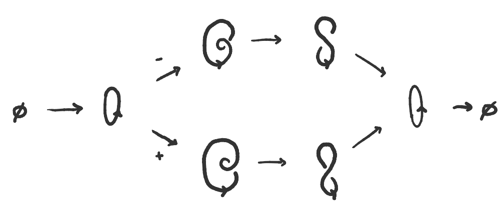
Talk: Open Problem Seminar

September 9, 2021 1:35 PM

Here are two projective planes in S^4

$P_\pm$: 

Not isotopic in S^4 (can be obstructed by normal Euler number $e(P_\pm) = \pm 2$)

via Katanaga-Saeki-Teragaito-Yamada '96

Kinoshita Conjecture ($\leq$'86 via Yoshikawa) Every knotted $\mathbb{RP}^2 \subset S^4$ is standard: SM/TOP isotopic to $P_\pm \# S$ where

$P_\pm$ is a standard $\mathbb{RP}^2 \hookrightarrow S^4$

S is a knotted 2-sphere

Many families of P^2's with $\pi_1(S^4 \setminus P) \not\cong \mathbb{Z}_2$ have been proven standard

Why is this of interest?

(1) knotting is cool (nice to understand any surface knotting)

(2) 2-knots are even cooler (says something weird about 2-knots)

(3) relation to Price and Gluck twists

if true $\Rightarrow$ homotopy S^4's from Price twisting along $P^2 \hookrightarrow S^4$

are the same as

htpy S^4's from Gluck twisting along $S^2 \hookrightarrow S^4$

(4) still open, and adjacent questions also open

some closed:

- relative case: $\exists$ Möbius bands M for $\overset{\text{slice}}{\text{knots}}$ K with $M \not\cong D^4 \# P_\pm$ [Swann-S (sm), Lipshitz-Sarkar (sm), HKMPS (top)]
- links: $\exists$ links of $\mathbb{RP}^2$'s that are not a link of knotted S^2's with local $P_\pm$'s [Yoshikawa 94]

(Open) Do there exist non-isotopic $\mathbb{RP}^2$'s with diffeo exteriors?

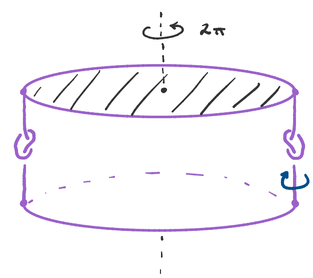
Aside 1 there can only be two [Price]

Aside 2 there exist non-iso S^2's ($S_0 \not\simeq S_1$) with diffeo exteriors [Cappell-Shaneson]

(Open) $S_0 \# P_\pm \not\simeq S_1 \# P_\pm$

Spun knot (Artin?)

- push tangle away from axis
- spin around axis
- fill boundary w/ disks



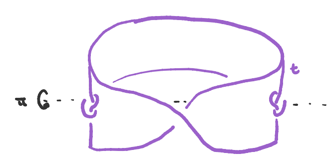
$= \text{spun}(t)$ (Artin)

$=: \text{spun}^n(t)$ (Zeeman)

Deform spun knot (Kamada '92)

- push tangle off axis
- spin and twist
  - tangle must be strongly invertible
  - produces one boundary component
- fill boundary w/ disk $=: \widetilde{\text{spun}}_\pm(t)$

(bounds a disk away from $M(t)$)



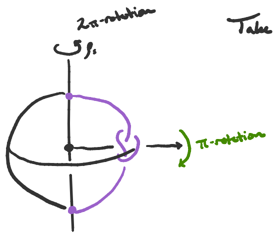
$+G$
$-G$

Thm (Price-Roseman) If t is a strongly-invertible tangle then $\widetilde{\text{spun}}(t) = \text{spun}(t) \# P_\pm$

Deform twist spun apply to $\text{spun}^n_\pm(t)$, giving $\widetilde{\text{spun}}^n_\pm(t)$

(Open) $\widetilde{\text{spun}}^n_\pm(t) \simeq \widetilde{\text{spun}}^{n+2}_\pm(t)$ (they have iso $\pi_1(S^4 \setminus\ )$'s)

(might only be open for trefoils)



Take a tangle t in $\mathbb{R}^3_+$

$\mathrm{spun}(t) := \bigcup_s \rho_s(t)$ knotted S^2

$M(t) := \bigcup_s \rho_s(\phi_s(t))$ Möbius band

$\widetilde{\mathrm{spun}}(t) :=$ closure of $M(t)$ knotted $\mathbb{RP}^2$