

# MS-E2112 - Multivariate statistical analysis - Home exercise 4

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## Problem 1a

### Exercise 4

→ Asymptotic & Sample breakdown point of sample median.

Let  $X_n = \{x_1, x_2, \dots, x_n\}$  where  $x_1, x_2, \dots, x_n$  are  $n$  independent and identically distributed observations from the distribution  $F_X$ .

Assume that  $m < n$  and replace  $x_1, x_2, \dots, x_m$  with  $x_1^*, x_2^*, \dots, x_m^*$ .

Let  $X_n^* = \{x_1^*, x_2^*, x_3^*, \dots, x_m^*, x_{m+1}, \dots, x_n\}$ .

Now the maximum bias

$$\max \text{Bias}(m, X_n, Q) = \sup_{x_1^*, x_2^*, \dots, x_m^*} d(Q(X_n), Q(X_n^*))$$

$$BP(Q, n) = \min_m \left\{ \frac{m}{n} \mid \max \text{Bias}(m, X_n, Q) = \infty \right\}$$

For odd case:-  $m = \frac{n+1}{2}$

$$B.P.(Q, n) = \frac{m}{n} = \frac{n+1}{2n}$$

$$\text{Asymptotic B.P.} = \lim_{n \rightarrow \infty} B.P.(Q, n)$$

$$= \lim_{n \rightarrow \infty} \left( \frac{1}{2} + \frac{1}{2n} \right)$$

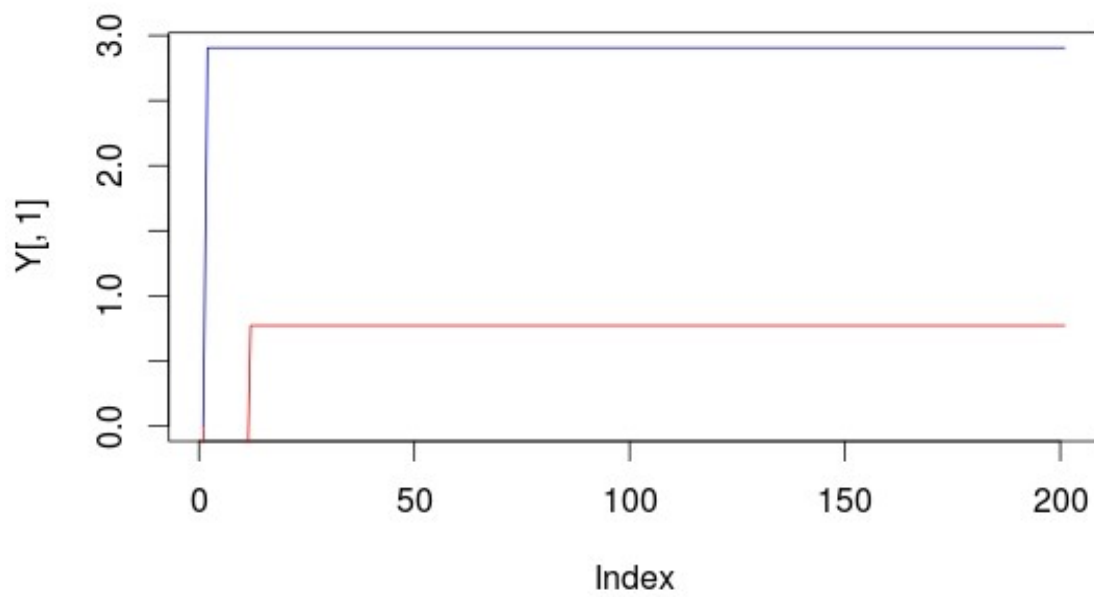
$$= \underline{\underline{\frac{1}{2}}}$$

For even case:  $m = \frac{n}{2}$

$$B.P.(Q, n) = \frac{m}{n} = \frac{n}{2n} = \underline{\underline{\frac{1}{2}}}$$

$$\begin{aligned}
 \text{Asymptotic B.P} &= \lim_{n \rightarrow \infty} \text{B.P}(g, n) \\
 &= \lim_{n \rightarrow \infty} \left( \frac{1}{2} \right) \\
 &= \underline{\underline{\frac{1}{2}}}
 \end{aligned}$$

Problem1b



The empirical influence function for the median is given in the above figure for bivariate data.

### Problem1c

As, the empirical influence for the median is clearly bounded (here the negative part of the plot is not available) it tells that median is a robust estimate.