Exercise 1

Problem 1: Introduction to R-studio

a) Learn how to change the working directory in R-studio. Furthermore, try the commands help() and help.search().

b) Calculate the affine transformation $\boldsymbol{y} = \boldsymbol{x}\boldsymbol{A}^{-1} + \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & 5 \\ -2 & 7 & 0 \\ 5 & -8 & -1 \end{pmatrix}, \qquad \boldsymbol{x}^T = \begin{pmatrix} 8 \\ -4 \\ 2 \end{pmatrix}, \qquad \boldsymbol{b}^T = \begin{pmatrix} 3 \\ 10 \\ -19 \end{pmatrix}$$

c) Use the package **mvtnorm**. Set seed 123 using the command **set.seed(123)**. Generate 100 observations from a two dimensional normal distribution with expected value $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Visualize the observations.

$$\boldsymbol{\mu} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \boldsymbol{\Sigma} = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}$$

d) Using the data from c), calculate the sample mean $\bar{\boldsymbol{x}}$ and the sample covariance matrix $\boldsymbol{S}_x$. Calculate the eigenvalues and eigenvectors from the matrix. Check that the following equations hold: $Tr\,(\boldsymbol{S}_x) = \lambda_1 + \lambda_2 + ... + \lambda_p$ and $Det\,(\boldsymbol{S}_x) = \lambda_1\lambda_2...\lambda_p$, where λ_i are the eigenvalues of $\boldsymbol{S}_x$. Note that the covariance matrix is always positive semidefinite.

e) Calculate the affine transformation $\boldsymbol{y}_i = \boldsymbol{A}\boldsymbol{x}_i + \boldsymbol{b}$, where

$$\boldsymbol{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix},$$

verify that $\bar{\boldsymbol{y}} = \boldsymbol{A}\bar{\boldsymbol{x}} + \boldsymbol{b}$ and $\boldsymbol{S}_y = \boldsymbol{A}\boldsymbol{S}_x\boldsymbol{A}^T$. What does affine equivariance mean in practice?

f) Read the file **Data1.txt** into your workspace. Create a function, that centers your data (removes the mean) and pairwise scatterplots the variables. Calculate the sample covariance and correlation matrices. Furthermore, calculate the eigenvalues and -vectors from the matrices.

Problem 2: The eigenvalues of a symmetric matrix

Prove that the eigenvalues of a symmetric matrix are real.

Home Exercise 1: Functions

a) Write your own R-function, that takes a matrix as an argument and returns the sample covariance matrix. Do not use the built-in function cov().

b) Write your own R-function that takes a covariance matrix $\boldsymbol{A}$ as an argument and it returns the square root of the inverse matrix such that $\boldsymbol{A}^{-\frac{1}{2}}\boldsymbol{A}^{-\frac{1}{2}} = \boldsymbol{A}^{\mathbf{-1}}$.

c) Use the functions from (a) and (b) and write a new function, that takes a covariance matrix as an argument and returns the Pearson's correlation matrix.