Hypothesis Testing

If we want to prove something in real world using Data then we perform Hypothesis Test.

- How to set up a hypothesis?
- How to perform an hypothesis test?
- How to draw conclusions?
- Statistical significance
- p-value
- Type I / II error
- And more topics…

Here starts the Inferential Statistics

* What is hypothesis Testing?

In hypothesis testing from a given population we take sample, then draw various inferences about the population.

H_0 = Null hypothesis
H_1 = Alternate hypothesis

Process:-

1. Stating Hypothesis
2. Level of Significance
3. Perform Tests
4. Draw Conclusions
   - Reject Hypothesis
   - Don't Reject Hypothesis

$P \rightarrow S$

$\mu_p \rightarrow \mu_s$

$\sigma_p \rightarrow \sigma_s$

① Stating Hypothesis

H_0

mean

Avg Bachelors Salary $\geq$ Avg masters Salary

$$\mu_B \geq \mu_m$$

proportion

$$p = 0.5$$

H_1

mean

Avg Bachelors Salary $<$ Avg masters Salary

$$\mu_B < \mu_m$$

proportion

$$p \neq 0.5$$

② Level of Significance

We need to have enough data in order to reject the Null hypothesis. This level of significance where after enough trials we decide whether to reject or not reject null hypothesis

It is explained in the testing phase with e.g

③ Testing

$H_0 =$ Coin is fair $p = 0.5$
$H_1 =$ Coin isn't fair $p \neq 0.5$

Lets toss a coin 5 times and note outcomes

Trial No	Outcome	Probability
1	H	0.5
2	H	0.25
3	H	0.125
4	H	0.06
5	H	P-value → 0.03

Assumed $\alpha = 0.05$ in this case

At this point we might believe that the coin isn't fair. Bse p-value has become lesser than α

④ Conclusion

Since $\alpha > 0.03$ we reject the null hypothesis

* Type I Error and Type II Error

	H_0 is true	H_0 is false
Reject H_0	Type I error probability = α	No error
Accept H_0	No error	Type II error probability = β

If we want to avoid Type I error then we may try to reduce α. However if we keep on increasing α then we may accept H_0, even it maybe false then this causes type II error. Same can happen vice-versa

$\alpha \downarrow$	$\beta \uparrow$
$\beta \downarrow$	$\alpha \uparrow$

$$\boxed{\text{Power of Statistical Test} = 1 - \beta}$$

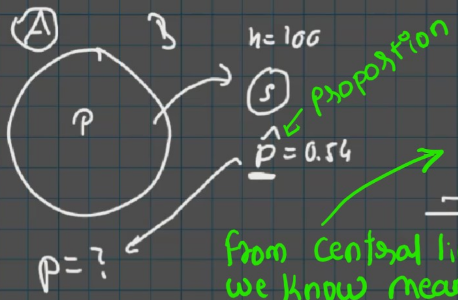
Now in order to prevent this we must increase the sample size. When sample size becomes equal to population size then chances of Type I & Type II errors becomes zero. However we cannot take sample size equal to population size as it's not feasable.

Therefore increasing the sample size in a optimum way depends on the type of test we are performing.

Various tests we can perform are discussed further.

* Confidence Interval and Margin of Error

Confidence interval and margin of error



From central limit Thm we know mean of m samples form a normal distn

$\mu + 2\sigma$

$\mu - 2\sigma$

95 % . $\hat{p}$ **is within** $2\sigma_s$

$\mu_s = p$

$\sigma_s = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{p \cdot (1-p)}}{\sqrt{n}}$

$\hat{\sigma}_s = \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} = \frac{\sqrt{0.54 \cdot 0.46}}{\sqrt{100}} \approx 0.05$

These values are fixed by domain expert

95 % confident :

$0.54 \pm 2 \cdot 0.05 \rightarrow$

0.44 and 0.64

$\alpha = 0.05$ $\beta = 0.1$ ← margin of error

And accordingly we select sample size

* Excursion (calculating sample size & power)

website :- http://powerandsamplesize.com/

We can calculate the appropriate sample size based on α & β values.

* Performing the Hypothesis Test

So let's assume we are a farmer and we have some chicken and we want to know if our eggs are smaller

or bigger than the rest of the eggs of the other farmers.

So let's assume in this example, we know that in the population of all eggs in general, the average

or the mean of the weight of this eggs is 53 grams.

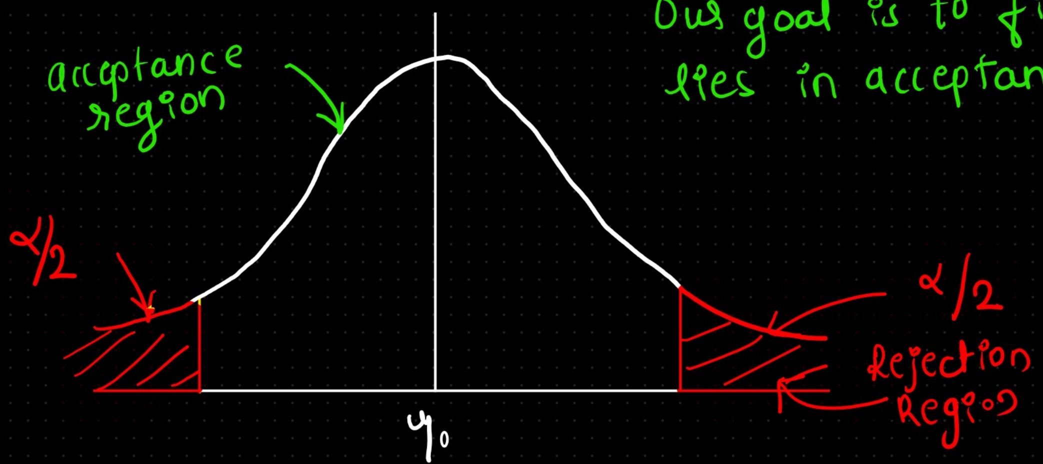
And we also know that the standard deviation is around five grams. $\alpha = 0.05$ $n = 100$, $\bar{x} = 54.5$

Sol^n → $\mu_0 = 53$, $n = 100$, $\sigma_0 = 5$, $\bar{x} = 54.50$, $\alpha = 0.05$

we will use two sided test as we don't know on which side the deviation will be

$$H_0 = \mu \leq 53$$
$$H_1 = \mu > 53$$

Our goal is to find if $\bar{x}$ lies in acceptance region



$$Z_{score} = \frac{\bar{x} - \mu_0}{\sigma_s}, \quad \sigma_s = \frac{\sigma_0}{\sqrt{n}} = \frac{5}{\sqrt{100}} = 0.5$$

$$Z_{score} = \frac{54.50 - 53}{0.5}, \quad Z_{score} = 3$$

from z-table probability $(P) = 0.9987$

$P(\bar{x} < 54.5) = 0.9987$

$(\alpha/2)$

$P_{value} = 1 - 0.9987 = 0.0013 < 0.025$

Therefore we reject the Null Hypothesis

* Proportion Testing :-

Steps followed :-

H_0 :- $p =$ $\quad$ H_a :- $p \neq$

H_0 :- $p \leq$ $\quad$ H_a :- $p >$

H_0 :- $p \geq$ $\quad$ H_a :- $p <$

{ P = Assumed proportion }

$$\boxed{\text{Proportion } (\hat{p}) = \frac{\text{No. of success}}{\text{Sample Size}}}$$

$$\hat{p} = \frac{S}{n}$$

{ $\hat{p}$ = Sample proportion }

$$\sigma_p = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\boxed{Z_{score} = \frac{\hat{p} - p}{\sigma_p}}$$

{ This z-score from z table will give us probability }

Let's solve a problem,

we read in a magazine that 25 percent of the people are vegetarians.

And we assume that in our country this is a lot different because we assume that in our country this would be less than 25 percent. 91 people out of these 400 people say that they are vegetarians.

So this is our strong assumption and therefore we want to challenge this null hypothesis and test against it.

→ $H_0 = p < 0.25$ $\quad$ $n = 400$

$H_a = p > 0.25$ $\quad$ $s = 91$

$$\hat{p} = \frac{91}{400} = 0.2275$$

We will choose $\alpha = 0.05$

$$\sigma_p = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.2275(1-0.2275)}{400}} = 0.021$$

$$Z = \frac{\hat{p} - p}{\sigma_p} = \frac{0.2275 - 0.05}{0.021} = (-1.07)$$

from Ztable $p = 0.8577$

Since z is negative we have to substract it from 1 to get our surged P value

$$P\text{value} = 1 - 0.8577 = 0.1423$$

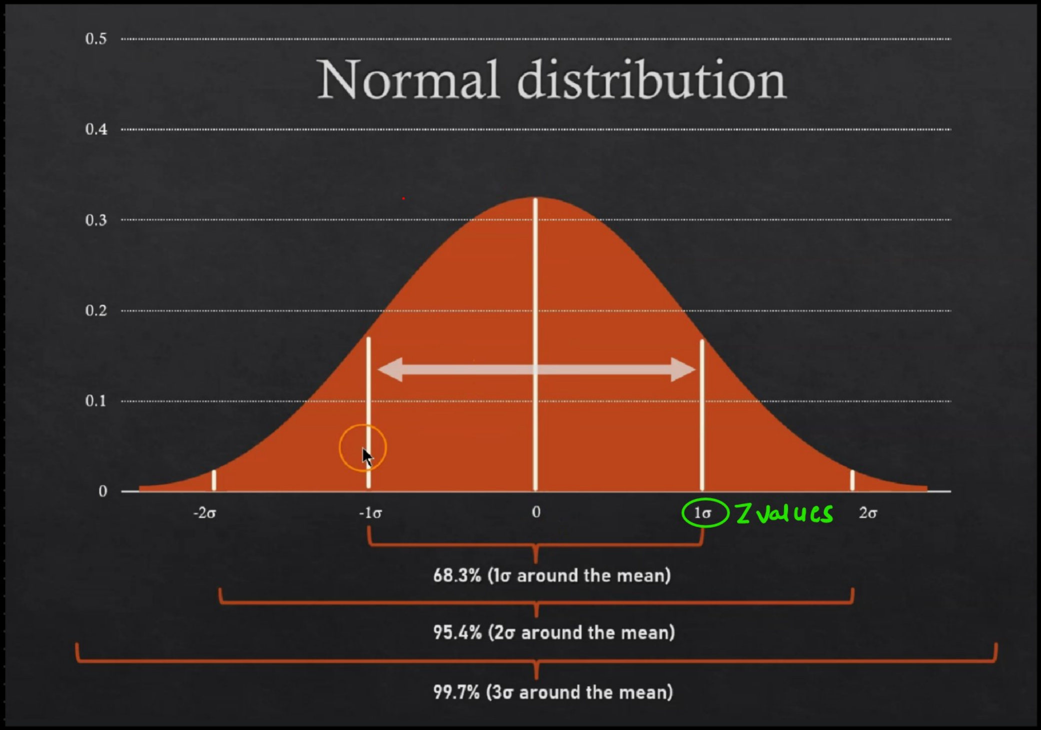
Now we compare this value with our α level

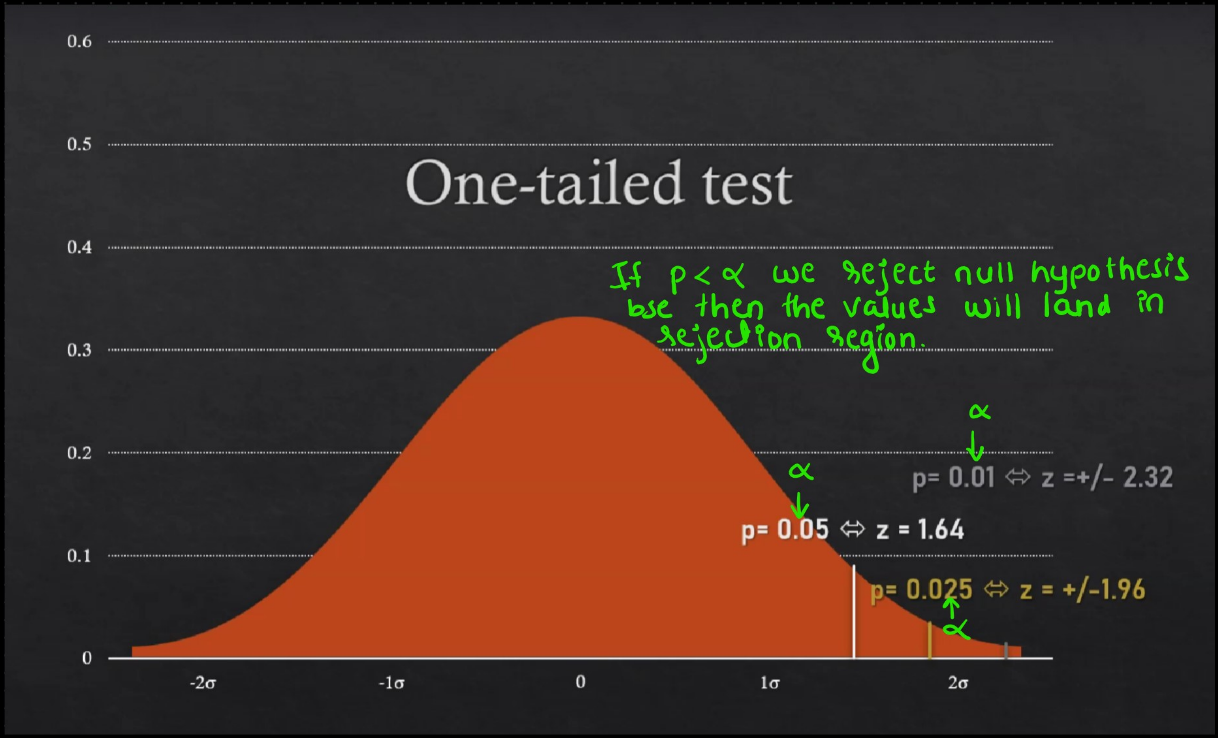
$$0.1423 > 0.05$$

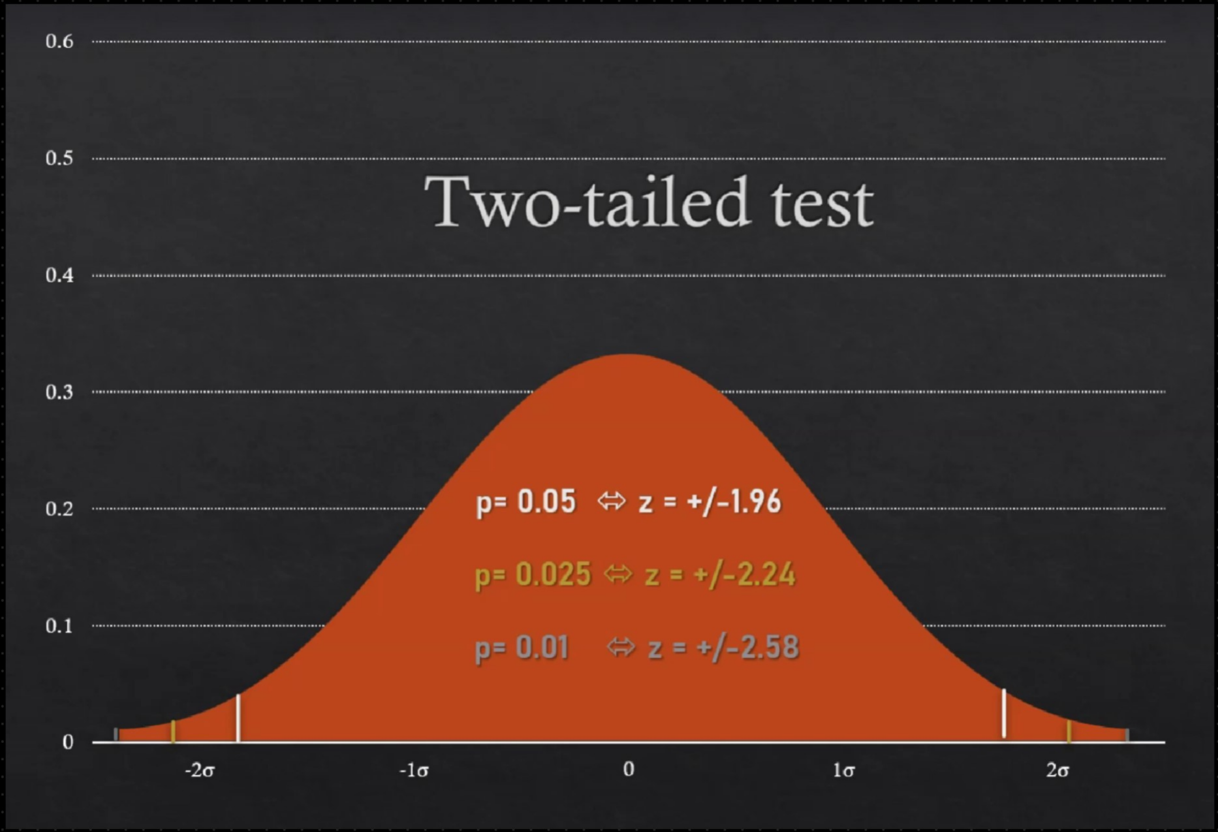
$$\therefore \; P\text{value} > \alpha$$

Since Pvalue is not smaller than α we failed to reject our null hypothesis

* Important P-Z pairs




0.6
0.5
One-tailed test
0.4
If p < α we reject null hypothesis
bse then the values will land in
rejection region.
0.3
α
0.2
p= 0.01 ⇔ z =+/- 2.32
α
p= 0.05 ⇔ z = 1.64
0.1
p= 0.025 ⇔ z = +/-1.96
α
0
-2σ
-1σ
0
1σ
2σ


0.6
0.5
Two-tailed test
0.4
0.3
0.2
p= 0.05 ⇔ z = +/-1.96
p= 0.025 ⇔ z = +/-2.24
0.1
p= 0.01 ⇔ z = +/-2.58
0
-2σ
-1σ
0
1σ
2σ