

# Table of Integration

1.  $\int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$
2.  $\int \frac{dx}{x} = \ln x$
3.  $\int e^x dx = e^x$
4.  $\int a^x dx = \frac{a^x}{\ln a}$
5.  $\int \sin x dx = -\cos x$
- 6.<sup>11</sup>  $\int \cos x dx = \sin x$
7.  $\int \frac{dx}{\sin^2 x} = -\cot x$
- 8.<sup>11</sup>  $\int \frac{dx}{\cos^2 x} = \tan x$
9.  $\int \frac{\sin x}{\cos^2 x} dx = \sec x$
10.  $\int \frac{\cos x}{\sin^2 x} dx = -\operatorname{cosec} x$
11.  $\int \tan x dx = -\ln \cos x$
12.  $\int \cot x dx = \ln \sin x$
13.  $\int \frac{dx}{\sin x} = \ln \tan \frac{x}{2}$
14.  $\int \frac{dx}{\cos x} = \ln \tan \left( \frac{\pi}{4} + \frac{x}{2} \right) = \ln (\sec x + \tan x)$
15.  $\int \frac{dx}{1+x^2} = \arctan x = \frac{\pi}{2} - \operatorname{arccot} x$
16.  $\int \frac{dx}{1-x^2} = \operatorname{arctanh} x = \frac{1}{2} \ln \frac{1+x}{1-x}$
17.  $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x = -\arccos x$
18.  $\int \frac{dx}{\sqrt{x^2+1}} = \operatorname{arsinh} x = \ln \left( x + \sqrt{x^2+1} \right)$
19.  $\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{arcosh} x = \ln \left( x + \sqrt{x^2-1} \right)$
20.  $\int \sinh x dx = \cosh x$
21.  $\int \cosh x dx = \sinh x$
- 22.<sup>11</sup>  $\int \frac{dx}{\sinh^2 x} = -\coth x$
23.  $\int \frac{dx}{\cosh^2 x} = \tanh x$
24.  $\int \tanh x dx = \ln \cosh x$
25.  $\int \coth x dx = \ln \sinh x$
26.  $\int \frac{dx}{\sinh x} = \ln \tanh \frac{x}{2}$

1.  $\int a f \, dx = a \int f \, dx$
2.  $\int [a f \pm b \varphi \pm c \psi \pm \dots] \, dx = a \int f \, dx \pm b \int \varphi \, dx \pm c \int \psi \, dx \pm \dots$
3.  $\frac{d}{dx} \int f \, dx = f$
4.  $\int f' \, dx = f$
5.  $\int f' \varphi \, dx = f \varphi - \int f \varphi' \, dx$  [integration by parts]
6.  $\int f^{(n+1)} \varphi \, dx = \varphi f^{(n)} - \varphi' f^{(n-1)} + \varphi'' f^{(n-2)} - \dots + (-1)^n \varphi^{(n)} f + (-1)^{n+1} \int \varphi^{(n+1)} f \, dx$
7.  $\int f(x) \, dx = \int f[\varphi(y)] \varphi'(y) \, dy$   $[x = \varphi(y)]$  [change of variable]
- 8.<sup>11</sup>  $\int (f)^n f' \, dx = \frac{(f)^{n+1}}{n+1}$   $[n \neq -1]$   
 For  $n = -1$   
 $\int \frac{f' \, dx}{f} = \ln f$
9.  $\int (a f + b)^n f' \, dx = \frac{(a f + b)^{n+1}}{a(n+1)}$
10.  $\int \frac{f' \, dx}{\sqrt{a f + b}} = \frac{2\sqrt{a f + b}}{a}$
11.  $\int \frac{f' \varphi - \varphi' f}{\varphi^2} \, dx = \frac{f}{\varphi}$
12.  $\int \frac{f' \varphi - \varphi' f}{f \varphi} \, dx = \ln \frac{f}{\varphi}$
13.  $\int \frac{dx}{f(f \pm \varphi)} = \pm \int \frac{dx}{f \varphi} \mp \int \frac{dx}{\varphi(f \pm \varphi)}$
14.  $\int \frac{f' \, dx}{\sqrt{f^2 + a}} = \ln \left( f + \sqrt{f^2 + a} \right)$
15.  $\int \frac{f \, dx}{(f+a)(f+b)} = \frac{a}{a-b} \int \frac{dx}{(f+a)} - \frac{b}{a-b} \int \frac{dx}{(f+b)}$   
 For  $a = b$   
 $\int \frac{f \, dx}{(f+a)^2} = \int \frac{dx}{f+a} - a \int \frac{dx}{(f+a)^2}$
16.  $\int \frac{f \, dx}{(f+\varphi)^n} = \int \frac{dx}{(f+\varphi)^{n-1}} - \int \frac{\varphi \, dx}{(f+\varphi)^n}$
17.  $\int \frac{f' \, dx}{p^2 + q^2 f^2} = \frac{1}{pq} \arctan \frac{q f}{p}$

18.  $\int \frac{f' dx}{q^2 f^2 - p^2} = \frac{1}{2pq} \ln \frac{qf - p}{qf + p}$
19.  $\int \frac{f dx}{1 - f} = -x + \int \frac{dx}{1 - f}$
20.  $\int \frac{f^2 dx}{f^2 - a^2} = \frac{1}{2} \int \frac{f dx}{f - a} + \frac{1}{2} \int \frac{f dx}{f + a}$
21.  $\int \frac{f' dx}{\sqrt{a^2 - f^2}} = \arcsin \frac{f}{a}$
22.  $\int \frac{f' dx}{af^2 + bf} = \frac{1}{b} \ln \frac{f}{af + b}$
23.  $\int \frac{f' dx}{f \sqrt{f^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{f}{a}$
24.  $\int \frac{(f'\varphi - f\varphi') dx}{f^2 + \varphi^2} = \arctan \frac{f}{\varphi}$
25.  $\int \frac{(f'\varphi - f\varphi') dx}{f^2 - \varphi^2} = \frac{1}{2} \ln \frac{f - \varphi}{f + \varphi}$