University of Tlemcen **Academic year 2023-2024**
Faculty of Sciences **(L1 ING-INF)** **Semester 2**
Department of Informatic **Algebra (First Year)**
Worksheet N°1/ "Polynomials and Rational Fractions"

Exercise 01: Let be the two polynomials:

$$P(X) = 2X^6 - 3X^4 + 5X^3 - 7X^2 + 2X - 1 \text{ and } Q(X) = X^3 + X^2 - 3X + 2.$$

(1) Make the Euclidean division of P on Q.

(2) Make the division according to the increasing powers in order 3 of P on Q.

(3) Make the Euclidean division of Q on P.

(4) Make the division according to the increasing powers in order 2 of Q on P.

Exercise 02: What is the remainder of the Euclidean division of $(X+1)^n - X^n - 1$ by:

$$(1) X^2 - 3X + 2 \quad (2) X^2 + X + 1 \quad (3) X^2 - 2X + 1.$$

Exercise 03: Determine the pgcd between the two polynomials in each case:

(1)

$$P(X) = X^4 - 3X^3 + X^2 + 4 \text{ and } Q(X) = X^3 - 3X^2 + 3X - 2.$$

(2)

$$P(X) = X^5 - X^4 + 2X^3 - 2X^2 + 2X - 1 \text{ and } Q(X) = X^5 - X^4 + 2X^2 - 2X + 1.$$

(3)

$$P(X) = X^n - 1 \text{ and } Q(X) = (X-1)^n, n \geq 1.$$

Exercise 04: Find two polynomials U and V of $\mathbb{R}[X]$ such as $AU + BV = 1$ where:

$$A(X) = X^7 - X - 1 \text{ and } B(X) = X^5 - 1.$$

Exercise 05: Find the partial fraction decompostion of each:

$$\begin{aligned} (1) f(x) &= \frac{x^4 + 1}{x(x+2)(x-1)^3(x^2+1)}. \\ (2) g(x) &= \frac{x^5 + 1}{(x+3)(x^2+x+1)}. \end{aligned}$$