University of Tlemcen **Academic year 2023-2024**
Faculty of Sciences **(L1 ING-INF)** **Semester 2**
Department of Informatic **Algebra (First Year)**

Worksheet N°2/ "Algebraic structures"

Exercise 01: In $E =]-1,1[$, we define $*$ by:

$$\forall (a,b) \in E^2, a * b = \frac{a+b}{1+ab}.$$

Show that $(E,*)$ is an abelian group.

Exercise 02: Let $E = \mathbb{R} \setminus \{-3\}$ be a set and $*$ is defined by :

$$\forall (a,b) \in E^2, a * b = ab + 3(a+b+2).$$

(1) verify that $*$ is a binary operation in E.

(2) Show that $(E,*)$ is an abelian group.

(3) Let f be the application:

$$\begin{array}{rcl} f & : & (\mathbb{R}^*, \cdot) \to (E,*) \\ x & \mapsto & f(x) = x - 3. \end{array}$$

Show that f is a group homomorphism. ($\cdot$ is the usual mutiplication)

Exercise 03: Let $(G,*)$ be an abelian group.

(1) If $H = \{x \in G : x = x^{-1}\}$, that is, H consists of all elements of G wich are their own inverses, prove that H is a subgroup of G.

(2) Let n be a fixed integer, and let $H = \left\{ x \in G : \underbrace{x * x * ... * x}_{n \text{ times}} = e \right\}$. prove that H is a subgroup of G.

Remark: We can write $\underbrace{x * x * ... * x}_{n \text{ times}} = x^n$.

Exercise 04: Let $\varphi : G \to H$ be a group homomorphism. The kernel of φ is defined to be the set:

$$\ker \varphi = \{g \in G / \varphi(g) = e_H\}.$$

Prove that $\ker \varphi$ is a subgroup of G.