Relative Khovanov-Jacobsson classes

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Bryn Mawr College

AIM Research Community in 4–dimensional Topology:
Current Events Seminar

14 April 2021

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2. Defining relative Khovanov-Jacobsson classes
3. Obstructing Boundary-Preserving Isotopy Classes
4. Obstructing Sliceness
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Link Cobordisms

Definition. A **link cobordism** $\Sigma\colon L_0 \to L_1$ is a smooth, compact, oriented, properly embedded surface $\Sigma \subset \mathbb{R}^3 \times [0,1]$ with boundary a pair ($i \in \{0,1\}$) of oriented links $L_i = \Sigma \cap (\mathbb{R}^3 \times \{i\})$.

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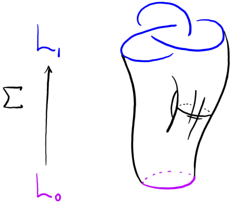

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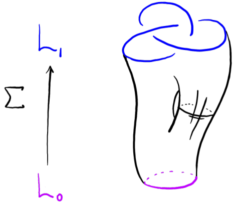

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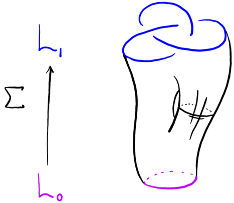

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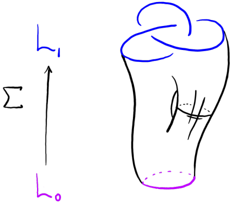

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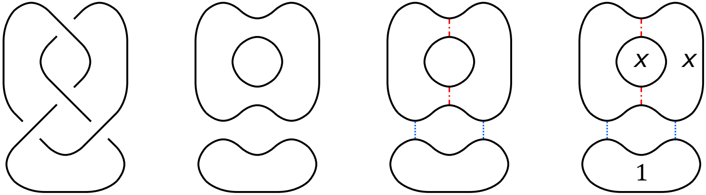

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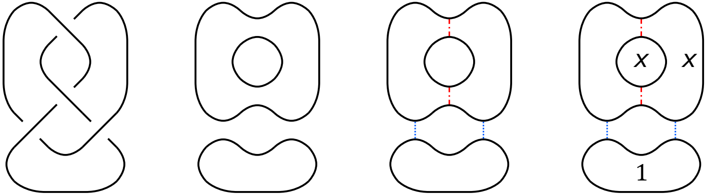

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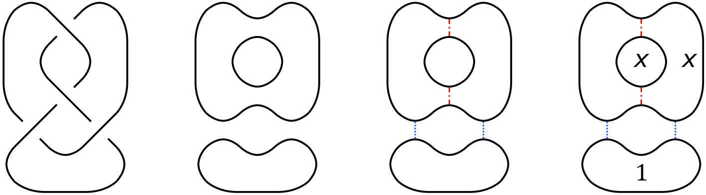

- the chain complex can be bigraded $\mathcal{C}\mathrm{Kh}^{h,q}(D)$
- different diagrams have isomorphic Khovanov homology, so $\mathrm{Kh}(L)$ means *choose a diagram D for L and consider* $\mathrm{Kh}(D)$

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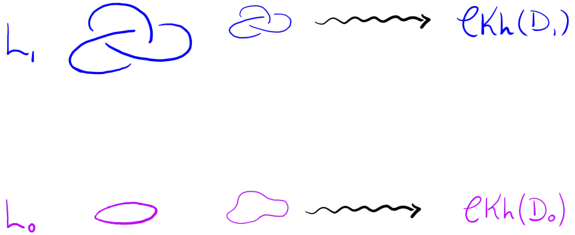

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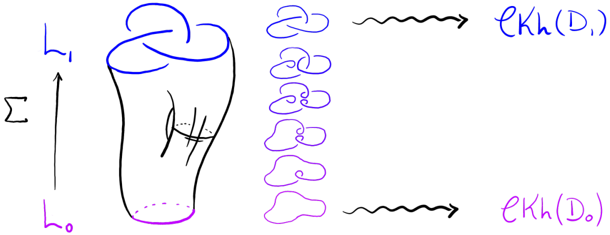

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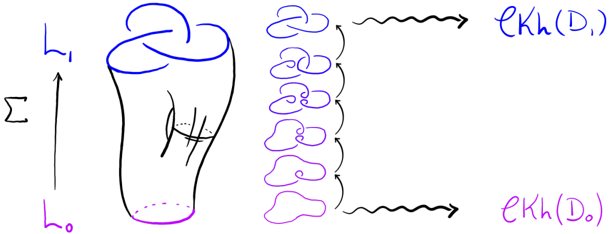

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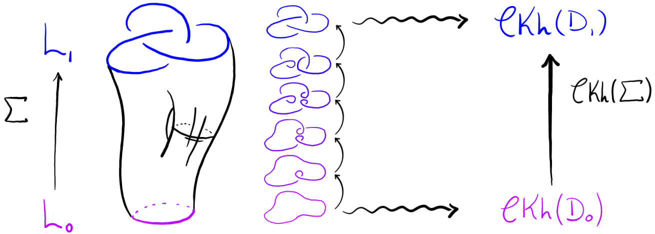

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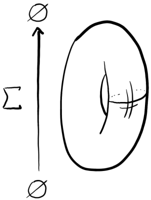

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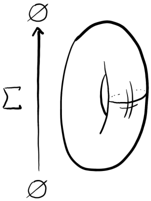

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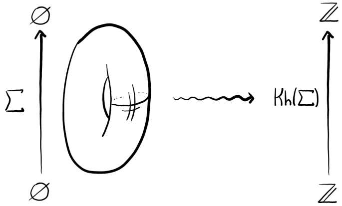

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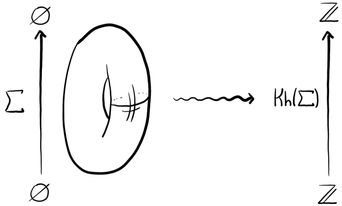

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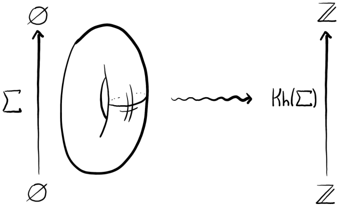

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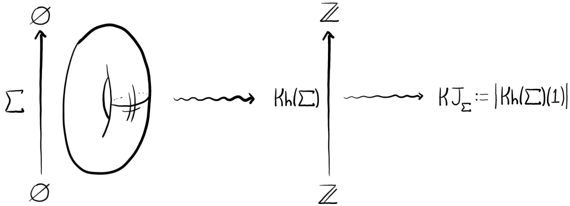

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c. $\mathsf{Kh}(\Sigma)$ is invariant, up to sign, under boundary-preserving isotopy of Σ, so the element

$$\mathsf{KJ}_\Sigma := |\mathsf{Kh}(\Sigma)(1)|$$

is also an invariant of the (ambient) isotopy class of Σ.

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Theorem (Rasmussen '05, Tanaka '05)

Khovanov-Jacobsson classes of connected Σ are determined by genus:

- *if* $g(\Sigma) = 1$, *then* $\mathsf{KJ}_\Sigma = 2$
- *if* $g(\Sigma) \neq 1$, *then* $\mathsf{KJ}_\Sigma = 0$

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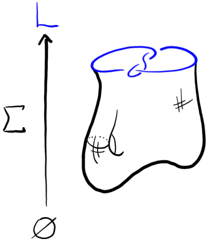

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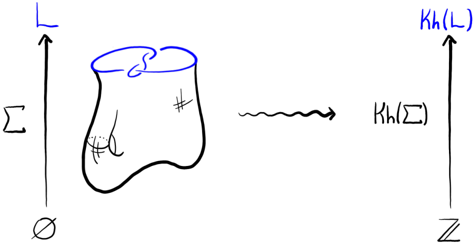

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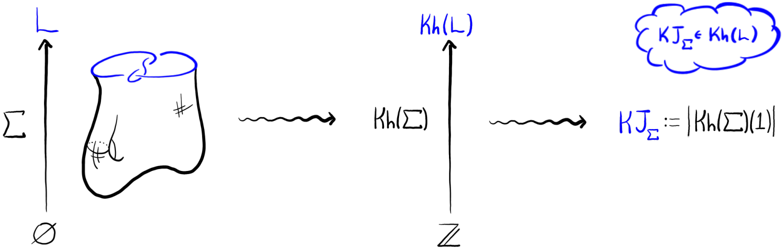

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c. Produce invariant KJ_Σ of the boundary-preserving isotopy class of Σ.

Relative Khovanov-Jacobsson Classes

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Will this relative invariant have the same fate as the absolute $(L = \emptyset)$ case?

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1. Background
2. Defining relative Khovanov-Jacobsson classes
3. Obstructing Boundary-Preserving Isotopy Classes
4. Obstructing Sliceness
5. Future work

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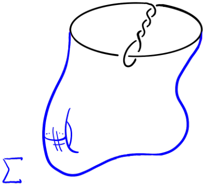

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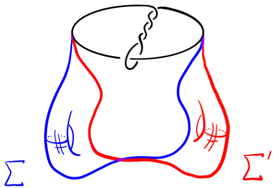

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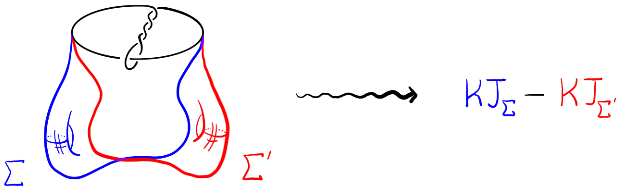

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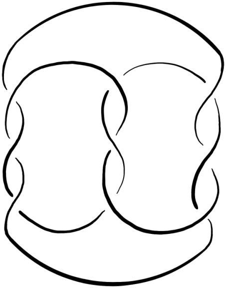

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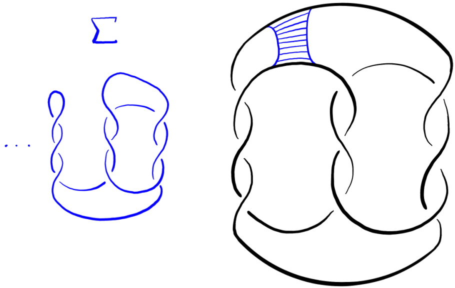

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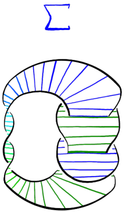

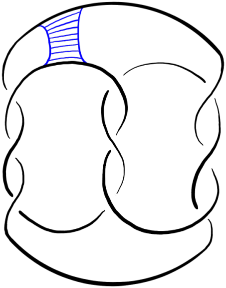

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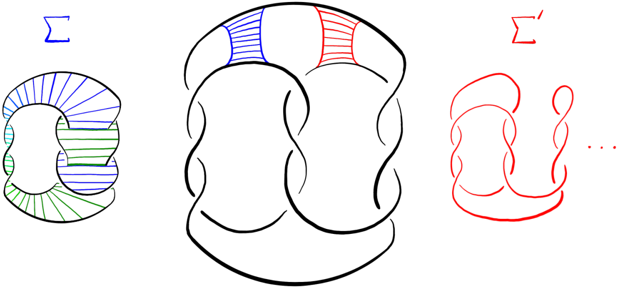

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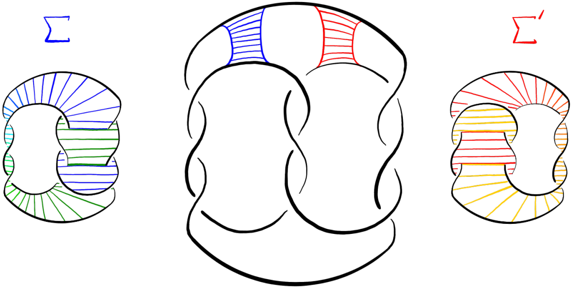

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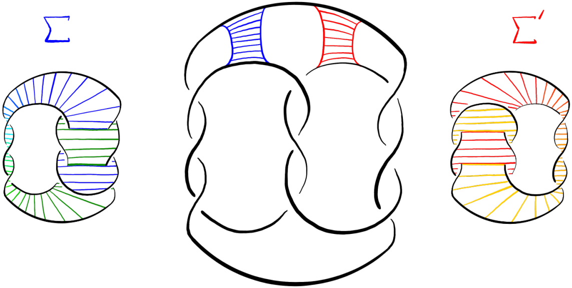

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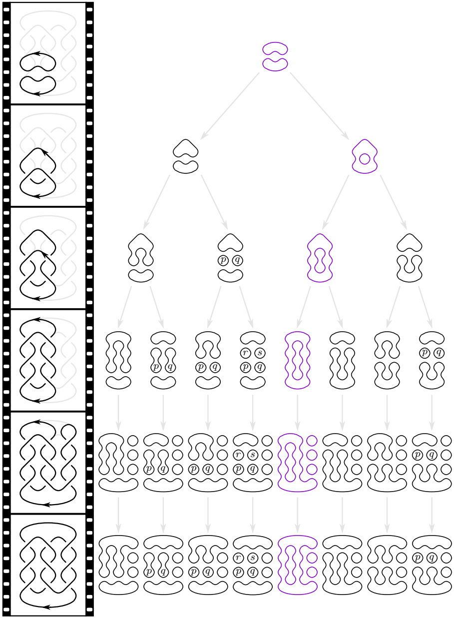

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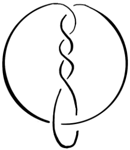

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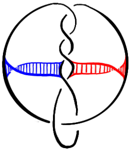

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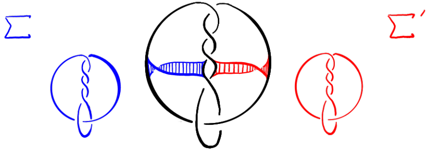

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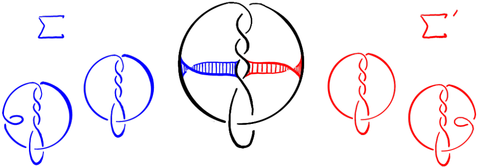

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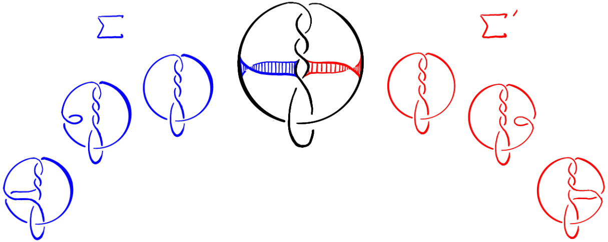

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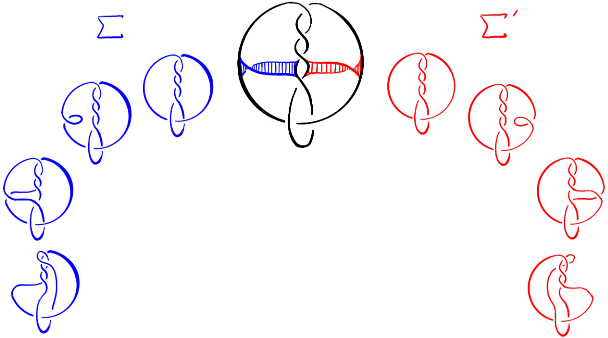

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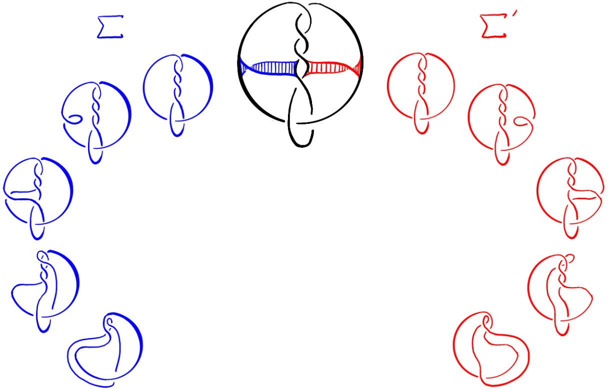

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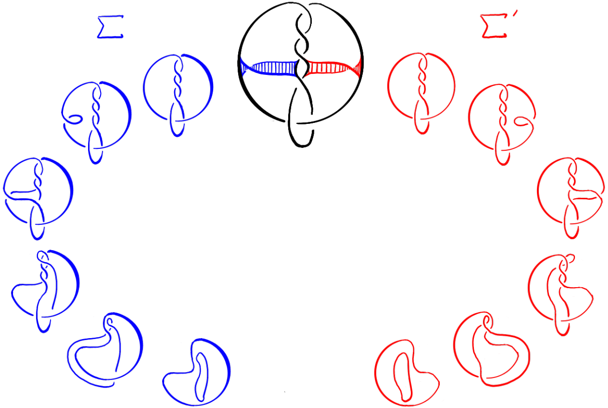

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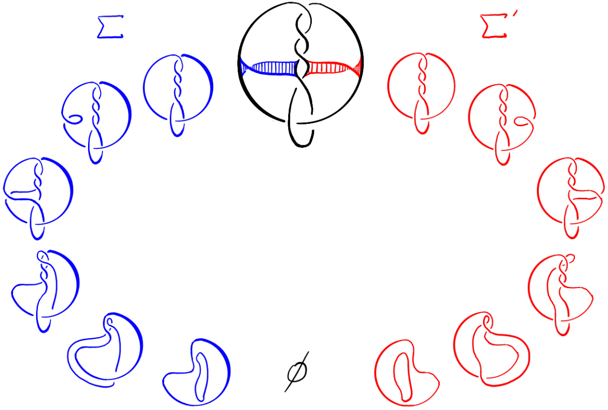

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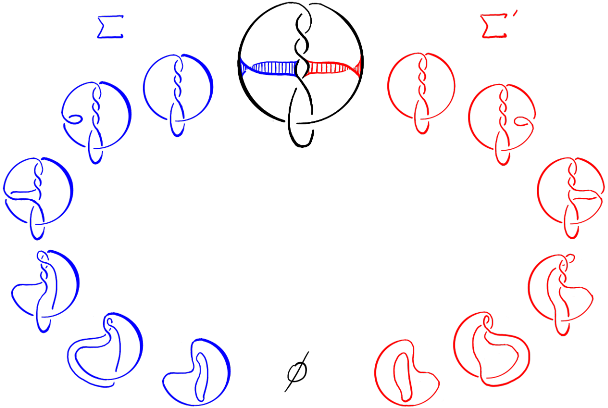

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- Can we distinguish other slices? Are there any other known/unknown examples to check?

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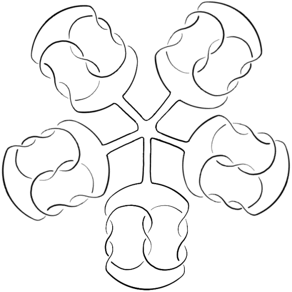

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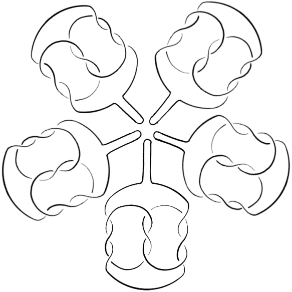

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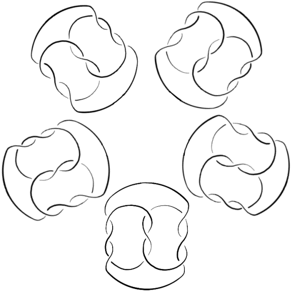

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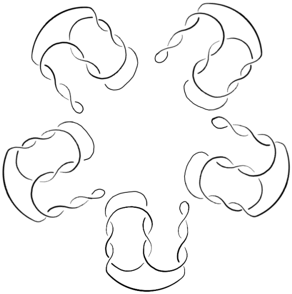

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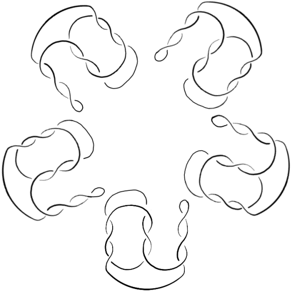

This was also shown in [4] through different techniques.

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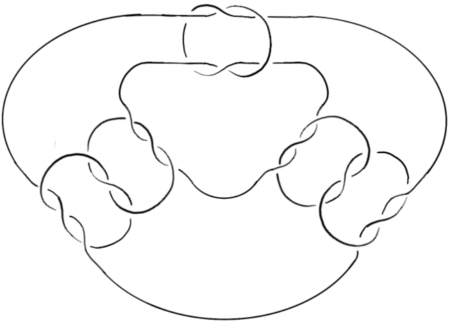

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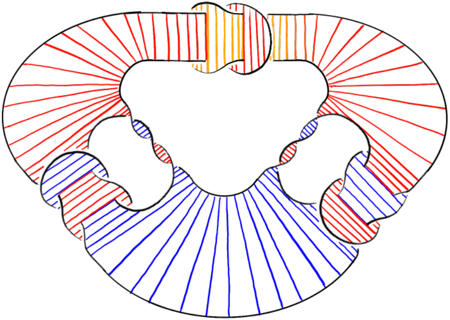

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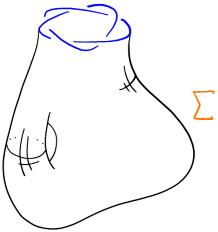

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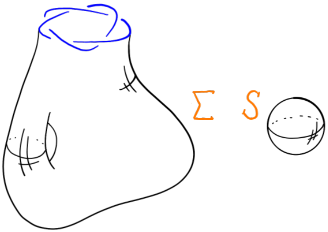

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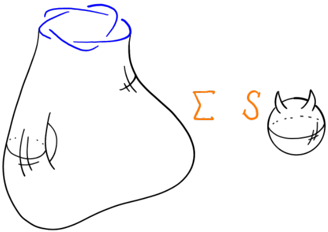

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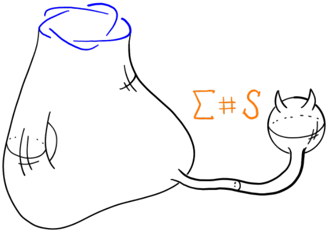

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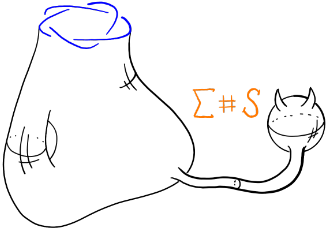

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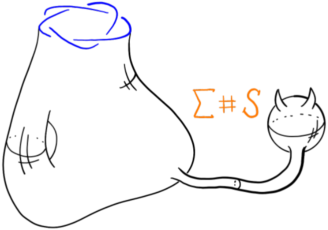

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That is, for a surface Σ and knotted 2-sphere S we have $\mathsf{KJ}_\Sigma = \mathsf{KJ}_{\Sigma \# S}$

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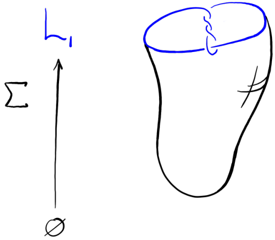
A note on ribbon disks

Suppose we have a ribbon disk $\Sigma\colon \emptyset \to K$ for some knot K.



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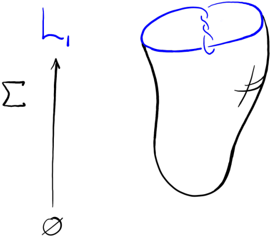

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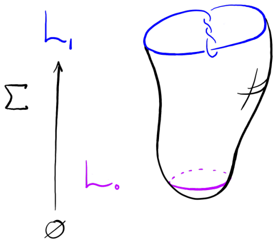

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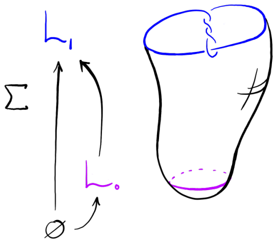

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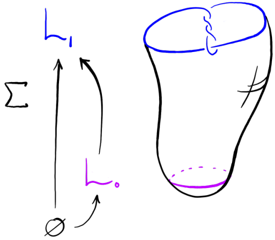

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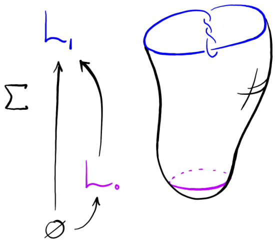

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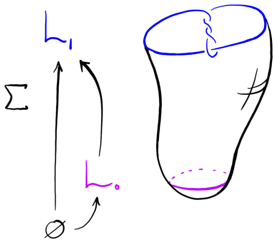

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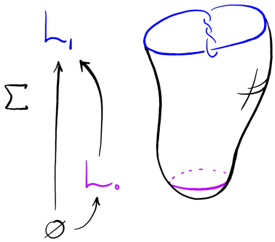

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Unfortunately, none can exist...

Obstructing Sliceness

Theorem (Swann '10)

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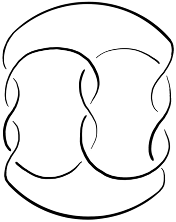
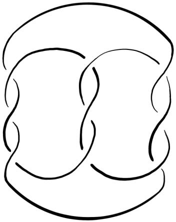
Note: there are classes of knots with 4-ball genus at most 1 (e.g. Whitehead doubles, unknotting number 1 knots)

Example: Three-stranded Pretzel Knots





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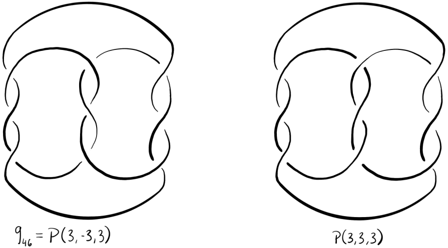

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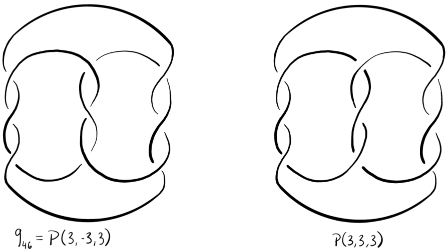

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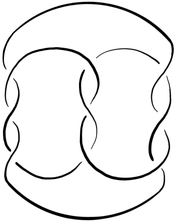

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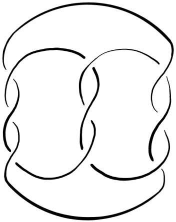

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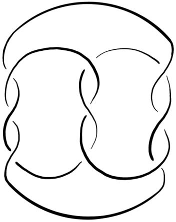

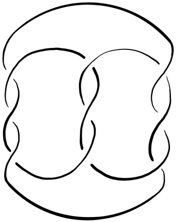

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Proof idea: relative class of Seifert surface is trivial (the cycle representing KJ_Σ is a boundary)

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Determining nontriviality of Khovanov classes

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If true, relative Khovanov-Jacobsson classes should be able to prove results similar to those from [3] about slice disks obtained from deform-spinning knots.

Thank You!

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