University of Tlemcen **Academic year 2023-2024**
Faculty of Sciences **(L1 ING-INF)**
Department of Informatic **Algebra (First Year)**
Worksheet N°3/ "Relations"

Exercise 01: (catch-up 22-23) The relation $\Re$ is defined in $\mathbb{R}^*$ as:

$$x\Re y \Longleftrightarrow x^2 - \frac{1}{x^2} = y^2 - \frac{1}{y^2}.$$

(1) Show that $\Re$ is an equivalence relation on $\mathbb{R}^*$.

(2) What is the equivalence class of $a \in \mathbb{R}^*$ for this equivalence relation.

(2) Find the quotient set.

Exercise 02: The relation $\Re$ is defined in $\mathbb{Z} \times \mathbb{N}^*$ as:

$$(x, y)\, \Re\, (x', y') \Leftrightarrow xy' - x'y = 0.$$

(1) Show that $\Re$ is an equivalence relation on $\mathbb{Z} \times \mathbb{N}^*$.

(2) Identify $cl\,((1,2))$ and $cl\,((-1,2))$.

Exercise 03 : Let E be a non-empty set and F a non-empty sub-set of E.

In $P(E)$ the power set of E, we defined $\Re$ by:

$$\forall\, (A, B) \in P(E) \times P(E), A\Re B \Leftrightarrow A \cap F = B \cap F.$$

(1) Prove that $\Re$ is an equivalence relation.

(2) What is the equivalence class of $\emptyset$.

(3) Have they: $E \in Cl\,(\emptyset)$? Justify.

(4) Find $Cl\,(E)$. Deduce $Cl\,(F)$.

Exercise 04: The relation Φ is defined in $\mathbb{N}^*$ as:

$$x\Phi y \Leftrightarrow \exists n \in \mathbb{N}^* \text{ such as: } x^n = y.$$

(1) Prove that $\Re$ is an order relation.

(2) Is it a total order? Justify.

(3) Let $A = \{1, 4, 8\}$. Determine if they exist, $\max A$ and $\min A$ for the order Φ.

Exercise 05: (Final exam 22-23) In $]1,+\infty[$, the relation $\Re$ is defined:

$$x\Re y \Leftrightarrow \frac{y}{y^2+1} \geq \frac{x}{x^2+1}$$

(1) Show that $\Re$ is an order relation in $]1,+\infty[$.

(2) Is it a total order? Justify.

(3) Let $A = \{2,7,8\}$.
Determine if they exist $\sup A$ and $\inf A$.

Exercise 06: In $\mathbb{R}^2$, the relation $\leq$ is defined:

$$(x,y) \leq (x',y') \Leftrightarrow x \leq x' \text{ and } y \leq y'.$$

(1) Show that it is an order relation. Is it a total order?

(2) Specify two lower bounds, two upper bounds, supremum and infimum of the part:

$$A = \{(1,2),(3,1)\}.$$

(3) Does part A have a Maximum - ? a minimum?

Sincere wishes you success (MESSIRDI BACHIR)