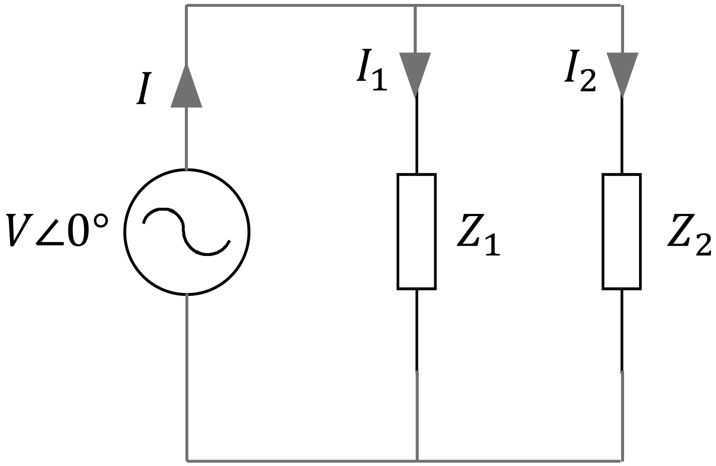
Basic Electrical Technology

[ELE 1051]

CHAPTER 3 – PARALLEL AC CIRCUITS

Impedance in parallel



Let $Z_1 = R_1 + jX_1$ $\quad Z_2 = R_2 + jX_2$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} \qquad I_1 = I * \frac{Z_2}{Z_1 + Z_2} \qquad I_2 = I * \frac{Z_1}{Z_1 + Z_2}$$

$Y_{eq} = Y_1 + Y_2$ $\quad$ *Y: Admittance*

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_1} = \frac{1}{(R_1 + jX_1)} * \frac{(R_1 - jX_1)}{(R_1 - jX_1)} = \frac{R_1}{(R_1^2 + X_1^2)} - j\frac{X_1}{(R_1^2 + X_1^2)} = G_1 - jB_1$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 + jX_2} = \frac{1}{(R_2 + jX_2)} * \frac{(R_2 - jX_2)}{(R_2 - jX_2)} = \frac{R_2}{(R_2^2 + X_2^2)} - j\frac{X_2}{(R_2^2 + X_2^2)} = G_2 - jB_2$$

G: Conductance $\quad$ *B: Susceptance*

$$\mathbf{Y_{eq} = (G_1 + G_2) - j(B_1 + B_2) = G_{eq} - jB_{eq}}$$

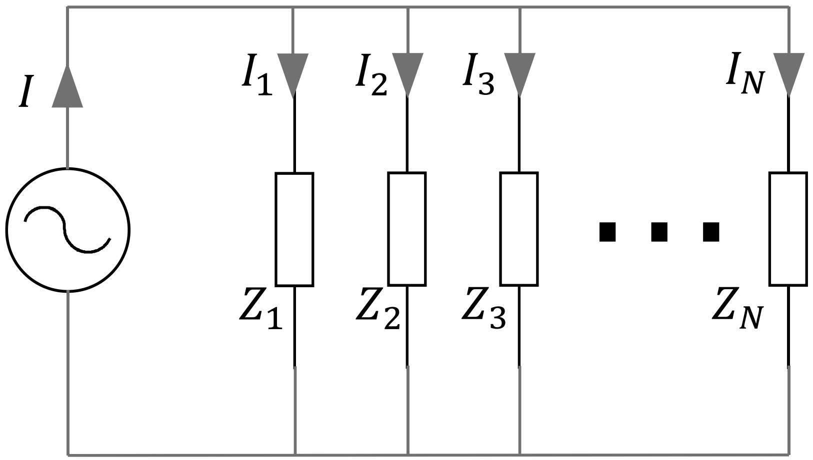
Impedance in parallel

For ‘N’ impedances connected in parallel,

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_N}$$

$$Y_{eq} = Y_1 + Y_2 + Y_3 + \cdots + Y_N$$

$$\mathbf{Y_{eq} = G_{eq} \pm \boldsymbol{j}B_{eq}}$$



$$I_1 = VY_1; I_2 = VY_2; I_3 = VY_3; \ldots \ldots I_N = VY_N$$

$$I = I_1 + I_2 + I_3 + \cdots + I_N = VY_{eq}$$

Network equations for AC circuits

KVL Equation

(Matrix form)

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

$$[V] = [Z][I]$$

KCL Equation

(Matrix form)

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1N} \\ \vdots & \ddots & \vdots \\ Y_{N1} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix}$$

$$[I] = [Y][V]$$

All the other theorems are applicable to the AC circuits

Crammers rule

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

Solution for the linear simultaneous equations above is as follows

Step 1: finding the determinant

$$\Delta = \begin{vmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{vmatrix}$$

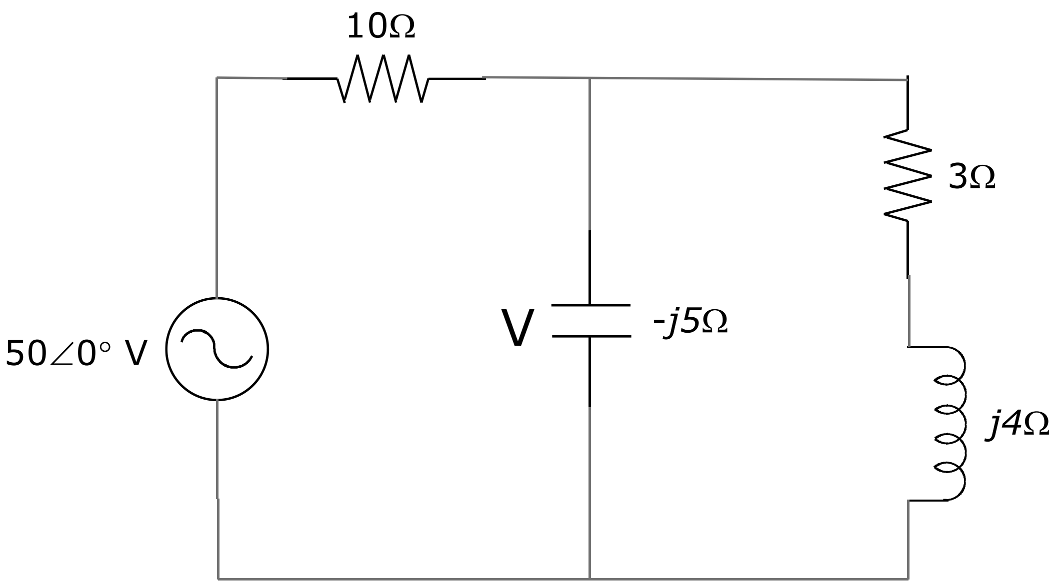
Step 2: finding the determinant after substituting first column with RHS column matrix

$$\Delta_1 = \begin{vmatrix} \boldsymbol{V_1} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ \boldsymbol{V_N} & \cdots & Z_{NN} \end{vmatrix}$$

Step 3 :Solution for I_1 $\quad I_1 = \dfrac{\Delta_1}{\Delta}$

Illustration 1

Assigning two mesh currents, find the voltage V across the capacitor in the following circuit

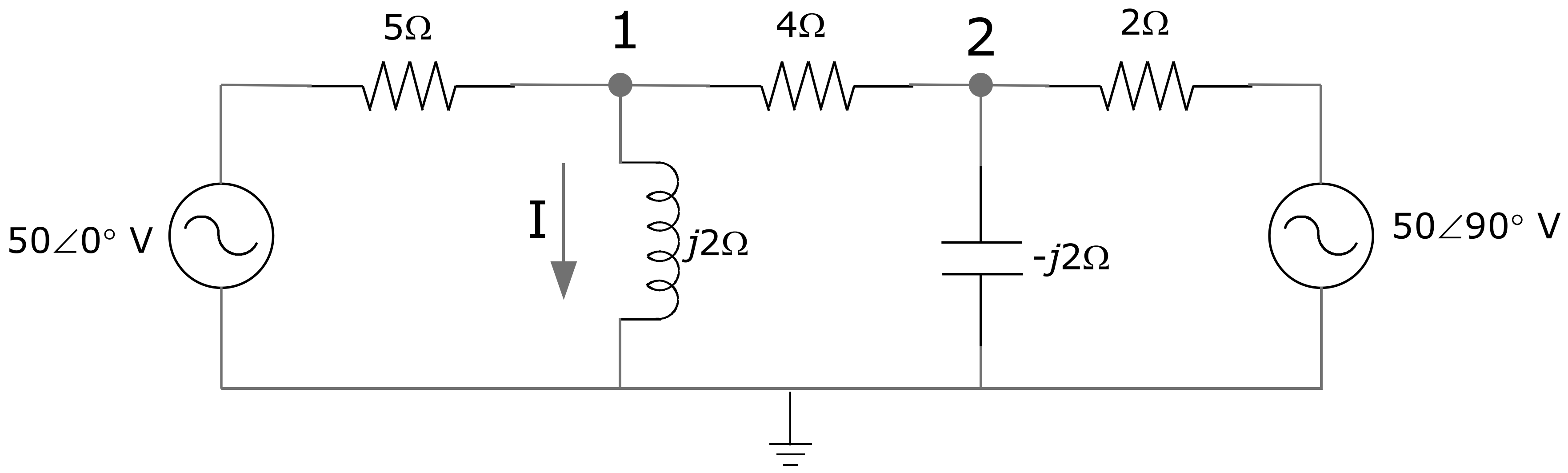


$Ans:$

$V = 22.36\angle -10.30°V$

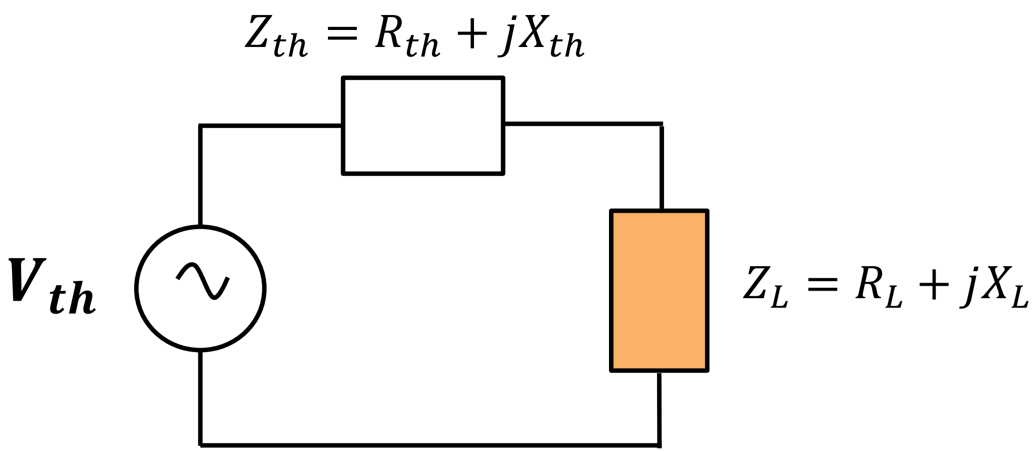
Illustration 2

Use node voltage method to obtain the current I in the network



$$Ans:$$
$$I = 12.38\angle -17.75°\ A$$

Maximum power transfer theorem



	Type of load	Condition of maximum power transfer
Case 1	Load is purely resistive	$R_L = \sqrt{R_{th}^2 + X_{th}^2}$
Case 2	Both R_L & X_L are variable	$Z_L = Z_{TH}^*$
Case 3	X_L is fixed & R_L is variable	$R_L = \sqrt{R_{th}^2 + (X_{th} + X_L)^2}$

Thank You!