EQUILIBRIUM OF COPLANAR CONCURRENT AND NON CONCURRENT FORCE SYSTEMS

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

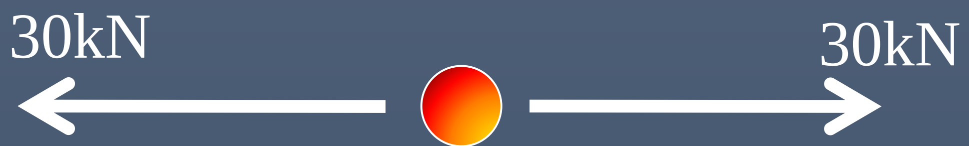
LECTURE 5

- Equilibrium of a particle
- Conditions for Equilibrium
- Space Diagram & Free Body Diagram (FBD)
- Application problems

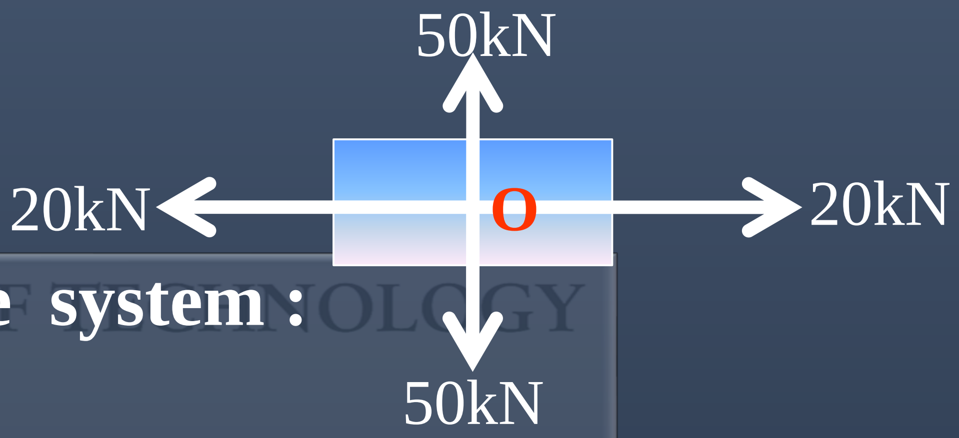
EQUILIBRIUM OF FORCE SYSTEMS

Equilibrium of a particle :

If the net effect of the given forces is zero, the particle is said to be in equilibrium.



A body is said to be in equilibrium when it does not have any translatory or rotatory motion in any direction.



Equilibrium of concurrent coplanar force system :

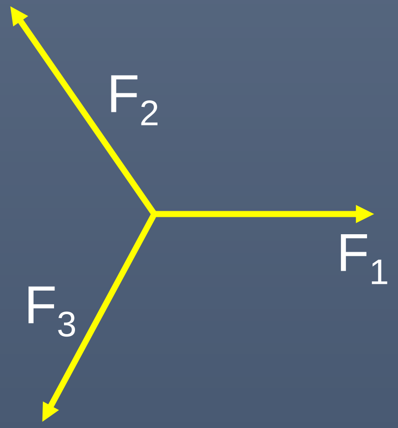
Equilibrium conditions:

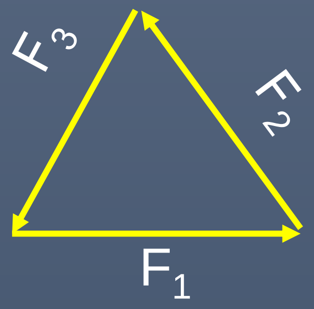
The algebraic sum of the components of the forces along each of three mutually perpendicular axes is zero.

In case of coplanar forces, $\Sigma F_X = 0$; $\Sigma F_Y = 0$

Triangle Law:

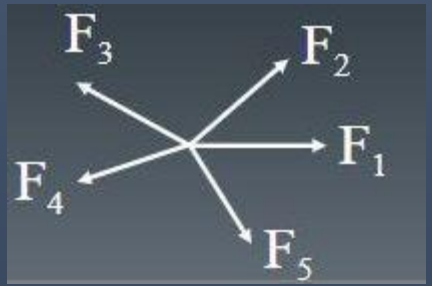
If three forces are in equilibrium, then, they form a closed triangle when represented in a Tip to Tail arrangement.

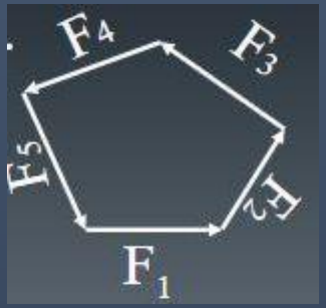




Polygonal Law:

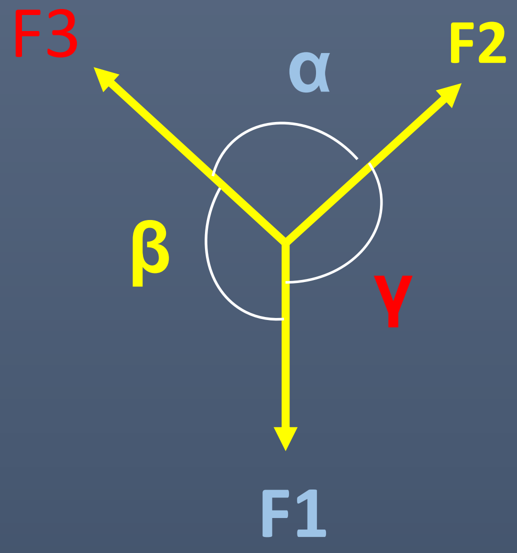
If more than three forces are in equilibrium, then, they form a closed polygon when represented in a Tip to Tail arrangement





Lami's theorem:

If a system of three "concurrent" forces is in equilibrium, then, each force of the system is proportional to sine of the angle between the other two forces (and constant of proportionality is the same for all the forces).



Note: While using Lami's theorem, all the three forces should be either directed away or all directed towards the point of concurrence.

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

Equilibrium of Non-concurrent Coplanar force system

When a body is in equilibrium, it has neither translatory nor rotatory motion in any direction.

Thus the resultant force R and the resultant couple M are both zero, and we have the **equilibrium equations** for two dimensional force system as

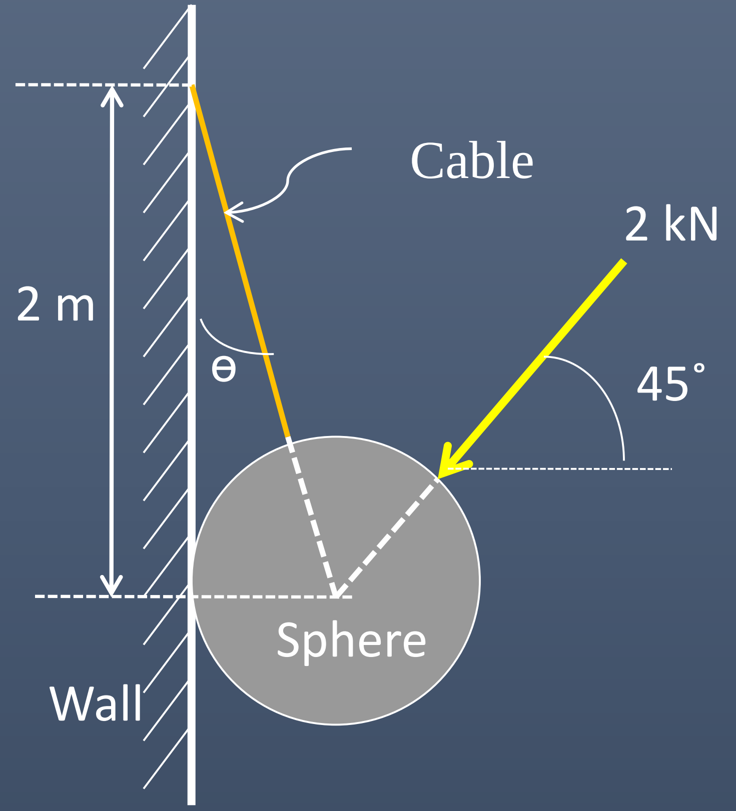
$$\sum F_x = 0; \qquad \sum F_y = 0 \quad \& \quad \sum M = 0$$

These requirements are both necessary and sufficient conditions for equilibrium.

Space and Free body diagrams

Space diagram :

The sketch showing the physical conditions of the problem, like, the nature of supports provided; size, shape and location of various bodies; forces applied on bodies, etc.,



Wt. of the sphere = 1 kN

Radius of the sphere = 0.5 m

Figure 2

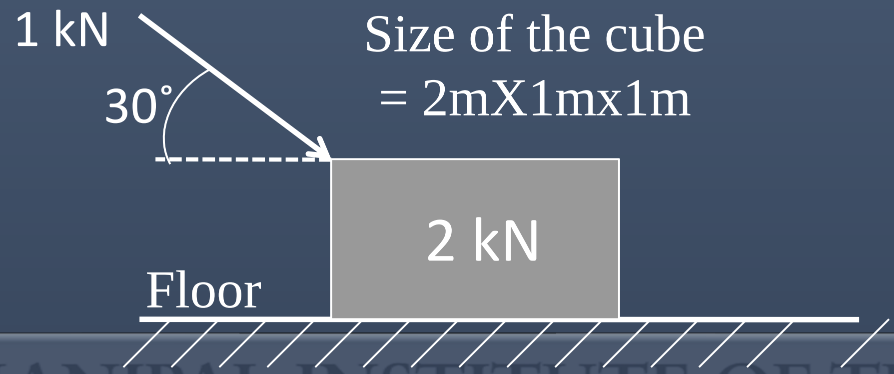


Figure 1

Free body diagram :

An isolated view of a body which shows only the external forces acting on the body

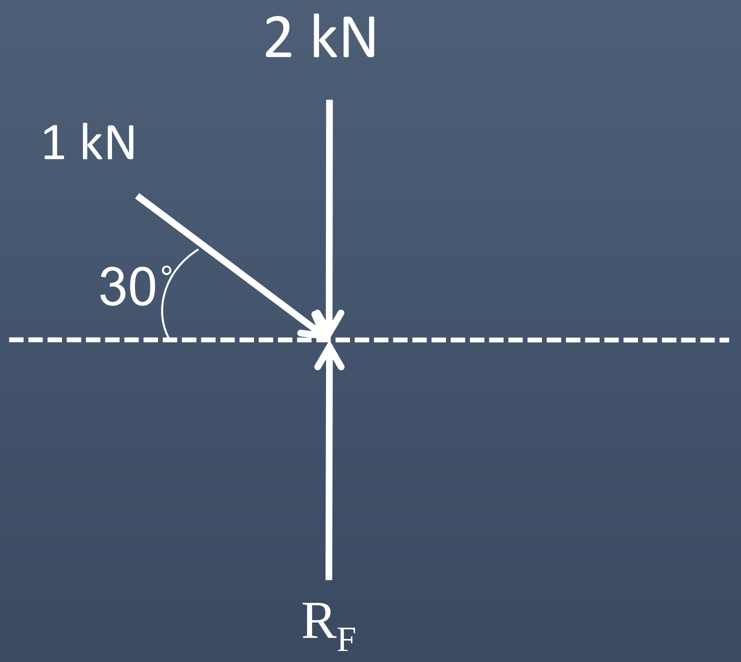


Figure 1

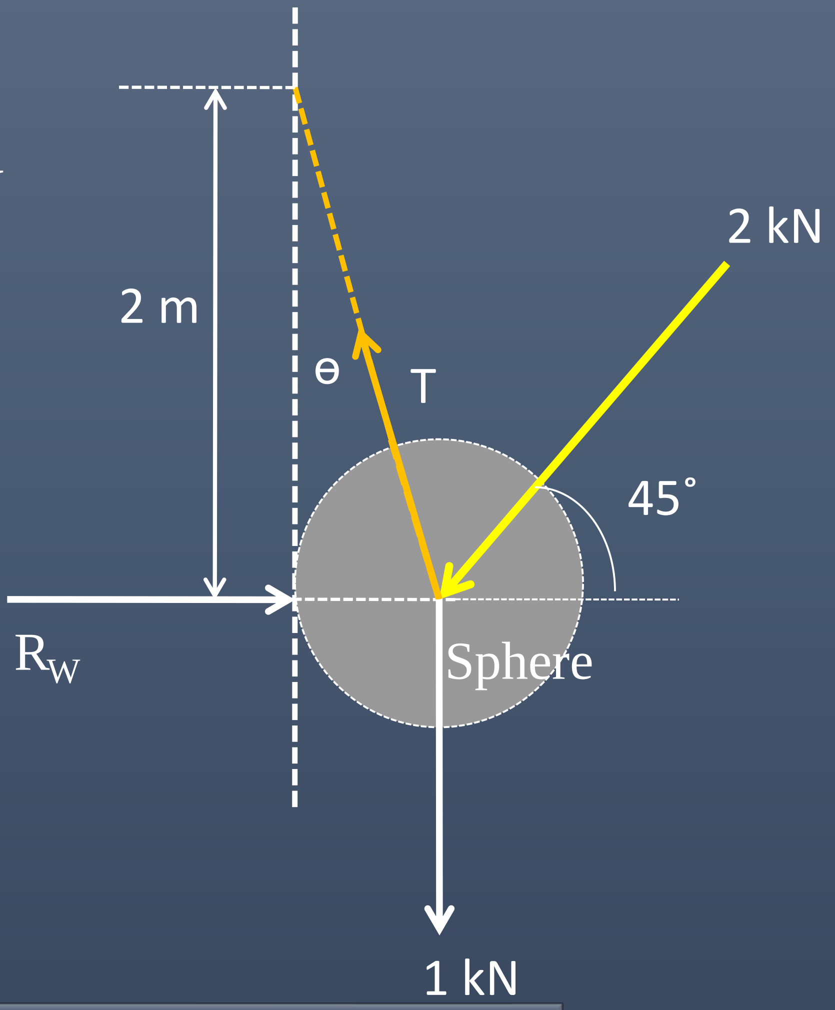


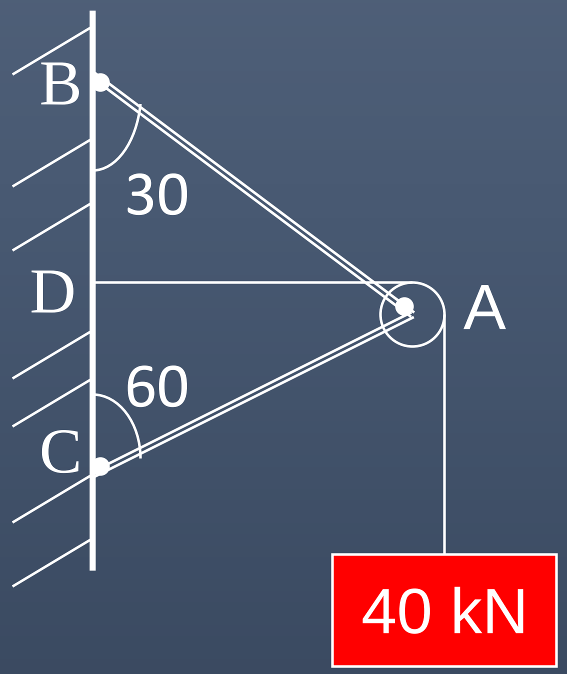
Figure 2

Few Guidelines for Drawing FBD:

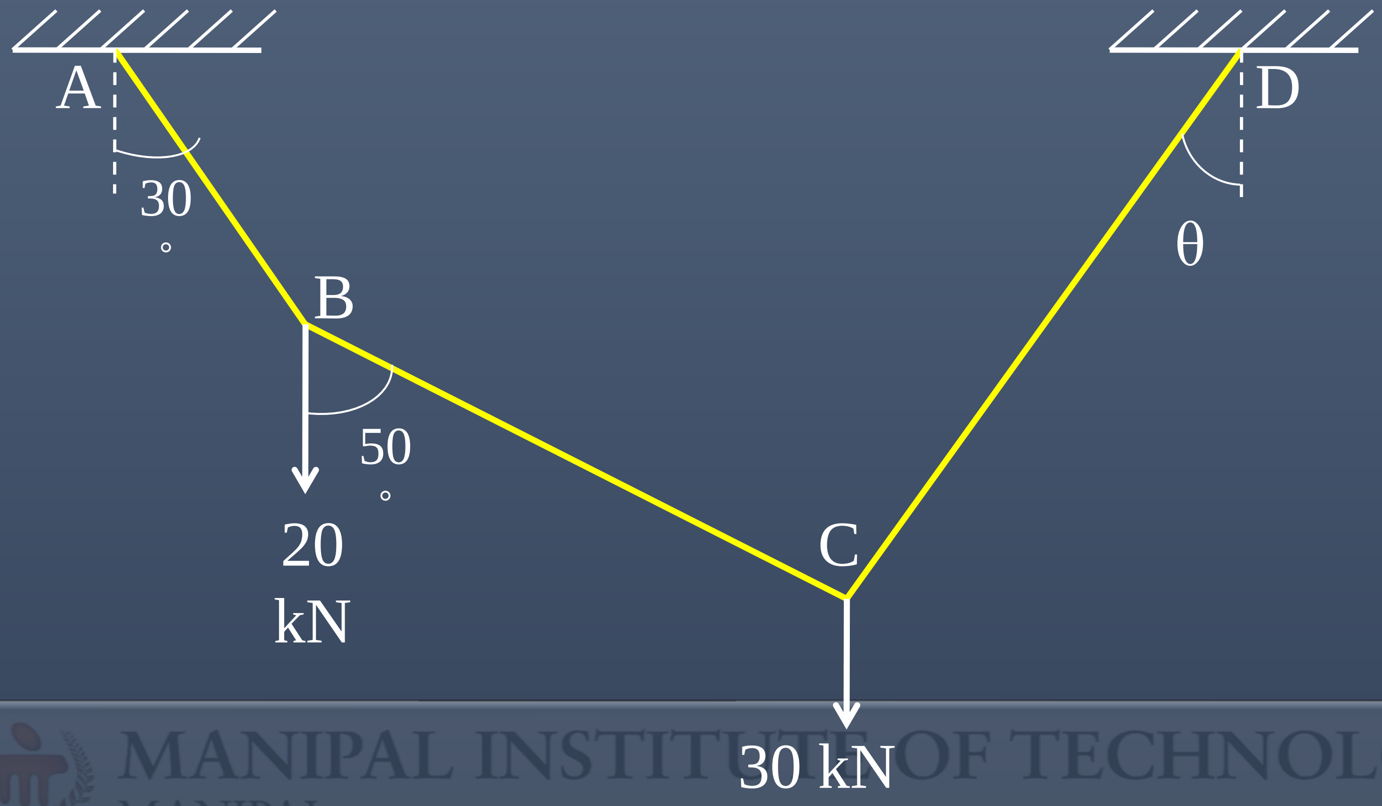
1. Tensile Force – vector directed away from the body
2. Compressive Force - vector directed towards the body
3. Reactions at smooth surfaces - normal to the surface and pointing towards the body
4. Forces in Link rods/connecting rods – along the axis (towards or away from the body)
5. Forces in Cables (Strings or Chords) - along the cable and directed away from the body (tension)

Application Problems

1. Determine the magnitude and nature of the forces in the bars AB and AC shown in figure. Neglect size and weight of the pulley.



2. Determine the tension in the strings AB, BC, CD and inclination of the segment CD to the vertical, in the system shown.



LECTURE 6

- Types of Beam supports & Support reactions
- Types of Beams
- Types on Loads on beam

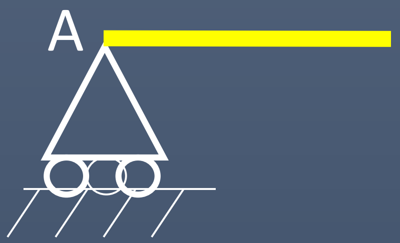
Types of Beam Supports & Support Reactions

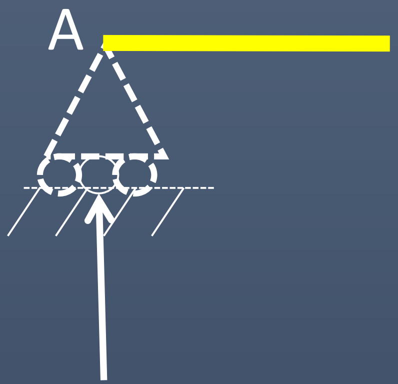
Beam: A beam is a structural element which is capable of withstanding load primarily by resisting bending.

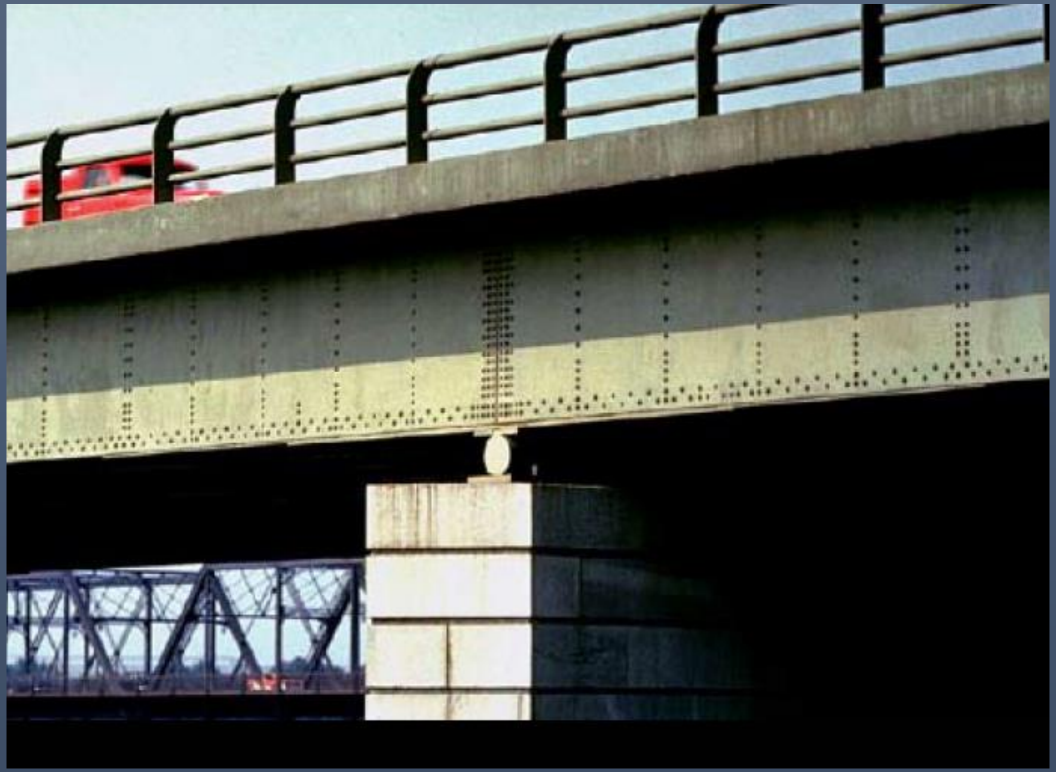
Supports: A structure is subjected to external forces and transfers these forces through the supports on to the foundation. Therefore the support reactions and the external forces together keep the structure in equilibrium.

1. Roller Support
2. Hinged or pinned support
3. Fixed or built in support

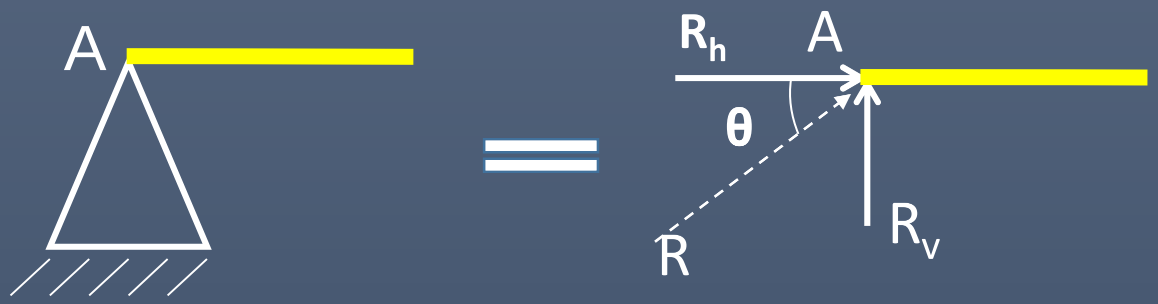
1. Roller Support

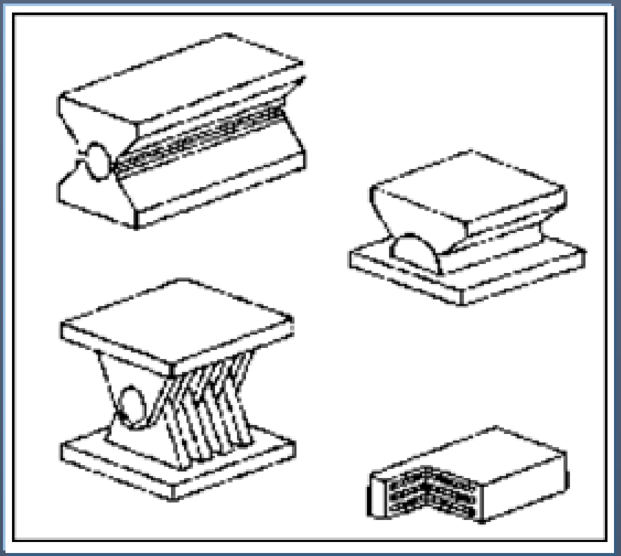


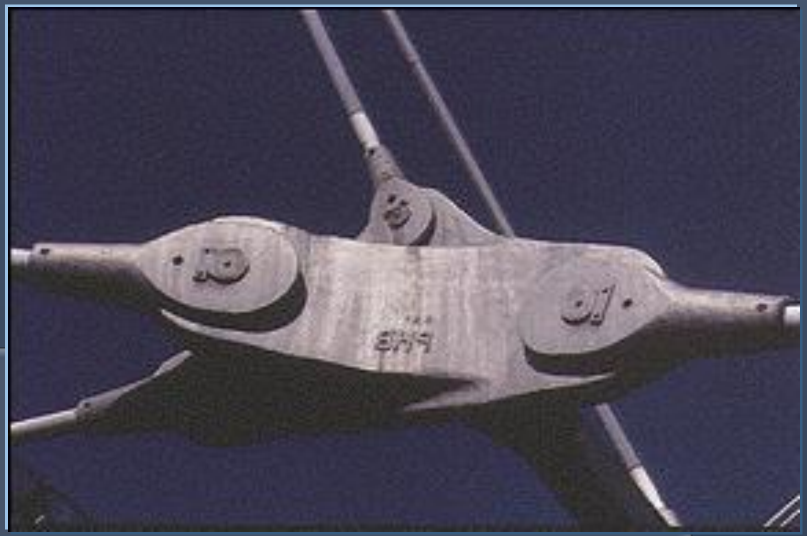


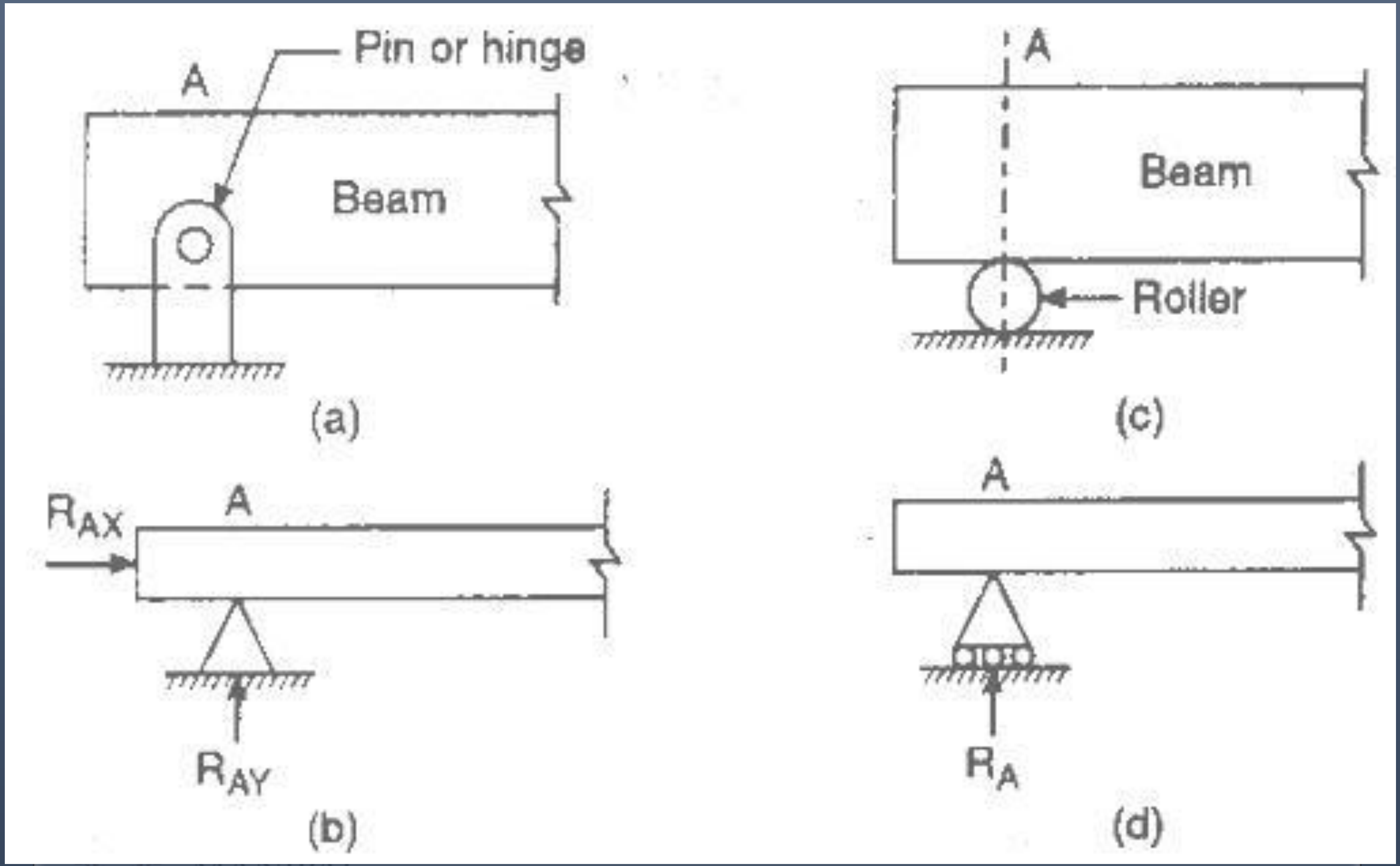


2. Hinged or pinned support

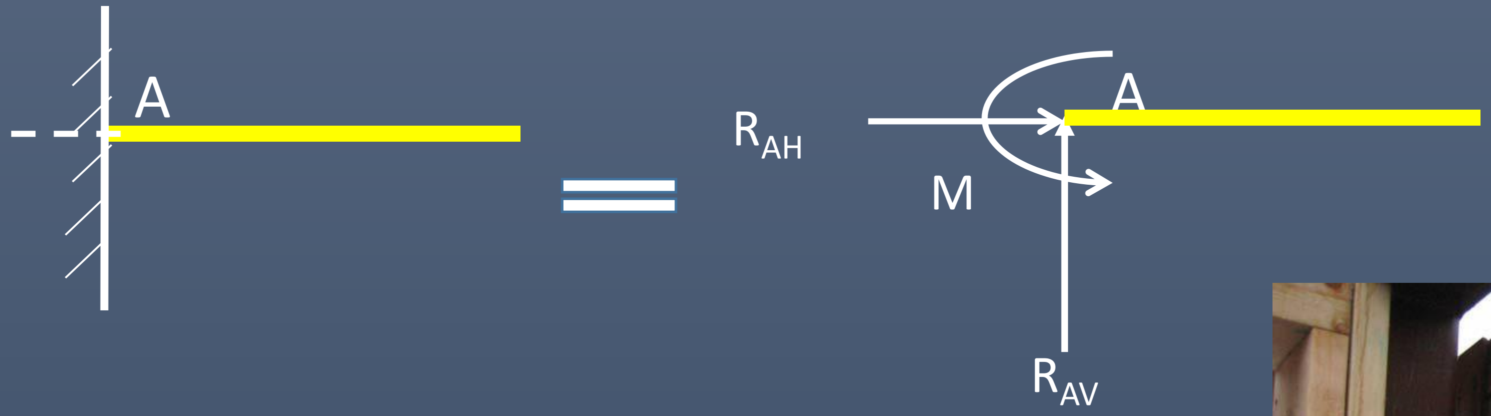


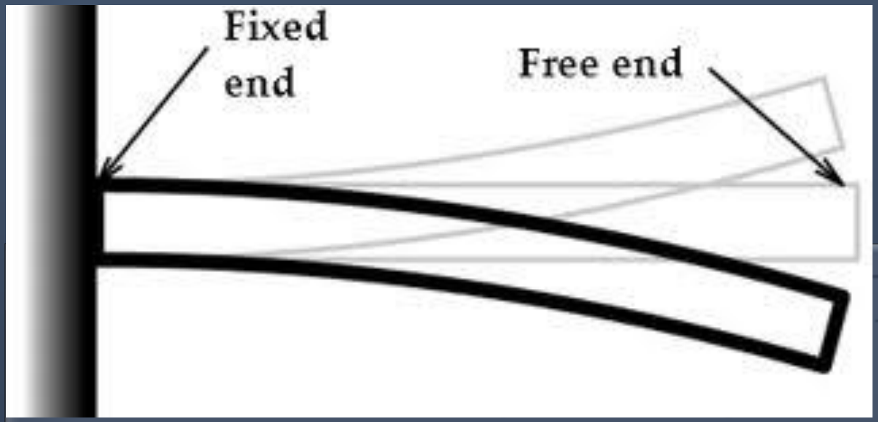





Pin or hinge
A
Beam
(a)
R_{AX}
A
R_{AY}
(b)
A
Beam
Roller
(c)
A
R_A
(d)

3. Fixed or Built-in Support

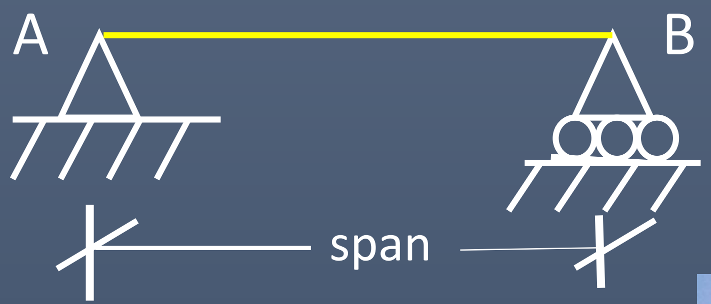


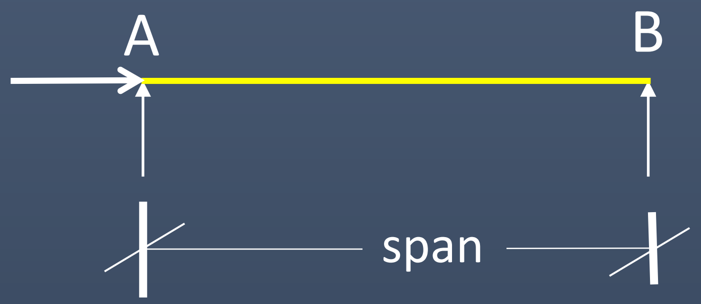


Types of Beams

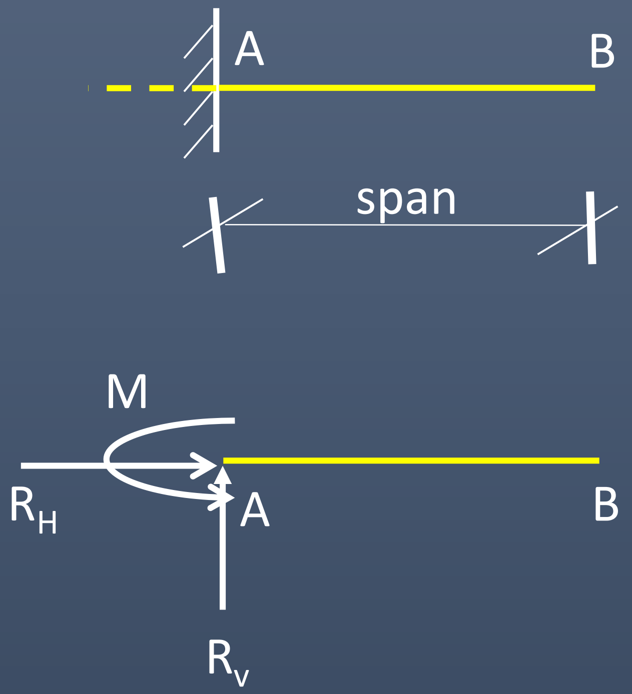
1. Simply supported beam
2. Cantilever beam
3. Overhanging beam

1. Simply Supported Beam

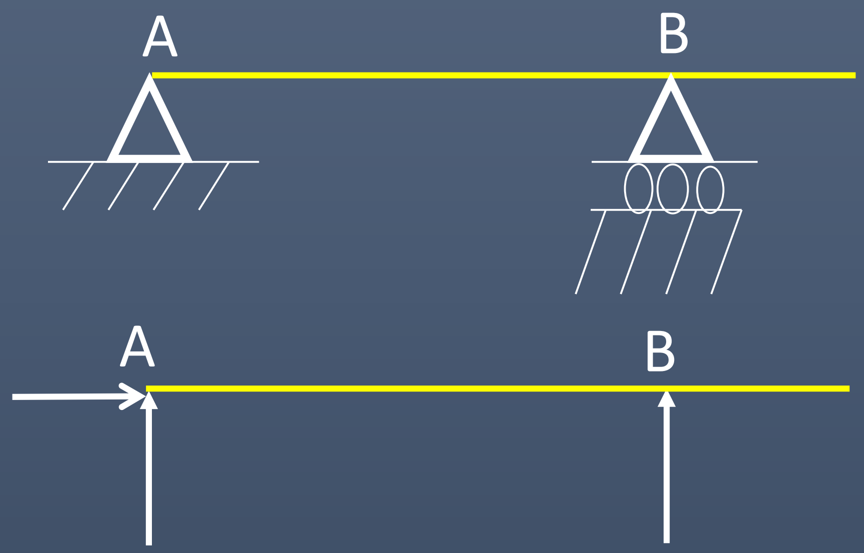




2. Cantilever Beam

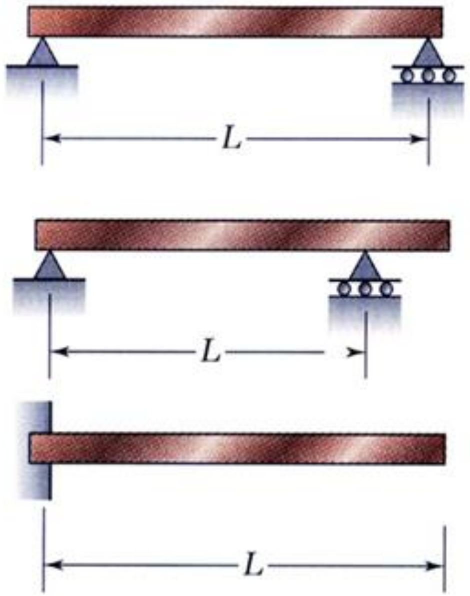


2. Overhanging Beam



Statically Determinate & Indeterminate Beams

Statically Determinate

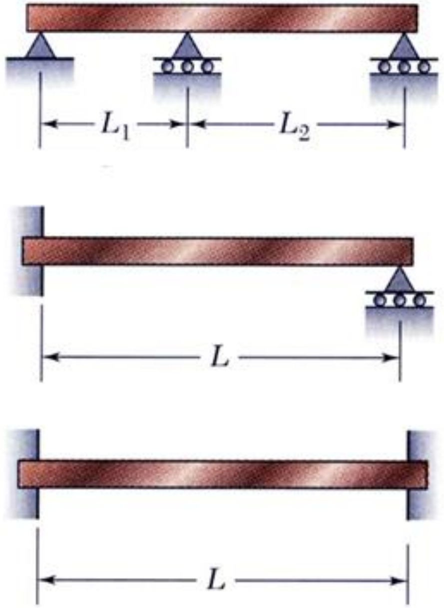


Simply Supported Beam

Overhanging Beam

Cantilever Beam

Statically Indeterminate



Continuous Beam

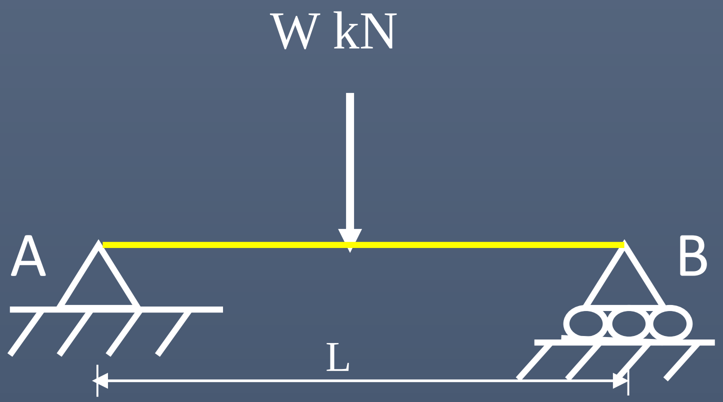
Propped Cantilever Beam

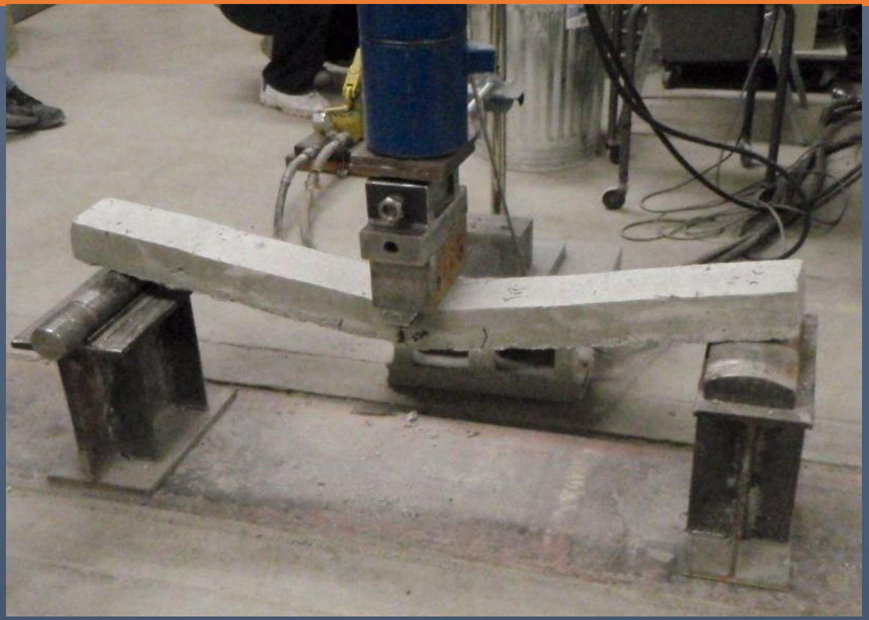
Fixed Beam

Loads on Beam

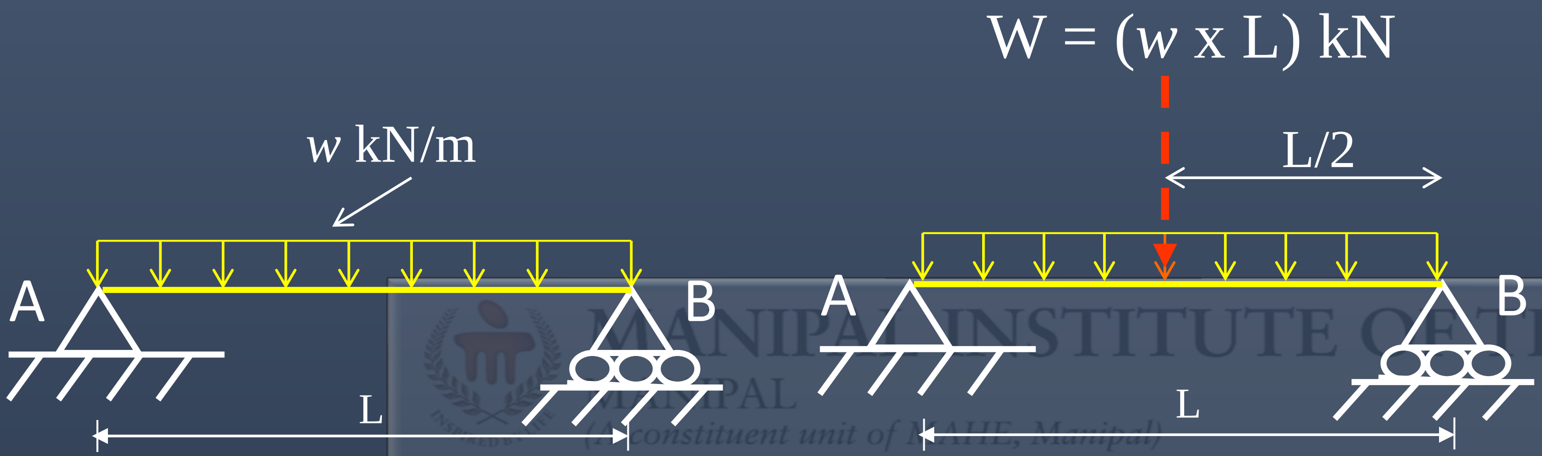
1. Concentrated or Point load
2. Uniformly distributed load (UDL)
3. Uniformly varying load (UVL)

1. Concentrated or Point Load



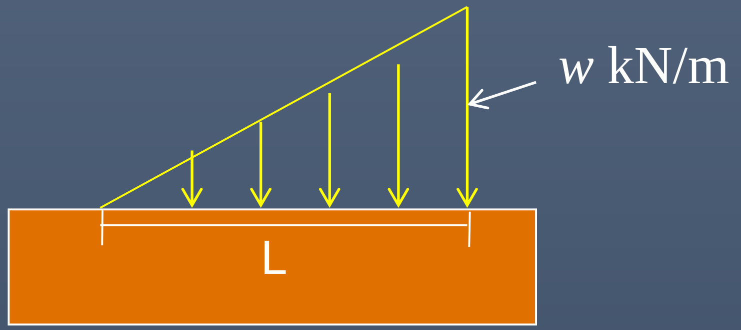


2. Uniformly Distributed Load (UDL)

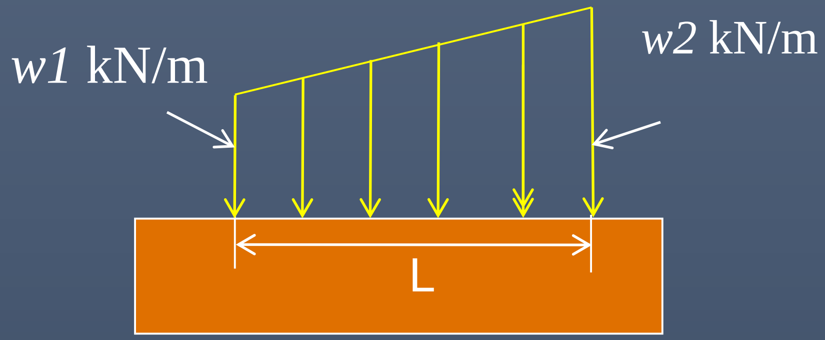


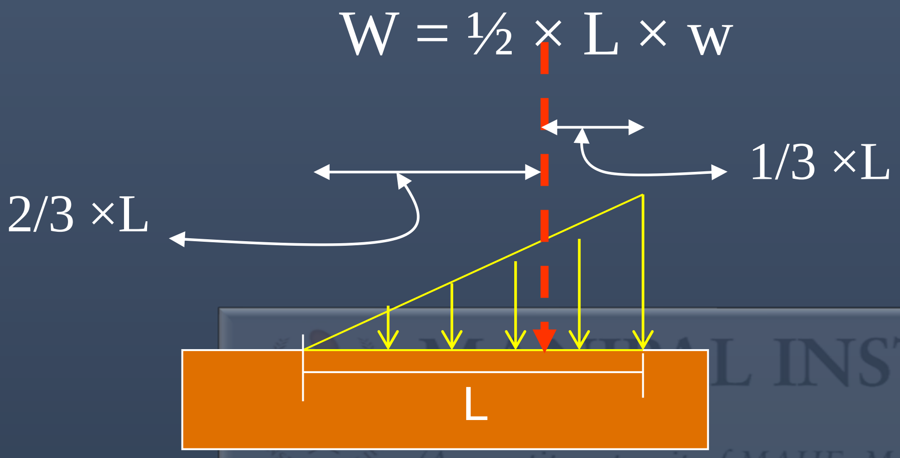
3. Uniformly Varying Load (UVL)

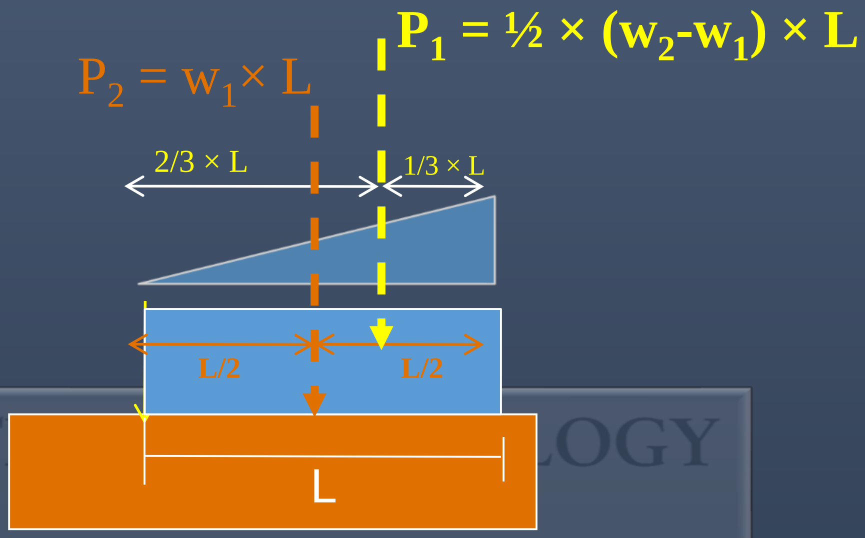
- Triangular Load



- Trapezoidal Load



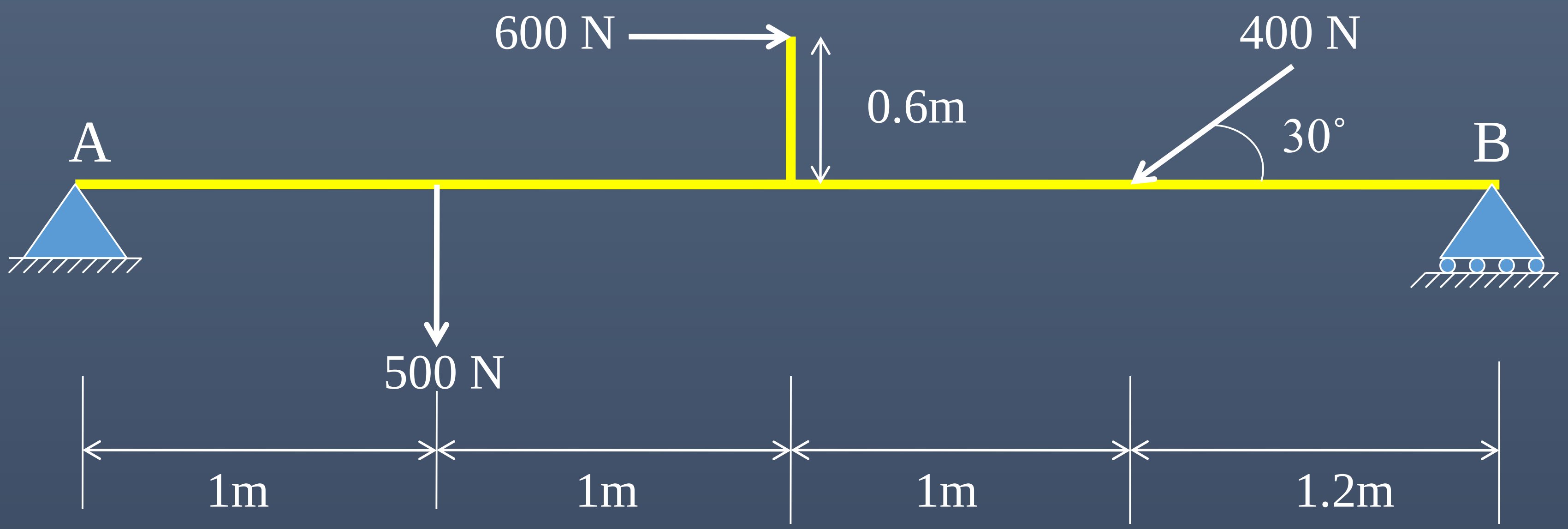




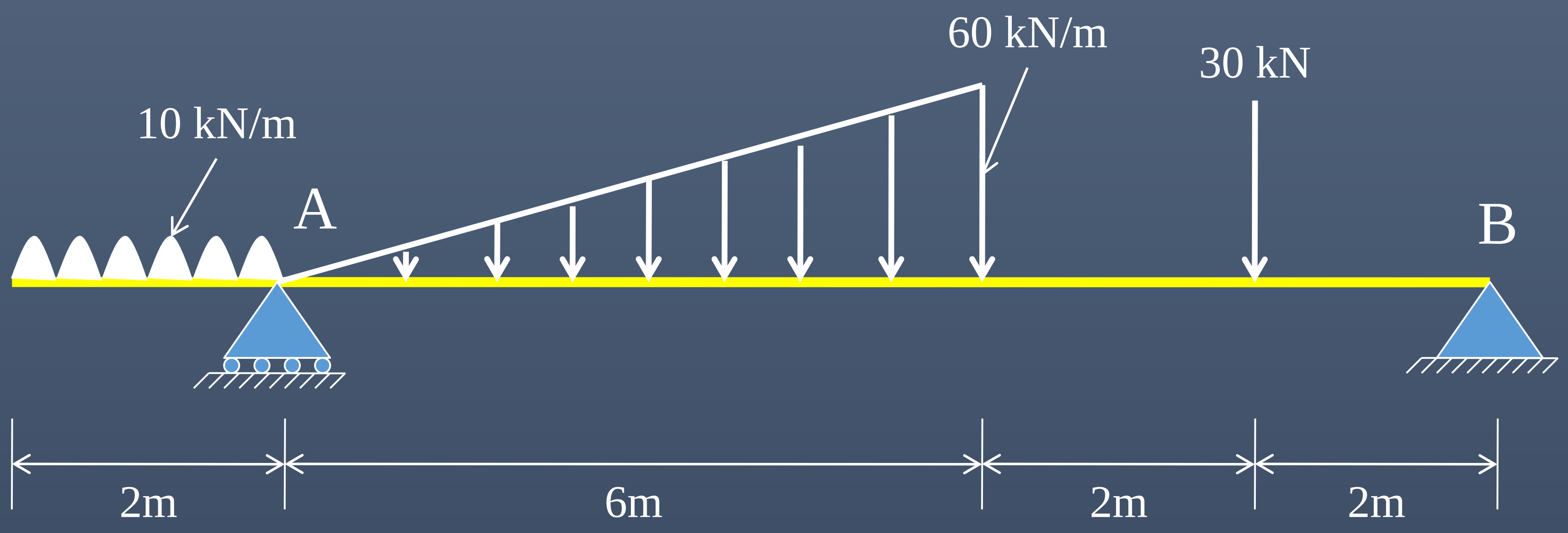
LECTURE 7

- Application Problems Continued…

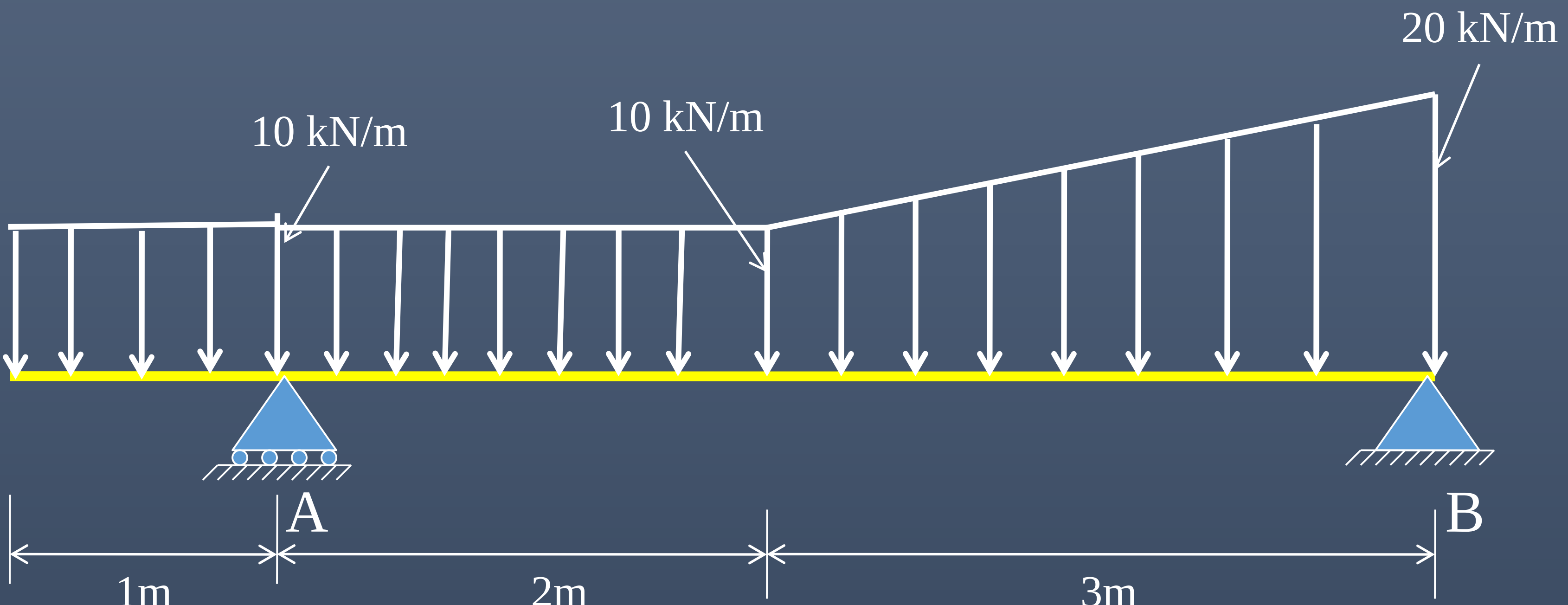
3. Find the support reactions for the beam loaded shown in figure.



4. An overhanging beam is on roller at A and is hinged at B. Determine the reactions at A and B.

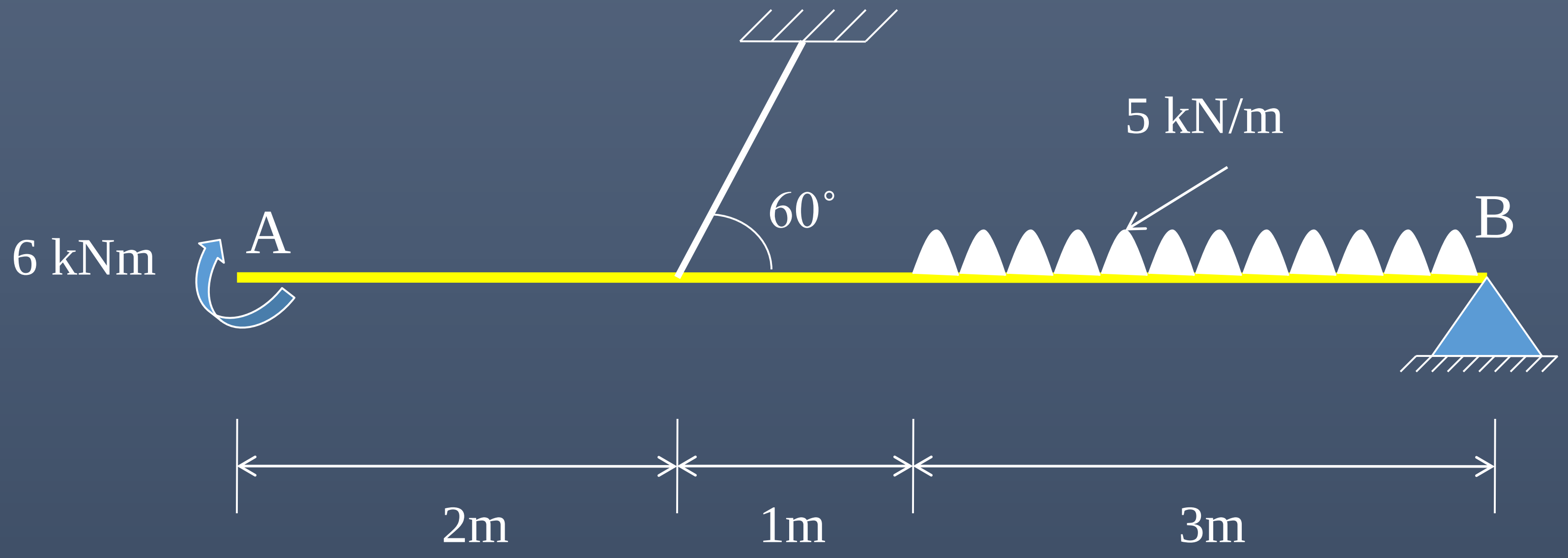


5. Determine the support reactions at A and B.

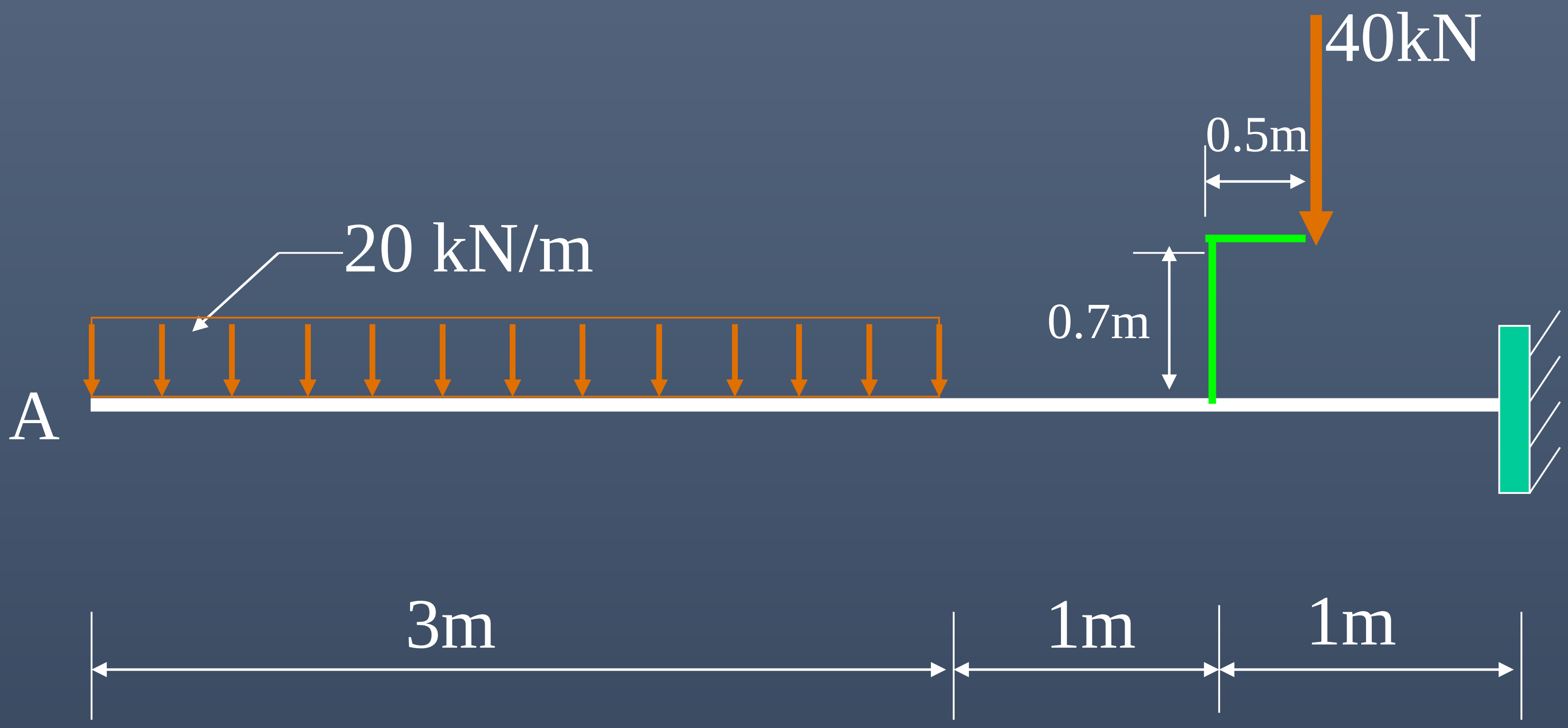


TUTORIAL 2

7. Find the tension in the rope and reaction at B.



8. Determine the reactions in the cantilever beam shown.



LECTURE 8

FRICTION

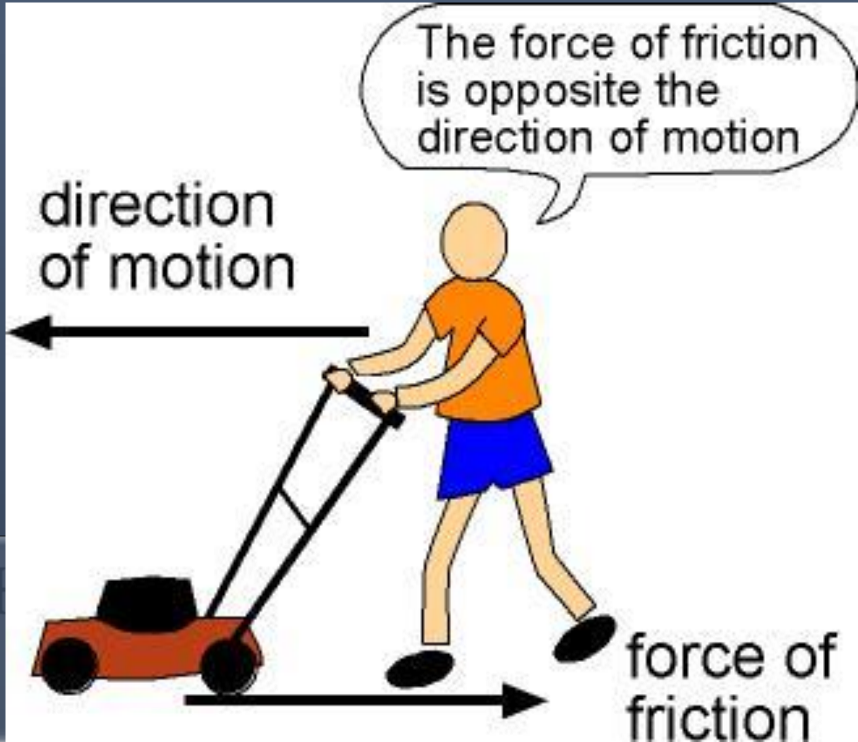
- Introduction
- Terms and Definitions
- Static, Limiting and Kinetic Friction
- Laws of dry friction
- Application Problems

Introduction

Force which opposes the movement or tendency of movement is known as frictional resistance or friction.

Defined broadly as the resistance that occurs between two bodies in contact when they tend to slide or roll relative to one another.

A frictional force is a shear force that acts tangent to the surface of contact between two bodies. This force opposes sliding motion between the bodies.



Plays very important role in the analysis of the performance of various mechanical devices.

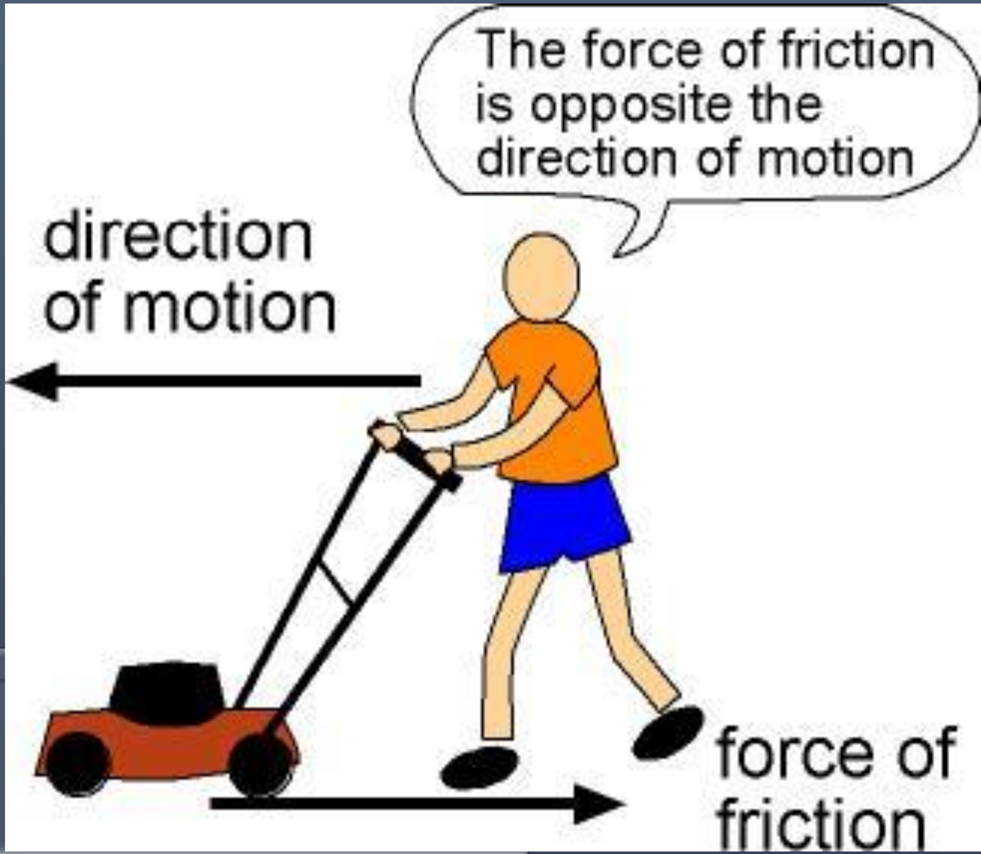
Friction – Beneficial and harmful

Advantages:

Walk, Traction, Brake, grind and polish

Disadvantages:

Power loss, wear and tear



Causes

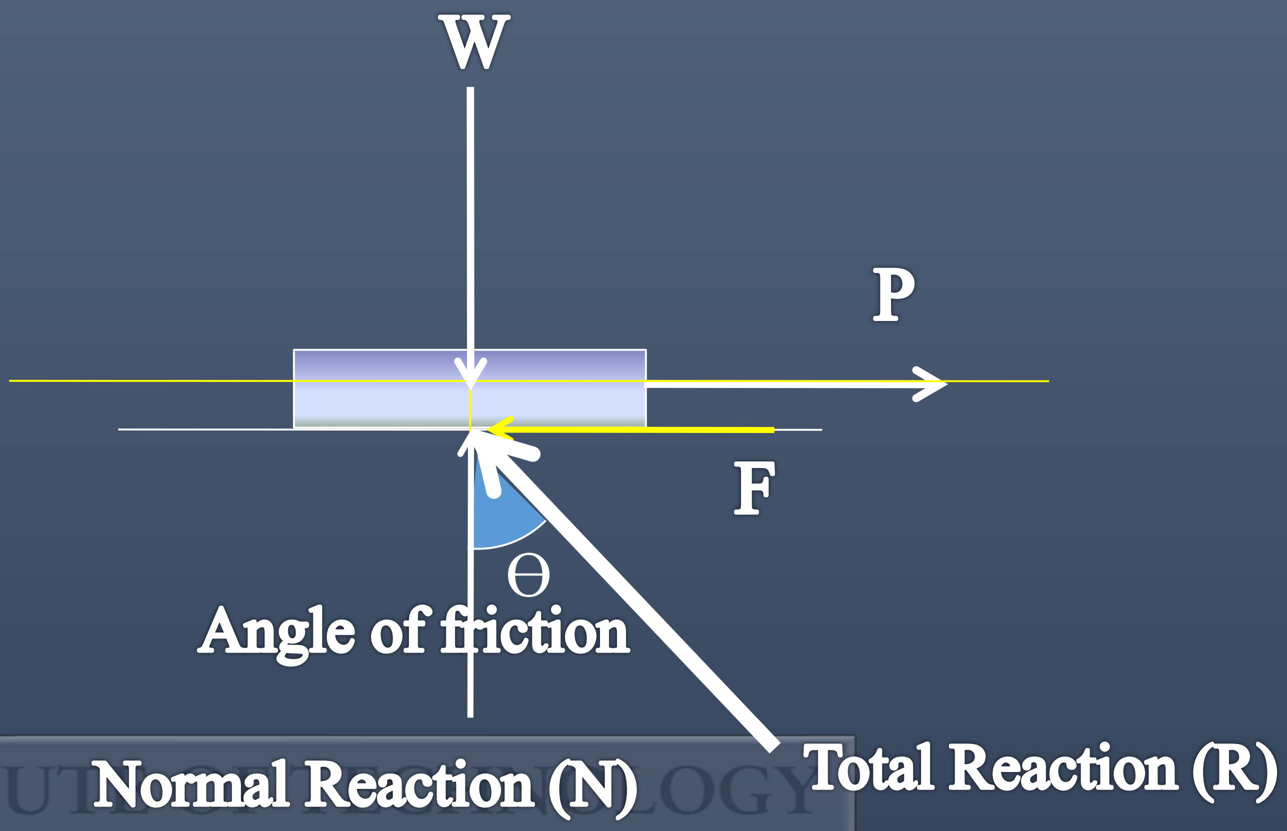
1. Microscopic irregularities (roughness) on surface

2. Adhesion b/w surfaces --- Molecular attraction (high for highly polished surfaces)

3. An indentation on one body caused by a second, harder body

Terms and Definitions

- Coefficient of friction (μ)
- Total reaction (R)
- Angle of Friction (Θ)
- Angle of limiting friction (ϕ)
- Angle of repose (α)



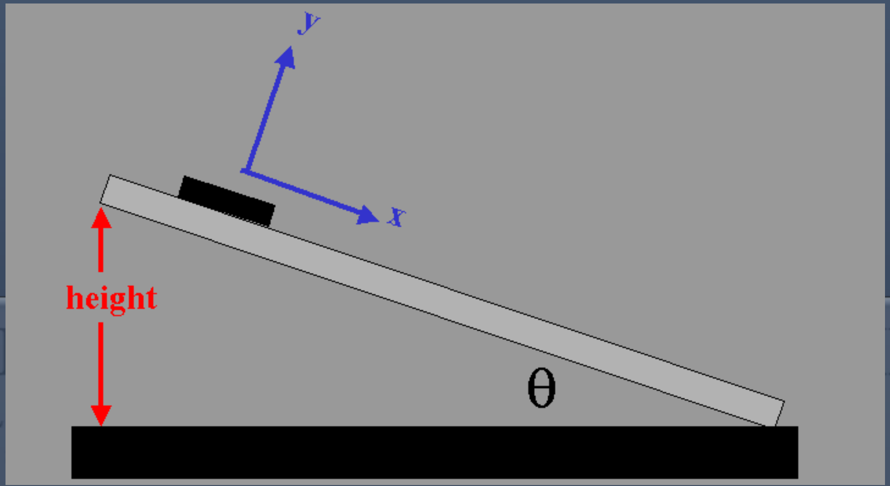
- **Coefficient of friction (μ)** = F_{max} / N

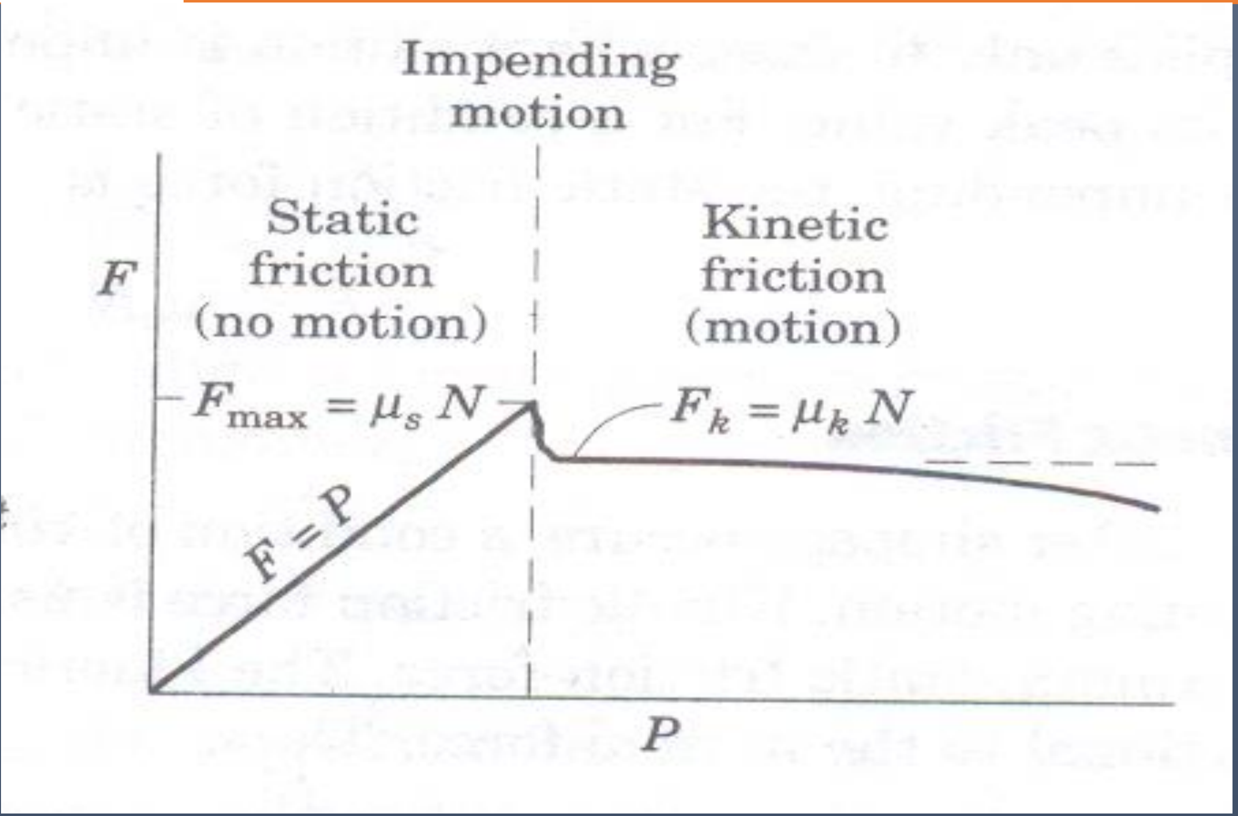
- **Total reaction :** Frictional force and the normal reaction could be replaced by a single force called **Total reaction.** **R** = $\sqrt{(F^2 + N^2)}$

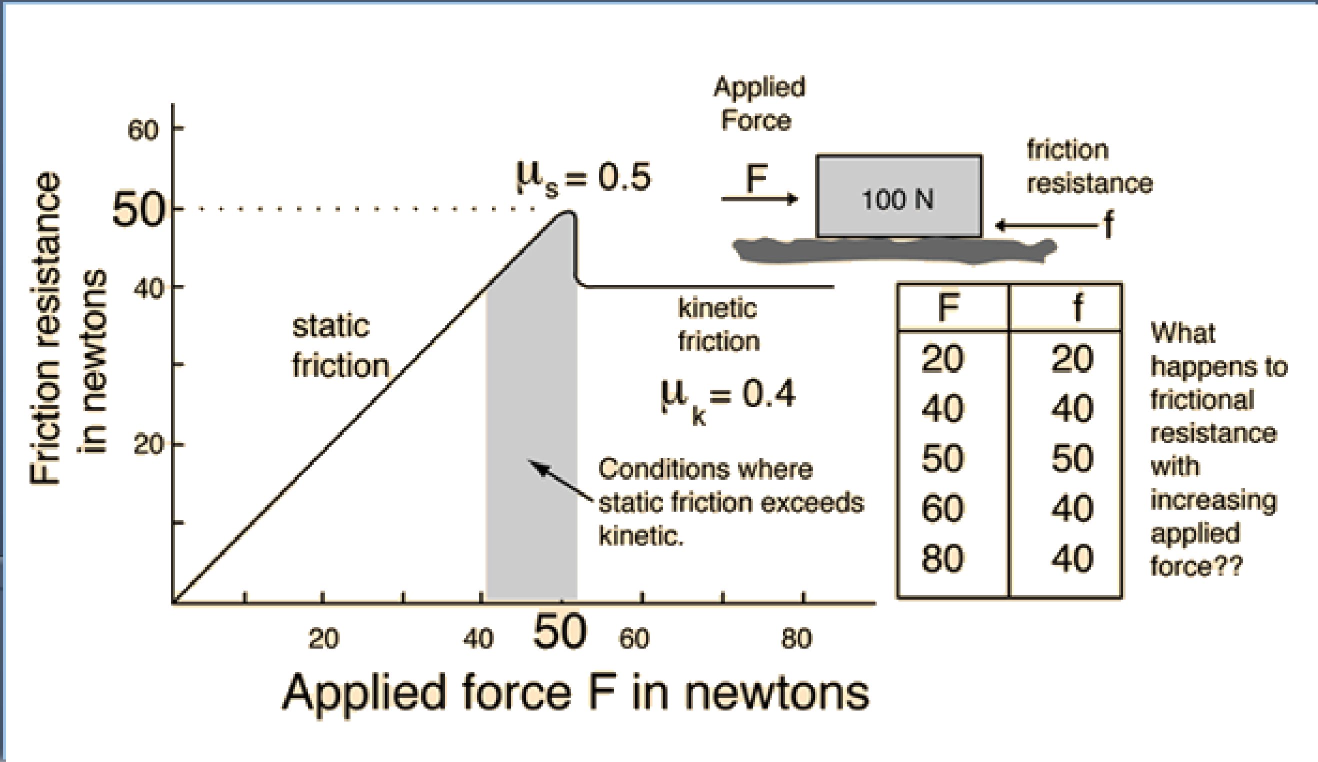
- **Angle of Limiting Friction (ϕ) :** An angle made by the total reaction (R) with the normal reaction (**N**) when F has reached F_{max}. **tanϕ = F_{max} / N**

- **Angle of Repose (α) :** The limiting angle up to which the grains repose (sleep/rest) is called the angle of repose of the granular material

- **Limiting Friction:** The maximum frictional force that can arise before an object begins to slide.
- **Static friction:** Body remains at rest when the applied force is less than the limiting friction.
- **Dynamic friction OR Kinetic friction:** Frictional resistance experienced by the body while in motion.



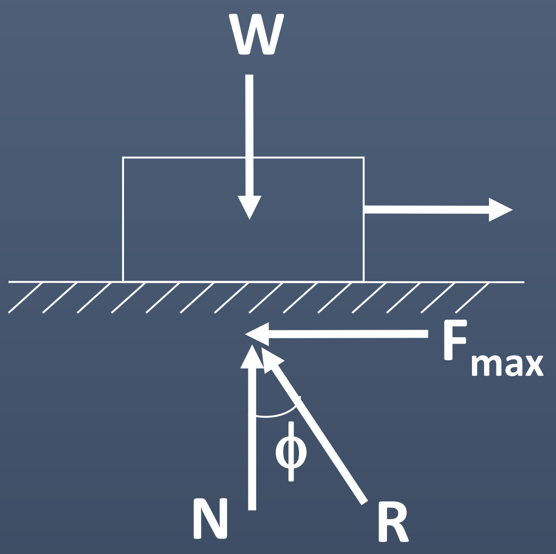

Impending motion
Static friction (no motion)
Kinetic friction (motion)
F
$F_{max} = \mu_s N$
$F_k = \mu_k N$
F = P
P

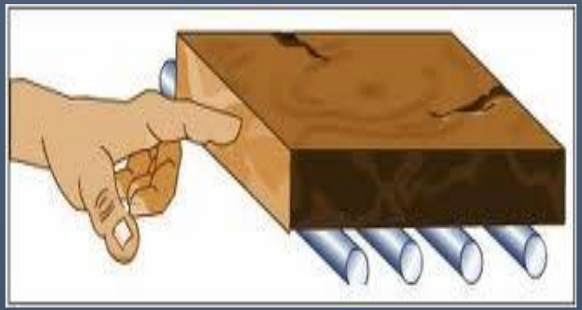

Friction resistance in newtons
60
50
40
20
$\mu_s = 0.5$
Applied Force
F
100 N
friction resistance
f
static friction
kinetic friction
$\mu_k = 0.4$
Conditions where static friction exceeds kinetic.
F f
20 20
40 40
50 50
60 40
80 40
What happens to frictional resistance with increasing applied force??
20
40
50
60
80
Applied force F in newtons

Kinetic/Dynamic Friction

Sliding friction → friction experienced when a body slides over another surface.

Rolling friction → friction experienced by a body when it rolls over a surface.





$$F \,\alpha\, N$$

$$F_{max} = \mu N$$

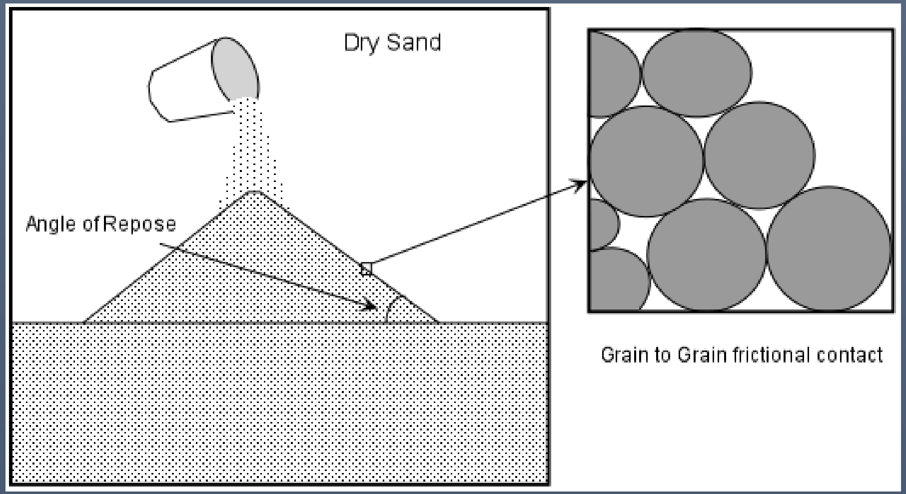
$$\mu = \frac{F_{max}}{N}$$

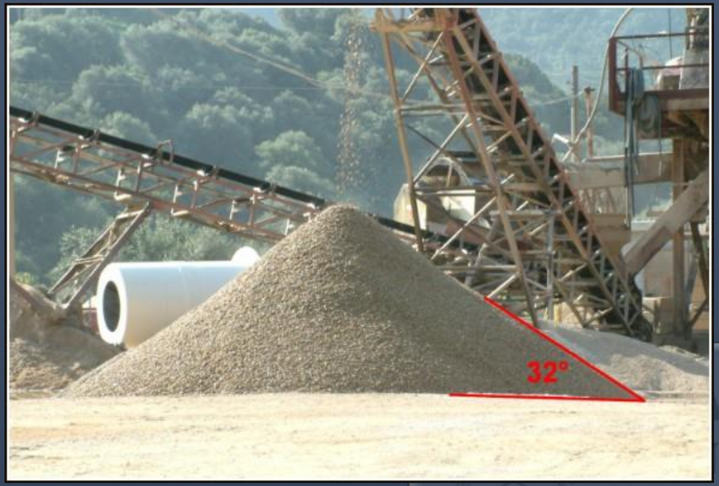
Where F_{max} = Limiting Friction

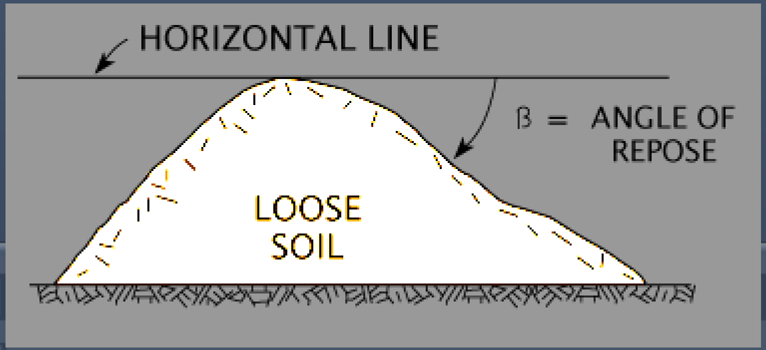
N = Normal Reaction between the contact surfaces

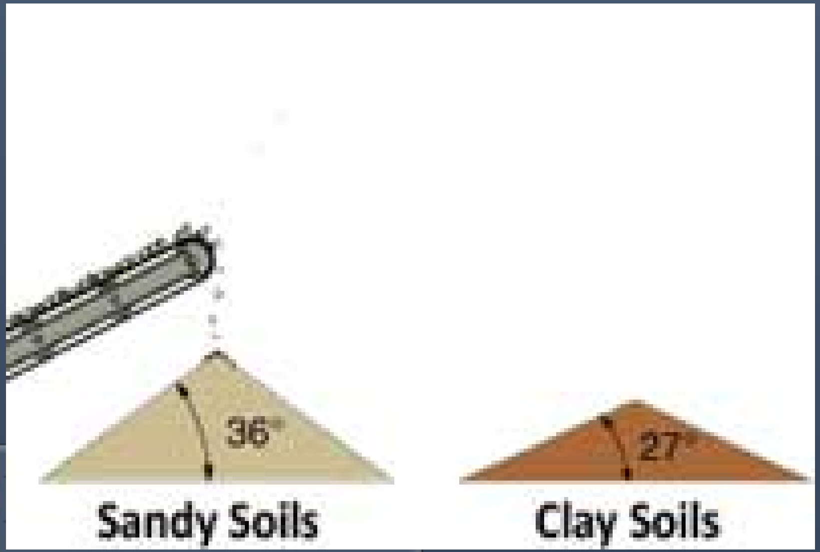
μ = Coefficient of friction

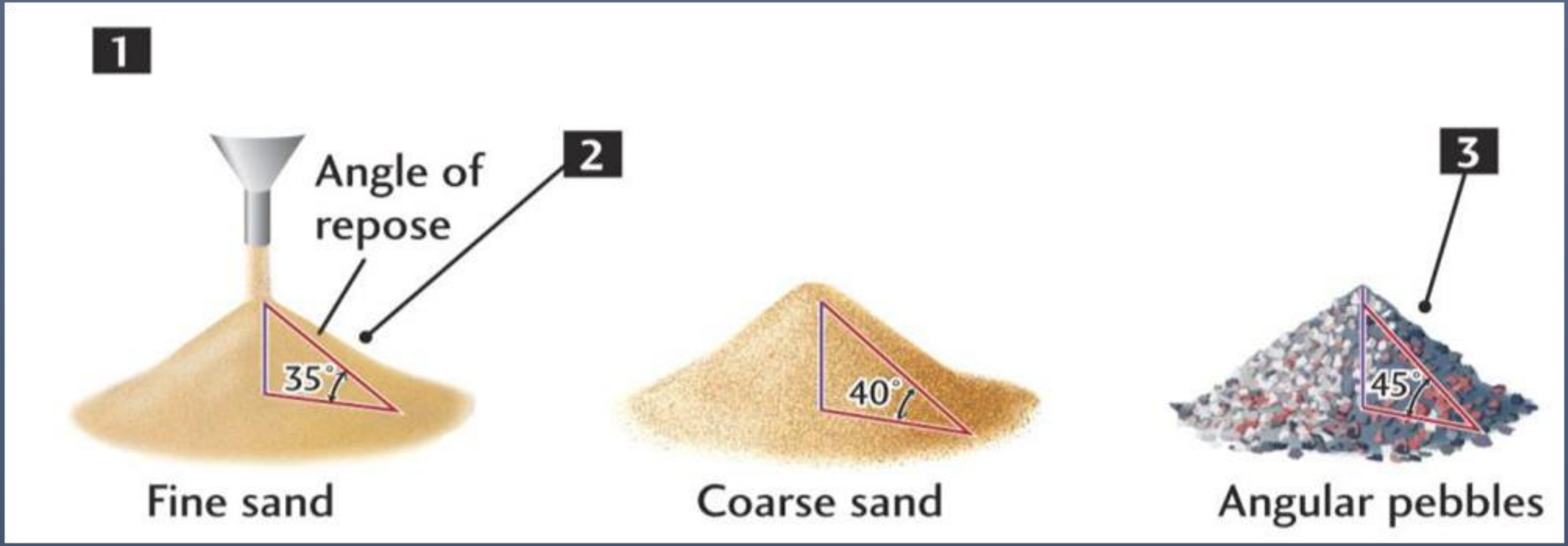
Angle of Repose (α) : limiting angle up to which the grains repose (sleep) is called the angle of repose of that granular material.









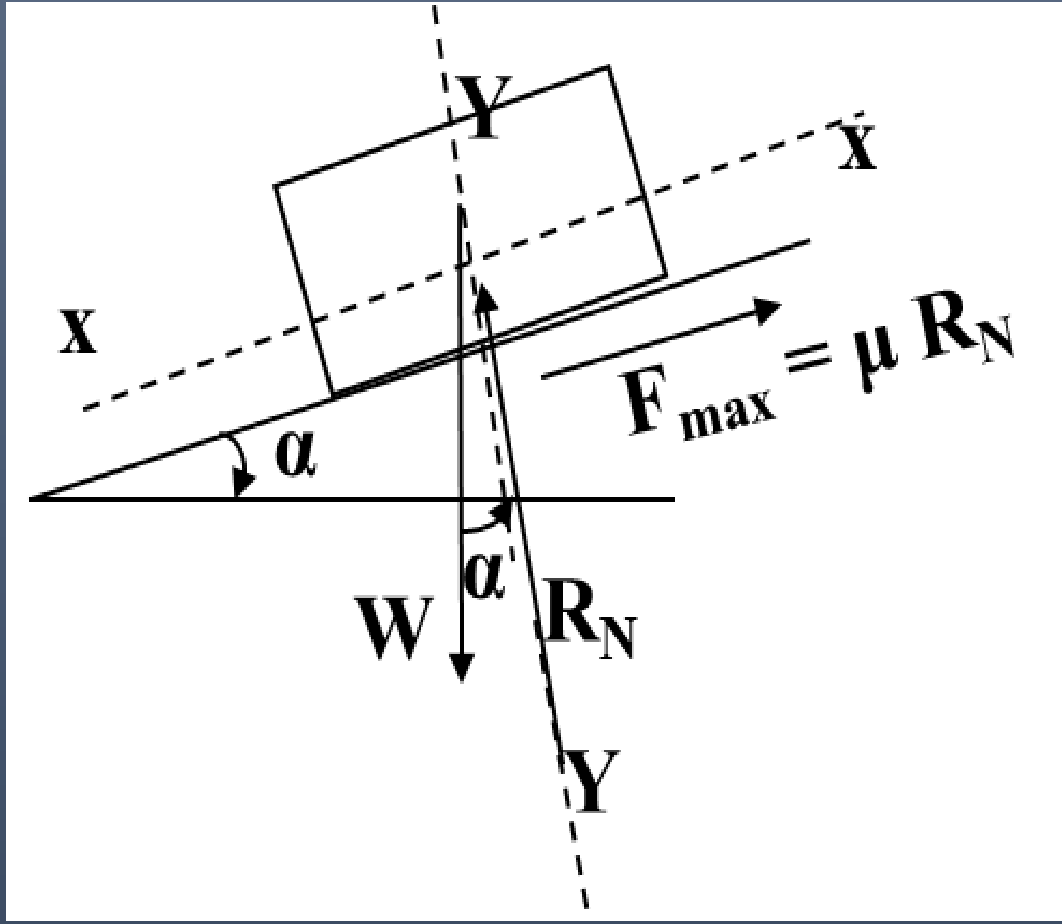


Significance of Angle of repose:

The angle that an inclined plane makes with the horizontal, when the body supported on the plane is on the verge of motion due to its self-weight is equal to the angle of repose.

Angle of repose is numerically equal to ***Angle of limiting friction.***

Significance of Angle of repose



$$\Sigma F_X = 0$$

$$F_{max} = W \sin \alpha \quad ------(1)$$

$$\Sigma F_Y = 0$$

$$R_N = W \cos \alpha \quad ------(2)$$

$$\frac{(1)}{(2)}$$

$$\tan \alpha = \frac{F_{max}}{R_N} = \mu = \tan \phi$$

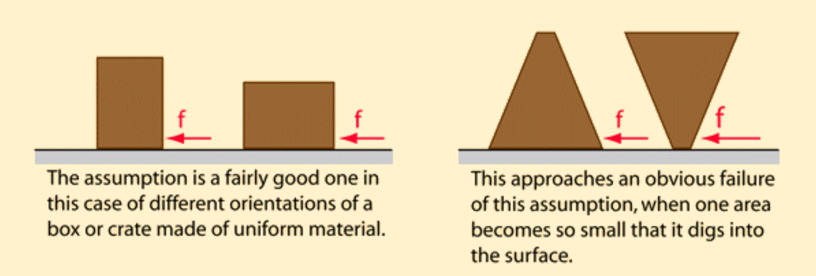
$$\alpha = \phi$$

Laws of dry Friction

1. The force of friction always acts in the direction opposite to the direction in which body tends to move.

2. The magnitude of limiting friction bears a constant ratio to the normal reaction between the two surfaces. (Experimentally proved)

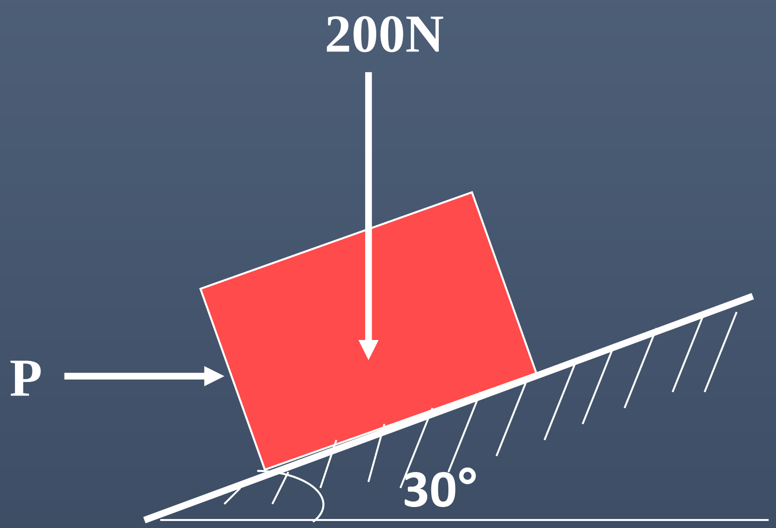
3. The force of friction is independent of the area of contact between the two surfaces.

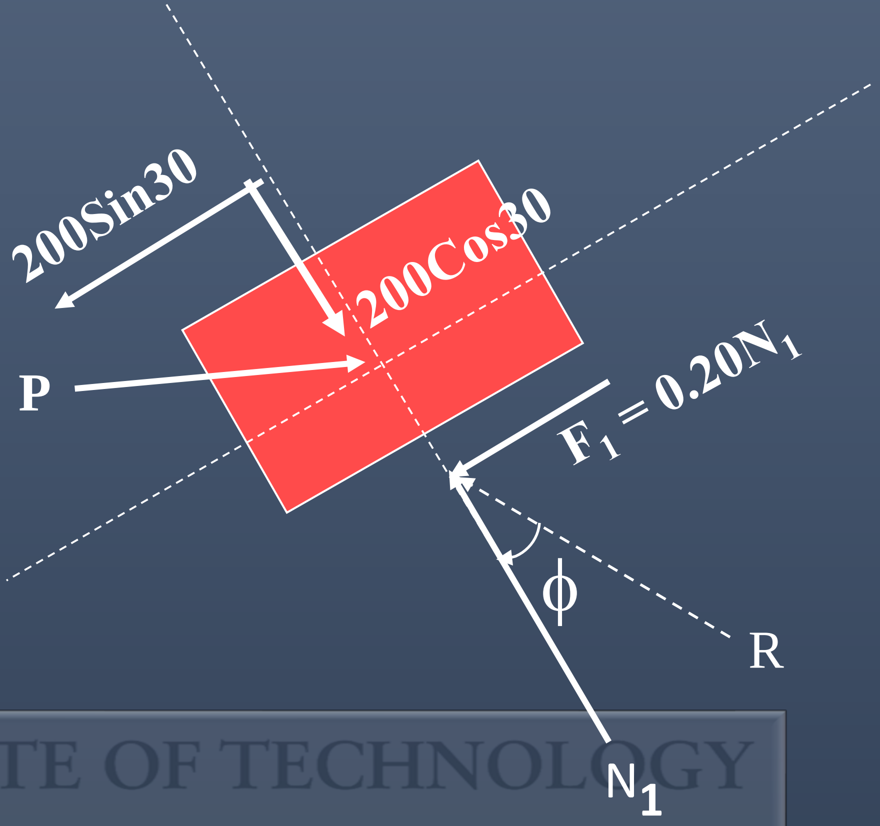


4. After the body starts moving, the dynamic friction comes into play, the magnitude of which is less than that of limiting friction and it bears a constant ratio with normal force. This ratio is known as **coefficient of dynamic friction**

Application Problems

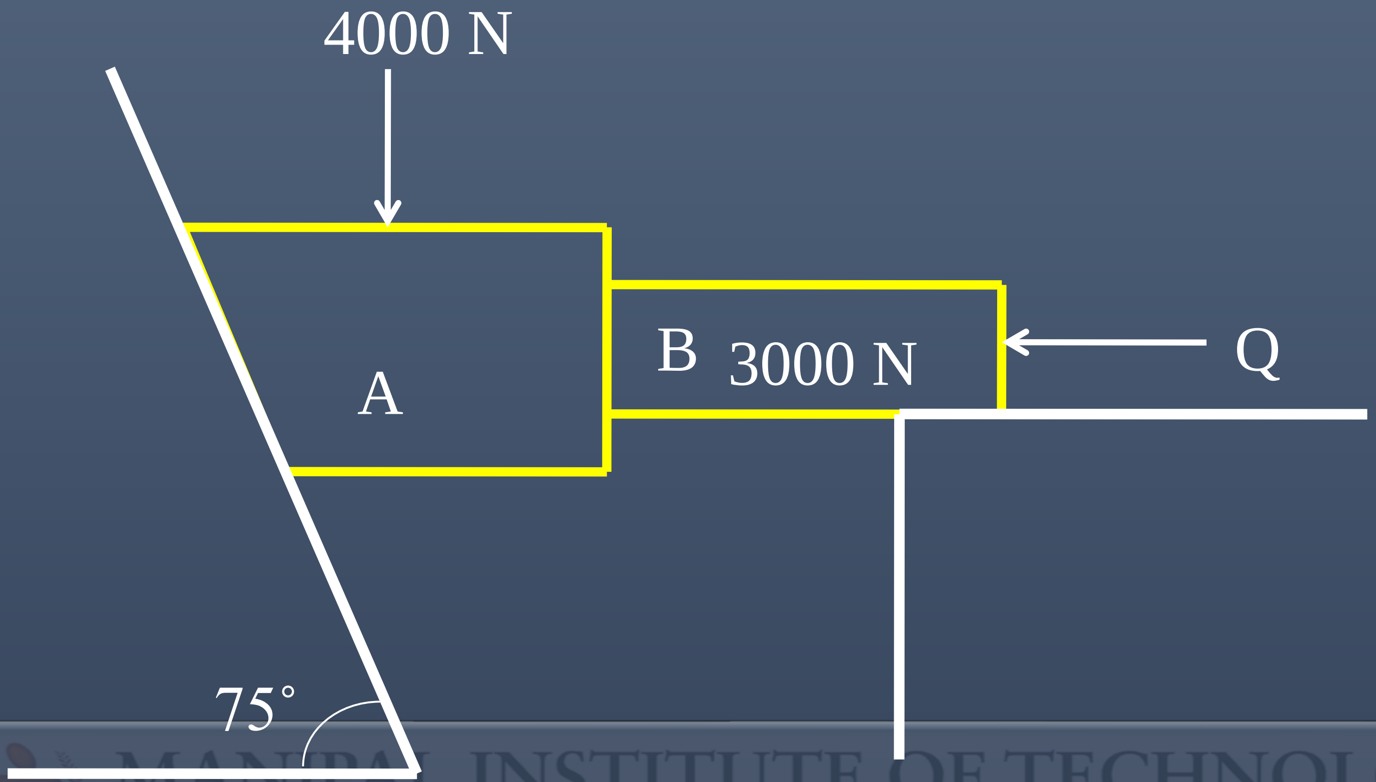
1. Compute the magnitude of **P** that will cause the motion to impend up the plane. Coefficient of friction, μ = 0.20





LECTURE 9

3. A 15° wedge is driven by a force 4000 N. Find force Q required to resist the motion of the wedge. Take μ=0.14 for all rubbing surfaces.



TUTORIAL 3

4. A ladder of length 5m weighing 500N is placed at 45° against a vertical wall. The coefficient of friction between the ladder and wall is 0.2 and ladder and ground is 0.5. If a man weighing 600N ascends the ladder, how high will he be when the ladder slips.

If a boy now stands on the bottom of the rung of the ladder what must be his least weight so that the man can go up to the top of the ladder.