Knot Traces and Sliceness

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MAA EPaDel Section Meeting
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Framed Knots

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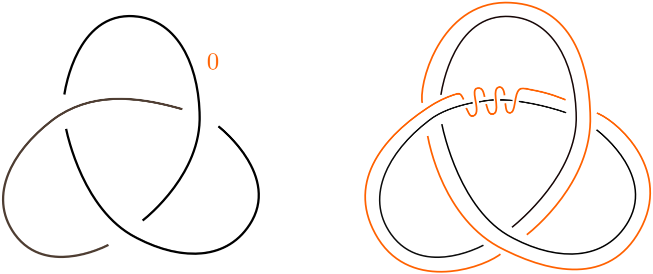
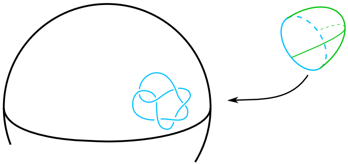


A 0-framed knot (left) with the 0-framed push-off (right).

The framing induces a push-off K' with linking number $\mathrm{lk}(K, K') = n$.

We want to use framed knots to build a 4-manifold by (essentially) attaching a disk to a knot in $S^3 = \partial B^4$. To visualize this, pretend we're down a dimension:



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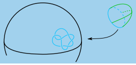

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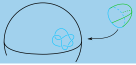

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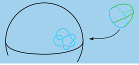

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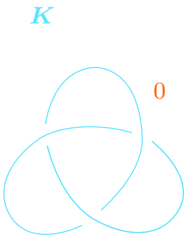

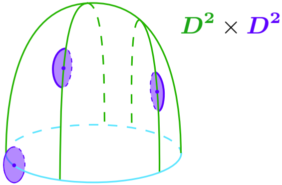

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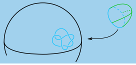

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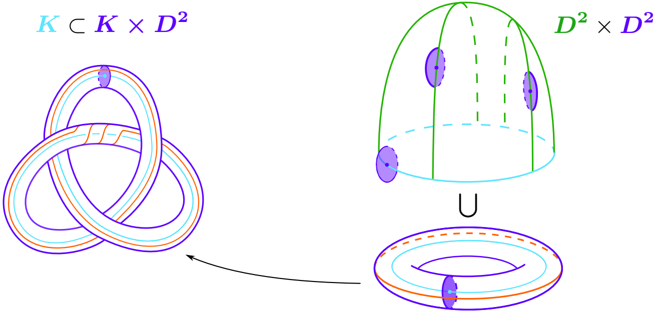

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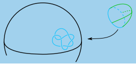

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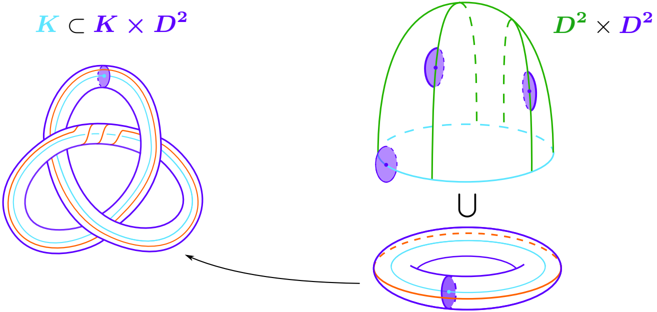

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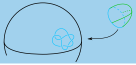

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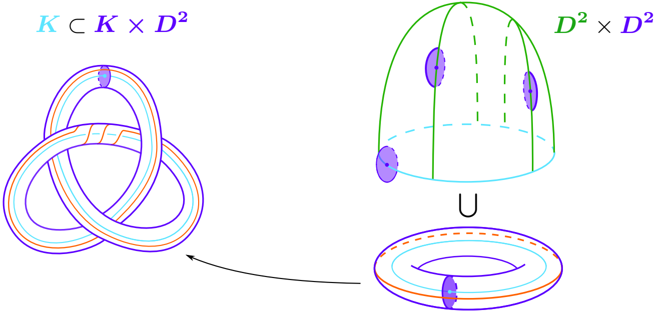

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Question. Do properties of K correspond to properties of $X(K)$, and conversely?

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Slice Knots

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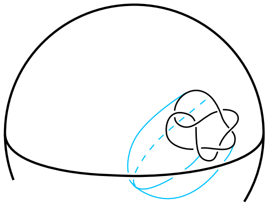

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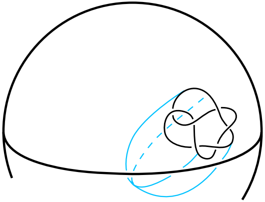

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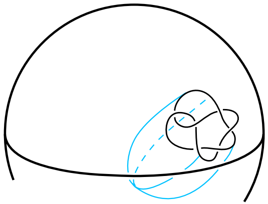

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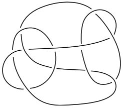

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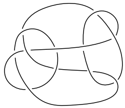

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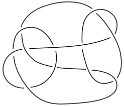

Proof Technique. Construct knot C' with $X(C) \cong X(C')$. Show C' is not slice.

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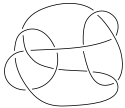
C 

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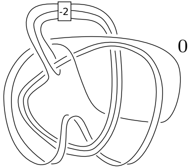
K' 

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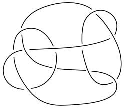

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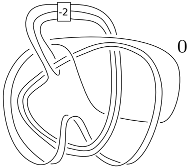
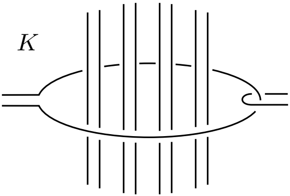


Remark. The construction of C' is known to work for unknotting $\#$ 1 knots.

A knot K with (positive) unknotting number 1 has a diagram:



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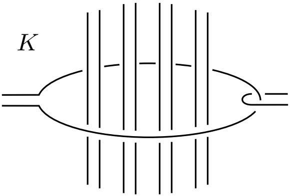
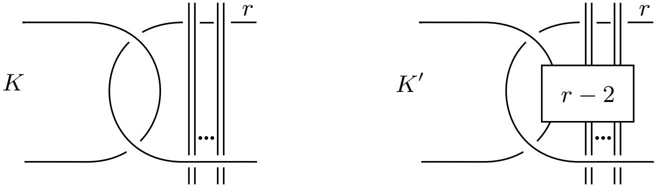
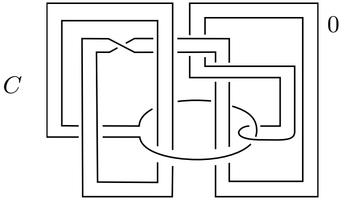
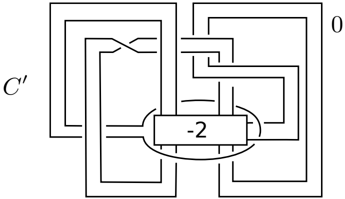


The associated knot K' then has the form:



New C and K'





Bibliography I

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Shake genus and slice genus,
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