Basic Electrical Technology

[ELE 1051]

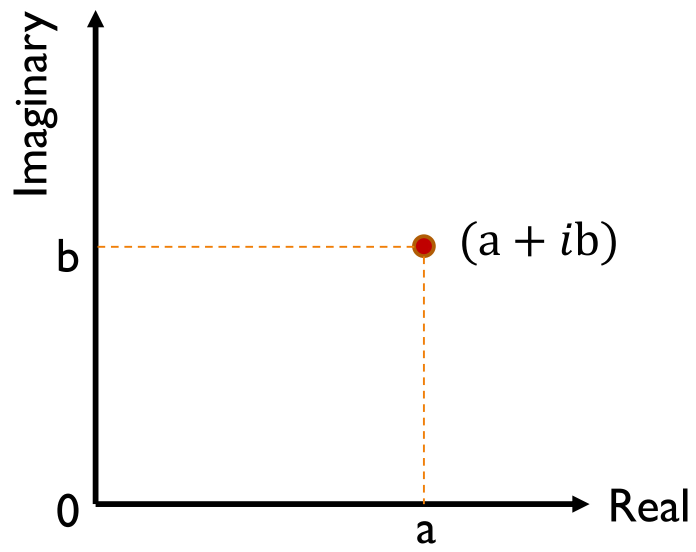
CHAPTER 3 - SINGLE PHASE AC CIRCUITS

(3.2)

Complex Number

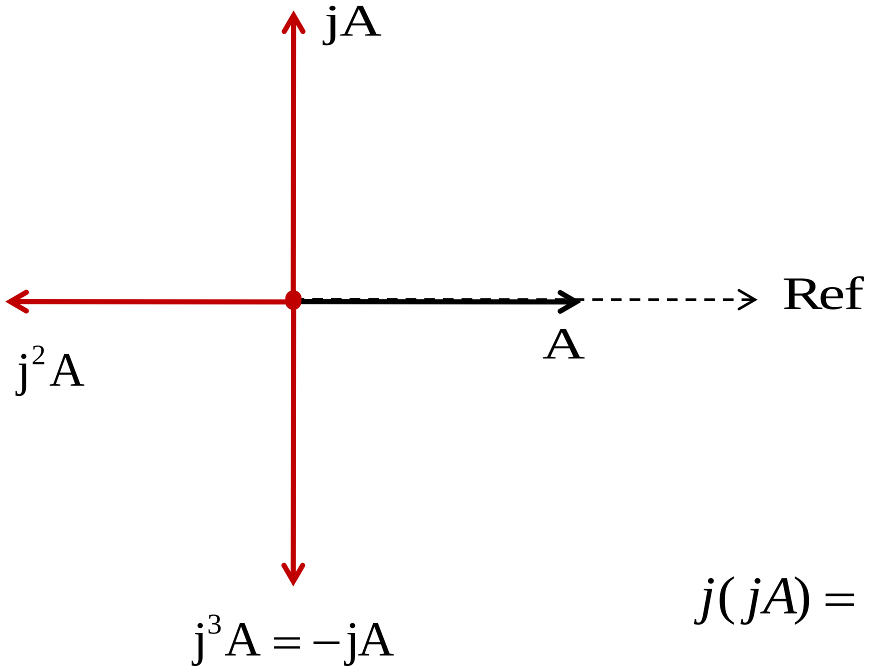
- A **complex number** is of the form **a + *i* b**

- Represented on complex plane as:



The operator ‘j’

$$j = 1\angle 90°$$



$$j(jA) = j^2 A = -A$$

$$\boxed{\text{Therefore,}\quad j^2 = -1; \quad j = \sqrt{-1}}$$

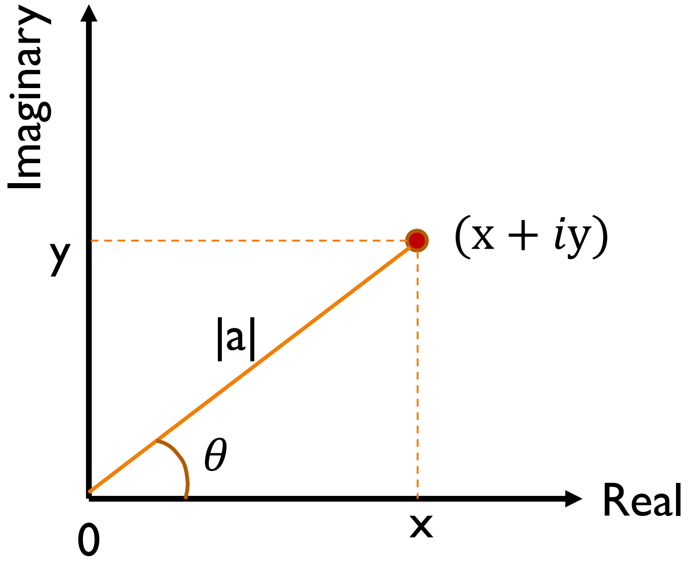
The operator ‘j’ rotates the given vector by 90 degrees in anti-clockwise direction

Rectangular ↔ Polar conversion

- **Rectangular to polar:**

$$|a| = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$



- **Polar to Rectangular:**

$$x = |a|\cos\theta$$

$$y = |a|\sin\theta$$

Representation of a complex number

- **Rectangular form:** $\mathbf{a = x \pm \mathit{j}y}$
- **Polar form:** $\mathbf{a = |\mathit{a}|\angle \pm \theta}$
- **Exponential form:** $\mathbf{a = |\mathit{a}|e^{\pm \mathit{j}\theta}}$
- **Trigonometric form:** $\mathbf{a = |a|(cos\theta \pm \mathit{j}sin\theta)}$

Representing AC

- Consider three sinusoidal signals $x(t), y(t)$ & $z(t)$ with same frequency

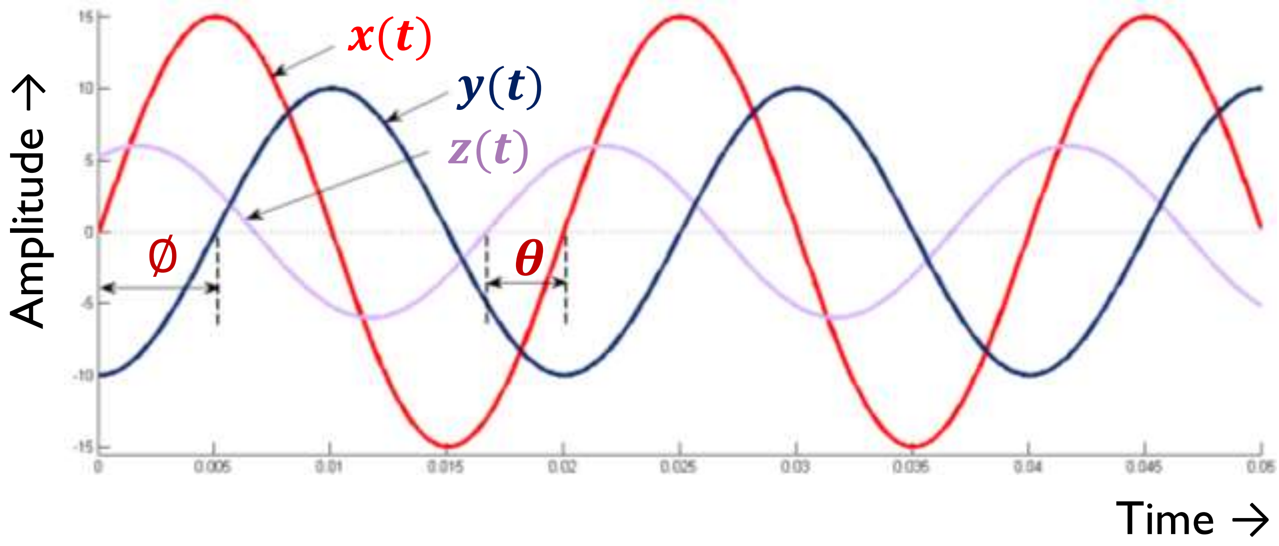
Mathematical Representation

$$x(t) = X_m sin(\omega t)$$

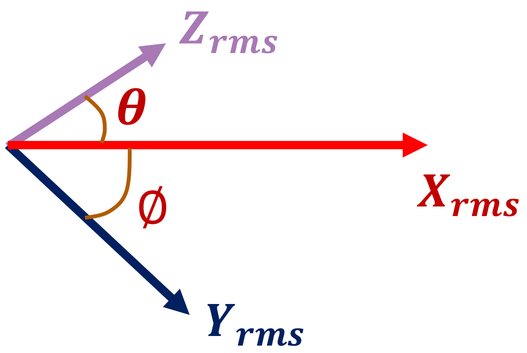
$$y(t) = Y_m \sin(\omega t - \emptyset)$$

$$z(t) = Z_m \sin(\omega t + \theta)$$

Graphical Representation

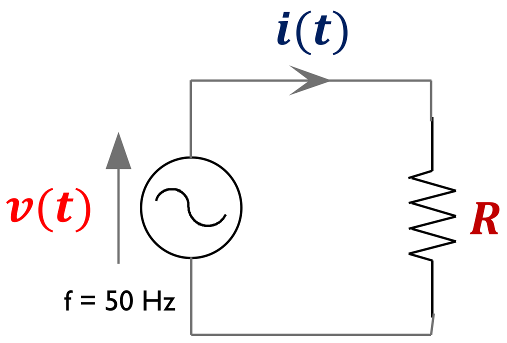


Phasor Representation



- Representing the relationship between sinusoidal signals with same frequency in graphical or mathematical form is tedious
- Phasor representation is often used

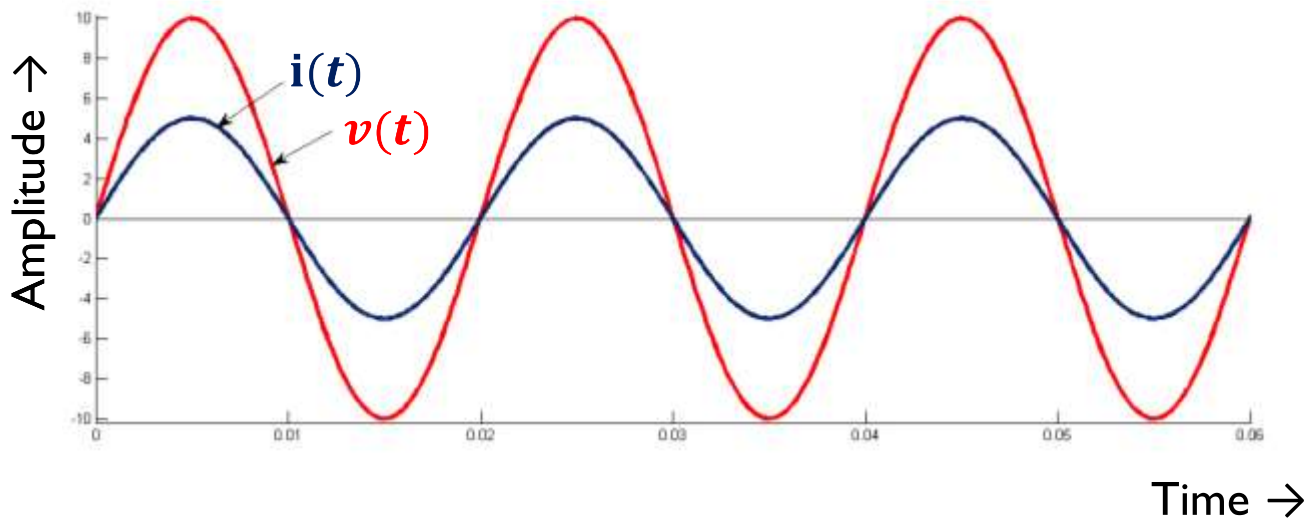
R circuit response with AC supply



$$i(t) = \frac{v(t)}{R}$$

'Current through the resistor is in phase with the voltage across it'

<u>*Graphical Representation*</u>

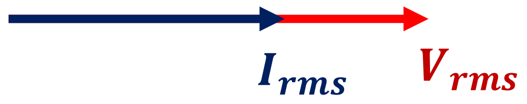


<u>*Mathematical Representation*</u>

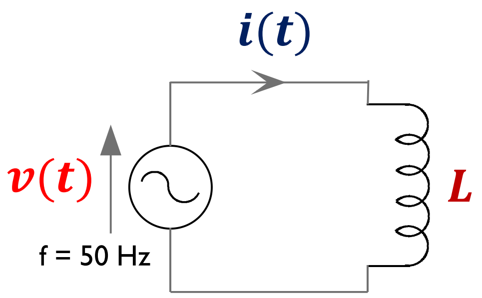
$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t)$$

<u>*Phasor Representation*</u>



L circuit response with AC supply



$$i(t) = \frac{1}{L}\int v(t)dt$$

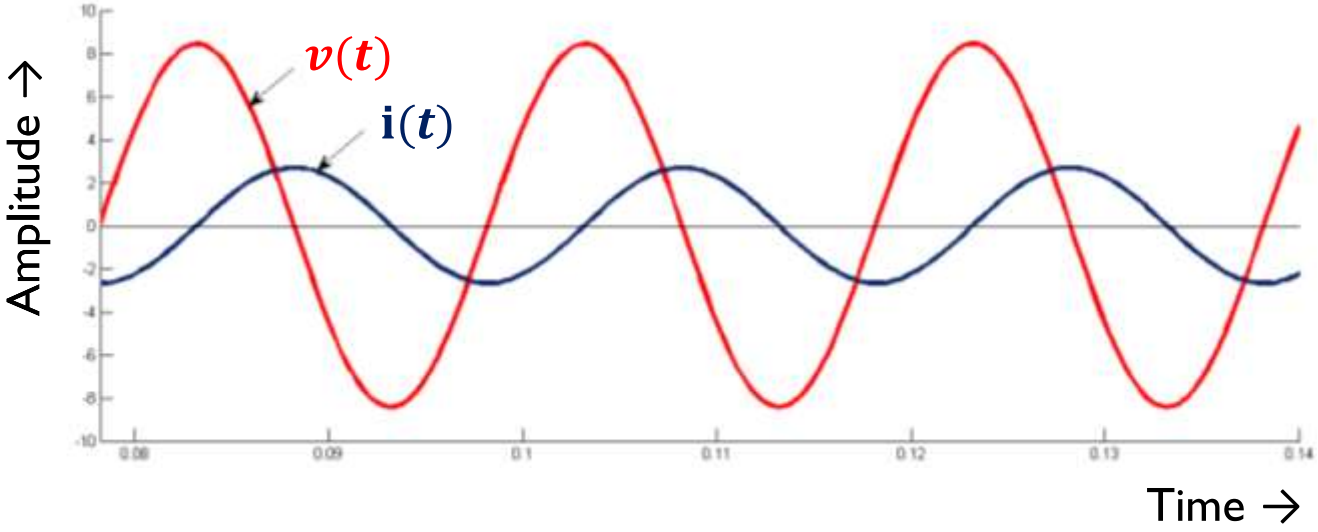
'Current through the inductor lags the voltage across it by 90°'

$\bar{V} = V\angle 0°$ $\quad\quad$ $\bar{I} = I\angle -90°$

$$\frac{\bar{\boldsymbol{V}}}{\bar{\boldsymbol{I}}} = \frac{\boldsymbol{V\angle 0°}}{\boldsymbol{I\angle -90°}} = \boldsymbol{jX_L} \quad where\ \frac{V}{I} = X_L$$

$\boldsymbol{X_L}$ *is called* ***Inductive Reactance***

<u>*Graphical Representation*</u>



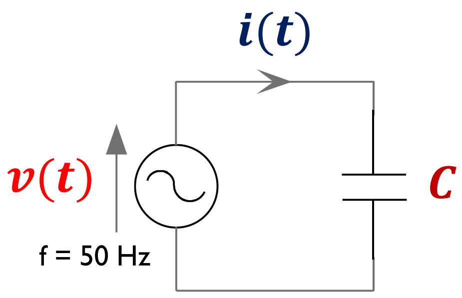
<u>*Mathematical Representation*</u>

$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t - 90°)$$

<u>*Phasor Representation*</u>

C circuit response with AC supply



$$i(t) = C\frac{dv(t)}{dt}$$

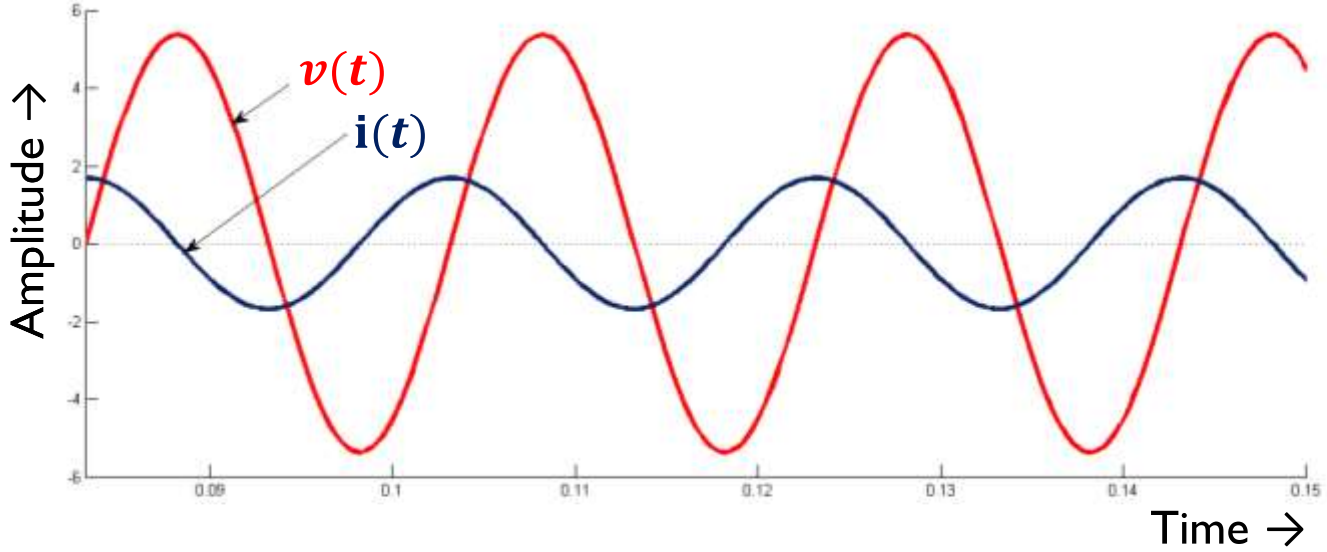
'Current through the capacitor leads the voltage across it by 90°'

$\bar{V} = V\angle 0°$ $\quad\quad$ $\bar{I} = I\angle 90°$

$$\frac{\bar{V}}{\bar{I}} = \frac{V\angle 0°}{I\angle 90°} = -jX_C \quad where\ \frac{V}{I} = X_C$$

X_C *is called* ***Capacitive Reactance***

<u>*Graphical Representation*</u>



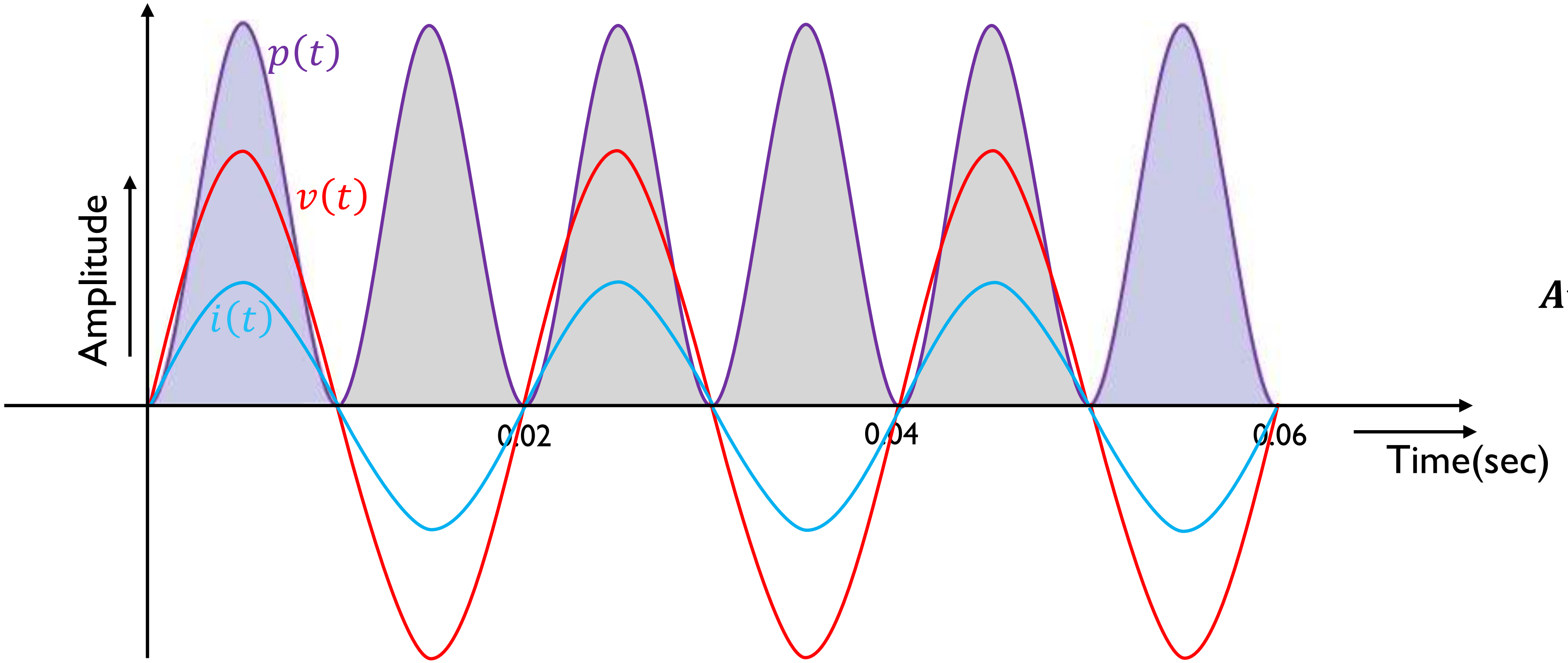
<u>*Mathematical Representation*</u>

$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t + 90°)$$

<u>*Phasor Representation*</u>

Power Associated - Pure Resistive Circuit



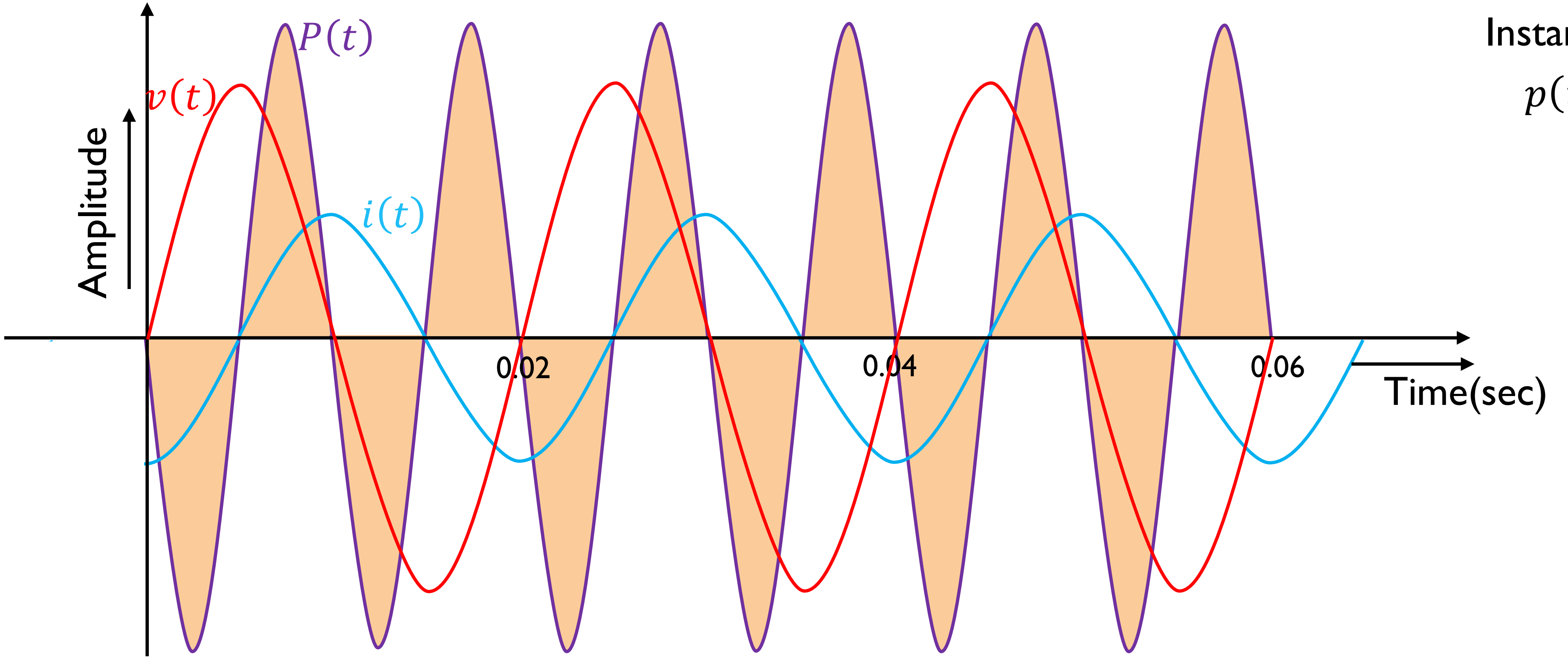
Instantaneous power,

$p(t) = v(t).\,i(t) = V_m I_m sin^2 \omega t$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{\boldsymbol{P_{avg} = \frac{V_m I_m}{2} = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R}}$$

Power Associated – Pure Inductive Circuit



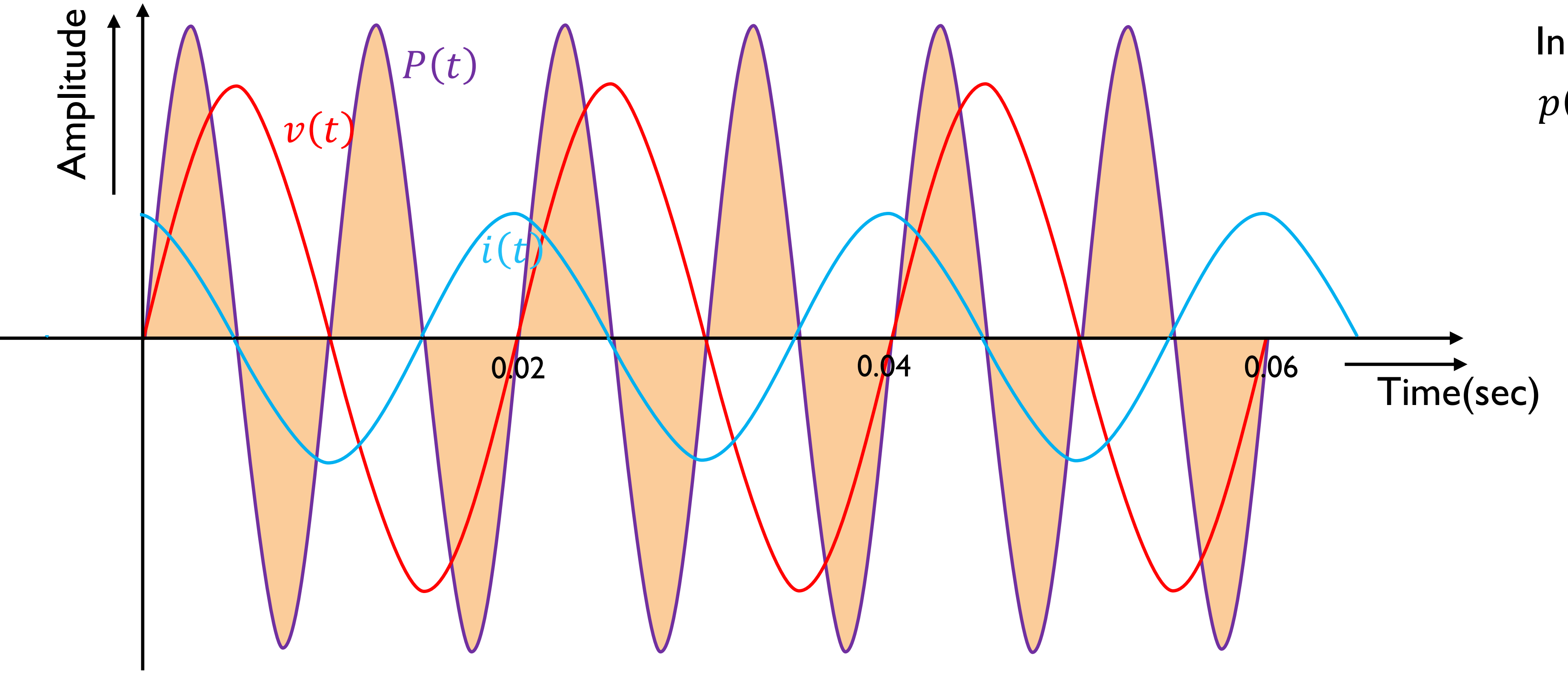
Instantaneous power,

$$p(t) = v(t).\,i(t)$$
$$= V_m I_m \sin \omega t \,.\, \sin(\omega t - 90\,°)$$
$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{P_{avg} = 0}$$

Power Associated – Pure capacitive Circuit



Instantaneous power,

$$p(t) = v(t).\, i(t)$$
$$= V_m I_m \sin \omega t \,.\, \sin(\omega t + 90\,°)$$
$$= \frac{V_m I_m}{2} \sin 2\omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{P_{avg} = 0}$$

Thank You!