Analysis Notes

Summer 2016

Text: Real Analysis by Frank Morgan

Sequences

Definition 1. A sequence (x_n) of real numbers converges to the real number x if for every $\varepsilon > 0$ there exists an index $N \in \mathbb{N}$ such that $n \geq N$ implies $|x_n - x| < \varepsilon$.

$$\forall \varepsilon > 0, \exists N \in \mathbb{N} \text{ s.t. } n \geq N \implies |x_n - x| < \varepsilon$$

Definition 2. A sequence (x_n) is bounded if there is a number M such that for all $n \in \mathbb{N}$, $|a_n| \leq M$.

Proposition 1. *Suppose that the sequence (x_n) converges to x. Then*

i. the limit is unique;

ii. the sequence is bounded.

Proof. For (i.) assume there are two limits and use the triangle inequality to show both limits are equal. For (ii.) fix ε, bound (x_n) after an index N by $1 + \varepsilon$, then set M to be the max of the first N elements and $1 + \varepsilon$. □

Proposition 2. *Suppose (a_n) converges to a and (b_n) converges to b. Then*

i. $ca_n \to ca$ *for* $c \in \mathbb{R}$;

ii. $a_n + b_n \to a + b$;

ii. $a_n b_n \to ab$;

iv. $a_n / b_n \to a/b$ *for nonzero sequence* b_n *and nonzero* b.