Exercise 13.2

Question 1:

Using laws of exponents, simplify and write the answer in exponential form:

(i) $3^2 \times 3^4 \times 3^8$ (ii) $6^{15} \div 6^{10}$

(iii) $a^3 \times a^2$ (iv) $7^x \times 7^2$

(v) $(5^2)^2 \div 5^3$ (vi) $2^5 \times 5^5$

(vii) $a^4 \times b^4$ (viii) $(3^4)^3$

(ix) $(2^{20} \div 2^{15}) \times 2^3$ (x) $8^t \div 8^2$

Answer 1:

(i) $3^2 \times 3^4 \times 3^8 = 3^{(2+4+8)} = 3^{14}$ $\quad \left[\because\ a^m \times a^n = a^{m+n}\right]$

(ii) $6^{15} \div 6^{10} = 6^{15-10} = 6^5$ $\quad \left[\because\ a^m \div a^n = a^{m-n}\right]$

(iii) $a^3 \times a^2 = a^{3+2} = a^5$ $\quad \left[\because\ a^m \times a^n = a^{m+n}\right]$

(iv) $7^x \times 7^2 = 7^{x+2}$ $\quad \left[\because\ a^m \times a^n = a^{m+n}\right]$

(v) $\left(5^2\right)^3 \div 5^3 = 5^{2\times3} \div 5^3 = 5^6 \div 5^3$ $\quad \left[\because\ \left(a^m\right)^n = a^{m\times n}\right]$

$= 5^{6-3} = 5^3$ $\quad \left[\because\ a^m \div a^n = a^{m-n}\right]$

(vi) $2^5 \times 5^5 = \left(2\times5\right)^5 = 10^5$ $\quad \left[\because\ a^m \times b^m = \left(a\times b\right)^m\right]$

(vii) $a^4 \times b^4 = \left(a\times b\right)^4$ $\quad \left[\because\ a^m \times b^m = \left(a\times b\right)^m\right]$

(viii) $\left(3^4\right)^3 = 3^{4\times3} = 3^{12}$ $\quad \left[\because\ \left(a^m\right)^n = a^{m\times n}\right]$

(ix) $\left(2^{20} \div 2^{15}\right)\times 2^3 = \left(2^{20-15}\right)\times 2^3$ $\quad \left[\because\ a^m \div a^n = a^{m-n}\right]$

$= 2^5 \times 2^3 = 2^{5+3} = 2^8$ $\quad \left[\because\ a^m \times a^n = a^{m+n}\right]$

(x) $8^t \div 8^2 = 8^{t-2}$ $\quad \left[\because\ a^m \div a^n = a^{m-n}\right]$

Question 2:

Simplify and express each of the following in exponential form:

(i) $\dfrac{2^3 \times 3^4 \times 4}{3 \times 32}$

(ii) $\left[\left(5^2\right)^3 \times 5^4\right] \div 5^7$

(iii) $25^4 \div 5^3$

(iv) $\dfrac{3 \times 7^2 \times 11^8}{21 \times 11}$

(v) $\dfrac{3^7}{3^4 \times 3^3}$

(vi) $2^0 + 3^0 + 4^0$

(vii) $2^0 \times 3^0 \times 4^0$

(viii) $\left(3^0 + 2^0\right) \times 5^0$

(ix) $\dfrac{2^8 \times a^5}{4^3 \times a^3}$

(x) $\left(\dfrac{a^5}{a^3}\right) \times a^8$

(xi) $\dfrac{4^5 \times a^8 b^3}{4^5 \times a^5 b^2}$

(xii) $\left(2^3 \times 2\right)^2$

Answer 2:

(i) $\dfrac{2^3 \times 3^4 \times 4}{3 \times 32} = \dfrac{2^3 \times 3^4 \times 2^2}{3 \times 2^5} = \dfrac{2^{3+2} \times 3^4}{3 \times 2^5}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$= \dfrac{2^5 \times 3^4}{3 \times 2^5} = 2^{5-5} \times 3^{4-3}$ $\left[\because a^m \div a^n = a^{m-n}\right]$

$= 2^0 \times 3^3 = 1 \times 3^3 = 3^3$

(ii) $\left[\left(5^2\right)^3 \times 5^4\right] \div 5^7 = \left[5^6 \times 5^4\right] \div 5^7$ $\left[\because \left(a^m\right)^n = a^{m \times n}\right]$

$= \left[5^{6+4}\right] \div 5^7 = 5^{10} \div 5^7$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$= 5^{10-7} = 5^3$ $\left[\because a^m \div a^n = a^{m-n}\right]$

(iii) $25^4 \div 5^3 = \left(5^2\right)^4 \div 5^3 = 5^8 \div 5^3$ $\left[\because \left(a^m\right)^n = a^{m \times n}\right]$

$= 5^{8-3} = 5^5$ $\left[\because a^m \div a^n = a^{m-n}\right]$

(iv) $\dfrac{3 \times 7^2 \times 11^8}{21 \times 11^3} = \dfrac{3 \times 7^2 \times 11^8}{3 \times 7 \times 11^3} = 3^{1-1} \times 7^{2-1} \times 11^{8-3}$ $\left[\because a^m \div a^n = a^{m-n}\right]$

$= 3^0 \times 7^1 \times 11^5 = 7 \times 11^5$

(v) $\dfrac{3^7}{3^4 \times 3^3} = \dfrac{3^7}{3^{4+3}} = \dfrac{3^7}{3^7}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$= 3^{7-7} = 3^0 = 1$ $\left[\because a^m \div a^n = a^{m-n}\right]$

(vi) $2^0 + 3^0 + 4^0 = 1+1+1 = 3$ $\left[\because a^0 = 1\right]$

(vii) $2^0 \times 3^0 \times 4^0 = 1\times1\times1 = 1$ $\left[\because a^0 = 1\right]$

(viii) $\left(3^0 + 2^0\right)\times 5^0 = \left(1+1\right)\times 1 = 2\times1 = 2$ $\left[\because a^0 = 1\right]$

(ix) $\dfrac{2^8 \times a^5}{4^3 \times a^3} = \dfrac{2^8 \times a^5}{\left(2^2\right)^3 \times a^3} = \dfrac{2^8 \times a^5}{2^6 \times a^3}$ $\left[\because \left(a^m\right)^n = a^{m\times n}\right]$

$= 2^{8-6} \times a^{5-2} = 2^2 \times a^2$ $\left[\because a^m \div a^n = a^{m-n}\right]$

$= \left(2a\right)^2$ $\left[\because a^m \times b^m = \left(a\times b\right)^m\right]$

(x) $\left(\dfrac{a^5}{a^3}\right)\times a^8 = \left(a^{5-3}\right)\times a^8 = a^2 \times a^8$ $\left[\because a^m \div a^n = a^{m-n}\right]$

$= a^{2+8} = a^{10}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

(xi) $\dfrac{4^5 \times a^8 b^3}{4^5 \times a^5 b^2} = 4^{5-5} \times a^{8-5} \times b^{3-2} = 4^0 \times a^3 \times b$ $\left[\because a^m \div a^n = a^{m-n}\right]$

$= 1\times a^3 \times b = a^3 b$ $\left[\because a^0 = 1\right]$

(xii) $\left(2^3 \times 2\right)^2 = \left(2^{3+1}\right)^2 = \left(2^4\right)^2$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$= 2^{4\times2} = 2^8$

Question 3:

Say true or false and justify your answer:

(i) $10 \times 10^{11} = 100^{11}$ (ii) $2^3 > 5^2$
(iii) $2^3 \times 3^2 = 6^5$ (iv) $3^0 = (1000)^0$

Answer 3:

(i) $10 \times 10^{11} = 100^{11}$

L.H.S. $10^{1+11} = 10^{12}$ and R.H.S. $(10^2)^{11} = 10^{22}$

Since, L.H.S. $\neq$ R.H.S.
Therefore, it is false.

(ii) $2^3 > 5^2$

L.H.S. $2^3 = 8$ and R.H.S. $5^2 = 25$
Since, L.H.S. is not greater than R.H.S.
Therefore, it is false.

(iii) $2^3 \times 3^2 = 6^5$

L.H.S. $2^3 \times 3^2 = 8 \times 9 = 72$ and R.H.S. $6^5 = 7,776$
Since, L.H.S. $\neq$ R.H.S.
Therefore, it is false.

(iv) $3^0 = (1000)^0$

L.H.S. $3^0 = 1$ and R.H.S. $(1000)^0 = 1$

Since, L.H.S. = R.H.S.
Therefore, it is true.

Question 4:

Express each of the following as a product of prime factors only in exponential form:

(i) 108×192 (ii) 270
(iii) 729×64 (iv) 768

Answer 4:

(i) 108×192

108 x 192 $= \left(2^2 \times 3^3\right) \times \left(2^6 \times 3\right)$

$= 2^{2+6} \times 3^{3+1}$

$= 2^8 \times 3^4$

2	**192**
2	96
2	48
2	24
2	12
2	6
3	3
	1

2	**108**
2	54
3	27
3	9
3	3
	1

(ii) 270

270 $= 2 \times 3^5 \times 5$

2	**270**
3	135
3	45
3	15
5	5
	1

(iii) 729 x 64

729 x 64 $= 3^6 \times 2^6$

2	**64**
2	32
2	16
2	8

2	4
2	2
	1

3	**729**
3	243
3	81
3	27
3	9
3	3
	1

(iv) 768

768 $= 2^8 \times 3$

2	**768**
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

Question 5:

Simplify:

(i) $\dfrac{\left(2^5\right)^2 \times 7^3}{8^3 \times 7}$

(ii) $\dfrac{25 \times 5^2 \times t^8}{10^3 \times t^4}$

(iii) $\dfrac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$

Answer 5:

(i) $$\frac{\left(2^5\right)^2 \times 7^3}{8^3 \times 7} = \frac{2^{5\times 2} \times 7^3}{\left(2^3\right)^3 \times 7}$$

$$= \frac{2^{10} \times 7^3}{2^9 \times 7}$$

$$= 2^{10-9} \times 7^{3-1} = 2 \times 7^2$$

= 2 x 49

= 98

(ii) $$\frac{25 \times 5^2 \times t^8}{10^3 \times t^4} = \frac{5^2 \times 5^2 \times t^8}{\left(5 \times 2\right)^3 \times t^4}$$

$$= \frac{5^{2+2} \times t^{8-4}}{2^3 \times 3^3}$$

$$= \frac{5^4 \times t^4}{2^3 \times 5^3}$$

$$= \frac{5^{4-3} \times t^4}{2^3}$$

$$= \frac{5t^4}{8}$$

(iii) $$\frac{3^5 \times 10^5 \times 25}{5^7 \times 6^5} = \frac{3^5 \times \left(2 \times 5\right)^5 \times 5^2}{5^7 \times \left(2 \times 3\right)^5}$$

$$= \frac{3^5 \times 2^5 \times 5^5 \times 5^2}{5^7 \times 2^5 \times 3^5}$$

$$= \frac{3^5 \times 2^5 \times 5^{5+2}}{5^7 \times 2^5 \times 3^5}$$

$$= \frac{3^5 \times 2^5 \times 5^7}{5^7 \times 2^5 \times 3^5}$$

$$= 2^{5-5} \times 3^{5-5} \times 5^{5-5}$$

$$= 2^0 \times 3^0 \times 5^0$$

= 1 x 1 x 1

= 1