MANIPAL ACADEMY OF HIGHER EDUCATION

FIRST SEMESTER B.TECH. EXAMINATIONS - JANUARY 2023

SUBJECT: PHY 1071 / PHY-1071 - ENGINEERING PHYSICS

SCHEME OF EVALUATION

FORMAT FOR SHARING QUESTION PAPER

Q. No.	Question and Scheme

1 (A)

Question:

i) Explain the term Poynting vector.

ii) Obtain an expression for radii of the mth order dark fringes in Newton's rings.[5]

Scheme:

The rate of transfer of energy by an electromagnetic wave is described by a vector $\vec{S}$, called the **Poynting vector,** which is defined by the expression

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

The magnitude of the Poynting vector represents the rate at which energy passes through a unit surface area perpendicular to the direction of wave propagation

1 Mark

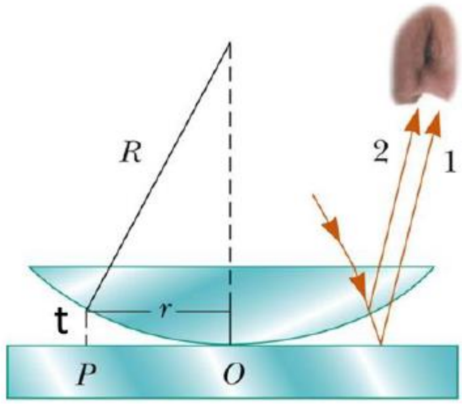


Figure: 1 Mark

Using the geometry shown in *Figure 1.8* (a), we can obtain expressions for the radii of the bright and dark rings in terms of the radius of curvature R and wavelength λ. For the thin air film trapped between the two glass surfaces as shown in the figure above, the condition for destructive (dark rings) interference is

$$2nt = m\lambda, \qquad m = 0, 1, 2, 3 \ldots$$

For air film, $n \approx 1$, $\therefore\ 2t = m\lambda$

1 Mark

From the above figure, $t = R - \sqrt{R^2 - r^2}$

$$t = R - R\left[1 - \left(\frac{r}{R}\right)^2\right]^{1/2}$$

1 Mark

Binomial theorem is, $(1+y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \dots\dots$

If $r/R << 1$, using binomial theorem and neglecting higher order terms,

$$t = R - R\left[1 - \frac{1}{2}\left(\frac{r}{R}\right)^2 + \dots\dots\right] \approx \frac{r^2}{2R}$$

Substituting the value of t in above equation, we get

$$r_{dark} \approx \sqrt{mR\lambda} \qquad (m = 0, 1, 2, \dots)$$

1 Mark

1 (B) **Question:**

Light of wavelength 500 nm is incident normally on a diffraction grating. If the third-order maximum of the diffraction pattern is observed at 32.0°, (a) what is the number of rulings per centimeter for the grating? (b) Determine the total number of primary maxima that can be observed in this situation.[3]

Scheme:

$d \sin\theta_{\text{bright}} = m\lambda \qquad m = 0, \pm1, \pm2, \pm3, \dots$

(a) For m = 3,

d = 3 x 500 x 10^{-9}/sin 32^o

d = 2.8 x 10^{-6} m = 2.8 x 10^{-4} cm

No. of rulings per cm = 1/d = 3532 2 Mark

(b) $\theta = 90^o$

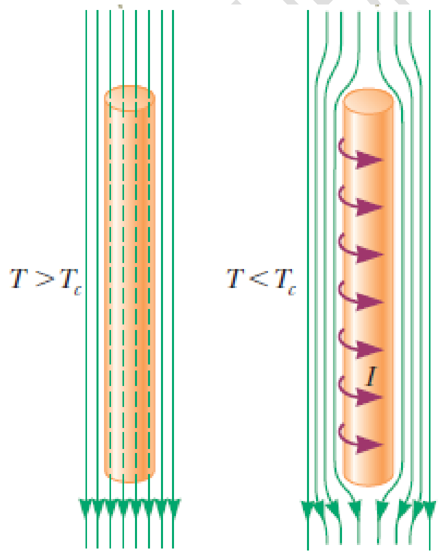
m = 5.6 = 5

No. of maxima = 5 + 1 + 5 = 11 1 Mark

1 (C) **Question:**

"A superconductor is not only a perfect conductor; it is also a perfect diamagnet" – Justify the statement.[2]

Scheme:



1 Mark

A superconductor expels magnetic fields by forming surface currents. Surface currents induced on the superconductor's surface produce a magnetic field that exactly cancels the externally applied field inside the superconductor. 1 Mark

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2 (A)

. **Question:**

i) Write the conditions for constructive and destructive interference of reflected light from a thin soap film in air, assuming normal incidence.

ii) With the help of a neat diagram state the Rayleigh's criterion for resolution. **[5M]**

Scheme:

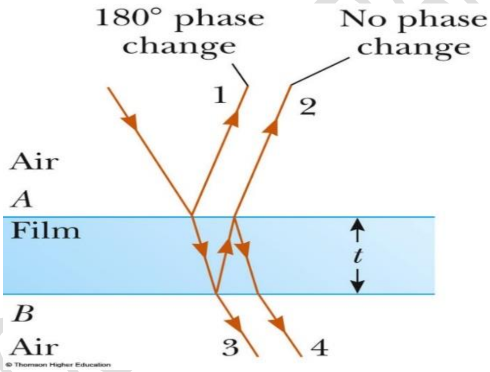
i.Consider a film of uniform thickness t and index of refraction n. Assume light rays traveling in air are nearly normal to the two surfaces of the film. If λ is the wavelength of the light in free space and n is the index of refraction of the film material, then the wavelength of light in the film is **1 mark**

$$2t = \left(m + \frac{1}{2}\right)\lambda_n \quad (m = 0, 1, 2, \ldots)$$

$$2nt = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \ldots)$$

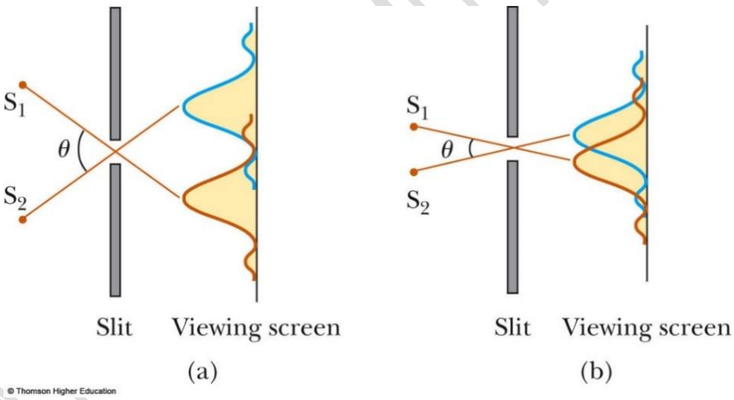
The condition for ***destructive*** interference n thin films is,

$$2nt = m\lambda \quad (m = 0, 1, 2, \ldots)$$



1 mark **1 Mark**

ii. When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved. This limiting condition of resolution is known as **Rayleigh's criterion.**



1+1 Mark

2 (B)

Question:

A material having an index of refraction of 1.30 is used as an antireflective coating on a piece of glass (n = 1.50). What should the minimum thickness of this film be to minimize reflection of 500-nm light? **[3M]**

Scheme:

There are a total of two phase reversals caused by reflection, one at the top and one at the bottom surface of the coating.

$$2nt = \left(m + \frac{1}{2}\right)\lambda \quad \text{so} \quad t = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n}$$

1.5 Mark

The minimum thickness of the film is therefore

$$t = \left(\frac{1}{2}\right)\frac{(500\text{ nm})}{2(1.30)} = \boxed{96.2\text{ nm}}$$

1.5 Mark

2 (C)	**Question:** An electron has a kinetic energy of 12 eV. The electron is incident upon a rectangular barrier of height 20 eV and thickness 1 nm. By what factor would the electron's probability of tunnelling through the barrier increase assuming that the electron absorbs all the energy of a photon with wavelength 546 nm? **[2M]** Given E = 12.0 eV = 12 X 1.6 X 10^{-19} J; U = 20.0 eV = 20 X 1.6 X 10^{-19} J L = 1.0 nm = 1.0 X 10^{-9} m Solving for C when E = 12.0 eV , we get $C = \frac{2X3.14X\sqrt{2X9.1X10^{-31}(20-12)X1.6X10^{-19}}}{6.63X10^{-34}}$ $C = 1.45X10^{10}$ /m 1 Mark Energy of the photon of wavelength 546 X 10^{-9} m = hc / λ = 3.64 X 10^{-19} J = 2.28 eV --------------- **½ M** Absorbing this photon energy, electron energy increases to E' = 14.28 eV. The value of C decreases to C' = 1.26 X 10^{10} /m. Factor of increase in electron's tunneling probability through the barrier, $= \frac{e^{-2C'L}}{e^{-2CL}} = e^{2L(C-C')} = e^{4.45} = 85.9$ 0.5 Mark
3 (A)	**Question:** Using the energy and momentum conservation, derive an expression for the wavelength of the scattered photon in a Compton effect experiment. [5] **Scheme:** Scheme: Let λ_o , $p_o = h/\lambda_o$ and $E_o = hc/\lambda_o$ be the wavelength, momentum and energy of the incident photon respectively. λ', $p' = h/\lambda'$ and $E' = hc/\lambda'$ be the corresponding quantities for the scattered photon. We know that, for the electron, the total **relativistic** energy $E = \sqrt{p^2c^2 + m^2c^4}$ **Kinetic** energy $K = E - m c^2$ And momentum $p = \gamma mv.$ where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ v and m are the speed and mass of the electron respectively. ½ MARK

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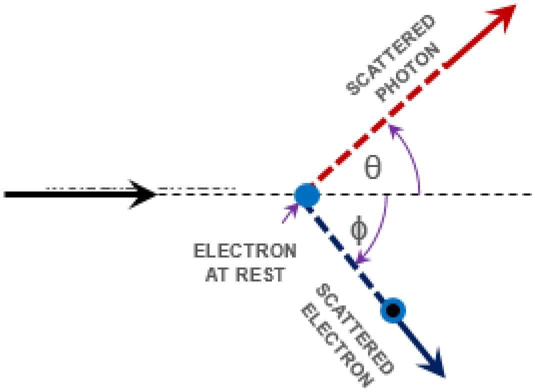

½ MARK

Figure: Quantum model for X-ray scattering from an electron

In the scattering process, the total energy and total linear momentum of the system must be conserved.

For conservation of energy we must have, $E_o = E' + K$

ie, $E_o = E' + (E - m c^2)$

Or $E_o - E' + m c^2 = E = \sqrt{p^2c^2 + m^2c^4}$ - ½ MARK

Squaring both the sides, $(E_o - E')^2 + 2(E_o - E')\, mc^2 + m^2c^4 = p^2c^2 + m^2c^4$

For conservation of momentum, x-component: $p_o = p' \cos\theta + p\cos\phi$

y-component: $0 = p'\sin\theta - p\sin\phi$ -

1 MARK

Rewriting these two equations

$$p_o - p'\cos\theta = p\cos\phi$$
$$p'\sin\theta = p\sin\phi$$

Squaring both the sides and adding,

$$p_o^2 - 2p_o p'\cos\theta + p'^2 = p^2$$

Substituting this p^2 in the equation :

$(E_o - E')^2 + 2(E_o - E')\, mc^2 = p^2c^2$, one gets

$$(E_o - E')^2 + 2(E_o - E')\, mc^2 = (p_o^2 - 2p_o p'\cos\theta + p'^2)c^2$$ -1 MARK

Substituting photon energies and photon momenta one gets

$$\left(\frac{hc}{\lambda_o} - \frac{hc}{\lambda'}\right)^2 + 2\left(\frac{hc}{\lambda_o} - \frac{hc}{\lambda'}\right) mc^2 = \left(\frac{hc}{\lambda_o}\right)^2 - 2\left(\frac{hc}{\lambda_o}\right)\left(\frac{hc}{\lambda'}\right)\cos\theta + \left(\frac{hc}{\lambda'}\right)^2$$

Simplifying one gets

$$\left(\frac{hc}{\lambda_o}\right)^2 - 2\left(\frac{hc}{\lambda_o}\right)\left(\frac{hc}{\lambda'}\right) + \left(\frac{hc}{\lambda'}\right)^2 + 2\,hc\left(\frac{1}{\lambda_o} - \frac{1}{\lambda'}\right) mc^2 = \left(\frac{hc}{\lambda_o}\right)^2 - 2\left(\frac{hc}{\lambda_o}\right)\left(\frac{hc}{\lambda'}\right)\cos\theta + \left(\frac{hc}{\lambda'}\right)^2$$

i.e., $-\frac{hc}{\lambda_o\lambda'} + \left(\frac{1}{\lambda_o} - \frac{1}{\lambda'}\right) mc^2 = -\frac{hc}{\lambda_o\lambda'}\cos\theta$

OR, $\left(\frac{\lambda' - \lambda_o}{\lambda_o\lambda'}\right) mc^2 = \frac{hc}{\lambda_o\lambda'}(1 - \cos\theta)$ -1 MARK

Compton shift:

$$\boldsymbol{\lambda' - \lambda_o = \frac{h}{mc}(1 - \cos\theta)}$$

½ MARK

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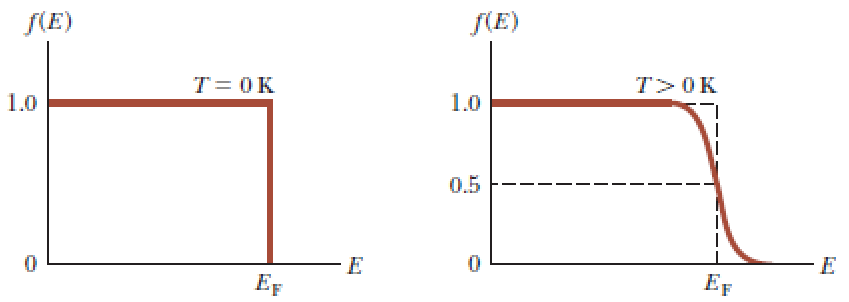
3 (B)	Question: Incident photons strike a photocathode having a work function of 2.26 eV, causing photoelectric emission. When a stopping potential of 2.69 V is imposed, there is no photocurrent. Find the wavelength of the incident photons and the speed of the most energetic photoelectrons. [3] **Scheme:** Scheme: $(hc/\lambda) = e\,\Delta VS + \varphi = 4.95$ eV $\Rightarrow \lambda = 251$ nm [1.5 mark] $e\,\Delta V_S = K_{max} = 2.69$ eV $\Rightarrow v_{max} = \mathrm{Sqrt}(2K_{max}/m) = 9.72 \times 10^5$ m/s [1.5 mark]
3 (C)	**Question:** The nucleus of an atom is of the order of 2.0×10^{-14} m in diameter. For an electron to be confined to a nucleus its de Broglie wavelength would have to be on this order of magnitude or smaller. What would be the total relativistic energy of the electron? [2] **Scheme:** The momentum of electron , $p_e = h / \lambda_e = 3.31 \times 10^{-20}$ kg.m/s [1 mark] Total energy, E, of the electron is given by $E^2 = p^2c^2 + m^2c^4 \Rightarrow E = 9.94 \times 10^{-12}$ J = 62.1 MeV [1 mark]
4 (A)	**Question:** Sketch schematically the plots of Fermi-Dirac distribution function for zero kelvin and for temperature above zero kelvin. Derive an expression for density-of-states. **[5M]** **Scheme:**  (a) (b) **[1M]** Based on the allowed states of a particle in a three dimensional box; derive the density-of-states function.

FORMATTING QUESTION PAPER

$$E_n = \frac{h^2}{8\,m\,L^2}\,n^2 = \frac{\hbar^2\pi^2}{2\,m\,L^2}\,n^2$$ Where n = 1, 2, 3, . . .

For a free-electron (mass m) in a metal cube of side L (three-dimensional box), the quantized energies are

$$E_n = \frac{\hbar^2\pi^2}{2\,m\,L^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$ n_x, n_y, n_z = quantum numbers.

---------½ **M**

Each allowed energy value is characterized by this set of quantum numbers (n_x, n_y, n_z) and the spin quantum number m_s (two values). Because of the macroscopic size L of the box, the energy levels for the electrons are very close together. As a result, the quantum numbers can be treated as continuous variables. In a three-dimensional quantum number space with axis representing n_x, n_y, n_z, the allowed energy states can be represented as dots located at positive integral values of the three quantum numbers.

……..½ M

Energy equation in 3D-box can be written as

$$n_x^2 + n_y^2 + n_z^2 = \frac{E}{E_o} = n^2 \quad \text{where } E_o = \frac{\hbar^2\pi^2}{2mL^2} \quad \text{and} \quad n = \sqrt{\frac{E}{E_o}}$$

---------½ **M**

This is the equation of a sphere of radius n. Thus, the number of allowed energy states having energies between E and E+dE is equal to the number of points in a spherical shell of radius n and thickness dn. The "volume" of this shell (NUMBER OF STATES FROM E TO E+dE) is

$$G(E)\,dE = (\tfrac{1}{8})(4\pi n^2 dn) = (\tfrac{1}{2})\pi n^2 dn$$ ---------½ **M**

Since all the n_x, n_y, n_z can have positive values only in an octant of the three-dimensional space.

The number of states per unit volume [v in normal space] per unit energy range is called density of states g (E)

$$g(E) = \frac{G(E)}{V}$$ ---------½ **M**

Replacing n by $\sqrt{\frac{E}{E_o}}$; we get $G(E)\,dE = \frac{1}{2}\pi\left(\frac{E}{E_o}\right)d\left[\left(\frac{E}{E_o}\right)^{1/2}\right]$

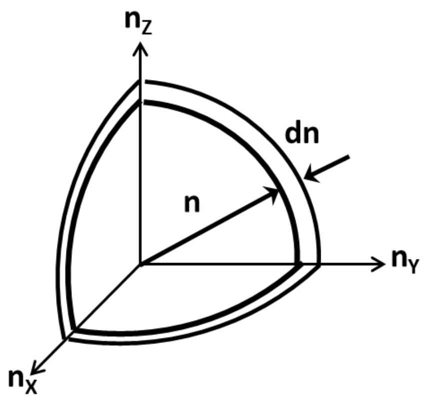
$$G(E)\,dE = \tfrac{1}{2}\pi\left(\frac{E}{E_o}\right)E_o^{-1/2}\,\tfrac{1}{2}E^{-1/2}\,dE = \tfrac{1}{4}\pi\,E_o^{-3/2}\,E^{1/2}\,dE$$ ---------½ **M**

$$G(E)\,dE = \tfrac{1}{4}\pi\left(\frac{\hbar^2\pi^2}{2mL^2}\right)^{-3/2} E^{1/2}\,dE$$

	$$G(E)\,dE = \frac{\sqrt{2}}{2}\frac{m^{3/2}L^3}{\pi^2\hbar^3}E^{1/2}\,dE \quad , \qquad L^3 = V$$ $$g(E)\,dE = \frac{G(E)}{V}\,dE = \frac{\sqrt{2}}{2}\frac{m^{3/2}}{\pi^2\hbar^3}E^{1/2}\,dE$$ To consider the spin states, each particle-in-a-box state should be multiplied by 2. $$\therefore \; g(E)\,dE = \frac{8\sqrt{2}\,\pi\, m^{3/2}}{h^3}E^{1/2}\,dE$$ ---------1 M Where g(E) is called the density-of-states function.
4 (B)	**Question:** Explain briefly the BCS theory of superconductivity in metals. Why all conductors are not superconductors? **[3M]** **Scheme:** • Cooper pair - behaves like a particle with integral spin (Bosons) • Bosons do not obey Pauli's exclusion principle – hence they can be in same quantum state at low temperature • Collision with lattice atoms is origin of resistance. But Cooper pair can't give up energy as they are in ground state. • Cooper pair can't gain energy – due to energy gap • Hence no interaction and no resistance **[2M]** Conductors possess low resistivity means good conducting properties do not create Cooper pairs – are not superconductors. **[1M]**
4 (C)	**Question:** For copper at 300 K, calculate the probability that a state with an energy equal to 99.0% of the Fermi energy is occupied. Fermi energy of copper is 7.05 eV. Mass of an electron is 9.1×10^{-31} Kg ; speed of light in vacuum = 3×10^{8} m/s; Planck's constant=6.63×10^{-34} Js; Avagadro number = 6.023×10^{23} / mol; Boltzmann constant=1.38×10^{-23} J/K **[2M]** **Scheme:** $$f(E) = \frac{1}{\exp\left(\frac{E-E_F}{kT}\right)+1}$$ $$= \frac{1}{e^{\frac{(7.075-6.98)eV}{k_B T}}+1}$$ $$= \frac{1}{e^{-2.732}+1}$$

	[1Mark] $= \dfrac{1}{0.0651+1}$ $= 0.938$ **[1Mark]**
5 (A)	Question: Based on the allowed states of a particle in a three-dimensional infinite potential well, 'box', derive the density-of-states function. [5] **Scheme**: The quantized energy of a particle (mass m) in a one dimensional box of length L are $$E_n = \frac{h^2}{8\,m\,L^2}n^2 = \frac{\hbar^2\pi^2}{2\,m\,L^2}n^2$$ where n = 1, 2, 3, . . . For a free-electron (mass m) in a metal cube of side L (three-dimensional box), the quantized energies can be taken as $$E_n = \frac{\hbar^2\pi^2}{2\,m\,L^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$ where n_x, n_y, n_z = quantum numbers. Each allowed energy value is characterized by this set of quantum numbers (n_x, n_y, n_z) and the spin quantum number m_s (two values). Because of the macroscopic size L of the box, the energy levels for the electrons are very close together. As a result, the quantum numbers can be treated as continuous variables. In a three-dimensional quantum number space with axis representing n_x, n_y, n_z, the allowed energy states can be represented as dots located at positive integral values of the three quantum numbers. - Energy equation in 3D-box can be written as $$n_x^2 + n_y^2 + n_z^2 = \frac{E}{E_o} = n^2 \qquad \text{where} \quad E_o = \frac{\hbar^2\pi^2}{2\,m\,L^2}$$ $$\text{and} \quad n = \sqrt{\frac{E}{E_o}}$$ 1 Mark This is the equation of a sphere of radius n. Thus, the number of allowed energy states having energies between E and E+dE is equal to the number of points in a spherical shell of radius n and thickness dn. The "volume" of this shell (NUMBER OF STATES FROM E TO E+dE) is $$G(E)\,dE = (1/8)(4\pi n^2 dn) = (1/2)\pi n^2 dn$$

FORMAT FOR SUBMITTING QUESTION PAPER



1 Mark

Since all the n_x, n_y, n_z can have positive values only in an octant of the three-dimensional space. The number of states per unit volume [v in normal space] per unit energy range is called density of states g (E)

Replacing n by $\sqrt{\frac{E}{E_o}}$; we get

$$G(E)\,dE = \tfrac{1}{2}\pi\left(\frac{E}{E_o}\right) d\left[\left(\frac{E}{E_o}\right)^{1/2}\right]$$

1 Mark

$$G(E)\,dE = \tfrac{1}{2}\pi\left(\frac{E}{E_o}\right) E_o^{-1/2}\,\tfrac{1}{2} E^{-1/2}\,dE = \tfrac{1}{4}\pi\, E_o^{-3/2}\, E^{1/2}\,dE$$

$$G(E)\,dE = \frac{\sqrt{2}}{2}\,\frac{m^{3/2}\,L^3}{\pi^2\,\hbar^3}\,E^{1/2}\,dE \quad , \qquad L^3 = V$$

$$G(E)\,dE = \tfrac{1}{4}\pi\left(\frac{\hbar^2\pi^2}{2mL^2}\right)^{-3/2} E^{1/2}\,dE$$

$$g(E) = \frac{G(E)}{V}$$

1 Mark

$$g(E)\,dE = \frac{G(E)}{V}\,dE = \frac{\sqrt{2}}{2}\,\frac{m^{3/2}}{\pi^2\,\hbar^3}\,E^{1/2}\,dE$$

To consider the spin states, each particle-in-a-box state should be multiplied by 2.

$$\therefore \quad g(E)\,dE = \frac{8\sqrt{2}\,\pi\, m^{3/2}}{h^3}\,E^{1/2}\,dE$$

where g(E) is called the density-of-states function

1 Mark

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5 (B)	Question: Consider a cube of gold (d = **1.00** mm) on an edge. Calculate the approximate number (N) of conduction electrons in this cube whose energies lie in the range E = **4.000** eV to E+ ΔE = **4.025** eV. Fermi energy for gold = **5.53** eV [3] Scheme: Volume of the gold cube: $V = d^3 = 1.00 \times 10^{-9}\ m^3$ Energy range: $\Delta E = 0.025\ eV$ $g(E) = \frac{8\sqrt{2}\pi m^{\frac{3}{2}}}{h^3} E^{\frac{1}{2}} = \frac{8\sqrt{2}\pi(9.11\times10^{-31}kg)^{\frac{3}{2}}}{(6.63\times10^{-34}J.s)^3}\sqrt{(4.00\ eV)(1.60\times10^{-19}J/eV)}$ $g(E) = 8.50 \times 10^{46}\ /\ J.m^3 = 1.36 \times 10^{28}\ /\ eV.m^3$ 2 Mark Since the energy levels are far below the fermi energy level, f(E) =1 Total number of electrons in the gold cube: $N = [g(E)]\ (\Delta E)\ (V)$ $= (1.36 \times 10^{28}\ /\ eV.m^3)\ (0.025eV)\ (1.00 \times 10^{-9}\ m^3)$ $N = 3.40 \times 10^{17}$ electrons 1 Mark
5 (C)	**Question:** What is top-down and bottom-up approach of nano material synthesis.[2] **Scheme:** The top down approach is physics friendly and deals with taking a bulk material and is crushed into fine particles using the processes such as mechanical alloying, laser ablation, sputtering etc. to produce the nanostructure. 1 Mark In the bottom up approach, individual atom or molecules are assembled or self-assembled to the desired size. 1 Mark

FORMAT FOR SUBMITTING QUESTION PAPER