Basic Electrical Technology

LECTURE – 01

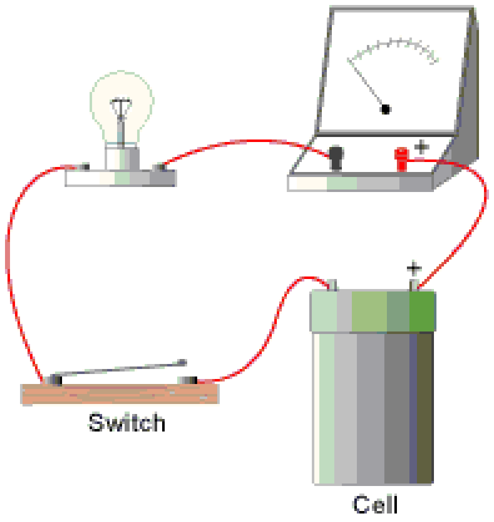
DC CIRCUIT ANALYSIS

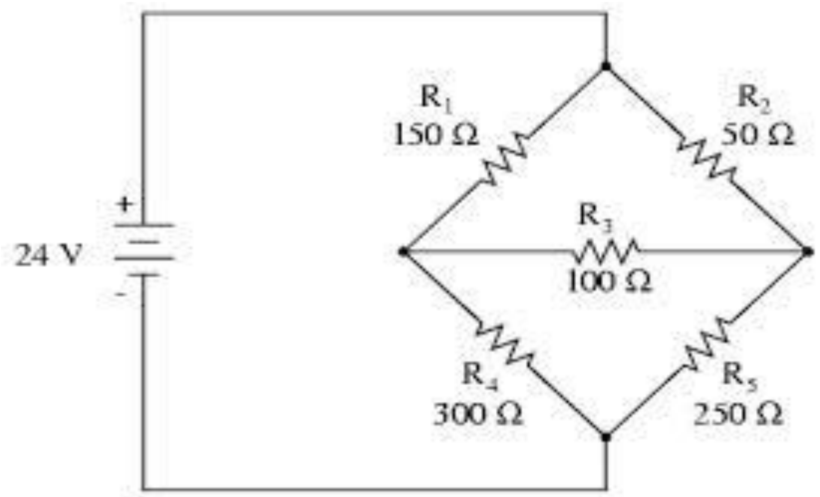
What is an Electric Circuit?

Definition:

“An interconnection of simple electrical devices with at least one closed path in which current may flow”

- Consists of a source of electrical energy; elements that either transform, dissipate, or store this energy; connecting wires.
- To prevent power overload, circuits often include fuse or circuit breaker.

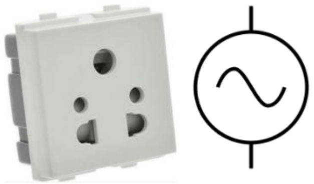


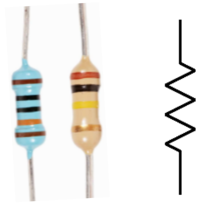


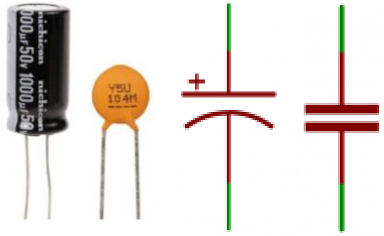
Circuit Elements

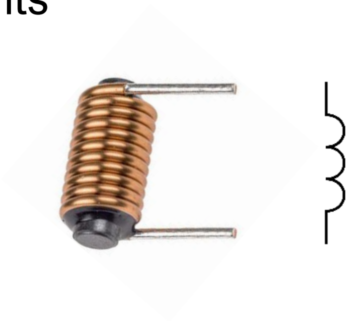
Active & Passive	• **Active Elements:** Voltage & Current Sources • **Passive Elements:** Resistor, Inductor, Capacitor
Linear & Non-linear Elements	• **Linear:** Resistor, Inductor, Capacitor • **Nonlinear:** Diode, LDR (Light Dependent Resistor), Thermistor, transistor
Unilateral & Bilateral Elements	• **Unilateral:** Diode, Transistor • **Bilateral** (same property in both the directions): Resistor, Inductor, Capacitor
Lumped & Distributed	• Lumped elements are simplified version of distributed elements

Our study is limited to **lumped linear bilateral** circuit elements







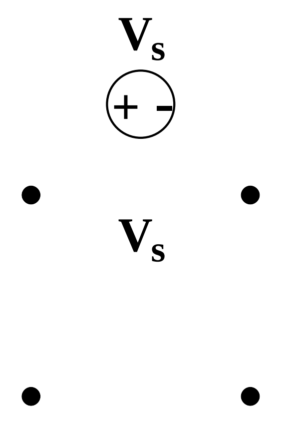


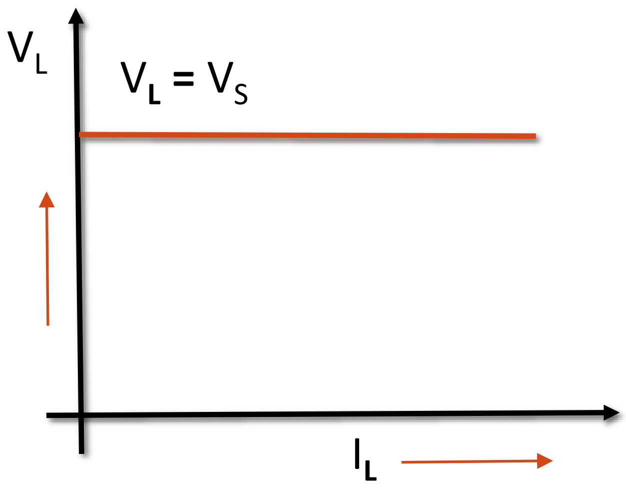
Active Elements - Sources

Voltage Source

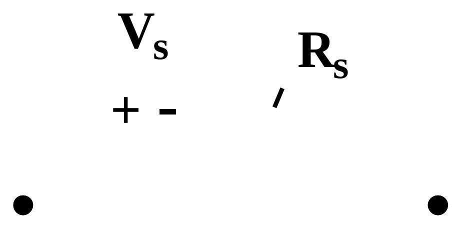
- **Ideal**
  - Maintains constant voltage irrespective of connected load
  - Internal resistance, $\mathbf{R_s = 0}$
- **Practical**
  - Terminal voltage changes based on the connected load
  - Internal resistance, $\mathbf{R_s \neq 0}$

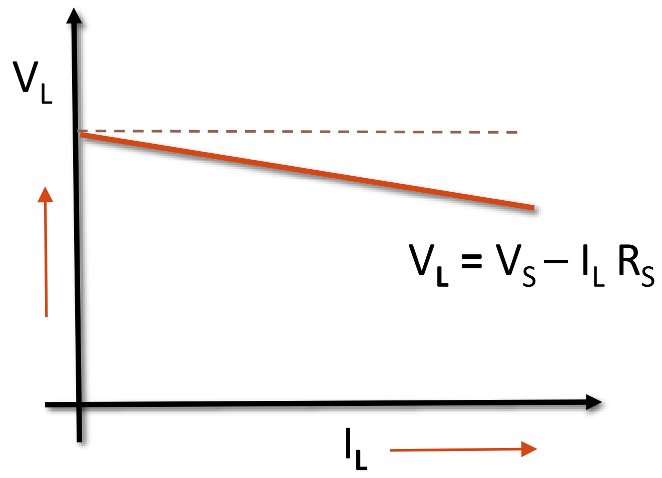
Ideal Voltage Source (DC)





Practical Voltage Source



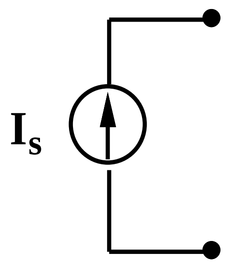


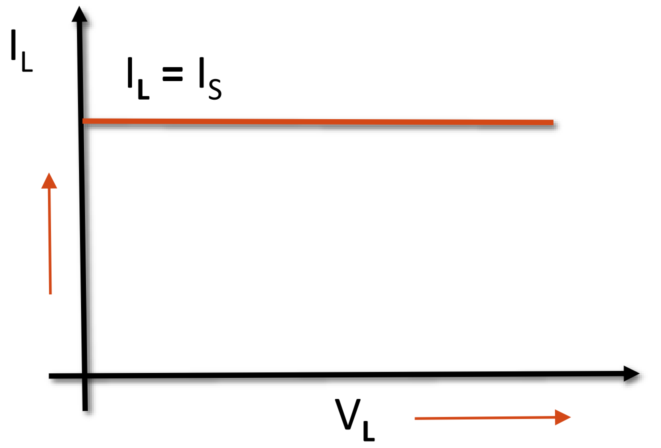
Active Elements - Sources

Current Source

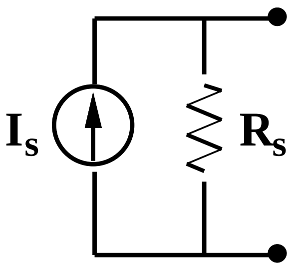
- **Ideal**
  - Maintains constant current irrespective of the load connected
  - Internal resistance, $\mathbf{R_s} = \infty$
- **Practical**
  - Output current changes based on the connected load
  - Internal resistance, $\mathbf{R_s} < \infty$

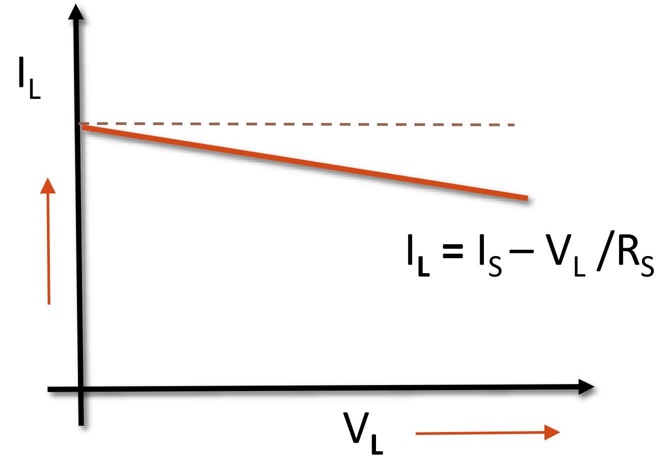
Ideal Current Source (DC)





Practical Current Source





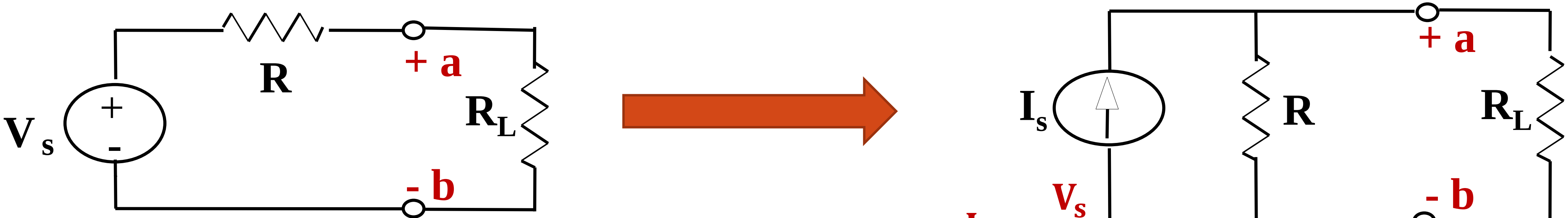
Source Transformation (A Network Reduction Technique)

Practical Voltage Source ⟷ **can be converted to** ⟷ **Practical Current Source**

Source Transformation

Practical Voltage Source

Practical Current Source



$$I_s = \frac{V_s}{R}$$

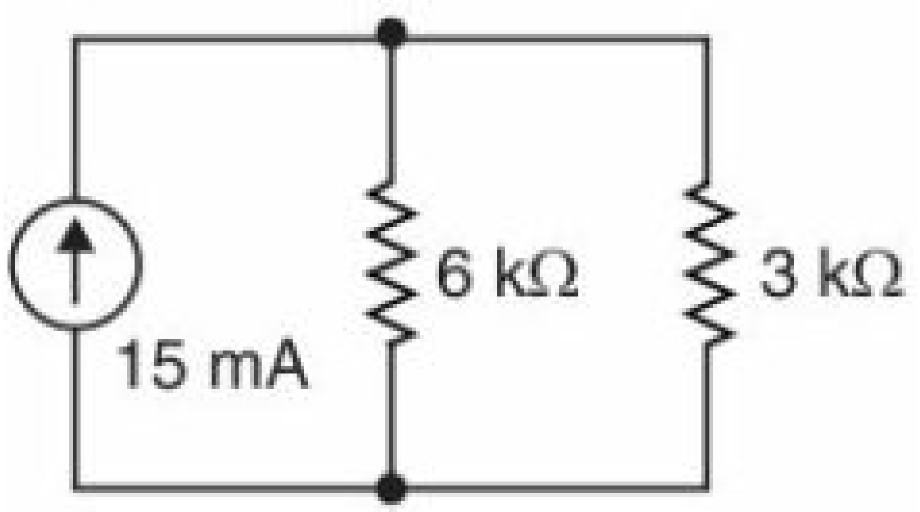
Source Transformation

Practical Voltage Source **Practical Current Source**

$$\mathbf{V_s = R \times I_s}$$

Illustration 1

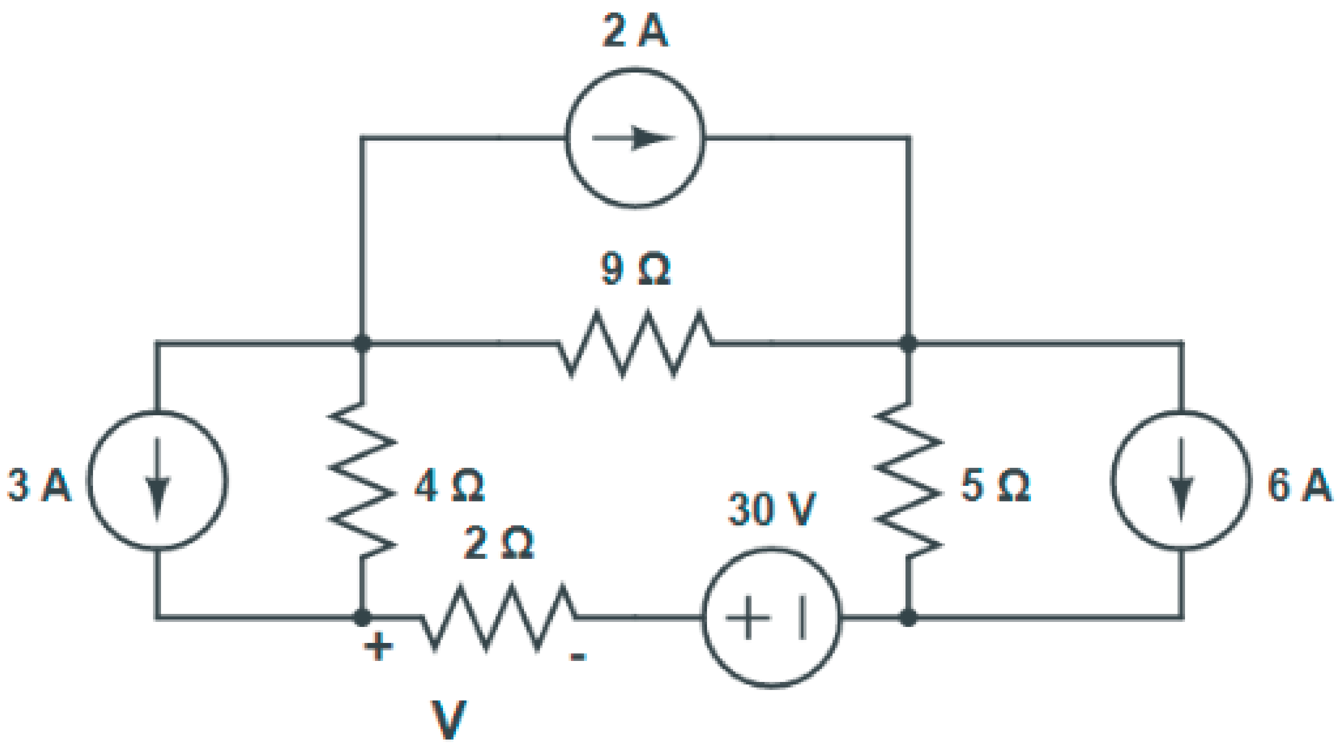
Find current in **6 KΩ** resistor by converting current source to a voltage source.



Ans: 5 mA

Illustration 2

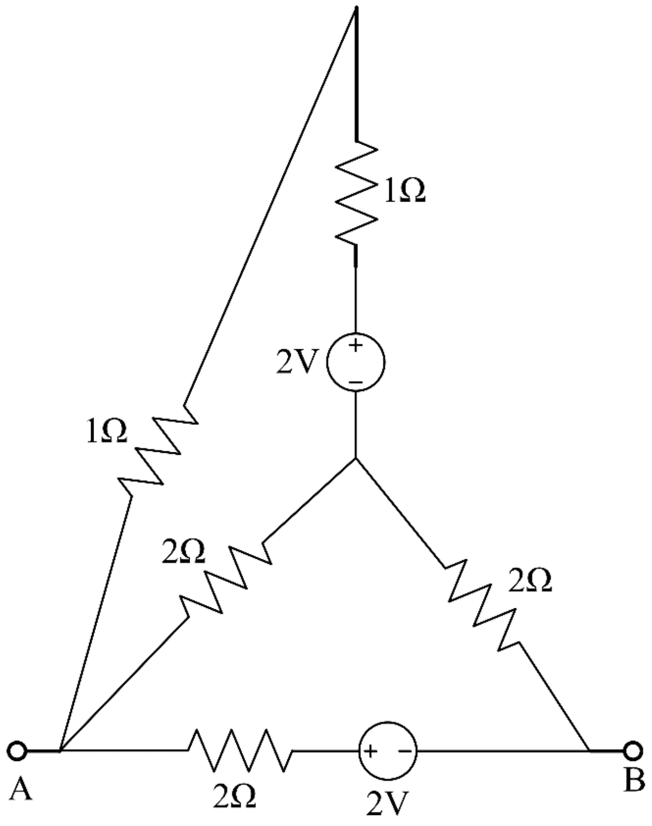
The value of voltage **‘V’** is:



Ans: - 6.6 Volts

Illustration 3

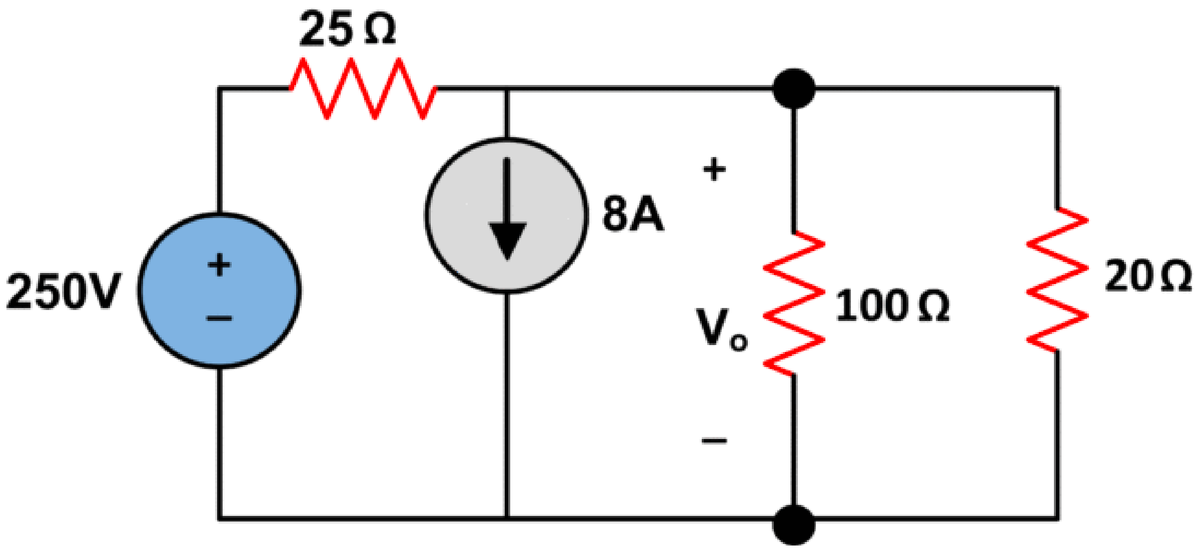
Reduce the following circuit to a current source in parallel with a resistor across the terminals **A & B.**



Ans. 1.33 A (from B to A) in parallel with 1.2 Ω

Self-Practice 1

Find $\mathbf{V_0}$ using source transformation.

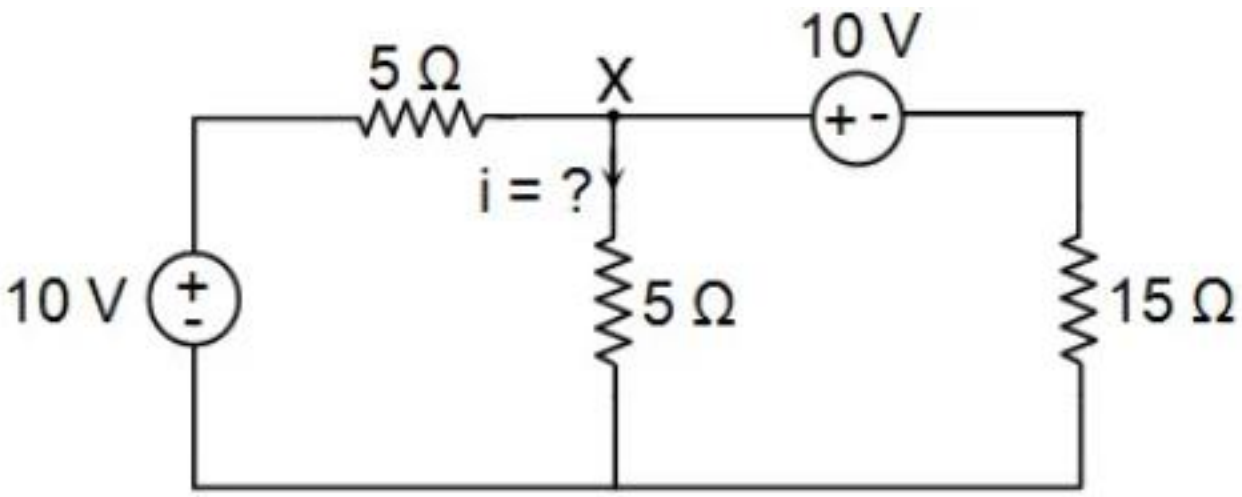


Ans: 20 V

Homework 1

Solve using source transformation.

(Final reduction should be into a single voltage source in series with a resistance)



Ans: 1.143 A

Basic Electrical Technology

LECTURE - 02

- ACTIVE & PASSIVE ELEMENTS

Resistor

Energy Consuming Element

Resistor

- **Passive electric device** that **dissipates energy**
- **Resistance:** Property that opposes the flow of current
  - Symbol: **R**
  - Unit: Ohms (Ω)
  - Power Consumed $= \boldsymbol{I^2 R}$

- **Conductance**
  - Reciprocal of resistance
  - Symbol: G
  - Unit – Siemens (S)
- **Resistivity or Specific Resistance**
  - $\mathbf{R} = \frac{\rho l}{A}$ $\boldsymbol{\rho}$ = resistivity l = length A = cross sectional area Unit: Ohm-meter
  - The factor in the resistance which takes into account the **nature of the material**
  - It is **temperature dependent**
  - The inverse of resistivity is called **conductivity, denoted by σ**

Effect of Temperature on Resistance

Metallic conductors (Example: Cu, Al)	If temperature is increased resistance increases	Positive temperature coefficient of resistance
Electrolytes, insulators (Example: glass, mica, rubber), **and semiconductors**	Resistance decreases with the increase in temperature	Negative temperature coefficient of resistance

- At temperature T_1, resistance is R_1
- Temperature is increased from T_1 to T_2, resistance becomes R_2
- Then, $\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$
- The constant α is known as **temperature coefficient of resistance**
- Unit: 1 / ^{o}C
- Defined as increase in resistance per unit original resistance per unit rise in temperature
- **Resistivity of metallic conductors also increases with the rise in temperature and vice-versa**

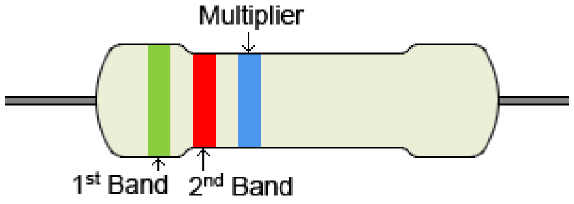
Effect of Temperature on Resistance

Typical Values of Electrical Resistivity (In Ohm-Meters) at 20^{o}C	
Aluminium	2.8×10^{-8}
Copper	1.7×10^{-8}
Gold	2.4×10^{-8}
Carbon (Graphite)	1×10^{-5}
Iron	1.0×10^{-7}
Nickel	7×10^{-8}
Silicon	6.4×10^{2}
Quartz	7×10^{17}

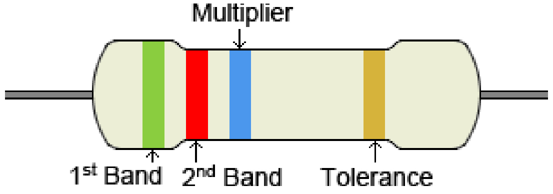
Temperature Coefficient of Resistance / ^{0}C (at 20^0 C)	
Aluminium	0.00429
Copper	0.00386
Gold	0.0034
Carbon (Graphite)	– 0 .0005
Iron	0.00651
Nickel	0.00641
Silicon	– 0.07
Quartz	

Resistor Value Determination

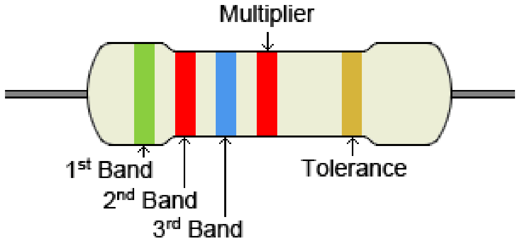
Ohm value and tolerance based on resistor color codes



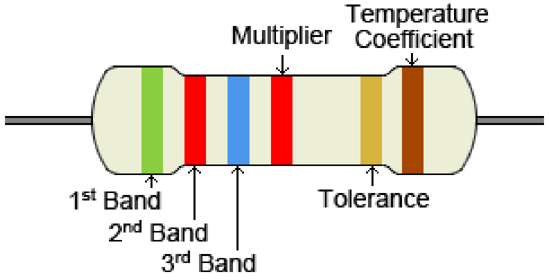
Color	1st, 2nd Band Significant Figures	Multiplier
Black	0	× 1
Brown	1	× 10
Red	2	× 100
Orange	3	× 1K
Yellow	4	× 10K
Green	5	× 100K
Blue	6	× 1M
Violet	7	× 10M
Grey	8	× 100M
White	9	× 1G
Gold		× 0.1
Silver		× 0.01



Color	1st, 2nd Band Significant Figures	Multiplier	Tolerance
Black	0	× 1	
Brown	1	× 10	±1% (F)
Red	2	× 100	±2% (G)
Orange	3	× 1K	±0.05% (W)
Yellow	4	× 10K	±0.02% (P)
Green	5	× 100K	±0.5% (D)
Blue	6	× 1M	±0.25% (C)
Violet	7	× 10M	±0.1% (B)
Grey	8	× 100M	±0.01% (L)
White	9	× 1G	
Gold		× 0.1	±5% (J)
Silver		× 0.01	±10% (K)



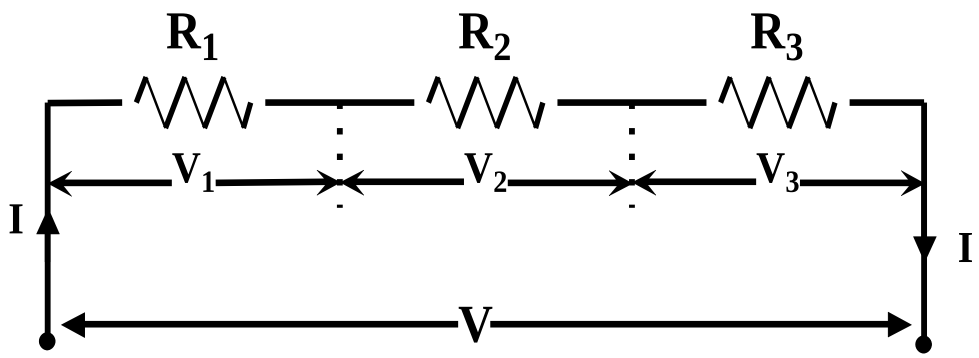
Color	1st, 2nd, 3rd Band Significant Figures	Multiplier	Tolerance
Black	0	× 1	
Brown	1	× 10	±1% (F)
Red	2	× 100	±2% (G)
Orange	3	× 1K	±0.05% (W)
Yellow	4	× 10K	±0.02% (P)
Green	5	× 100K	±0.5% (D)
Blue	6	× 1M	±0.25% (C)
Violet	7	× 10M	±0.1% (B)
Grey	8	× 100M	±0.01% (L)
White	9	× 1G	
Gold		× 0.1	±5% (J)
Silver		× 0.01	±10% (K)



Color	1st, 2nd, 3rd Band	Multiplier	Tolerance	Temperature Coefficient
Black	0	× 1		250 ppm/K(U)
Brown	1	× 10	±1% (F)	100 ppm/K(S)
Red	2	× 100	±2% (G)	50 ppm/K (R)
Orange	3	× 1K	±0.05%(W)	15 ppm/K (P)
Yellow	4	× 10K	±0.02%(P)	25 ppm/K (Q)
Green	5	× 100K	±0.5% (D)	20 ppm/K (Z)
Blue	6	× 1M	±0.25%(C)	10 ppm/K (Z)
Violet	7	× 10M	±0.1% (B)	5 ppm/K (M)
Grey	8	× 100M	±0.01% (L)	1 ppm/K (K)
White	9	× 1G		
Gold		× 0.1	±5% (J)	
Silver		× 0.01	±10% (K)	

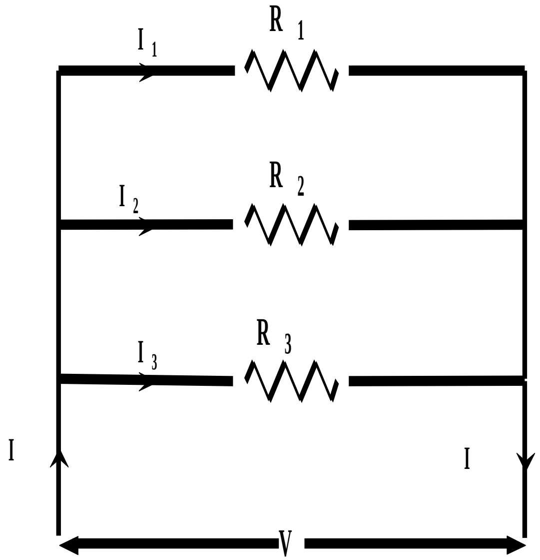
Resistors

Series connection of Resistors



- Current (I) in the all the resistors remains same
- $V = V_1 + V_2 + V_3$
- $R_{eq} = R_1 + R_2 + R_3$

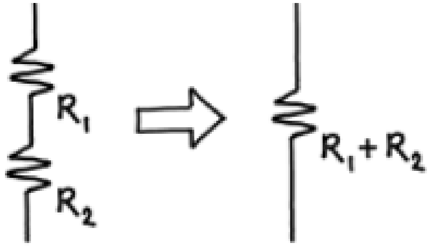
Parallel connection of Resistors



- Voltage (V) is same
- $I = I_1 + I_2 + I_3$
- $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Resistors – Voltage and Current Division

Series Resistors

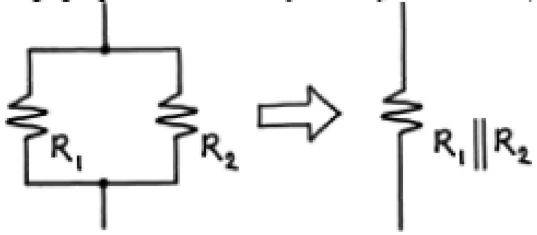


Equivalent resistance $= R_1 + R_2$

Voltage Divider

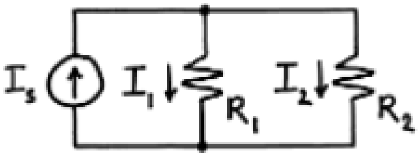
$V_1 = \frac{R_1}{R_1 + R_2} V_s \qquad V_2 = \frac{R_2}{R_1 + R_2} V_s$

Parallel Resistors



Equivalent resistance $= R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2}$

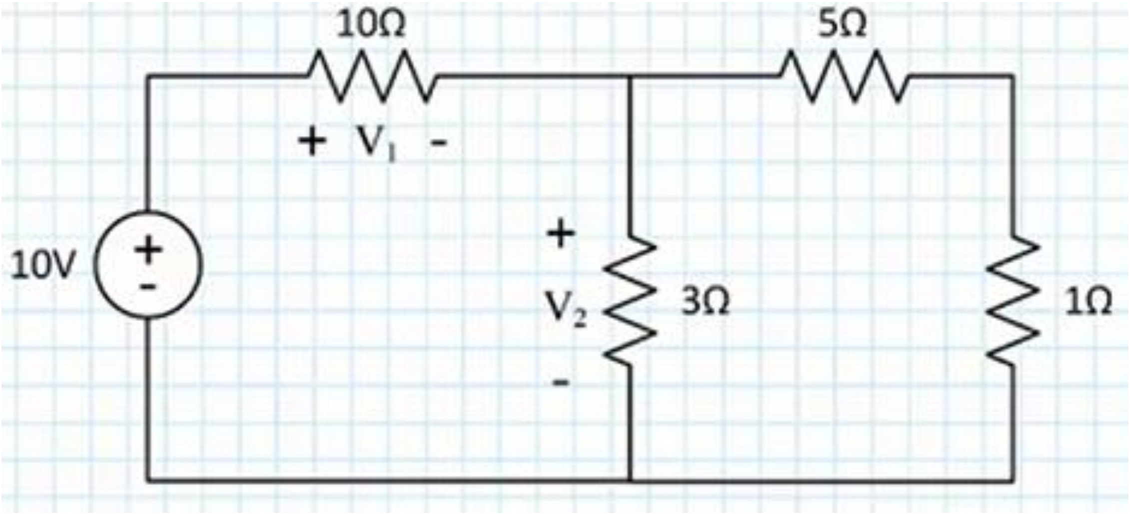
Current Divider



$I_1 = \frac{R_2}{R_1 + R_2} I_s \qquad I_2 = \frac{R_1}{R_1 + R_2} I_s$

Illustration 1

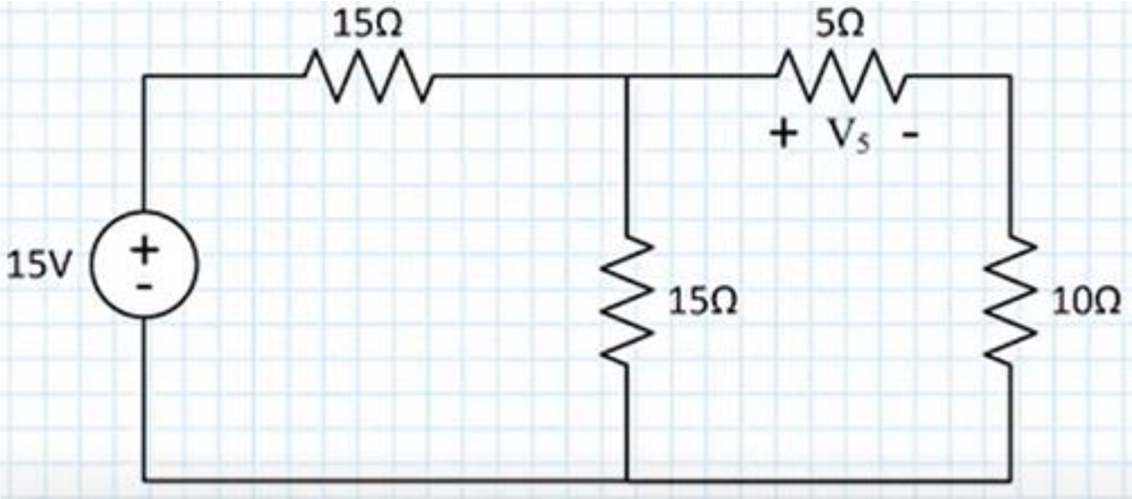
Find voltage $\mathbf{V_1}$ and $\mathbf{V_2}$ as marked in the given circuit using voltage division rule.



Ans: 8.333 V and 1.667 V

Illustration 2

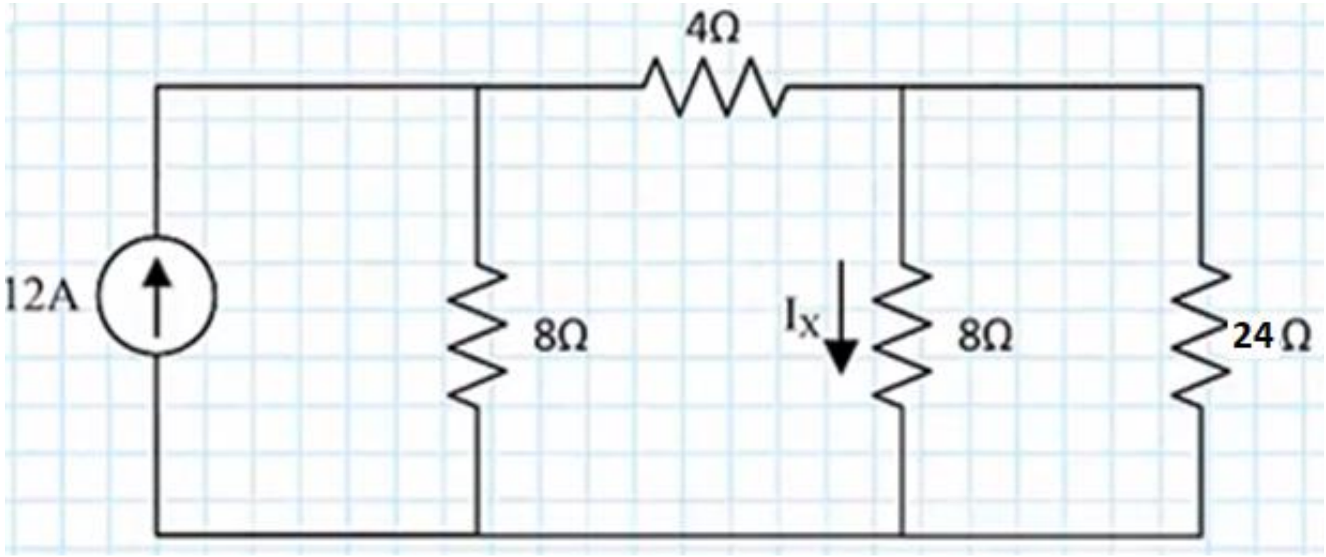
Find voltage $\mathbf{V_5}$ as marked in the given circuit using voltage division rule.



Ans: 1.667 V

Illustration 3

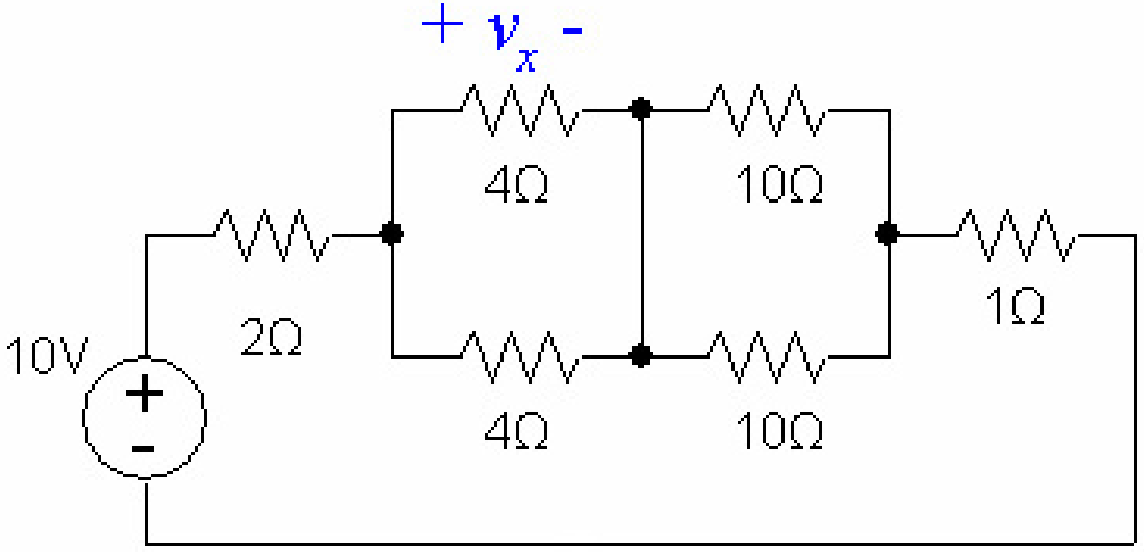
Find current $\mathbf{I_x}$ as marked in the given circuit using current division rule.



Ans: 4 A

Self-Practice 1

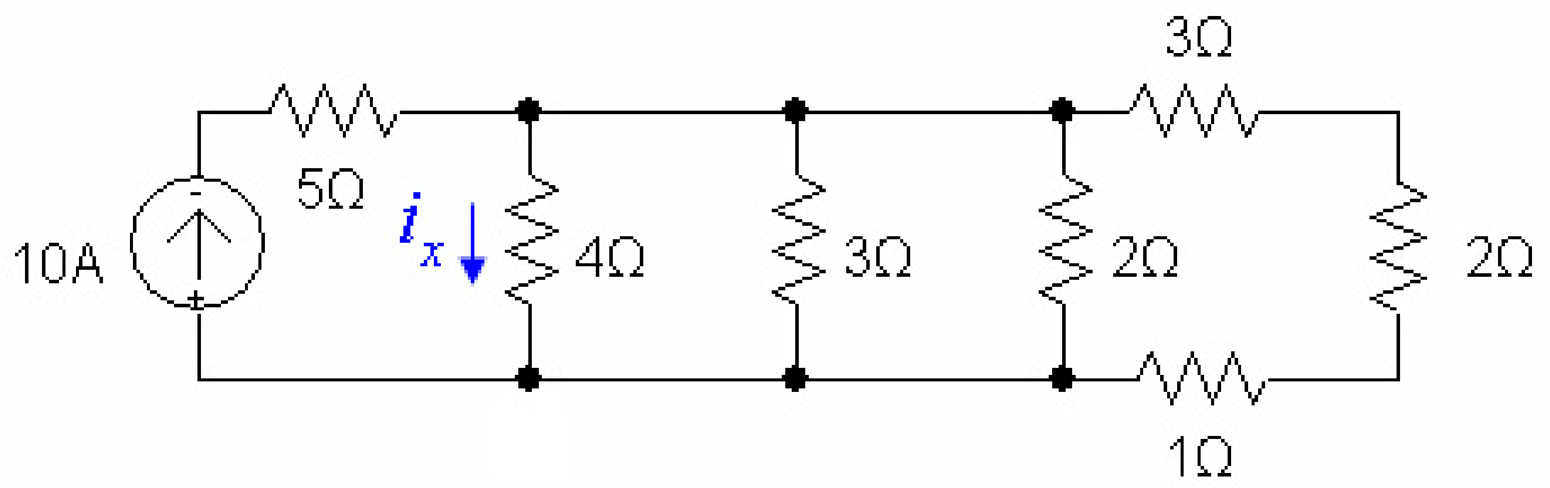
Use voltage division to find $\mathbf{v_x}$ in the circuit



Ans: 2 V

Self-Practice 2

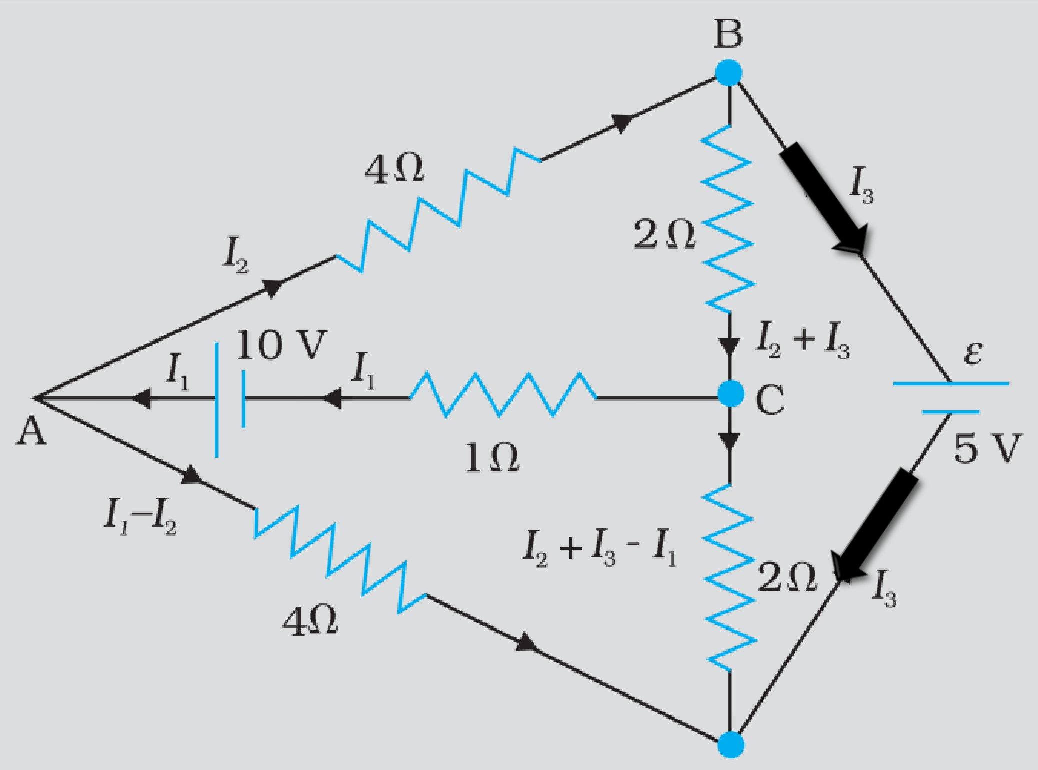
Use current division to find $\mathbf{i_x}$ in the circuit



Ans: 2 A

Source: Delivering or absorbing power?

- A battery is discharging (delivering power/energy) if,
  - Current coming out from positive (+) terminal
- A battery is charging (absorbing power/energy) if,
  - Current flowing into positive (+) terminal
- When current flows through a resistor,
  - Power is dissipated

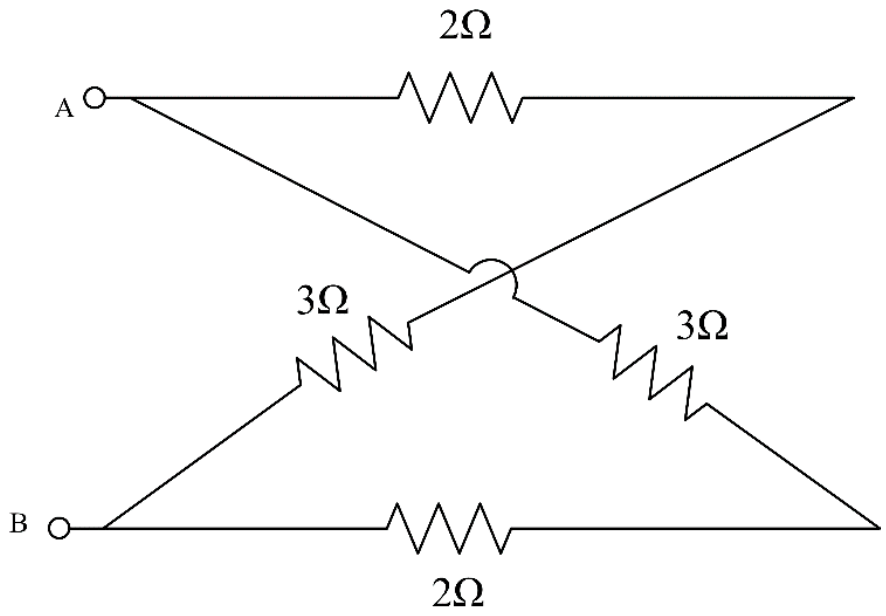


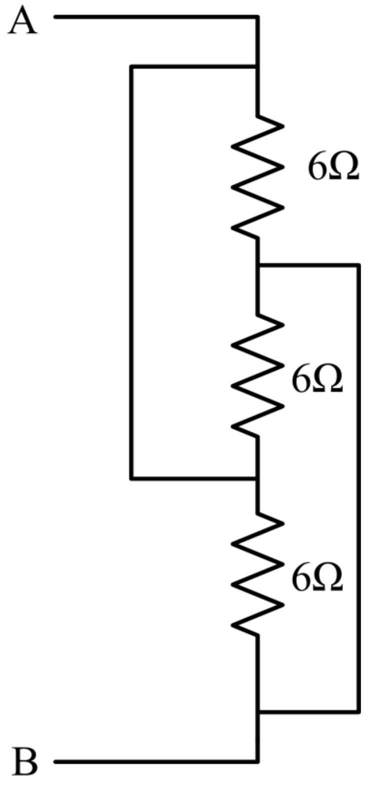
10 V battery is discharging

5 V battery is charging

Homework 1

Find the equivalent resistance of the networks across terminals A & B.





Ans: 2.5 Ω

Ans: 2 Ω

Basic Electrical Technology

LECTURE – 03

DC CIRCUIT ANALYSIS – PASSIVE ELEMENTS: L AND C

Inductor

Energy Storage Element

Inductor

- **Passive** electric device that **stores energy in its magnetic field** when current flows through it.

Inductance (also known as Self-Inductance, Unit: Henry or H)

- The property of a coil that opposes any change in the amount of current flowing through it.
- This property is due to the self-induced emf in the coil itself by the changing current.
- Self-inductance does not prevent the current from changing, it serves only to delay the change.

$$e = -N\frac{d\phi}{dt} = -N\frac{d\phi}{di}\cdot\frac{di}{dt} = -L\frac{di}{dt}$$

$$L = N\frac{d\phi}{di}$$

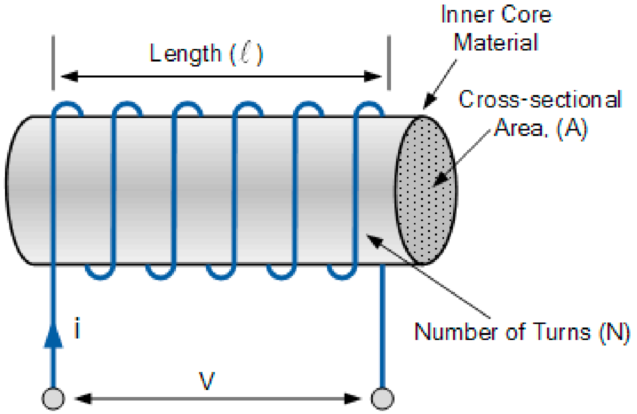
- The greater the self-induced emf, the greater the self-inductance of the coil and hence larger is the opposition to the changing current.
- Inductance of the coil depends upon: Shape and number of turns, relative permeability (μ_r) of the material surrounding the coil, and the speed at which the magnetic field changes

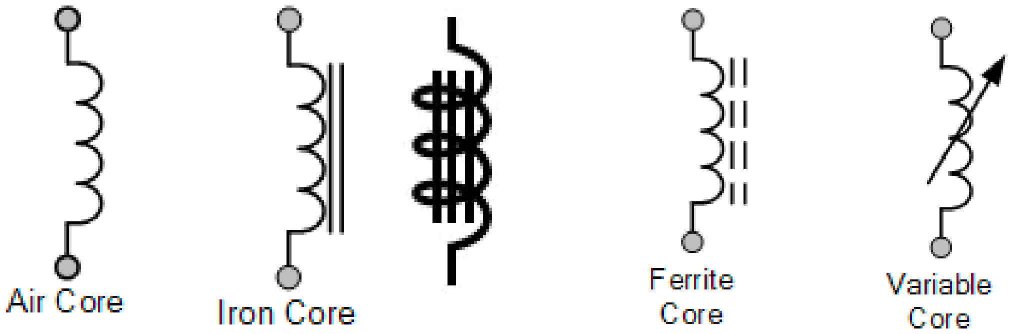
(i) $\mathbf{e = L\frac{di}{dt}}$ (ii) $N\phi = LI$ (iii) $\mathbf{L = \frac{N^2}{Reluctance} = \frac{N^2\mu_0\,\mu_r\,A}{l}}$

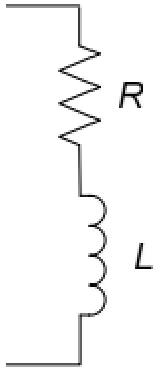
Inductor

Types of Inductors

Air-core inductor (fixed inductor)	- **Linear B-H curve, so L is the same no matter what current is in the coil.** - Since μ_r of air is 1, the values of L obtained are very low
Iron-core inductor (variable inductor)	- **Ferromagnetic core** - Provides much higher value of L (as μ_r of ferromagnetic material is very large) - **B-H curve is not linear, so inductance will vary with current.**
Practical Inductor	- All inductors are coil that have some winding resistance.

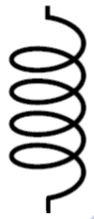
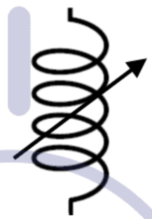
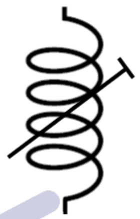
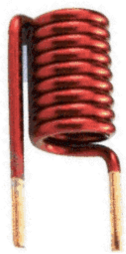
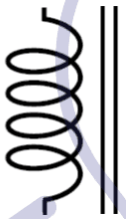
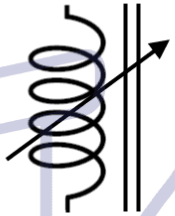
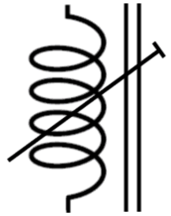
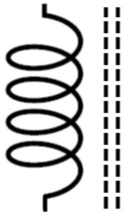
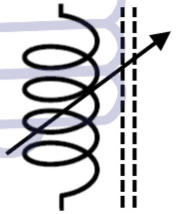
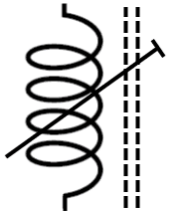






Practical Inductor

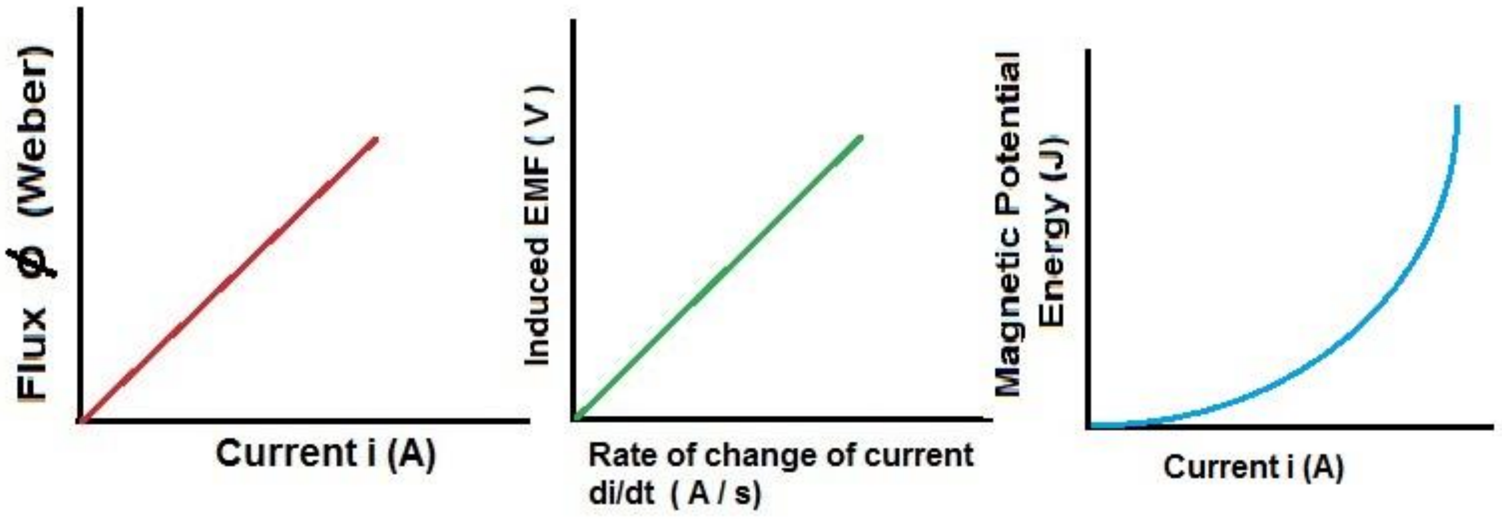
Inductor

Inductor	Fixed	Variable	Pre-set	Shape
Air Core				
Iron Core				
Ferrite Core				

Energy Stored in an Inductor

- Instantaneous power,

$$\mathbf{p = \ v_L . i = L\, i \frac{di}{dt}}$$



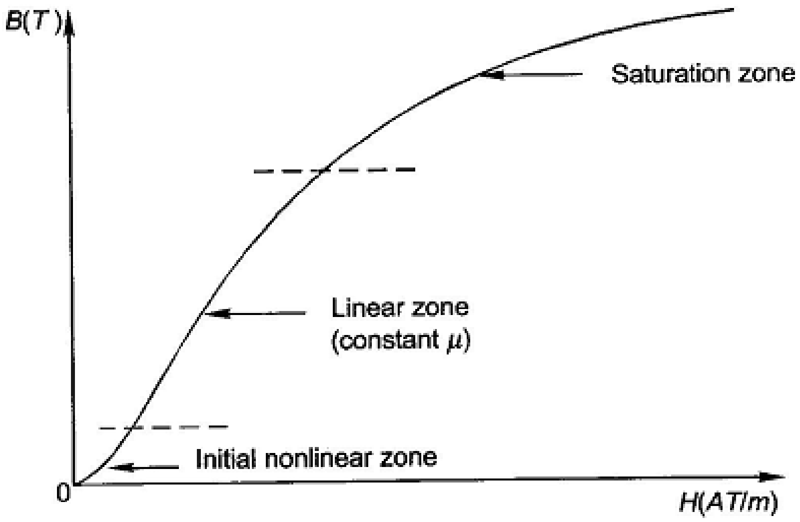
- Energy absorbed in ‘**dt**’ time is

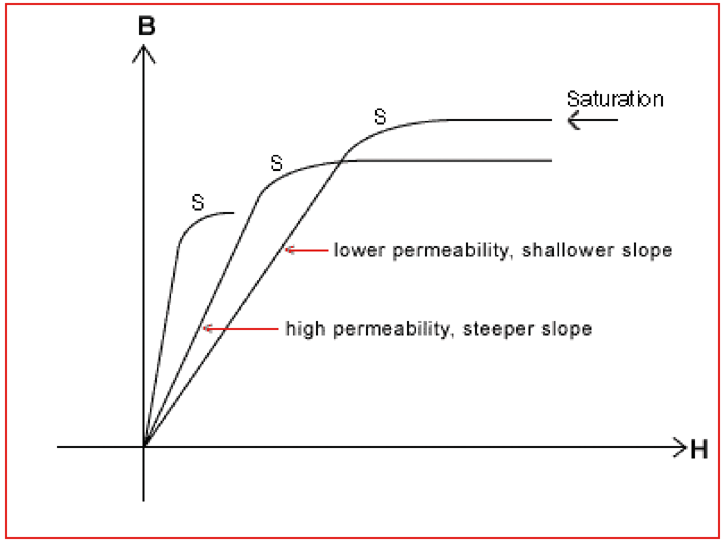
$$\mathbf{dw = L\, i\, di}$$

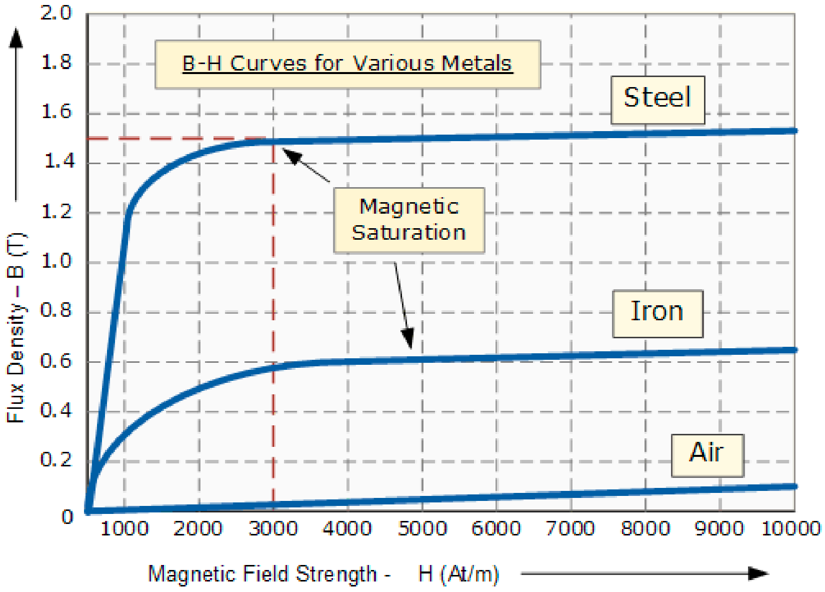
- Energy absorbed by the magnetic field when current increases from $\mathbf{0}$ to $\mathbf{I}$ amperes, is

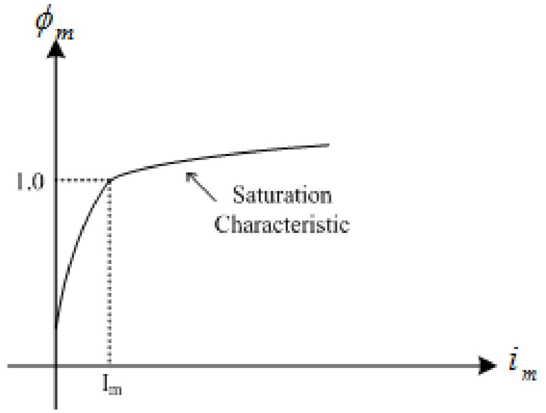
$$\mathbf{W = \ \int_0^I L\, i\, di = \frac{1}{2} L\, I^2}$$

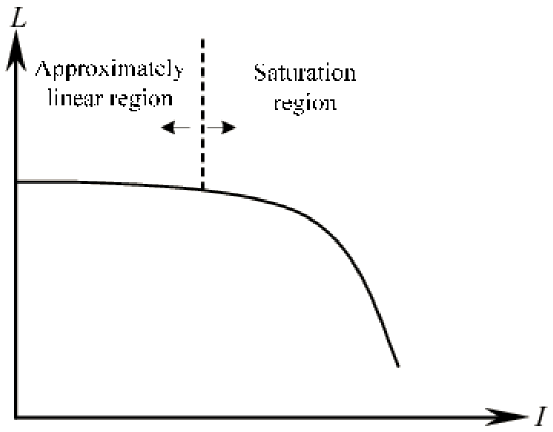
Inductance and Saturation Current





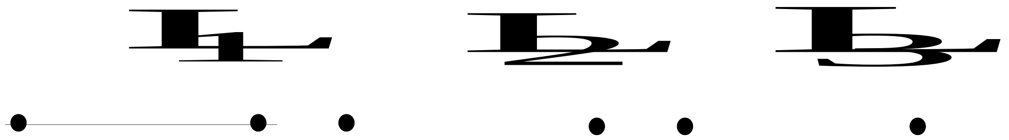






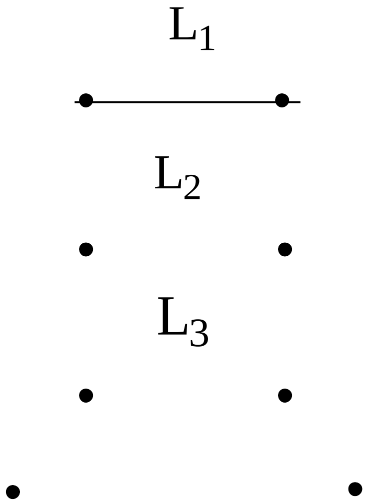
Equivalent Inductance

Inductors in series



$$\mathbf{L_{eq} = L_1 + L_2 + \dots\dots + L_n}$$

Inductors in Parallel



$$\mathbf{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots\dots . + \frac{1}{L_n}}$$

Capacitor

Energy Storing Element

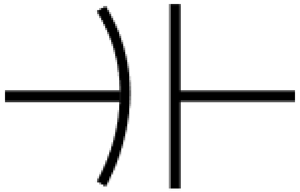
Capacitors

- **Passive electric device** that **stores energy in the electric field** between a pair of closely spaced conductors

- **Capacitance**: Property which opposes the rate of change of voltage
  - Symbol: **C**
  - Unit: Farad (F)

- The capacitive current is proportional to the rate of change of voltage across it

$$\mathbf{i_c = C\frac{dv_c}{dt}}$$



- Charge stored in a capacitor whose plates are maintained at constant voltage:

$$\mathbf{Q = CV}$$

Terminologies

- Electric field strength,

$$\mathbf{E} = \frac{\mathbf{V}}{\mathbf{d}} \text{ volts/m}$$

- Electric flux density,

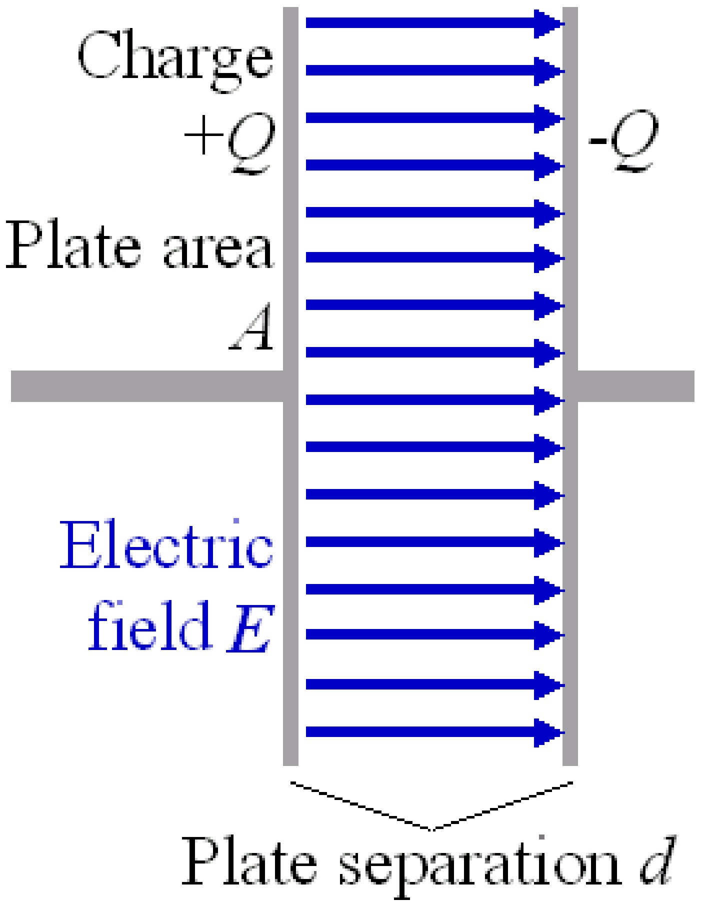
$$\mathbf{D} = \frac{\mathbf{Q}}{\mathbf{A}} \ \mathbf{C/m^2}$$

- Permittivity of free space,

$$\boldsymbol{\varepsilon_0} = \mathbf{8.854 \times 10^{-12}\ F/m}$$

- Relative permittivity, $\boldsymbol{\varepsilon_r}$
- Capacitance of parallel plate capacitor

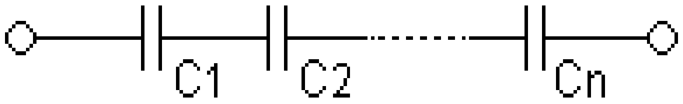
$$\mathbf{C} = \frac{\boldsymbol{\varepsilon_0 \varepsilon_r} \mathbf{A}}{\mathbf{d}}$$



Equivalent Capacitance

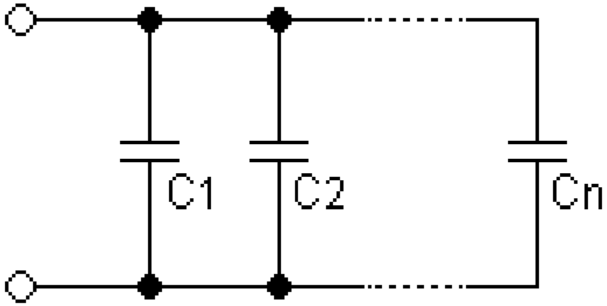
Capacitors in Series

$$\frac{1}{C_{eq}} = \frac{1}{C1} + \frac{1}{C2} + \ \ldots\ldots + \frac{1}{Cn}$$



Capacitors in Parallel

$$C_{eq} = C_1 + C_2 + \ \ldots\ldots + C_n$$



Energy stored in a Capacitor

- Instantaneous power

$$\mathbf{p} = \mathbf{v_c} \times \mathbf{i} = \mathbf{C}\,\mathbf{v_c}\,\frac{\mathbf{dv_c}}{\mathbf{dt}}$$

- Energy supplied during ‘$\mathbf{dt}$’ time is:

$$\mathbf{dw} = \mathbf{C}\,\mathbf{v_c}\,\mathbf{dv_c}$$

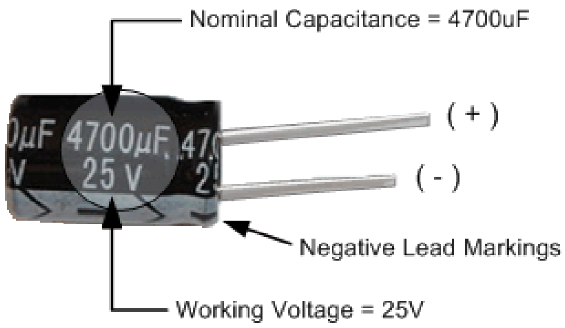
- Energy stored in the electric field when potential rises from $\mathbf{0}$ to $\mathbf{V}$ volts is,

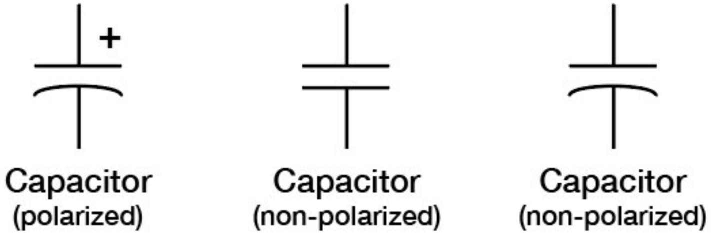
$$\mathbf{W} = \int_0^V \mathbf{C}\,\mathbf{v_c} \mathbf{dv_c} = \frac{1}{2}\mathbf{CV^2}\ \textbf{Joules}$$

Types of Capacitors and their Applications

Electrolytic Capacitor

- Used when large capacitor values are required
- Used generally in the DC power supply circuit
- Typical values ranges from 1 μF up to 47,000 μF





Mica Capacitor

- Mica is the dielectric material
- A stable, reliable, low loss capacitor of small value
- Used in high-frequency applications
- Typical values: under 100 nF

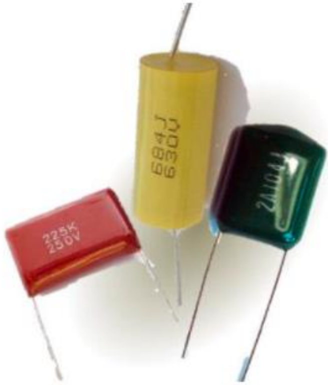
Paper Capacitor

- Typical values: Ranges from 0.001 to 2 μF
- Used in electronic noise filtering, signal processing applications, etc.

Types of Capacitors and their Applications

Film Capacitor

- Thin plastic is used as dielectric material
- It is heat resistant and used in aerospace and military technology
- Typical values: ranging from 1 nF to 30 µF



Ceramic Capacitor

- Ceramic is used as dielectric material
- Applications: Power circuit breakers, induction furnaces, also printed circuit boards in electronics
- Typical values: ranging from 1 nF to 1 µF

Non-Polarized Capacitor

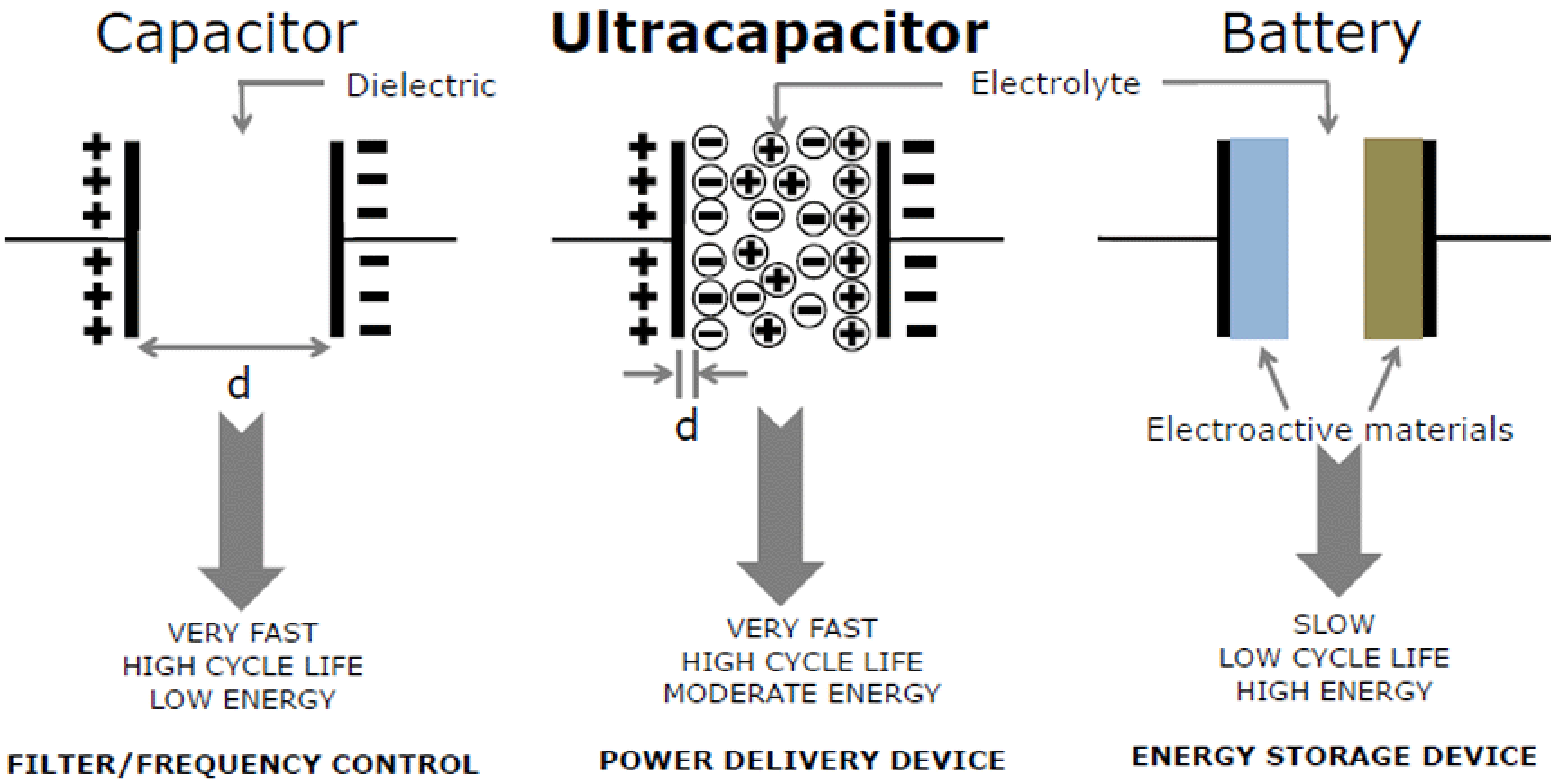
- It can be of two types: plastic foil or electrolytic.
- Used in AC applications with signal or power supply
- Typical values ranges from 1 µF up to 47,000 µF

Types of Capacitors and their Applications

Ultracapacitor (Also known as Supercapacitor or Electrochemical Capacitor)

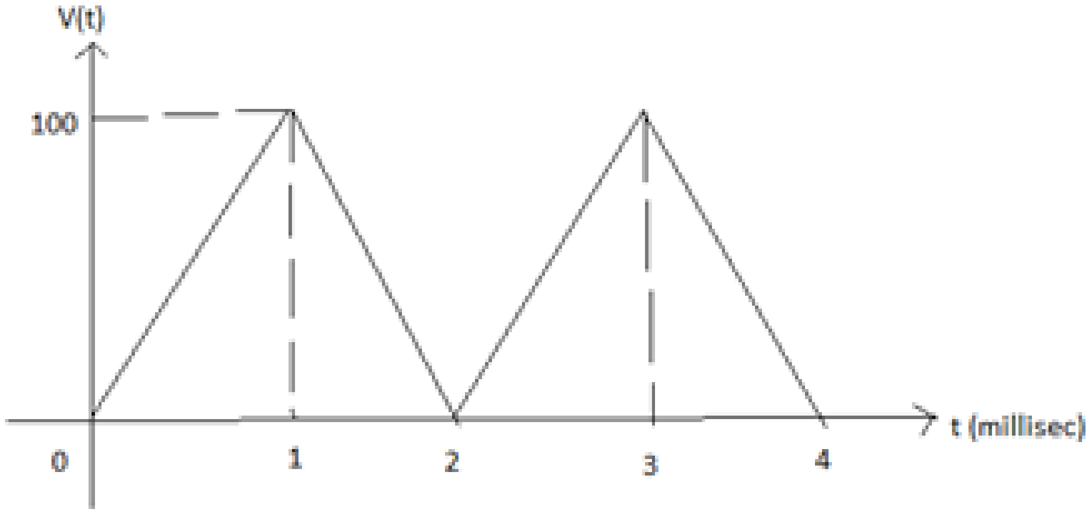
- Store energy electrostatically
- Much higher energy storage density (high-capacity capacitor) than a normal capacitor
- Can release energy much quicker than the battery and can be used many times over without degradation
- Possess high energy density, short charging cycle, and a wide range of operating temperatures
- Applications: Ranges from large-scale energy storage to small portable/wearable watches
- Has the potential to replace or augment battery and fuel cell systems in many areas of technology
- Rated in Farads - typically be found in the 100 F to 500 F range (individual cells)

Capacitor Vs Supercapacitor Vs Battery



Exercise 1

The following voltage is applied to a capacitor of **50 μF.** Determine the current for **0 – 1 ms** duration.



A) - 5 A

B) 5 mA

C) 5 A

D) 1 A

Hint: $i_c = C\frac{dV_c}{dt}$

Ans: 5 A

Exercise 2

A **100 mH** inductor is supplied with a voltage of $\mathbf{v(t) = 25\ e^{-5t}\ V}$**.** Determine the inductor current.

Hint: $i_L = \frac{1}{L}\int_0^t v\,dt$

Ans: - 50 e^{-5t} A

Mesh Current Analysis

A NETWORK ANALYSIS TECHNIQUE

LECTURE 04

Introduction

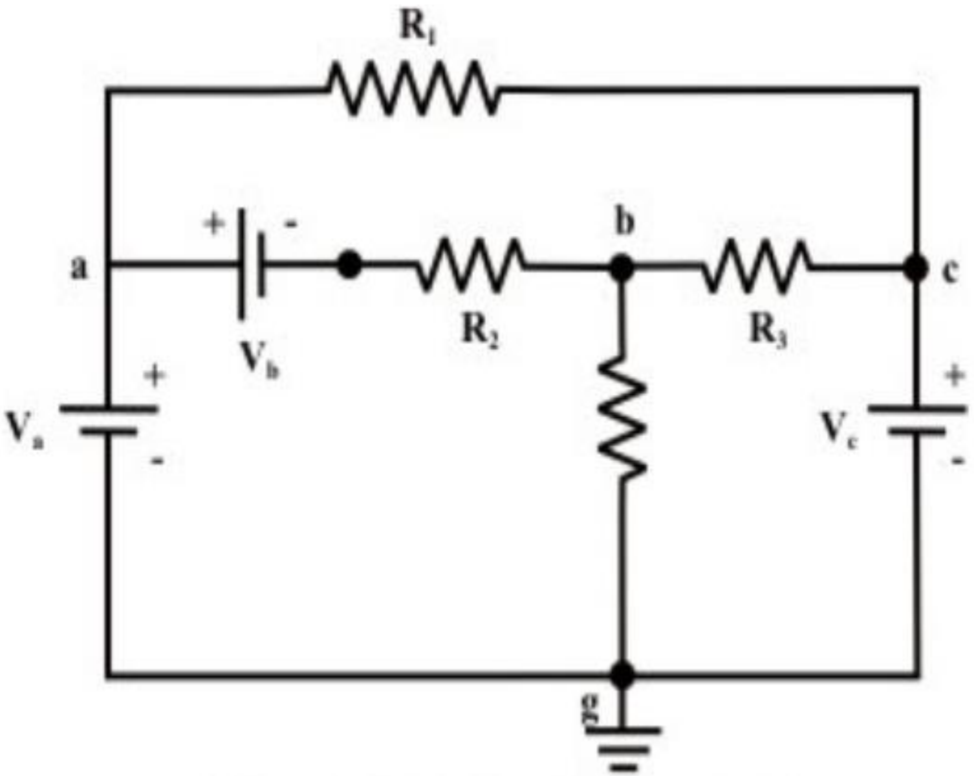
Kirchhoff's Voltage Law (KVL)

- In any closed circuit or mesh, the

  Algebraic sum of electromotive forces (emf) + Algebraic sum of voltage drops = 0

Loop

- Any closed path of electrical network
- Inside loops: a-b-g-a, b-c-g-b & a-b-c-a
- Outside loops: a-c-g-a & a-b-c-g-a



Mesh

- A loop which do not contain any other loop inside it. Same as inside loops above.
- a-b-g-a, b-c-g-b & a-b-c-a

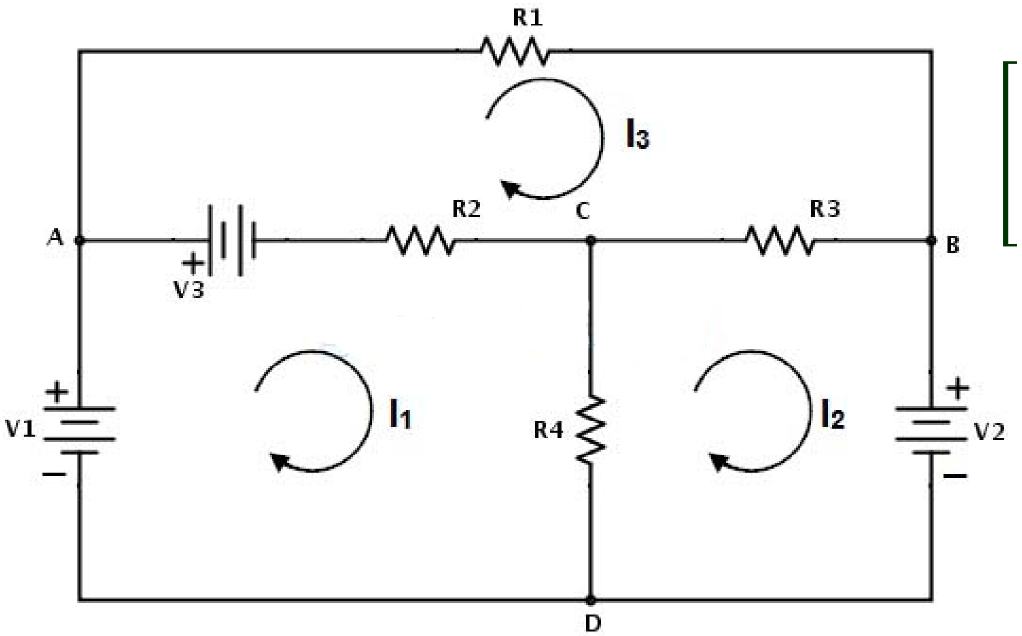
Mesh Current Analysis Method

Also known as Maxwell's Mesh Current Method

Steps

- Transform all the current sources present in the circuit to voltage sources
- Mark different currents in all the independent meshes of the given network
- Write KVL equations for these independent meshes
- Solve for the currents

Mesh Current Analysis – Matrix Equivalent



$$\begin{bmatrix} (R_2 + R_4) & -(R_4) & -(R_2) \\ -(R_4) & (R_3 + R_4) & -(R_3) \\ -(R_2) & -(R_3) & (R_1 + R_2 + R_3) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} (V_1 - V_3) \\ -V_2 \\ V_3 \end{bmatrix}$$

$$(\text{Resistance matrix}) \cdot (\text{The vector of mesh currents}) = (\text{The vector of the sum of source voltages around each mesh})$$

- Matrices can be formed by inspection and can be solved by Cramer's rule
- All self-resistances will always be positive and all mutual-resistances will always be negative
- No circuit branch can carry more than two currents
- Resistance matrix is symmetric
- Convert all practical current sources to equivalent voltage sources

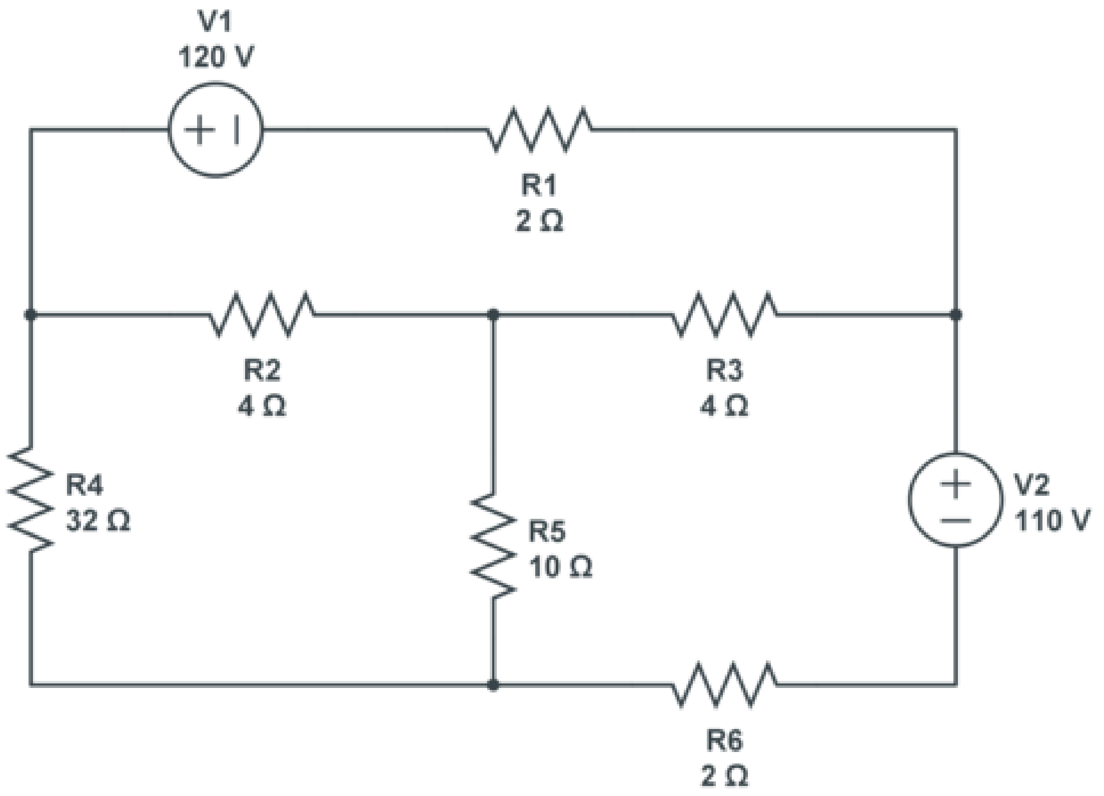
Illustration 1

a) Write matrix form of network equation and solve for currents by Cramer's rule.

b) Find the power dissipated in the **10 Ω** resistor.

c) Self Practice: Write KVL equations for independent meshes and solve for currents to find $\mathbf{P_{10\,\Omega}}$

Case 1: No requirement of source conversion

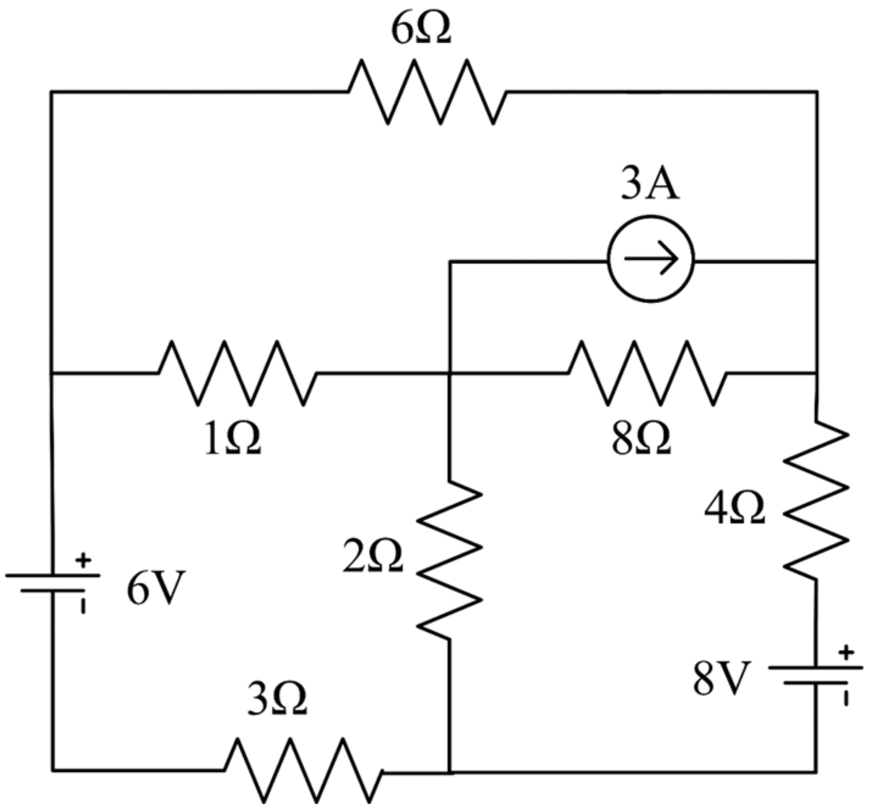


Ans: 1000 W

Illustration 2

Determine the power consumed by the **2 Ω** resistor using mesh current analysis.

Case 2: Source conversion is required and can be done

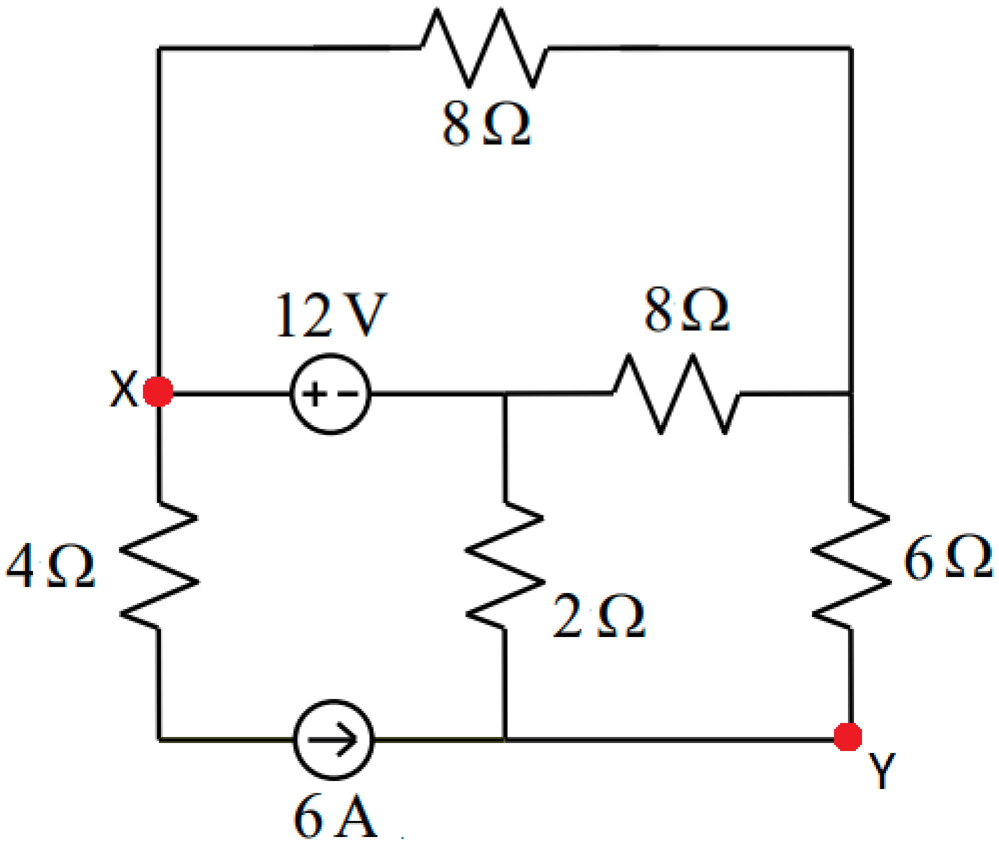


Ans: 0.3283 W

Illustration 3 (Supermesh)

Determine $\mathbf{V_{XY}}$

Case 3 A: When current source is present in the perimeter of any individual mesh

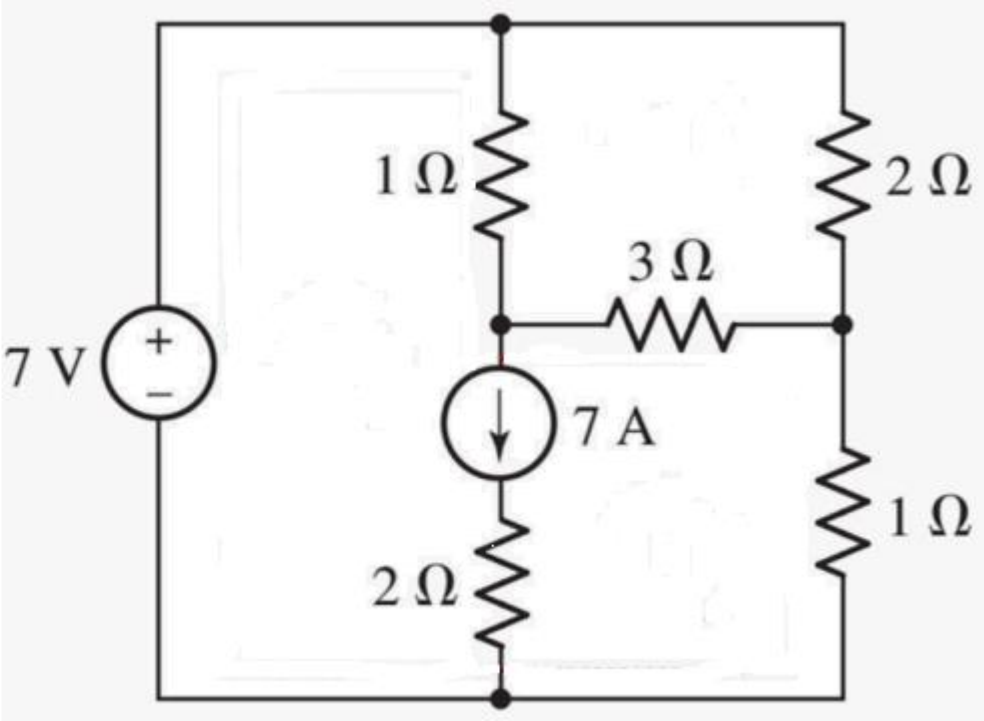


Ans: 1 V

Illustration 4 (Supermesh)

Determine the power supplied by the **7 V** and **7 A** current source.

Case 3 B: When current source is present in the middle of any individual mesh



Ans: P_{7V} = 63 W (supplied), P_{7A} = 94.5 W (supplied)

Illustration 5

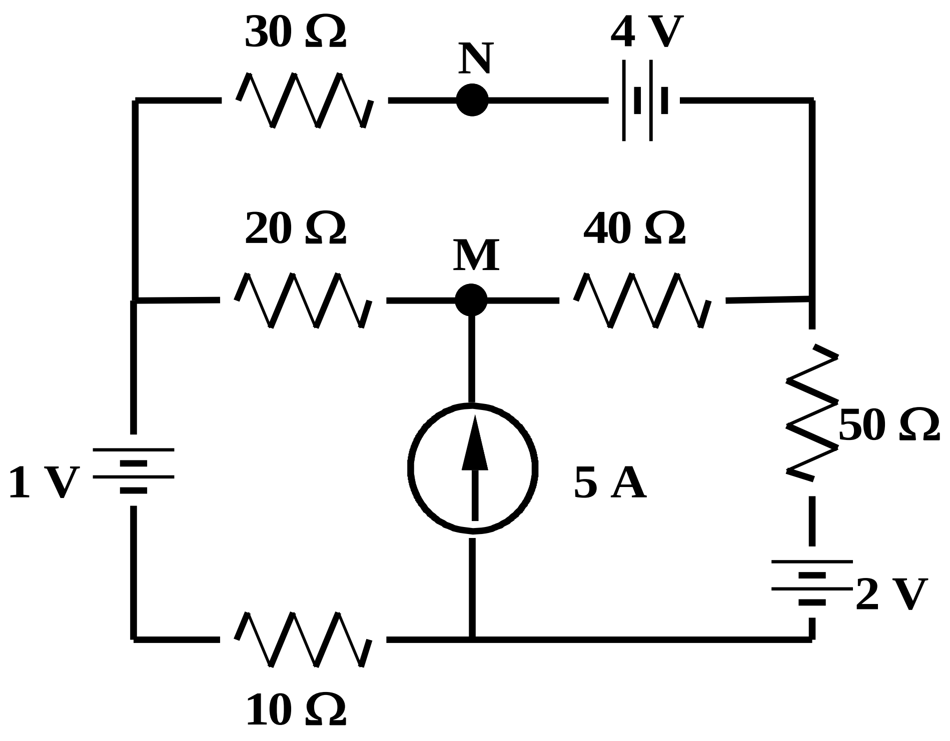
Realize the network defined by mesh current equation

$$\begin{bmatrix} 30 & -20 & 0 \\ -20 & 50 & -20 \\ 0 & -20 & 20 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -20 \end{bmatrix}$$

Case 4: Network realization from matrix form of equations.

Self-Practice 1

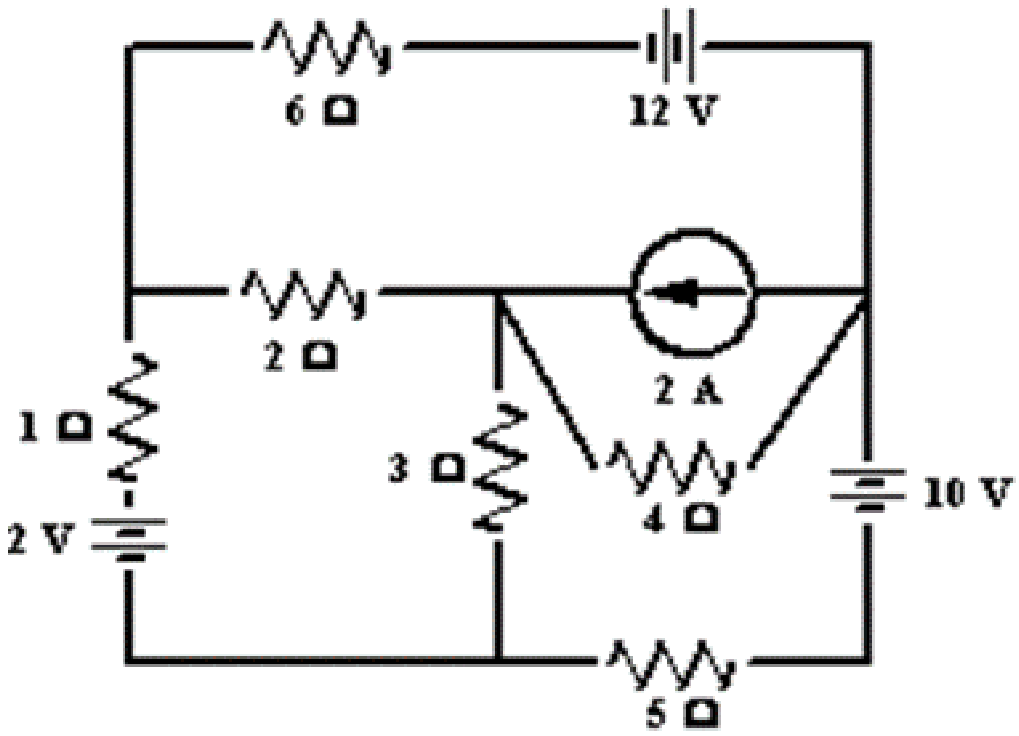
Find the power supplied by the **5 A** current source. Also, determine the voltage between the points **M & N.**



Ans: $P_{5\,A}$ = 556.5 W and V_{MN} = 55.8 V

Self-Practice 2

$P_{3\Omega}$?



Ans: 4.889 W

Node Voltage Analysis

A NETWORK ANALYSIS TECHNIQUE

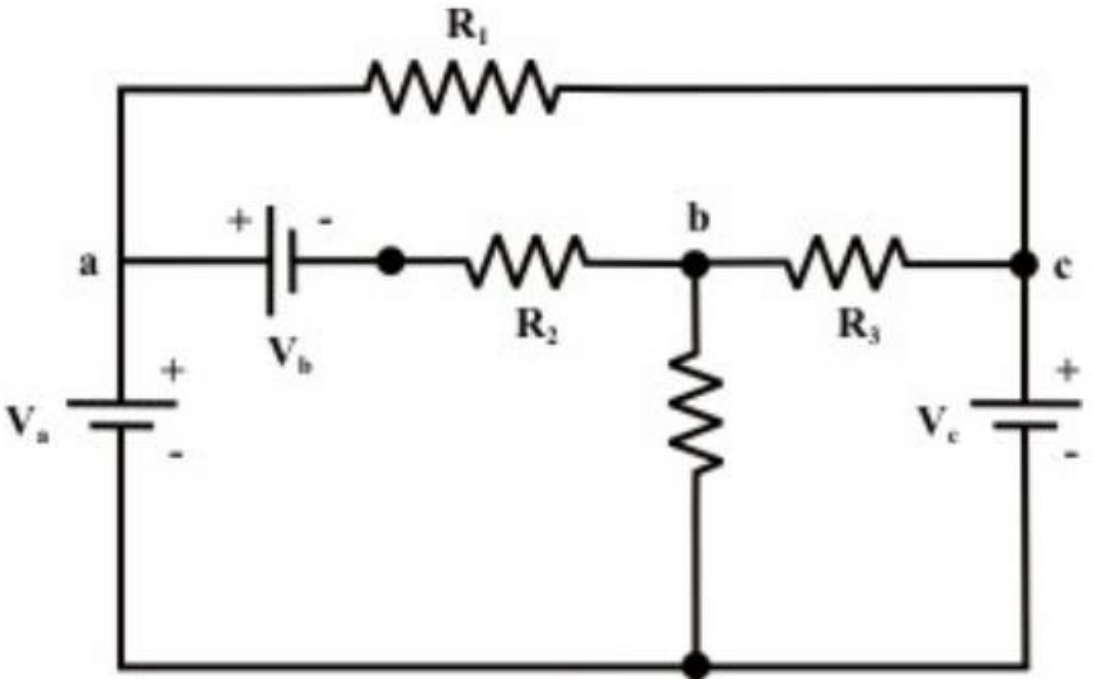
LECTURE: 05

Introduction

Kirchhoff's Current Law (KCL)

- At any node (or junction) in a circuit, the algebraic sum of currents entering and leaving the node at any instant of time must be equal to zero
- At any junction or node

  Incoming currents = Outgoing current



Node

- A point in an electric circuit where 2 or more elements are connected

Branch

- A conducting path/connection between two adjoining nodes/junction points in a circuit containing circuit elements.

Node Voltage Analysis Method

- Also known as **Nodal Analysis** or **Node to Datum Analysis**
- Convert all the practical voltage sources to current sources
- Identify nodes in the circuit
- One of the nodes is taken as the **reference (datum) node**
- Assign a voltage to each of the remaining nodes
- Write **KCL** equations for all the nodes (excluding the reference node)
- Solve for voltages
- **Advantage:** Minimum number of equations need to be written to determine the unknown quantities.
- Particularly suitable for networks having **many parallel circuits with common ground connected** such as electronic circuits.

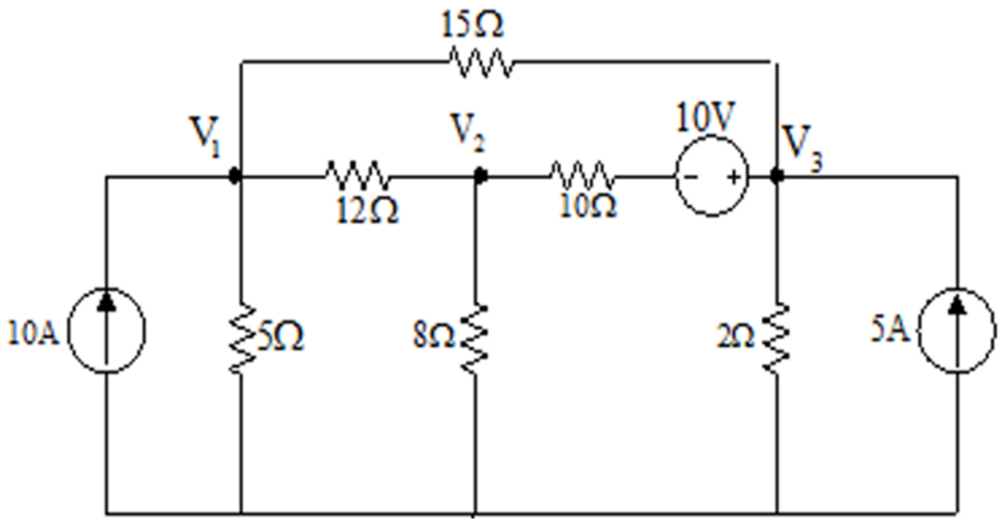


Illustration 1: Nodal Analysis - Matrix Equivalent

KCL at Node 1: $\frac{V_1}{2}+\frac{V_1-V_2}{2}+\frac{V_1-V_3}{10}-28=0$ or $\left(\frac{11}{10}\right)V_1-\left(\frac{1}{2}\right)V_2-\left(\frac{1}{10}\right)V_3=28$

KCL at Node 2: $\frac{V_2}{5}+\frac{V_2-V_1}{2}+\frac{V_2-V_3}{1}=0$ or $-\left(\frac{1}{2}\right)V_1+\left(\frac{17}{10}\right)V_2-\left(\frac{1}{1}\right)V_3=0$

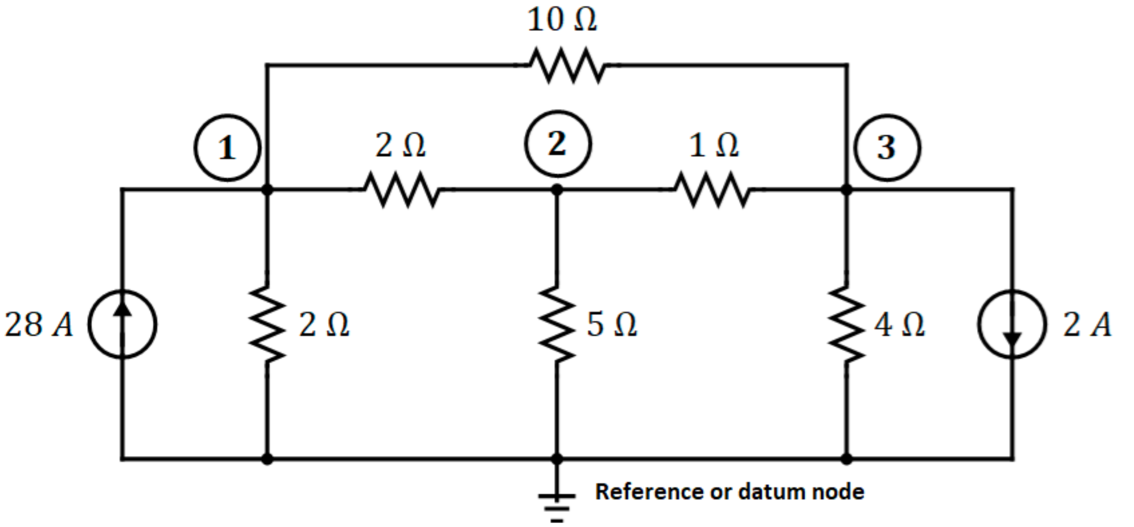
KCL at Node 3: $\frac{V_3-V_1}{10}+\frac{V_3-V_2}{1}+\frac{V_3}{4}+2=0$ or $-\left(\frac{1}{10}\right)V_1-\left(\frac{1}{1}\right)V_2+\left(\frac{27}{20}\right)V_3=-2$

$$\mathbf{G}=\frac{\mathbf{1}}{\mathbf{R}}$$

$$\begin{bmatrix} \frac{11}{10} & -\frac{1}{2} & -\frac{1}{10} \\ -\frac{1}{2} & \frac{17}{10} & -\frac{1}{1} \\ -\frac{1}{10} & -\frac{1}{1} & \frac{27}{20} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 28 \\ 0 \\ -2 \end{bmatrix}$$

Case 1: No requirement of source conversion

$$\begin{bmatrix} \left(\frac{1}{2}+\frac{1}{2}+\frac{1}{10}\right) & -\frac{1}{2} & -\frac{1}{10} \\ -\frac{1}{2} & \left(\frac{1}{2}+\frac{1}{1}+\frac{1}{5}\right) & -\frac{1}{1} \\ -\frac{1}{10} & -\frac{1}{1} & \left(\frac{1}{4}+\frac{1}{1}+\frac{1}{10}\right) \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 28 \\ 0 \\ -2 \end{bmatrix}$$



$V_1 = 36\text{ V}$

$V_2 = 20\text{ V}$

$V_2 = 16\text{ V}$

Nodal Analysis - Matrix Equivalent

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

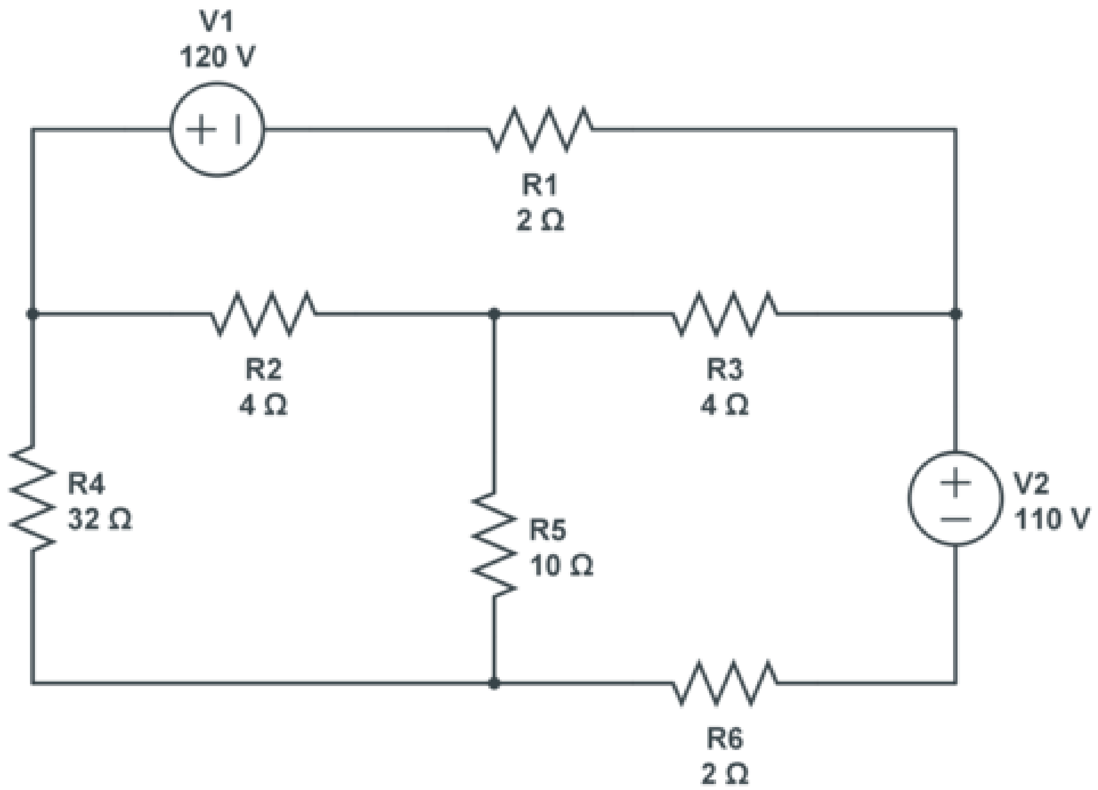
(Conductance or Admittance matrix) * (Vector of node voltages)
= (Vector of source currents entering each node)

- Conductance matrix is **symmetric**
- **Convert all practical voltage sources to equivalent current sources**

Illustration 2

Using nodal analysis find $\mathbf{P_{10\,\Omega}}$

Case 2: Source conversion is required and can be done



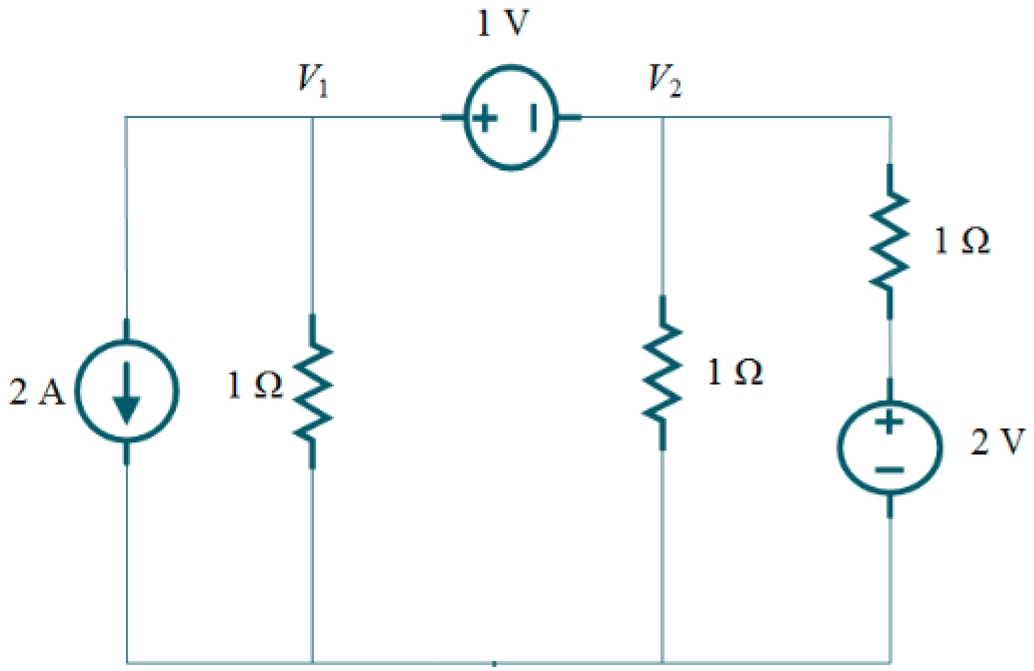
Ans: 1000 W

Illustration 3 (Supernode)

Find nodal voltages $\mathbf{V_1}$ and $\mathbf{V_2}$

Case 3: When source conversion is not possible.

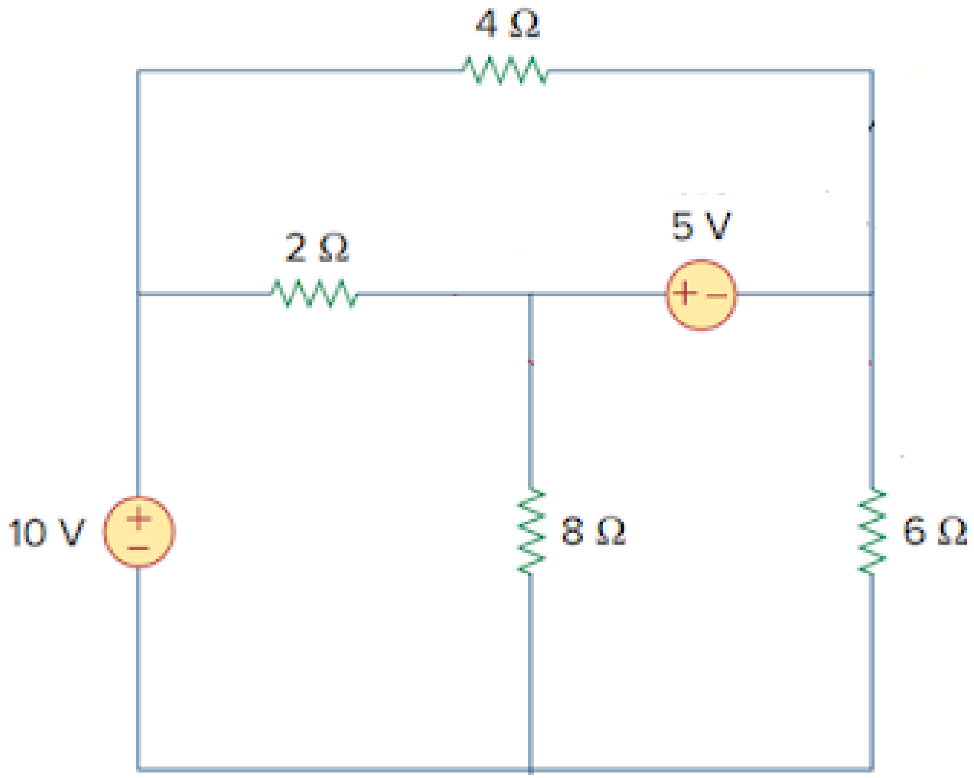
SUPERNODE: It is the presence of a voltage source between two nodes



Ans: $V_1 = 2/3$ V and $V_2 = -1/3$ V

Illustration 4

Determine the power supplied by the two sources using node voltage method.



Ans:

V_1 = 10 V, V_2 = 9.2 V, V_3 = 4.2 V

$P_{10\text{ V}}$ = 18.5 W (supplied), $P_{5\text{ V}}$ = 3.75 W (supplied)

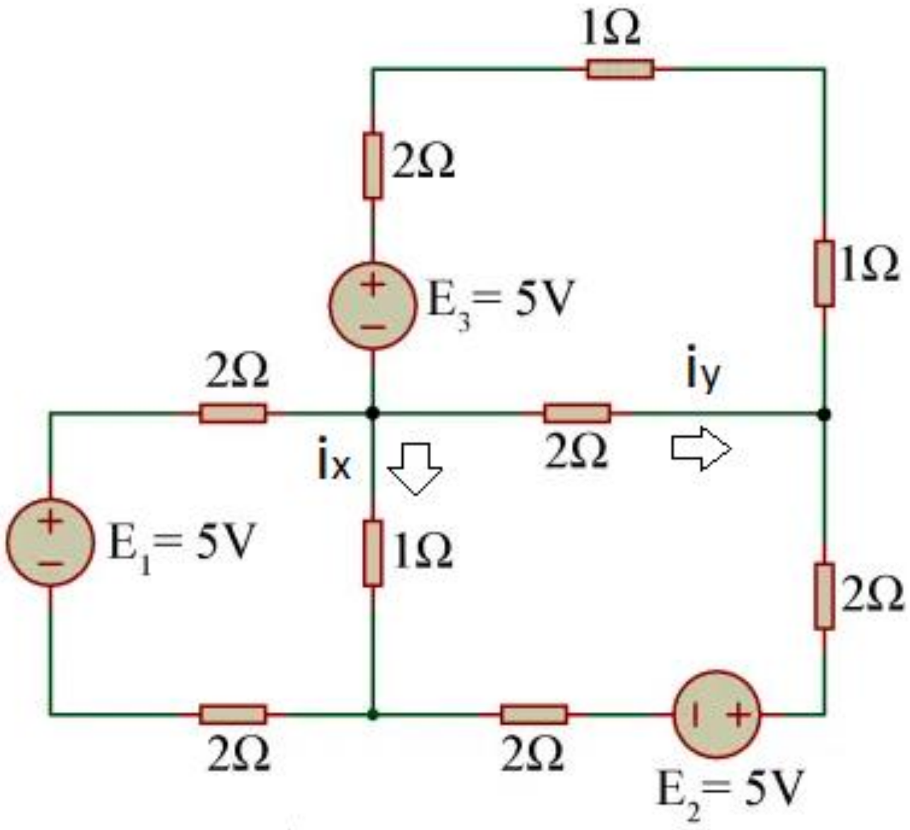
Illustration 5

Realize the network defined by node voltage equation

$$\begin{bmatrix} 1.55 & -0.5 & -0.05 \\ -0.5 & 0.875 & -0.25 \\ -0.05 & -0.25 & 0.4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}$$

Self-Practice 1

Using nodal analysis find $\mathbf{i_x}$ and $\mathbf{i_y}$



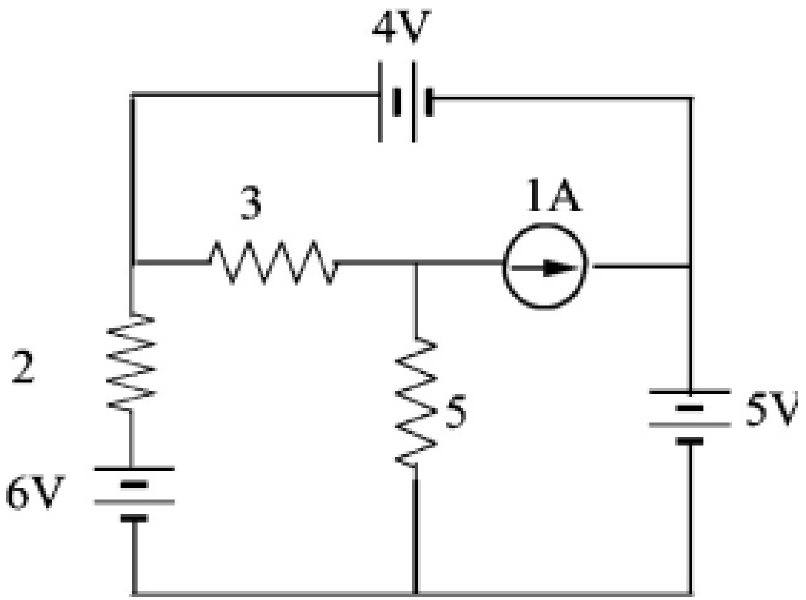
Ans:

i_x= 1.304 A, i_y = - 2.478 A

Node voltages: 1.304 V and 3.478 V

Self-Practice 2

Determine the power consumed by **5 Ω** resistor by node voltage analysis. All resistances are in Ohms.



Ans: $P_{5\,\Omega} = 2.8125$ W

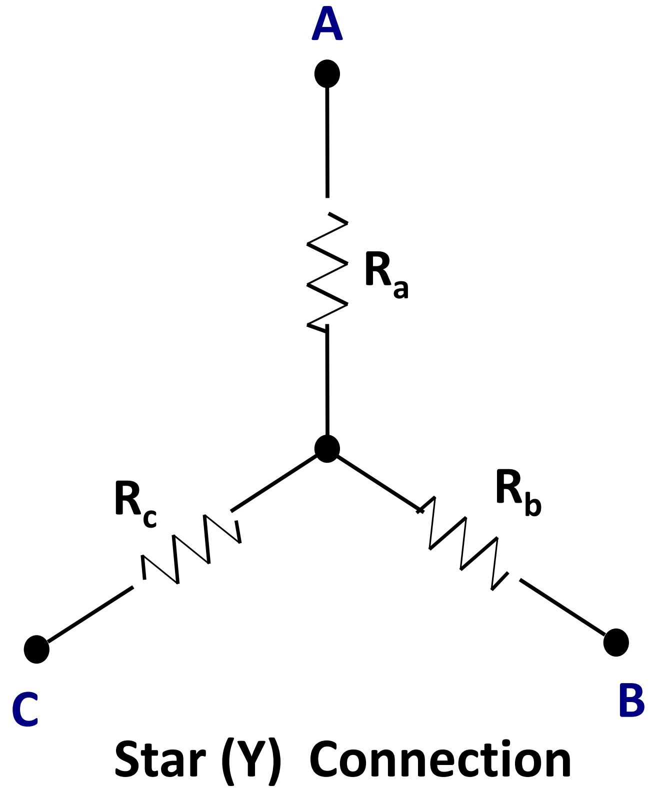
Star - Delta and Delta - Star Transformation

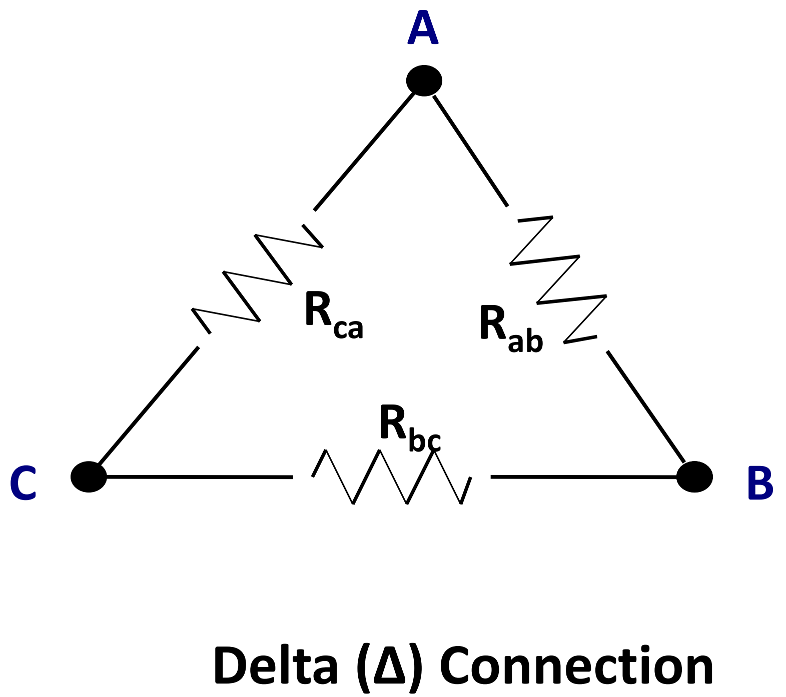
$\mathrm{Y} - \Delta$ and $\Delta - \mathrm{Y}$

A NETWORK REDUCTION TECHNIQUE

LECTURE: 06

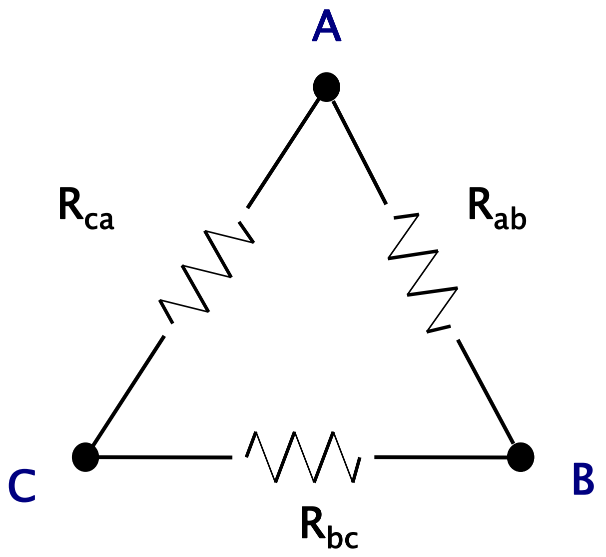
Star (Y) & Delta (Δ) Connections





Link for the formula derivation:
https://nptel.ac.in/content/storage2/courses/108105053/pdf/L-06(GDR)(ET)%20((EE)NPTEL).pdf

Delta (Δ) – Star (Y) Transformation

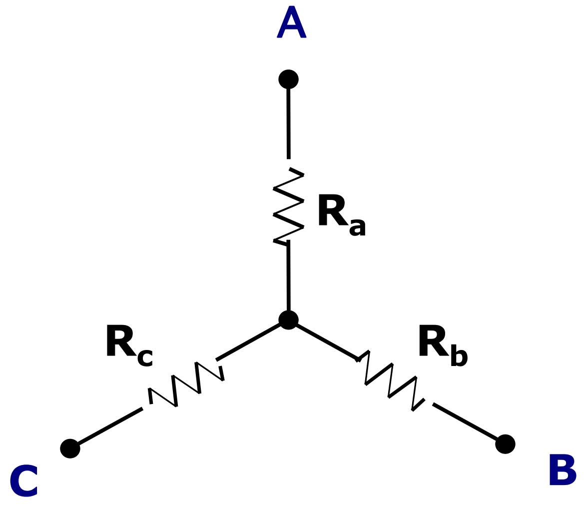


Δ to Y Transformation

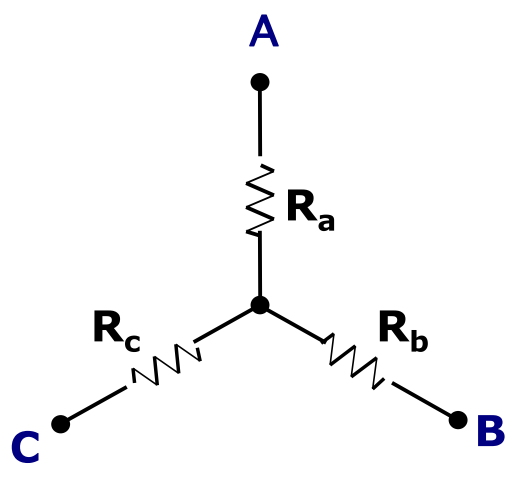
$$R_a = \frac{R_{ab}\,R_{ca}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{ab}\,R_{ca}}{\sum R_\Delta}$$

$$R_b = \frac{R_{bc}\,R_{ab}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{bc}\,R_{ab}}{\sum R_\Delta}$$

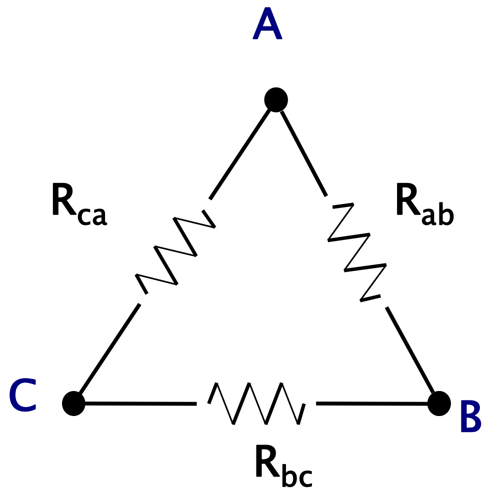
$$R_c = \frac{R_{ca}\,R_{bc}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{ca}\,R_{bc}}{\sum R_\Delta}$$



Star (Y) - Delta (Δ) Transformation



Y to Δ Transformation

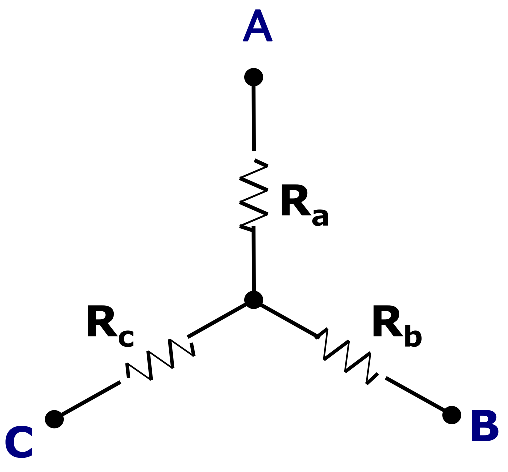


$$R_{ab} = R_a + R_b + \frac{R_a R_b}{R_c} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

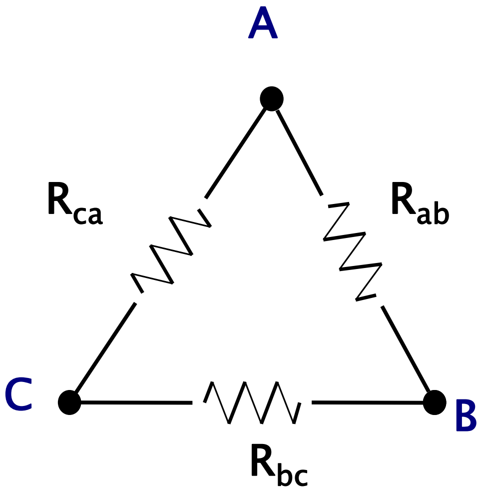
$$R_{bc} = R_b + R_c + \frac{R_b R_c}{R_a} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$$R_{ca} = R_c + R_a + \frac{R_a R_c}{R_b} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

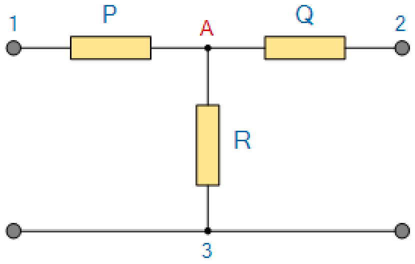
Balanced Star (Y) –Delta (Δ)



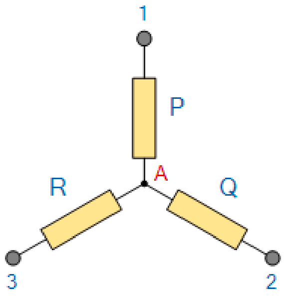
Balanced Y



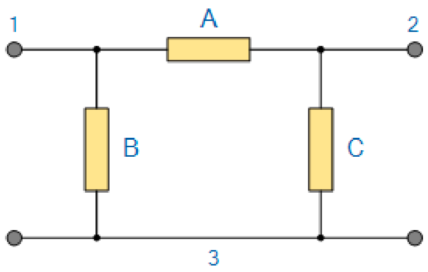
Balanced Δ



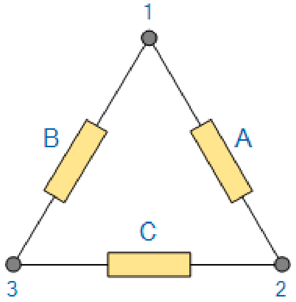
T-Network



Star-Network



Pi-Network



Delta-Network

$$\mathbf{R_Y = R_a = R_b = R_c}$$

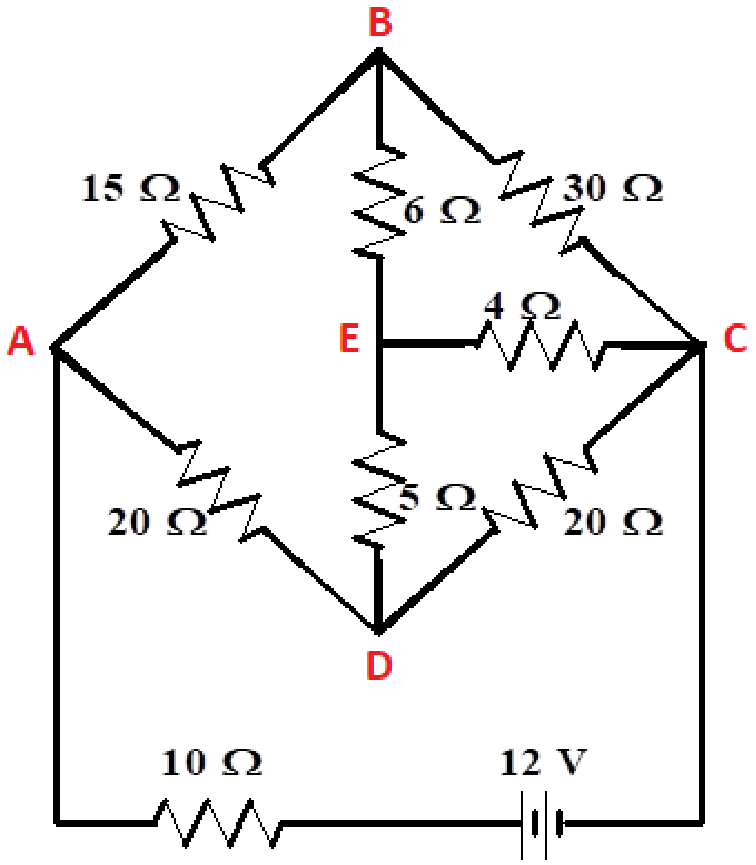
$$\mathbf{R_\Delta = R_{ab} = R_{bc} = R_{ca}}$$

$$\mathbf{R_\Delta = 3\ R_Y}$$

Converting Δ network into Y and vice-versa often simplifies the network and makes it possible to apply series-parallel circuit techniques

Illustration 1

Determine $\mathbf{P_{10\,\Omega}}$



Starting Step:

Approach 1: E as Y point and Y to Δ of R_B, R_C, R_D

Self-Practice Approaches:

Approach 2: Δ to Y of B-E-C setup

Approach 3: Δ to Y of D-E-C setup

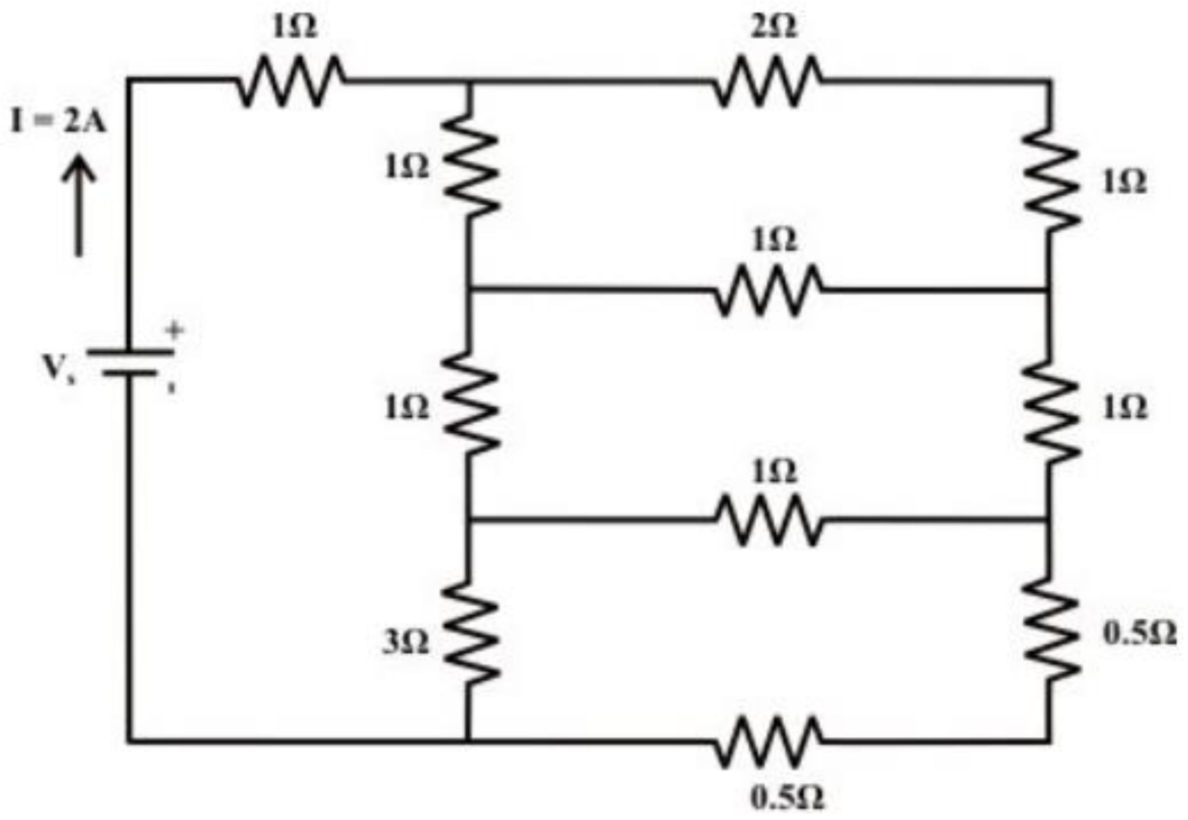
Approach 4: B as Y point and Y to Δ of R_A, R_E, R_C

Approach 5: D as Y point and Y to Δ of R_A, R_E, R_C

Ans: $P_{5\,\Omega}$ = 2.71845 W

Self-Practice 1

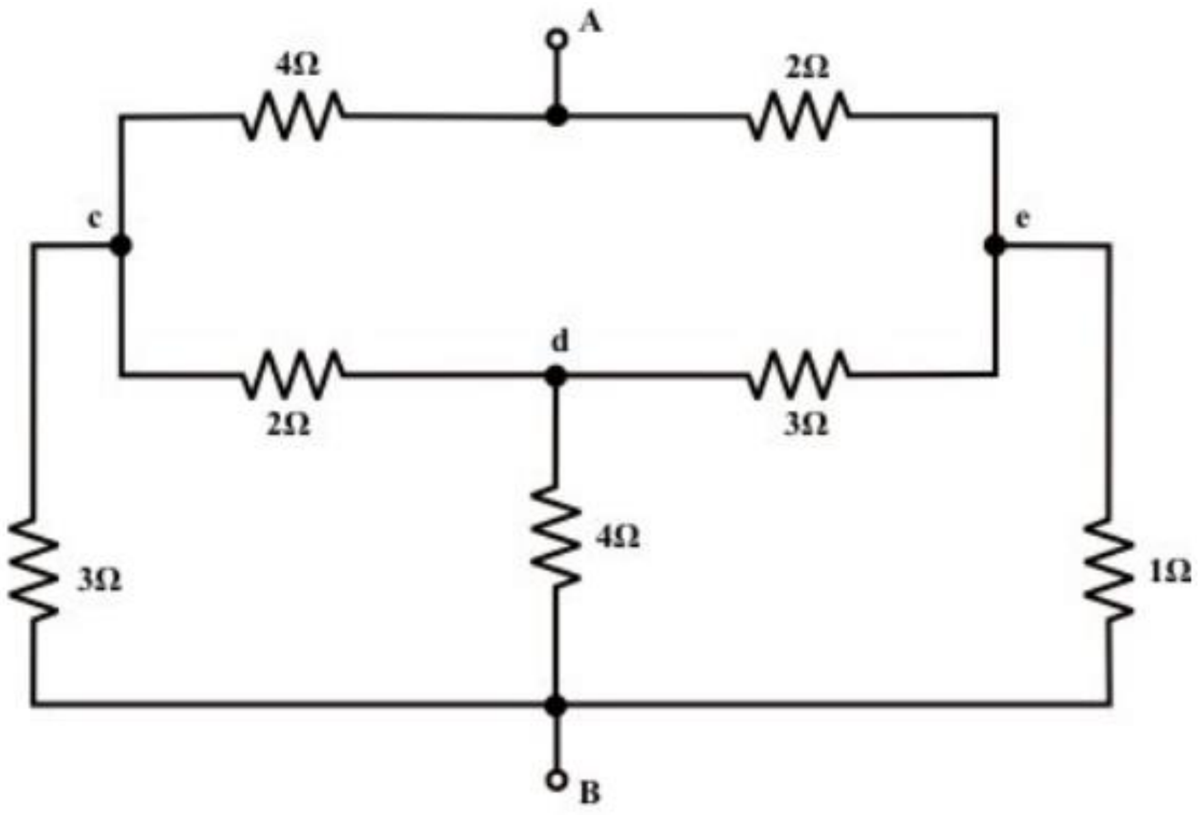
Determine the source voltage $\mathbf{V_s}$ that delivers the **2 A** in the circuit as shown



Ans: V_S = 6.2 V

Self-Practice 2

Determine equivalent resistance across the terminals **A** and **B**.



Ans: 2.21 Ω

Network Theorems

SUPERPOSITION THEOREM

LECTURE: 07

Terminologies Used

- Linear element: V-I characteristics are linear. Example: R, L, and C
- Non-linear element: V-I characteristics are non-linear. Example: Diode
- Bi-lateral element: Property does not change with the direction of operation. Example: R, L, and C
- Unilateral element: Property changes with the direction of operation. Example: Diode
- Linear Circuit: Circuit with linear elements only.
- Bi-lateral circuit: Circuit with bi-lateral elements only.
- Response: The output of the network. Example: Currents and voltages at various points in a circuit
- Excitation: The independent sources in the circuit. Example: Current and voltage sources

Superposition Theorem

- In **any linear, bi-lateral** network, the **total response** is the **sum** of partial responses.
- In any linear, bilateral, resistive network, with more than one generator (current sources and voltage sources), the current through or voltage across any element of the circuit is the algebraic sum of individual currents or voltages caused by the separate independent sources acting alone, with all other independent sources being replaced by their internal resistances (ideal voltage sources being replaced by a short circuit and ideal current sources being replaced by open circuit).

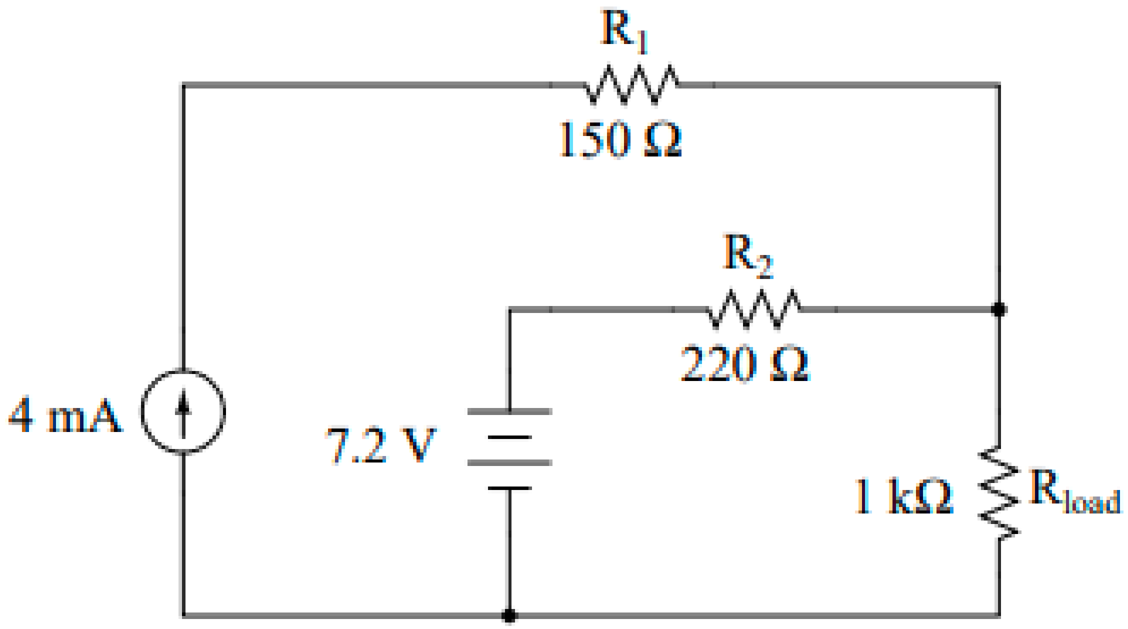
Procedure to Apply Superposition Theorem

1. Draw the circuit with passive elements only.
2. Place one of the sources in its position.
3. Replace the other sources with their internal resistances.
   a. **Ideal voltage source by a short circuit**
   b. **Ideal current source by an open circuit**
4. Find the response using one of the methods, i.e., network reduction, mesh current, node voltage methods, star-delta, etc.
5. Repeat the procedure for all the sources.
6. Add the responses due to individual sources.

Illustration 1

Apply the **Superposition** theorem to calculate the amount of current through the load resistor.

Self-Practice: Verify the result using 1) Mesh analysis, and 2) Nodal analysis

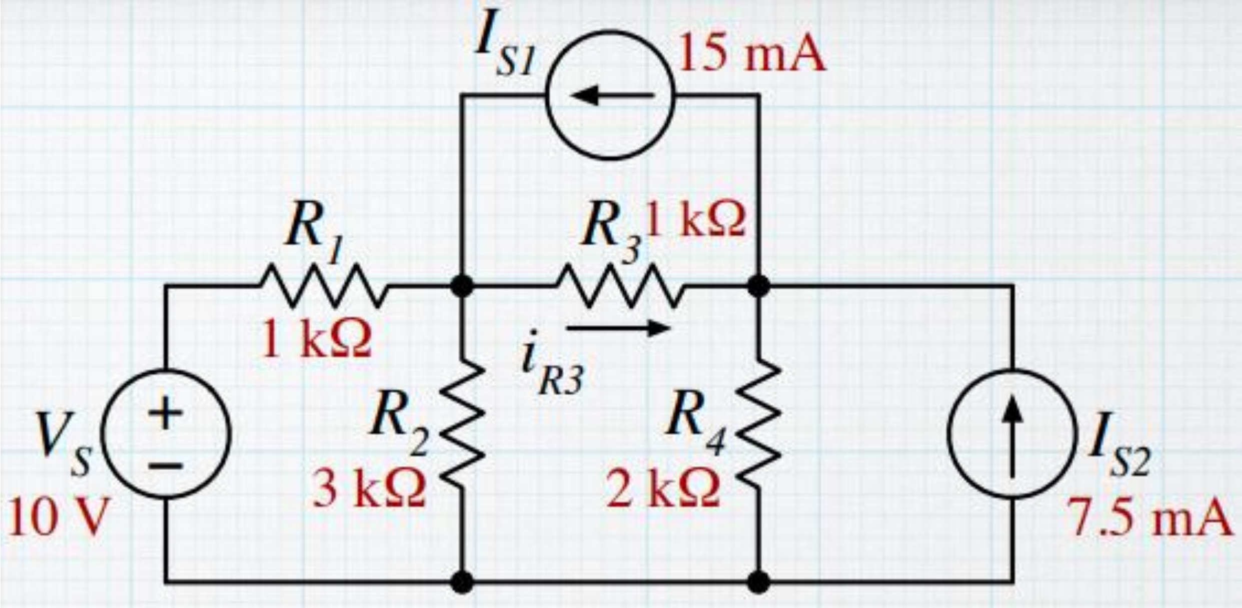


Ans: 6.623 mA

Illustration 2

In the circuit, find $\mathbf{I_{R3}}$ using the **Superposition** method.

Self-Practice: Verify the result using Nodal analysis



Ans:

Due to source V_S: 2 mA

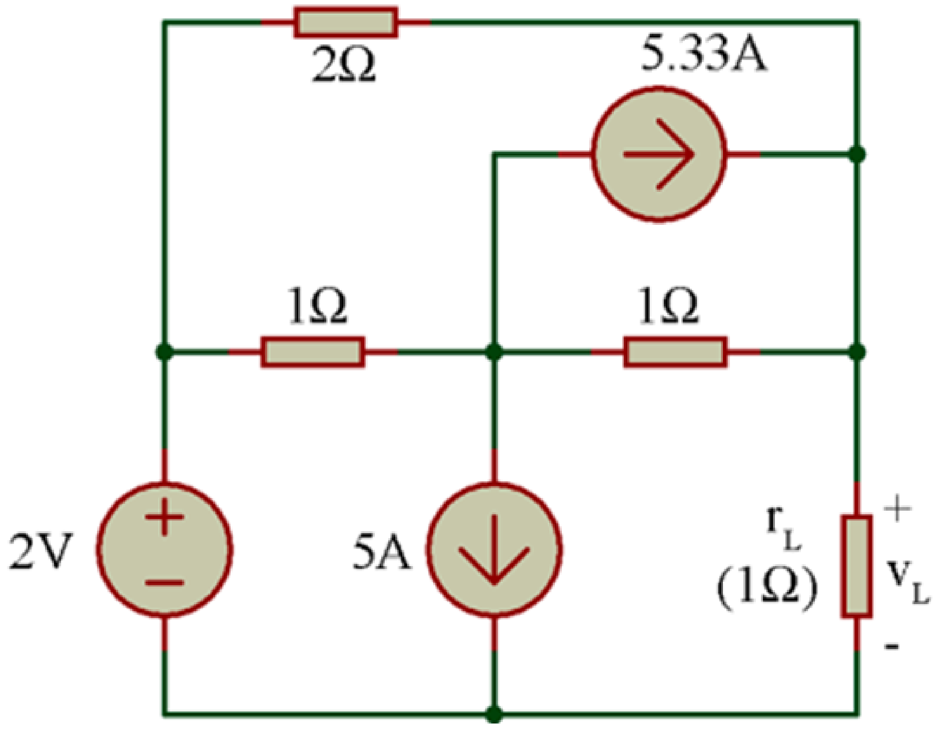
Due to source I_{S1}: 11 mA

Due to source I_{S2}: - 4 mA

Total: 9 mA

Illustration 3

Find $\mathbf{V_L}$ in the circuit using the Superposition principle.



Answer:

Due to source 2 V: 1 V

Due to source 5 A: - 1.25 V

Due to source 5.33 A: 1.3325 V

Total: 1.0825 V

Limitations of superposition Theorem

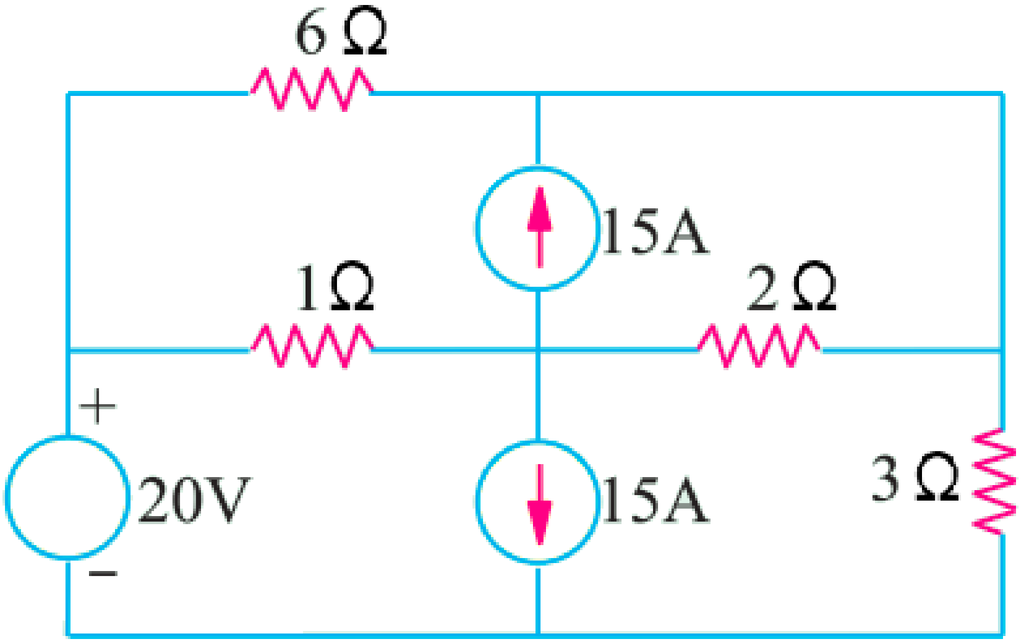
- **Doesn't work for power computation which involves**
  - product of voltage and current,
  - the square of current or
  - the square of the voltage

  which are non-linear mathematical operations

- The superposition principle can be used to find the power dissipated in a resistor provided computation of power is performed after finding the actual current through or voltage across the resistor. It is **not permissible** to find the power by adding all the power dissipation computed when each source is acting along.

Self-Practice 1

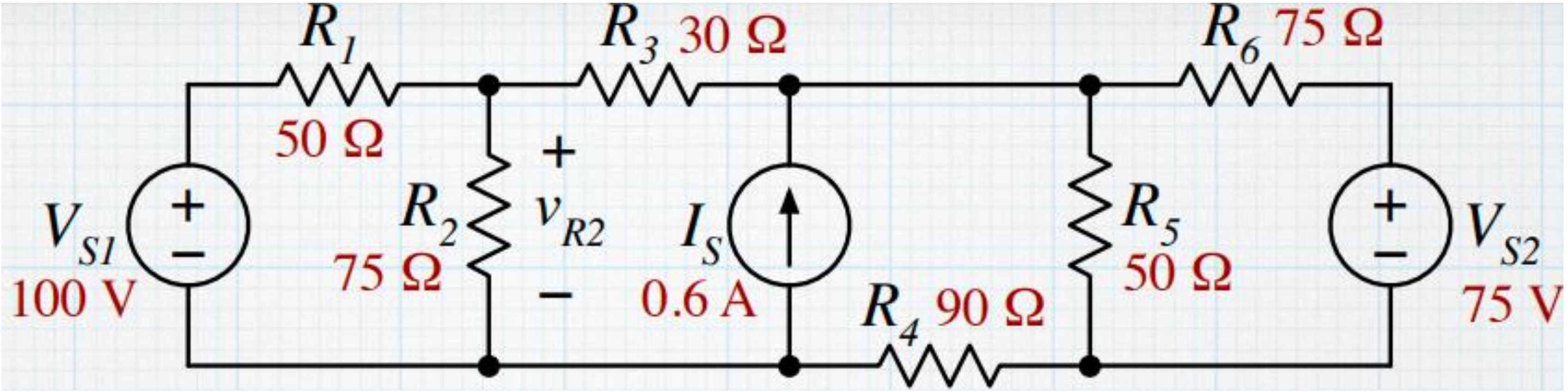
Use **Superposition** theorem to calculate $\mathbf{V_{3\Omega}}$



Ans: 18 V

Self-Practice 2

Applying **Superposition** principle, find $\mathbf{V_{R2}}$



Answer:

Due to source V_{S1}: 50 V

Due to source V_{S2}: 5 V

Due to source IS: 18 V

Total: 73 V

Self-Practice 3

Solve Illustration 2 of node voltage analysis (Illustration 1 of mesh current analysis) using Superposition principle.

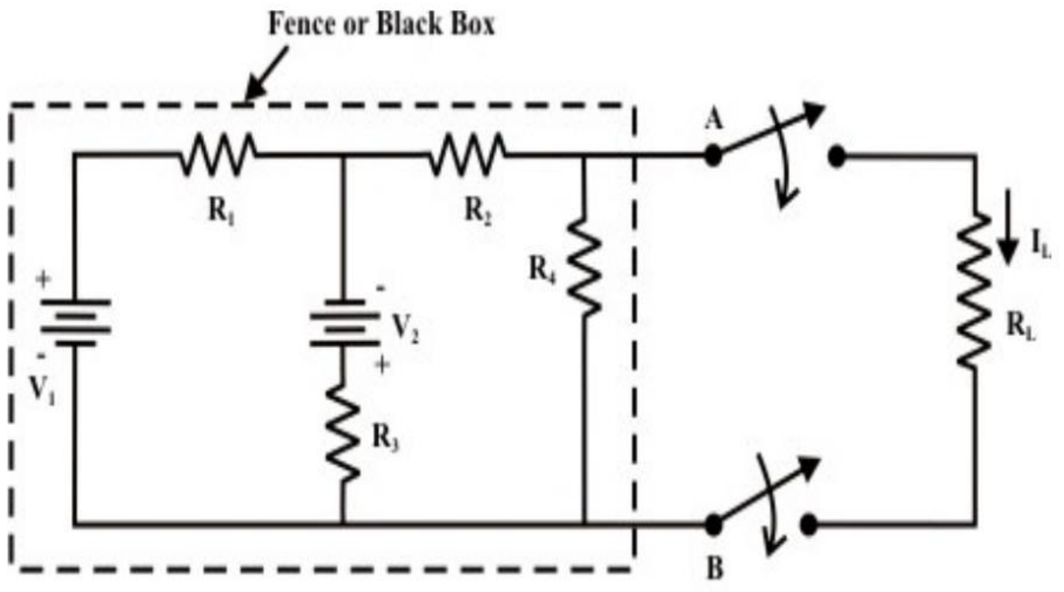
Network Theorems

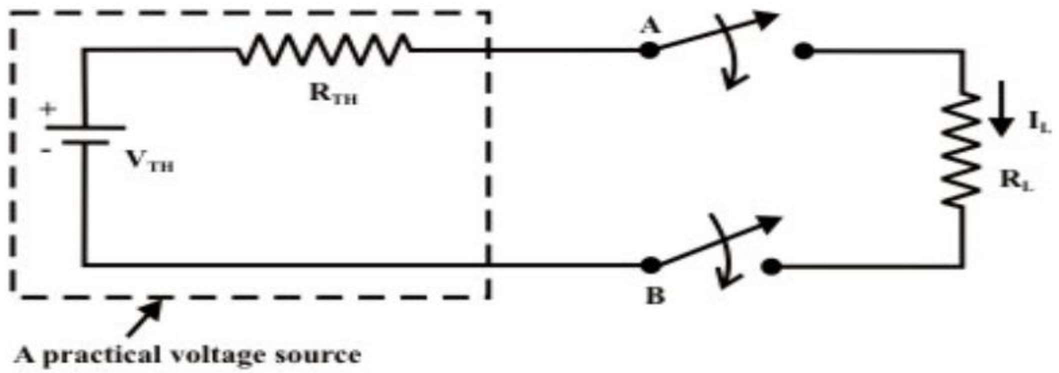
THEVENIN’S THEOREM

LECTURE: 08

Why Thevenin's Theorem

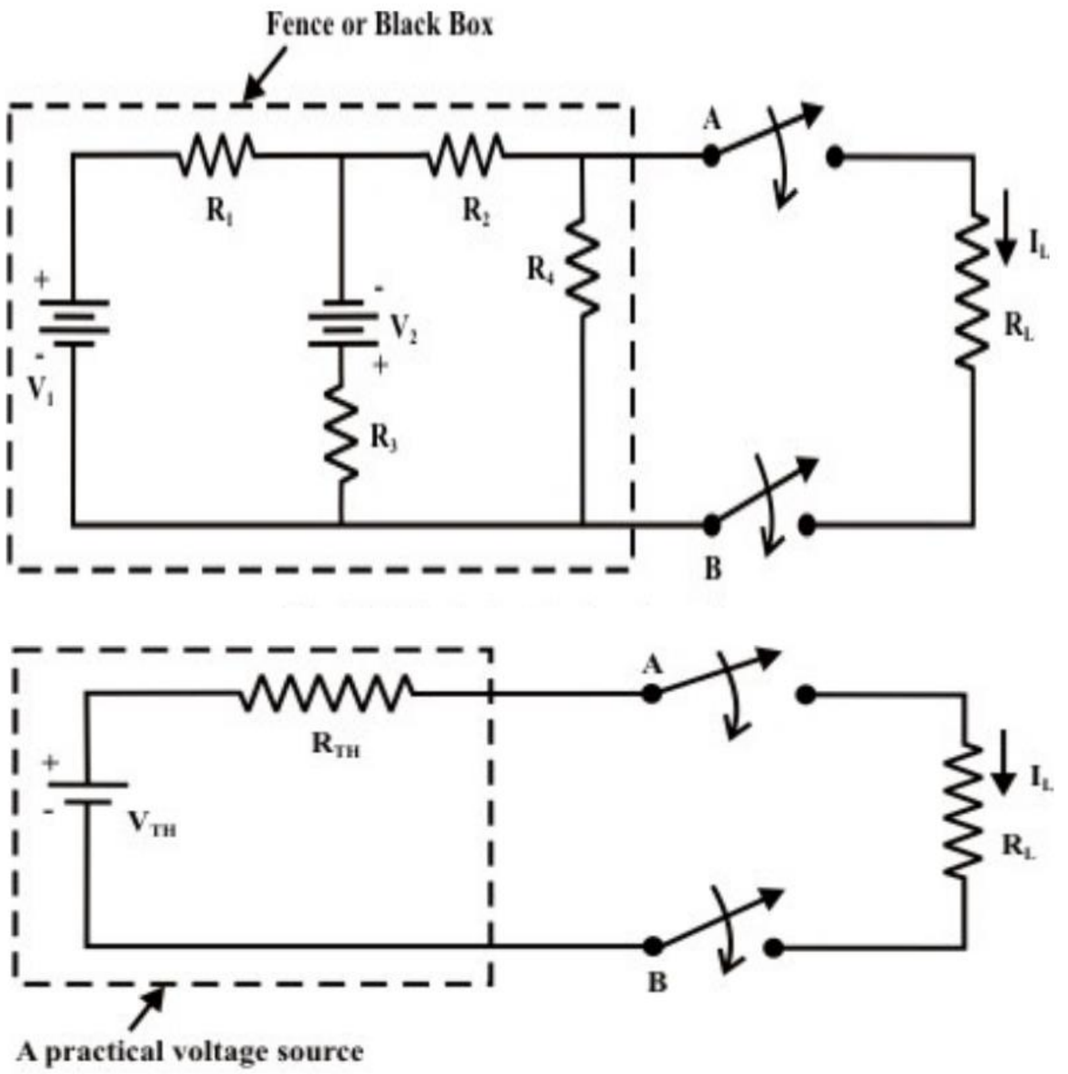
- In many applications, a network may contain **a variable component or element** while other elements in the circuit are kept constant.
- If the solution for current or voltage or power in any component of the network is desired, in such cases the whole circuit need to be analyzed each time with the change in component value.
- In order to avoid such repeated computation, it is desirable to introduce a method that will not have to be repeated for each value of the variable component.
- For the circuit shown,
    - Mesh current method needs 3 equations to be solved
    - Node voltage method requires 2 equations to be solved





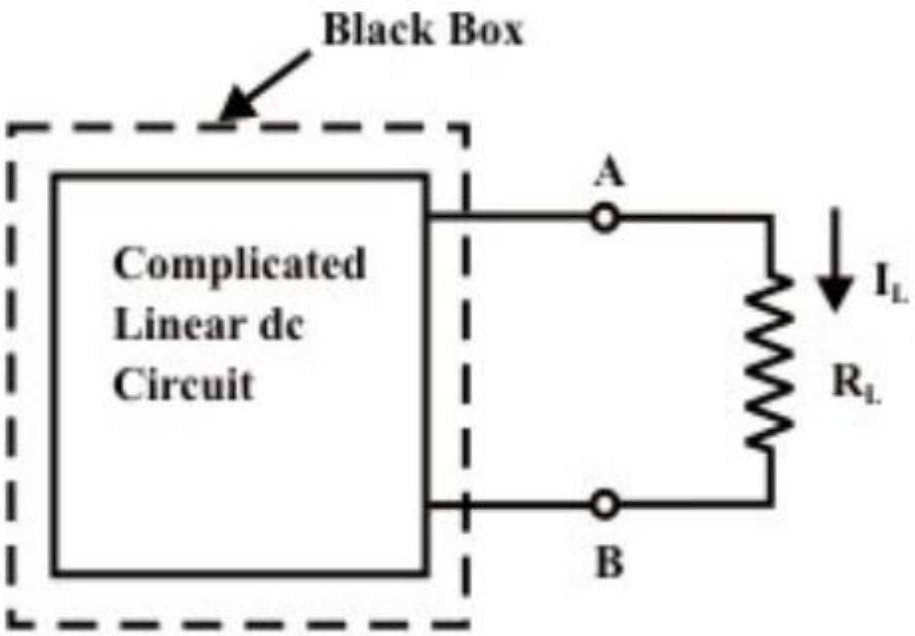
Definition of Thevenin's Theorem

- Any circuit consisting of **linear, bilateral resistances and sources** (current sources and voltage sources) can be replaced by a single voltage source (called **Thevenin's equivalent voltage, V_{Th}**) in series with one resistance (called **Thevenin's equivalent resistance, R_{Th}**) across the terminals of interest.
- Thevenin's equivalent voltage, **V_{Th}**, is the **open-circuit voltage** across the terminals of interest.
- Thevenin's equivalent resistance, **R_{Th}**, is the net resistance across the terminals of interest with all sources being replaced by their internal resistances **(ideal voltage sources should be short-circuited and ideal current sources should be open-circuited)**.



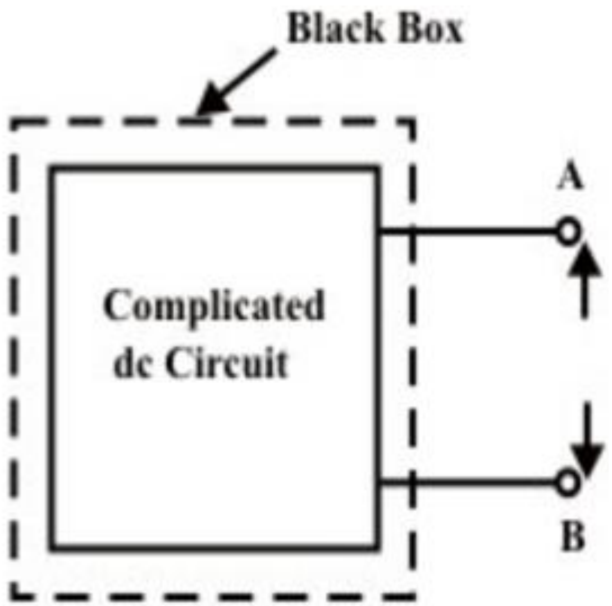
Procedure of Thevenin's Theorem

Suppose: Find I_L through R_L



Step 1: Disconnect load resistance, R_L

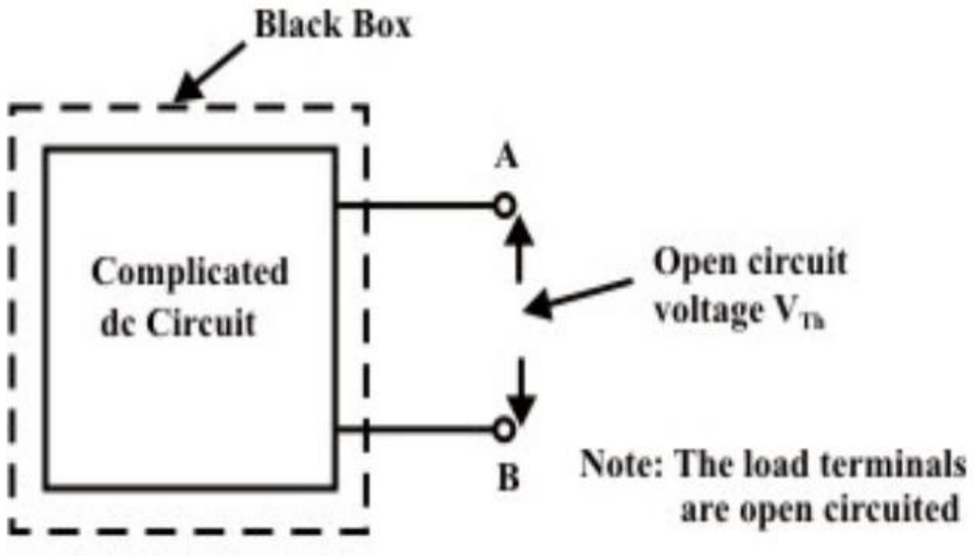
- Remove the load
- Keep the terminals open-circuited as shown below.



Procedure of Thevenin's Theorem

Step 2: Find Thevenin's voltage, V_{Th}

- Apply mesh current or node voltage or any network reduction technique
- Find the voltage across the open-circuited terminals.

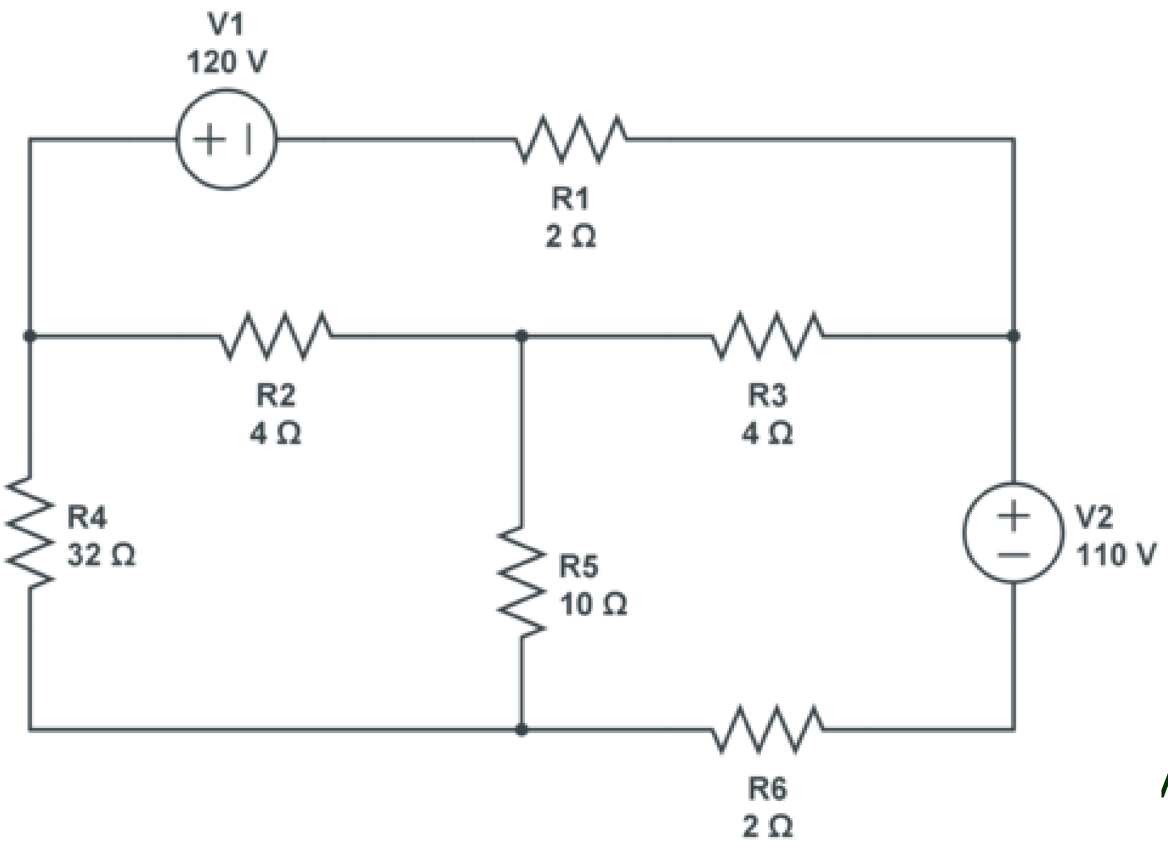


Step 3: Find Thevenin's resistance, R_{Th}

- Keep the load terminals open.
- Replace all the sources with their internal resistances.
- Ideal voltage sources should be short-circuited (just replace them with plain wire)
- Ideal current sources should be open-circuited (just remove them)
- Find the equivalent resistance across open-circuited load terminals.

Illustration 1

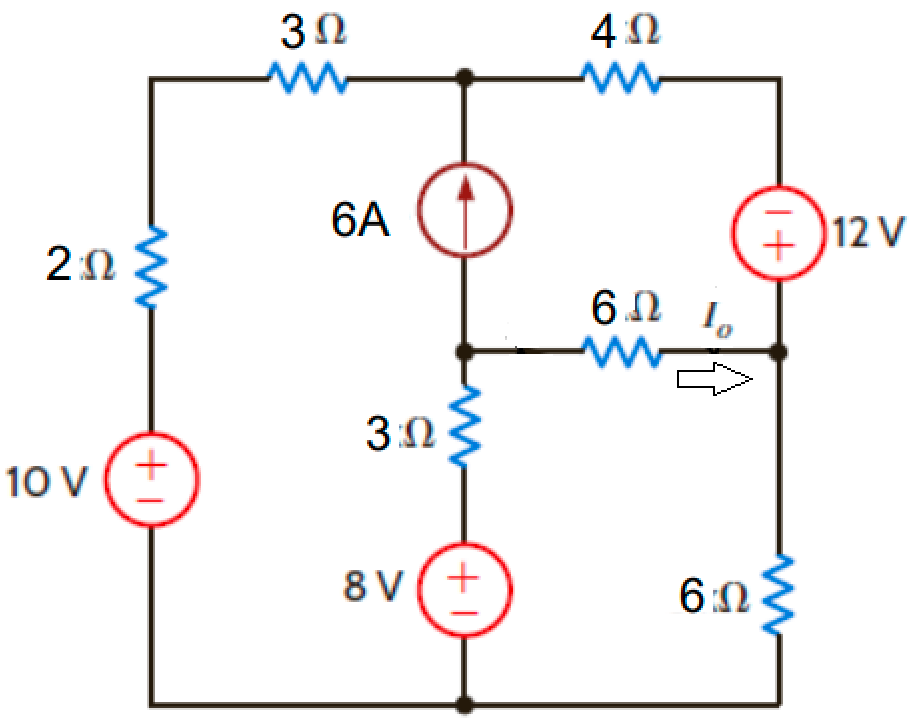
Using **Thevenin's** theorem find the power dissipated in **10 Ω** resistor.



Ans: V_{TH} = 141.797 V, R_{Th} = 4.179 Ω, $P_{10\Omega}$ = 1000 W

Illustration 2

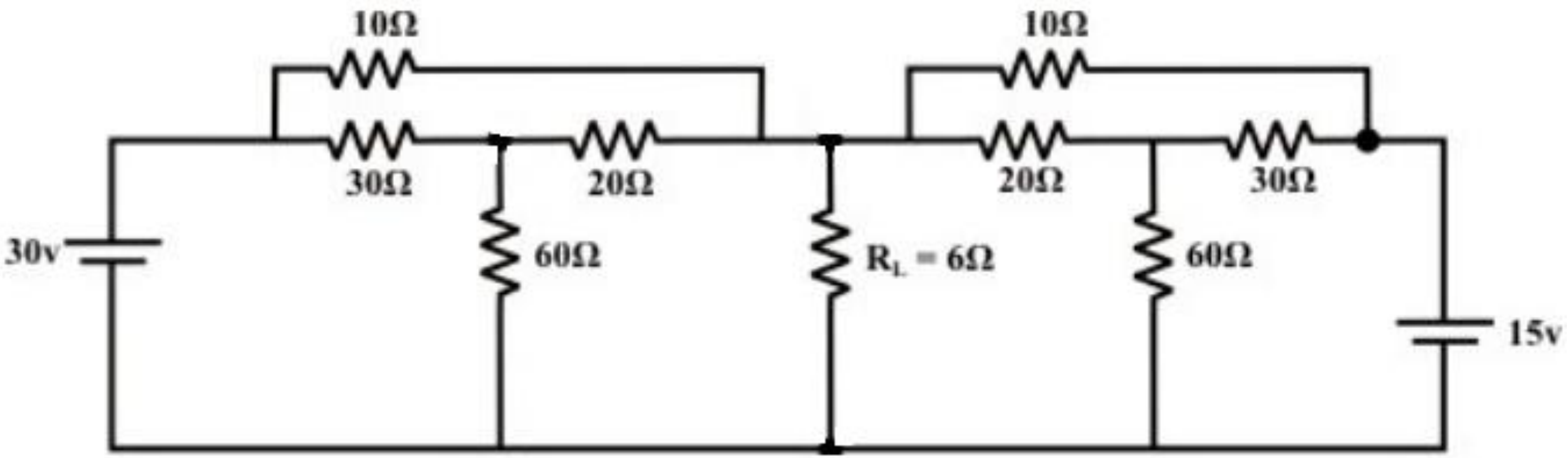
Determine current $\mathbf{I_0}$ using **Thevenin's** theorem.



Ans: V_{TH} = - 30.8022 V, R_{Th} = 6.6 Ω , I_0 = - 2.44 A

Illustration 3

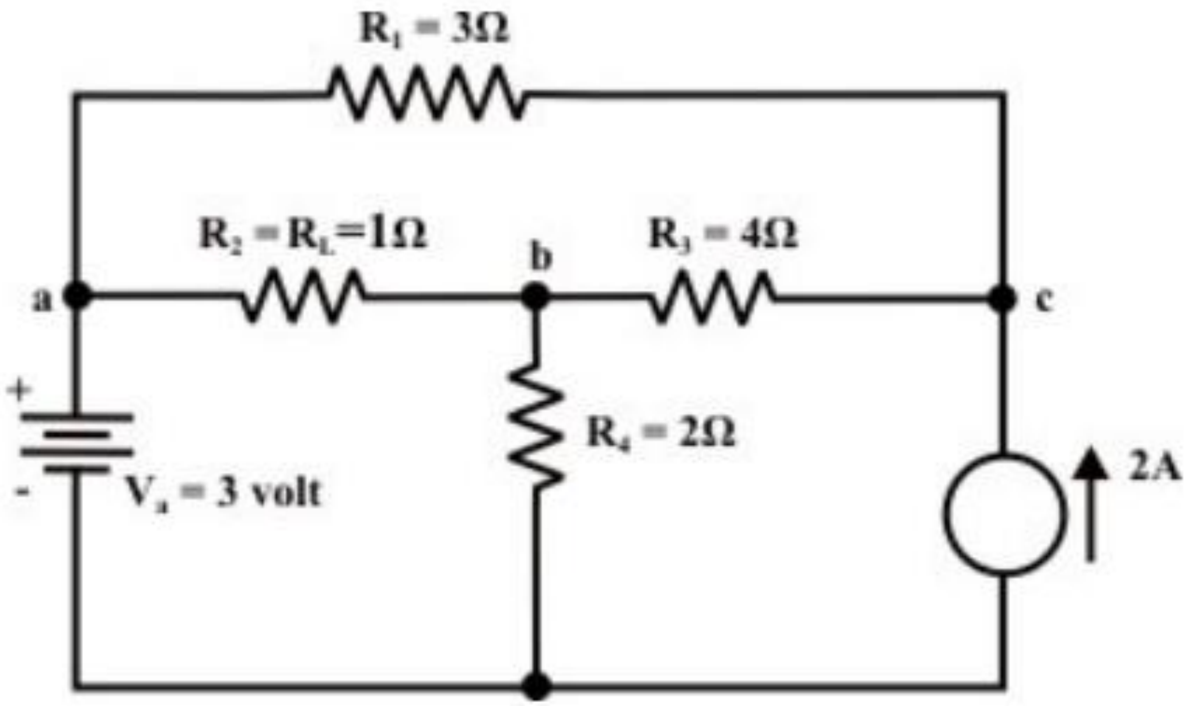
Determine the current $\mathbf{I_L}$ through the resistor $\mathbf{R_L = 6\ \Omega}$ using Thevenin's equivalent circuit



Ans: V_{TH} = 21.236 V, R_{Th} = 4.045 Ω , I_L = 2.114 A

Self-Practice 1

Determine the current through the resistor $\mathbf{R_L} = \mathbf{R_2} = \mathbf{1\ \Omega}$ using Thevenin's theorem.



Ans: $V_{TH} = 1$ V, $R_{Th} = 1.555\ \Omega$, $I_L = 0.39$ A

Network Theorems

MAXIMUM POWER TRANSFER THEOREM

LECTURE: 09

Maximum Power Transfer Theorem

- In **any linear, bilateral dc circuit**, maximum power will be transferred from a source to load when the load resistance is made equal to the internal resistance of the supply (source) as viewed from the load terminals with load removed and all emf sources replaced by their internal resistances.

OR

- The maximum power is obtained from a linear circuit at a given pair of terminals when terminals are loaded by the Thevenin's resistance (R_{Th}) of the circuit.

Proof

Consider the Thevenin's equivalent circuit of a network

$$I_L = \frac{V_{Th}}{R_{Th}+R_L}$$

$$P_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

- For a given source generated voltage V_{Th} and R_{Th} are constant
- Therefore, power delivered to load R_L depends upon R_L

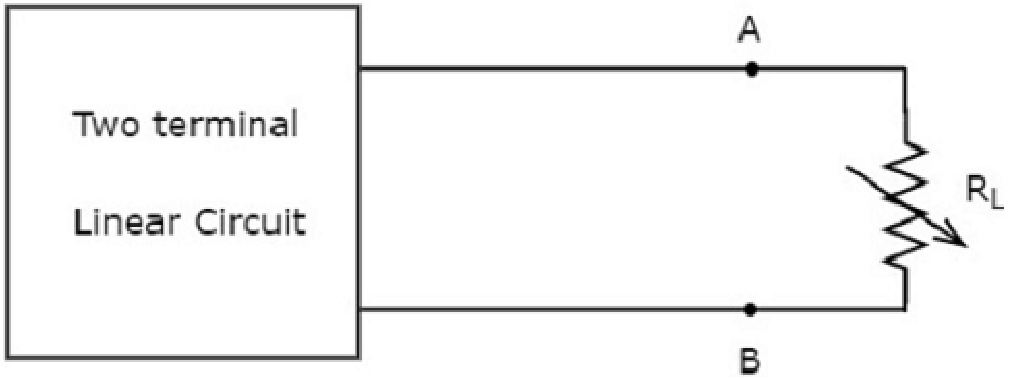
For P_L to be maximum, $\frac{dP_L}{dR_L} = 0$

which yields, $\mathbf{R_L = R_{Th}}$

Load resistance = Internal resistance of source

Maximum Power,

$$P_{L-max} = \left(\frac{V_{Th}}{R_{Th}+R_{Th}}\right)^2 R_{Th} = \frac{V_{Th}^2}{4R_{Th}}$$



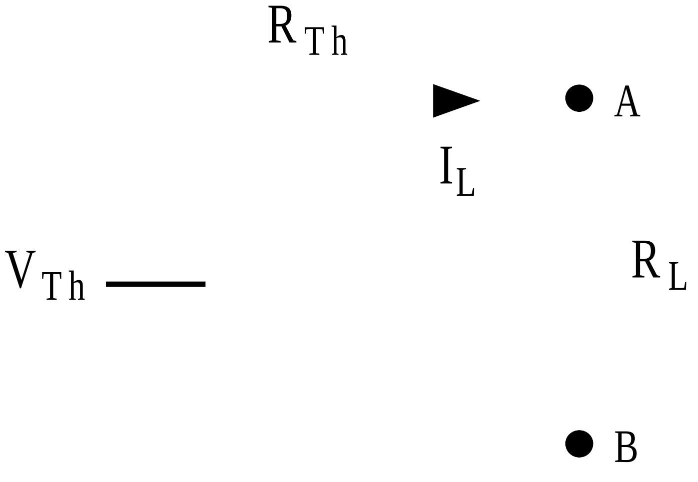
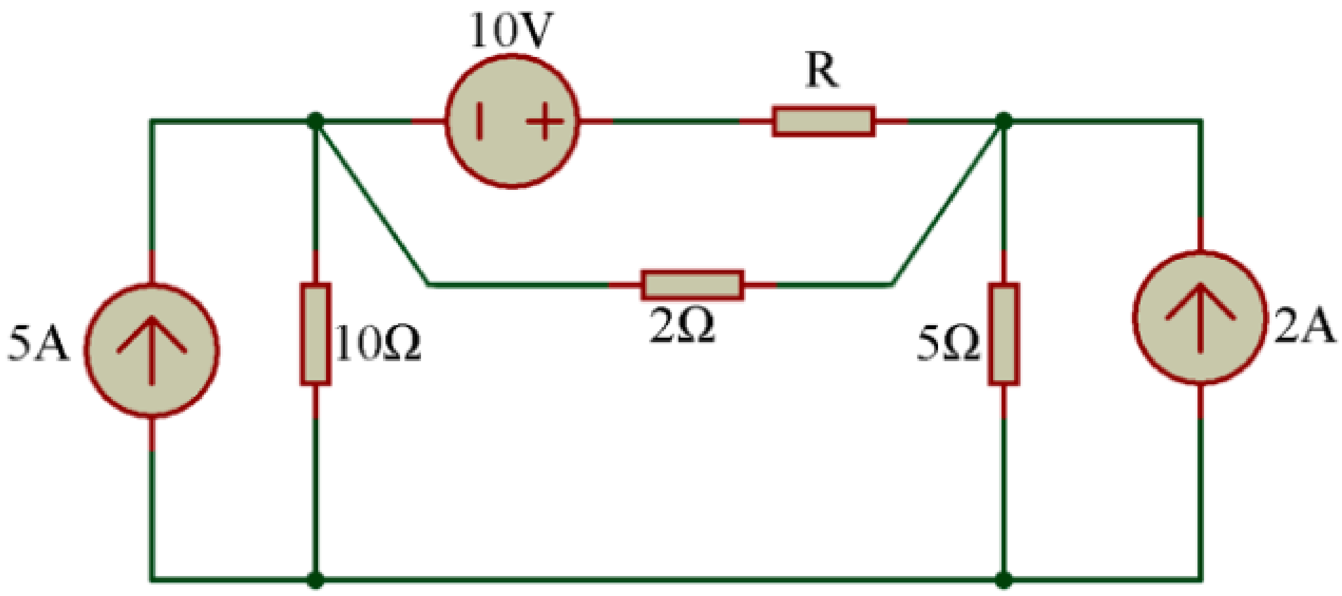


Illustration 1

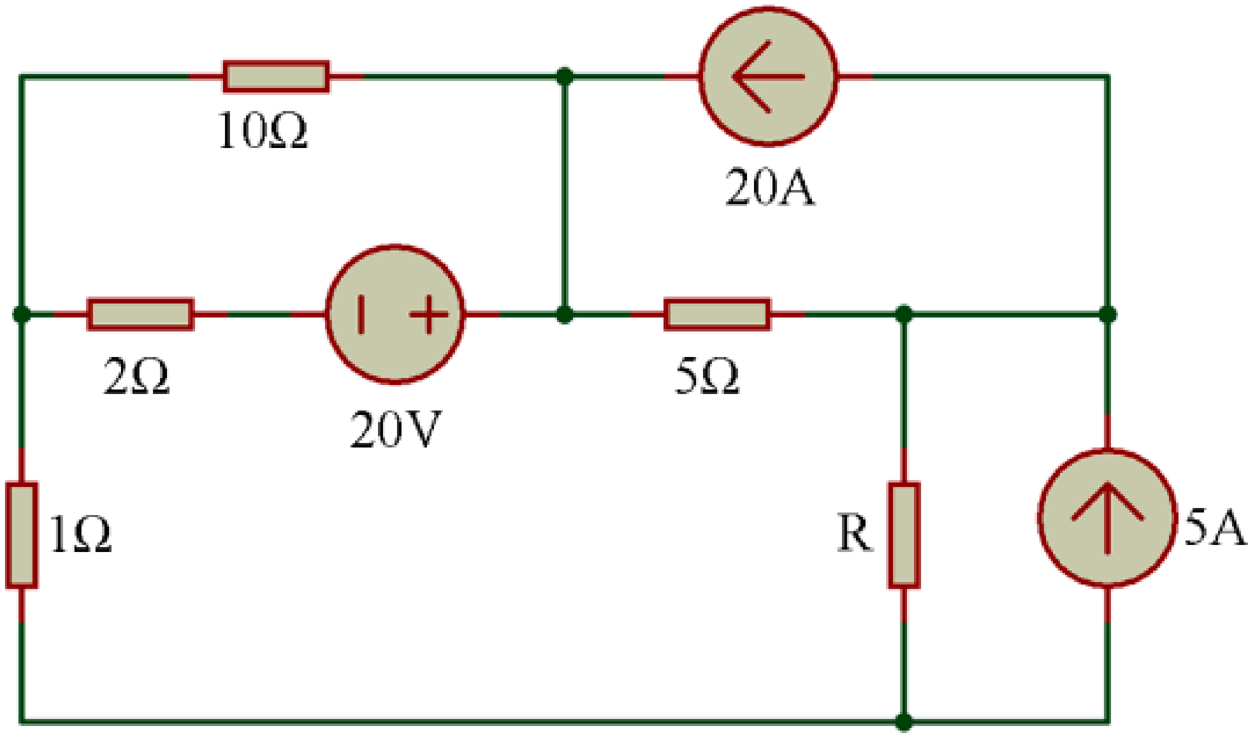
Obtain maximum amount of power transfer in **R** from the source.



Ans: V_{Th} = 14.706 V, R_{Th} = 1.765 Ω, P_{Max} = 30.637 W

Illustration 2

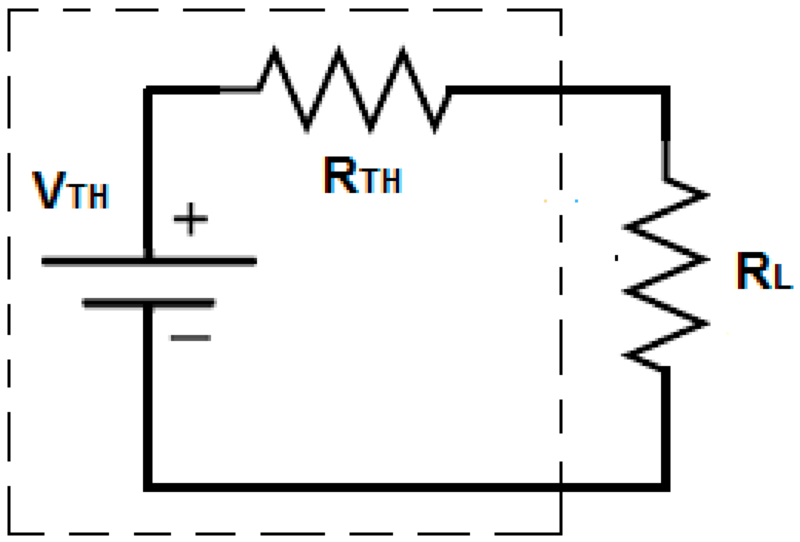
Determine the maximum power transfer to **R** in the circuit shown.



Ans: $V_{Th} = -45$ V, $R_{Th} = 7.667\ \Omega$, $P_{Max} = 66.03$ W

Illustration 3

What percent of maximum power (possible maximum power) is delivered to $\mathbf{R_L}$ in the figure when $\mathbf{R_L = 2R_{Th}}$?



Ans: 88.89 % of P_{max}

Note

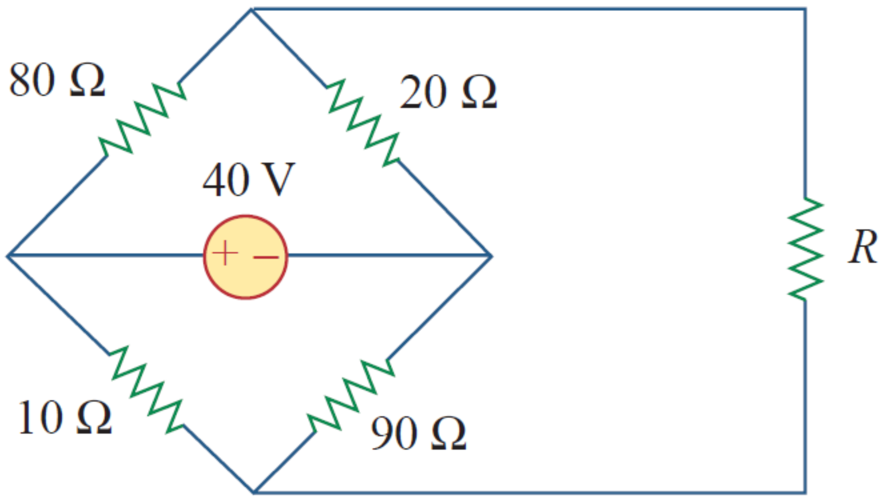
- This theorem states **how to choose** (so as to maximize power transfer) **the load resistance, once the source resistance is given**, not the opposite.
- It **does not** say how to choose the source resistance, once the load resistance is given.
- Given a certain load resistance, the source resistance that maximizes the power transfer is always zero, regardless of the value of the load resistance.
- **Applications:** Electronics and communication networks – the goal is either to receive or transmit maximum power (though at reduced efficiency). Here, the power involved is only a few milliwatts or microwatts.

Self-Practice 1

The variable resistor **R** in the figure below is adjusted until it absorbs the maximum power from the circuit.

(a) Calculate the value of **R** for maximum power.

(b) Determine the **maximum power** absorbed by **R.**



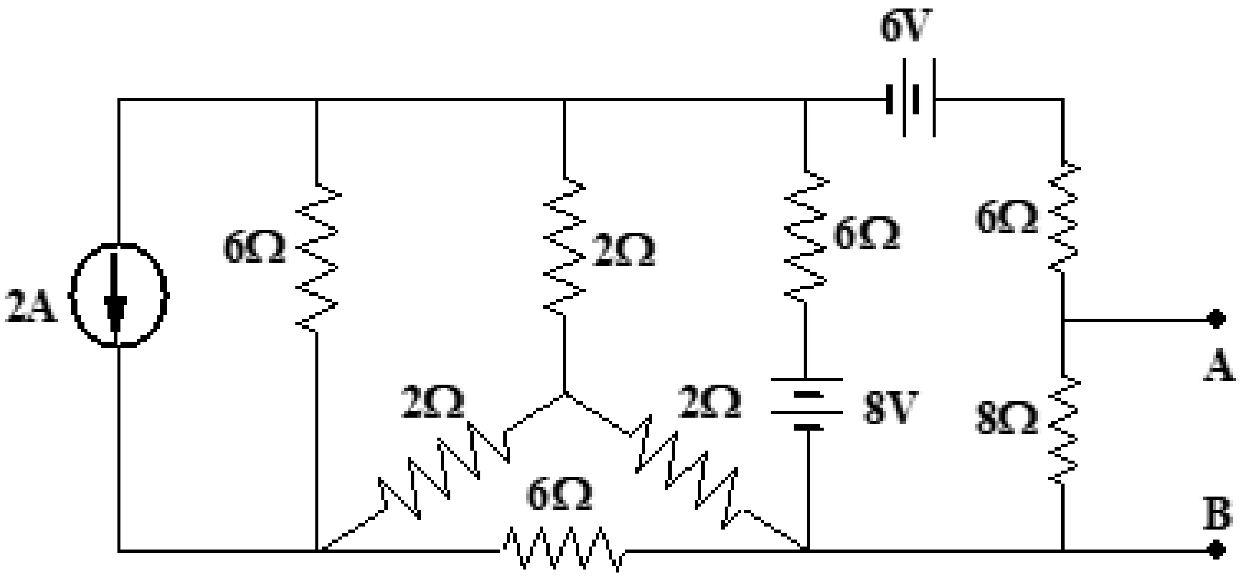
Ans:

R = 25 Ω

$P_{max} = \frac{V_{Th}^2}{4R_L} = \frac{28^2}{4\times25} = 7.84$ W

Self-Practice 2

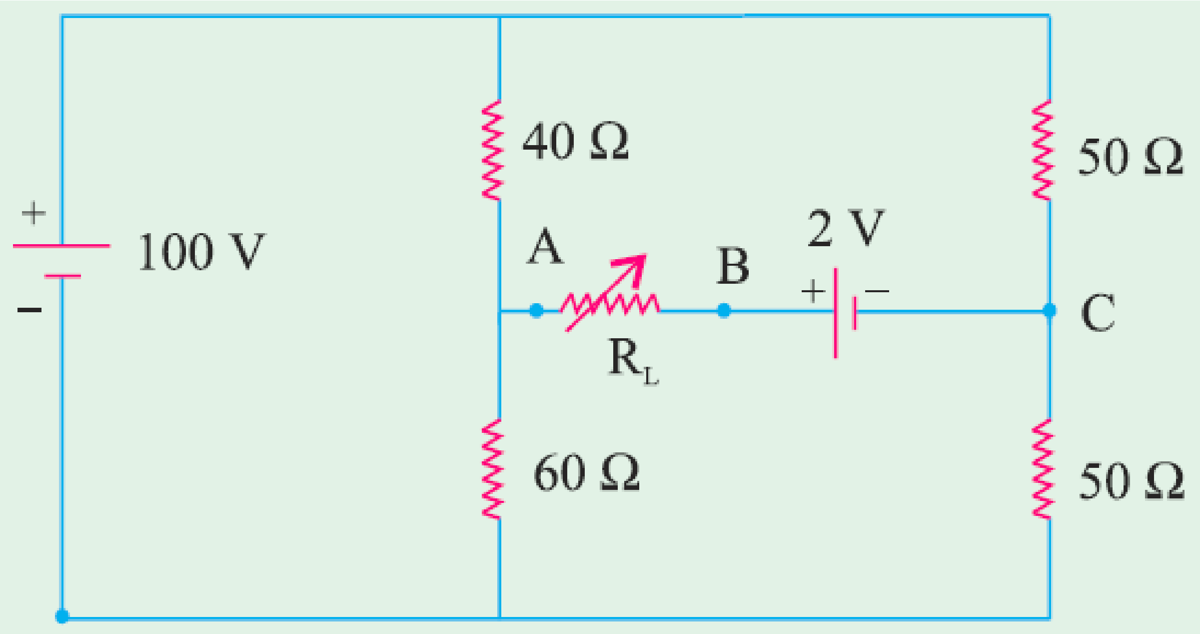
Determine the value of load resistance to be connected across terminals **A & B** such that maximum power is transferred to the load. Also find the maximum power.



Ans: $R_L = 4\ \Omega$, $V_{Th} = 3.333$ V, $P_{Max} = 0.6943$ W

Self-Practice 3

Find the maximum power in $\mathbf{R_L}$ which is variable in the circuit.



Ans: V_{Th} = 8 V, R_L= R_{Th}= 49 Ω, P_{Max} = 0.3265 W

Basic Electrical Technology

DC TRANSIENT ANALYSIS

DC Transient Analysis

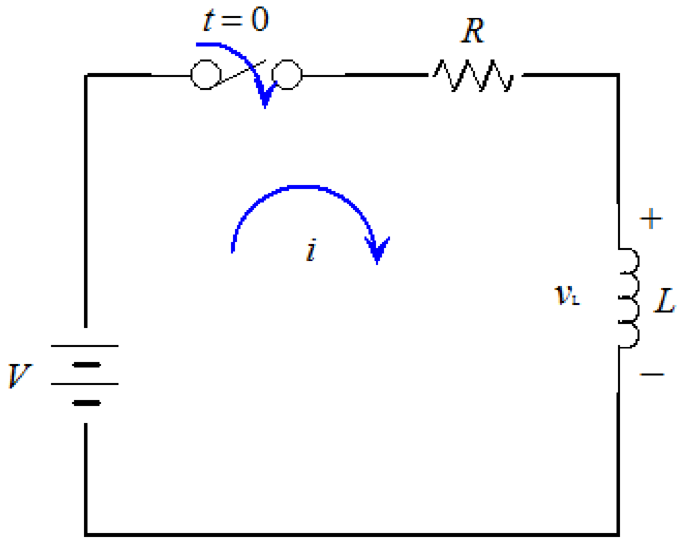
TRANSIENT BEHAVIOUR OF R–L CIRCUIT

LECTURE: 10

Closing and Breaking an Inductive Circuit

- Consider an inductive circuit shown.
- When the switch is closed, the current increases gradually and takes some time to reach the final value.
- Suppose at any instant, the current is **i** and is increasing at the rate of **di/dt**
- Then,

$$\mathbf{V = v_R + v_L = iR + L\frac{di}{dt}}$$



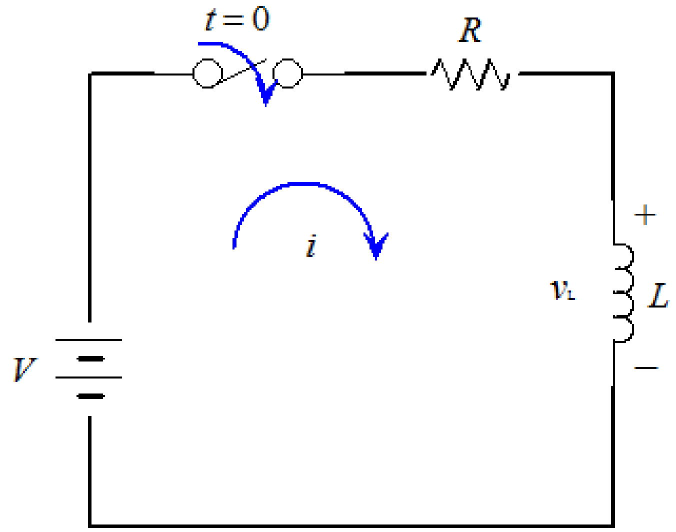
- After some time **di/dt becomes zero** and so does the self-induced emf $\mathbf{v_L = L\frac{di}{dt}}$
- At this stage, the **current attains the final fixed value (maximum) I** given by:

$$\mathbf{V = IR + 0 \quad or \quad I = \frac{V}{R}}$$

- Thus, when a dc circuit containing inductance is switched on, the current takes some time to reach the final value, $\mathbf{I = \frac{V}{R}}$
- Similarly, when an inductive circuit is opened, the current does not jump to zero but falls gradually.
- In either case, the **delay in change depends upon the values of L and R**.

Growth (Rise) of Current in an Inductive Circuit

- Consider an inductive circuit shown.
- When the switch is closed, the current rises from zero to final value, $\mathbf{I} = \frac{\mathbf{V}}{\mathbf{R}}$ in small time **t**.
- Suppose at any instant, the current is **i** and is increasing at the rate of **di/dt**
- Then,



$$V = v_R + v_L = iR + L\frac{di}{dt}$$

$$V - iR = L\frac{di}{dt} \quad \text{or} \quad \frac{di}{V - iR} = \frac{dt}{L}$$

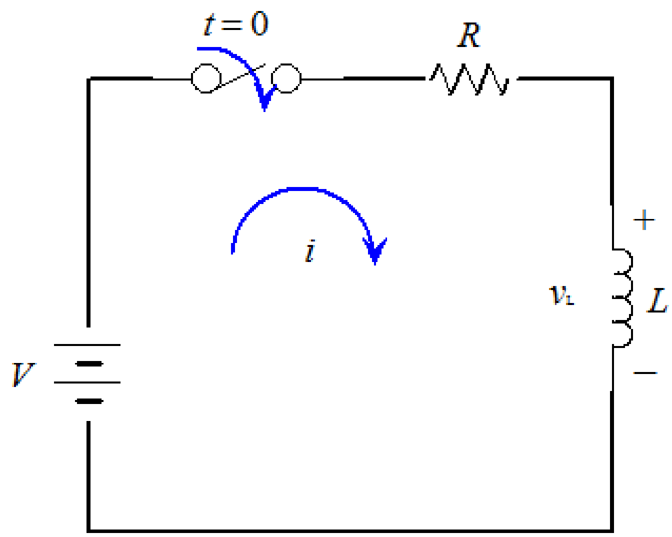
$$\frac{-R}{V - iR}di = -\frac{R}{L}dt$$

Integrating both sides, we get,

$$\log_e(V - iR) = -\frac{R}{L}t + K$$

To find K, **we have i = 0 at t = 0**, which gives $\mathbf{K = \log_e V}$

Growth (Rise) of Current in an Inductive Circuit



Therefore, $$\log_e(V - iR) = -\frac{R}{L}t + \log_e V$$

$$\log_e \frac{V - iR}{V} = -\frac{R}{L}t \quad \text{or} \quad \frac{V - iR}{V} = e^{-Rt/L}$$

$$\mathbf{i = \frac{V}{R}\left(1 - e^{-Rt/L}\right) \quad or \quad i = I\left(1 - e^{-Rt/L}\right)}$$

Voltage across the inductance: $\mathbf{v_L = V\,e^{-Rt/L}}$

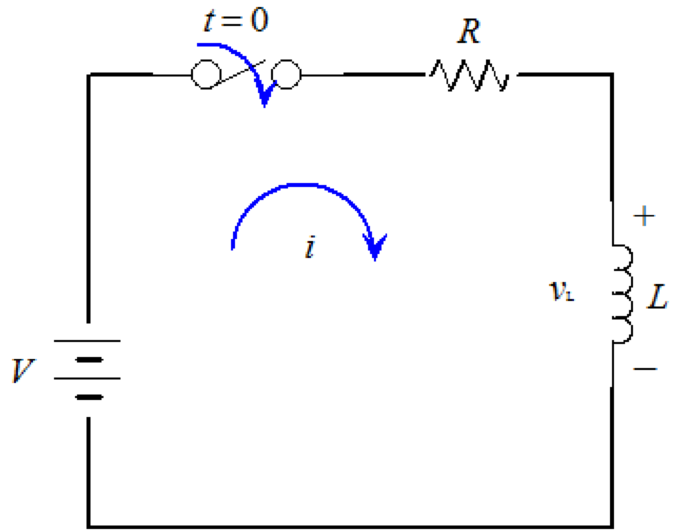
Thus, **the rise of current follows an exponential law.**

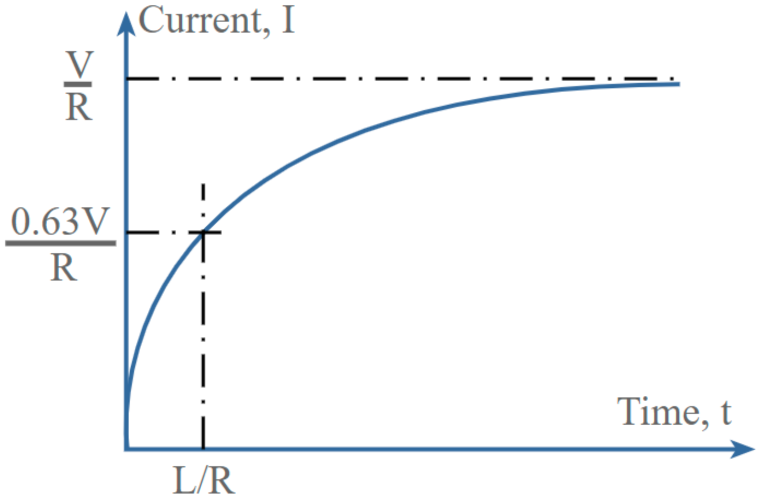
- Theoretically, the current will reach its final value $I = \frac{V}{R}$ in an infinite time. However, practically it reaches this value in a short time.
- **Initial rate of rise of current is:**

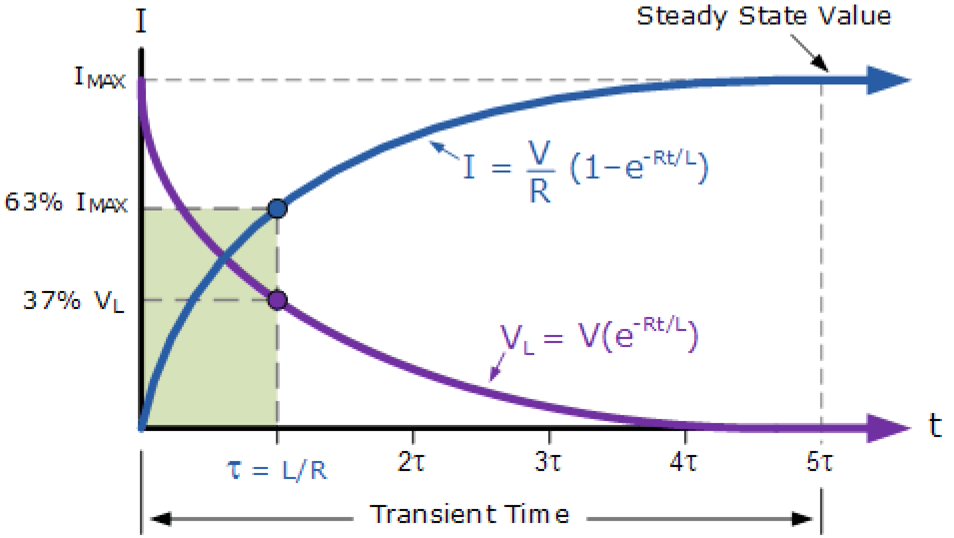
(the instant the switch is closed, i = 0 and hence $V = L\frac{di}{dt}$)

$$\frac{\mathbf{di}}{\mathbf{dt}} = \frac{\mathbf{V}}{\mathbf{L}} \quad A/s$$

Growth (Rise) of Current in an Inductive Circuit







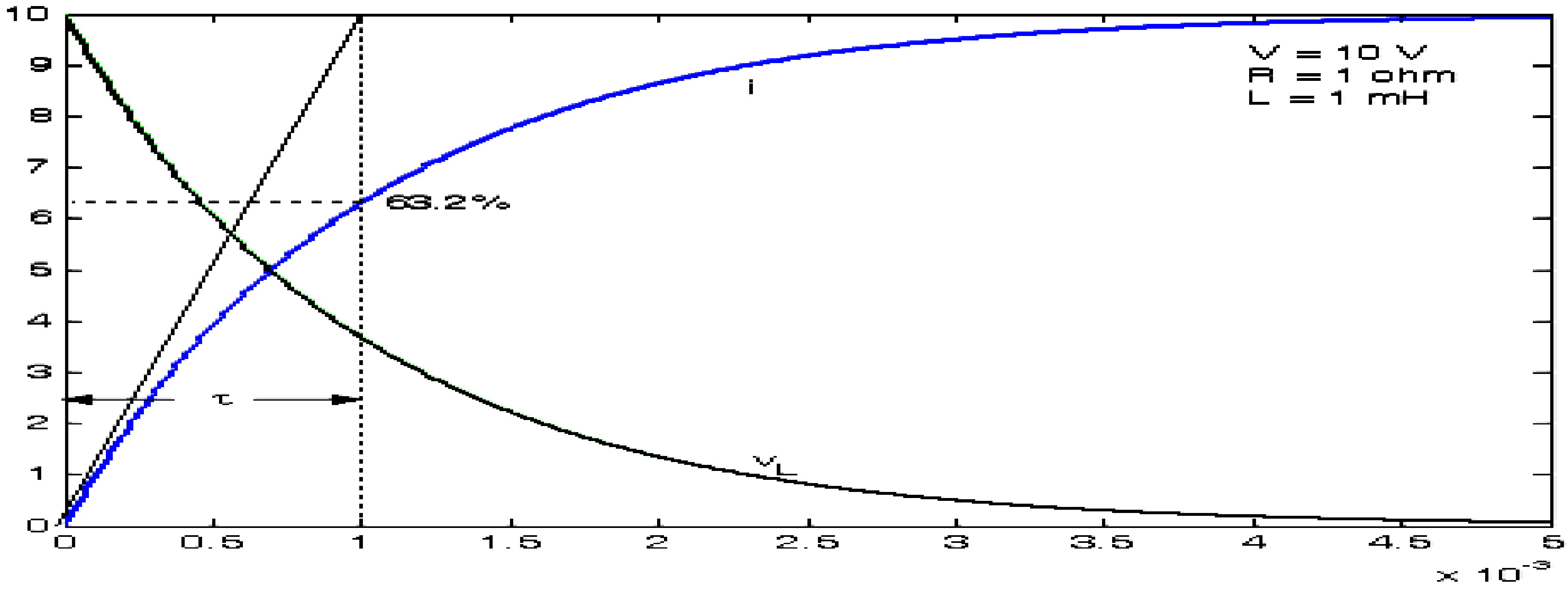
Time Constant (τ) = $\frac{\mathbf{L}}{\mathbf{R}}$ seconds

- The quantity $\frac{\mathbf{L}}{\mathbf{R}}$ is called the time constant of the circuit and affects the rise of current in the circuit.
- **It has the dimensions of time** so that the exponent of e (i.e., $^{-Rt}/_{L}$) is a number.
- If time interval, $t = \tau \left(\text{or } \frac{L}{R}\right)$, then, $\mathbf{i = I\left(1 - e^{-Rt/L}\right) = 0.632\ I}$
- Hence, time constant can be defined as the time required for the current to reach **0.632 (or 63.2%) of its final steady state value.**

Growth (Rise) of Current in an Inductive Circuit

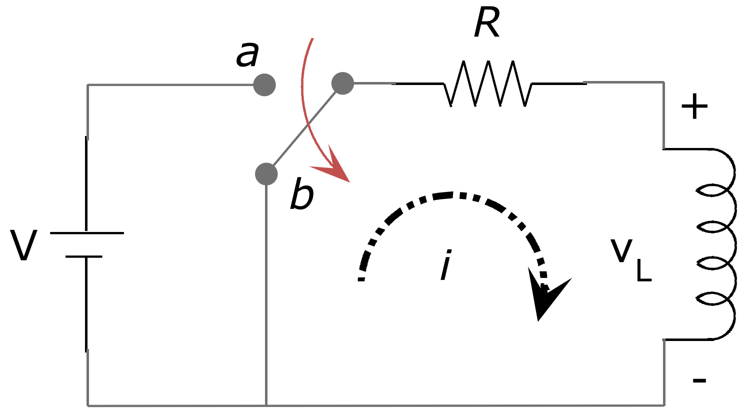
- Time constant can also be defined as the time taken by the current through the inductor to reach its final steady-state value, had the initial rate of rise been maintained constant.

$$\tau = \frac{L}{R}$$



Decay (Fall) of Current in an Inductive Circuit

- Consider an inductive circuit shown. When the switch is thrown to **Position a**, the current in the circuit starts rising and attains the final value $\mathbf{I = \frac{V}{R}}$ in small **time t** as discussed previously.
- If now switch is thrown to **Position b**, the current in R-L circuit does not cease immediately but gradually reduces to zero.
- Suppose at any instant, the current is **i** and is decreasing at the rate of **di/dt**, then,



$$\mathbf{0 = v_R + v_L = iR + L\frac{di}{dt}}$$

or $$\frac{di}{i} = -\frac{R}{L}t$$

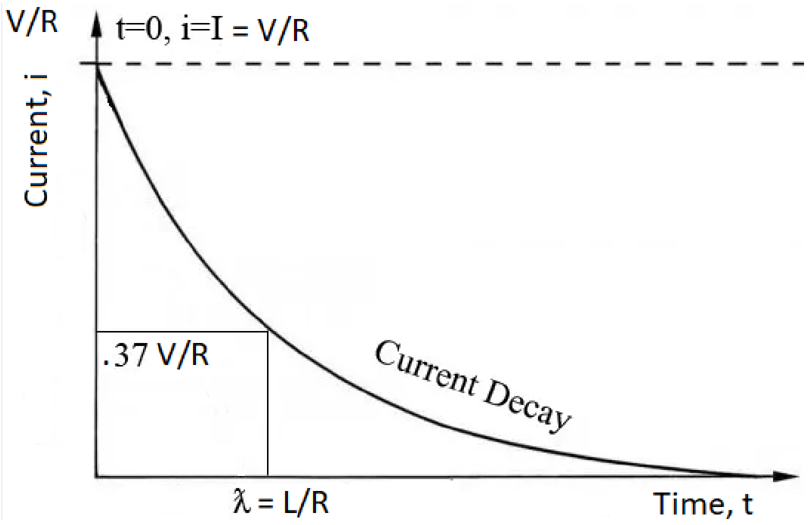
Integrating both sides, we get,

$$\log_e i = -\frac{R}{L}t + K$$

To find **K, we have** $\mathbf{i = I\ (= \frac{V}{R})}$ **at t = 0, which gives** $\mathbf{K = \log_e I}$

Therefore, $$\log_e i = \frac{-R}{L}t + \log_e I$$

$$\mathbf{i = I\ e^{-Rt/L}}$$ Also, $$\mathbf{v_L = -V\,e^{-Rt/L}}$$



Decay of Current in an Inductive Circuit

Time Constant (τ) = $\frac{\mathbf{L}}{\mathbf{R}}$ seconds

Time taken by the current to fall to **0.37 (or 37%)** of its final steady value $\left(\mathbf{I} = \frac{\mathbf{V}}{\mathbf{R}}\right)$ while decaying.

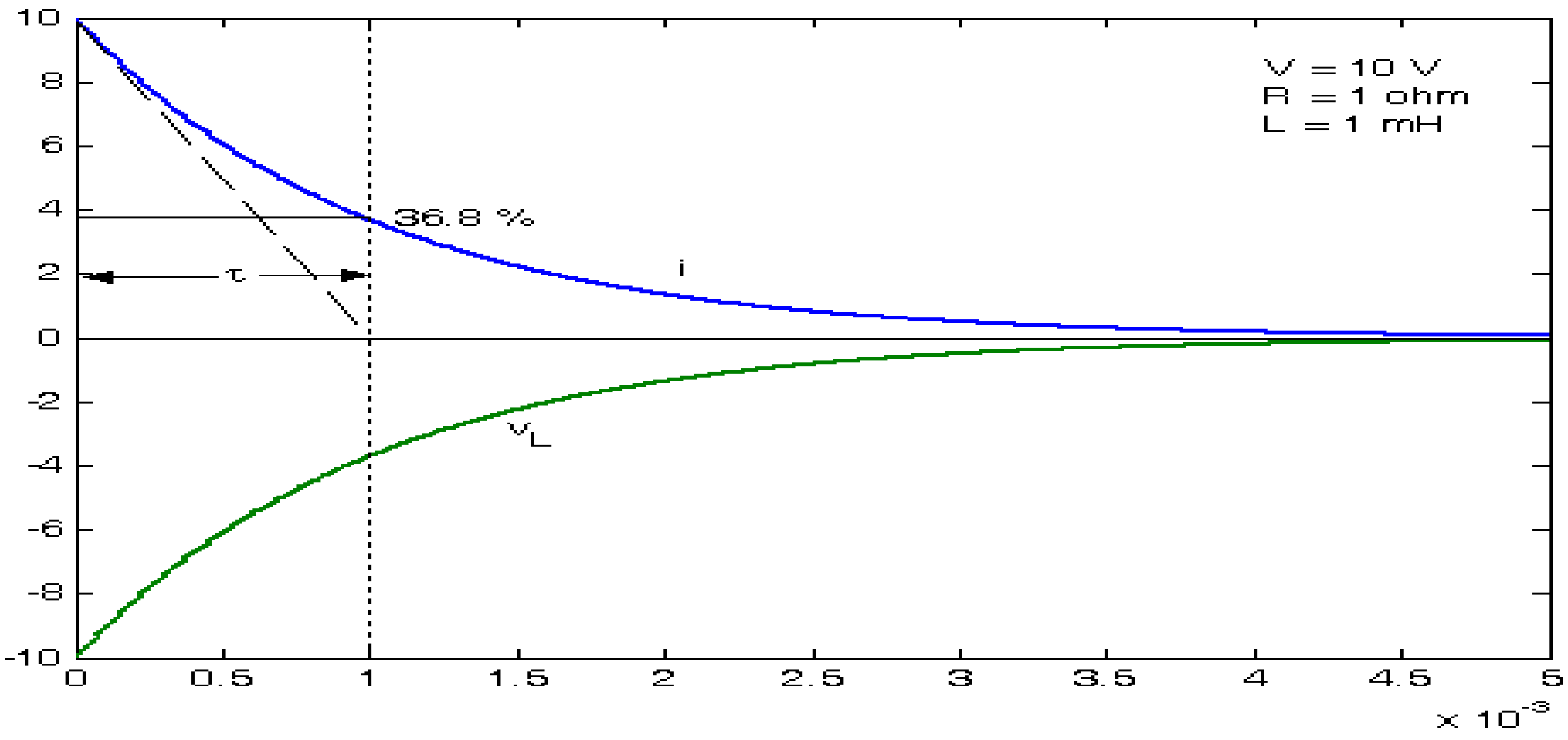


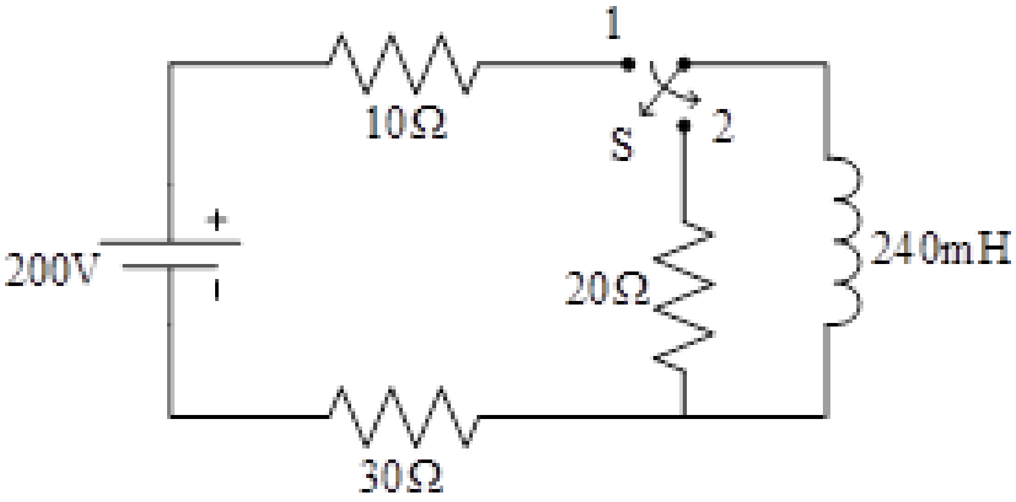
Illustration 1

In the given network, initially the switch **S** was at **position 1** and then shifted to **position 2** at **t = 15 ms**.

a) Determine the inductor current **i (t)** and the voltage across inductor $\mathbf{v_L}$ **(t)** at **t = 12 ms** and **t = 20 ms**

b) Determine the time taken by inductor current to reach the value **2.5 A**

c) Sketch the inductor **current i (t)** for **0 ≤ t ≤ 25 ms**



Ans: (a) 12 ms: i (t) = 4.3233 A, V_L(t) = 27.067 V

15 ms: i (t) = 4.5896 A, V_L(t) = 16.417 V

20 ms: i (t) = 3.02563 A, V_L(t) = - 10.82275 V

25 ms: i (t) = 1.9946 A

(b) t = 4.1588 ms (Rise)

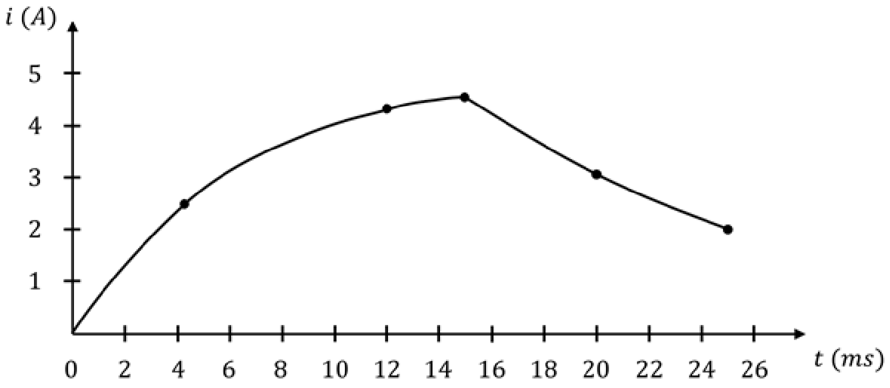


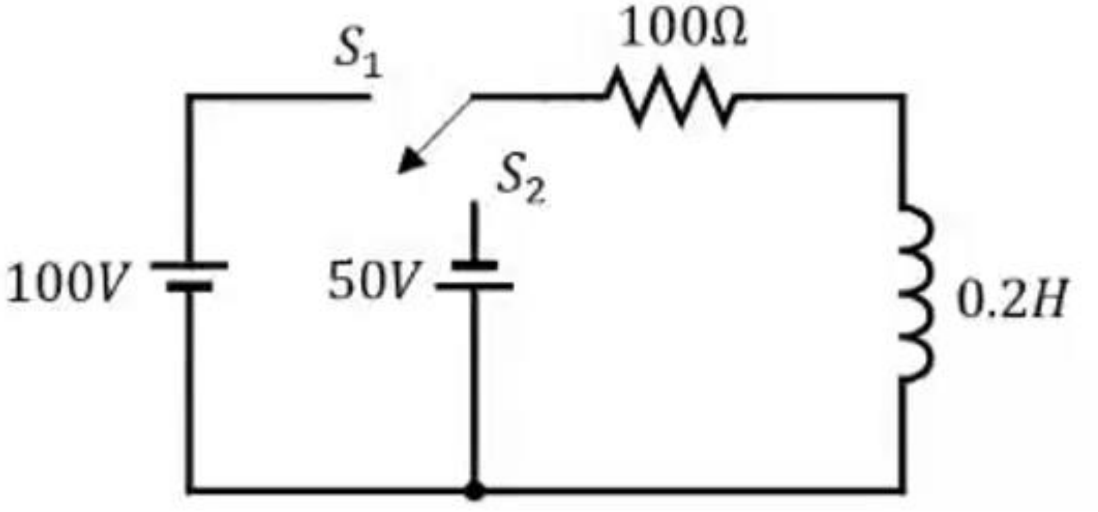
Illustration 2

An R-L series circuit is designed for a steady current of **250 mA**. A current of **120 mA** flows in the circuit at an instant **0.1 s** after connecting the supply voltage. Calculate i) the **time constant** of the circuit ii) the time from closing the circuit at which the circuit current has reached **200 mA**.

Ans: i) Time constant = 0.1529 s ii) t= 0.2461 s

Illustration 3

For the initially relaxed circuit shown, the switch is closed on to **position S_1** at time **$t = 0$** and changed to **position S_2** at time **$t = 0.5$ ms**. Obtain the equation for inductor current and sketch the transients for both intervals.



Solution

For $0 < t < 0.5ms$,

The growth in inductor current will be governed by,

$$i_L(t) = \frac{V}{R}\left(1 - e^{-Rt/L}\right) \quad \text{... eq. 1}$$

where,

V = 100 V; I = V/R = 1 A

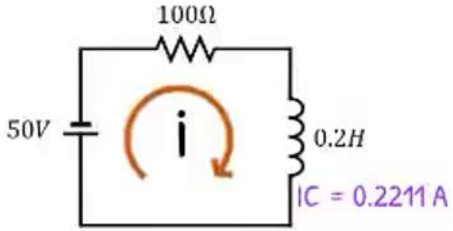
At t = 0.5ms,

$$i_L(t) = \frac{100}{100}\left(1 - e^{-100\times0.5\times10^{-3}/0.2}\right)$$

$$i_L(t) = 0.2211\ A$$

For t > 0.5ms,

The circuit can be re-drawn as,



Assuming clockwise flow of current and applying KVL,

$$-V - Ri - L\frac{di}{dt} = 0 \quad \text{... eq. 2}$$

To derive current expression, assume, at t=0, the circuit is as shown above.

Re-arranging eq. 2,

$$\Rightarrow \frac{di}{I + i} = -\frac{R\,dt}{L}$$

integrating,

$$\ln(I + i) = -\frac{Rt}{L} + K \quad \text{... eq. 3}$$

at t = 0, i = 0.2211

Substituting this in eq. 3,

$\Rightarrow$ K = ln (I + 0.2211)

Substituting this 'K' in eq. 3, we get expression for current 'i' as,

$$\boldsymbol{i(t) = I\left(e^{-Rt/L} - 1\right) + 0.2211e^{-Rt/L}} \quad \text{... eq. 4}$$

where,

V = 50 V; I = V/R = 0.5 A

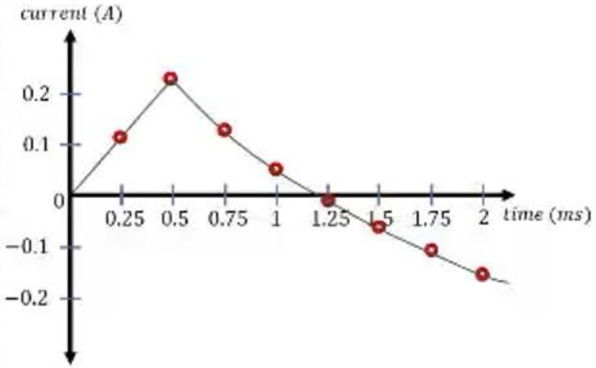
Finding zero crossing

During t > 0.5ms, there will be an instant of time, when inductor will not have energy to maintain the original current direction (top to bottom).

The current becomes zero at this instant & then current will grow in opposite direction.

This instant is found by setting LHS = 0 in eq. 4

Time	Current (i)	
0.25 ms	0.1175	governed by eq. 1
0.5 ms	0.2211	
0.75 ms	0.1363	
1 ms	0.0615	
1.23 ms	0	governed by eq. 4
1.25 ms	− 4.395m	
1.5 ms	− 0.0626	
1.75 ms	− 0.1141	
2 ms	− 0.1593	



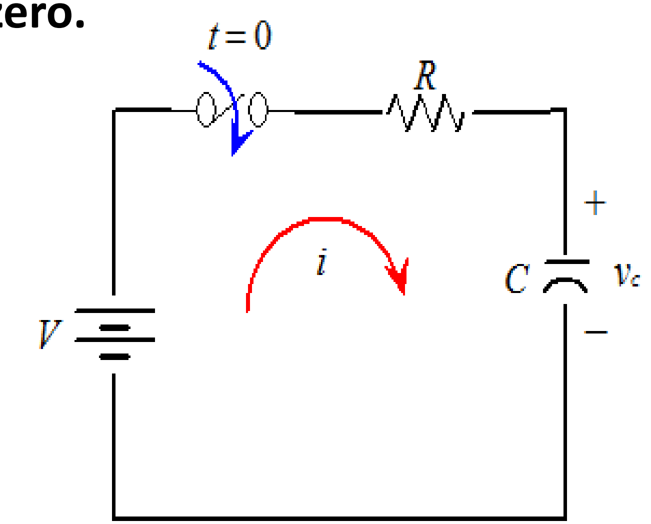
DC Transient Analysis

TRANSIENT BEHAVIOUR OF R–C CIRCUIT

LECTURE: 11

Charging of a Capacitor Through a Resistor

- When the switch is closed, the capacitor starts charging up and a charging current flows in the circuit.
- The charging current is maximum at the instant of switching and decreases gradually as the voltage across the capacitor increases.
- When the capacitor is charged to applied voltage **V**, the charging current becomes **zero.**



1. At switching instant

- The entire voltage **V** is dropped across R and the charging current is maximum
- Voltage across capacitor = 0
- Charge on capacitor = 0
- Initial charging current, $\mathbf{I_m = \frac{V}{R}}$

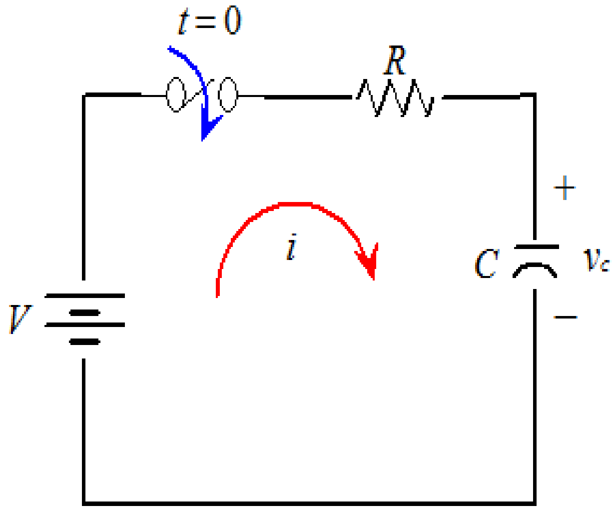
2. At any instant

- After having closed the switch, the charging current starts decreasing and the voltage across the capacitor gradually increases.
- Let at any instant during charging:

i = charging current **v = pd across C** **q = charge on capacitor = Cv**

Charging of a Capacitor Through a Resistor

(a) Voltage across capacitor



According to KVL, $V = v + iR$

$$V = v + CR\frac{dv}{dt} \qquad \left[i = \frac{dq}{dt} = \frac{d(Cv)}{dt} = \frac{Cdv}{dt}\right]$$

$$-\frac{dv}{V-v} = -\frac{dt}{RC}$$

Integrating both sides, $\mathbf{\log_e(V - v) = -\frac{t}{RC} + K}$

Initial condition: At the instant of closing the switch, **t = 0** and **v = 0**, which gives $\mathbf{K = \log_e V}$

Therefore, $\log_e(V - v) = -\frac{t}{RC} + \log_e V$ or $\log_e\frac{(V-v)}{V} = -\frac{t}{RC}$

$$\mathbf{v = V\left(1 - e^{-t/RC}\right)}$$

- This is the expression for variation of voltage across the capacitor w.r.t time.
- As **t increases,** the term $\mathbf{e^{-t/RC}}$ **gets smaller** and **voltage (v) across capacitor gets larger**

Charging of a Capacitor Through a Resistor

(b) Charge on capacitor

q = charge at any time t Q = final charge

Therefore, $v = \frac{q}{C}$ and $V = \frac{Q}{C}$

$$\frac{q}{C} = \frac{Q}{C}\left(1 - e^{-t/RC}\right) \quad \text{or} \quad \mathbf{q = Q\left(1 - e^{-t/RC}\right)}$$

(c) Charging Current

We have, $V - v = iR$ and also $V - v = Ve^{-t/RC}$

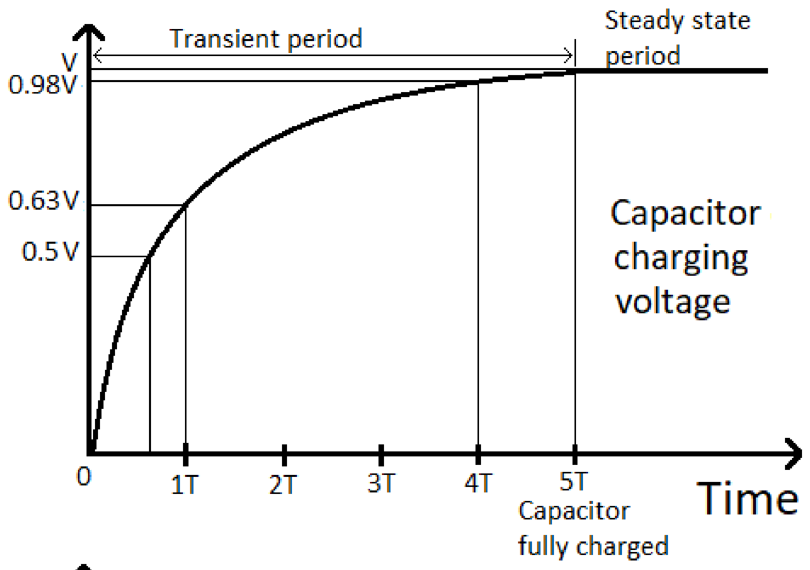
Therefore, $iR = Ve^{-t/RC}$ or $\mathbf{i = \frac{V}{R}e^{-t/RC}}$ or $\mathbf{i = I_m e^{-t/RC}}$

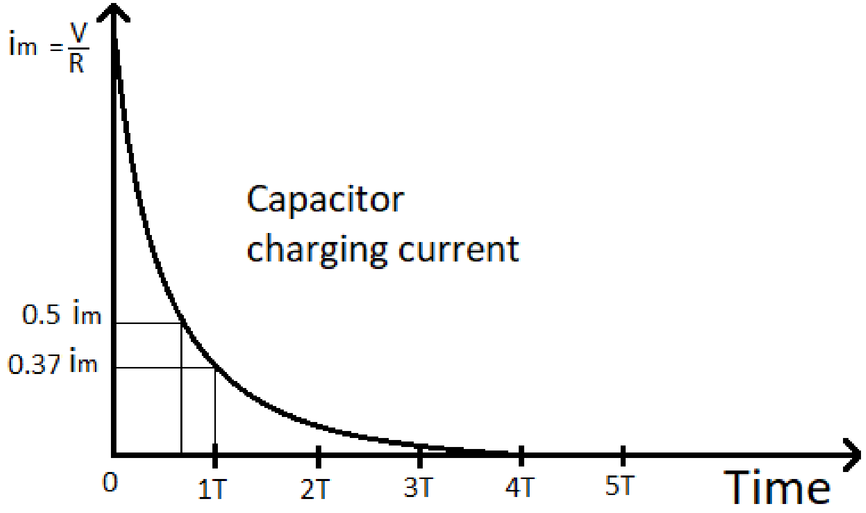
(d) Rate of rise of voltage across capacitor

We have, $V - v = CR\frac{dv}{dt}$

At the instant the switch is closed, v = 0, therefore, $V = CR\frac{dv}{dt}$

Hence initial rate of rise of voltage across capacitor is: $\mathbf{\frac{dv}{dt} = \frac{V}{CR}}$ volts





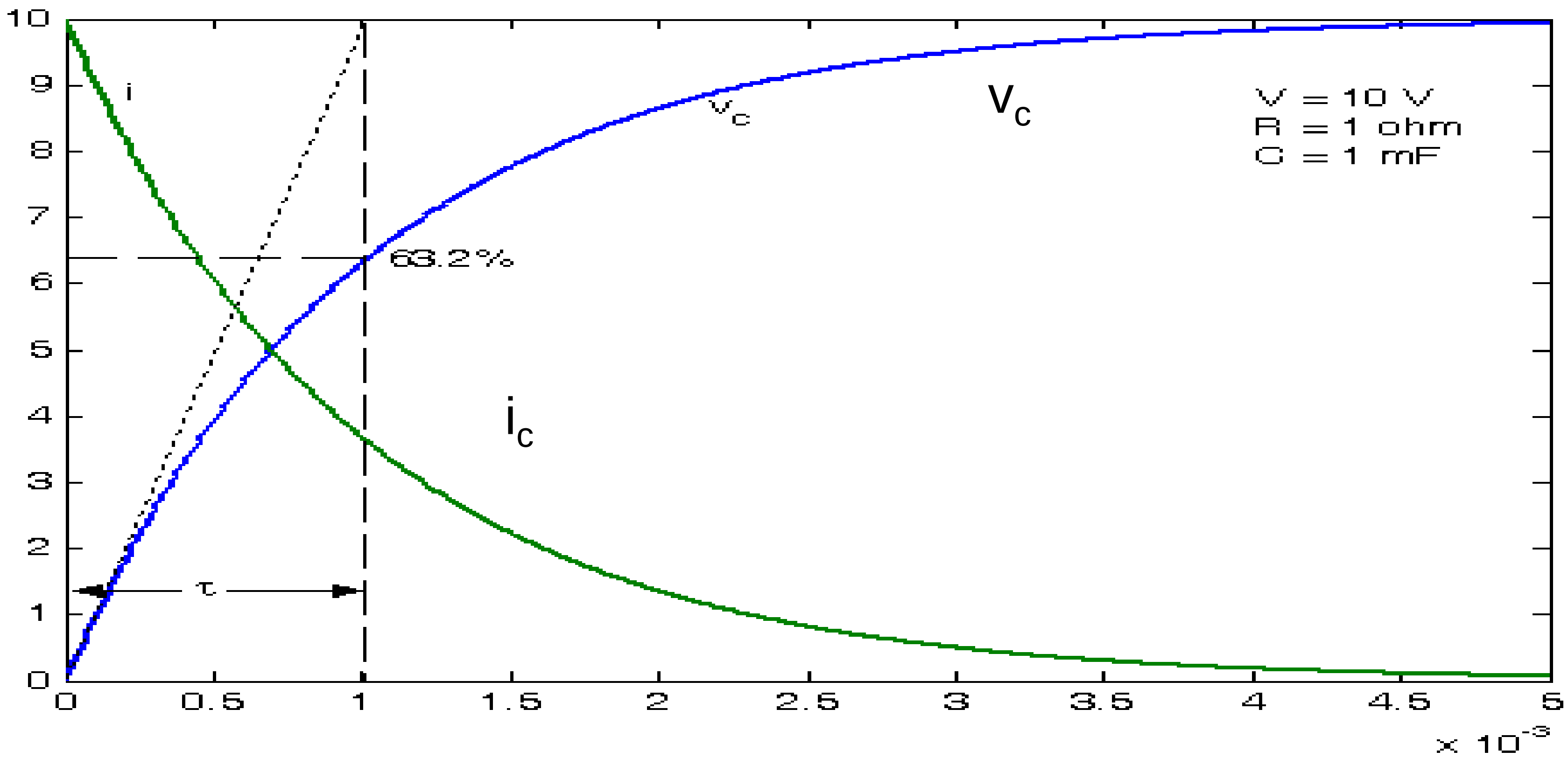
Charging of a Capacitor Through a Resistor

Time Constant (τ) = RC seconds

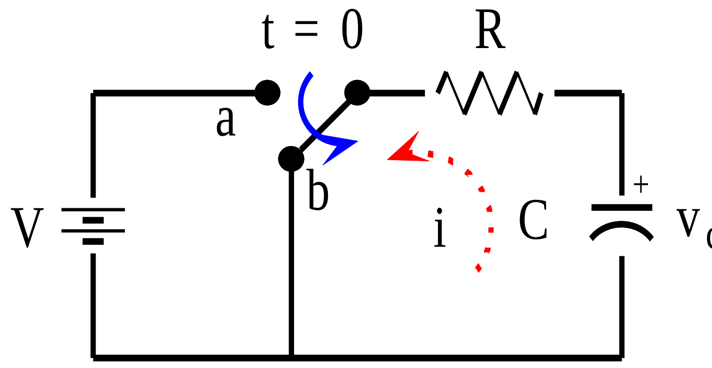
- The quantity **RC** is called the time constant of the circuit and affects the charging (or discharging) time.
- **It has the dimensions of time** so that the exponent of e (i.e., $^{-t}/_{RC}$) is a number.
- If time interval, $\mathbf{t = \tau}\ (\mathbf{or\ RC}),\ \text{then},\ \mathbf{v = V}\left(\mathbf{1} - e^{-t/t}\right) = \mathbf{0.632\ V}$
- Hence, time constant can be defined as the time required for the capacitor voltage to reach **0.632 (or 63.2 %) of its final steady state value, V**
- If time interval, $\mathbf{t = \tau}\ (\mathbf{or\ RC}),\ \text{then},\ \mathbf{i = I_m e^{-t/t} = 0.37\ I_m}$
- Hence, time constant can be defined as the time required for the charging current to fall to **0.37 (or 37 %) of its initial maximum value, $\mathbf{I_m}$**

Charging of a Capacitor Through a Resistor

Time Constant (τ) may also be defined as the time required for the capacitor voltage to rise to its final steady-state value V if it continued rising at its initial rate **(V/RC).**



Discharging of a Capacitor Through a Resistor



- Consider capacitor charged to a pd of **V** volts (through **Position a** of switch). When the switch is thrown to **Position b**, the voltage across the capacitor starts decreasing .
- The discharge current rises instantaneously to a value of $\mathbf{I_m = \frac{V}{R}}$ and then decays gradually to zero.
- Let at any instant during discharging:

i = discharging current   v = pd across C   q = charge on capacitor = C v

By KVL, $0 = \mathrm{v} + \mathrm{RC}\frac{\mathrm{dv}}{\mathrm{dt}}$ or $\frac{\mathrm{dv}}{v} = -\frac{\mathrm{d}t}{\mathrm{RC}}$

Integrating both sides, we get $\log_e \mathrm{v} = -\frac{\mathrm{t}}{\mathrm{RC}} + \mathrm{K}$

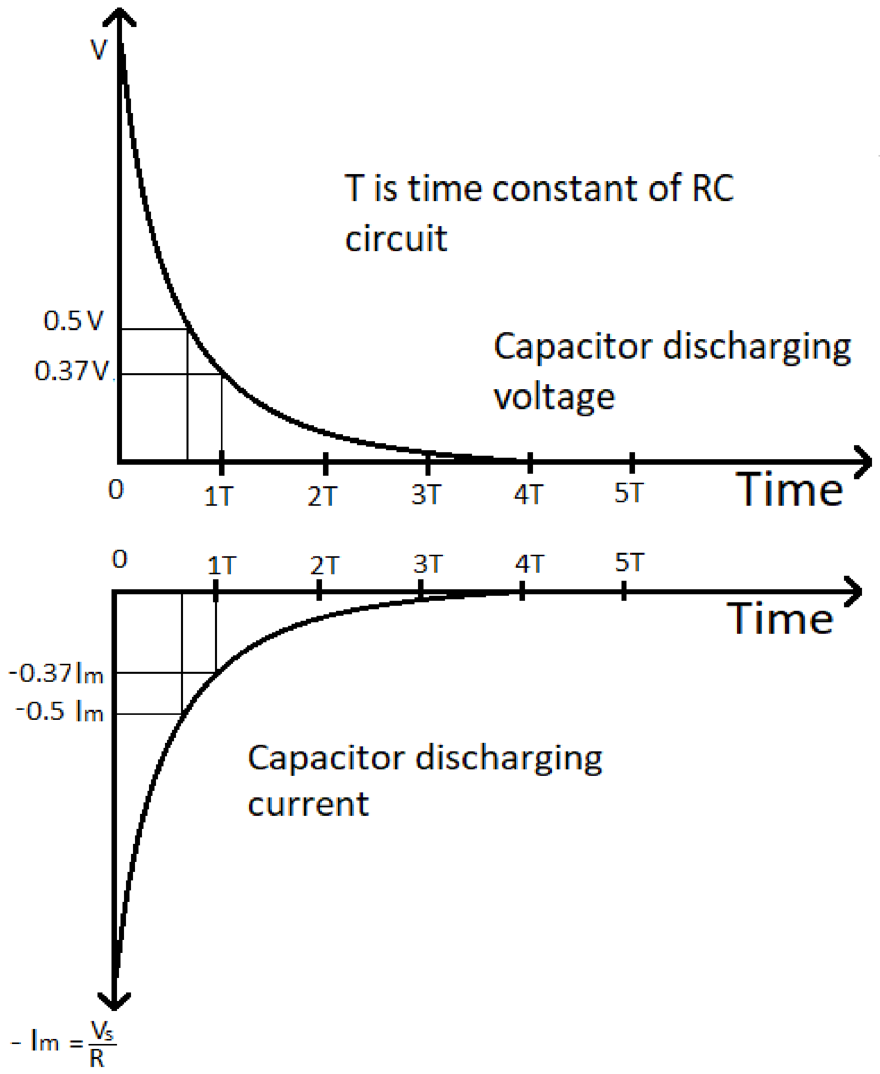
Initial condition: At the instant of closing the switch, **t = 0** and **v = V** which gives $\mathbf{K = \log_e V}$

Therefore, $\log_e \mathrm{v} = -\frac{\mathrm{t}}{\mathrm{RC}} + \log_e \mathrm{V}$ or $\log_e \frac{v}{V} = -\frac{\mathrm{t}}{\mathrm{RC}}$

$\mathbf{v = V\, e^{-t/RC}}$ Similarly, $\mathbf{q = Q\, e^{-t/RC}}$ and $\mathbf{i = -I_m\, e^{-t/RC}}$

Negative sign indicates that the discharging current flows in the opposite direction of charging current.

Discharging of a Capacitor Through a Resistor



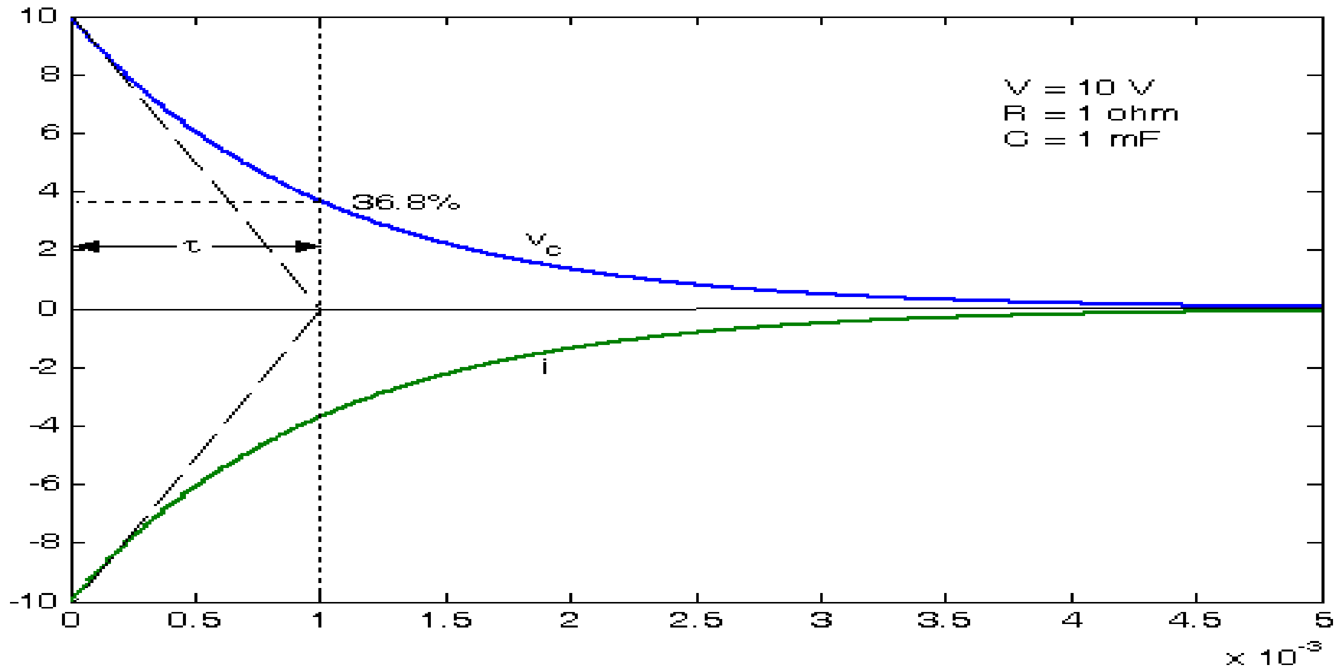


Illustration 1

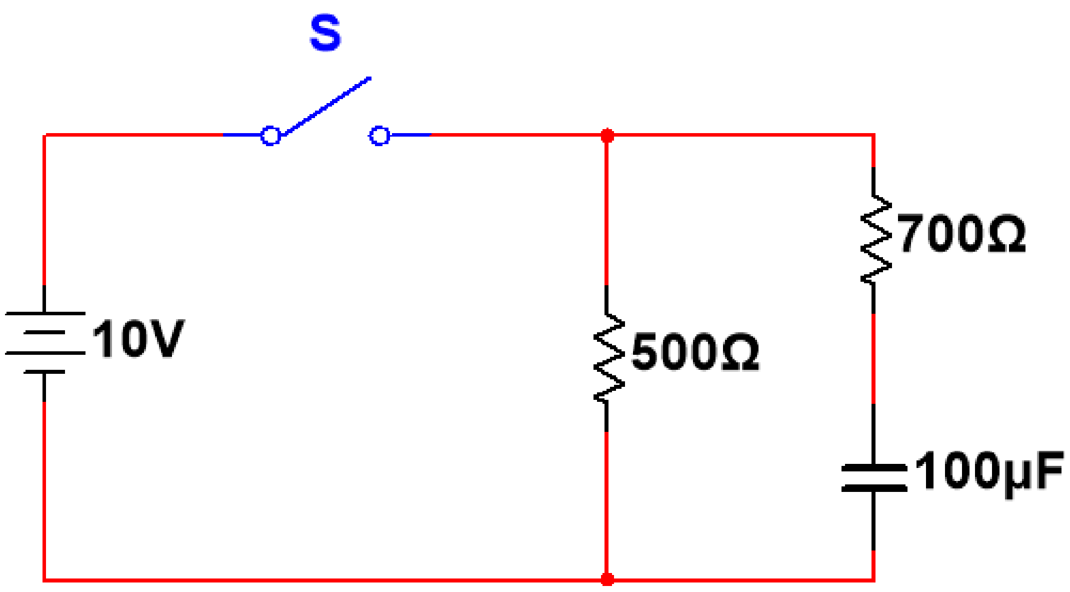
An **8 μF** capacitor is connected in series with a **0.5 MΩ** resistor, across a **200 V** dc supply through a switch. At **t = 0 s**, the switch is turned on. Calculate

i. Time constant of the circuit
ii. Initial charging current.
iii. Time taken for the potential difference across the capacitor to grow to **160 V**.
iv. Current & potential difference across the capacitor **4.0 seconds** after the switch is turned on.

Ans: (i) 4 s, (ii) 400 μA , (iii) 6.44 s (iv) 126.42 V & 147.15 μA

Illustration 2

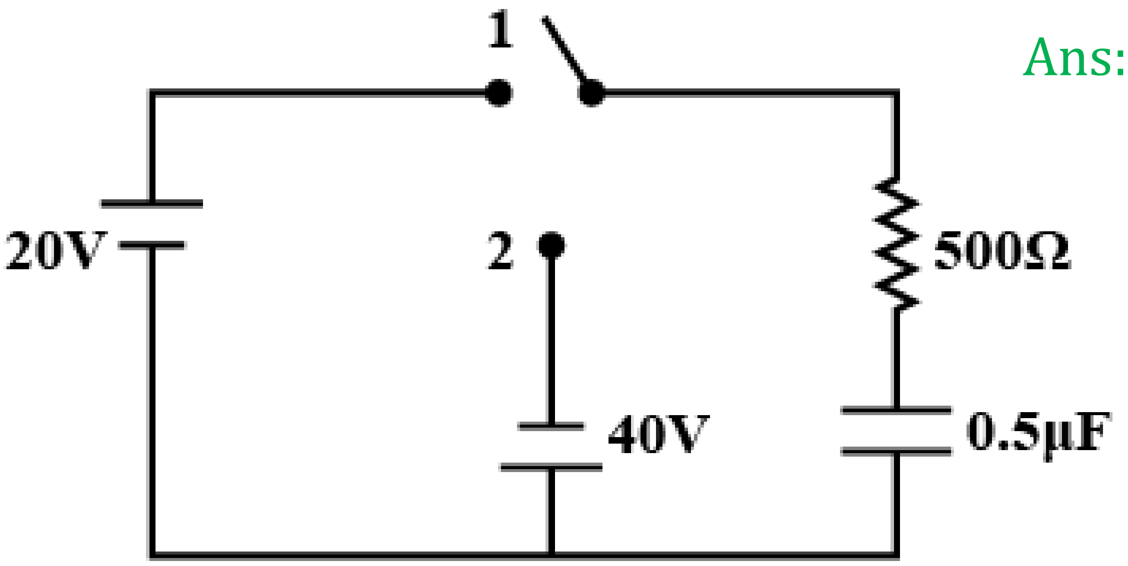
For the circuit shown, the switch **S** is closed at **t = 0 s**. Determine how long will it take, after the switch is closed, for the total current drawn from the supply to reach **25 mA**.



Ans: t = 73.5 ms

Illustration 3

In the network shown below, the switch is closed to **Position 1** at **t = 0**, and then moved to **2** after one time constant, at **t = 250 µs**. Obtain the current for **t > 0** and also sketch it.



$$\text{Ans: } i = \begin{cases} 40\, e^{-4000t} \quad \text{mA} \quad (0 < t < \tau) \\ -105.28\, e^{-4000(t-0.00025)} \quad \text{mA} \quad (t > \tau) \end{cases}$$