Exercise 6.4

Question 1:

Is it possible to have a triangle with the following sides?

(i) 2 cm, 3 cm, 5 cm
(ii) 3 cm, 6 cm, 7 cm
(iii) 6 cm, 3 cm, 2 cm

Answer 1:

Since, a triangle is possible whose sum of the lengths of any two sides would be greater than the length of third side.

(i) 2 cm, 3 cm, 5 cm

$2 + 3 > 5$ No
$2 + 5 > 3$ Yes
$3 + 5 > 2$ Yes

This triangle is not possible.

(ii) 3 cm, 6 cm, 7 cm

$3 + 6 > 7$ Yes
$6 + 7 > 3$ Yes
$3 + 7 > 6$ Yes

This triangle is possible.

(iii) 6 cm, 3 cm, 2 cm

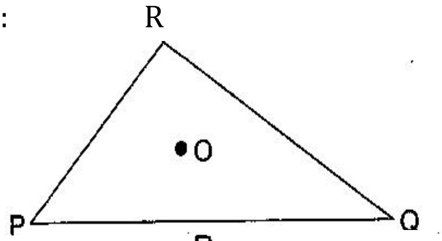
$6 + 3 > 2$ Yes
$6 + 2 > 3$ Yes
$2 + 3 > 6$ No

This triangle is not possible.

Question 2:

Take any point O in the interior of a triangle PQR. Is:

(i) OP + OQ > PQ ?
(ii) OQ + OR > QR ?
(iii) OR + OP > RP ?



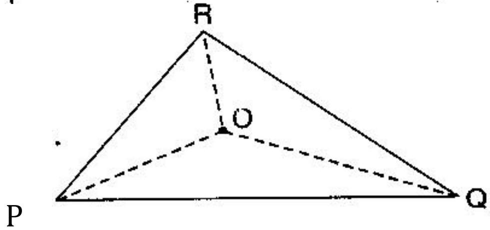
Answer 2:

Join OR, OQ and OP.

(i) Is OP + OQ > PQ ?
Yes, POQ form a triangle.

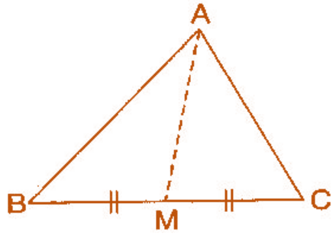
(ii) Is OQ + OR > QR ?
Yes, RQO form a triangle.

(iii) Is OR + OP > RP ?
Yes, ROP form a triangle.



Question 3:

AM is a median of a triangle ABC. Is AB + BC + CA > 2AM? (Consider the sides of triangles ΔABM and ΔAMC.)



Answer 3:

Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side.

Therefore, In ΔABM, $AB + BM > AM$ ... (i)

In ΔAMC, $AC + MC > AM$ ... (ii)

Adding eq. (i) and (ii),

$$AB + BM + AC + MC > AM + AM$$

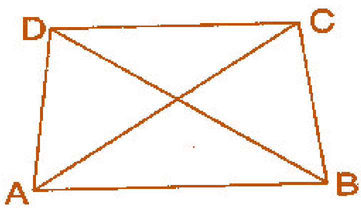
$$\Rightarrow \quad AB + AC + (BM + MC) > 2AM$$

$$\Rightarrow \quad AB + AC + BC > 2AM$$

Hence, it is true.

Question 4:

ABCD is a quadrilateral. Is AB + BC + CD + DA > AC + BD?



Answer 4:

Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side.

Therefore, In ΔABC, $AB + BC > AC$ ....(i)

In ΔADC, $AD + DC > AC$ ....(ii)

In ΔDCB, $DC + CB > DB$ ....(iii)

In ΔADB, $AD + AB > DB$ ....(iv)

Adding equations (i), (ii), (iii) and (iv), we get

$AB + BC + AD + DC + DC + CB + AD + AB > AC + AC + DB + DB$

$\Rightarrow \quad (AB + AB) + (BC + BC) + (AD + AD) + (DC + DC) > 2AC + 2DB$

$\Rightarrow \quad 2AB + 2BC + 2AD + 2DC > 2(AC + DB)$

$\Rightarrow \quad 2(AB + BC + AD + DC) > 2(AC + DB)$

$\Rightarrow \quad AB + BC + AD + DC > AC + DB$

$\Rightarrow \quad AB + BC + CD + DA > AC + DB$

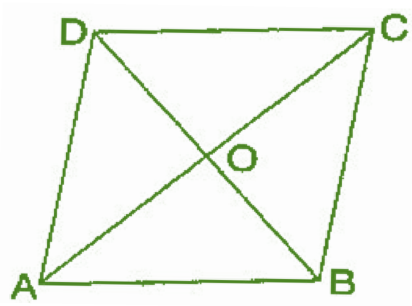
Hence, it is true.

Question 5:

ABCD is quadrilateral. Is $AB + BC + CD + DA < 2\ (AC + BD)$?

Answer 5:

Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side.



Therefore, In ΔAOB, $\quad AB < OA + OB \qquad$ ....(i)

In ΔBOC, $\quad BC < OB + OC \qquad$ ....(ii)

In ΔCOD, $\quad CD < OC + OD \qquad$ ....(iii)

In ΔAOD, $\quad DA < OD + OA \qquad$ ....(iv)

Adding equations (i), (ii), (iii) and (iv), we get

$AB + BC + CD + DA < OA + OB + OB + OC + OC + OD + OD + OA$

$\Rightarrow \quad AB + BC + CD + DA < 2OA + 2OB + 2OC + 2OD$

$\Rightarrow \quad AB + BC + CD + DA < 2[(AO + OC) + (DO + OB)]$

$\Rightarrow \quad AB + BC + CD + DA < 2(AC + BD)$

Hence, it is proved.

Question 6:

The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

Answer 6:

Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side.

It is given that two sides of triangle are 12 cm and 15 cm.

Therefore, the third side should be less than $12 + 15 = 27$ cm.

And also the third side cannot be less than the difference of the two sides.

Therefore, the third side has to be more than $15 - 12 = 3$ cm.

Hence, the third side could be the length more than 3 cm and less than 27 cm.