V5D2 - Selected Topics in Topology

Slice knots and knot concordance

Summer 2023, MPIM/Uni Bonn
Tuesdays 10:15-12:00 at MPIM lecture hall

Course website [1]
NextCloud [2]
Zoom [3]

Instructors

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Course description

This course is about slice knots and knot concordance in the topological and smooth categories. Our goal is to talk about the basics of this subject, highlighting open problems, numerous invariants and obstructions ranging from purely topological to smooth, instructive examples and constructions, applications to other interesting questions, etc.

Audience and Prerequisites

The course is aimed at advanced bachelor's students, master's students, PhD students, and researchers who are not experts, including those not in low-dimensional topology. The prerequisites are point-set topology and some algebraic topology. Some experience with knot theory will be helpful, but the important details will be summarized/recalled throughout the semester.

Homework, Exams, and Evaluation

Homework will be assigned throughout the semester, however, it will not be graded. An oral exam will be held at the end of the semester, covering definitions, theorem statements, and the homework problems $\pm\varepsilon$. Students are encouraged to prepare for the oral exam by consistently working on the homework problems throughout the semester and by attending problem sessions. The exam will be conducted at the MPIM by both instructors and the teaching assistant, and it will last approximately 30 minutes. The structure of the exam can be changed to accommodate exceptional circumstances (e.g., a virtual exam due to illness).

[1] https://imsundberg.github.io/concordance/
[2] https://nextcloud.mpim-bonn.mpg.de/s/d7jwzTGgP4iYNMM
[3] https://www.zoom.us/j/65367172480

Virtuals

Although the core of the class will be held in person, the lectures will also be streamed on Zoom and recordings will be made available on NextCloud. Both the stream and recording will be password protected via a password given in class.

Course Materials

There is no official textbook for the course, however, many resources are available on the course website and on NextCloud. This includes a running collection of lecture notes and homeworks.

Problem Sessions

Office hours will be on Monday 10 am-11 am at MPIM B27. In addition, solutions to the homework problems will be posted on the website.

Course Outline

Week	Date	Topic
1	4/4	Motivation, applications, and course overview
2	11/4	Definitions and examples
3	18/4	Algebraic concordance (I)
4	25/4	Algebraic concordance (II), Casson-Gordon invariants (I)
5	2/5	Casson-Gordon invariants (II), twisted Alexander polynomials
6	9/5	Filtrations
7	16/5	Alexander ploynomial and topological sliceness
8	23/5	Slice-Bennequin inequalities
–	–	*Pentecost Holiday*
9	6/6	Donaldson obstructions
10	13/6	Khovanov homology (I)
11	20/6	Khovanov homology (II)
12	27/6	Heegaard Floer homology
13	4/7	Miscellaneous topics
14	11/7	TBD