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Worksheet N°2
The complex numbers
LMD 1^{st} year 2024-2025

Exercise 1

1. Write in the "algebraic" form $(a+ib)$ the following complex numbers

$$\frac{1}{2+2i}, \qquad i(1+i)(1-i)^2, \qquad \frac{2+5i}{1-i}+\frac{2-5i}{1+i}$$

2. Write in the polar $(r(\cos\theta+i\sin\theta))$ and the exponential polar form $(re^{i\theta})$, the following complex numbers and there conjugate

$$\frac{1}{2+2i}, \qquad \sqrt{3}+i, \qquad -1+i\sqrt{3} \qquad (\textbf{Optional}).$$

3. Prove that
$$\frac{\sqrt{2}(\cos(\frac{\pi}{12})+i\sin(\frac{\pi}{12}))}{1+i}=\frac{\sqrt{3}-i}{2}.$$
$$(1-i)\times(\cos(\frac{\pi}{5})+i\sin(\frac{\pi}{5}))\times(\sqrt{3}-i)=2\sqrt{2}\times(\cos(\frac{13\pi}{60})-i\sin(\frac{13\pi}{60})). \qquad (\textbf{Optional})$$

4. Linéarize the following expressions $(\cos x)^3$ and $((\sin x)^4$ (**Optional**)).

Exercise 2
Let $a=\sqrt{3}+i$ and $b=\sqrt{3}-1+i(\sqrt{3}+1)$ be two complex numbers,

1. Check that $b=(1+i)a$.
2. Deduce that $|b|=2\sqrt{2}$ and $\arg(b)=\frac{5\pi}{12}$ $[2\pi]$.
3. Deduce from the above that: $\cos(\frac{5\pi}{12})=\frac{\sqrt{6}-\sqrt{2}}{4}$.

Exercise 3

1. Find the squar roots for a complex number

$$-1, \qquad i, \qquad 1+i, \qquad \frac{\sqrt{3}+i}{2} \qquad (\textbf{Optional})$$

2. Find $z\in\mathbb{C}$ such that

$$z^2-(3+4i)z-1+5i=0, \qquad z^2=\frac{\sqrt{3}}{2}+i\frac{1}{2}, \qquad z^3+8=0, \qquad z^4+i=0.$$

Exercise 4

1. Let 'f' be a function defined from $\mathbb{C}$ to $\mathbb{C}$, by

$$\forall z \in \mathbb{C}, \qquad z \neq -i, \qquad f(z) = \frac{1-z}{1-iz}$$

   (a) Find $z \in \mathbb{C}$ such that $f(z) \in \mathbb{R}$, and $f(z) \in i \times \mathbb{R}$.

2. (**Optional**) Determine in each case, the set of points $M(x, y)$, with affix $z = x + iy$ such that:

$$|z - (2 - i)| = \sqrt{2}, \qquad |z - 1 - 2i| = |z + 2 - i|\,, \qquad |\overline{z} - 2i| = |z + 2|\,.$$