The Ins and Outs of Eversions

GSAW Research Symposium

Isaac M. Craig

2 April 2019

Bryn Mawr College

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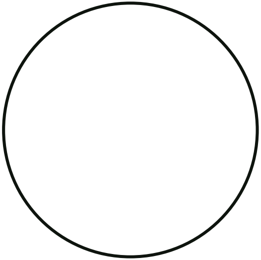

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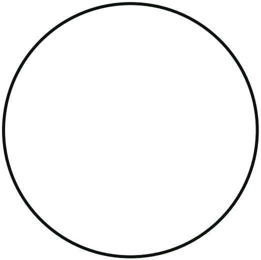

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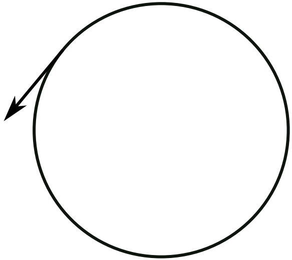

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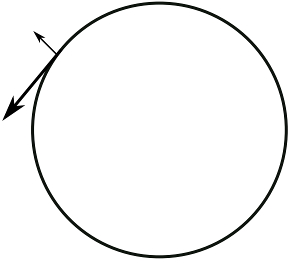

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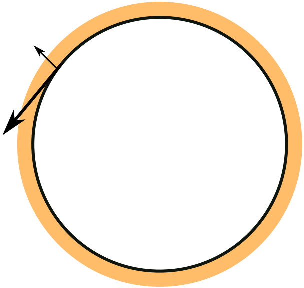

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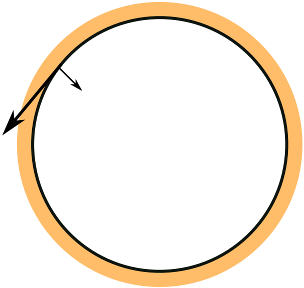

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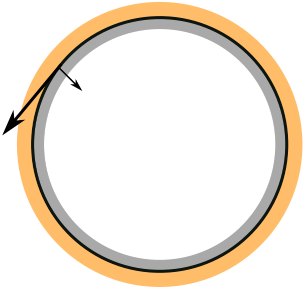

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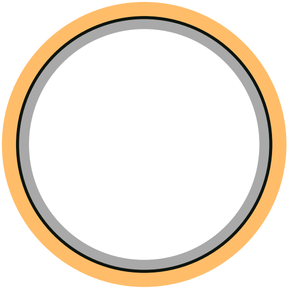

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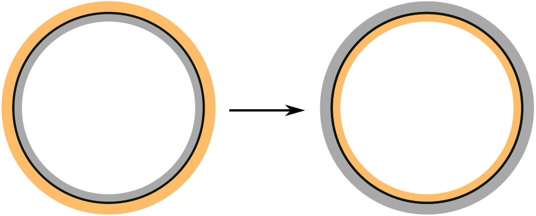

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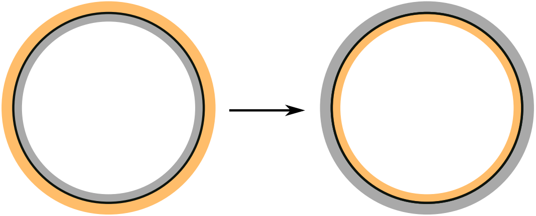

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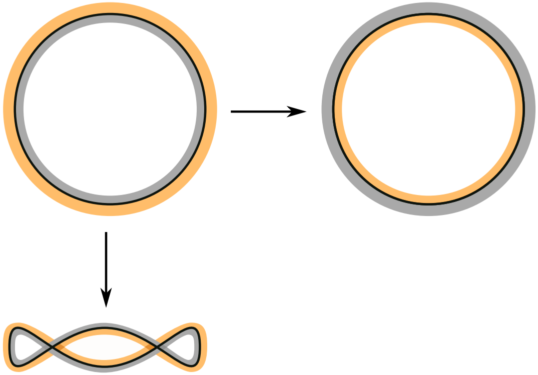

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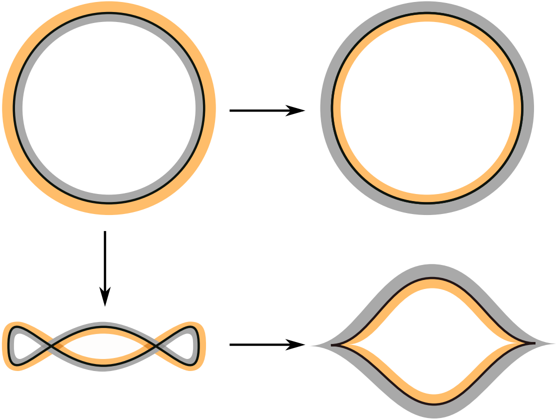

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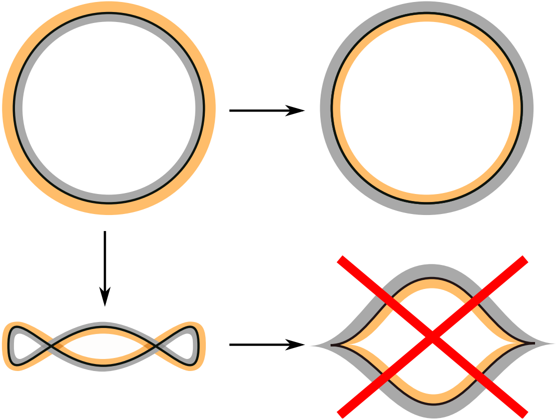

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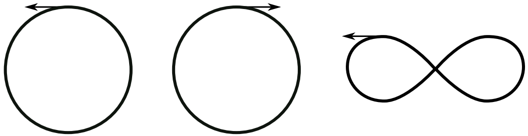

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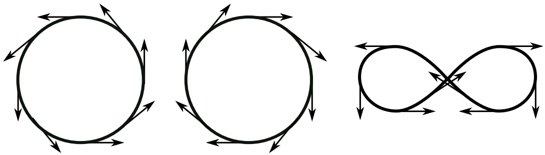

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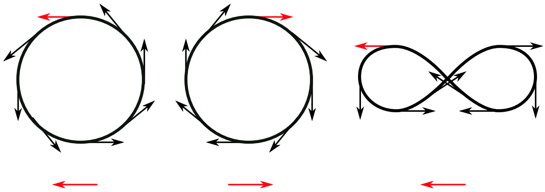

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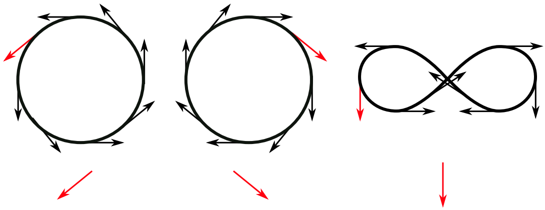

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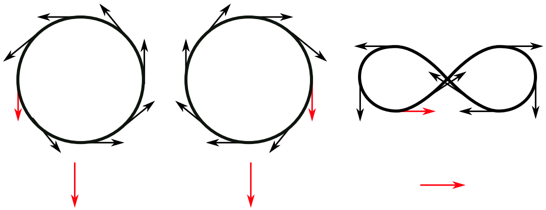

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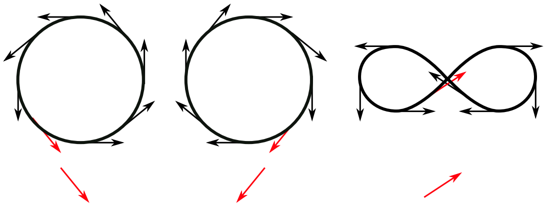

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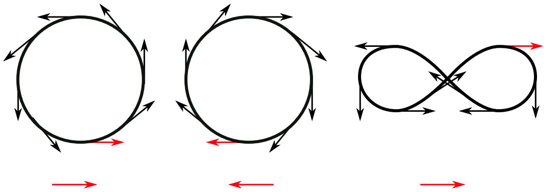

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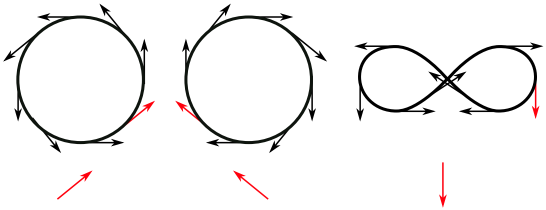

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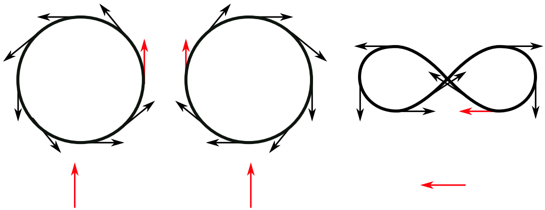

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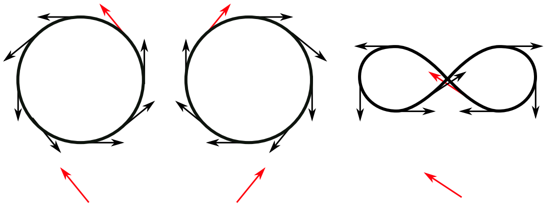

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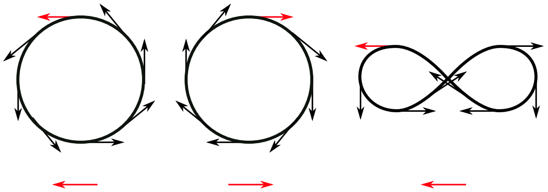

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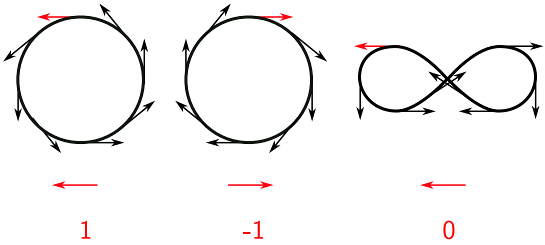

Can other shapes be everted?

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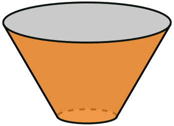

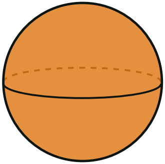

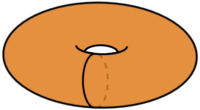

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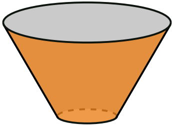

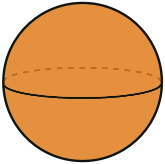

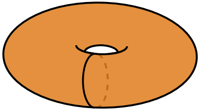

Click here to watch the sphere be everted!
Click here for the torus!

Thank you!