INSTITUTION of EMINENCE

Basic Electrical Technology

[ELE 1051]

Three Phase AC Circuits

L22 ,L23– Star & Delta Connected Balanced Loads & Unbalanced loads

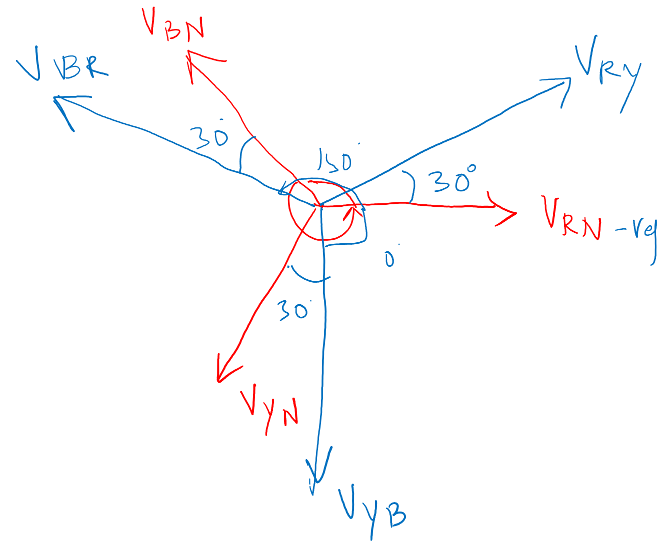
Topics Covered

- *Analysis of balanced/unbalanced star/delta connected loads with 3 phase excitation.*
- *Phase and line voltage/current relations.*
- *Neutral shift and circulating currents with unbalanced loads.*

RECAP

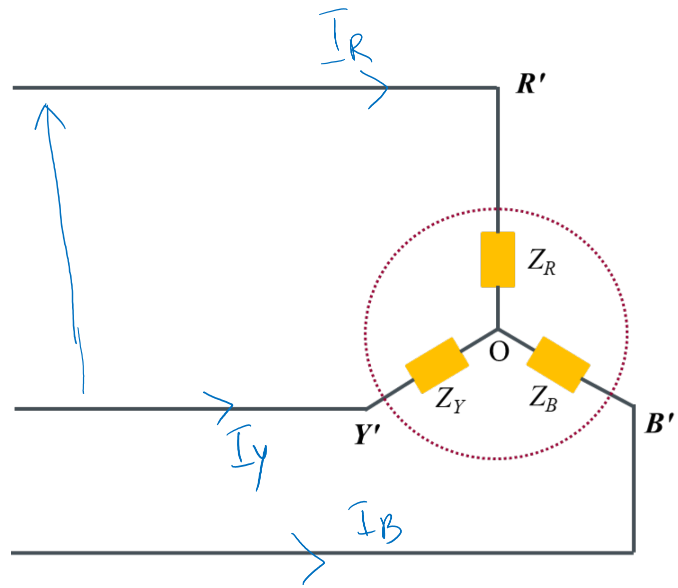
✓

Phase Voltages,	Line Voltages
$\hat{V}_{RN} = V_m\ Sin(\omega t)$	$\hat{V}_{RY} = \sqrt{3} \times V_m\ Sin(\omega t + 30)$
$\hat{V}_{YN} = V_m\ Sin(\omega t - 120°)$	$\hat{V}_{YB} = \sqrt{3} \times V_m\ Sin(\omega t - 90)$
$\hat{V}_{BN} = V_m\ Sin(\omega t - 240°)$	$\hat{V}_{BR} = \sqrt{3} \times V_m\ Sin(\omega t + 150)$



- ✓ Sum of all three Phase voltages = Zero
- ✓ Sum of all three Line Voltages = Zero
- ✓ Line Voltage = √3 (Phase Voltage)
- ✓ Phase Sequence RYB & RBY
- ✓ 3 Phase supply can be 3 wire or 4 wire
- ✓ 3 Phase load can be Star or Delta

Balanced and Unbalanced Load



Balanced Load – All the three phase impedances are same

$$Z_R = Z_Y = Z_B = R \pm JX$$

So, the Phase Currents will be same in all three phases

Unbalanced Load – All the three phase impedances are not equal. So the phase currents will be not be same.

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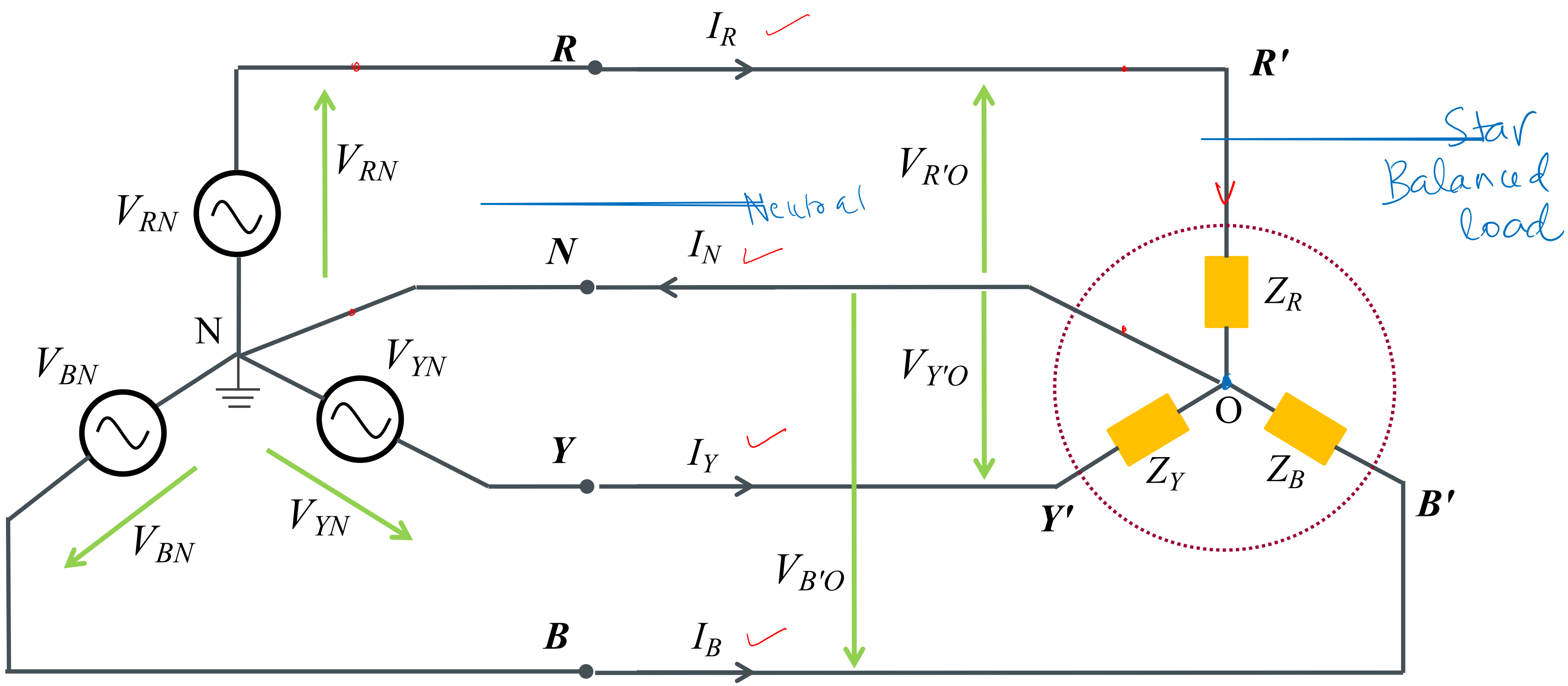
3φ, 4 Wire System with Y Load

Consider the 3 phase star load connected to a 4 wire balanced source.



Supply Balanced

Phase Voltages of Load,

$$\hat{V}_{R'O} = \hat{V}_{RN}$$

$$\hat{V}_{Y'O} = \hat{V}_{YN}$$

$$\hat{V}_{B'O} = \hat{V}_{BN}$$

$I_R = \frac{V_{R'O}}{Z_R}$ $I_Y = \frac{V_{Y'O}}{Z_Y}$

$I_B = \frac{V_{B'O}}{Z_B}$ Same

Neutral Current: $I_{ph} = I_{line}$

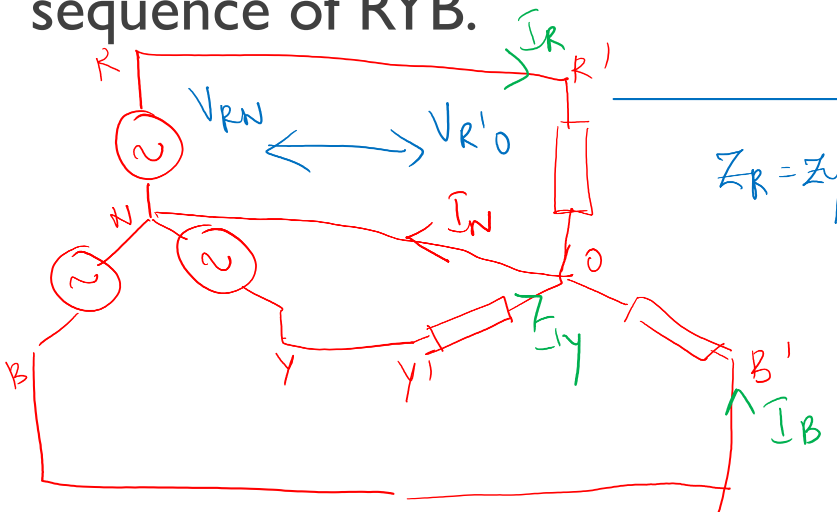
$$\hat{I}_N = \hat{I}_R + \hat{I}_Y + \hat{I}_B$$

$$\hat{I}_N = 0; \text{(If } Z_R = Z_Y = Z_B = Z\angle\theta°\text{)}$$

Illustration 01

3 similar coils each having resistance of 20Ω and an inductance of 0.05H are connected in star to a 3 phase 4 wire 50Hz 400V supply. Calculate the line and phase voltages and currents of the load. Also draw the phasor diagram taking V_{RN} as the reference. Consider phase sequence of RYB.

LINE



$$Z = 20 + j(2\pi \times 50 \times 0.05)$$

$$Z_R = Z_Y = Z_B = Z = 20 + j15.70 = 25.426\angle 38.13^\circ\ \Omega$$

$$V_{RY} = 400\angle 30^\circ \qquad V_{R'0} = V_{RN} = 230.94\angle 0^\circ$$

$$V_{YB} = 400\angle -90^\circ \qquad V_{YN} = 230.94\angle -120^\circ$$

$$V_{BR} = 400\angle 150^\circ \qquad V_{BN} = 230.94\angle -240^\circ$$

$$V_L = \sqrt{3}V_{ph}$$

$$400 = \sqrt{3}\,V_{ph}$$

$$V_{ph} = \frac{400}{\sqrt{3}} = 230.94\,V$$

$$I_R = \frac{V_{R'0}}{Z_R} = \frac{230.94\angle 0^\circ}{25.426\angle 38.13^\circ} = 9.08\angle -38.13^\circ\ A$$

$$I_Y = \frac{V_{Y'0}}{Z_Y} = \frac{230.94\angle -120^\circ}{25.426\angle 38.13} = 9.08\angle -158.13^\circ\ A$$

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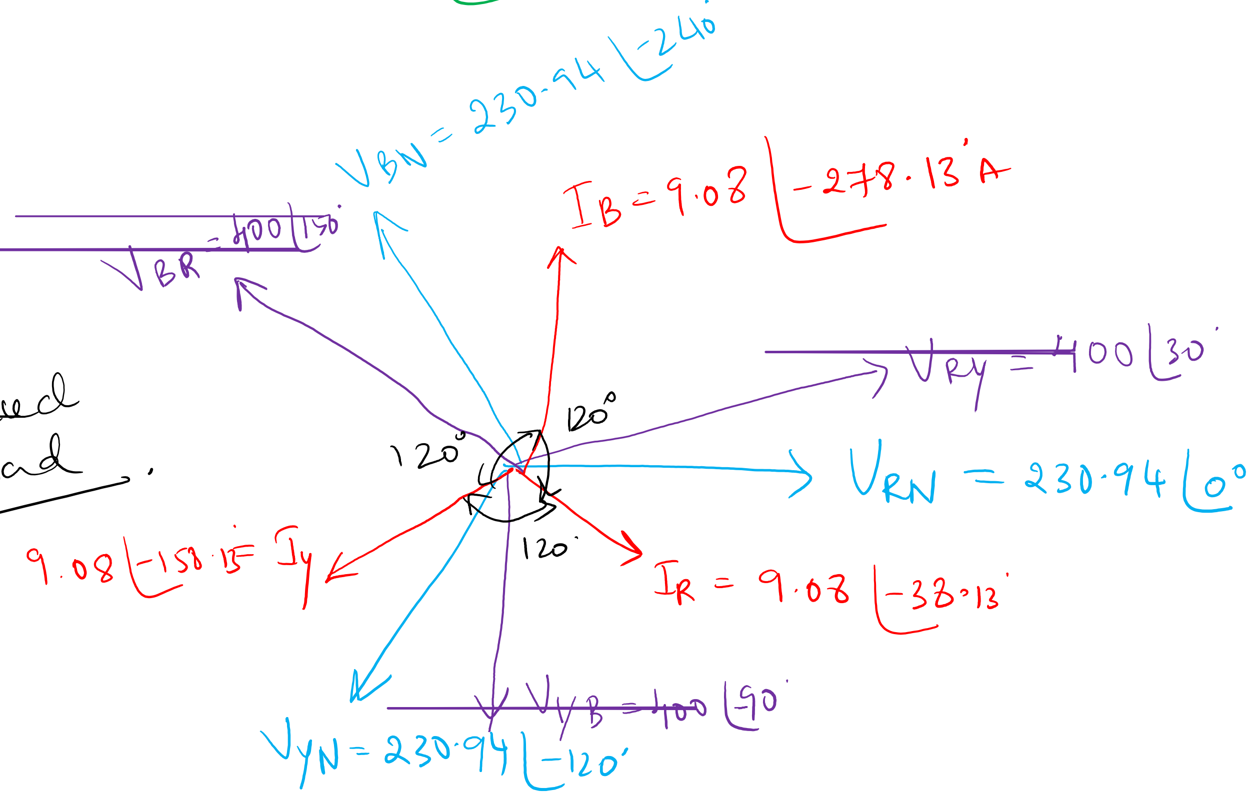
$$I_B = \frac{V_{B'O}}{Z_B} = \frac{230.94\angle -240^\circ}{25.426\angle +38.13} = 9.08\angle -278.13^\circ \text{ A}$$

~~Balanced load~~.



$\boxed{I_N = 0}$

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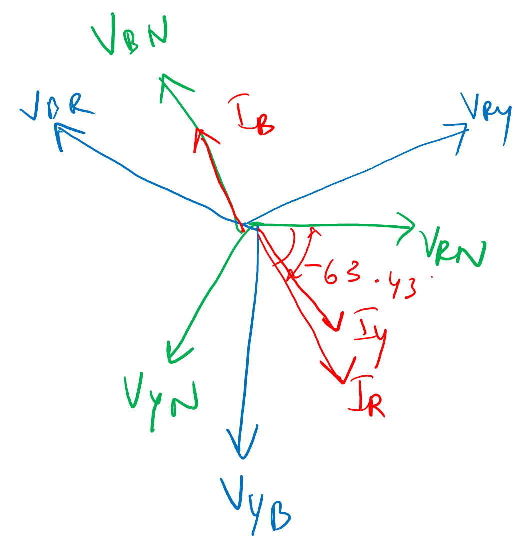
Illustration 02

If the impedances are Z_R = 10+j20Ω, Z_Y = 15-j30Ω and Z_B = 50Ω in Illustration 01, find the current neutral current of the circuit.

$$I_R = \frac{V_R \angle 0}{Z_R} = \frac{230.94\angle 0^\circ}{10+j20} = 10.328\angle -63.43^\circ \text{ A}$$

$$I_Y = \frac{V_Y \angle 0}{Z_R} = \frac{230.94\angle -120^\circ}{15-j30} = 6.885\angle -56.56 \text{ A}$$

$$I_B = \frac{V_B \angle 0}{Z_B} = \frac{230.94\angle -240^\circ}{50} = 4.618\angle 120^\circ \text{ A}$$



R — I inphase V — $I_L = I_{ph}$

$$I_N = I_R + I_Y + I_B = 12.565\angle -60.93 \text{ A}$$

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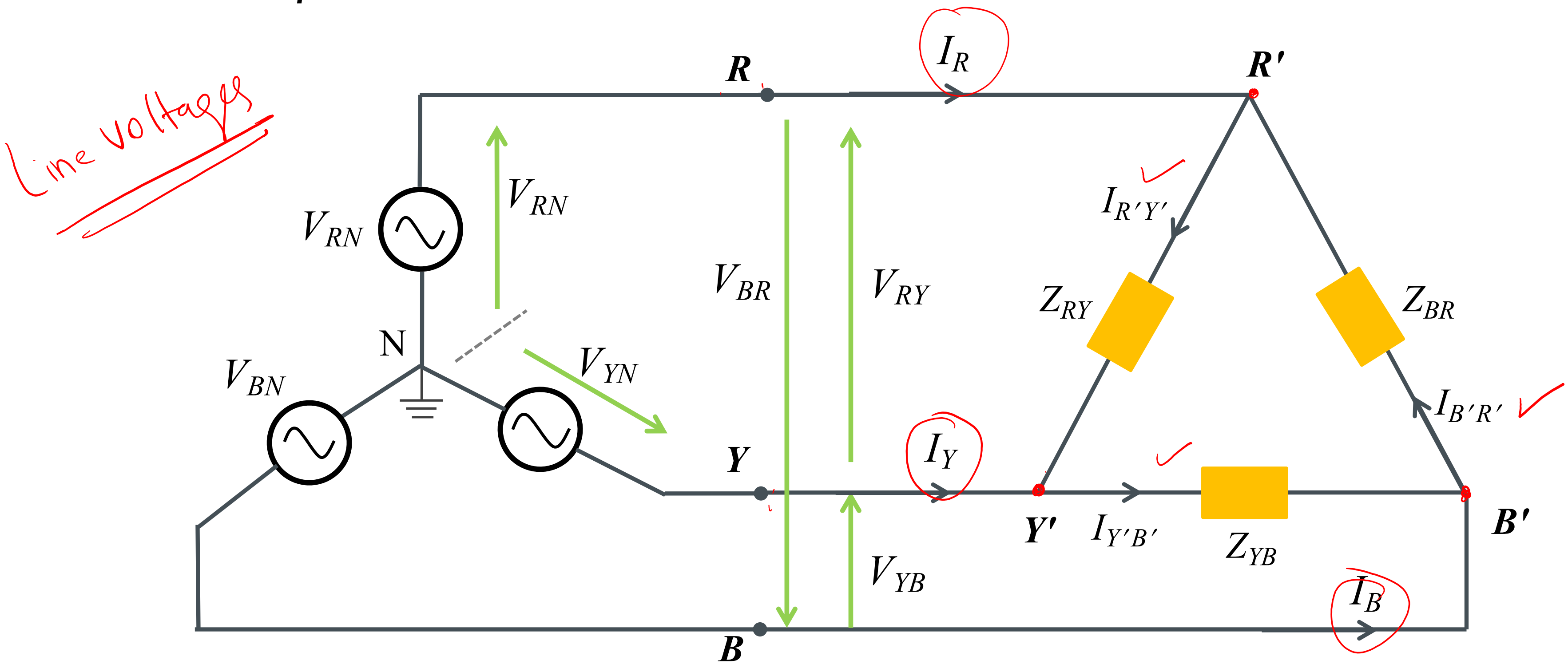
3φ , 3 Wire System with Δ Load

Consider the 3 phase Delta load connected to a 3 wire balanced source.



Phase Currents,

$$\hat{I}_{R'Y'} = \frac{\hat{V}_{RY}}{\bar{Z}_{RY}} \qquad \hat{I}_{Y'B'} = \frac{\hat{V}_{YB}}{\bar{Z}_{YB}} \qquad \hat{I}_{B'R'} = \frac{\hat{V}_{B'R'}}{\bar{Z}_{BR}}$$

Line Currents,

$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'}$$
$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'}$$
$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'}$$

Balanced Δ Load

If $Z_R = Z_Y = Z_B = Z\angle\theta$

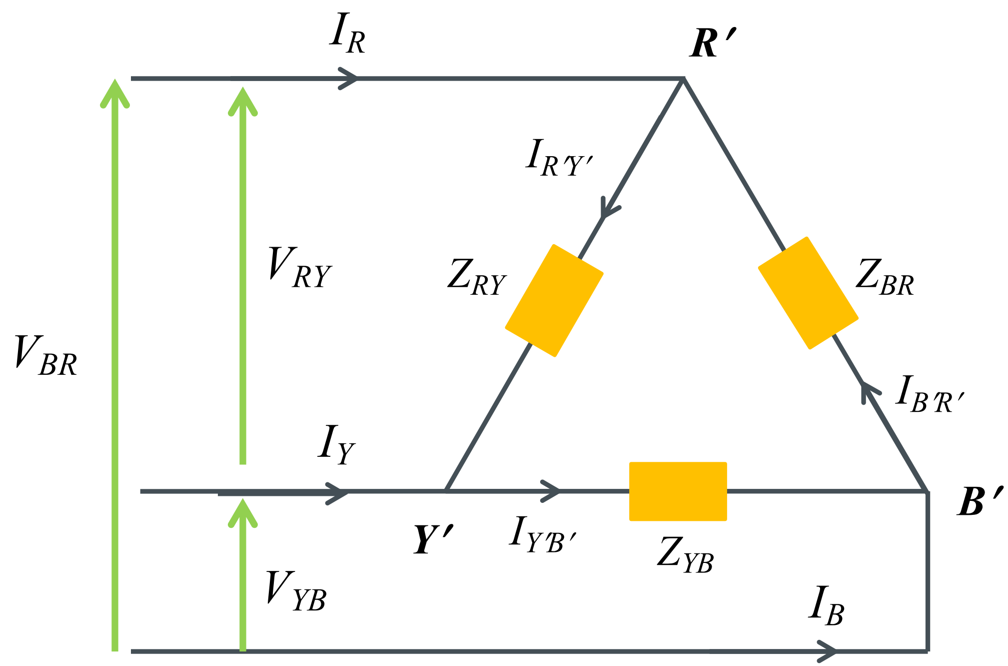
Then, $|I_{R'Y'}| = |I_{Y'B'}| = |I_{B'R'}| = I_{Ph}$

Phase Currents:

$$\hat{I}_{R'Y'} = I_{Ph}\angle 0°$$

$$\hat{I}_{Y'B'} = I_{Ph}\angle -120°$$

$$\hat{I}_{B'R'} = I_{Ph}\angle +120°$$



Line Currents:

$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'}$$

$$= I_{Ph}\angle 0° - I_{Ph}\angle +120° = \sqrt{3}\times I\angle -30°$$

$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'} = \sqrt{3}\times I_{Ph}\angle -150°$$

$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'} = \sqrt{3}\times I_{Ph}\angle 90°$$

Unbal

Iph differe..

~~Neutral line~~ doesnot exist

Illustration-3

Three loads, Z_{RY}=50+j40 Ω, Z_{YB}=100 Ω and Z_{BR}=80-j60 Ω are connected in Delta across a balanced 3 phase, 415V, 50 Hz supply. Determine

Line voltage

(i) Phase Currents

(ii) Line Currents and hence draw the complete phasor diagram.

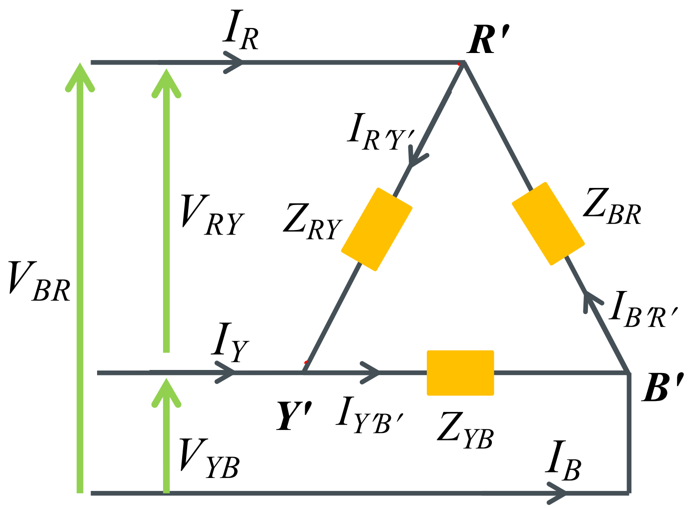
Assume a phase sequence of RYB.

$\hat{V}_{RY} = 415\angle 0°$ (Reference Voltage)

$V_{YB} = 415\angle -120° \; V$

$V_{BR} = 415\angle -240° \; V$

$I_{RY'} = \frac{V_{RY}}{Z_{RY}} = \frac{415\angle 0°}{50+j40} = 6.48\angle -38.65 \; A$



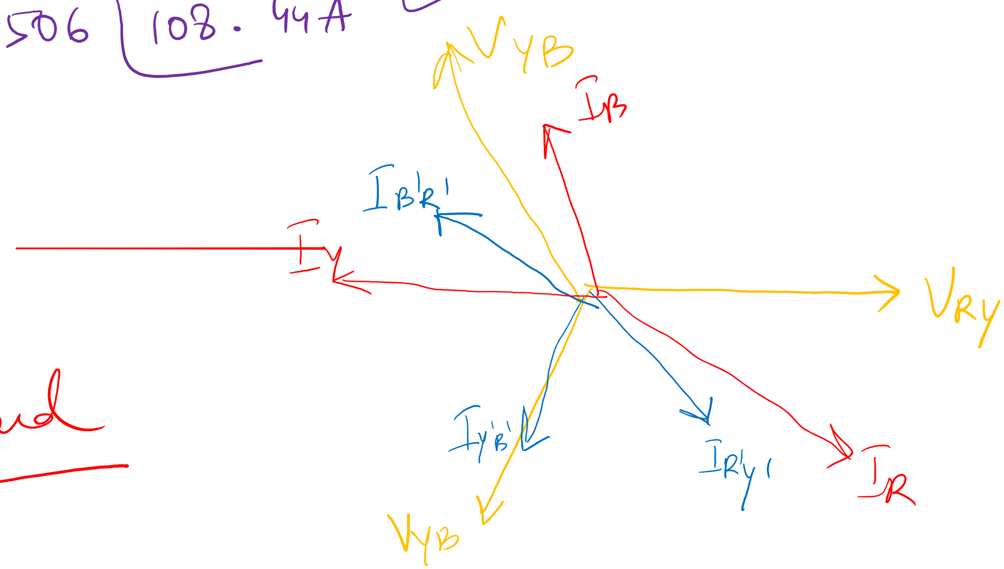
$$I_{R'Y'} = 6.48\angle{-38.65}$$

$$I_{Y'B'} = \frac{415\angle{-120^\circ}}{100} = 4.15\angle{-120^\circ}\ A, \quad I_{B'R'} = \frac{415\angle{-240^\circ}}{80-j60} = 4.15\angle{156.87}\ A$$

$$I_R = I_{R'Y'} - I_{B'R'} = 10.537\angle{-32.61}\ A$$

$$I_Y = I_{Y'B'} - I_{R'Y'} = 7.149\angle{176.35}\ A$$

$$I_B = I_{B'R'} - I_{Y'B'} = 5.506\angle{108.44}\ A$$



Unbalanced

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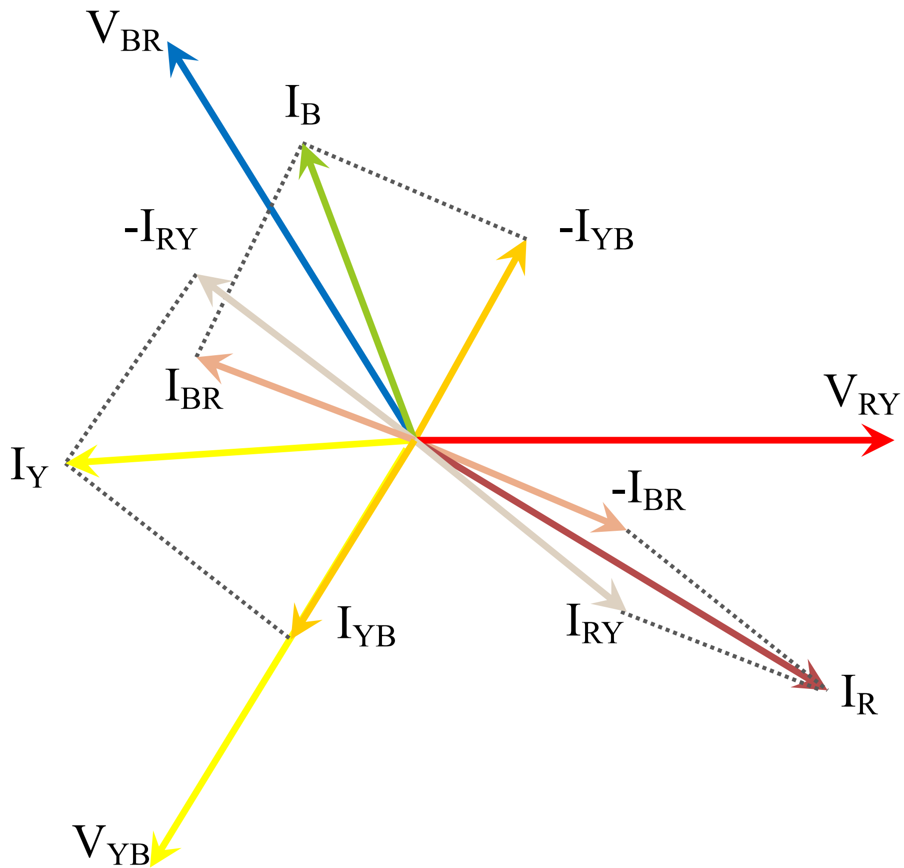
Illustration-2…

(ii) Phasor Diagram,



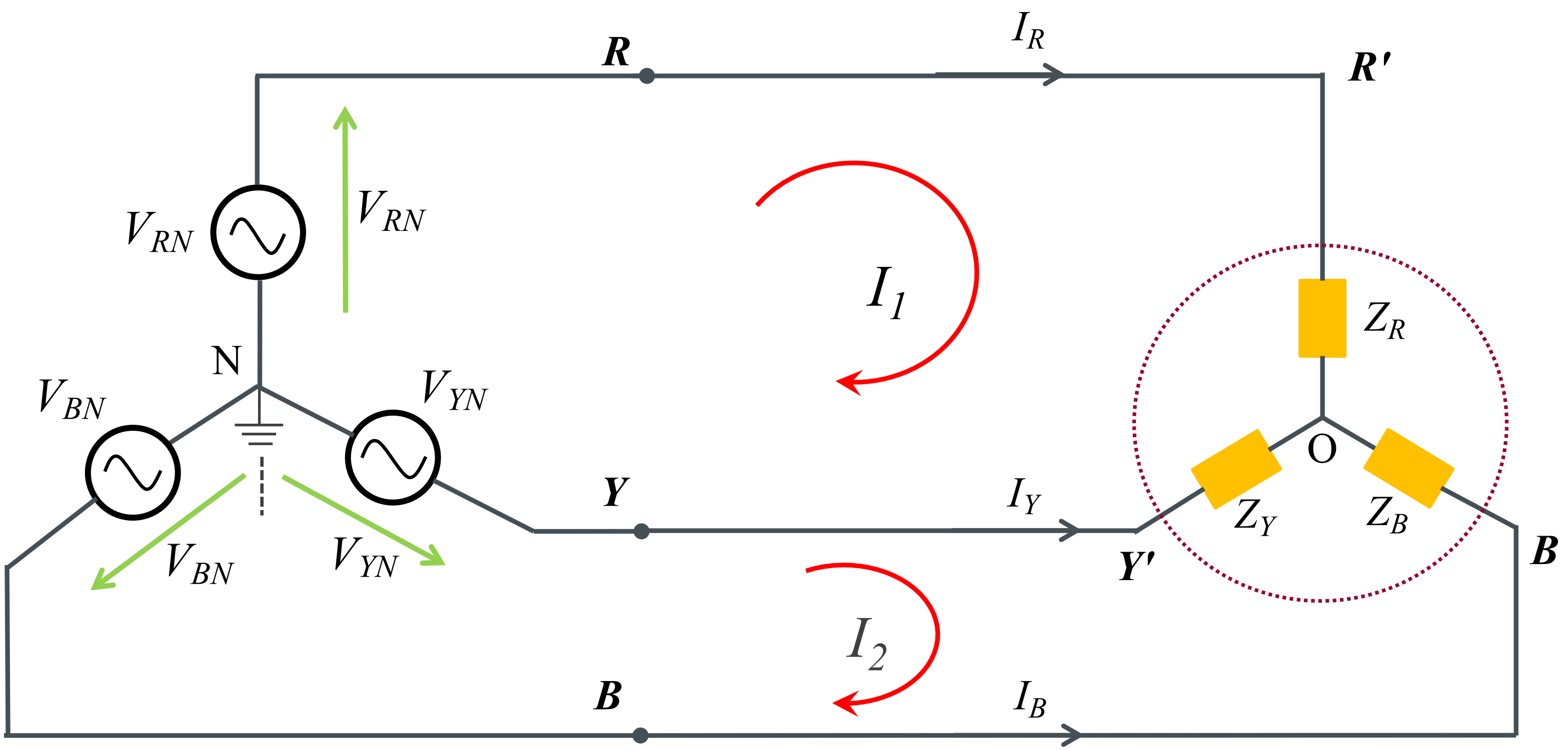
RECAP of L22

3 Phase 4 Wire – Star Connected

3 Phase 3 Wire – Delta Connected

3φ System with Y Connected Load

Consider the 3 phase star load connected to a 3 wire balanced source. Two mesh currents $\hat{I}_1$ and $\hat{I}_2$ are assumed to flow as shown in Fig.



Writing mesh equations,

$$\begin{bmatrix} \bar{Z}_R + \bar{Z}_Y & -\bar{Z}_Y \\ -\bar{Z}_Y & \bar{Z}_Y + \bar{Z}_B \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} \hat{V}_{RN} - \hat{V}_{YN} \\ \hat{V}_{YN} - \hat{V}_{BN} \end{bmatrix}$$

Loop Current Analysis…

Using Cramer's Rule,

$$\hat{I}_1 = \frac{\begin{vmatrix} \hat{V}_{RN} - \hat{V}_{YN} & -\bar{Z}_Y \\ \hat{V}_{YN} - \hat{V}_{BN} & \bar{Z}_Y + \bar{Z}_B \end{vmatrix}}{\begin{vmatrix} \bar{Z}_R + \bar{Z}_Y & -\bar{Z}_Y \\ -\bar{Z}_Y & \bar{Z}_Y + \bar{Z}_B \end{vmatrix}} \qquad \hat{I}_2 = \frac{\begin{vmatrix} \bar{Z}_R + \bar{Z}_Y & \hat{V}_{RN} - \hat{V}_{YN} \\ -\bar{Z}_Y & \hat{V}_{YN} - \hat{V}_{BN} \end{vmatrix}}{\begin{vmatrix} \bar{Z}_R + \bar{Z}_Y & -\bar{Z}_Y \\ -\bar{Z}_Y & \bar{Z}_Y + \bar{Z}_B \end{vmatrix}}$$

The line currents are determined using the following equations:

$$\hat{I}_R = \hat{I}_1$$

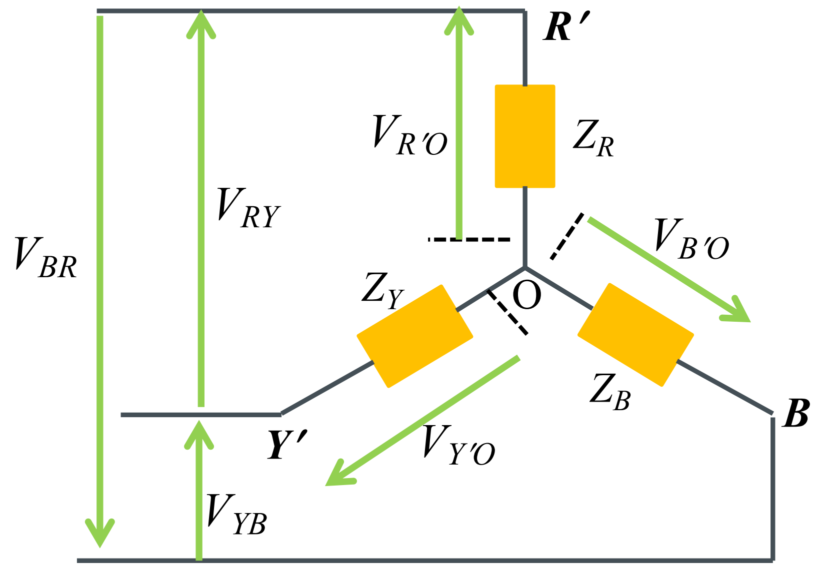
$$\hat{I}_Y = \hat{I}_2 - \hat{I}_1$$

$$\hat{I}_B = -\hat{I}_2$$

Balanced Star Connected Load

If $Z_R = Z_Y = Z_B = Z\angle\theta$

Then, $|V_{R'O}| = |V_{Y'O}| = |V_{B'O}| = V_{Ph}$



Phase Voltages:

$$\hat{V}_{R'O} = V_{Ph}\angle 0°$$

$$\hat{V}_{Y'O} = V_{Ph}\angle -120°$$

$$\hat{V}_{B'O} = V_{Ph}\angle +120°$$

Line Voltages:

$$\hat{V}_{RY} = \hat{V}_{R'O} - \hat{V}_{Y'O}$$

$$= V_{Ph}\angle 0° - V_{Ph}\angle -120° = \sqrt{3}\times V_{Ph}\angle 30°$$

$$\hat{V}_{YB} = \hat{V}_{Y'O} - \hat{V}_{B'O} = \sqrt{3}\times V_{Ph}\angle -90°$$

$$\hat{V}_{BR} = \hat{V}_{B'O} - \hat{V}_{R'O} = \sqrt{3}\times V_{Ph}\angle 150°$$

Illustration-1

Three loads $Z_R = 10\angle 0^0\ \Omega$; $Z_Y = 15\angle -30^0\ \Omega$ and $Z_B = 20\angle 45^0$ are connected in star across a balanced, 3 phase, 400 V, RYB supply. Determine (a) line currents (b) Phase Voltages (c) Neutral shift voltage, V_{ON}.

Solution:

The three phase load is supplied with a balanced supply of 400V, hence the line voltages appearing across the load are:

$$\hat{V}_{RY} = 400\angle 0° \text{ (Reference Voltage)}$$

$$\hat{V}_{YB} = 400\angle -120°$$

$$\hat{V}_{BR} = 400\angle +120°$$

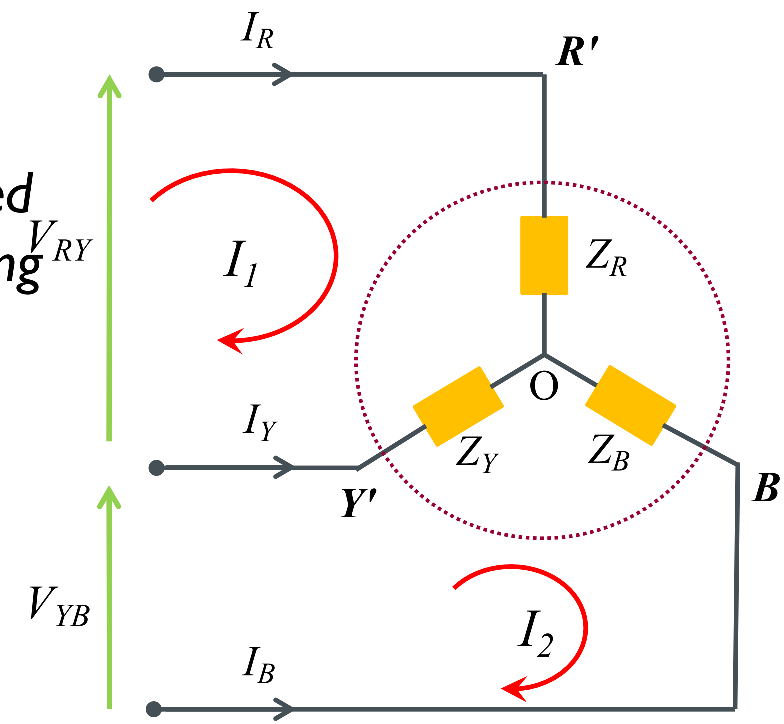


Illustration-I …

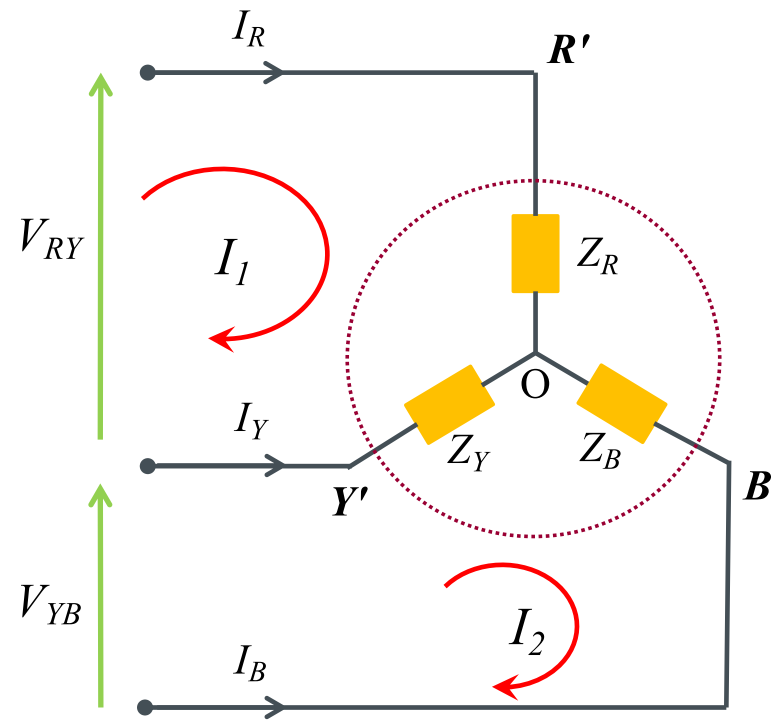
Writing Mesh Equation in Matrix form,

$$\begin{bmatrix} 10\angle 0 + 15\angle -30 & -15\angle -30 \\ -15\angle -30 & 15\angle -30 \ + 20\angle 45 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 400\angle\ 0° \\ 400\angle\ -120° \end{bmatrix}$$

Using Cramer's rule,

$$\hat{I}_1 = \mathbf{9.783\angle -17.87}\ A$$

$$\hat{I}_2 = \mathbf{16.69\angle -116.63}\ A$$



(i) The line currents are

$$\hat{I}_R = \hat{I}_1 = \mathbf{9.783\angle -17.87}\ A$$

$$\hat{I}_Y = \hat{I}_2 - \ \hat{I}_1 = \mathbf{20.59\angle -144.63}\ A$$

$$\hat{I}_B = -\hat{I}_2 = \mathbf{16.69\angle 63.37}\ A$$

Illustration-I …

(ii) Phase Voltages are determined using the following equations.

$$\hat{V}_{R'O} = \hat{I}_R \times \bar{Z}_A = \mathbf{97.83\angle -7.87\ V}$$

$$\hat{V}_{Y'O} = \hat{I}_Y \times \bar{Z}_B = \mathbf{308.85\angle -174.63\ V}$$

$$\hat{V}_{B'O} = \hat{I}_B \times \bar{Z}_C = \mathbf{338\angle 108.37\ V}$$

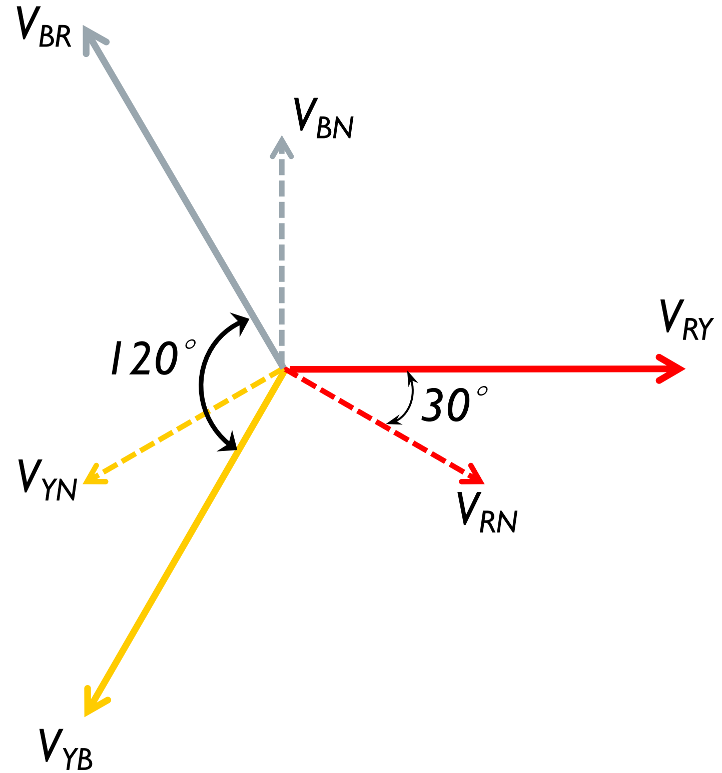
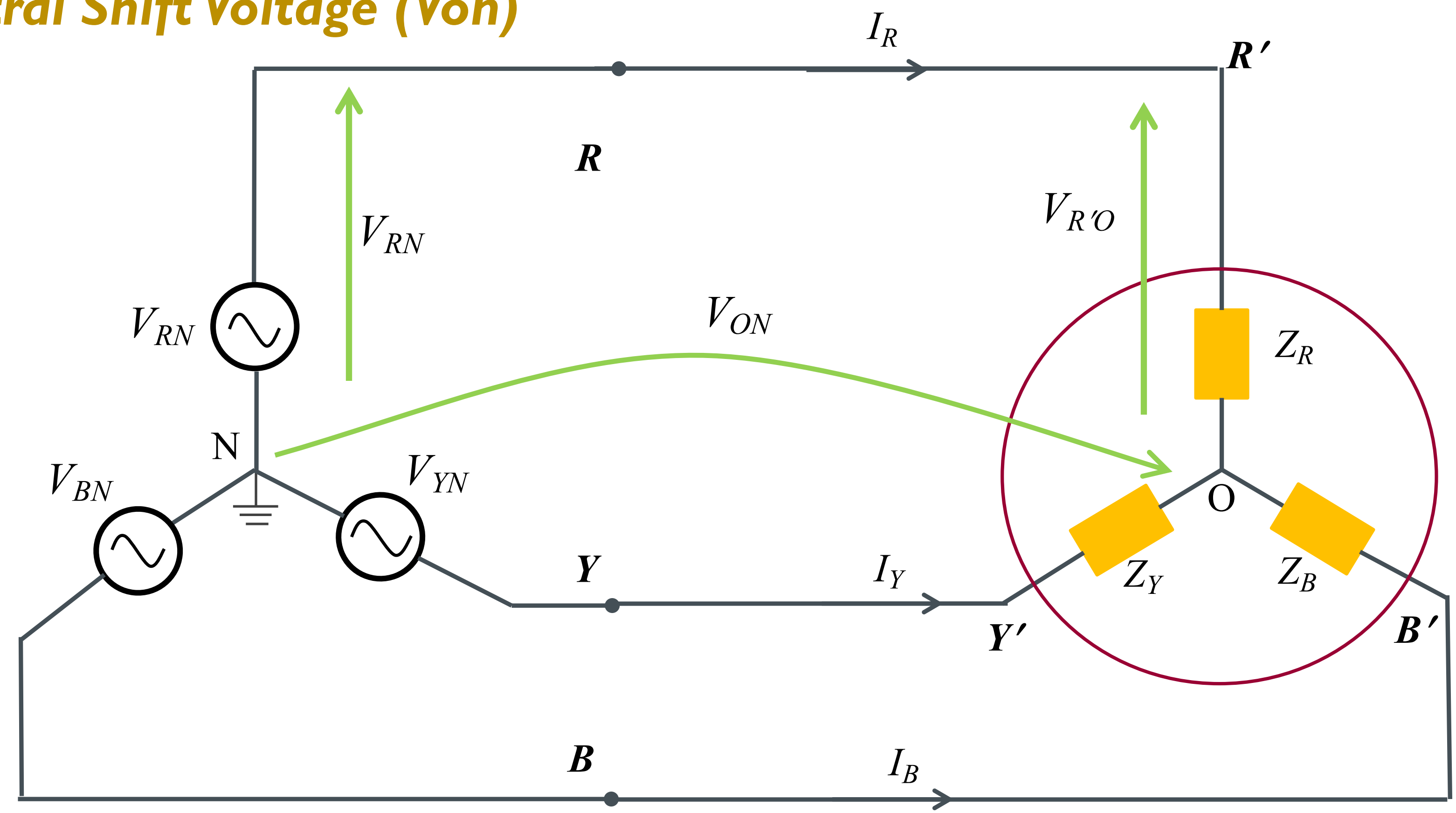


Illustration-I ...

(c)Neutral Shift Voltage (Von)



Applying KVL, $\hat{V}_{RN} - \hat{V}_{R'O} - \hat{V}_{ON} = 0$

$$\hat{V}_{ON} = \hat{V}_{RN} - \hat{V}_{R'O} = 136.84\angle - 38.63° \, V$$

Summary

Analysis of balanced/unbalanced three phase star/delta connected load with 3 phase balanced excitation is performed.

- *For Balanced Star connected load, the line voltage = $\sqrt{3}$ x phase voltage.*
- *For Balanced Delta connected load, the line current = $\sqrt{3}$ x phase current.*