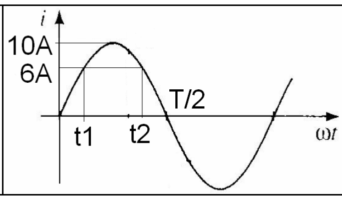
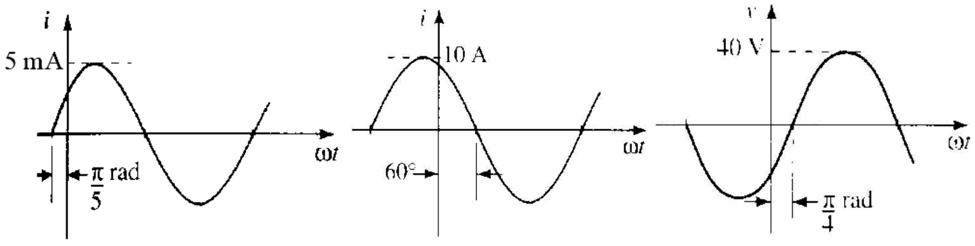
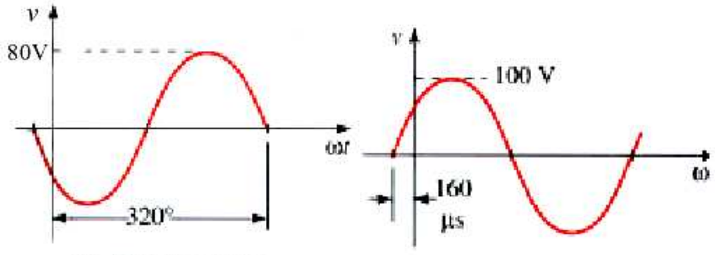
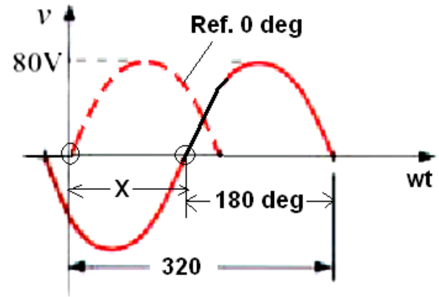
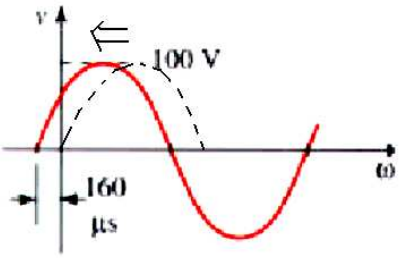
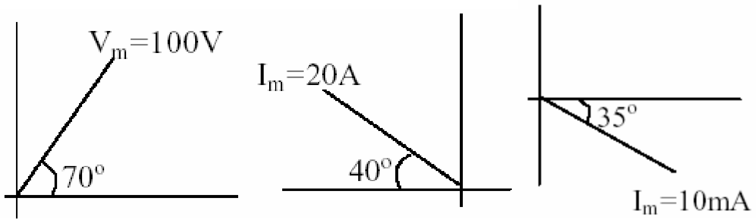
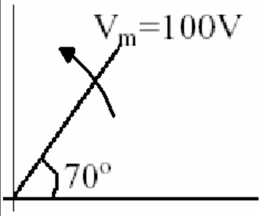
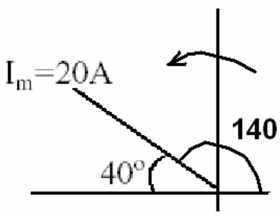
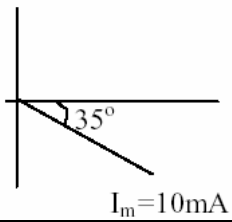
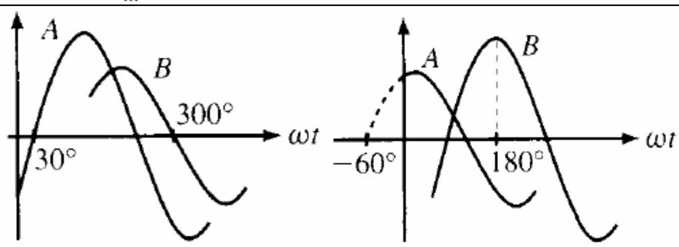
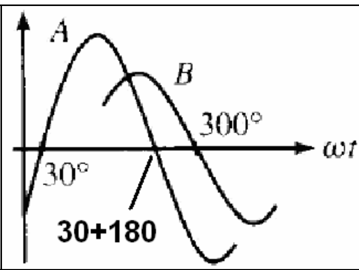
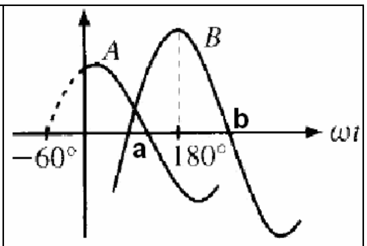
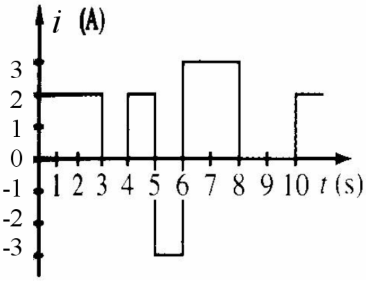
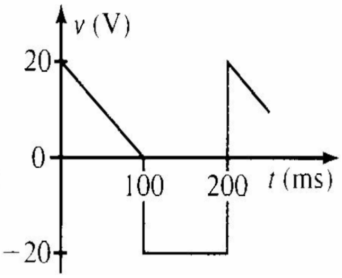
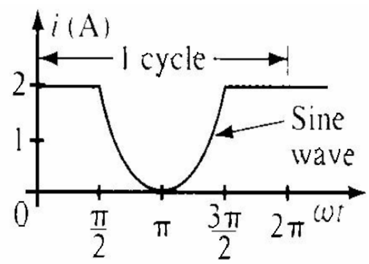
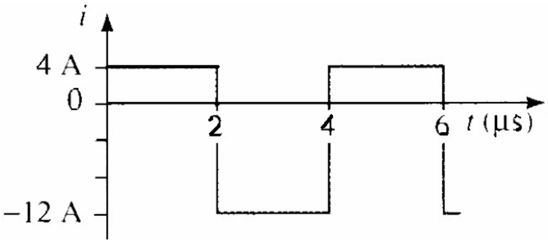
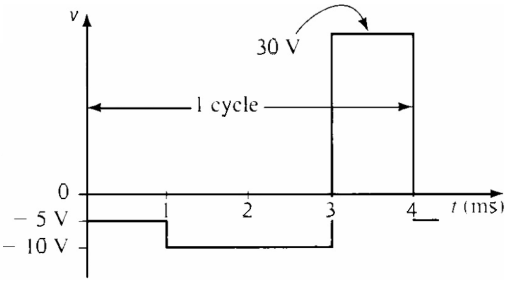
TUTORIAL 1 – AC FUNDAMENTALS

1.	An alternating current i is represented by: i = 10sin942t ampere. Determine: (a) the frequency; (b) the period; (c) the time taken from t=0 for the current to reach 6 A for a first and second time; [150 Hz; 6.67 ms; 0.68 ms; 2.66 ms]
1 soln	(a) $942 = \omega = 2\pi f \Rightarrow f = 942/2\pi = 150$ Hz (b) $T = 1/f = 1/150 = 6.67$ms (c) $6 = 10 \sin 942\, t_1 \Rightarrow [\sin^{-1} 0.6]/942 = t_1$ $t_1 = 0.68$ms $t_2 = T/2 - t_1 =$ 6.67/2 – 0.68=2.66ms 
2.	2. Determine equations for sine waves with the following: (a) V_m = 170 V, f = 60 Hz (b) I_m = 40 μA, T = 10 ms (c) T = 120 μs, v = 10 V at t = 12 μs [v = 170sin377t V; i= 40sin628t μA; v = 17sin(52.4x10^3t) V]
2 soln	(a) In general $v(t) = V_m \sin \omega t$, $\omega = 2\pi f = 2\pi\, 60 = 377$ $v(t) = 170 \sin 377\, t$ [V] (b) $i(t) = I_m \sin \omega t$, $\omega = 2\pi f = 2\pi/T = 2\pi / 10\text{x}10^{-3} = 628$ $i(t) = 40 \sin 628\, t$ [μA] (c) $\omega = 2\pi f = 2\pi/T = 2\pi/ 120\text{x}10^{-6} = 52.4 \text{ x } 10^3$ $v(t) = V_m \sin \omega t \Rightarrow 10 = V_m \sin 52.4 \text{ x } 10^3 \text{x}12\text{x}10^{-6}$ $\Rightarrow 10 = V_m \sin 0.6288 \Rightarrow V_m = 10 / 0.588 = 17$ $\Rightarrow v(t) = 17 \sin 52.4 \text{ x } 10^3\, t$ [V]
3	Determine f, T and amplitude for each of the following: (a) v = 75sin200πt (b) i = 8sin300t [100 Hz, 0.01 s, 75 V; 47.7 Hz, 20.9 ms, 8 A]
3 soln	(a) Amplitude = 75, 2πf = 200π⇒f= 100Hz⇒T = 1/f = 0.01s (b) Amplitude = 8A, 2πf = 300⇒f=47.7Hz⇒T = 1/f =20.9ms
4.	Given $v = 5\sin(\omega t + 45^o)$. If ω = 20π rad/s, what is v at t = 20, 75 and 90 ms? [4.46 V; -3.54 V; 0.782 V]
4 soln	45=π/4 $v(t) = 5\sin(20\pi t + \pi/4)$ At t=20ms $v(20ms) = 5\sin(20\pi 20\text{x}10^{-3} + \pi/4) = 4.46$ [V] At t=75ms $v(75ms) = 5\sin(20\pi 75\text{x}10^{-3} + \pi/4) = -3.54$ [V] At t=90ms $v(90ms) = 5\sin(20\pi 90\text{x}10^{-3} + \pi/4) = 0.782$ [V]
5.	Write equations for the following waveforms. Express the phase angle in degrees.  (a) ω = 1000 rad/s (b) T = 50 ms (c) f = 900 Hz [5sin(1000t + 36^o) mA; 10sin(40πt + 120^o) A; 4sin(1800πt - 45^o) V]
5 soln	(a) $i(t) = I_m \sin(\omega t + \phi) \Rightarrow$The waveform is shift to the left by π/5 Maximum = 5mA ⇒I_m =5, π/5 [rad] =π/5 /π x 180 = 36 [deg] $i(t) = 5 \sin(1000t + 36^o)$ (b) $i(t) = I_m \sin(\omega t + \phi) \Rightarrow$The waveform is shift to the left by 180-60 ⇒ϕ = 120 [deg], $\omega = 2\pi f = 2\pi/T = 2\pi/ 50\text{x}10^{-3} = 40\pi$ Maximum = 10A ⇒$I_m \Rightarrow i(t) = 10 \sin(40\pi\, t + 120^o)$ (c) $v(t) = V_m \sin(\omega t + \phi) \Rightarrow$The waveform is shift to the right by π/4 ⇒π/4 = π/4 /π x180 = 45 [deg], ω = 2πf = 2π900 = 1800π Maximum = 40V ⇒$V_m = 40 \Rightarrow v(t) = 40 \sin(1800\pi\, t - 45^o)$
6.	Write equations for the following waveforms. Express the phase angle in degrees. [80sin(2x10^5πt – 140^o) V; 100sin(5236t + 48^o) V]

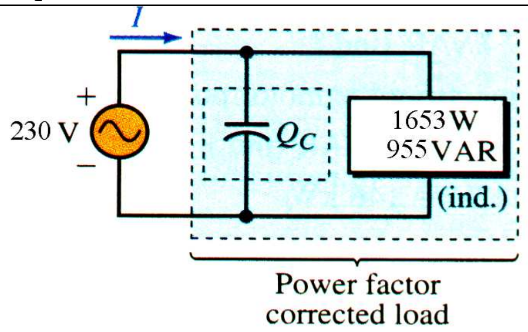
	 (a) Period = 10 µs (b) f = 833.3 Hz	
6(a) soln	(a) Since the curve shift to right hand side by x=320 – 180 = 140 Phase lag 140 deg, T = 10 x 10^{-6} , $\omega = 2\pi f = 2\pi/T = 2\pi / 10\text{x}10^{-6}$ $\omega = 2\pi \text{ x}10^{-5}$ $\Rightarrow v(t) = 80 \sin (2\pi \text{ x}10^{-5}t – 140^o)$	
6(b) soln	(b) The curve is shift to the left by 160µs f = 833.3 Hz, $\omega = 2\pi f = 5236$, T =1/f=1200 µs Phase lead = 160µs / 1200 µs x360=48 deg $\Rightarrow v(t) = 100 \sin (5236t +48^o)$	
7.	Determine the equation for *v(t)* or *i(t)* for each of the following phasor.  [100sin(ωt + 70°) V; 20sin(ωt + 140°) mA; 10 sin(ωt – 35°) mA]	
7 soln	Rotating vector rotates in anti-clockwise direction, all phase angle measure with respect to the horizontal x-axis At time t=0, initial phase angle = 70 Magnitude is 100V, $\Rightarrow v(t) = 100 \sin(\omega t + 70^o)$	
7 soln	At time t = 0, initial phase angle=180-40=140 Magnitude = 20A $20\sin(\omega t + 140^o)$ A  	At time t = 0, initial phase angle=-35 Magnitude = 10mA $10 \sin(\omega t – 35^o)$ mA 
8.	For the following waveforms, determine the phase differences. Which waveform leads?	 (a) (b)

<table>
<tr><td>8
soln</td><td>(a)
B is lagging with respect to A
300 – (30+180) = 90</td><td></td><td>(b) B is lagging with respect to A
Point a
= 180 - 60=120
Point b
= 180 + 90=270
270 – 120 = 150</td><td></td></tr>
<tr><td>9.</td><td colspan="4">Compute the average values of the following waveforms. [1.1 A; -5 V; 1.36 A]
</td></tr>
<tr><td>9</td><td colspan="2">**(a) Total Area Under the Curve = A**
A=(2x3)+(1x2)-(1x3)+(2x3)=11
Average Voltage = A / 10 = 11/10=1.1
(b) Period = T = 200
Total Area under the curve from 0 to 200
A=(100x20/2)-(100x20)=-1000
Average Voltage=-1000/200=-5V</td><td colspan="2">**(c) Period=T=2π**
A=Area under the curve=2x2π - A_1
A_1=Area under a sine wave for half cycle
Average Value of a half cycle sine wave is 0.637 x Max Value
A_1=2x0.637xπ=1.274π
A=4π-1.274π=2.726π
Average = 2.726π/(2π)=1.363</td></tr>
<tr><td>10.</td><td colspan="4">Determine the rms values of each for the following.
(a) A 12V battery   (b) -24sin(ωt + 73°) mA
(c) 10 + 24sinωt V   (d) 45 – 27cos2ωt V
[12 V; 17 mA; 19.7 V; 48.9 V]</td></tr>
<tr><td>10</td><td colspan="4">**(a) DC voltage 12V, RMS = 12**
(b) AC Amplitude = -24⇒ RMS = 24/√2=16.9mA
(c) DC component = 10, AC Amplitude = 24
Average Square = $10^2+24^2/2$= 388 ⇒ RMS=√388=19.7V
(d) DC component = 45, AC Amplitude=27
Average Square = $45^2+27^2/2$= 2389.5 ⇒ RMS=√2389.5=48.9V</td></tr>
<tr><td>11.</td><td colspan="4">Compute the rms values for the following waveforms.

[8.94 A; 16.8 A]</td></tr>
<tr><td>**11**
Soln</td><td colspan="4">**(a) A=Total Area Under the $(Current)^2$ Curve**
A=$(4)^2 2+(-12)^2 2$=320
Mean Square= 320/4=80
Root Mean Square = √80=8.94
(b) A = Total Area Under the V^2 Curve
A=$(-5)^2 1+(-10)^2 2+(30)^2 1$=1125
Mean Square=1125/4=281.25
Root Mean Square=√281.25=16.8</td></tr>
</table>

TUTORIAL 2 – AC SINGLE PHASE

1.	Determine the resistance and the series connected inductance or capacitance for each of the following impedances (a) 12 + j5, (b) – j40 and (c) 30∠60°. Assume a frequency of 50 Hz.
1. Soln	(a) $Z = R + jX_L$ Hence, $12 + j5$ shows that **the resistance is 12 ohms** and the inductive reactance is 5 ohms. $X_L = 2\pi f L$, $L = \dfrac{X_L}{2\pi f} = \dfrac{5}{2\pi(50)} = 0.016\ H$ i.e. **the inductance is 16 mH**. **$X_L = \omega L = 2\pi f L$** (b) $Z = R - jXc$ Hence, $0 - j40$ shows that **the resistance is zero** and the capacitive reactance is 40 ohms. $X_C = \dfrac{1}{2\pi f C}$ $2\pi fL$, $C = \dfrac{1}{2\pi f X_C} = \dfrac{10^6}{2\pi(50)(40)}\ \mu F = 79.6\ \mu F$ i.e. **the capacitance is 79.6 μF.** **$X_C = 1/[\omega C] = 1/[2\pi f C]$** (c) $30\angle 60^o = 30\,(\cos 60^o + j \sin 60^o) = 15 + j25.98$. From equation (4), $15 + j25.98$ shows that **the resistance is 15 ohms** and the inductive reactance is 25.98 ohms. $X_L = 2\pi f L$, $L = \dfrac{X_L}{2\pi f} = \dfrac{25.98}{2\pi(50)} = 0.0827\ H$ i.e. **the inductance is 82.7 mH** **Z = R + Xj if X > 0 ⇒ Inductive ⇒ X = 2πf L** **If X < 0 ⇒ Capacitive ⇒ $X_C = 1/[2\pi f C]$**
2.	The impedance of an electrical circuit is 30 – j50 ohms. Determine (a) the resistance, (b) the capacitance, (c) the modulus of the impedance and (d) the current flowing, when the circuit is connected to a 240 V, 50 Hz supply.
2. Soln	**Modulus (Magnitude) = |Z|** **$Z = \|Z\| \angle \tan^{-1}(X/R)$** (a) Since $Z = R - jXc$, **the resistance is 30 ohms.** (b) Since $Z = R - jXc$, the capacitive reactance is 50 ohms. $X_C = \dfrac{1}{2\pi f C}$, $C = \dfrac{1}{2\pi f X_C} = \dfrac{10^6}{2\pi(50)(50)}\ \mu F = 63.7\ \mu F$ i.e. **the capacitance is 63.7 μF.** (c) The modulus of the impedance $\|Z\| = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + (-50)^2} = \underline{\mathbf{58.31\ ohms}}$ (d) The circuit phase angle $\phi = \arctan\left(\dfrac{X_C}{R}\right) = \arctan\left(\dfrac{50}{30}\right) = 59^o\ 2'$ Since $Z = R - jXc$, this angle is in the fourth quadrant, i.e. $-59^o\ 2'$. Thus an alternative way of expressing the impedance is $Z = 58.31 \angle -59^o\ 2'$ The current flowing, $I = \dfrac{V}{Z} = \dfrac{240\angle 0^o}{58.31\angle -59^o 2'} = \underline{\mathbf{4.116\angle 59^o\ 2'}}$ *(Since the voltage is 240 volts, it is 240 + j0 volts in rectangular form, and 240∠0° in polar form.)*

3.	A series connected electrical circuit has a resistance of 32 ohms and an inductance of 0.15H. It is connected to a 200V, 50Hz supply. Determine (a) the inductive reactance, (b) the impedance in rectangular and polar forms, (c) the current and the circuit phase angle, (d) the voltage drop across the resistor and (e) the voltage drop across the inductor.
3. Soln	(a) Inductive resistance $X_L = 2\pi fL = 2\pi(50)(0.15) =$ **<u>47.1 ohms</u>** (b) Impedance $Z = R + jX_L$ i.e. $Z = 32 + j47.1$ Thus, $\lvert Z \rvert = \sqrt{32^2 + 47.1^2} = 57$ ohms and the circuit phase angle $\phi = \arctan\left(\frac{47.1}{32}\right) = 55^o\ 48'$ Thus **$Z = 57\angle 55^o\ 48'$ ohms, in polar form.** (c) Current $I = \frac{V}{Z} = \frac{200\angle 0^o}{57\angle 55^o 48'} = \underline{\mathbf{3.51\angle -55^o\ 48'}}$ i.e. **the current is 3.51 A, lagging V by $55^o\ 48'$** (d) The voltage drop across the 32 ohm resistor, $V_R = IR = (3.51\angle -55^o\ 48')(32) = \underline{\mathbf{112.3\angle -55^o\ 48'}}$ **<u>volts</u>** (e) The voltage drop across the 0.15 H inductor, $V_L = IX_L = (3.51\angle -55^o\ 48')(47.1\angle 90^o) = \underline{\mathbf{165.3\angle 34^o\ 12'}}$ **<u>volts</u>**
4.	A 240 V, 50 Hz voltage is applied across a series connected circuit having a resistance of 12 ohm, an inductance of 0.10 H and a capacitance of 120 μ F. Determine the current flowing in the circuit.
4. Soln	 R + jX_L -j X_C R + j (2π f L) -j /(2π f C) Inductive resistance $X_L = 2\pi fL = 2\pi(50)(0.10) = 31.4\ \Omega$ Capacitive reactance $X_C = \frac{1}{2\pi fC} = \frac{10^6}{2\pi(50)(120)} = 26.5\ \Omega$ Impedance $Z = R + j(X_L - X_C) = 12 + j\ (31.4 - 26.5) = 12 + j\ 4.9$ In polar form, $Z = \sqrt{12^2 + 4.9^2}\arctan\left(\frac{4.9}{12}\right) = 13\ \angle 22^o\ 13'$ ohms Hence, current $I = \frac{V}{Z} = \frac{240\angle 0^o}{13\angle 22^o 13'} = 18.5\ \angle -22^o\ 13'$ amperes i.e. **the current flowing is 18.5 amperes, lagging the voltage by $22^o\ 13'$.**
5.	Determine the resistance R and series inductance L (or capacitance C) for each of the following impedances, assuming the frequency to be 50 Hz. (a) 4 + j7 (b) 3 – j2 (c) j10 (d) – j200 (e) 15 $\angle\pi/3$ (f) 6 $\angle -45^o$ [(a) R = 4 ., L = 22.3 mH (b) R = 3 ., C = 1592 μF (c) R = 0, L = 31.8 mH (d) R = 0, C = 15.92 μF (e) R = 7.5 ., L = 41.3 mH (f) 4.243Ω, C = 750.3μF]
5 Soln	(a) $4 + j7 = R + jX \Rightarrow R = 4, X > 0 \Rightarrow$ Inductive , $X = L\omega = L\ 2\pi f = L\ 100\pi$ $\Rightarrow L = 7 / (100\pi) = 0.0223 = 22.3$mH (b) $3 - j2 = R + jX \Rightarrow R = 3, X < 0 \Rightarrow$ Capacitive, $X = 1/(C\omega) = 1/(C\ 2\pi f) = 1/(C100\pi)$ $\Rightarrow C = 1 / (2x100\pi) = 0.00159F = 1592\mu F$ (c) $10j = R + jX \Rightarrow R = 0, X > 0 \Rightarrow$ Inductive , $X = L\omega = L\ 2\pi f = L\ 100\pi$ $\Rightarrow L = 10 / (100\pi) = 31.8$mH (d) -j200, $X < 0 \Rightarrow$ Capacitive, $X = 1/(C\omega) = 1/(C\ 2\pi f) = 1/(C100\pi)$

	$\Rightarrow C = 1 / (200x100\pi)=15.92\mu F$ (e) $15\angle\pi/3 = 15\cos(\pi/3)+j\ 15\sin(\pi/3)=7.5+13j \Rightarrow R = 7.5$ $X = L\omega = L\ 2\pi f = L\ 100\pi \Rightarrow L = 13 / (100\pi)=41.3mH$ (f) $6\angle -45^o = 6\cos(-45)+j\ 6\sin(-45)= 4.24-4.24j$ $R=4.24\Omega$, $X = 1/(C\omega) = 1/(C\ 2\pi f) = 1/(C100\pi) \Rightarrow C = 1 / (4.24x100\pi)=750.3\mu F$
6.	An alternating voltage of 100 V, 50 Hz is applied across an impedance of (20 – j30)ohms. Calculate: (a) the resistance, (b) the capacitance, (c) the current, and (d) the phase angle between current and voltage. [(a) 20 . (b) 106.1 μF (c) 2.774 A (d) 56^o 19']
7.	Two voltages are represented by *(15 + j10)* and *(12 – j4)* volts. Determine the magnitude of the resultant voltage when these voltages are added. [27.66 V]
8.	Two impedances, $Z_1=(2 + j6)$ ohms and $Z_2=(5 – j2)$ohms, are connected in series to a supply voltage of 100*V*. Find the magnitude of the current and its phase angle relative to the voltage.[*12.40A;29^o 45' lag*]
9.	A resistance of 45 ohms is connected in series with a capacitor of 42 μ F. If the applied voltage is 250V, 50Hz determine: (a) the capacitive reactance; (b) the impedance; (c) the current, and its phase relative to the applied voltage; (d) the voltage across the resistance; and (e) the voltage across the capacitance. [*(a) 75.79 . (b) 88.14 . (c) 2.836 A at 59o18' leading (d) 127.6 V (e) 214.9 V*]
10	A single –phase motor takes 8.3A at a power factor of 0.866 lagging when connected to a 230V, 50Hz supply. Two similar capacitors are connected in parallel with each other to form a capacitance bank. This capacitance bank is now connected in parallel with the motor to raise the power factor to unity. Determines the capacitance of each capacitor. [28.7uF]
10. Soln	VA = 230 x 8.3 = 1909 VA Real Power = 1909 x 0.866=1653.2 W Phase angle = $\cos^{-1}0.866=30$ Reactive Power = 1909 x sin 30 = 954.6 VAR For Unity Power Factor $Qc = 954.6 = V^2/ X_c \Rightarrow 230^2/954.6 = X_c$ $X_c=55.42\Omega = 1/ (2\pi f\ C)=1/(2\pi 50\ C)$ Capacitance of Cap. Bank = 57.44μF Capacitance of Each Capacitor = 57.44/2=28.7 μF 
11	(a) A single-phase load of 5kW operates at a power factor of 0.6 lagging. It is proposed to improve this power factor to 0.95 lagging by connecting a capacitor across the load. Calculate the kVA rating of the capacitor. [5.02kVA] (b) Give reasons why it is to a consumer's economic advantage to improve his power factor with respect to the supply, and explain the fact that the improvement is rarely made to unity in practice.
12	A 25kVA single-phase has a power factor of 0.8 lag. A 10kVA capacitor is connected for power-factor correction. Calculate the input apparent power in kVA taken form the mains and its power factor when the motor is (a) on half load; (b) on full load. Sketch a phasor diagram for each case. [10.3 kVA, 0.97 leading; 20.6kVA, 0.97 lagging]
13	A single-phase motor takes 50A at a power factor of 0.6 lagging from a 250V, 50Hz supply. What value of capacitance must be a shunting capacitor have to raise the overall power factor to 0.9 lagging ? How does the installation of the capacitor affect the line and motor currents? [324uF]
14	A 240V, single-phase supply feeds the following loads (a) incandescent lamps taking a current of 8A at unity power factor, (b) fluorescent lamps taking a current of 5A at 0.8 leading power factor (c) motor taking a current of 7A at 0.75 lagging power factor. Sketch the phasor diagram and determine the total current, active power and reactive power taken from the supply and the overall power factor. [17.35A, 4140W, 390 var, 0.996 lagging]
15	The load taken from an ac supply consists of (a) a heating load of 15kW (b) a motor load of 40kVA at 0.6 power factor lagging (c) a load of 20kW at 0.8 power factor lagging. Calculate the total load from the supply (in kW and kVA) and its power factor. What would be the kvar rating of a capacitor to bring the power factor to unity and how would the capacitor be connected. [59kW, 75.5kVA, 0.78 lagging; 47kvar in parallel]
16	A cable is required to supply a welding set taking a current of 225A at 110V alternating current, the average power factor being 0.5 lagging. An available cable has a rating of 175A and it is decided to use this cable by installing a capacitor across the terminal of the welding set. Find (a) the required capacitor

<table>
<tr><td></td><td colspan="2">current and reactive power to limit the cable current to 175A (b) the overall power factor with the capacitor in circuit. [60.8A, 6.7 kvar, 0.643 lag]</td></tr>
</table>

TUTORIAL 3 – AC THREE PHASE

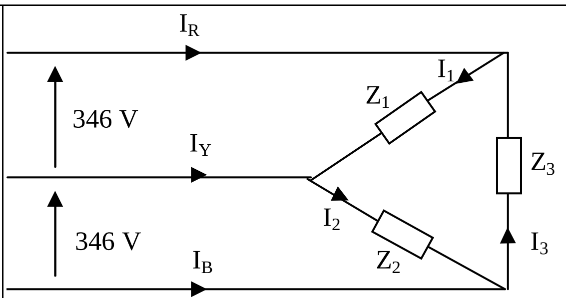
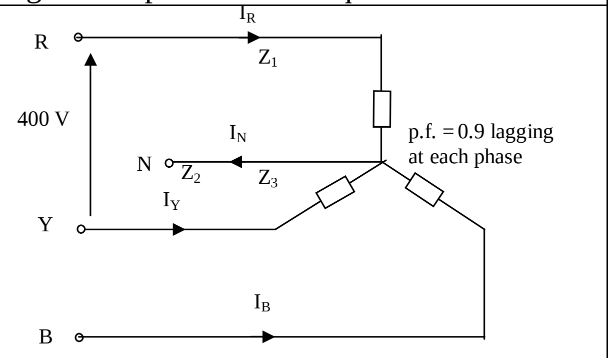
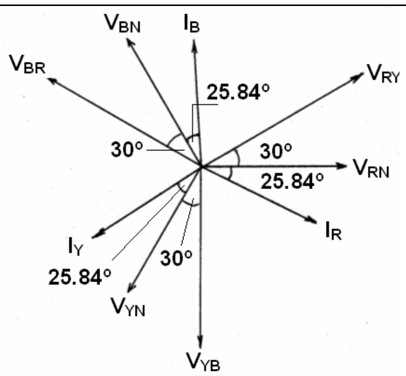
<table>
<tr><td>1.</td><td>A delta load is connected as shown in fig. 1.
Calculate I_P and I_L of the circuit.
$Z_1 = 100 \angle 0° \ \Omega$
$Z_2 = 20 + j60 \ \Omega$
$Z_3 = -j106 \ \Omega$
I_P : Phase Current I_1, I_2, I_3
I_L : Line Current I_R, I_Y, I_B</td><td>

Fig. 1</td></tr>
<tr><td>1.
Soln</td><td colspan="2">Take V_{RY} as reference.
Phase currents:
$I_1 = \mathbf{V_{RY}/Z_1} = 346\angle 0° / 100\angle 0° = 3.46\angle 0°$ A
$I_2 = \mathbf{V_{YB}/Z_2} = 346\angle -120° / (20 + j60) = 5.47\angle -(120 + 71.57)° = 5.47\angle 168.43°$A (-191.57°=168.43°)
$I_3 = \mathbf{V_{BR}/Z_3} = \mathbf{346\angle -240° / -j106} = 346\angle 120° / 106\mathbf{\angle 270°} = 3.26 \angle -150°$ A
Line currents:
$I_R = I_1 - I_3 = 3.46\angle 0° - 3.26 \angle -150° = 6.5\angle 14.54°$ A
$I_Y = I_2 - I_1 = 5.47\angle 168.43° - 3.46\angle 0° = 8.89\angle 173°$ A
$I_B = I_3 - I_2 = 3.26 \angle -150° - 5.47\angle 168.43° = 3.72\angle -47.08°$ A</td></tr>
<tr><td>2.</td><td colspan="2">A factory has the following load with <u>power factor of 0.9 lagging in each phase</u>. Red phase 40 A, yellow phase 50 A and blue phase 60 A. If the supply is 400V, three-phase, four-wire, calculate the current in the neutral and the total power. Draw a phasor diagram for phase and line quantities.</td></tr>
<tr><td>2
Soln</td><td>$|I_R| = 40$ A
$|I_Y| = 50$ A
$|I_B| = 60$ A
Find I_N and $P_{3\phi}$. , (400 /√3)=230.9
Take V_{RN} as reference,
$V_{RN} = 230.9\angle 0°$ V, $V_{YN} = 230.9\angle -120°$ V,
$V_{BN} = 230.9\angle 120°$ V
Power angle at each phase,
$\phi = \cos^{-1} 0.9 = 25.84°$</td><td>
</td></tr>
<tr><td></td><td>Therefore,
$I_R = 40\angle -25.84°$ A,
$I_Y = 50\angle -145.84°$ A,
$I_B = 60\angle 94.16°$ A
$I_N = 40\angle -25.84° + 50\angle -145.84° + 60\angle 94.16°$
$= 17.31\angle 124.16°$ A #
$P_{3\phi} = P_R + P_Y + P_B = V_R I_R \mathbf{\cos\phi_R} + V_Y I_Y \mathbf{\cos\phi_Y} + V_B I_B \mathbf{\cos\phi_B}$
$= V_P(I_R + I_Y + I_B)\mathbf{\cos\phi_P} = (400/\sqrt{3}) \times (40+50+60) \times (0.9)$
=31.18kW</td><td>
</td></tr>
</table>

TUTORIAL 4 – AC THREE PHASE

1 A 380V 3-phase power supply with phase sequence RYB is supplying power to an unbalance 3-phase star load as shown in fig Q1.

Given $Z_1=5+5j$ $Z_2=2-j10$ $Z_3=5+j20$.

Take **V_{RY} as reference**. Determine:

(a) the magnitude and phase angle of the voltage at the star point;
(b) the magnitudes and phase angles of the three line currents;
(c) the total active and reactive powers of the load;
(d) the overall power factor of the load; and,
(e) draw the phasor diagram of the three-phase load.

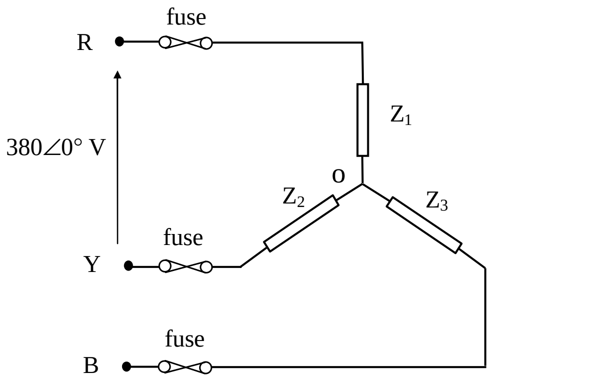


figure Q1

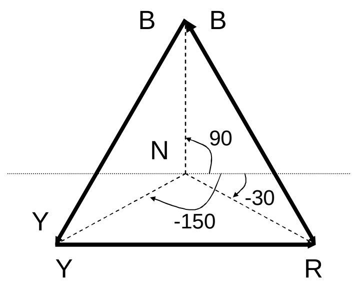
1 Soln

Take V_{RY} as reference.

$$V_{RY} = 380\angle 0° \text{ V}$$
$$V_{YB} = 380\angle -120° \text{ V}$$
$$V_{BR} = 380\angle 120° \text{ V} \qquad \mathbf{380/\sqrt{3}=219.4}$$
$$V_{RN} = 219.4\angle -30° \text{ V}$$
$$V_{YN} = 219.4\angle -150° \text{ V}$$
$$V_{BN} = 219.4\angle 90° \text{ V}$$

By Millman's Therom

$$V_{on} = \frac{V_{RN}Y_1 + V_{YN}Y_2 + V_{BN}Y_3}{Y_1 + Y_2 + Y_3} \text{ where:}$$



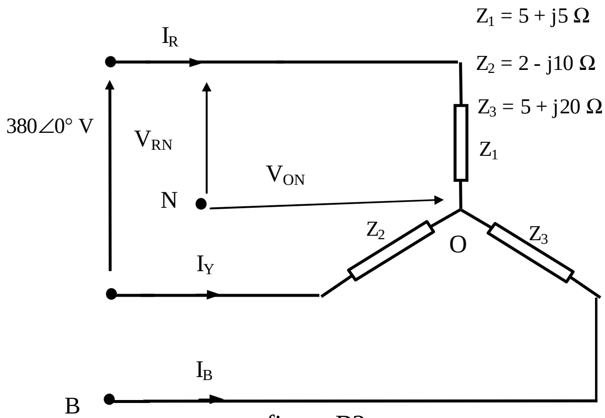


figure B2

$$Y_1 = \frac{1}{Z_1} = \frac{1}{5+j5} = 0.1414\angle -45° \text{ s} \qquad Y_2 = \frac{1}{Z_2} = \frac{1}{2-j10} = 0.0981\angle 78.7° \text{ s}$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{5+j20} = 0.0485\angle -76.0° \text{ s}$$

$$V_{on} = \frac{(219.4\angle -30°)(0.1414\angle -45°) + (219.4\angle -150°)(0.0981\angle 78.7°) + (219.4\angle 90°)(0.0485\angle -76.0°)}{0.1414\angle -45° + 0.0981\angle 78.7° + 0.0485\angle -76.0°}$$

$$\therefore \quad I_R = (V_{RN} - V_{on}).Y_1 = (219.4\angle -30° - 384.46\angle -40.9°)(0.1414\angle -45°) \text{ A} = 24.61\angle 80.3° \ A$$
$$I_Y = (V_{YN} - V_{on}).Y_2 = (219.4\angle -150° - 384.46\angle -40.9°)(0.0981\angle 78.7°) \text{ A} = 49.14\angle -117.8° \text{ A}$$
$$I_B = (V_{BN} - V_{on}).Y_3 = (219.4\angle 90° - 384.46\angle -40.9°)(0.0485\angle -76.0°) \ A = 26.85\angle 45.7° \ A$$

Total active power, $P_T = |I_R|^2.R_{Z_1} + |I_Y|^2.R_{Z_2} + |I_B|^2.R_{Z_3}$

$= (24.61)^2.(5) + (49.14)^2.(2) + (26.85)^2.(5)$ W $= 11.46$ kW #

Total reactive power, $Q_T = |I_R|^2.X_{Z_1} + |I_Y|^2.X_{Z_2} + |I_B|^2.X_{Z_3}$

$= (24.61)^2.(+5) + (49.14)^2.(-10) + (26.85)^2.(+20)$ Var $=$ -6.7 kVar (capacitive) #

The overall power factor, $p.f. = \dfrac{P_T}{\sqrt{P_T^2 + Q_T^2}} = \dfrac{11.46}{\sqrt{(11.46)^2 + (6.7)^2}} = 0.863 \quad leading$

2 Referred to figure Q1, if the fuse in the Yellow line is blown, determine the new values of:

(a) the voltage at the star point; and,

	(b) the three line currents.
2 Soln	If fuse on Yellow is blown, the star point voltage is: **(Z$_2$ becomes OPEN Circuit, $Z_2=\infty$, $Y_2=0$)** $V_{on}=\dfrac{V_{RN}Y_1+V_{YN}Y_2+V_{BN}Y_3}{Y_1+Y_2+Y_3}=\dfrac{(219.4\angle-30°)(0.1414\angle-45°)+(219.4\angle-150°)(0)+(219.4\angle 90°)(0.0485\angle-76.0°)}{0.1414\angle-45°+0+0.0485\angle-76.0°}$ $V_{on}=178.5\angle-3.41° \quad V$
	If fuse on Yellow is blown, I_Y = 0 A, and $I_B = -I_R$ $I_R=\dfrac{V_{RB}}{Z_1+Z_3}=\dfrac{(380\angle-60°)}{(5+j5)+(5+j20)}$ A $I_R=14.1\angle-128.1°\ A_{\#}$; $I_Y=0\ A_{\#}$; $I_B=-I_R=14.1\angle 51.9°\ A_{\#}$ $I_B=-14.1\angle-128.1°\ A=14.1\angle(180°-128.1°)A=14.1\angle 51.9° A$ $-1=1\angle 180°$

TUTORIAL 5 – INTRODUCTION TO ELECTRICAL MACHINES, TRANSFORMER

1	(i) State two advantages and two disadvantages of using auto-transformer compared with the two-winding transformer.
1 Soln	Advantages of auto-transformer 1. Less copper is used, when compared with 2 winding transformer. 2. The weight and volume of transformer is less. 3. The auto-transformer has a higher efficiency. 4. The auto-transformer suffers less voltage variation. 5. The auto-transformer provides continuously variable output voltage. Disadvantages of auto-transformer 1. The auto-transformer primary and secondary have a common neutral connection, so it cannot be used as an isolation transformer. 2. The secondary short-circuit current will be larger. 3. A break in the secondary winding stops the transformer action and the full primary voltage will be applied to the secondary circuit.
2	(i) State the losses associated with the iron core of a transformer. (ii) Explain the method(s) used to reduce the losses you stated in part (a)(i)
2 Soln	(1) There are hysteresis and eddy current losses. (2) Hysteresis loss can be reduced by using material with small remanent flux or narrow hysteresis loop, since the area of the hysteresis represent losses per cycle of magnetization. Eddy current can be reduced by using laminated iron core. Induced emf is reduced to 1/N of the total if N metal sheets are used. At the same time, the resistance path is increased N times due to the reduction in the thickness.
3	A 200kVA,6600/400V, 50Hz single-phase transformer has 80 turns on the secondary. Calculate (a) the approximate values of the primary and secondary currents; (b) the approximate number of primary turns; (c) the maximum value of the flux.(30.3A, 1320 turns, 0.0225 Wb)
3 Soln	$V_1/V_2=N_1/N_2 \Rightarrow 6600/400 = N_1/80$ $\Rightarrow N_1$ = 6600 / 400 x 80 = 1320 I_1 = 200 x 1000 / V_1 = 200 x 1000 / 6600 = 30.3A I_2 = 200 x 1000 / V_2 = 200 x 1000 / 400 = 500A V_1=4.44 N_1 f $\Phi \Rightarrow \Phi$ = 6600 / (4.44 x 50 x 1320) = 22.5mWb
4	The primary winding of a single-phase transformer is connected to a 230V, 50Hz supply. The secondary winding has 1500 turns. If the maximum value of core flux is 0.00207Wb, determine : (a) the number of turns on the primary winding; (b) the secondary induced voltage; (c) the net cross-sectional core area if the flux density has a maximum value of 0.465 Tesla. (500, 690V, 4450mm^2)
4. Soln	(a) V_1 = 4.44 N_1 f $\Phi \Rightarrow N_1 = V_1/(4.44 f \Phi)$ = 230 / (4.44 x 50 x 0.00207) = 500 turns (b) V_2 = 4.44 N_2 f Φ = 4.44 x 1500 x 50 x 0.00207 = 689V (c) 0.00207 = B A = 0.465 x A $\Rightarrow$A=4451 mm^2
5	The primary of a transformer has 500 turns and is supplied at a voltage of 2000V r.m.s. at a frequency of 50Hz. Estimate the maximum value of the flux through the core. (0.018Wb)

5. Soln	2000 r.m.s 2000 = 4.44 x N_1 f Φ = 4.44 x 500 x 50 Φ ⇒Φ=0.018
6	The primary of a transformer has 1000 turns and produces a maximum flux of 0.03Wb alternating at 50Hz in the iron core. The secondary winding has 35 turns. Estimate the r.m.s. values of primary and secondary e.m.f.'s on the assumption that the flux change is sinusoidal. (6660V, 233V)
6. Soln	V_1 = 4.44 N_1 f Φ = 4.44 x 1000 x 50 x 0.03 = 6660 V_2 = 4.44 N_2 f Φ = 4.44 x 35 x 50 x 0.03 = 233V
7	A single-phase transformer has a primary voltage of 2000V, a secondary voltage of 440V and a full load output of 20kVA. The secondary winding has 130 turns. Calculate the number of primary turns and the primary and secondary full load currents, neglecting losses. (591, 10A, 45.5A)
7. Soln	I_1 = 20 x 1000 / 440 = 45.45A I_2 = 20 x 1000 / 2000 = 10A V_1 / V_2 = N_1 / N_2 ⇒ N_1 = V_1 / V_2 x N_1 = 2000 / 440 x 130 = 591
8	A 4-pole, 3-phase induction motor is energized from a 60Hz supply, and is running at a load condition for which the slip is 0.03. Determine (a) the speed of the rotation field; (b) the rotor speed; (c) the rotor frequency. [Ans (a) 1800 r.p.m (b) 1746 r.pm. (c) 1.8 Hz]
8. Soln	(a) Speed of Rotating Field = Frequency / pole-pair = 60 / 2 = 30 rev/s = 1800 rpm (b) Rotor Speed = N_r = (1-s) N_s = (1-0.03) x 1800 = 1746 rpm (c) Rotor frequency = slip x stator frequency = s f = 0.03 x 60 = 1.8Hz
9	The frequency of the e.m.f. induced in the rotor of an 3-phase, 6-pole induction motor is found to have 180 cycles/min. The motor is connected to a 50Hz, 440V supply, calculate (a) the speed of the motor; (b) the percentage slip of the motor. [Ans (a) 940 r.p.m(b) 0.06]
9. Soln	(a) 180 cycles/min = 3 cycles / sec ⇒ Rotor frequency = f_r = 3Hz 3 = s x stator frequency = s 50 ⇒ s = 0.06 (b) N_s = f / p = 50 / 3 = 16.67 rev/s = 1000 rpm Speed of the rotor = (1-s) x N_s = 1000 x (1-0.06) = 940 rpm