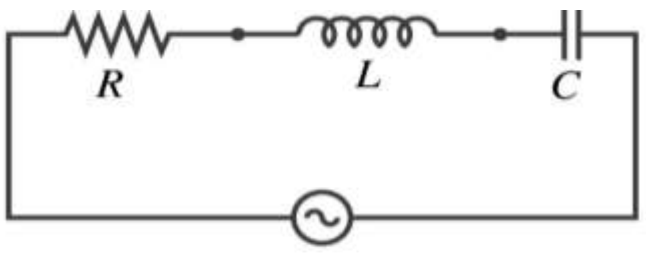
Basic Electrical Technology

[ELE 1051]

CHAPTER 3 - RESONANCE

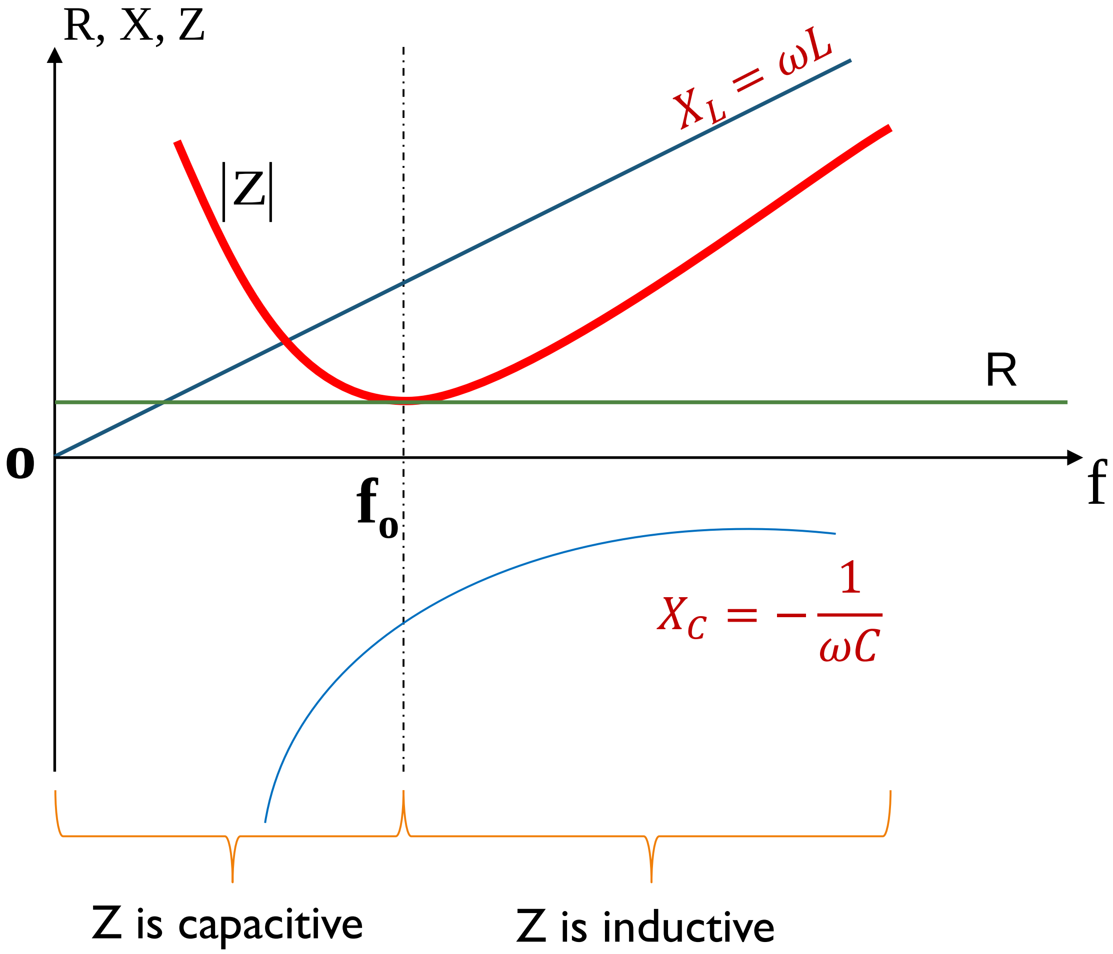
Series Resonance



v(t), variable frequency

$$Z = R + j(X_L \sim X_C)$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



'f_0 is called the resonant frequency'

Series Resonance

- **When series RLC circuit is at resonance,**
  - Current is in phase with voltage
  - Circuit power factor is unity
  - $X_L = X_C$
  - $Z = R$

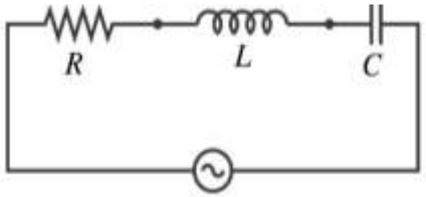
- **Resonant frequency for a series RLC circuit is obtained as follows:**

$$Imaginary\ part\ of\ Z_{eq} = 0$$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}\ hertz}$$

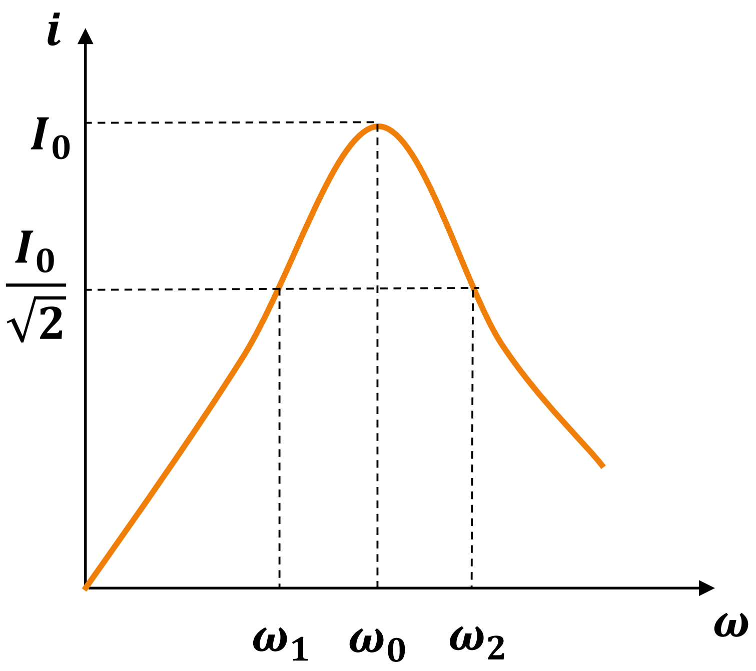


v(t), variable frequency

$Z = R + j(X_L \sim X_C)$

Current vs. Frequency in RLC Series Circuit

Variation of current with frequency



$$I_0 = I_{max} = \frac{V_{rms}}{R}$$

- **Half Power Frequency**

  'Frequency at which the power is half of the power at resonant frequency'

$$Power = \frac{1}{2} I_0^2 R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R$$

$$At\ \omega_1\ and\ \omega_2, I = \frac{I_0}{\sqrt{2}}$$

$$\omega_1 = Lower\ half\ power\ frequency$$

$$\omega_2 = Upper\ half\ power\ frequency$$

$$\mathbf{Bandwidth = \omega_2 - \omega_1}$$

In practice the curve of $|I|$ against ω is not symmetrical about the resonant frequency

Half Power Frequency

$$Impedance\ at\ \omega_1 and\ \omega_2, |Z| = {V_0}\Big/{\frac{I_0}{\sqrt{2}}} = \sqrt{2}R$$

Below Resonant frequency $\omega_0, |X_C| > |X_L|$

At ω_1,

$$\sqrt{R^2 + (X_C - X_L)^2} = \sqrt{2}R$$

$$X_C - X_L = R$$

$$\frac{1}{\omega_1 C} - \omega_1 L = \mathrm{R}$$

$$\boxed{\boldsymbol{\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}}$$

Above Resonant frequency $\omega_0, |X_L| > |X_C|$

At ω_2,

$$\sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2}R$$

$$X_L - X_C = R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = \mathrm{R}$$

$$\boxed{\boldsymbol{\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}}$$

$$\boldsymbol{\omega_2 \omega_1 = \frac{1}{LC} = \omega_0^2}$$

$$\boldsymbol{\omega_2 - \omega_1 = \frac{R}{L}}$$

Quality Factor for series circuit

- At resonance, V_C and V_L can be very much greater than applied voltage

$$|V_C| = |I|X_C = \frac{V.X_C}{\sqrt{R^2 + (X_L - X_C)^2}}$$

At resonance, $X_L = X_C$

$$V_C = \frac{V}{R}X_C$$

$$V_C = \frac{V}{\omega_0 CR} = \boldsymbol{Q}V$$

Q is termed the Q factor or voltage magnification

- High value of Q can lead to component damage
- Careful design necessary
- Larger the value of Q, more symmetrical the curve appears about the resonant frequency

$$\boxed{\boldsymbol{Q = \frac{1}{\omega_0 CR} = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}}}$$

$$\boldsymbol{Q = \frac{Resonant\ frequency}{Bandwidth}}$$

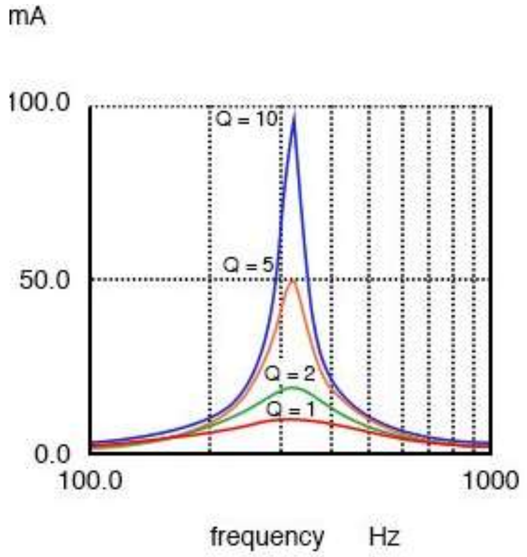


Illustration 1

A circuit having a resistance of 4Ω and inductance of 0.5H and a variable capacitance in series, is connected across a 100V, 50Hz supply. Calculate:

a) The capacitance to give resonance
b) The voltages across the inductor and the capacitor
c) The Q factor of the circuit

$a)\ C = 20.26\mu F$

b) V_C = 3926.99 V

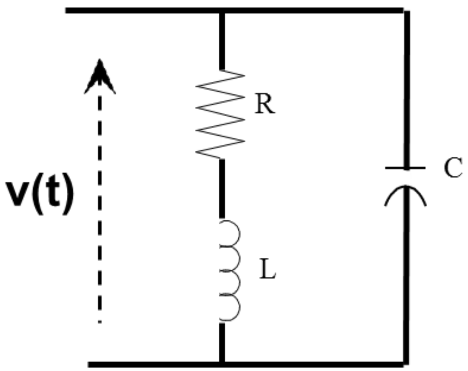
c) Q-factor = 39.26

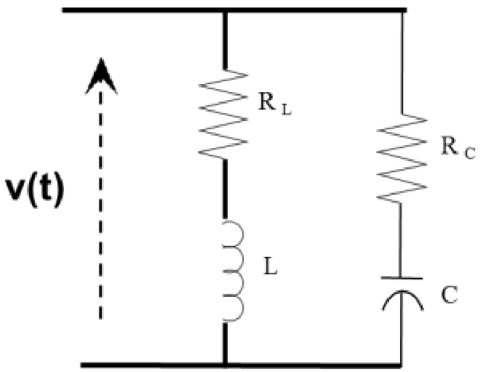
Illustration 2

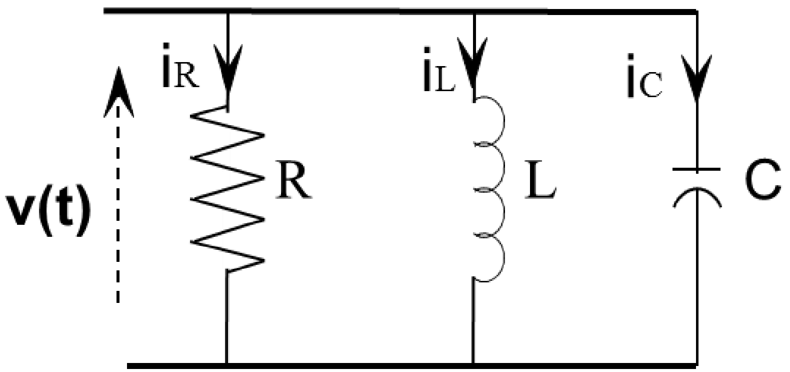
The bandwidth of a series resonant circuit is 500 Hz. If the resonant frequency is 6000 Hz, what is the value of Q? If R =10 Ω, what is the value of the inductive reactance at resonance? Calculate the inductance and capacitance of the circuit

$$C = 0.22\mu F$$

Resonance in parallel circuits







Steps to obtain the expression of resonant frequency in parallel circuits

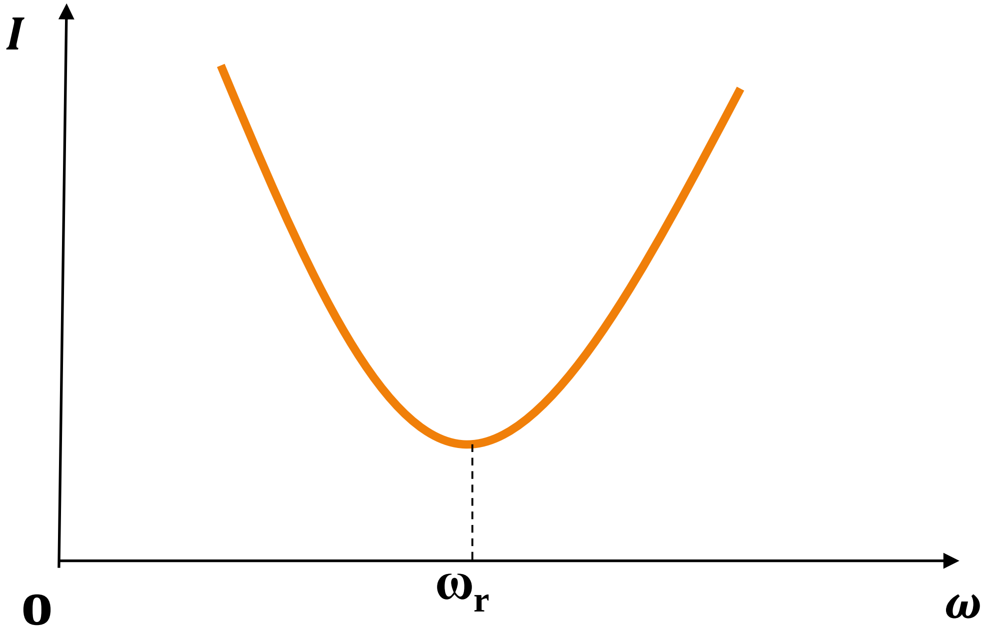
- Obtain the net admittance of the circuit ; $Y_{eq} = y_1 + y_2 + \cdots$

$$\mathbf{Y_{eq} = G_{eq} \pm jB_{eq}}$$

- Equate the imaginary part (susceptance) to zero; $B_{eq} = 0$ and obtain the expression of ω_r

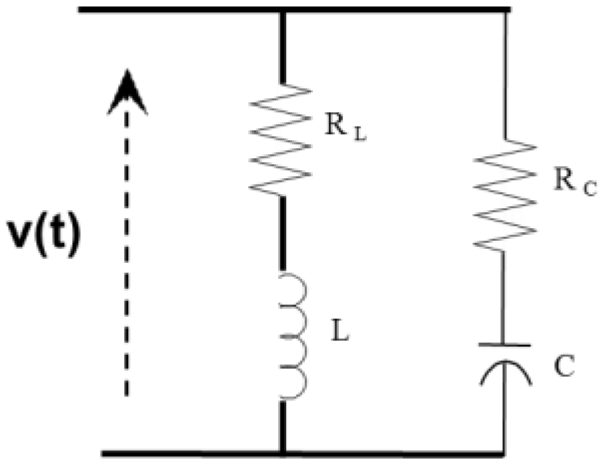
The expression for resonant frequency depends on circuit configuration

Current vs. Frequency in parallel Circuits



- At resonance
  - Impedance is maximum
  - Resultant current minimum

Parallel resonance circuits



$$y_{eq} = \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_c}$$

$$y_{eq} = \frac{R_C - jX_c + R_L + jX_L}{(R_L R_C + X_L X_c) - j(R_L X_C - X_L R_C)}$$

Rationalizing ;

$$y_{eq} = \frac{\left((R_L R_C + X_L X_c) + j(R_L X_C - X_L R_C)\right)(R_C - jX_c + R_L + jX_L)}{(R_L R_C + X_L X_c)^2 + (R_L X_C - X_L R_C)^2}$$

Separating the real & imaginary terms ;

$$y_{eq} = \frac{1}{(R_L R_C + X_L X_c)^2 + (R_L X_C - X_L R_C)^2}(R_L^2 R_C + R_L R_C^2 - R_L X_C^2 - X_L^2 R_C) + \boldsymbol{j}(R_L^2 X_C + X_C X_L^2 - X_L R_C^2 - X_C^2 X_L)$$

Parallel resonance circuits

Equating the imaginary part to zero;

$$B_{eq} = 0 \;;$$

$$R_L^2 X_C + X_C X_L^2 - X_L R_C^2 - X_C^2 X_L = 0$$

Solving for ω_0;

$$\boldsymbol{\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 C - L}{R_C^2 C - L}}}$$

If $R_L = R_c$:

$$\boldsymbol{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

Exercise 1

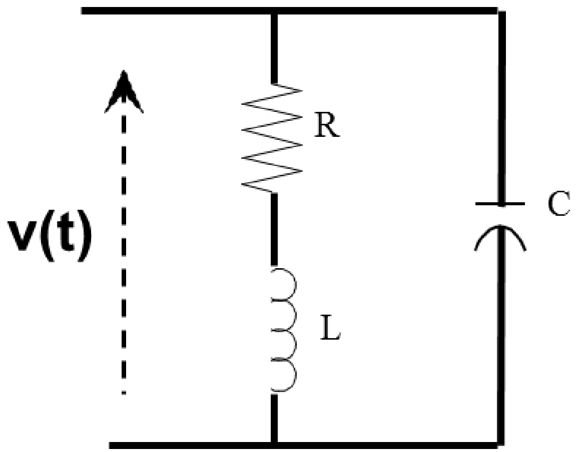
A parallel circuit with an RL series branch (R = 20 Ω and L = 50 mH) and an RC series branch (R = 10 Ω and C = 100 μF) are connected to a variable frequency voltage source. Find at what frequency the circuit will resonate?

Ans:

$$\omega_c = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 C - L}{R_C^2 C - L}} = 223.6067\, rad/sec = 35.58\, Hz$$

Exercise 2

Obtain the expression for resonant frequency for the given parallel circuit



Thank You!