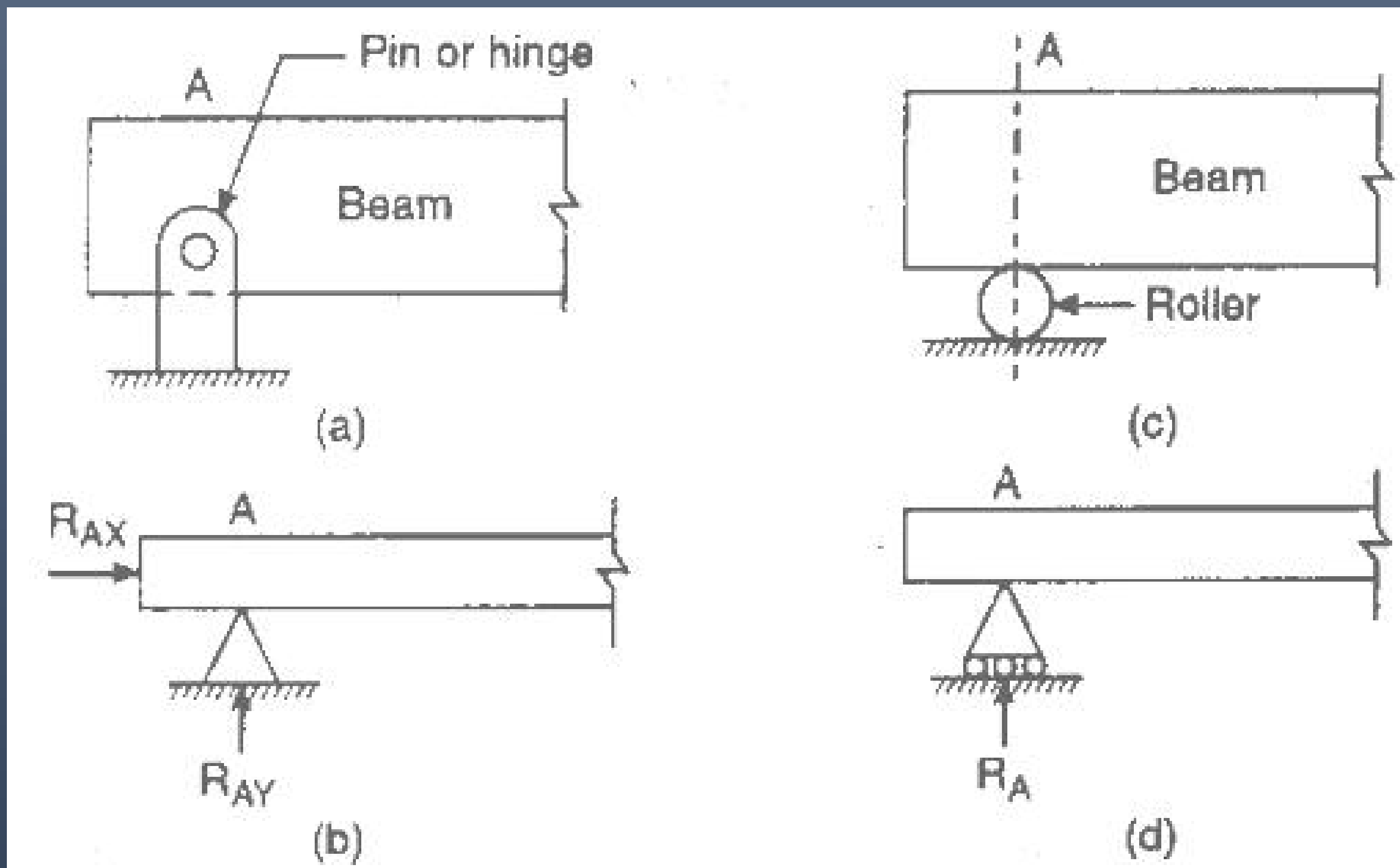


2. Hinged or pinned support

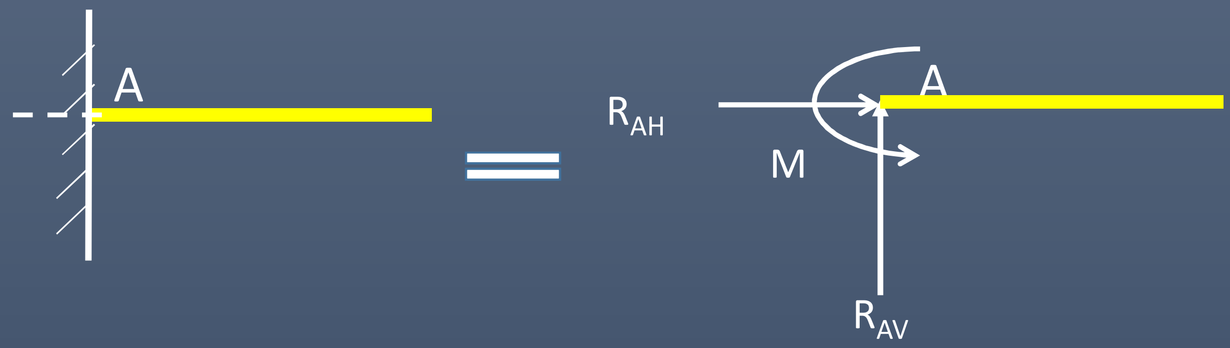


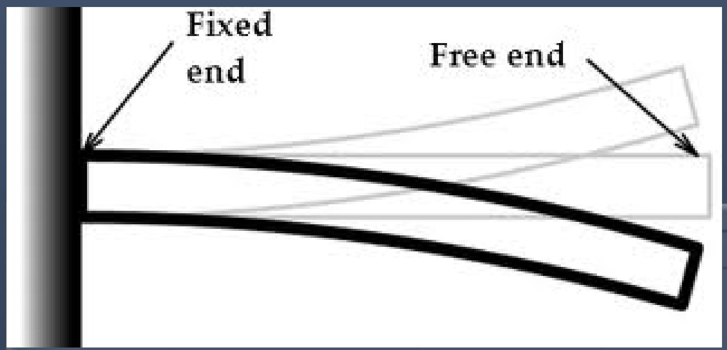





Pin or hinge
A
Beam
(a)
R_{AX}
A
R_{AY}
(b)
A
Beam
Roller
(c)
A
R_A
(d)

3. Fixed or Built-in Support

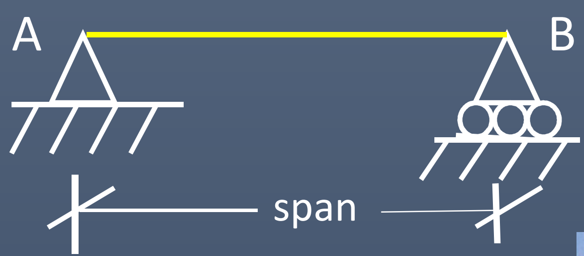


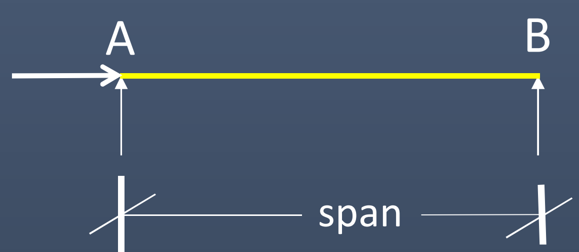


Types of Beams

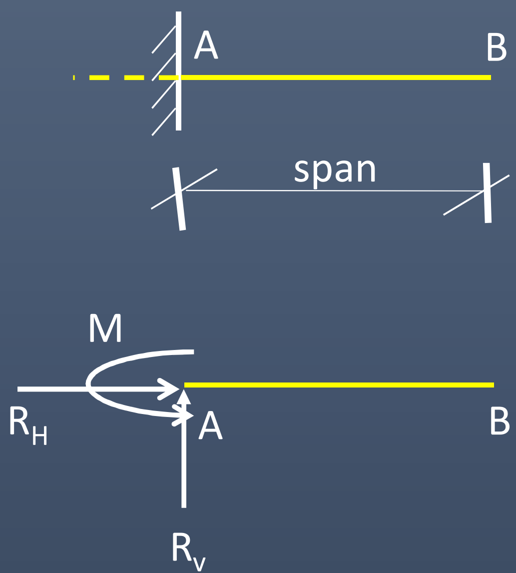
1. Simply supported beam
2. Cantilever beam
3. Overhanging beam

1. Simply Supported Beam

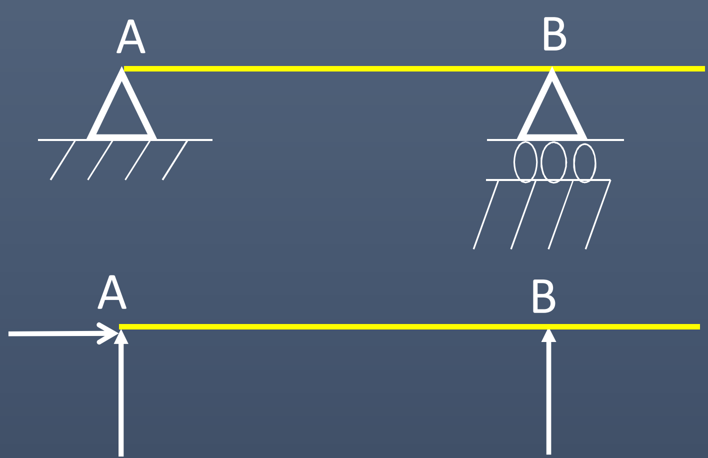




2. Cantilever Beam

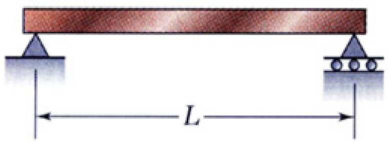


2. Overhanging Beam

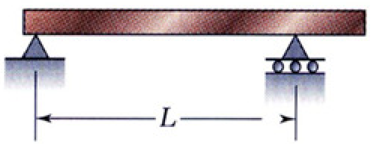


Statically Determinate & Indeterminate Beams

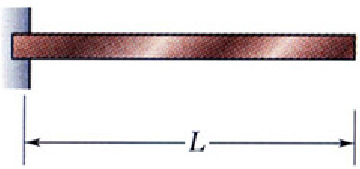
Statically Determinate



Simply Supported Beam

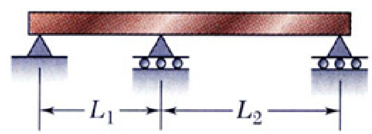


Overhanging Beam

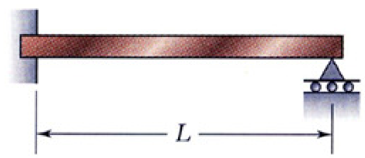


Cantilever Beam

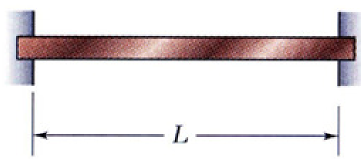
Statically Indeterminate



Continuous Beam



Propped Cantilever Beam

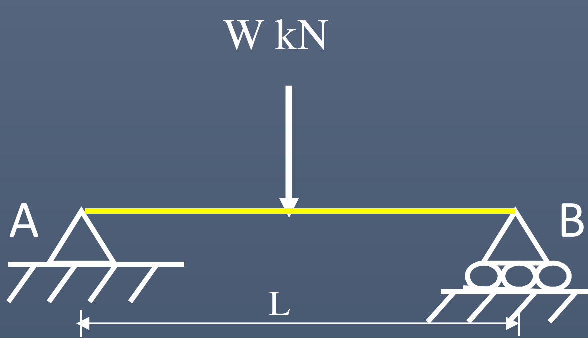


Fixed Beam

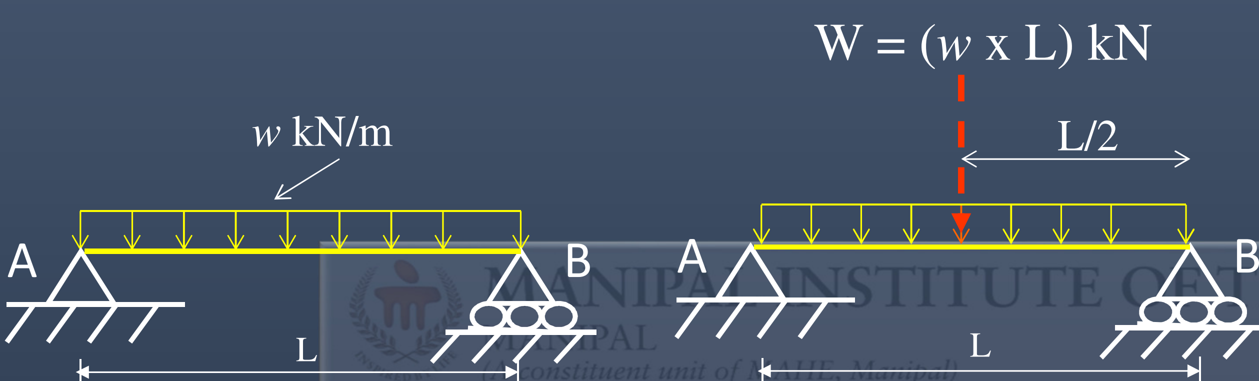
Loads on Beam

1. Concentrated or Point load
2. Uniformly distributed load (UDL)
3. Uniformly varying load (UVL)

1. Concentrated or Point Load

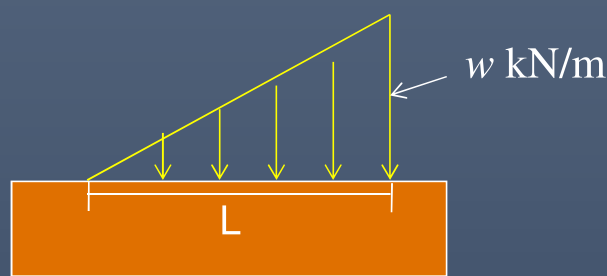


2. Uniformly Distributed Load (UDL)

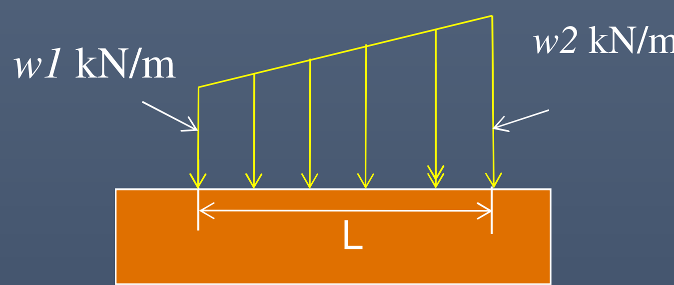


3. Uniformly Varying Load (UVL)

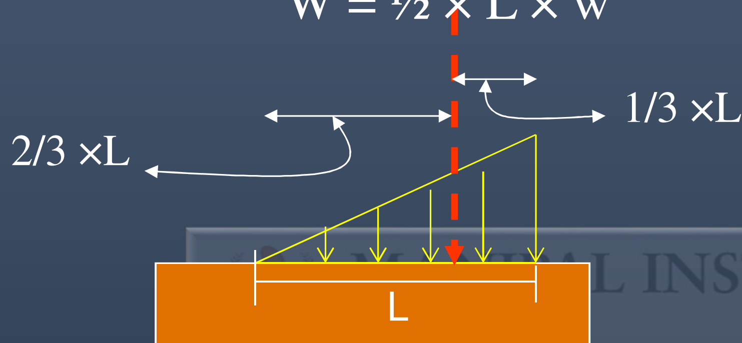
- Triangular Load



- Trapezoidal Load

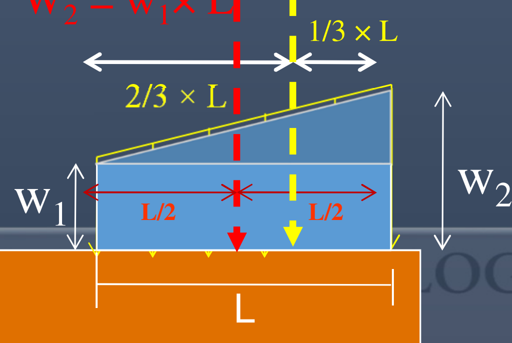


$W = ½ \times L \times w$



$W_2 = w_1 \times L$

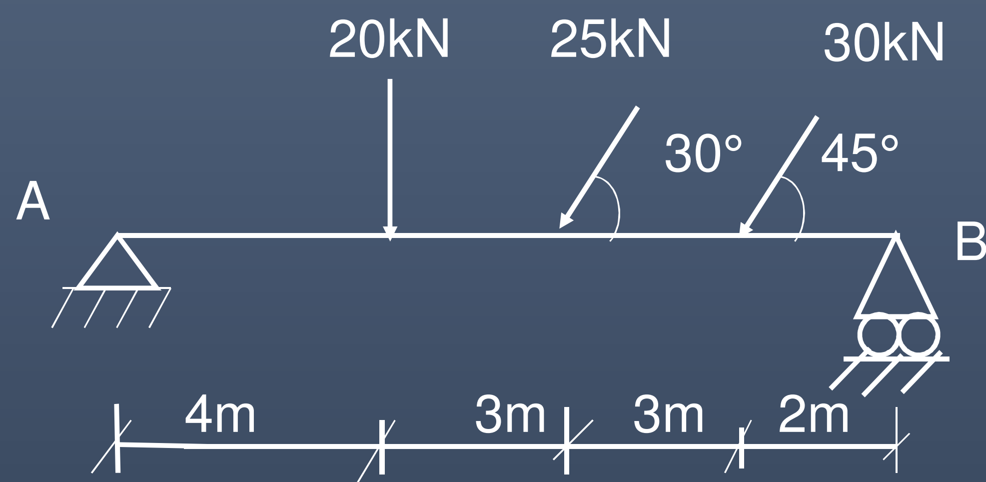
$W_1 = ½ \times (w_2 - w_1) \times L$



LECTURE 7

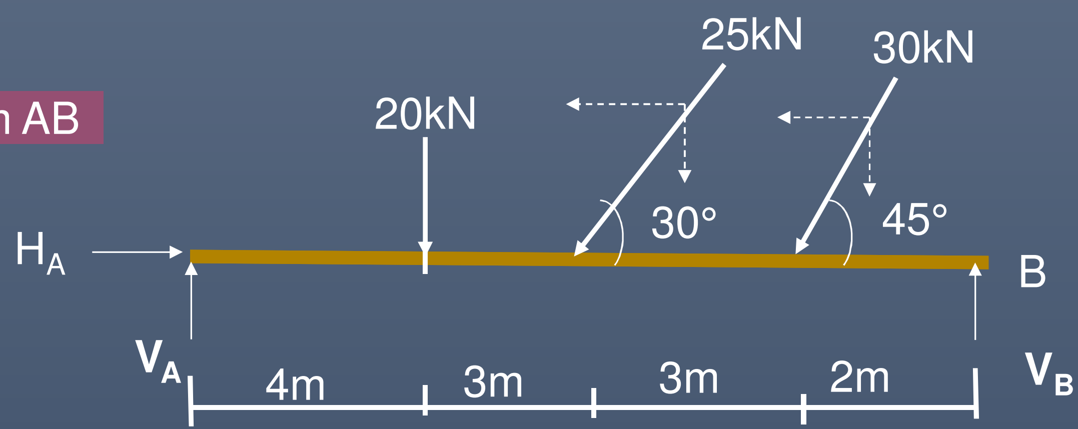
- Application Problems Continued…

3. A beam AB of span 12m shown in the figure is hinged at A and is on rollers at B. Determine the reactions at A and B for the loading shown.



Solution

FBD of Beam AB



$\sum Fx = 0$ $\quad H_A - 25\cos 30 - 30\cos 45 = 0$

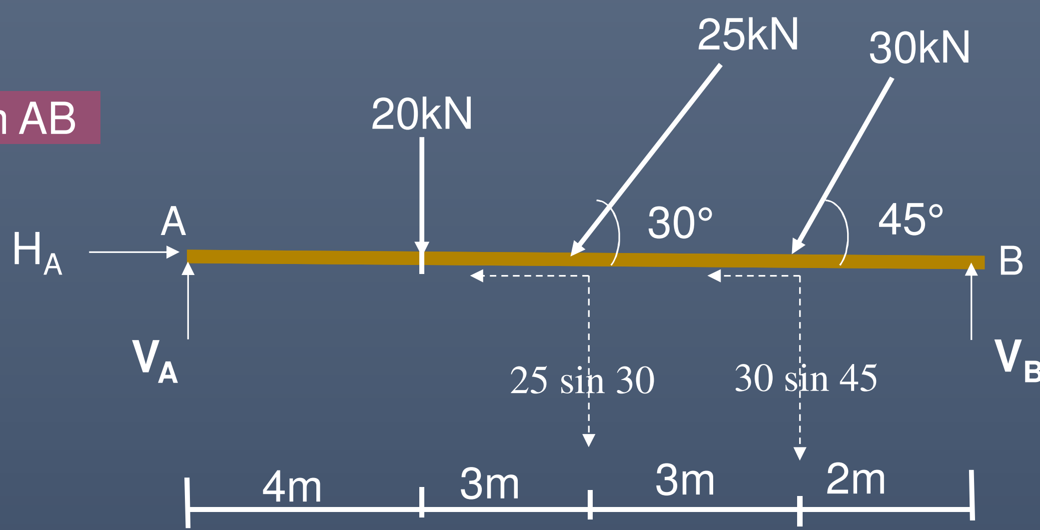
$\sum Fy = 0$ $\quad V_A - 20 - 25 \sin 30 - 30 \sin 45 + V_B = 0$

$\sum M_A = 0$ $\quad -20\times4 - 25 \sin 30\times7 - 30 \sin 45\times10 + V_B\times12 = 0$

+

Solution

FBD of Beam AB



$$\circlearrowleft + \ \Sigma M_A = 0$$

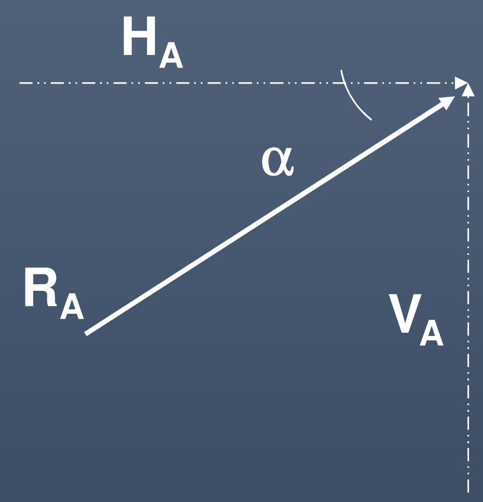
$$0 = -20\times4 - 25\sin30\times7 - 30\sin45\times10 + V_B\times12$$

Solution(contd.)

H_A=42.86kN, V_A=22.07kN, V_B=31.64kN

$$R_A = \sqrt{\left(H_A^2 + V_A^2\right)}$$

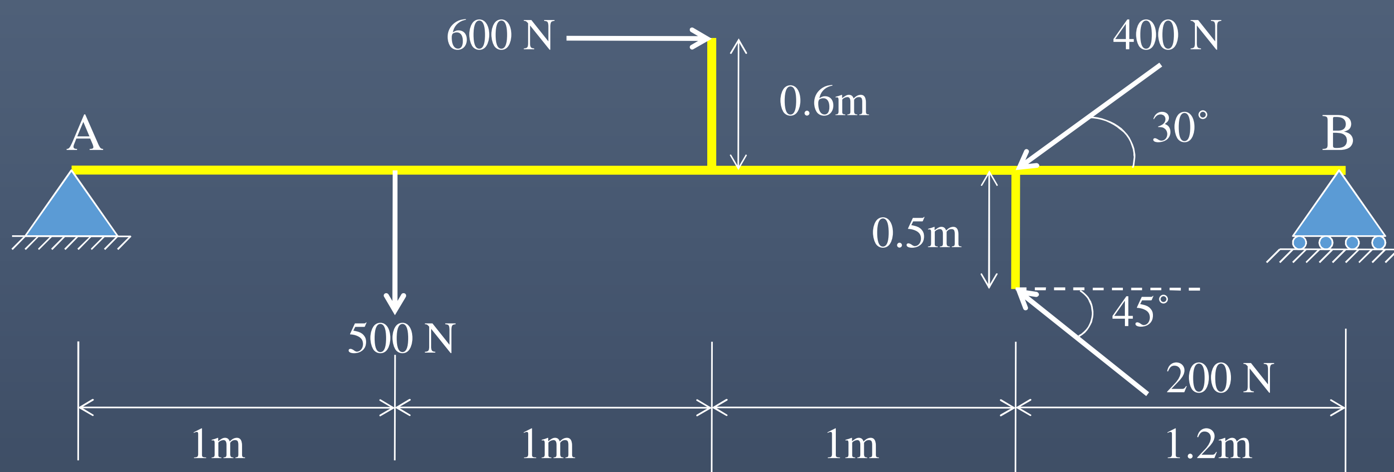
$$\alpha = \tan^{-1}\left(\frac{V_A}{H_A}\right)$$

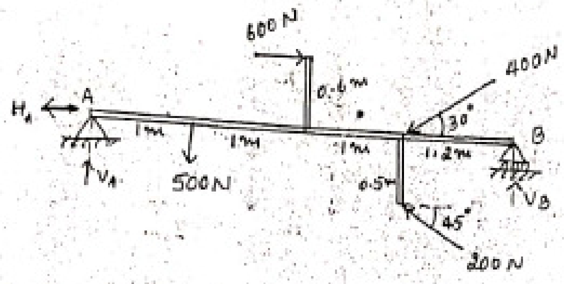


R_A= 48.21 kN

α = 27.25°

4. Find the support reactions for the beam loaded shown in figure.





$\xrightarrow{+} \sum F_x = 0$

$600 - 400\cos 30 - 200\cos 45 - H_A = 0$

$H_A = \underline{112.17\ N}$

$+\uparrow \sum F_y = 0$

$V_A - 500 - 400\sin 30 + 200\sin 45 + V_B = 0.$

$V_A + V_B = 558.6\ N.$

$+\circlearrowleft \sum M_A = 0.$

$(500\times 1) + (600\times 0.6) + (400\sin 30\times 3) + (200\cos 45\times 0.5) - (200\sin 45\times 3)$

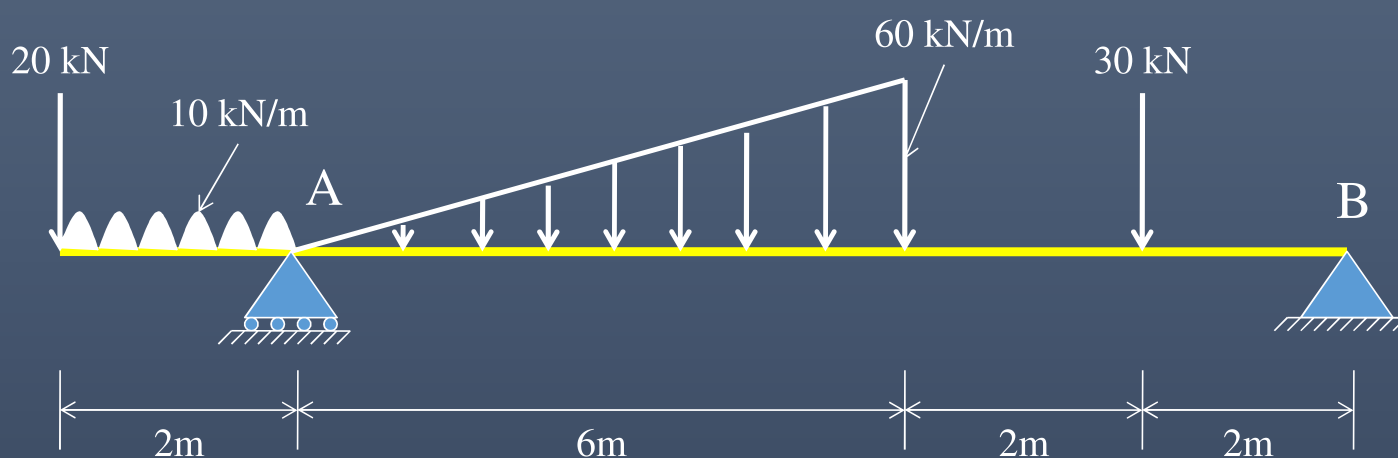
$- V_B \times 4.2 = 0.$

$4.2\, V_B = 1106.45$

$V_B = 263.44\ N.$

$\therefore V_A = \underline{295.2N}$

5. An overhanging beam is on roller at A and is hinged at B. Determine the reactions at A and B.



$\overset{+}{\rightarrow}\sum F_x = 0.$

$-H_B = 0.$

$\therefore \underline{H_B = 0}$

$+\uparrow \sum F_y = 0$

$-20-20+V_A - \left(\frac{1}{2}\times 60\times 6\right) - 30 + V_B = 0.$

$V_A + V_B = 250.$

$+\circlearrowleft \sum M_A = 0$

$(-20\times 2) - (10\times 2\times 1) + \left(\frac{1}{2}\times 60\times 6\times 4\right) + (30\times 8) - V_B\times 10 = 0.$

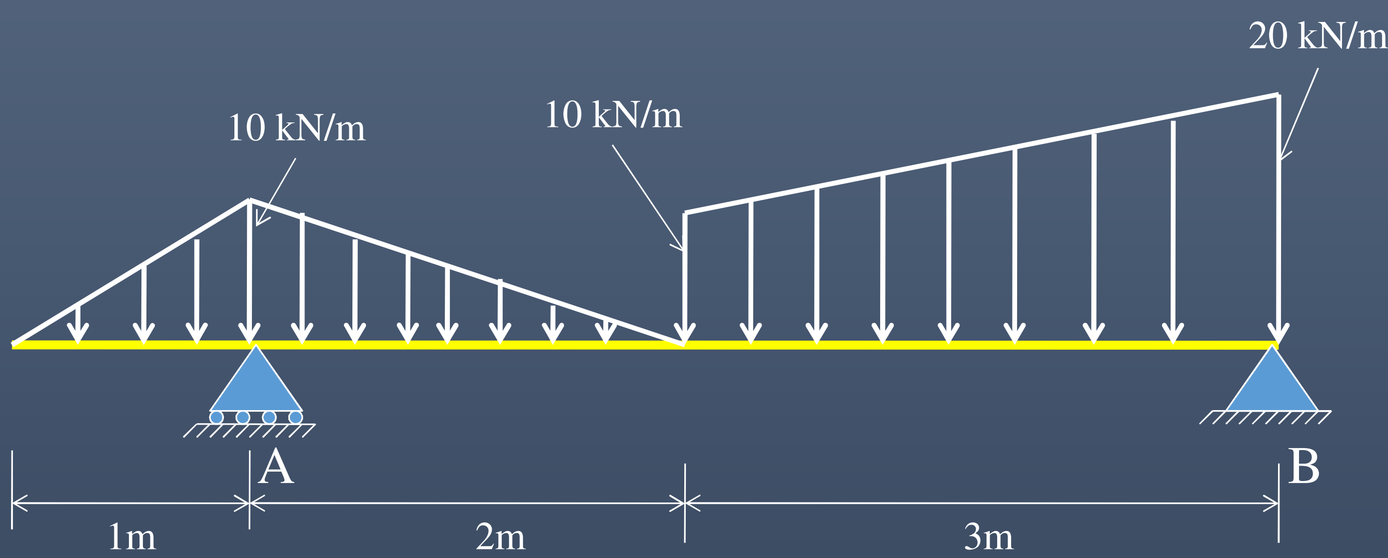
$910 - 10V_B = 0.$

$\therefore V_B = 90\,N.$

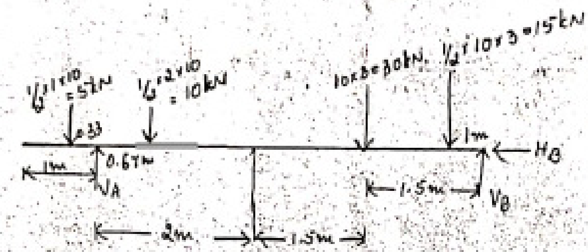
$\therefore V_A = 160\,N.$

$\underline{R_A = 160N \;\&\; R_B = 90\,N}$

6. Determine the support reactions at A and B.



FBD ⇒



$\xrightarrow{+} \sum F_x = 0.$

$H_B = 0.$

$+\uparrow \sum F_y = 0$

$-5 + V_A - 10 - 30 - 15 + V_B = 0.$

$V_A + V_B = 60.$

$+\circlearrowleft \sum M_A = 0.$

$(-5 \times 0.33) + (10 \times 0.67) + (30 \times 3.5) + (15 \times 4) - (V_B \times 5) = 0.$

$170.05 - 5V_B = 0.$

$V_B = 34.01$ kN

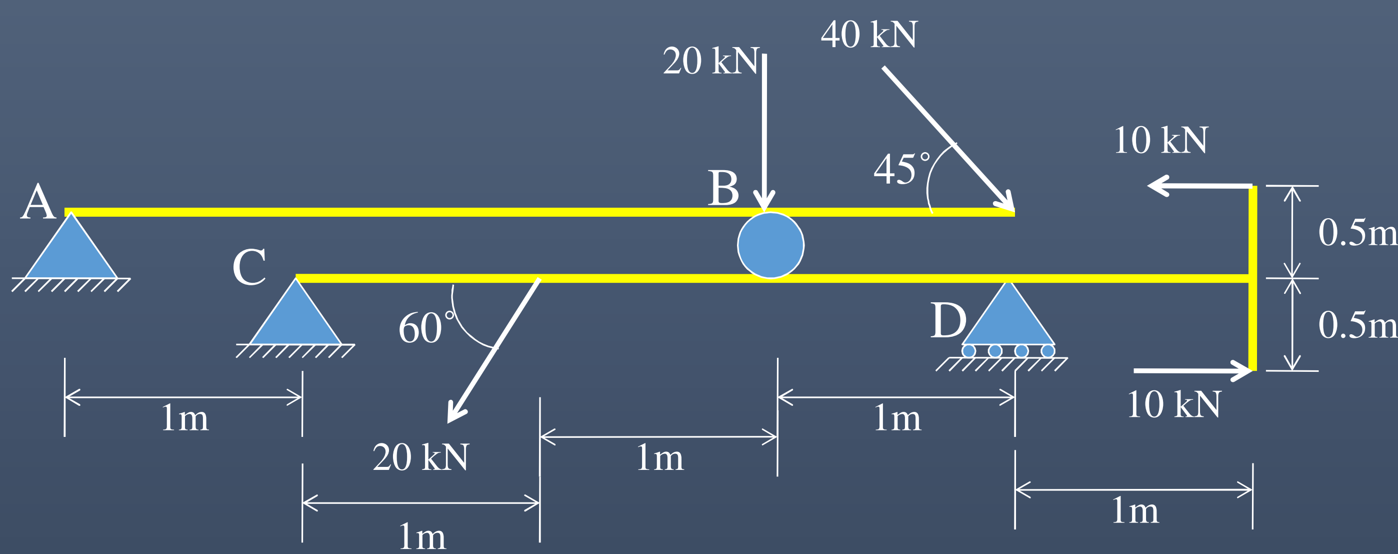
$\therefore V_A = 25.99$ kN

TECHNOLOGY

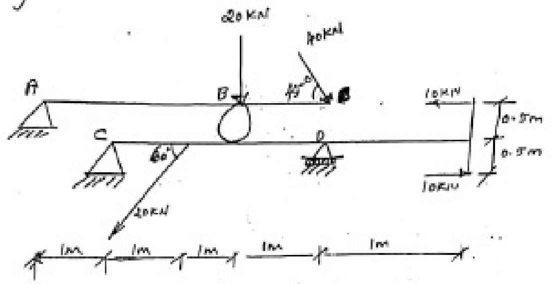
TUTORIAL 2

- Application Problems

6. Determine the reactions at A, C and D due to the forces acting on the beam.

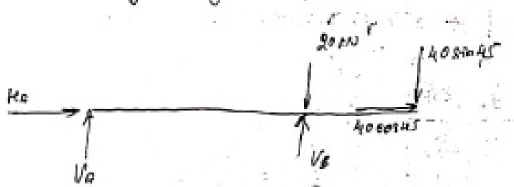


forces acting on the beam



Sol

Free body diagram of AB :



$\xrightarrow{+ve}$ $\Sigma F_x = 0$

$H_A + 40\cos 45 = 0$

$\boxed{H_A = 28.28\ kN\ (\leftarrow)}$

+ve↑ $\Sigma F_y = 0$

$V_A + V_B = 20 + 40\sin 45°$

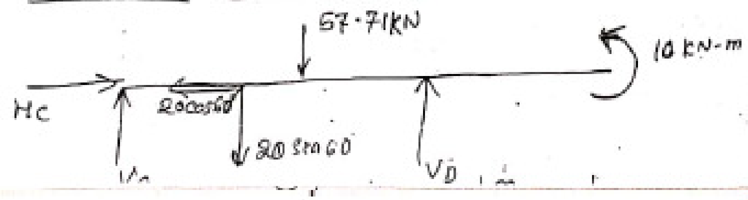
$\boxed{V_A + V_B = 48.28\ kN}$ — ①

$\Sigma M_A = 0$ ↻ +ve

$\Rightarrow (-)V_B \times 3 + 20 \times 3 + 40\sin 45 \times 4 = 0$

$\boxed{V_B = 57.71\ kN}$ ∴ $\boxed{V_A = -9.43\ kN}$

Consider FBD of CD Beam



$H_C = 20\cos 60° = \underline{10\ kN}$

+ve↑ $\Sigma F_y = 0$

$V_C + V_D - 57.71kN - 20\sin 60° = 0$

$\boxed{V_C + V_D = 75.03\ kN}$ — ①

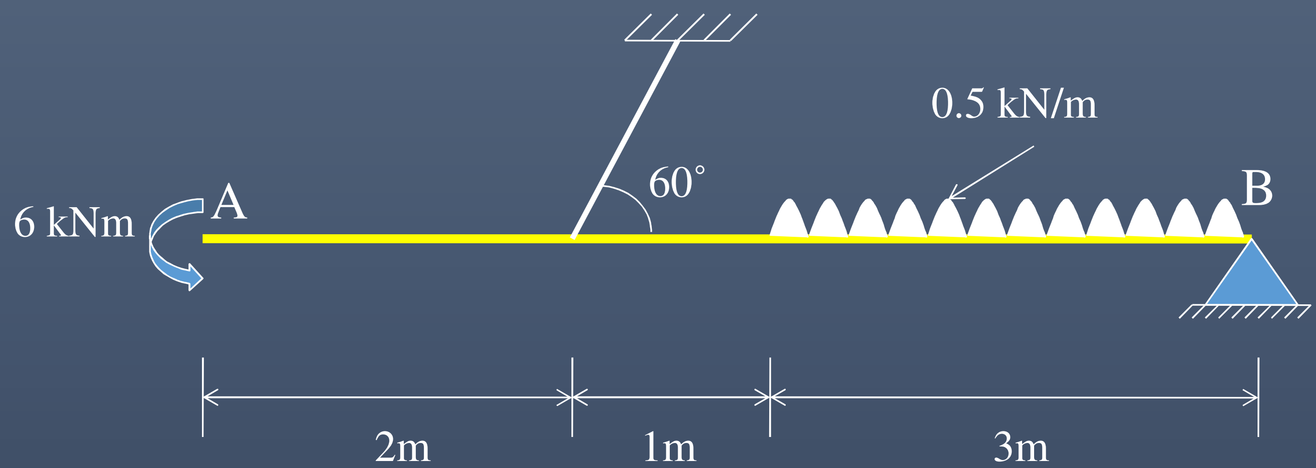
↺ +ve $\Sigma M_C = 0$

$20\sin 60 \times 1 + 57.71 \times 2 - V_D \times 3 - 10 = 0$

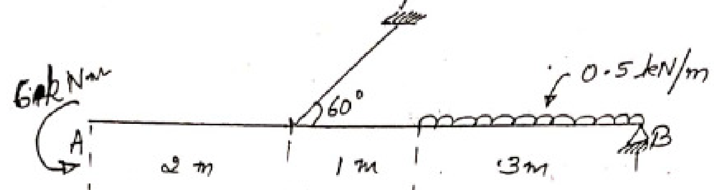
$\boxed{V_D = 40.91\ kN}$

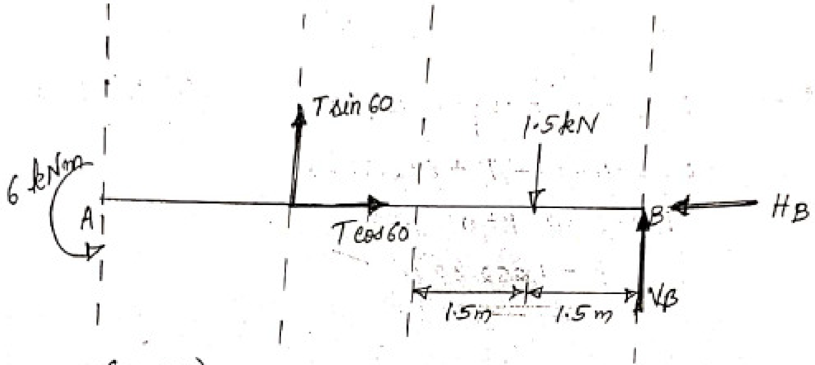
$\boxed{V_C = 34.12\ kN}$

7. Find the tension in the rope and reaction at B.



10. Find the tension in the rope and reaction at B.





$(\rightarrow +ve)$

$\Sigma F_x = 0$

$T \cos 60 - H_B = 0$

$\Rightarrow H_B = T \cos 60 \rightarrow ①$

$\Sigma F_y = 0 \ (\uparrow +ve)$

$T \sin 60 - 1.5 + V_B = 0 \rightarrow ②$

$\Sigma M_B = 0$ (↺ +ve)

$-6 + (T \sin 60 \times 4) - (1.5 \times 1.5) = 0$

$-8.25 + 3.464\, T = 0$

$T = \frac{8.25}{3.464} = \underline{\underline{2.38 \text{ kN}}}$

Substitute in ①,

$H_B = 2.38 \cos 60 = \underline{\underline{1.19 \text{ kN}}}$

Substitute T in ②. $2.38 \sin 60 - 1.5 + V_B$

$N_2 + T \sin 30 - 2000 = 0$

$N_2 + 800 \sin 30 - 2000 = 0$

$\underline{\underline{N_2 = 1600 \text{ N}}}$

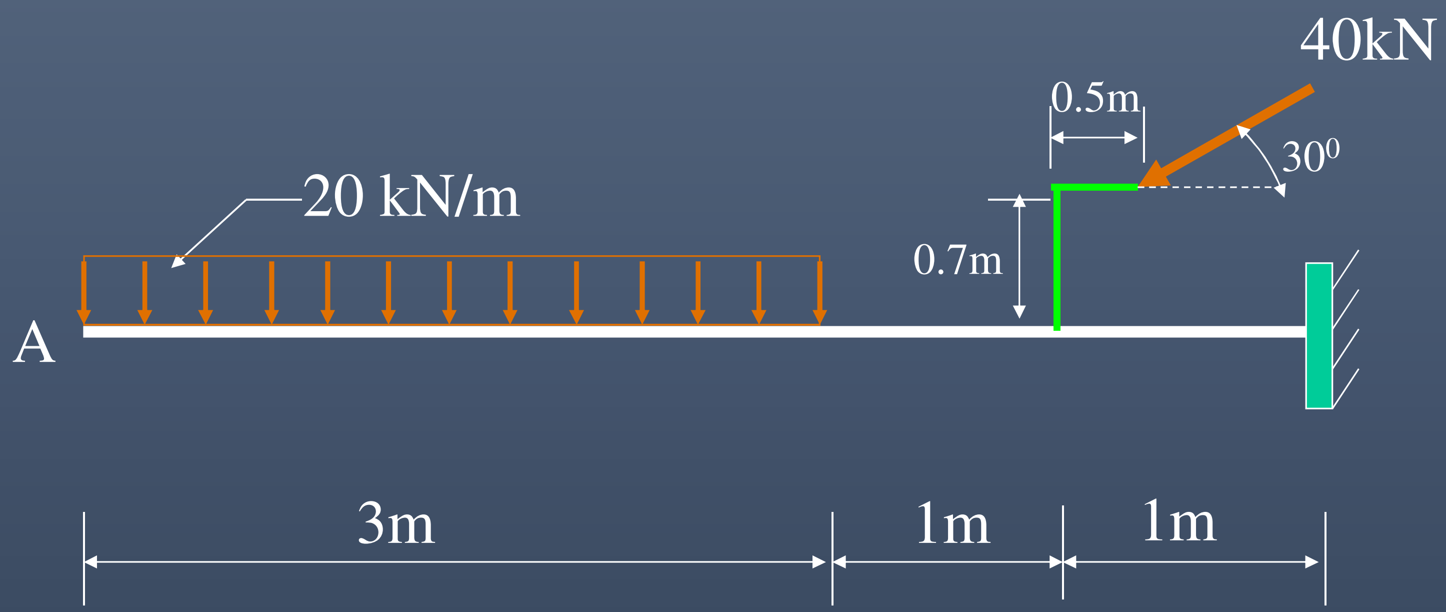
eqn ③ $\Rightarrow \mu N_2 - P + 800 \cos 30 = 0$

$(0.35 \times 1600) - P + 692.82 = 0$

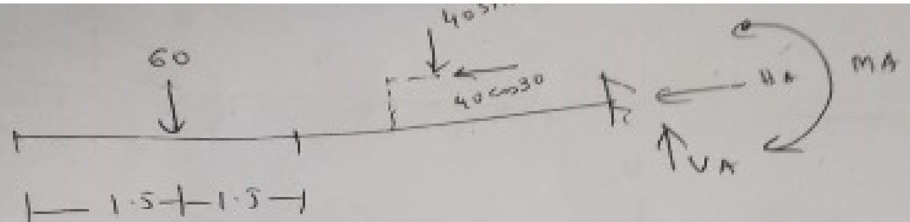
$1{,}252.82 - P = 0$

$\underline{\underline{P = 1{,}252.82 \text{ N}}}$

8. Determine the reactions in the cantilever beam shown.



Solution:



$\Sigma F_x = 0$

$-H_A - 40\cos 30 = 0 \qquad H_A = -34.641\ \text{kN}$

$\Sigma F_y = 0$

$V_A = 60 + 40\sin 30 = 80\ \text{kN}$

$M_A \circlearrowleft^{+} = 0$

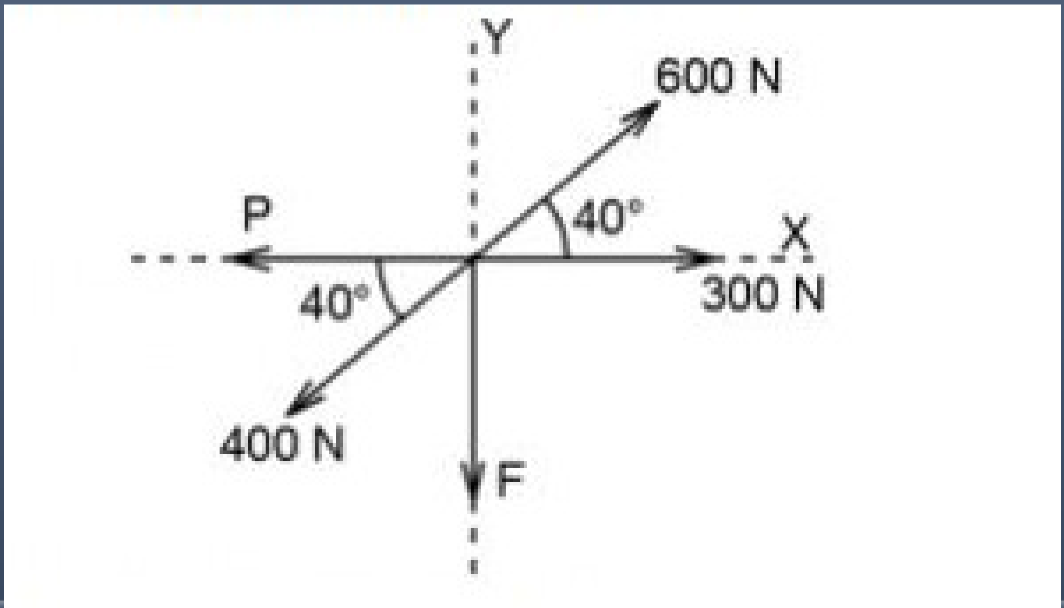
$-60(3.5) - 40\cos 30\,(0.7) - 40\sin 30\,(0.5) + M$

$-210 - 24.25 - 10 + M_A = 0$

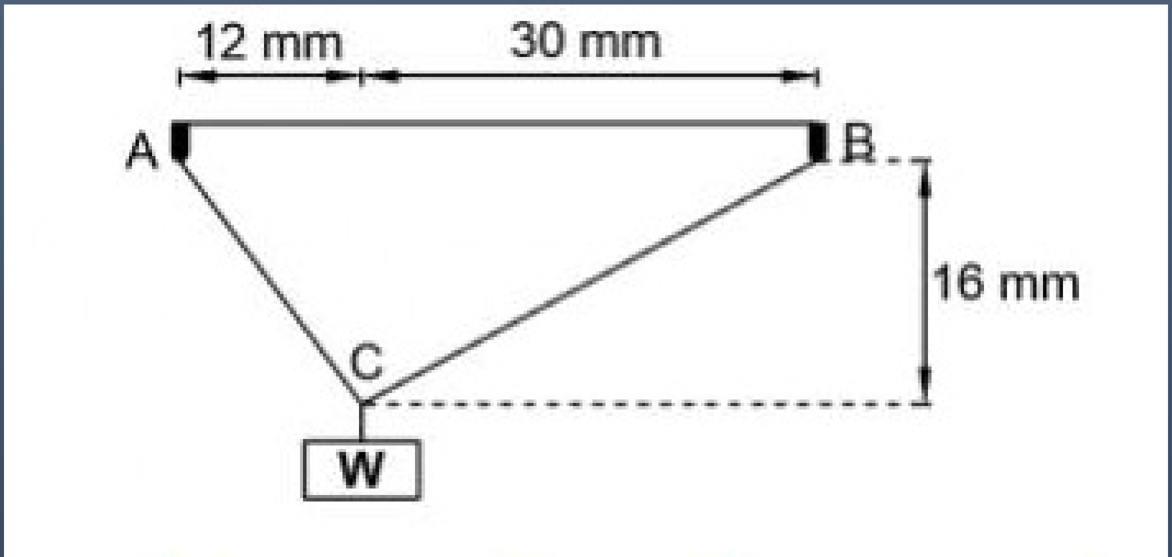
$M_A = 244.25\ \text{kN-m}$

TUTORIAL (Additional)

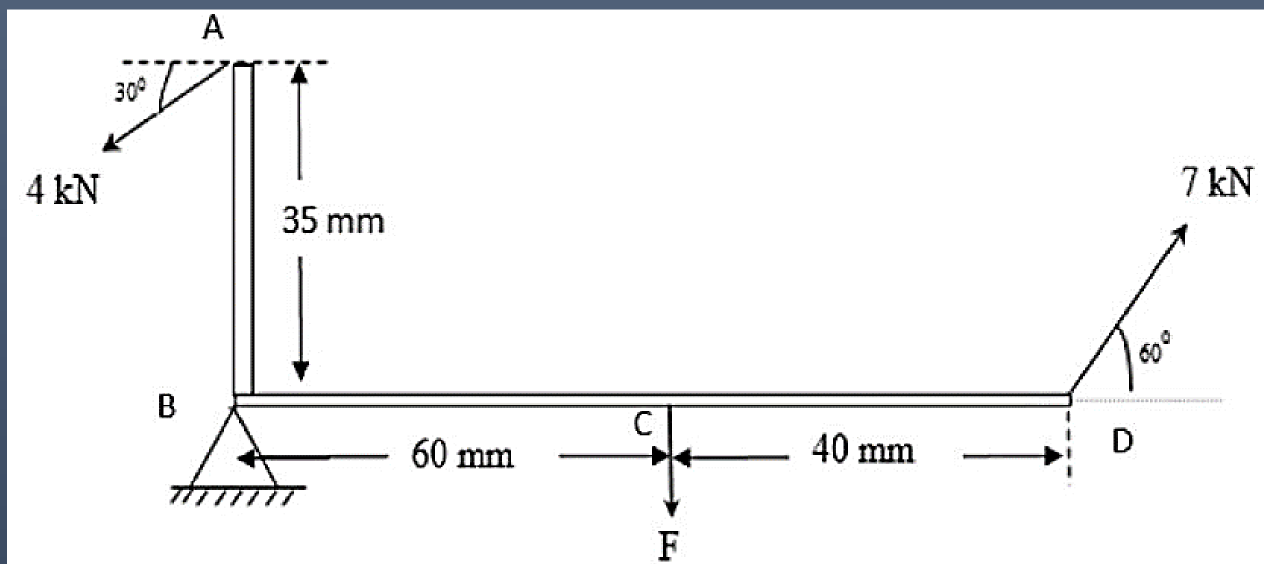
1. The figure shows the concurrent force system acting at a joint of a bridge truss. Determine the values of P and F required to maintain equilibrium of forces.



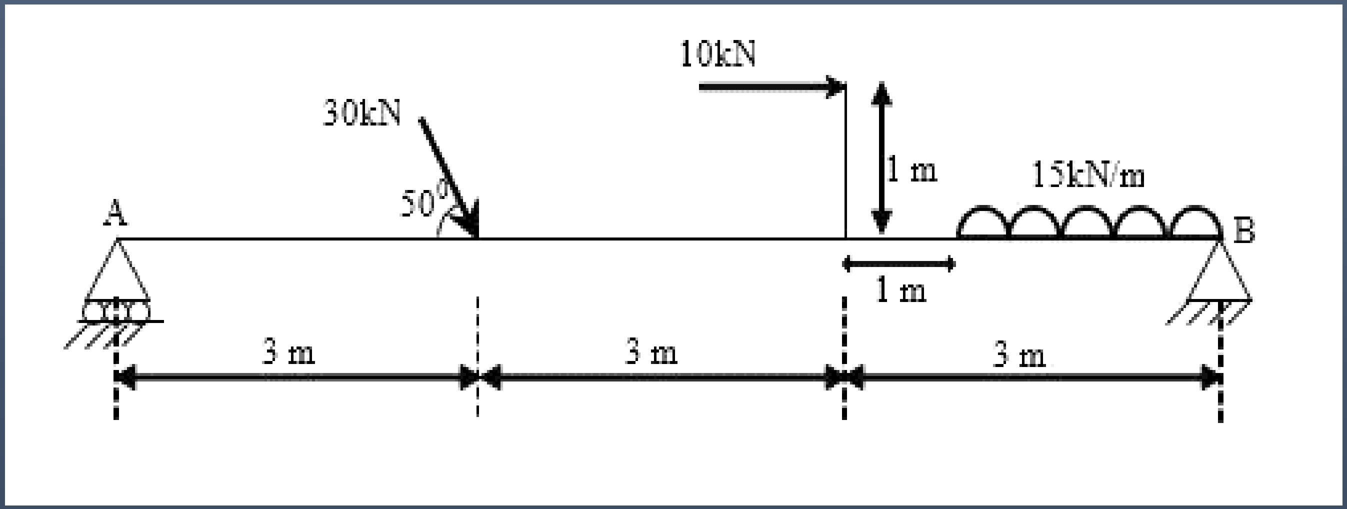
2. Two cables tied together at C are loaded with a weight W = 190 N as shown in figure. Determine the tension in the cable AC and BC to maintain equilibrium.



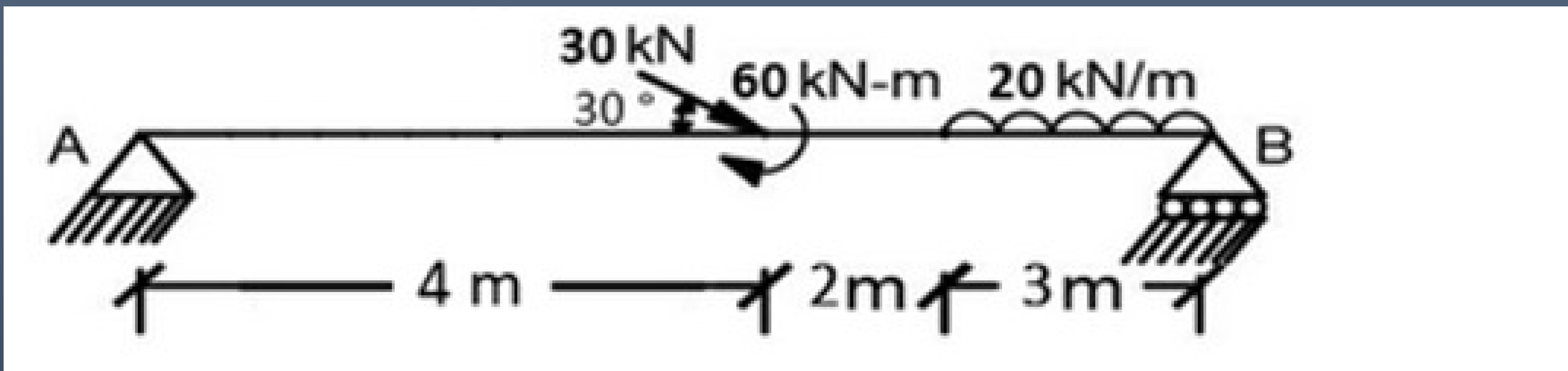
3. A beam ABCD, hinge supported at B is subjected to loads as shown in the figure. Calculate force 'F' and reactions at support B. (2 marks)



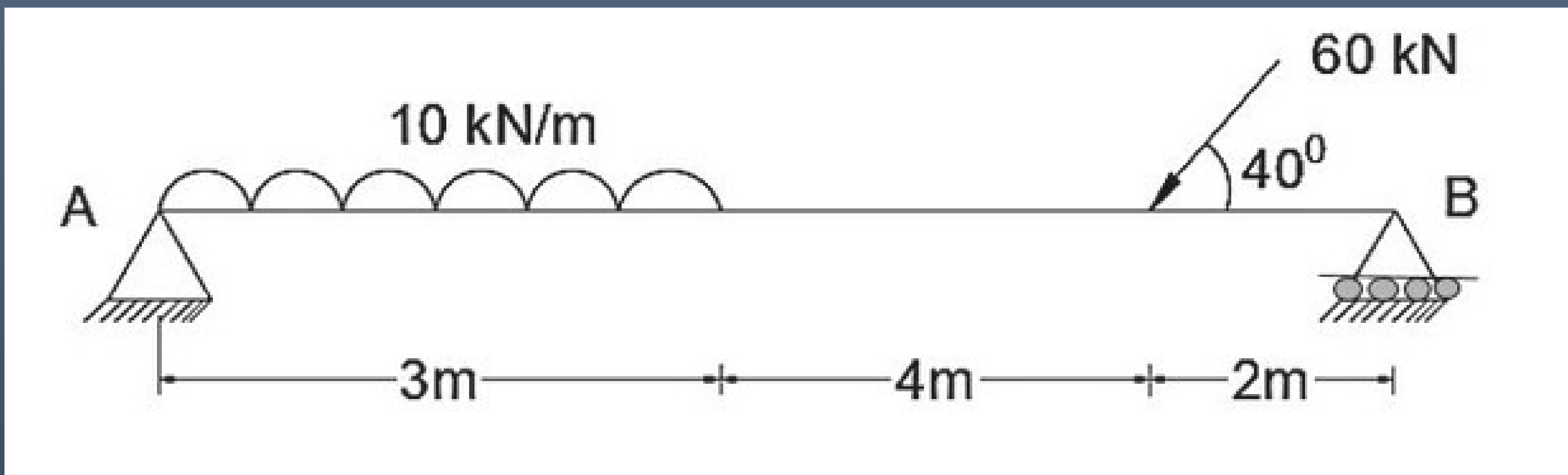
4. Determine the reactions at the supports A and B for the beam loaded as shown in the figure. (4 marks)



5. Determine the reactions at the supports A and B for the beam loaded as shown in the figure. (4 marks)



6. Determine the reactions at the supports A and B for the beam loaded as shown in the figure. (4 marks)



LECTURE 8

FRICTION

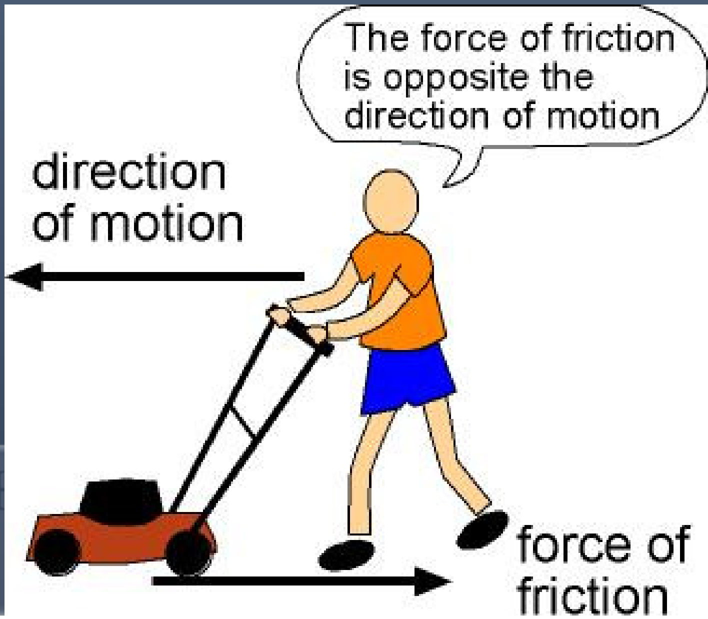
- Introduction
- Terms and Definitions
- Static, Limiting and Kinetic Friction
- Laws of dry friction
- Application Problems

Introduction

Force which opposes the movement or tendency of movement is known as frictional resistance or friction.

Defined broadly as the resistance that occurs between two bodies in contact when they tend to slide or roll relative to one another.

A frictional force is a shear force that acts tangent to the surface of contact between two bodies. This force opposes sliding motion between the bodies.



Plays very important role in the analysis of the performance of various mechanical devices.

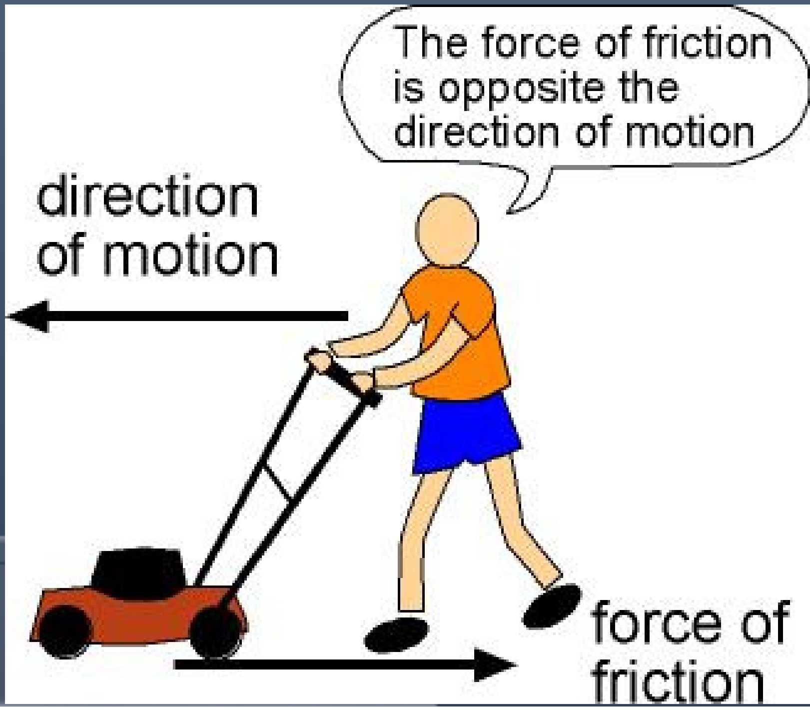
Friction – Beneficial and harmful

Advantages:

Walk, Traction, Brake, grind and polish

Disadvantages:

Power loss, wear and tear



Causes

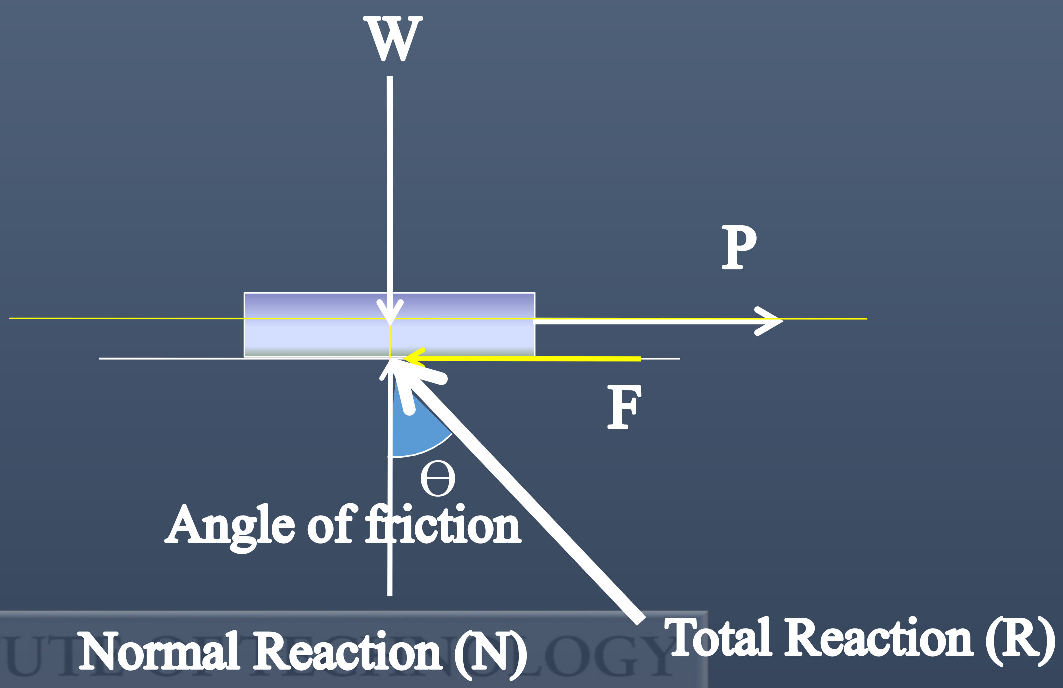
1. Microscopic irregularities (roughness) on surface

2. Adhesion b/w surfaces --- Molecular attraction (high for highly polished surfaces)

3. An indentation on one body caused by a second, harder body

Terms and Definitions

- Coefficient of friction (μ)
- Total reaction (R)
- Angle of Friction (Θ)
- Angle of limiting friction (ϕ)
- Angle of repose (α)



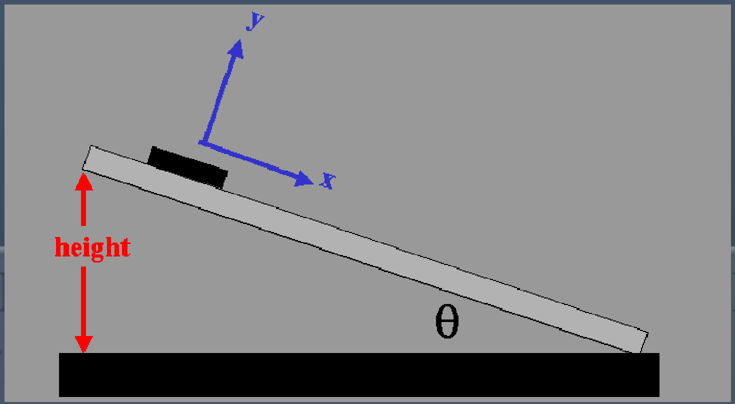
- **Coefficient of friction (μ)** = F_{max} / N

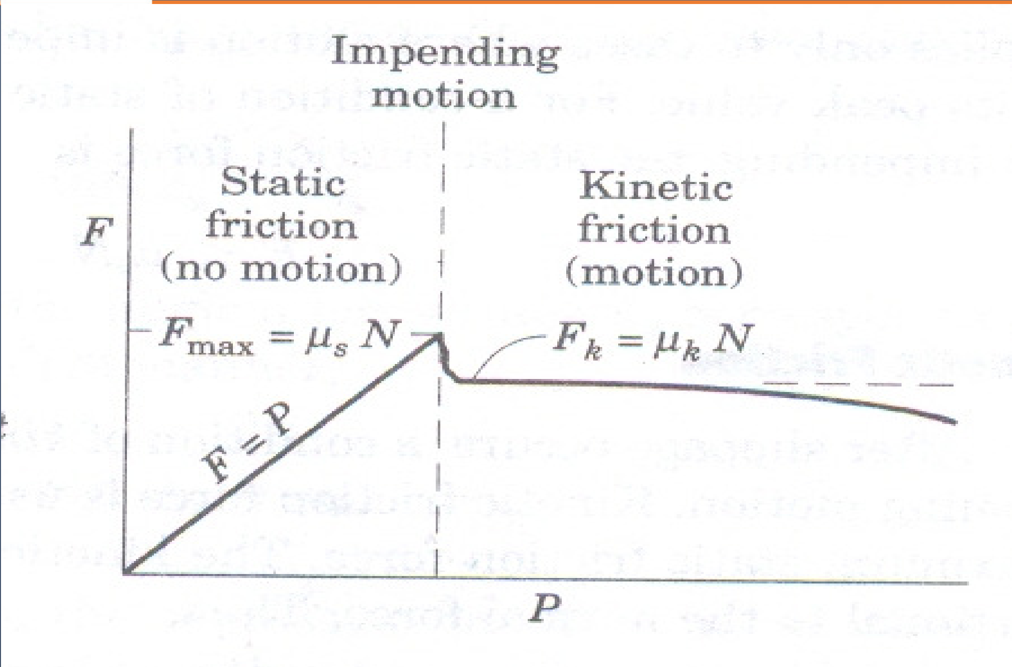
- **Total reaction :** Frictional force and the normal reaction could be replaced by a single force called **Total reaction.** **R** $= \sqrt{(F^2 + N^2)}$

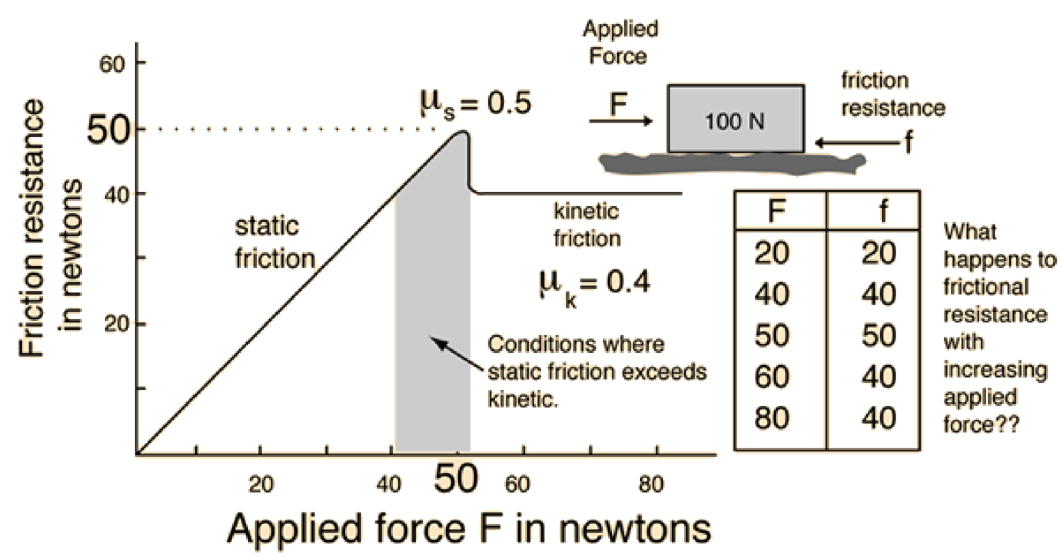
- **Angle of Limiting Friction (ϕ) :** An angle made by the total reaction (R) with the normal reaction (**N**) when F has reached F_{max}. $\tan\phi = F_{max} / N$

- **Angle of Repose (α) :** The limiting angle up to which the grains repose (sleep/rest) is called the angle of repose of the granular material

- **Limiting Friction:** The maximum frictional force that can arise before an object begins to slide.
- **Static friction:** Body remains at rest when the applied force is less than the limiting friction.
- **Dynamic friction OR Kinetic friction:** Frictional resistance experienced by the body while in motion.



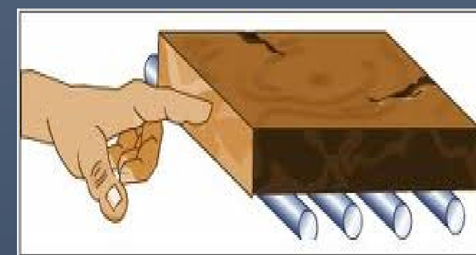

Impending motion
Static friction (no motion)
Kinetic friction (motion)
F
$F_{max} = \mu_s N$
$F_k = \mu_k N$
F = P
P

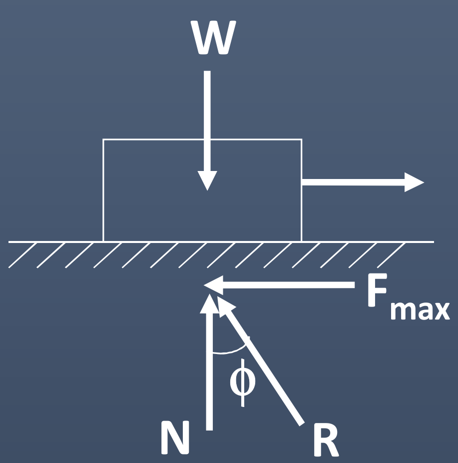

Applied Force
F
100 N
friction resistance
f
$\mu_s = 0.5$
$\mu_k = 0.4$
static friction
kinetic friction
Conditions where static friction exceeds kinetic.
Friction resistance in newtons
Applied force F in newtons
60
50
40
20
20
40
50
60
80
F f
20 20
40 40
50 50
60 40
80 40
What happens to frictional resistance with increasing applied force??

Kinetic/Dynamic Friction

Sliding friction → friction experienced when a body slides over another surface.

Rolling friction → friction experienced by a body when it rolls over a surface.





$$F \propto N$$

$$F_{max} = \mu N$$

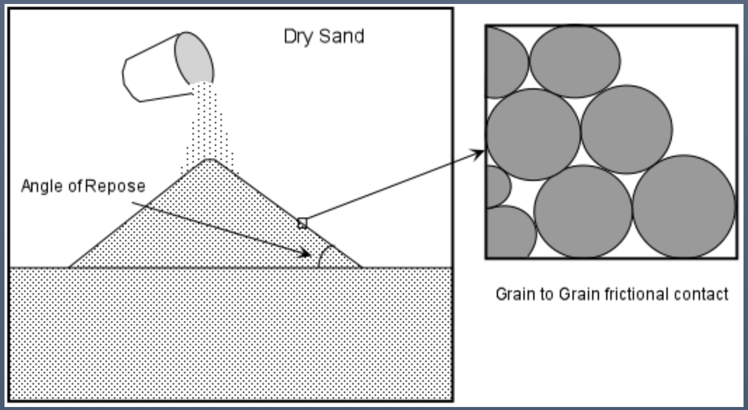
$$\mu = \frac{F_{max}}{N}$$

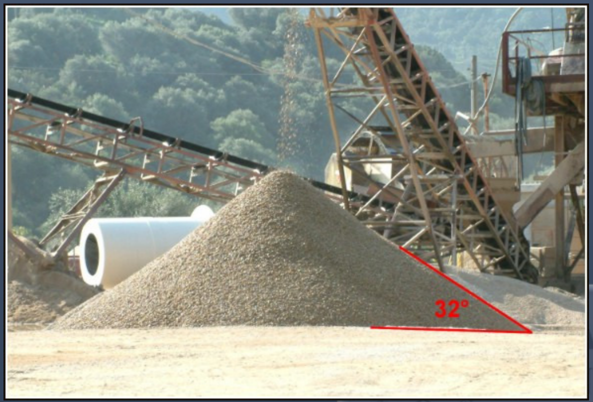
Where F_{max} = Limiting Friction

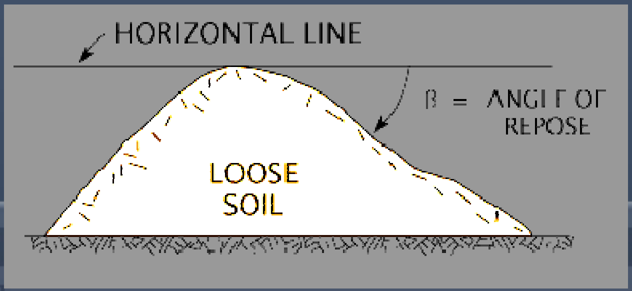
N= Normal Reaction between the contact surfaces

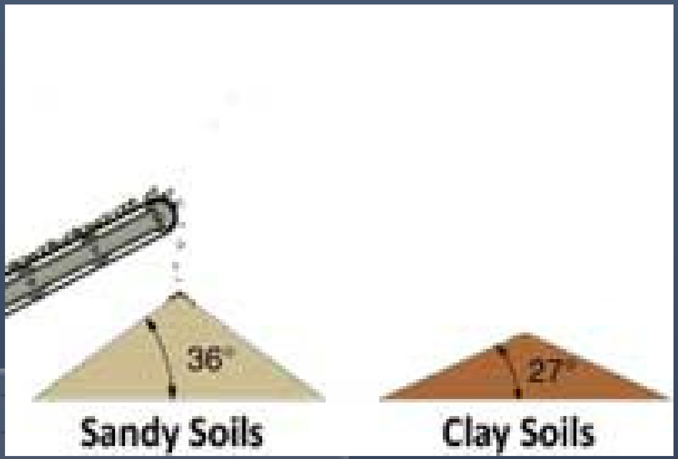
μ = Coefficient of friction

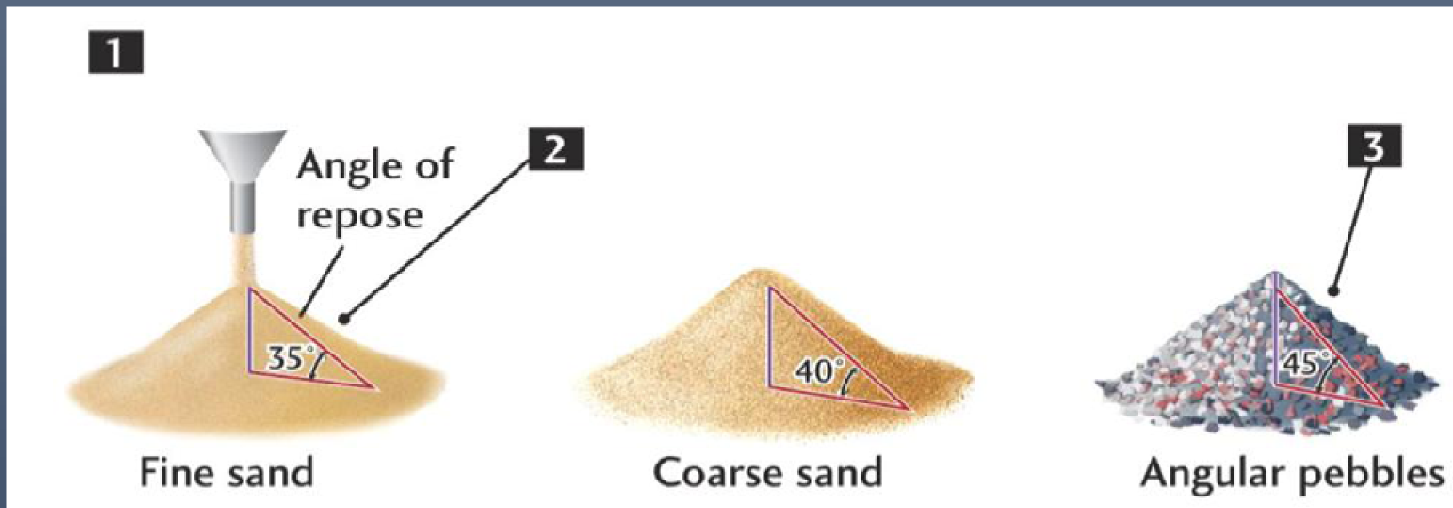
Angle of Repose (α) : limiting angle up to which the grains repose (sleep) is called the angle of repose of that granular material.









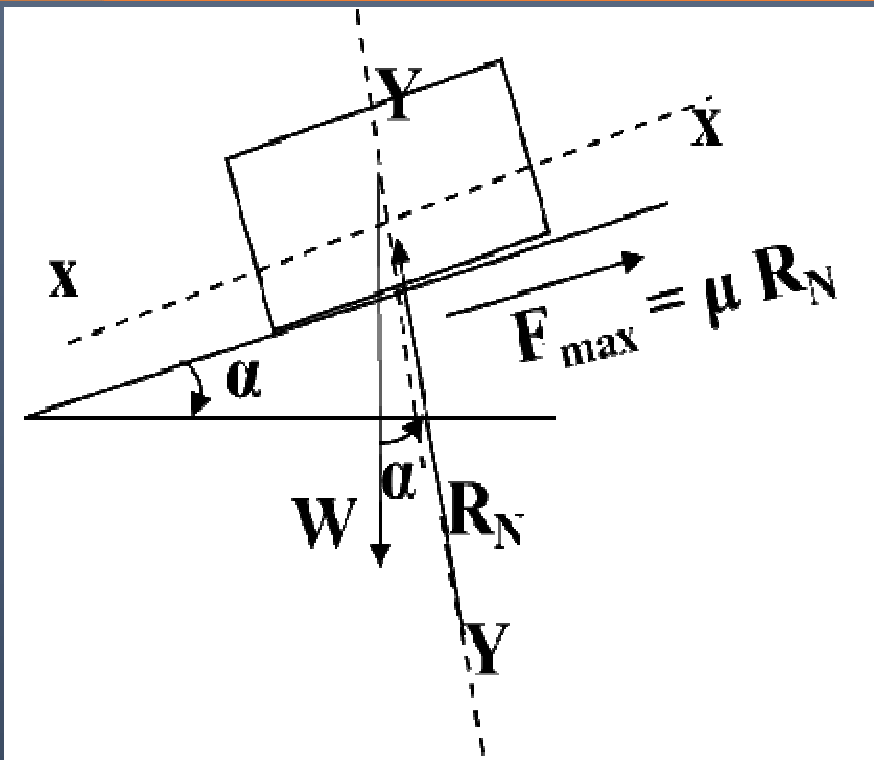


Significance of Angle of repose:

The angle that an inclined plane makes with the horizontal, when the body supported on the plane is on the verge of motion due to its self-weight is equal to the angle of repose.

Angle of repose is numerically equal to ***Angle of limiting friction.***

Significance of Angle of repose



$$\Sigma F_X = 0$$

$$F_{max} = W \sin \alpha \text{ - - - - - - } (1)$$

$$\Sigma F_Y = 0$$

$$R_N = W \cos \alpha \text{ - - - - - - } (2)$$

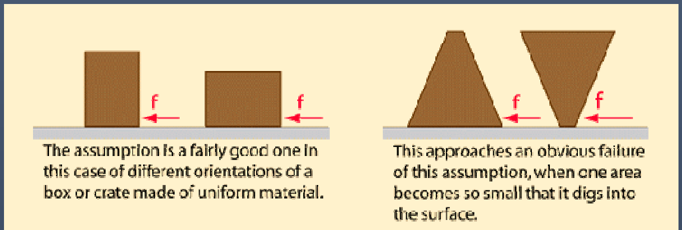
$$\frac{(1)}{(2)}$$

$$\tan \alpha = \frac{F_{max}}{R_N} = \mu = \tan \phi$$

$$\alpha = \phi$$

Laws of dry Friction

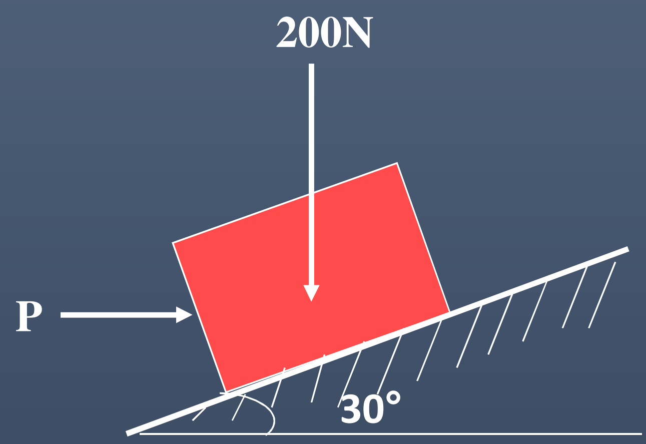
1. The force of friction always acts in the direction opposite to the direction in which body tends to move.
2. The magnitude of limiting friction bears a constant ratio to the normal reaction between the two surfaces. (Experimentally proved)
3. The force of friction is independent of the area of contact between the two surfaces.

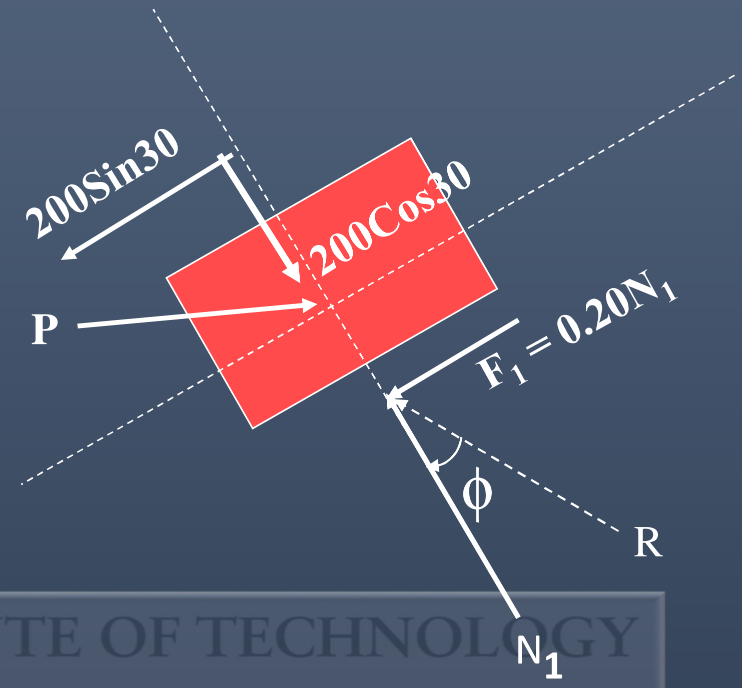


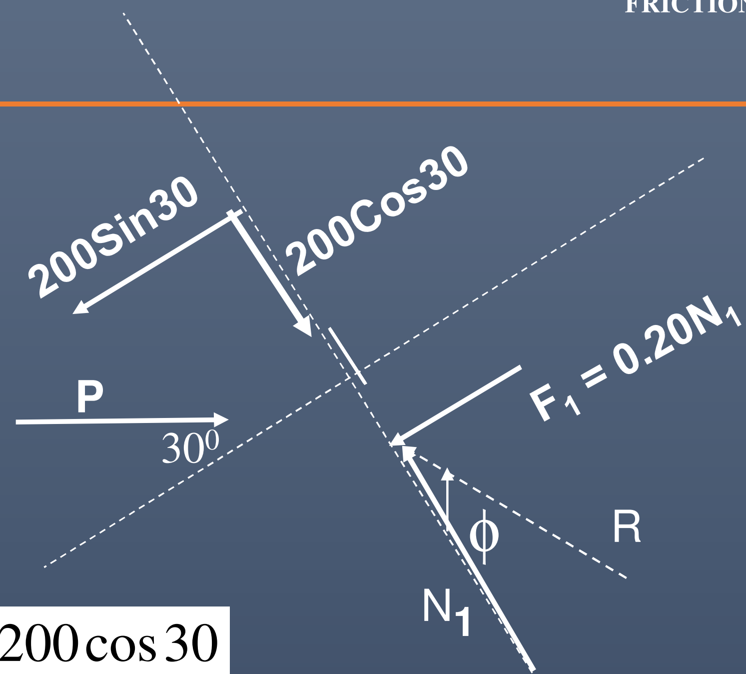
4. After the body starts moving, the dynamic friction comes into play, the magnitude of which is less than that of limiting friction and it bears a constant ratio with normal force. This ratio is known as **coefficient of dynamic friction**

Application Problems

1. Compute the magnitude of **P** that will cause the motion to impend up the plane. Coefficient of friction, $\mu = 0.20$





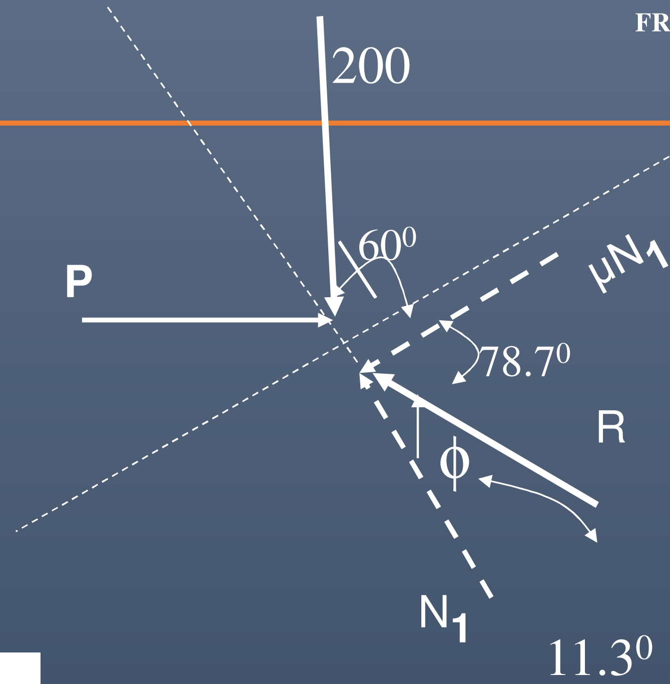


$$\sum Fx = 0 = +P\cos 30 - 0.2N_1 - 200\cos 30$$
$$\sum Fy = 0 = N_1 - P\sin 30 - 200\cos 30$$
$$P = 175.8N$$

OR

$\tan \phi = \mu = 0.20$

$\therefore \ \phi = 11.3°$



$$\frac{200}{\sin 131.3} = \frac{P}{\sin 138.7} = \frac{R}{\sin 90}$$

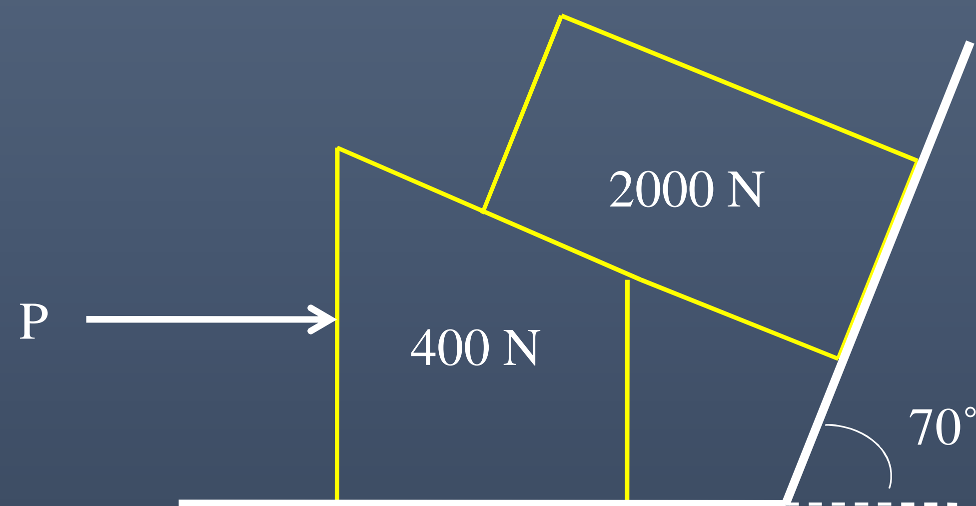
P = 175.7

LECTURE 9

- Application Problems

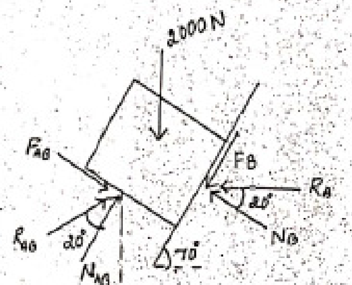
Application Problems

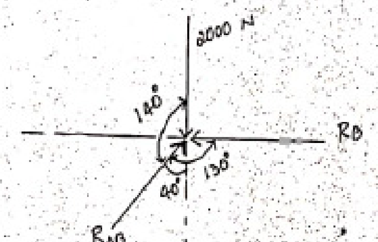
2. Determine the force P required to start the 400 N wedge to the right. Take $\Phi=20^\circ$ for all contact surfaces.



Home

FBD of 2000 N Block



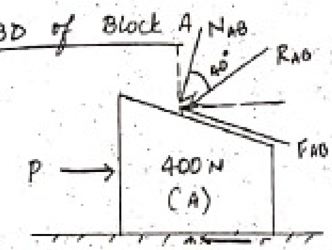


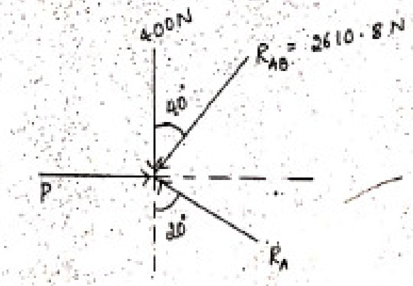
$$\frac{2000}{\sin 130} = \frac{R_{AB}}{\sin 90} = \frac{R_B}{\sin 140}$$

$$R_B = \frac{2000 \times \sin 140}{\sin 130} = 1678.8 \text{ N}$$

$$R_{AB} = \frac{2000 \sin 90}{\sin 130} = 2610.8 \text{ N}$$

FBD of Block A





$\xrightarrow{+} \sum F_x = 0$

$$P - R_A \cos 70 - 2610 \cos 50 = 0.$$

$$P - R_A \cos 70 - 1677.68 = 0 \longrightarrow (1)$$

$+\uparrow \sum F_y = 0$

$$-400 + R_A \sin 70 - 2610 \sin 50 = 0.$$

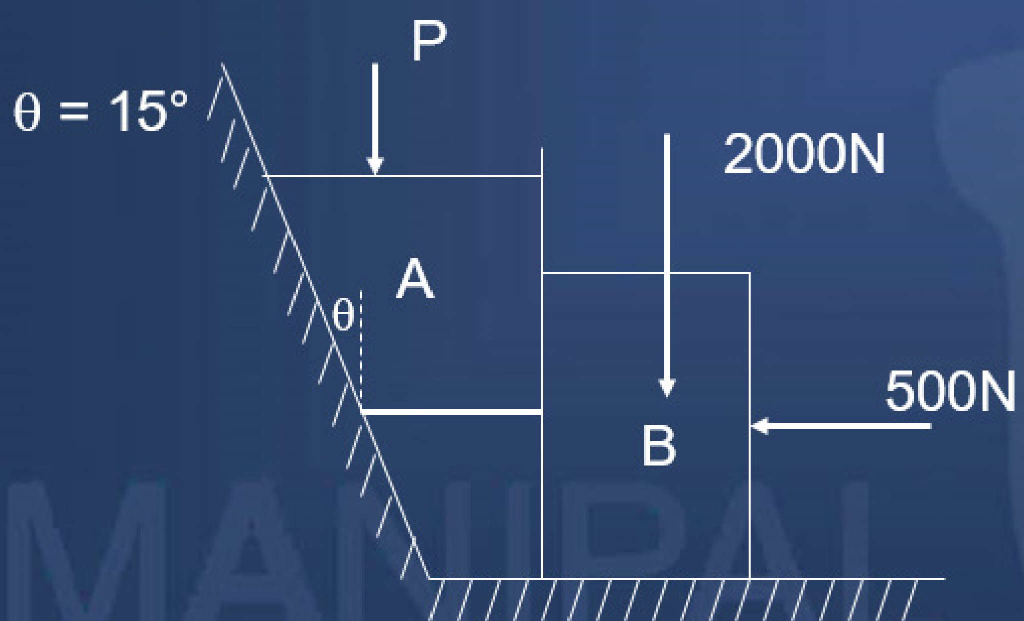
$$R_A \sin 70 = 2399.4.$$

$$R_A = 2553.4 \text{ N}$$

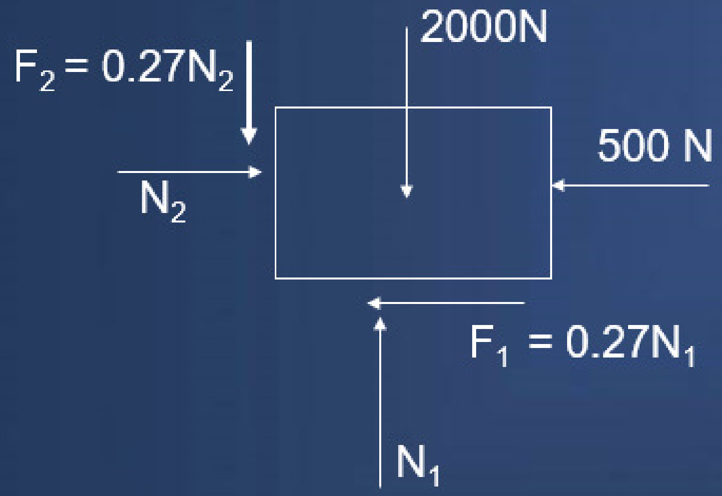
$$(1) \Rightarrow \quad P - 2553.4 \cos 70 - 1677.68 = 0.$$

$$P = 2551 \text{ N}$$

3. Determine the force **P** required just to start the wedge **A** shown in the figure. Angle of limiting friction between all contact surfaces is 15°.



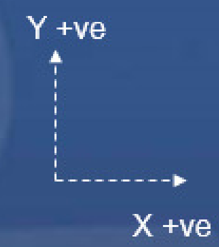
FBD of B



$\phi = 15°$

$\tan\phi = \mu$

$\therefore\ \mu = 0.27$



$\Sigma F_x = 0$

$N_2 - 500 - 0.27N_1 = 0$

$N_2 = 500 + 0.27N_1$ ---------(1)

$\Sigma F_y = 0$

$N_1 - 2000 - 0.27N_2 = 0$

$0.27N_2 = N_1 - 2000$

$N_2 = 3.70N_1 - 7407.41$ ------(2)

From (1) & (2)

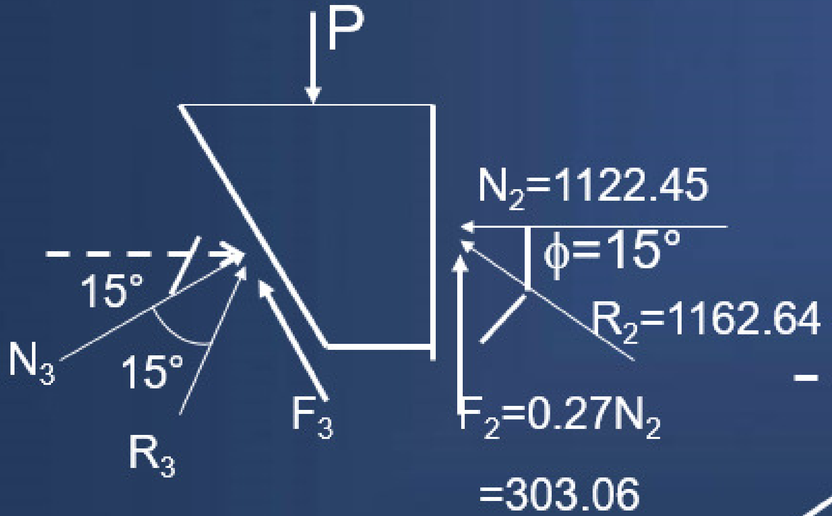
$500 + 0.27N_1 = 3.70N_1 - 7407.41$

$3.43N_1 = 7907.41$

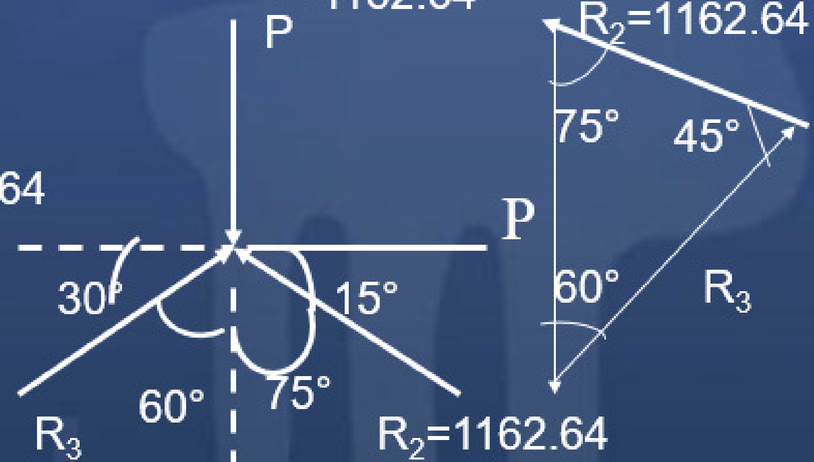
$\therefore\ N_1 = 2305.37\text{ N}$

$\therefore N_2 = 1122.45\text{N}$

FBD of Wedge A



$$R_2 = \sqrt{1122.45^2 + 303.06^2}$$
$$= 1162.64$$

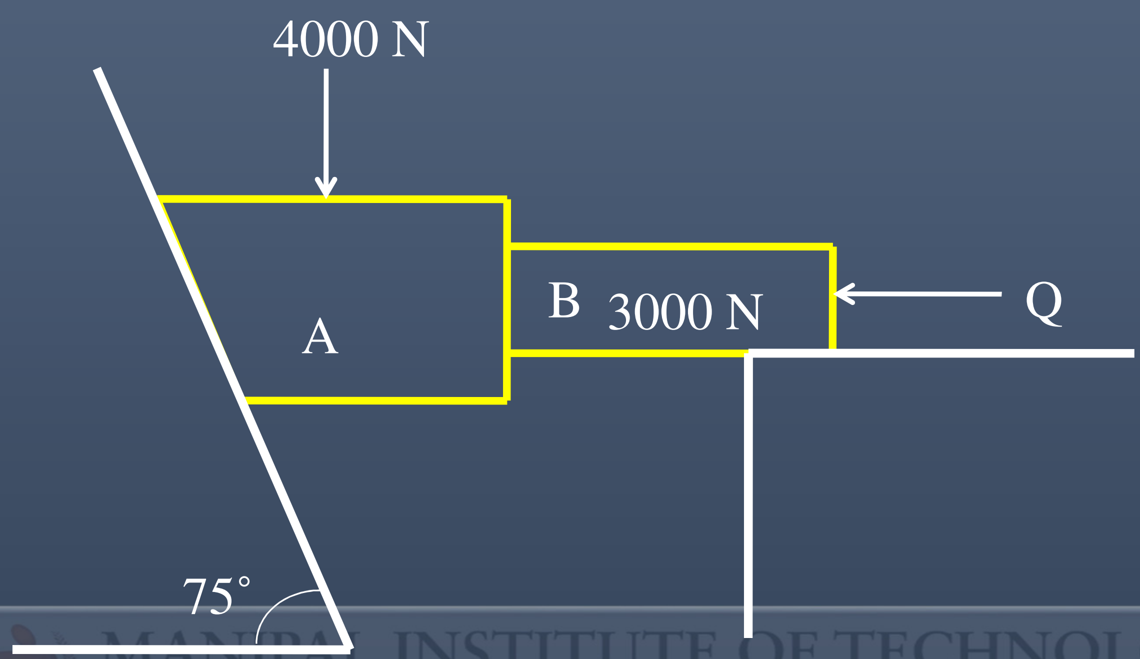


P/Sin45

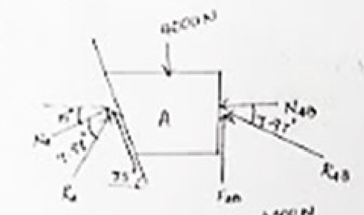
= 1162.64/ Sin60

$\therefore$ P = 949.29N

4. A 15° wedge is driven by a force 4000 N. Find force Q required to resist the motion of the wedge. Take μ=0.14 for all rubbing surfaces.



FBD of A.



$\mu = 0.14$

$\tan\phi = 0.14$

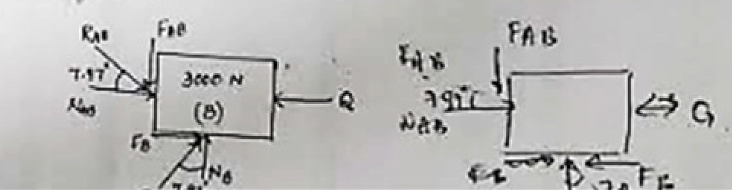
$\phi = 7.97°$

$$\frac{4000}{\sin 149.06} = \frac{R_A}{\sin 97.97} = \frac{R_{AB}}{\sin 112.97}$$

$$R_A = 4000\,\frac{\sin 97.97}{\sin 149.06} = \underline{7704.8\ N}$$

$$R_{AB} = 4000\,\frac{\sin 112.97}{\sin 149.06} = \underline{7163.10\ N}$$

FBD of B →



$\overrightarrow{\sum F_x} = 0$

$R_{AB}\cos 7.97 + R_B \sin 7.97 - Q = 0.$

$7163.1 \cos 7.97 + R_B \sin 7.97 - Q = 0.$

$7093.9 + R_B \sin 7.97 - Q = 0 \longrightarrow (1).$

$+\uparrow \sum F_y = 0$

$-R_{AB}\sin 7.97 - 3000 + R_B \cos 7.97 = 0.$

$-7163.1 \sin 7.97 - 3000 + R_B \cos 7.97 = 0.$

$-3993.2 + R_B \cos 7.97 = 0.$

$R_B = 4032.14\ N.$

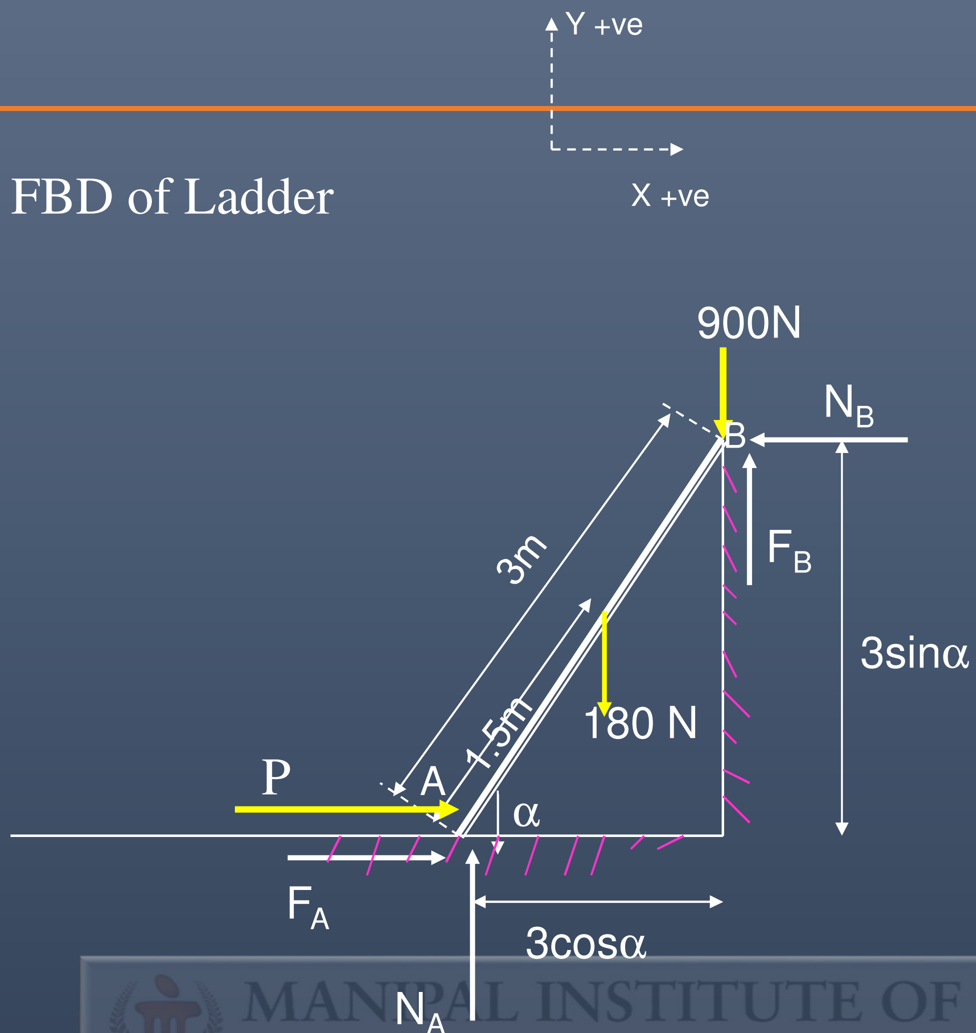
$(1) \Rightarrow 7093.9 + 4032.14 \sin 7.97 - Q = 0.$

$Q = \cancel{7652.97\ N}\ \ \underline{6534.8\ N}$

TUTORIAL 3

4. An uniform ladder 3m in length and weighing 180N is placed against a wall with its end **A** at the floor and the other end **B** on the wall, ladder **AB** making 60° with the floor. Coefficient of friction between the wall and ladder is 0.25 and between floor and ladder is 0.35. In addition to the self weight, the ladder has to support a person weighing 900N at its top **B**. To prevent slipping, a force **P** is applied horizontally at A at the level of the floor. Find the minimum force **P** required for this condition. Find also the minimum angle **α** at which the above ladder with the person at the top should be placed to prevent slipping without the horizontal force **P**.

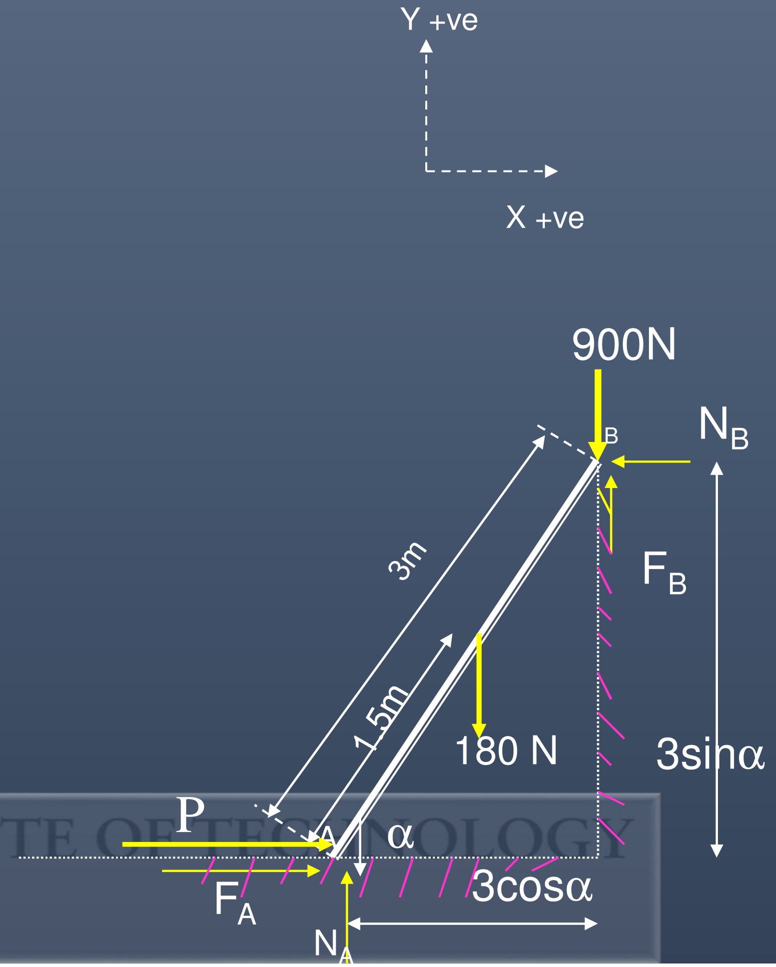
FBD of Ladder



a) When $\alpha = 60°$

$$\Sigma F_x = 0, \quad F_A+P-N_B=0$$
$$0.35N_A+P-N_B=0$$
$$N_B=P+0.35N_A \text{---------}(1)$$

$$\Sigma F_y=0, \quad N_A-180-900+F_B=0$$
$$N_A+0.25N_B=1080 \text{----}(2)$$



$$\Sigma M_B = 0$$

$$= 0.35N_A \times 3\sin 60 + P \times 3\sin 60$$

$$+ 180 \times 1.5\cos 60$$

$$- N_A \times 3\cos 60$$

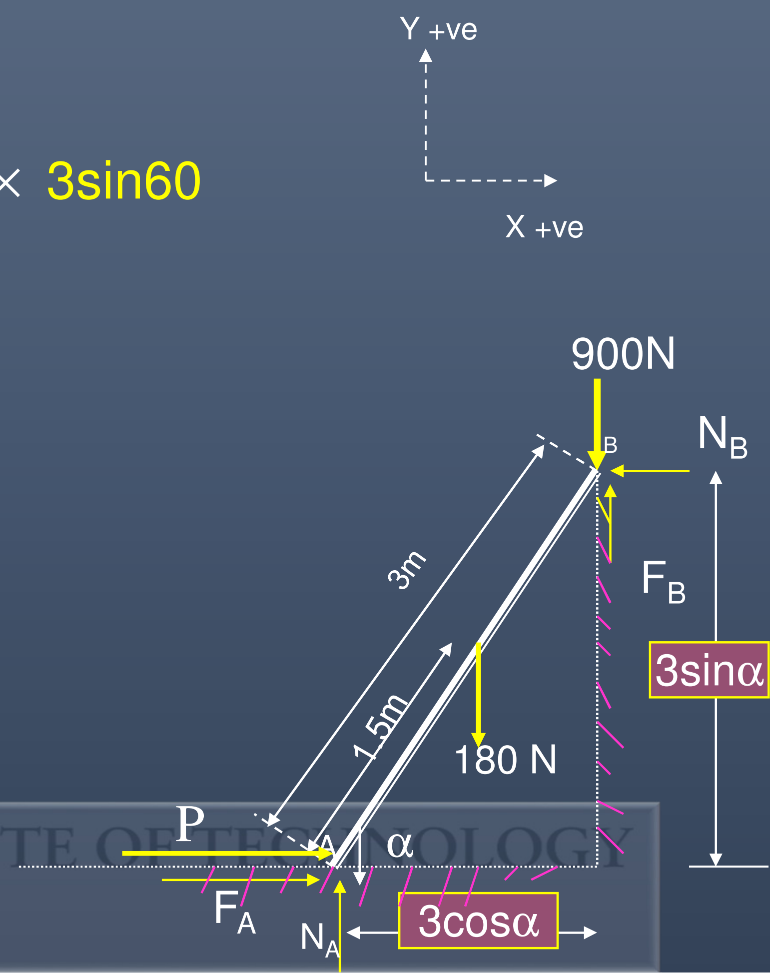
$$= 0 \qquad \text{--------}(3)$$

From(1),(2)&(3)

$N_B = 499.16N$

$N_A = 955.21N$

$\therefore P = 164.80N$



Y +ve

X +ve

(b) Force P is removed, α=?

$\Sigma F_x=0$

$F_A - N_B = 0 \quad 0.35N_A - N_B = 0 \quad 0.35N_A = N_B$---(1)

$\Sigma F_y=0$

$N_A - 180 - 900 + F_B = 0 \quad N_A + 0.25N_B = 1080$-----(2)

$\Sigma M_B=0$

$0.35N_A \times 3\sin\alpha + 180 \times 1.5\cos\alpha - N_A \times 3\cos\alpha = 0$----(3)

From(1), (2) &(3)

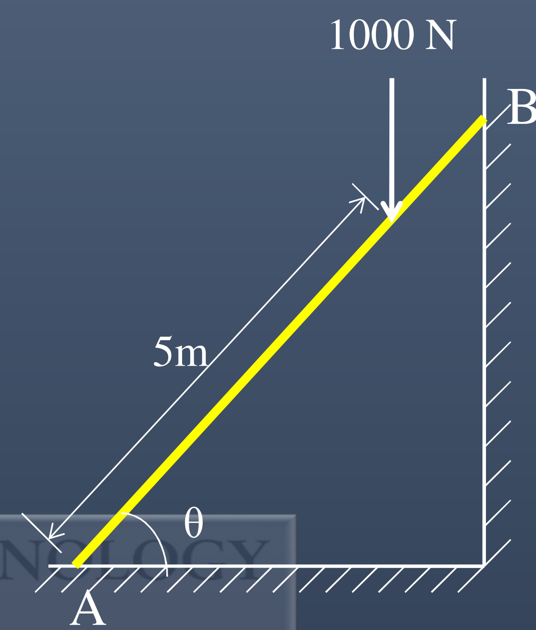
$\alpha=68.95°$

5. A ladder 6m long is supported as shown in figure. Coefficient of friction between floor and ladder is 0.5 and between wall is 0.25 and it supports a vertical load of 1000N.

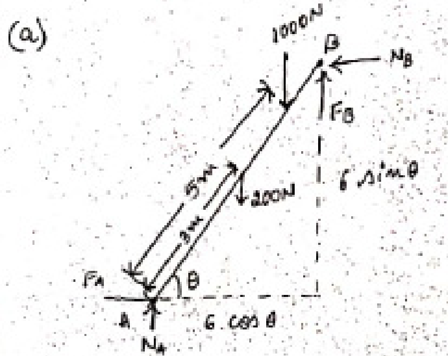
a) Determine the least value of θ at which the ladder may be placed without slipping.

b) Find the reaction at A and B.

Take weight of the ladder as 200N.



(a)



$F_B = 0.25 N_B$

$F_A = 0.5 N_A$

$\xrightarrow{+} \sum F_x = 0$

$F_A - N_B = 0$

$F_A = N_B \Rightarrow 0.5 N_A = N_B$

$N_A = 2 N_B$

$+\uparrow \sum F_y = 0$

$N_A - 200 - 1000 + F_B = 0$

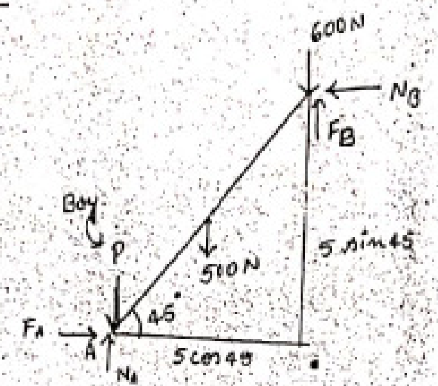
$N_A - 1200 + 0.25 N_B = 0$

$2 N_B - 1200 + 0.25 N_B = 0$

$N_B = 533.33\ N$

$N_A = 1066.67\ N$

case (b)



$F_A = 0.5 N_A$

$F_B = 0.2 N_B$

$\xrightarrow{+} \sum F_x = 0$

$F_A - N_B = 0$

$F_A = N_B$

$0.5 N_A = N_B \Rightarrow N_A = 2 N_B$

$+\uparrow \sum F_y = 0$

$N_A - P - 500 - 600 + F_B = 0$

$2 N_B - P - 1100 + 0.2 N_B = 0$

$2.2 N_B - P = 1100 \longrightarrow (1)$

$$+\circlearrowleft \sum M_A = 0$$

$$500 \times 2.5 \cos 45 + 600 \times 5 \cos 45 - 0.2 N_B \times 5 \cos 45 - N_B \times 5 \sin 45 = 0.$$

$$3005.2 - 4.24 N_B = 0.$$

$$N_B = \underline{708.3\ N}$$

$$N_A = 2N_B = 2 \times 708.3 = 1416.6\ N$$

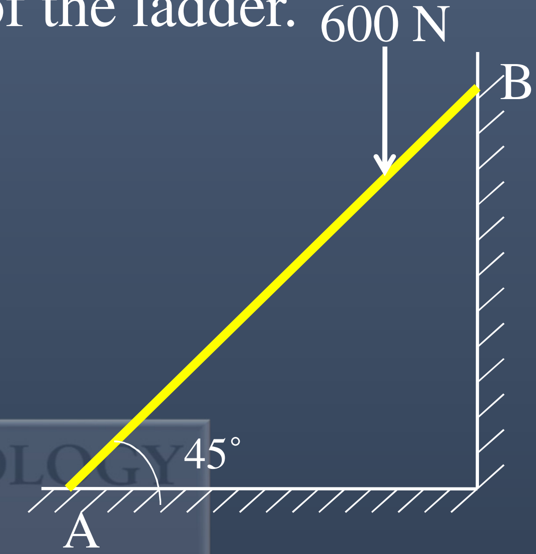
$$① \Rightarrow 2.2 \times 708.3 - 1100 - P = 0.$$

$$P = \underline{458.26\ N}$$

6. A ladder of length 5m weighing 500N is placed at 45˚ against a vertical wall. The coefficient of friction between the ladder and wall is 0.2 and ladder and ground is 0.5. If a man weighing 600N ascends the ladder, how high will he be when the ladder slips.

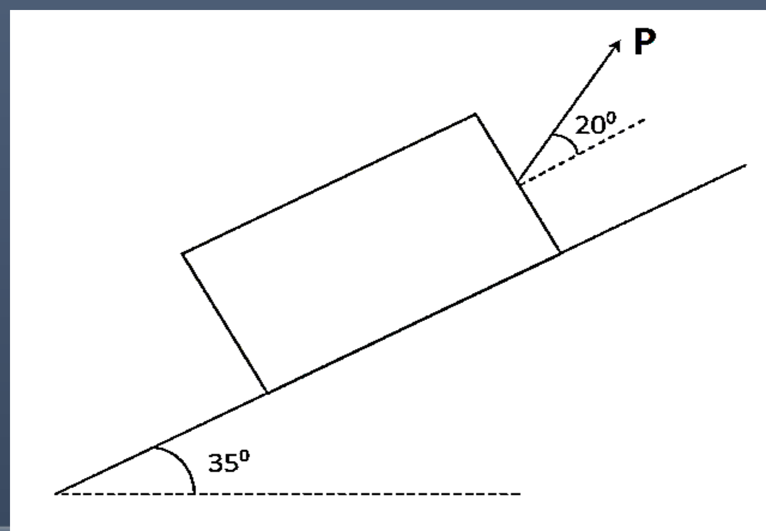
If a boy now stands on the bottom of the rung of the ladder what must be his least weight so that the man can go up to the top of the ladder.

[Ans.: (a) x = 2.92m (b) W_{boy} = 458N]

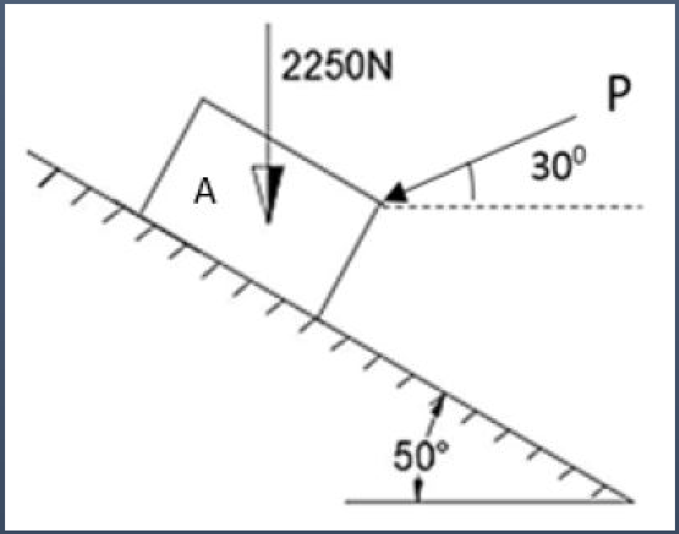


TUTORIAL (Additional)

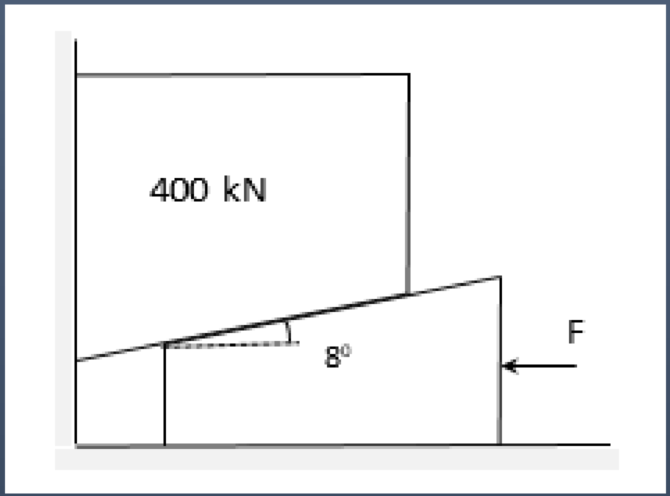
1. A block weighing 200 kN is resting on an inclined plane and is acted upon by force P as shown in the figure. If the coefficient of friction between the inclined plane and block is 0.3, calculate force P required to impend the block up the plane. (3 marks)



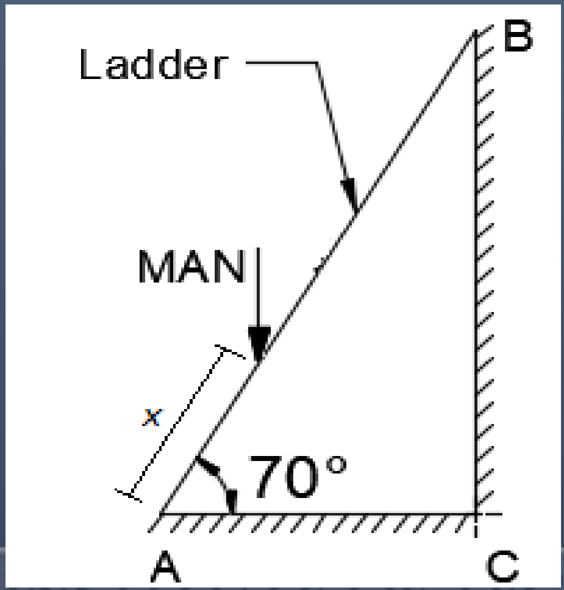
2. Block A shown in figure is in contact with 50° inclined plane. The coefficient of friction between plane and block is 0.25. Compute the value of force 'P' necessary to just prevent the motion of the block down the plane. (2 marks)



3. A block of weight 400 kN is lifted by a wedge as shown in the figure. Calculate force 'F' required to rise the block. Consider angle of limiting friction as 19° at all contact surfaces. (4 marks)



4. A man weighing 750 N starts to climb 7m long ladder weighing 250 N. Determine distance ‘x’ indicated in the figure when the ladder starts to slip. The coefficient of friction for all rubbing faces is 0.30. (3 marks)



5. Two blocks A and B are resting against a wall and the floor as shown in figure. Find the minimum value of horizontal force P applied to the resist the motion of the block A. Given coefficient of friction between all contact surfaces is 0.2.