# Laboratory Session 6

Computer Vision - Università Degli Studi di Genova

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***Abstract*—The aim of this lab is to estimate the fundamental Matrix F of a pair of stereo images. Therefore the 8-Points algorithm is implemented.**

***Index Terms*—Stereopsis, Fundamental Matrix, 8-Points-Algorithm**

## I. Introduction

Two different pairs of images were given, each pair showing a scene from a slightly shifted perspective, as usual in stereovision. For each image pair, a set of correspondence points was given, in order to focus only on the estimation of F, without taking into account possible errors in determining the correspondences. Fig. 1 shows the first scene with its 15 correspondence points (red crosses), fig. 2 shows the second scene with 13 correspondences. These points are used as a input to the 8-Points-Algorithm to estimate the fundamental matrix F, which allows determining the epipolar lines corresponding to each point of interest.

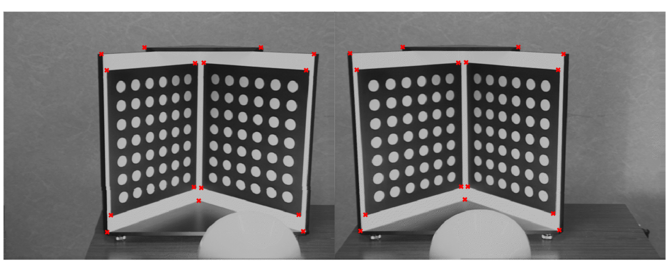

Fig. 1: Scene ”Mire” shown from two perspectives with 15 point correspondences. Source: [2]

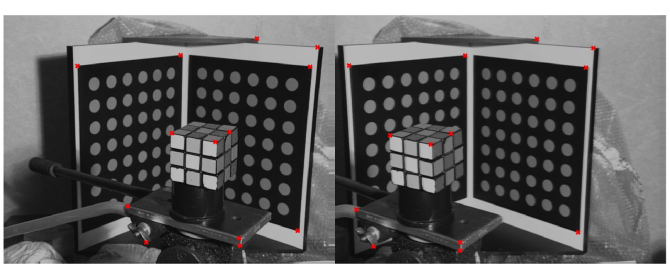

Fig. 2: Scene ”Rubik” shown from two perspectives with 13 point correspondences. Source: [2]

## II. Procedure

The 8-Points-Algorithm is applied to both image pairs in two different versions: The first version of the 8-Points-Algorithm finds the fundamental matrix just based on the pairs of points directly, the second version of the algorithm normalizes the points before computing F in order to increase precision of F.

### A. Version 1

The first step to obtain the matrix F is to define the matrix A, s.t

$$A * f = 0 \tag{1}$$

where f is a column vector containing the elements of the fundamental matrix F. In detail, A and f have the following form

$$A = \begin{bmatrix} x_1'x_1 & x_1'y_1 & x_1' & y_1'x_1 & y_1'y_1 & y_1' & x_1 & y_1 & 1 \\ x_2'x_2 & x_2'y_2 & x_2' & y_2'x_2 & y_2'y_2 & y_2' & x_2 & y_2 & 1 \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ x_n'x_n & x_n'y_n & x_n' & y_n'x_n & y_n'y_n & y_n' & x_n & y_n & 1 \end{bmatrix} \tag{2}$$

$$f = \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ ... \\ f_{33} \end{bmatrix} \tag{3}$$

In order to solve Eq. 1, a SVD (Singular Value Decomposition) of matrix A is performed. Because the system is overdetermined, there are multiple solutions to this linear equation. We can choose a solution from the nullspace of A, which is contained in the last columns of V. We pick the last column of V as f and reshape the vector into a 3x3 Matrix. There is one constraint for Matrix F which still needs to be met: $det(F) = 0$. This is why we need to enforce rank 2 instead of rank 3 which will generally be the case. That can be accomplished by applying again SVD to F and setting the last element to zero: $D' = D$ with $D(3,3) = 0$. Then the new F' with rank 2 will be $F' = UD'V^T$ After having computed F, the epipolar lines of each point can be displayed in the other image of the stereo image pair.

### B. Version 2

As stated before, now in order to find the fundament matrix F, the set of points given is first normalized for eac pair of images before executing the 8-Points-Algorithm. Th normalization can be seen as a single transformation whe all the points are translated and scaled in the same way, a shown in Eq. 4. Then, after the 8-Points-Algorithm resolve the fundamental matrix obtained is now denormalized bac following the Eq. 5.

$$T = \begin{bmatrix} \frac{\sqrt{2}}{m_d} & 0 & -\frac{\sqrt{2}}{m_d}c_x \\ 0 & \frac{\sqrt{2}}{m_d} & -\frac{\sqrt{2}}{m_d}c_y \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$F = (T')^T \tilde{F} T \tag{5}$$

Besides from these transformations, the procedure follows the same line as the previous version, where a new matrix A -given by the Eq. 2- is computed but now using the normalized points.

## III. Results and Analysis

The script can be executed using the following command in MATLAB:

*Lab6();*

All the output images are the stored in the subdirectory *output/*.

### A. Version 1

After running the scripts developed for version 1 of the algorithm, Fig. 3 and Fig. 4 were obtained. It can be seen that, for both pairs of images, the points given in the left image were correctly identified to the ones in the right image. This is said because, the color lines shown represent the epipolar lines which should pass -as close as possible in practice- through the same given point in both images. This also means that the fundamental matrix obtained, shown in the Eq. 6 for the Mire scene and in the Eq. 7 for the Rubik scene, for each pair of images does a good job in correlating each point in the left image to its corresponding point in the right image.

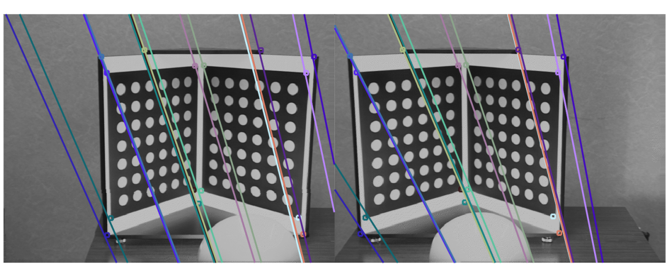

Fig. 3: Scene "Mire" with epipolarlines from version 1 of algorithm (no normalization). Source: Own elaboration

$$F = \begin{bmatrix} -0.0000 & 0.000 & -0.0034 \\ -0.0000 & 0.000 & 0.0023 \\ 0.0050 & -0.0019 & -1.000 \end{bmatrix} \tag{6}$$

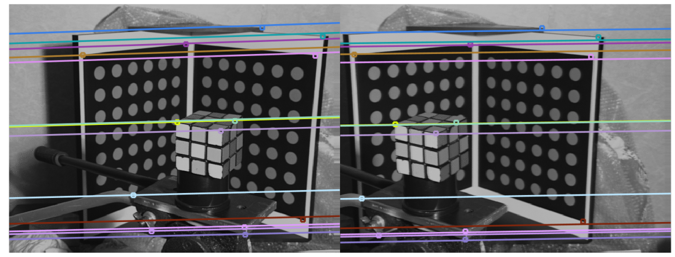

Fig. 4: Scene "Rubik" with epipolarlines from version 1 of algorithm (no normalization). Source: Own elaboration

$$F = \begin{bmatrix} -0.0000 & 0.0000 & 0.0013 \\ 0.0000 & -0.0000 & 0.1180 \\ -0.0041 & -0.1190 & 0.9858 \end{bmatrix} \tag{7}$$

However, a visual confirmation is not enough to conclude that the algorithm is working properly, and thus for the fundamental matrix estimated to be correct, Eq. 8 must be satisfied, which comes from the epipolar constraints definition. In Eq. 9 the results from this constraint obtained for the Mire scene is shown and in Eq.10 for the Rubik scene.

$$(x')^T F x = 0 \tag{8}$$

$$(x')^T F x = \begin{matrix} 0.0072 & -0.0018 & 0.0073 \\ 0.0100 & 0.0227 & 0.0253 \\ 0.0252 & 0.0547 & 0.0246 \\ 0.0401 & 0.0427 & 0.0008 \\ 0.0192 & 0.0115 & 0.0010 \end{matrix} \tag{9}$$

$$(x')^T F x = \begin{matrix} 0.0905 & -0.7632 & -0.7653 & -0.1886 \\ -0.0934 & -0.2757 & -0.2190 & -0.6220 \\ -0.4213 & -0.1686 & -0.8453 & -0.0050 \\ -0.1218 & & & \end{matrix} \tag{10}$$

It can be seen that even though the values obtained are not completely equal to zero, they are close by up to two decimals. The closer these values are to zero, the better the estimation of the fundamental matrix for the pair of images is. This result shows that a better estimation of F could be obtained, as shown in the next section. Finally, the epipoles obtained are shown in Eq. 11 for the Mire scene and in Eq.12 for the Rubik scene.

$$\begin{aligned} \text{Left epipoles } &= \begin{bmatrix} 0.4487 & 0.8937 & 0.0005 \end{bmatrix} \\ \text{Right epipoles } &= \begin{bmatrix} 0.4091 & 0.9125 & 0.0003 \end{bmatrix} \end{aligned} \tag{11}$$

$$\begin{aligned} \text{Left epipoles } &= \begin{bmatrix} 0.9999 & -0.0113 & 0.0000 \end{bmatrix} \\ \text{Right epipoles } &= \begin{bmatrix} 0.9994 & -0.0346 & -0.0000 \end{bmatrix} \end{aligned} \tag{12}$$

### B. Version 2

After running the scripts developed for version 1 of the algorithm, Fig. 5 and Fig. 6 were obtained. Following the same reasoning as before, it can be noticed how the epipolar lines passes through the identified points. However, also notice that the epipolar lines for the Fig. 5 are way more horizontal than those in the Fig. 3. This is result of the normalization transformation applied to the points before executing the 8-Points-Algorithm, which results in a conditioned number of the matrix A and in a fundamental matrix estimate that is invariant to point transformations. Therefore, a better F estimate should be expected where the epipolar constrains are met in a better way than the non-normalized estimate. The fundamental matrices obtained are shown in Eq.13 for the Mire scene and in Eq.14 for the Rubik scene.

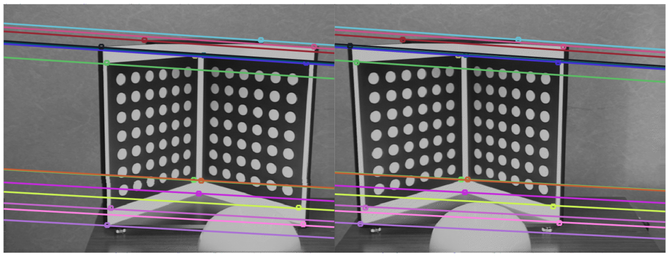

Fig. 5: Scene "Mire" with epipolarlines from version 2 of algorithm (with normalization). Source: Own elaboration

$$F_n = \begin{bmatrix} -0.0000 & 0.0000 & 0.0003 \\ -0.0000 & 0.0000 & 0.0042 \\ -0.0003 & -0.0042 & -0.0356 \end{bmatrix} \tag{13}$$

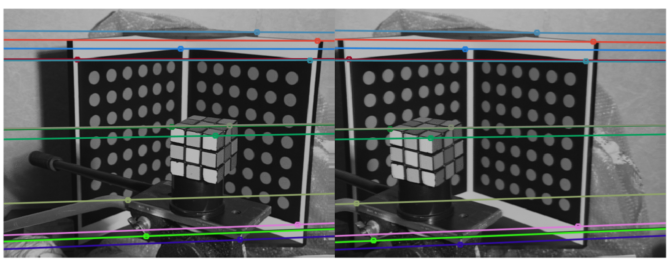

Fig. 6: Scene "Rubik" with epipolarlines from version 2 of algorithm (with normalization). Source: Own elaboration

$$F_n = \begin{bmatrix} -0.0000 & -0.0000 & 0.0001 \\ 0.0000 & -0.0000 & -0.0044 \\ -0.0000 & 0.0044 & -0.0071 \end{bmatrix} \tag{14}$$

Now the epipolar constrain of Eq. 8 should be evaluated for the new fundamental matrices obtained. In Eq. 15 the results from this constraint obtained for the Mire scene are shown and in Eq.16 for the Rubik scene.

$$(x')^T F_n x = \begin{array}{ccc} 0.0094, & -0.0004, & -0.0000, \\ 0.0050, & 0.0031, & 0.0097, \\ -0.0048, & 0.0072, & 0.0030, \\ 0.0010, & 0.0092, & 0.0043, \\ 0.0021, & 0.0099, & 0.0069 \end{array} \tag{15}$$

$$(x')^T F_n x = \begin{array}{cccc} 0.030, & 0.0012, & 0.0023, & -0.0020, \\ -0.0014, & 0.0011, & -0.0012, & 0.0013, \\ 0.0035, & -0.0010, & 0.0012, & 0.0015, \\ 0.0014 & & & \end{array} \tag{16}$$

It can be noticed how these new fundamental matrices estimates obtained shown in Eq. 15 and in Eq. 16 are much more closer to zero than the ones obtained using the non-normalized version of the algorithm (Eq. 11 and Eq. 12). This means that these normalized fundamental matrices are a better estimation for matching the identified points from one image to the other. Finally, the epipoles obtained are shown in Eq. 17 for the Mire scene and in Eq. 18 for the Rubik scene.

$$\begin{aligned} \text{Left epipoles } &= \begin{bmatrix} 0.9975 & 0.0711 & 0.0000 \end{bmatrix} \\ \text{Right epipoles } &= \begin{bmatrix} 0.9977 & 0.0672 & 0.0000 \end{bmatrix} \end{aligned} \tag{17}$$

$$\begin{aligned} \text{Left epipoles } &= \begin{bmatrix} -0.9999 & -0.0168 & -0.0001 \end{bmatrix} \\ \text{Right epipoles } &= \begin{bmatrix} -1.0000 & -0.0095 & -0.0001 \end{bmatrix} \end{aligned} \tag{18}$$

## IV. Conclusions

- The 8-Points Algorithm effectively estimate the fundamental matrix F of a stereo image pair.
- It is advisable to normalized the identified points before applying the algorithm for a more robust F estimation.
- The epipolar constraints should be checked in order to measure the reliability of the F estimated.

## References


[1] Noceti, N. Stereopsis, 2020. Retrieved from: https://2019.aulaweb.unige.it/pluginfile.php/256819/mod_resource/content/3/lecture11_stereo1.pdf

[2] Noceti, N. Lab Session n.6, 2020. Retrieved from: https://2019.aulaweb.unige.it/mod/assign/view.php?id=67442