UNIVERSITY OF TLEMCEN
FACULTY OF SCIENCES
COMPUTER SCIENCE DEPARTMENT

GUIDED TUTORIALS N°4

Exercise 1

Formalize the given sentences using predicate logic:

1. Every even natural number greater than 2 is the sum of two prime numbers.
2. A student is a football-player if and only if he is athletic and disciplined.
3. There is no smallest positive real number.
4. Any student's absence of any exam implies his exclusion.
5. All non-athletic students are honest except one.

Exercise 2

Find two interpretations I_1 and I_2 such that the formula below is true in I_1 and false in I_2.

$$\forall x\ \forall y\ Q(g(x,y), g(y,y), z)$$

Exercise 3

The following three formulas express that the binary predicate p is reflexive, symmetric, and transitive. Demonstrate that none of these formulas is a logical consequence of the two others.

1. $\forall x\ p(x,x)$
2. $\forall x\ \forall y\ (p(x,y) \Rightarrow p(y,x))$
3. $\forall x\ \forall y\ \forall z\ (p(x,y) \wedge p(y,z) \Rightarrow p(x,z))$

Exercise 4

Provide, when it exists, a unifier for each pair of atoms (A_1, A_2).

- $A_1 = p(x, g(x), z)$ et $A_2 = p(f(y), g(f(b)), h(y))$
- $A_1 = p(x, h(b), h(x))$ et $A_2 = p(f(g(y)), y, h(f(g(h(a)))))$

Exercise 5

Convert the following formulas into prenex form:

1. $(\forall x\ \exists y\ R(x,z,y)) \Rightarrow (\exists x\ \forall y\ \exists t\ S(x,z,t))$
2. $((\exists x\ A(x) \Rightarrow \exists y\ B(y)) \Rightarrow \exists z\ C(z)) \Rightarrow \exists t\ D(t)$
3. $(\forall x\ \exists y\ \forall t\ R(x,z,t)) \Rightarrow (\exists x\ \forall y\ \exists t\ S(x,z,t))$
4. $\exists x\ \forall y (P(x) \wedge Q(y)) \Rightarrow \forall x\ \forall y\ \neg R(x,y)$

Convert the following formulas into Skolem normal form:

1. $\forall x\ \forall z\ \exists y\ \exists w\ (\forall t\ P(x,y,z,t) \Rightarrow \exists t\ Q(w,t))$
2. $\exists x\ \exists y\ P(x,y) \wedge \forall x\ \neg P(x,x)$
3. $\forall x\ P(x) \wedge \forall x\ (P(x) \Rightarrow \exists R(x,y))$