

Exercise 01: Let $E = \{x, y, z\}$ and $F = \{-2, 2\}$.

- (1) Identify the power set (ensemble des parties): $\wp(E)$, $\wp(F)$ and $\wp(\wp(F))$.
- (2) Determine 3 example of partition of the set E .
- (3) Add the following symbols: \in, \notin, \subset or $\not\subset$.

$x \dots E$	$\{y\} \dots E$	$3 \dots F$	$\{3\} \dots F$
$x \dots \wp(E)$	$\{y\} \dots \wp(E)$	$\emptyset \dots \wp(E)$	$\{\emptyset\} \dots \wp(E)$
$\{3\} \dots \wp(F)$	$\{-2, 1\} \dots \wp(F)$	$\{-2\} \dots \wp(F)$	$\{\{-2\}\} \dots \wp(F)$
$\{y\} \dots \wp(\wp(E))$	$\{\{y\}\} \dots \wp(\wp(E))$	$\emptyset \dots \wp(\wp(F))$	$\{\emptyset\} \dots \wp(\wp(F))$

Exercise 02: Let E be a nonempty set, A, B and C three sub-sets of E . Prove that:

(1)
$$A \subset B \Leftrightarrow C_E^B \subset C_E^A \Leftrightarrow A \cup B = B.$$

(2)
$$C_E^{A \cup B} = C_E^A \cap C_E^B.$$

(3)
$$A \cap B = A \cap C \Leftrightarrow A \cap C_E^B = A \cap C_E^C$$

(b) Give two assertions equivalent to the following assertion:

$$x \in (A - B) \Rightarrow x \notin A \cap B.$$

Say if this implication is true or false?

Exercise 03 : Let f and g be two defined applications of \mathbb{R} in \mathbb{R} such as:

$$f(x) = x^2 - 3x + 3 \text{ et } g(x) = \frac{x^2 - 5}{x^2 + 2}.$$

- (1) f and g are they injective? surjective? (Method of definitions)
- (2) Say if the following propositions are true or false?
 - (a) $f(\{0\}) = 3$, (b) $0 \in f^{-1}(\{3\})$, (c) $g^{-1}(0) = \frac{-5}{2}$,
 - (d) $g^{-1}(\{0\}) = \{\frac{-5}{2}\}$ and (e) $g^{-1}(\{10\}) = \emptyset$.
- (3) Find $f([0, 1])$, $f^{-1}([0, 1])$, $f(\mathbb{R})$, $g([0, 1])$.

Exercise 04: Let f defined from E in F by : $f(x) = \frac{3x}{x^2+x-2}$.

- (1) Find E for f to be an application.
- (2) Study the application f and draw its table of variations.
- (3) Say if f is injective and if it is surjective from E in \mathbb{R} ? (Don't forget to write the definitions and the rationale for your answer).
- (4) Otherwise give examples from the table of variations where f is bijective.
- (5) (Additional) Answer the same questions to: $g(x) = \frac{2}{\sqrt{(x+4)^2+1}} - 3$.
- (6) (Additional) Answer the same questions to: $h(x) = \frac{2}{\sqrt{x^2-1}}$.

Exercise 05: (Additional) Let $a, b, c, d \in \mathbb{R}^*$ and let f be defined as follows:

$$\begin{aligned} f & : A \rightarrow B \\ x & \mapsto f(x) = \frac{ax+c}{bx+d}. \end{aligned}$$

How should the greatest unknowns A and B and other constants be selected so that f is:

- (1) An application? (2) injective? (3) surjective? et (4) bijective?

Sincere wishes you success (MESSIRDI BACHIR)