University of Tlemcen **Academic year 2023-2024**
Faculty of Sciences **(L1 ING-INF)**
Department of Informatic **Algebra (First Year)**

Worksheet N°1/ "Logic - Modes of Reasoning"

Exercise 01: Solve in $\mathbb{R}$ the following equations and inequations:

(1) $x^2 - 3x + 1 = 0$	(2) $x^2 - 2\sqrt{3}x + 3 = 0$	(3) $2x^2 - 4x + 6 = 0$
(4) $x^2 - 3x + 1 > 0$	(5) $x^2 - 3x + 1 \leq 0$	(6) $x^2 - 3x + 6 > 0$
(7) $x^2 - 2\sqrt{3}x + 3 > 0$	(8) $\sqrt{x^2 - 3x} > 2$	(9) $\sqrt{x^2 + x - 2} > 1$

Exercise 02: Let $P, Q,\ R$ be three assertions.

(1) Draw up the truth table of the following assertion:

$$(A) : \left[(P \wedge Q) \Leftrightarrow \overline{R}\right] \Rightarrow \left[\bar{Q} \vee (Q \Rightarrow P)\right].$$

(2) Without using the truth table let us show that this proposition is true.

$$\left(Q \wedge \overline{R}\right) \Rightarrow \left[(P \Rightarrow R) \vee \left(P \wedge \bar{R}\right)\right].$$

(3) Say if the following assertions are true or false and write their negation.

$$(a)\ \forall x \in \mathbb{R}, x < 5 \Rightarrow x^2 < 25.$$

$$(b)\ \forall x, y \in \mathbb{R}, x > y \Rightarrow x^2 > y^2.$$
$$(c)\ \forall x, y \in \mathbb{R}^+, x < y \Rightarrow \sqrt{x} < \sqrt{y}.$$

$$(d) : (\forall x \in \mathbb{R})\,(\exists y \in \mathbb{Z})\,; (3x + y \leq 0).$$

$$(e)\ \forall x, y \in \mathbb{R}, \exists n \in \mathbb{N}, nx > y.$$

$$(f) : (\exists x \in \mathbb{R})\,(\forall y \in \mathbb{N})\,; -5x + 2y > 1.$$

$$(g)\ \forall x \in \mathbb{R}, \exists y \in \mathbb{R}^*, x^2 + 2xy + 3 > 0.$$

Exercise 03 : Prove that:

(1) $\forall n \in \mathbb{N}^*, \sqrt{n^2 + 1}$ is not an integer.

(2) (Additional) $\forall n \in \mathbb{N}^*, \sqrt{n + 1} + \sqrt{n}$ is not an integer.

(3) $\forall n \in \mathbb{N}^*, 7^n + 6n - 1$ is a multiple of 12.

(4) $\forall n \in \mathbb{N}^*, 2^{n-1} \leq n!$, with $n! = n\,(n-1)\,(n-2) \times ... \times 1$ and $0! = 1$.

(5)

$$\sum_{k=1}^{n} k^2 = \frac{n\,(n+1)\,(2n+1)}{6}.$$

<u>Exercise 04 :</u>

(1) Let a and p be two natural integers, prove that :

$$(p \text{ prime integer and } p \text{ divide } a^2) \Rightarrow p \text{ divide } a.$$

(2) (a) If p is a prime integer then $\sqrt{p}$ is an irrational number.

(b) deduce that, $\sqrt{2}+\sqrt{3}$ is irrational number.

(3) Prove that:

$$\sqrt{2}+\sqrt{3}+\sqrt{6} \notin \mathbb{Q}.$$

Sincere wishes you success (MESSIRDI BACHIR)