1 Definitions and properties

(1) Let x and y be two real numbers. A complex numbers is an ordred pair (x, y) or noted $z = x + iy$ (said algebraic writing). x is the real part noted $\operatorname{Re} z$ and y is the imaginary part noted $\operatorname{Im} z$ and i is the imaginary number with the property $(i)^2 = -1$. The set of the complex numbers is noted by $\mathbb{C}$. If $y = 0$ then z is a real numbers and if $x = 0$ then z is pure imaginary numbers. Note that if: $z = x + iy$ and $z' = x' + iy'$, then:

(i) $-z = (-x) + i(-y) = -x - iy$.

(ii) $z + z' = (x + x') + i\,(y + y')$.

(iii) $z \times z' = (xx' - yy') + i\,(xy' + yx')$.

(2) In the plane reported to an orthonormed frame, the point M of abscissa x and ordinate y is the image of the complex number z. It is said again that M has for affix z.

(3) Modulus - Argument

a) The modulus of z is the norm of vector $\overrightarrow{OM}$. It is referred to as:

$$r = \operatorname{mod}(z) = |z| = \left\| \overrightarrow{OM} \right\|,$$

We have:

$$x = r\cos\theta \text{ and } y = r\sin\theta$$

where ■ is the angle between the x-axis and the $\overrightarrow{OM}$ vector.

The result is:

$$r = \sqrt{x^2 + y^2},$$

hence the trigonometric or polar script:

$$z = r\,(\cos\theta + i\sin\theta)\,, (z \neq 0)\,.$$

As we can call writing: $z = re^{i\,\theta}$ by exponential writing.

The argument of a complex number is the angle θ with $-\pi < \theta \leq \pi$ and it's denoted by $\arg z$ such that for $x \neq 0, \tan\theta = \frac{y}{x}$.

The convention for complex numbers is to use radians as the measure for angles.

Remark 1

$$\begin{aligned} x^2 + 1 &= 0 \Rightarrow x^2 = -1 = i^2 \Rightarrow x = i \;\; or \; x = -i. \\ x^2 + 4 &= 0 \Rightarrow x^2 = -4 = i^2 4 \Rightarrow x = 2i \;\; or \; x = -2i. \\ x^2 + 9 &= 0 \Rightarrow x^2 = -9 = i^2 9 \Rightarrow x = 3i \;\; or \; x = -3i. \end{aligned}$$

$$\begin{aligned} x^2 + x + 1 &= 0 \Rightarrow \triangle = -3 = 3i^2 \\ z_1 &= \frac{-1 - i\sqrt{3}}{2} \ and \ z_2 = \frac{-1 + i\sqrt{3}}{2}. \end{aligned}$$