Basic Electrical Technology

2. Magnetic Circuits & Electromagnetism

LECTURE **

Parallel Magnetic Circuits

Parallel Magnetic Circuit

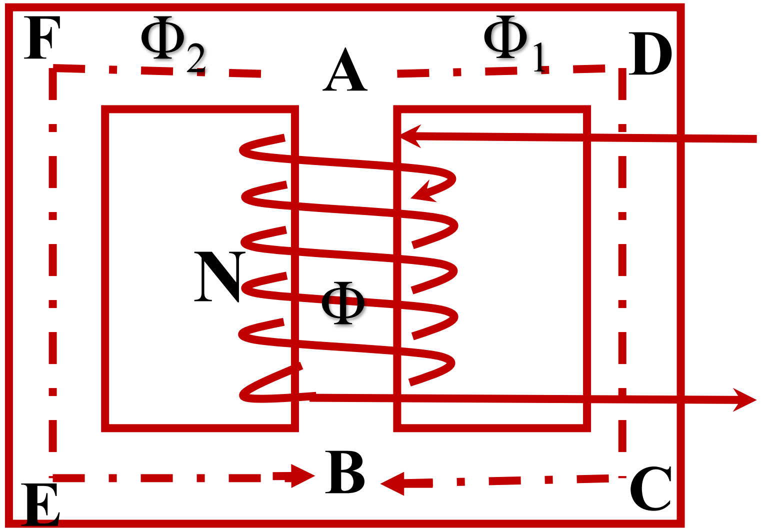
- **More than one path for flux**
- Can be compared to a parallel electric circuit which has more than one path for electric current

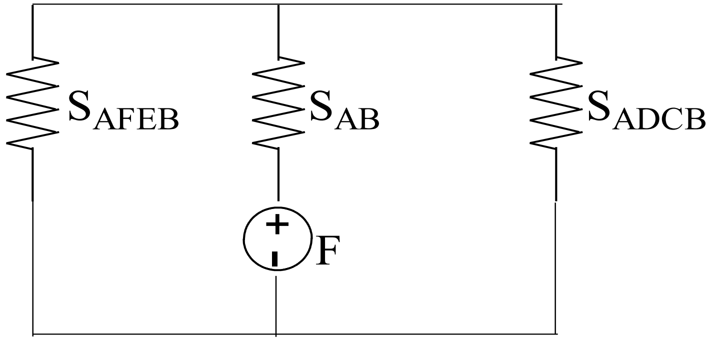
$$\Phi = \Phi_1 + \Phi_2$$

$$S_{AB} = \frac{L_{AB}}{\mu_0 \, \mu_{rAB} \, A_{AB}}$$

$$S_{ADCB} = \frac{L_{ADCB}}{\mu_0 \, \mu_{rADCB} \, A_{ADCB}}$$

$$S_{AFEB} = \frac{L_{AFEB}}{\mu_0 \, \mu_{rAFEB} \, A_{AFEB}}$$





Analogous Electrical Circuit

Parallel Magnetic Circuit

$(MMF)_{Total} = (MMF)_{AB} + (MMF)_{ADCB}$

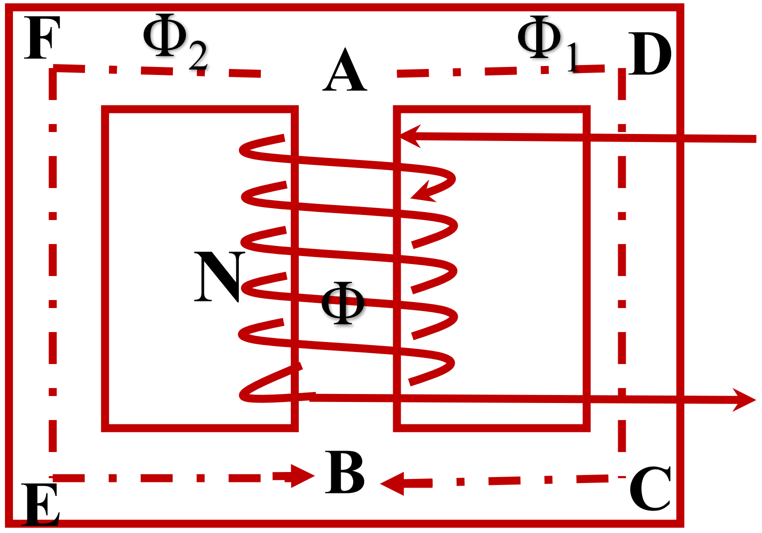
OR

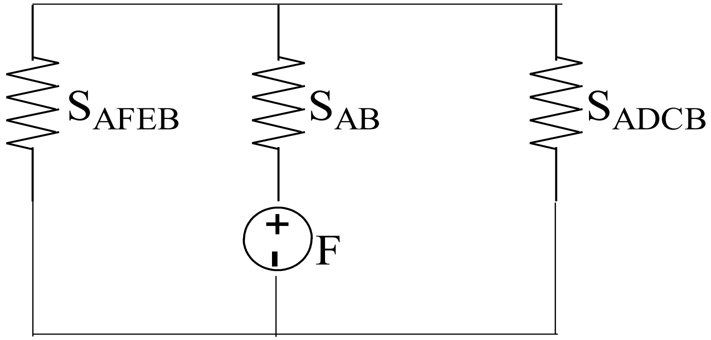
$(MMF)_{Total} = (MMF)_{AB} + (MMF)_{AFEB}$

$(MMF)_{Total} = \Phi\, S_{AB} + \Phi_1\, S_{ADCB}$

OR

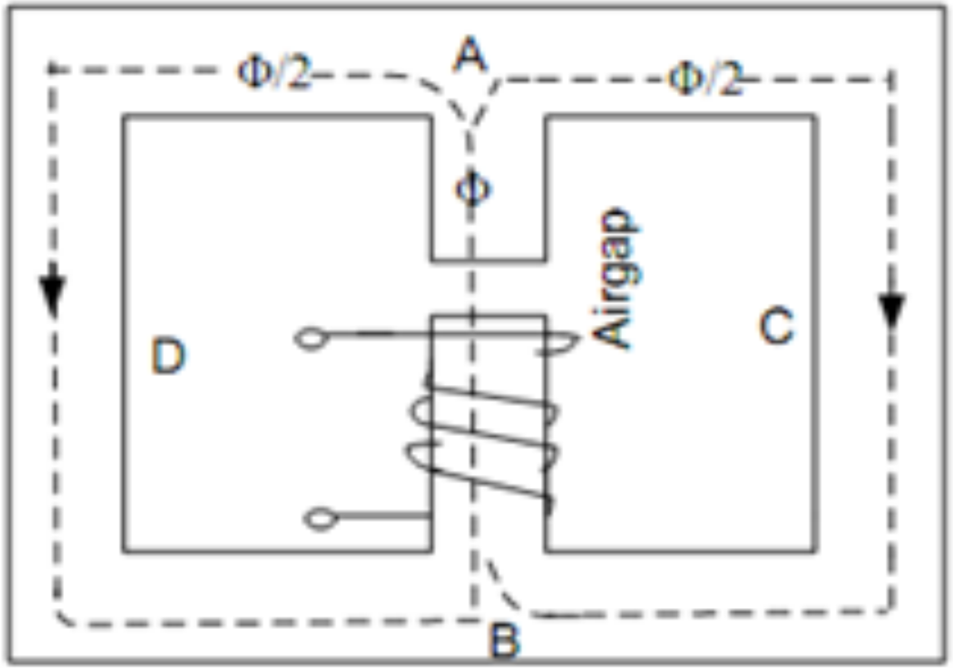
$(MMF)_{Total} = \Phi\, S_{AB} + \Phi_2\, S_{AFEB}$

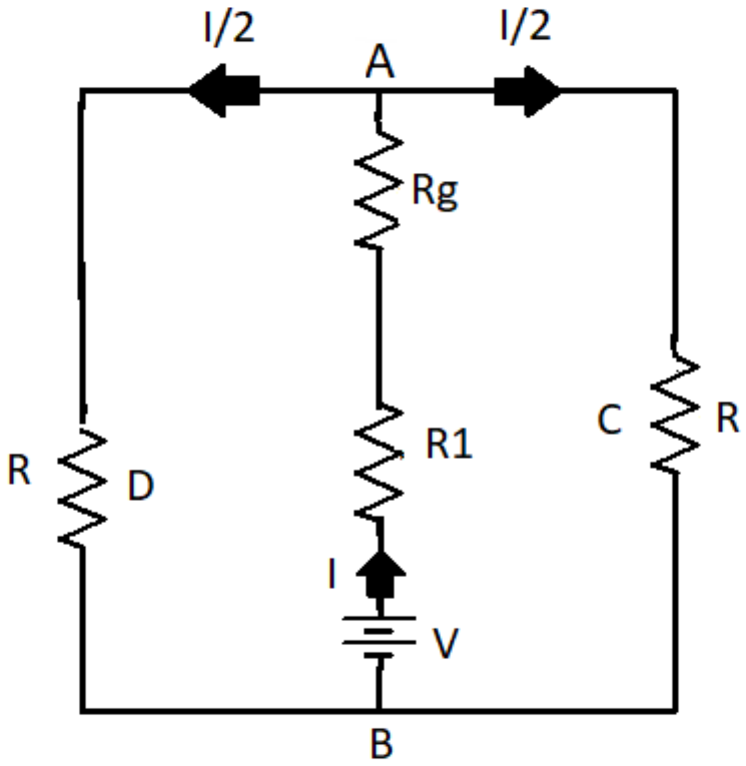




Analogous Electrical Circuit

Parallel Magnetic Circuit with Air Gap



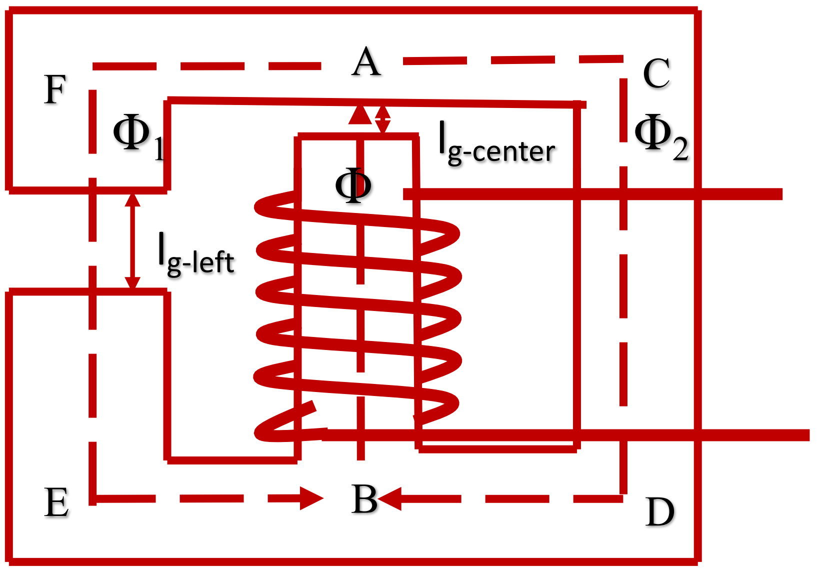


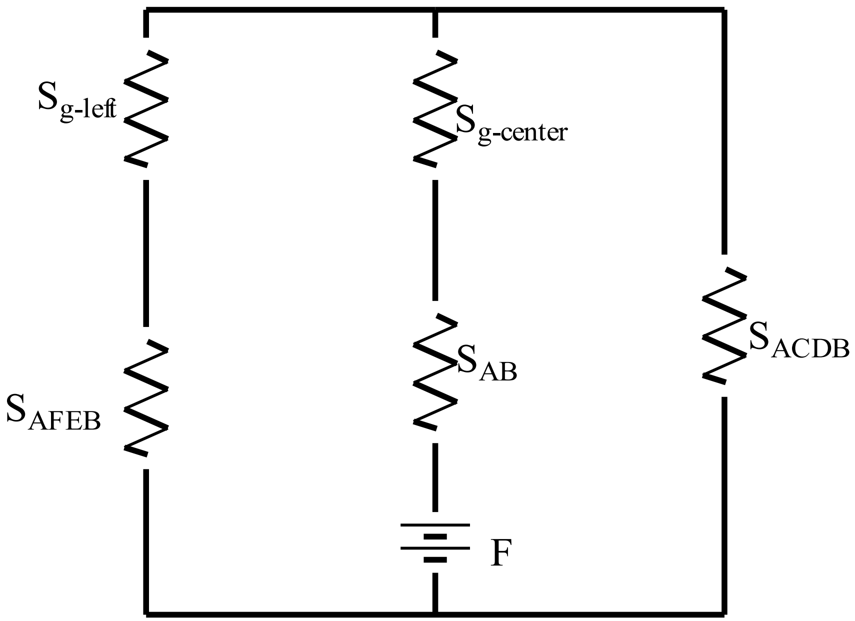
MMF required for this circuit would be the sum of:

(i) that required for central limb (air-gap + core material) and,

(ii) that required for either of two parts (not both)

Parallel Magnetic Circuit with Air Gap



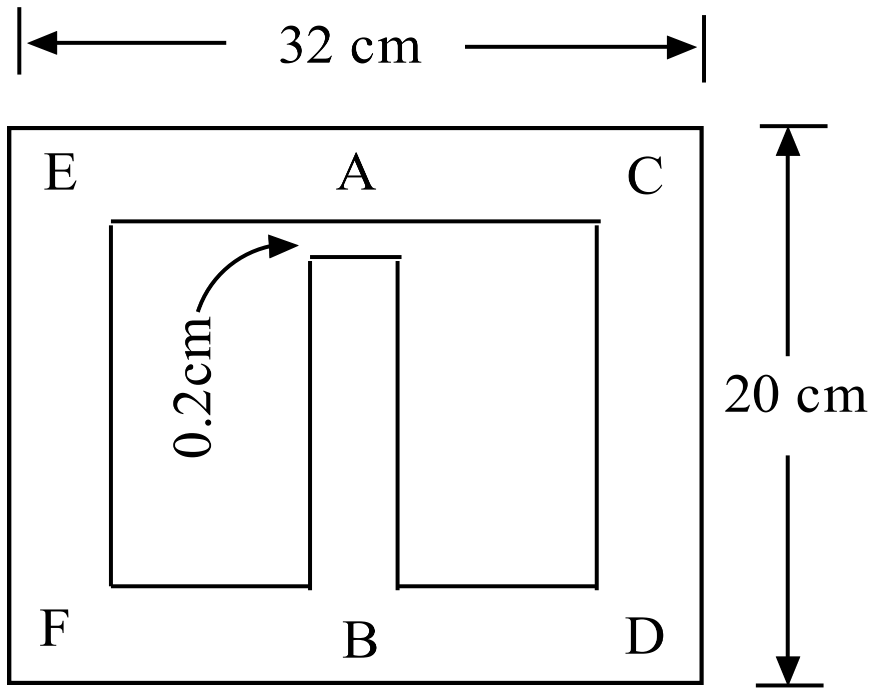


$$S_{AFEB} = \frac{(L_{AFEB} - L_{gleft})}{\mu_0 \, \mu_{rAFEB} \, A_{AFEB}} \qquad S_{AB} = \frac{(L_{AB} - L_{gcenter})}{\mu_0 \, \mu_{rAB} \, A_{AB}} \qquad S_{ACDB} = \frac{L_{ACDB}}{\mu_0 \, \mu_{rACDB} \, A_{ACDB}}$$

$$S_{g-center} = \frac{L_{gcenter}}{\mu_0 \, A_{g-center}} \qquad S_{g-left} = \frac{L_{gleft}}{\mu_0 \, A_{g-left}}$$

Illustration 6

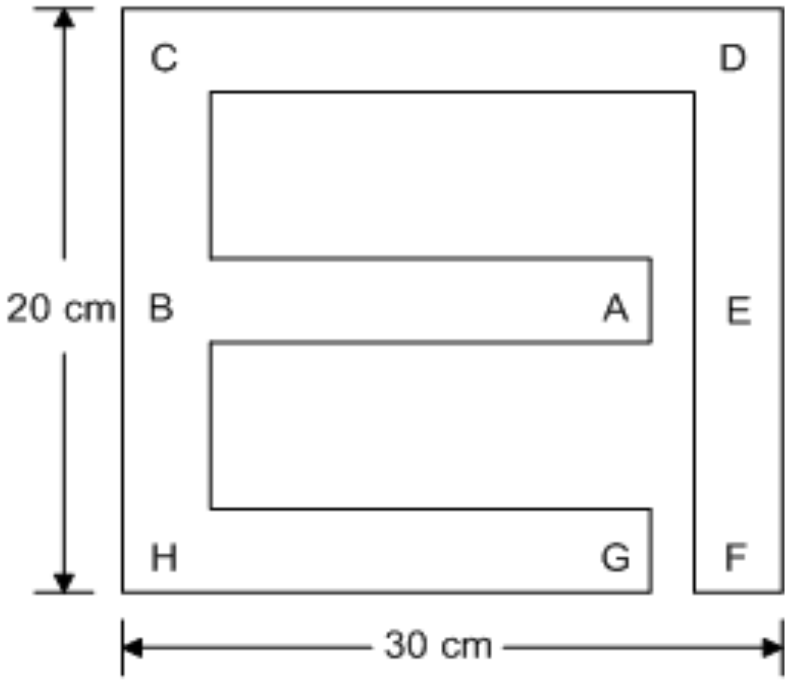
A coil carrying a current of **2.8 A** is wound on the **left limb** of the cast steel symmetrical frame of uniform square cross section of **16 cm^2** as shown. Calculate the number of turns in the coil to produce a flux of **1.8 mWb** in the air gap of **0.2 cm** length. The relative permeability of cast steel is **1200.**



Ans: 1481 Turns

Illustration 7

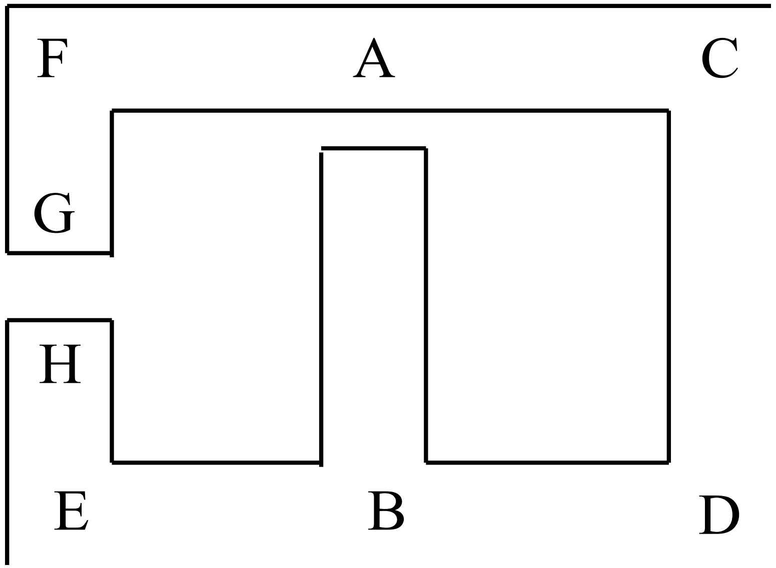
The magnetic circuit shown is made of a material having relative permeability of **2000.** The limb **AB** is wound with **1000** turns. Find the current through the coil to produce a flux of **4 mWb** in the limb AB. The length of each air gap is **2 mm** and the square cross-sectional area of the frame is **9 cm²**.



Ans: 8.2554 A

Self-Practice 3

The magnetic circuit shown is made of a material having relative permeability of **2000.** The central limb is wound with **1000 turns** and has an air gap of length **of 2 mm.** The side limb air gap is **8 mm**. Calculate the current required to set up a flux of **2.6 mWb** in the central limb. Mean lengths of various sections are as follows: **AB = 24 cm**, **ACDB = AFGHEB = 60 cm.** Cross sectional area of the structure is **10 cm^2.**

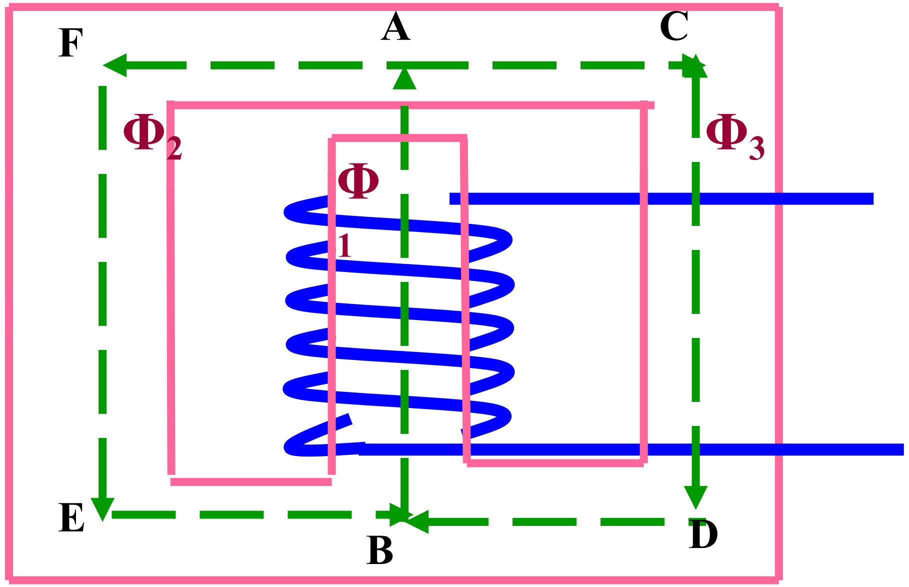


Ans: 4. 98 A

Self-Practice 4

A **710** turns coil is wound on the central limb of the cast steel symmetrical frame of uniform cross section **16 cm²** is as shown. Calculate the current required to produce a flux of **1.8 mWb** in an air gap of **0.2 cm** length. Given $\mathbf{L_{AFEB} = L_{ACDB} = 25\ cm}$, $\mathbf{L_{AB} = 12.5\ cm}$. The magnetization details is as follows.

H	300	500	600	700	900	1092
B	0.1	0.45	0.562	0.775	1	1.125



Ans: 2.92 A

Basic Electrical Technology

2. Magnetic Circuits & Electromagnetism

LECTURE **

Electromagnetic Induction and Coupled Circuits

Faraday’s Laws of Electromagnetic Induction

First Law

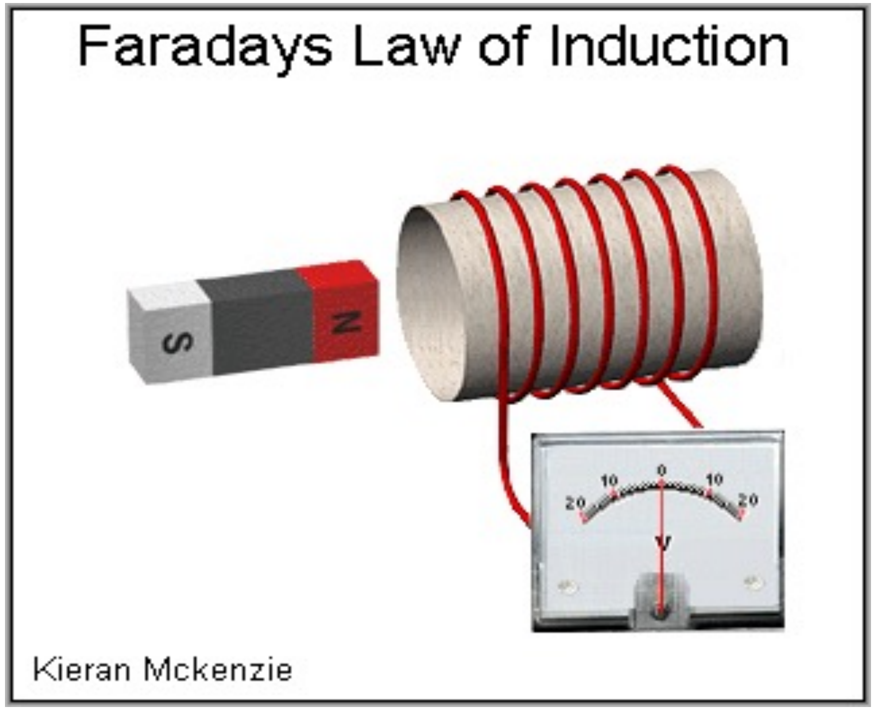
Whenever the magnetic field linking with a conductor changes, an EMF will be induced in that conductor

Second Law

The magnitude of the induced EMF is proportional to the rate of change of the magnetic flux linking the conductor

$$\mathbf{e = N\frac{d\phi}{dt}}$$

N = number of turns in the coil



Lenz's Law

The electro-magnetically induced emf always acts in such a direction to set up a current opposing the motion or change of flux responsible for inducing the emf.

$$e = -N\frac{d\phi}{dt}$$

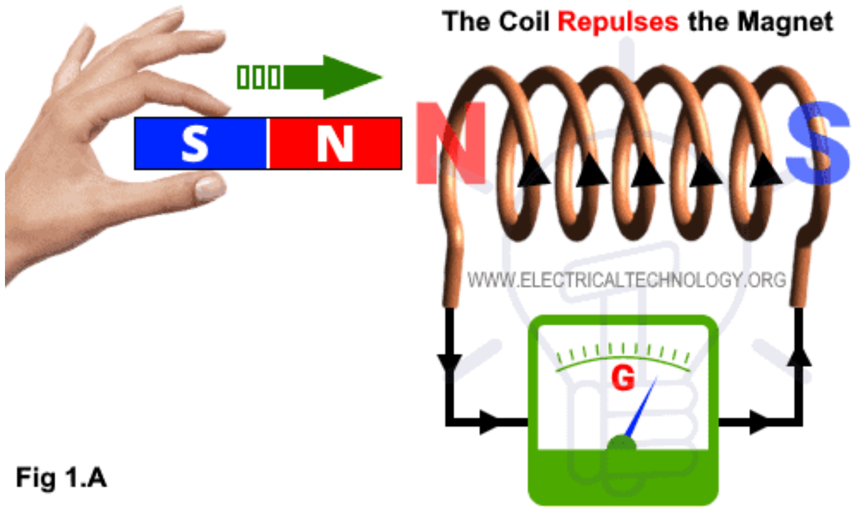


Fig 1.A

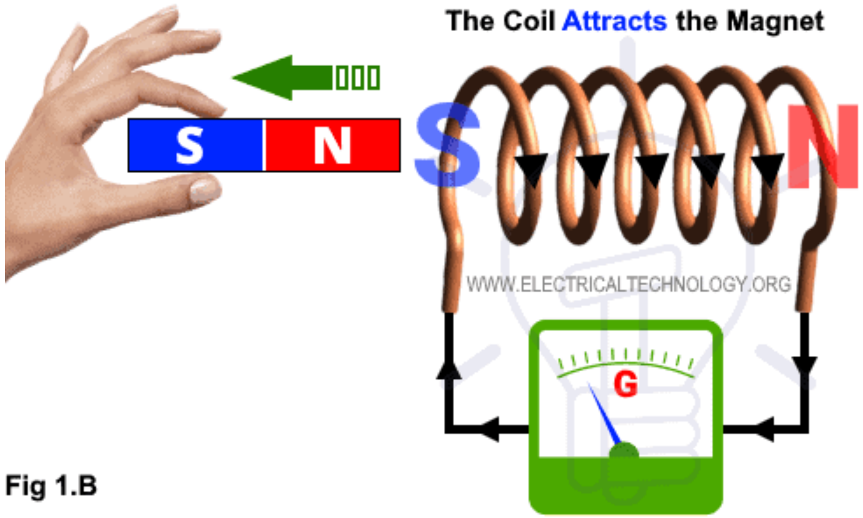
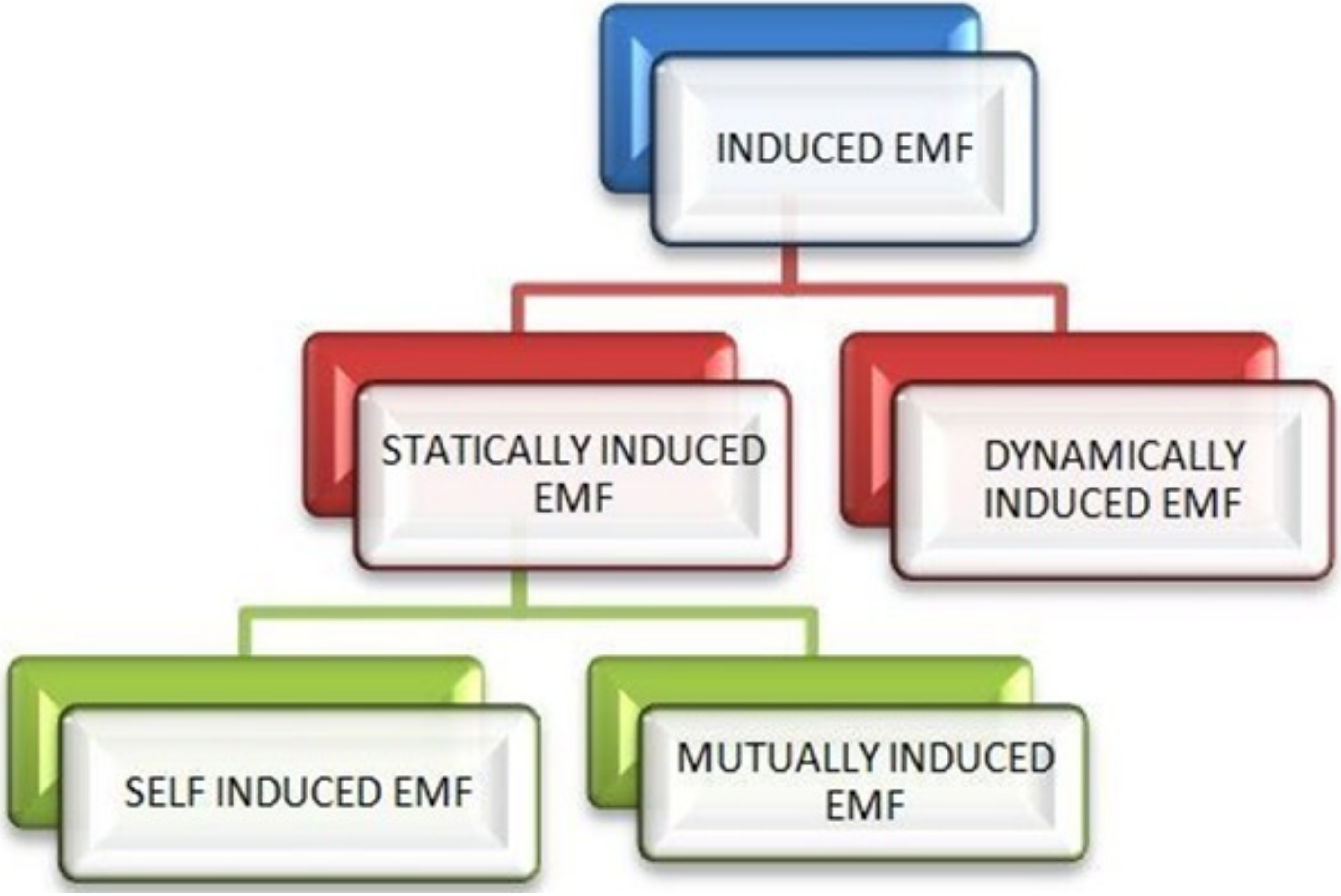


Fig 1.B

Fleming’s Right Hand Rule

If the first, second and the thumb of the right hand are held at right angles to each other,
first finger indicates the direction of the magnetic flux
and
thumb finger indicates the direction of motion of the conductor relative to the magnetic field,
then
the second finger represents the direction of induced EMF.

Types of Induced EMF



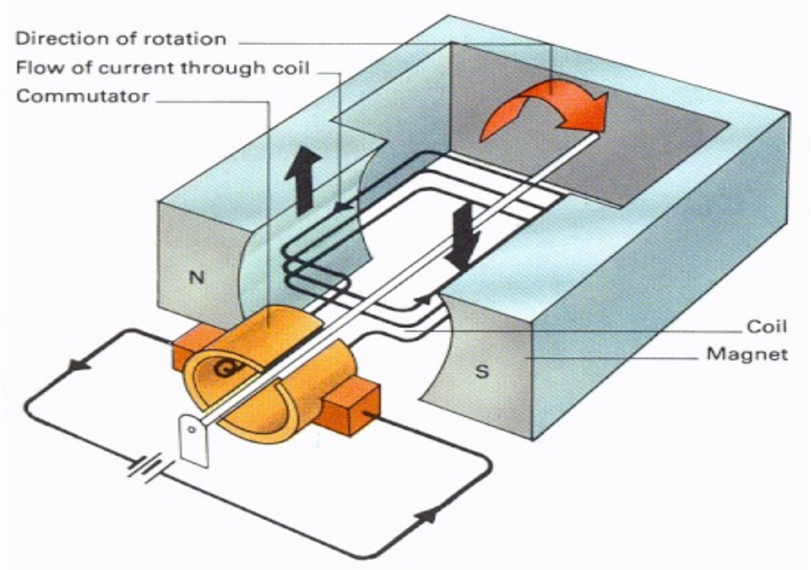
Types of Induced EMF

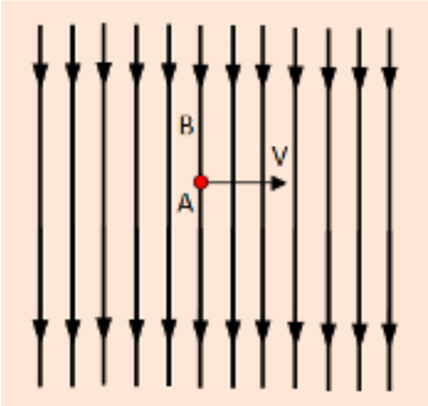
Dynamically Induced EMF

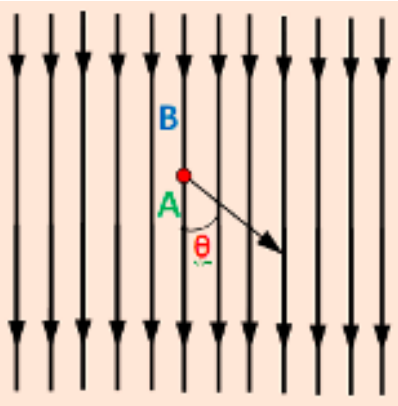
- The voltage induced in the conductor due to **relative motion of conductor and magnetic field**

$$e = B\,l\,v\,\mathrm{Sin}\theta$$

- Either conductor or magnetic field is moving
- The magnetic field system is kept stationary, and the conductor is moving, or the magnetic field system is moving, and the conductor is stationary.
- **Example: Principle of electric generator**



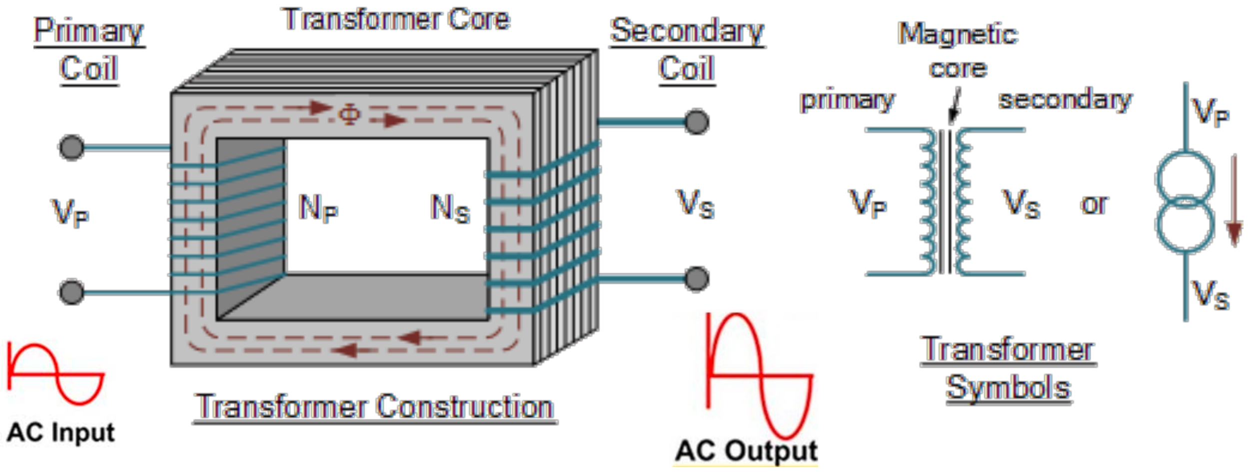




Types of Induced EMF

Statically induced EMF

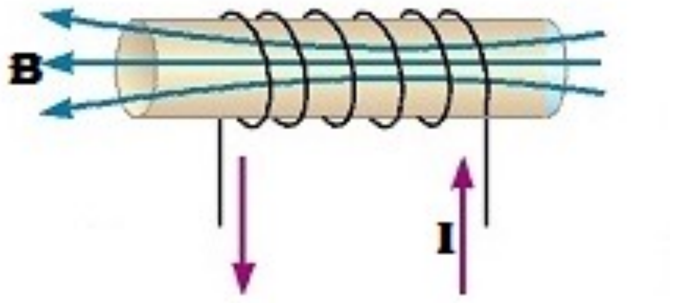
- The voltage induced in the conductor due to change in the magnetic field
- **Conductor is stationary**
- **Magnetic field is changing in a stationary magnetic system**
- The change of flux produced by the field system linking with the coil is obtained by changing the electric current in the field system.
- **Example: Transformer**



Types of Statically Induced EMF

Self Induced EMF

The induced emf in a coil proportional to the rate of the change of the magnetic flux passing through it due to its own current.



$$\mathbf{e = -L\frac{di}{dt}}$$

Self Inductance (L)

The proportionality constant is called the **Self Inductance, L**. Unit is **Henry (H)**

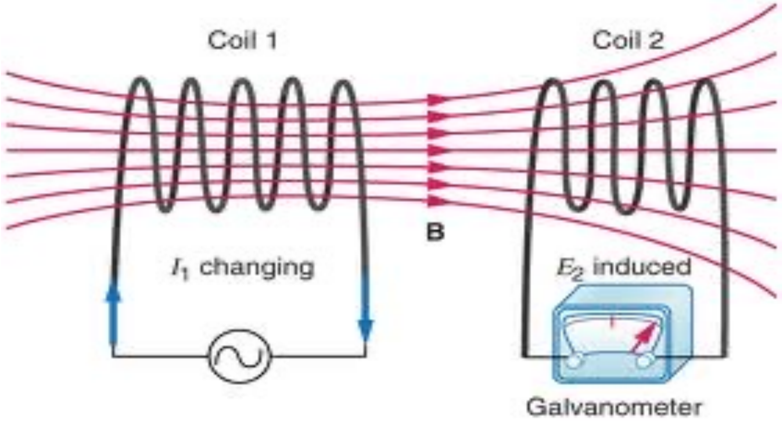
$$\mathbf{e = -N\frac{d\phi}{dt} = -L\frac{di}{dt}} \qquad \mathbf{where, L = N\frac{d\phi}{di}}$$

(i) $e = L\frac{di}{dt}$ (ii) $N\,\phi = L\,I$ (iii) $L = \frac{N^2}{\text{Reluctance}} = \frac{N^2 \mu_0\, \mu_r\, A}{l_{length}}$

Types of Statically Induced EMF

Mutually Induced EMF

The induced emf in a coil due to the change of flux produced by the change of current in the nearby coil



Mutual Inductance (M)

This proportionality constant is called the **Mutual Inductance, M.** Unit is **Henry (H)**

If coil 1 is excited: Mutually induced emf e_2 in Coil 2,

$$\mathbf{e_2 = N_2 \frac{d\phi_{12}}{dt} = N_2 \frac{d\phi_{12}}{di_1} \times \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}}\text{, } \textbf{Mutual Inductance, } \mathbf{M_{21} = N_2 \frac{d\phi_{12}}{di_1}}$$

If coil 2 is excited: Mutually induced emf e_1 in Coil 1,

$$\mathbf{e_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 \frac{d\phi_{21}}{di_2} \times \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}}\text{, } \textbf{Mutual Inductance, } \mathbf{M_{12} = N_1 \frac{d\phi_{21}}{di_2}}$$

Coupling Coefficient (k)

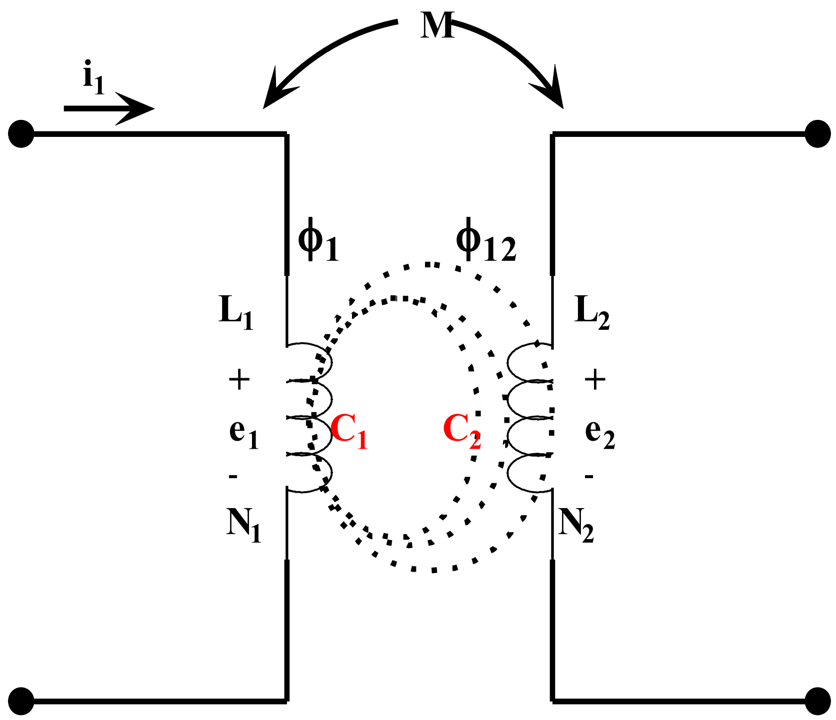
Gives an idea about the **degree of magnetic coupling** between two coils.

$$M_{21} = N_2 \frac{d\phi_{12}}{di_1} = M_{12} = N_1 \frac{d\phi_{21}}{di_2} = M$$

where, $\phi_{12} = k\,\phi_1; \quad \phi_{21} = k\,\phi_2$

$$M^2 = \left(N_2 k \frac{d\phi_1}{di_1}\right)\left(N_1 k \frac{d\phi_2}{di_2}\right) = k^2 L_1 L_2$$

$$\mathbf{k = \frac{M}{\sqrt{L_1 L_2}}}$$



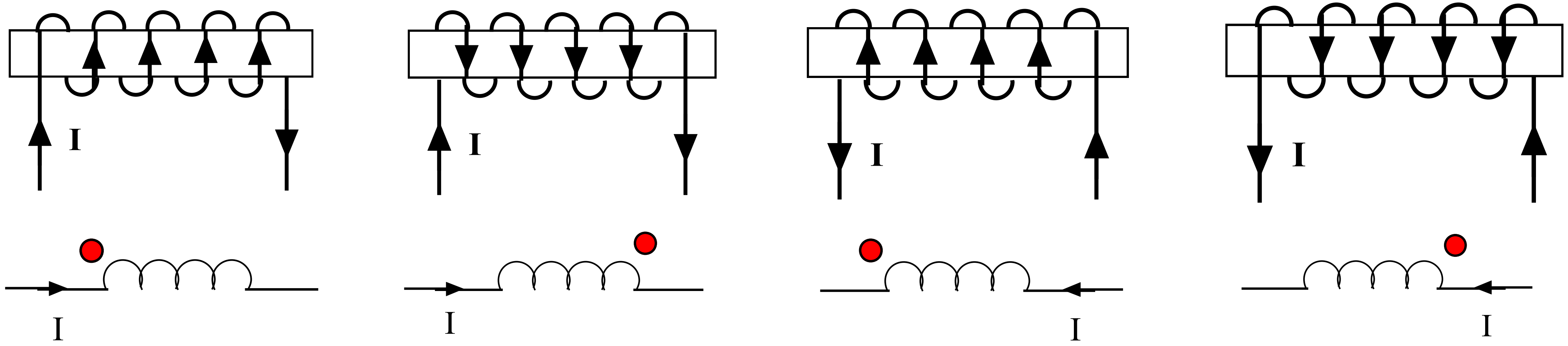
Coupled Circuits

Polarity of mutually induced EMF depends on

- current direction
- physical construction of the coils

Obtaining the dotted equivalent: Right Hand Grip Rule

Place the dot at the terminal directed by the thumb



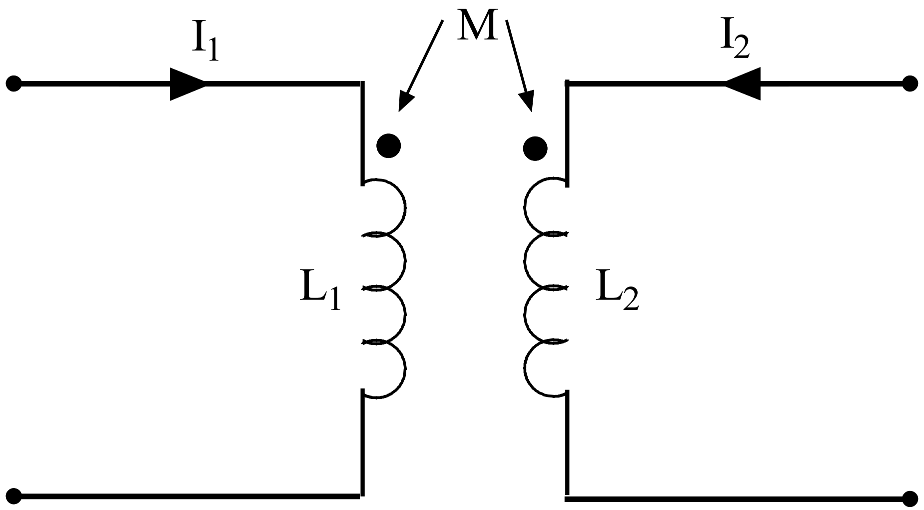
Dot Rule for coupled coils

Dot Rule helps in determining the sign of mutually induced emf without going into the details of physical construction

Dot Rule

- ✓ If **currents enters (or leaves) the dotted terminals in both the coils,** the sign of mutually induced EMF is same as that of sign of self-induced EMF. **(Additive Coupling)**
- ✓ If the **current enters the dotted terminal in one coil and leaves the dotted terminal in the other coil,** the sign of mutually induced emf is opposite to that of sign of self induced emf. **(Subtractive Coupling)**

Additive Coupling: (Fluxes are aiding)

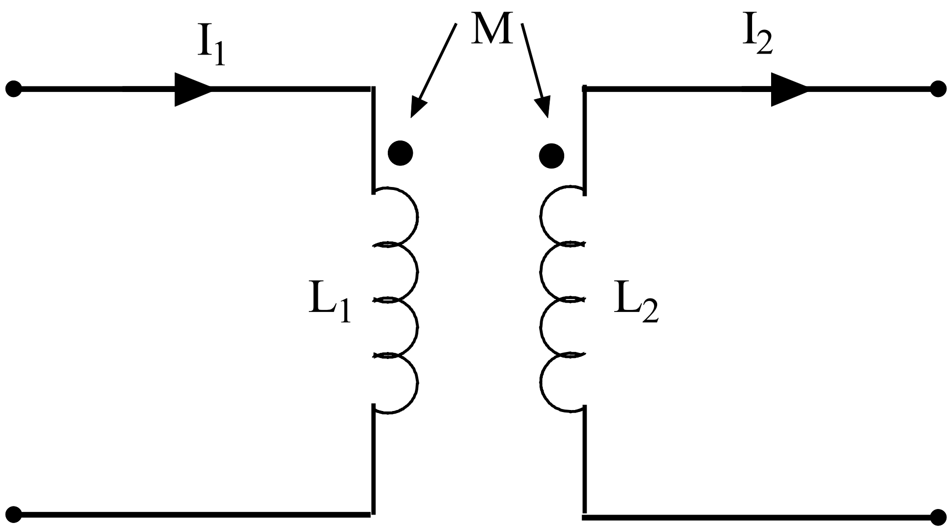


Self induced emf in L_1 = $$L_1 \frac{di_1}{dt}$$

Mutually induced emf in L_1 = $$M \frac{di_2}{dt}$$

Total induced emf in L_1 = $$L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

Subtractive Coupling: (Fluxes are opposing)

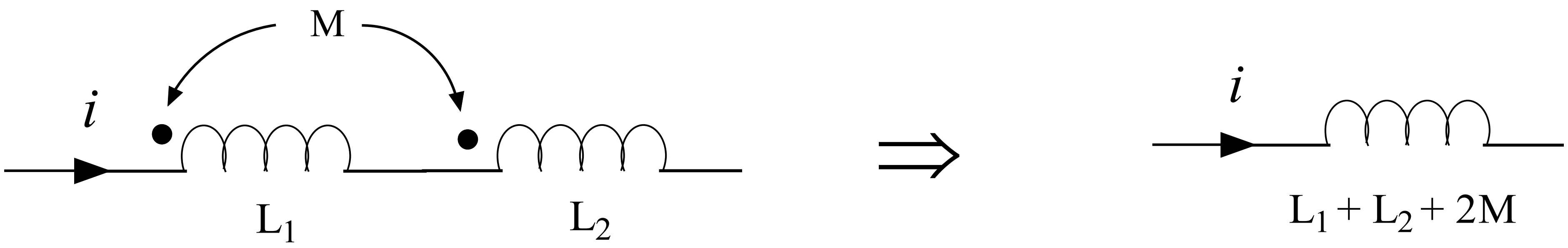


Self induced emf in L_1 = $$L_1 \frac{di_1}{dt}$$

Mutually induced emf in L_1 = $$M \frac{di_2}{dt}$$

Total induced emf in L_1 = $$L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

Coupled Coils in Series - Aiding



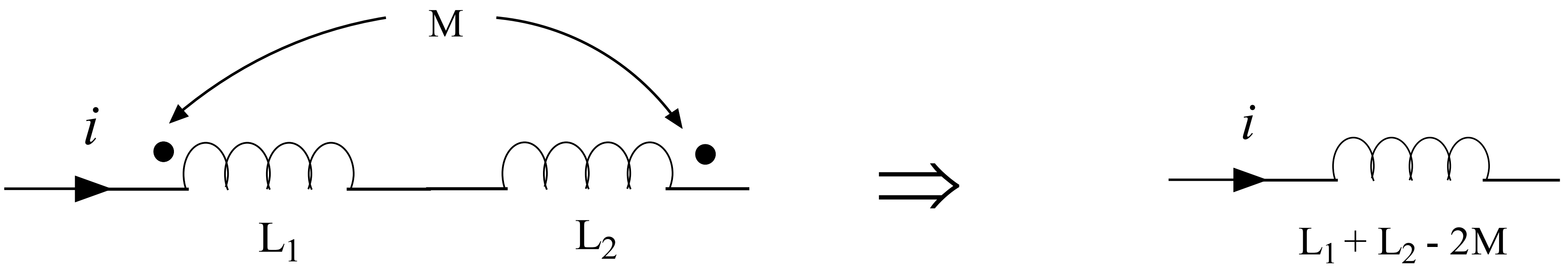
$$\mathbf{e_1 = L_1 \frac{di}{dt} + M \frac{di}{dt}}$$

$$\mathbf{e_2 = L_2 \frac{di}{dt} + M \frac{di}{dt}}$$

$$\mathbf{e = L_{eq} \frac{di}{dt} = e_1 + e_2 = (L_1 + L_2 + 2M) \frac{di}{dt}}$$

$$\mathbf{L_{eq} = L_1 + L_2 + 2M}$$

Coupled Coils in Series - Opposing



$$\mathbf{e_1 = L_1 \frac{di}{dt} - M \frac{di}{dt}}$$

$$\mathbf{e_2 = L_2 \frac{di}{dt} - M \frac{di}{dt}}$$

$$\mathbf{e = L_{eq} \frac{di}{dt} = e_1 + e_2 = (L_1 + L_2 - 2M) \frac{di}{dt}}$$

$$\mathbf{L_{eq} = L_1 + L_2 - 2M}$$

Inductances in Series:

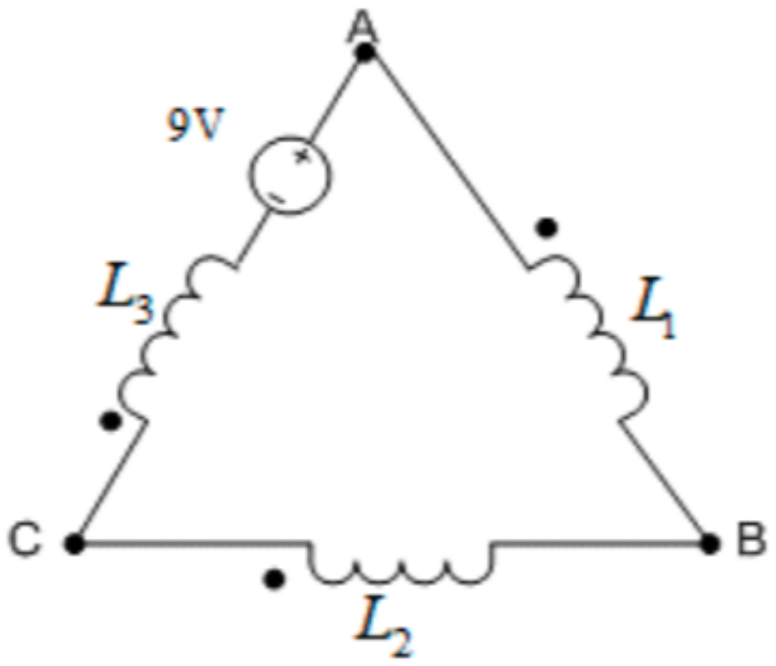
$$L_{eq} = L_1 + L_2 + \cdots + L_n$$

Inductances in Parallel:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_3}$$

Illustration 8

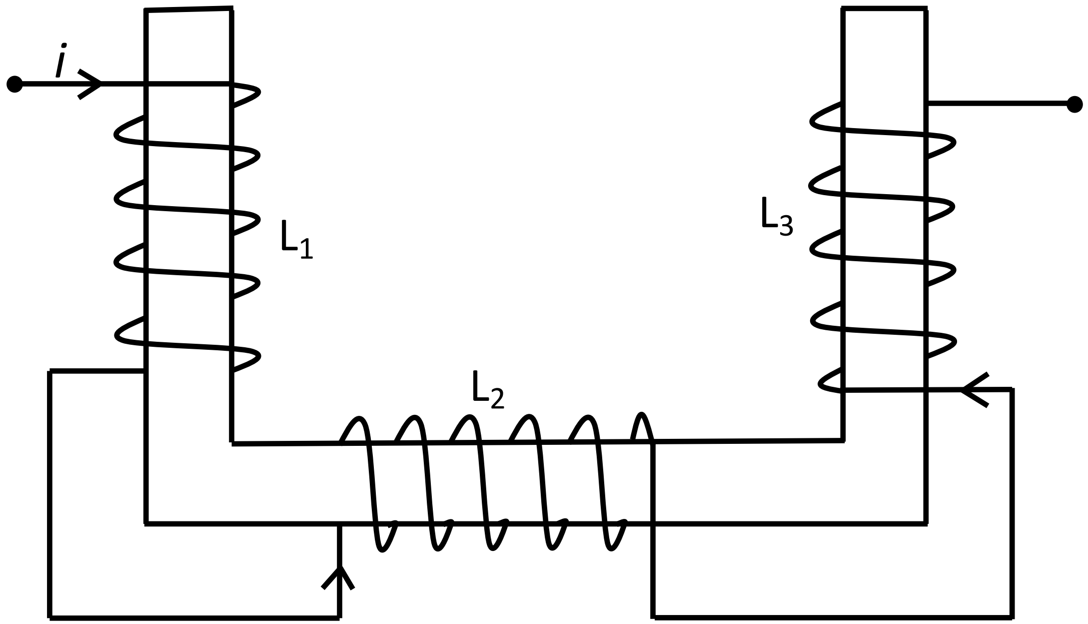
Three magnetically coupled inductive coils having the following data are connected as shown. **L_1 = 0.1 H; L_2 = 0.2 H; L_3 = 0.4 H; k_{12} = 0.4; k_{23} = 0.5; k_{31} = 0.5.** Find the equivalent inductance of the circuit.



Ans: 0.5041 H

Illustration 9

Three magnetically coupled inductive coils having the following data are connected in series as shown in Figure. **L_1 = 0.3 H; L_2 = 0.6 H; L_3 = 0.8 H** and the coefficients of coupling are, **k_{12} = 0.8; k_{23} = 0.75; k_{31} = 0.5.** Draw the dotted equivalent circuit of the figure, also find the equivalent inductance of the circuit.



Ans: 0.472 H

Illustration 10

Two coils with inductances in the ratio **4:1** have a coupling coefficient **k = 0.6.** When these coils are connected in series aiding the equivalent inductance is **44.4 mH.** Find **L_1, L_2**, and **M.**

Ans: L_1 = 6 mH, L_2 = 24 mH, and M = 7.2 mH

Illustration 11

When two identical coils are connected in series, the inductance of the combination is found to be **80 mH.** When the connections to one of the coils are reversed, a similar measurement indicates **20 mH.** Find the mutual inductance and coupling co-efficient between the coil.

Ans: 0.015 H and 0.6

Self-Practice 1

Two similar coils have a coupling coefficient of **0.4.** When they are connected in series aiding, the equivalent inductance is **560 mH.** Calculate:

a) Self-inductance of both the coils.

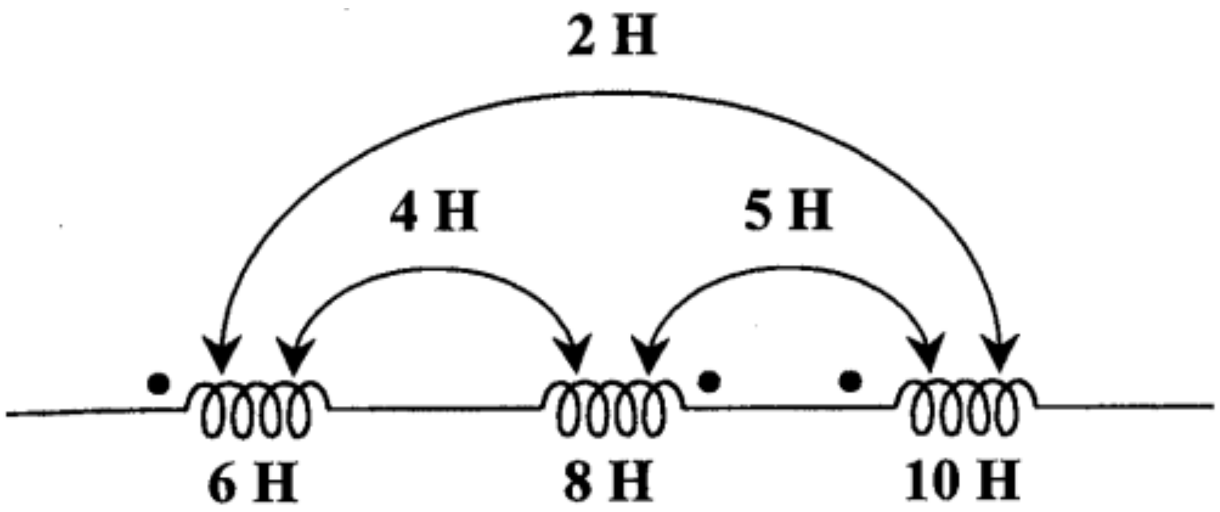
b) Total inductance when the coils are connected in series opposition.

c) Total energy stored due to a current of **3A** when the coils are connected in series opposition.

Ans: a) 0.2 H, b) 0.24 H, and c) 1.08 J

Self-Practice 2

For the three coupled coils, calculate the total inductance.



Ans: 10 H