# Knot Traces and Sliceness

Isaac M. Craig
Joint with Paul Melvin

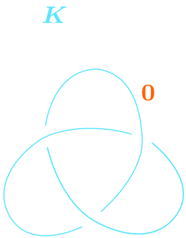
Bryn Mawr College

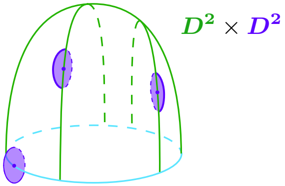
GSTGC
30 March 2019

## Knot Traces

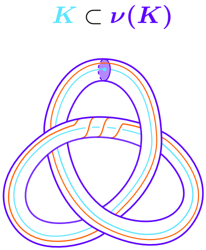
To a framed knot $K \subset S^3 = \partial B^4$, we associate a $4$-manifold $X(K)$ by attaching a framed $2$-handle to $B^4$ along $K$ according to its framing:

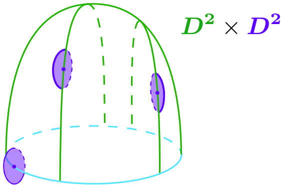
To a framed knot $K \subset S^3 = \partial B^4$, we associate a 4-manifold $X(K)$ by attaching a framed 2-handle to $B^4$ along $K$ according to its framing:



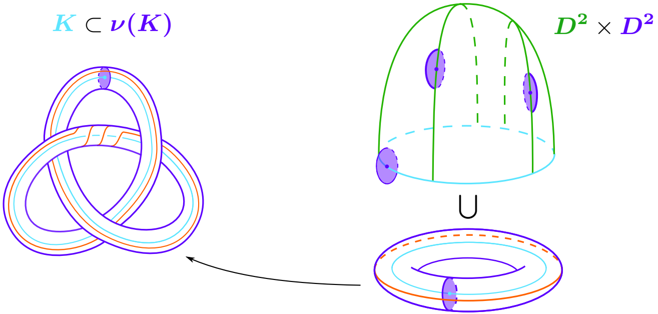



To a framed knot $K \subset S^3 = \partial B^4$, we associate a 4-manifold $X(K)$ by attaching a framed 2-handle to $B^4$ along $K$ according to its framing:

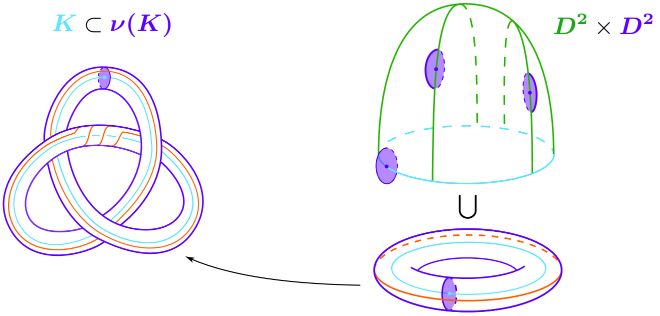


To a framed knot $K \subset S^3 = \partial B^4$, we associate a 4-manifold $X(K)$ by attaching a framed 2-handle to $B^4$ along $K$ according to its framing:

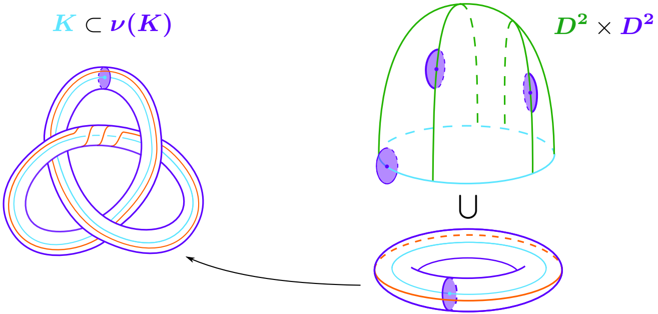


To a framed knot $K \subset S^3 = \partial B^4$, we associate a $4$-manifold $X(K)$ by attaching a framed $2$-handle to $B^4$ along $K$ according to its framing:




The associated $4$-manifold $X(K)$ to an $n$-framed knot $K$ is called its $\boldsymbol{n}$**-trace**.

To a framed knot $K \subset S^3 = \partial B^4$, we associate a $4$-manifold $X(K)$ by attaching a framed $2$-handle to $B^4$ along $K$ according to its framing:

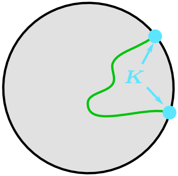



The associated $4$-manifold $X(K)$ to an $n$-framed knot $K$ is called its **$n$-trace**.

*Question*. Do properties of $K$ correspond to properties of $X(K)$, and conversely?

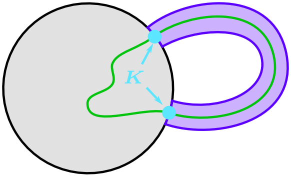
# Slice Knots

A knot $K \subset S^3 = \partial B^4$ is **slice** if it bounds a smoothly embedded disk $D^2 \hookrightarrow B^4$.



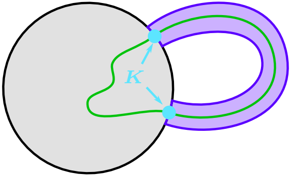
# Slice Knots

A knot $K \subset S^3 = \partial B^4$ is **slice** if it bounds a smoothly embedded disk $D^2 \hookrightarrow B^4$.



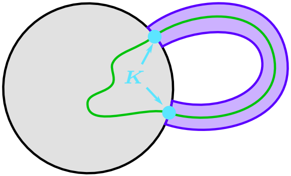
# Slice Knots

A knot $K \subset S^3 = \partial B^4$ is **slice** if it bounds a smoothly embedded disk $D^2 \hookrightarrow B^4$.




*Remark*. The core of the $2$-handle and slice-disk form a smoothly embedded $S^2$.

A knot $K \subset S^3 = \partial B^4$ is **slice** if it bounds a smoothly embedded disk $D^2 \hookrightarrow B^4$.

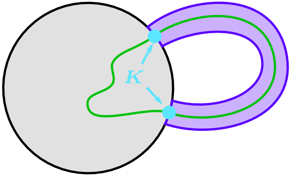



*Remark*. The core of the $2$-handle and slice-disk form a smoothly embedded $S^2$.

**Theorem**. Let $K$ and $K'$ be knots with diffeomorphic traces $X(K) \cong X(K')$. Then $K$ is slice if and only if $K'$ is slice.

# Slice Knots

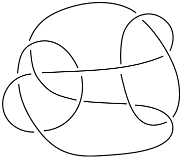
A knot $K \subset S^3 = \partial B^4$ is **slice** if it bounds a smoothly embedded disk $D^2 \hookrightarrow B^4$.




*Remark*. The core of the $2$-handle and slice-disk form a smoothly embedded $S^2$.

**Theorem**. Let $K$ and $K'$ be knots with diffeomorphic traces $X(K) \cong X(K')$. Then $K$ is slice if and only if $K'$ is slice.

**Theorem**. (Piccirillo [1]) The Conway knot is not slice.



# The Conway Knot

**Theorem**. (Piccirillo [1]) The Conway knot $C$ is not slice.

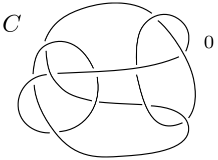
# The Conway Knot

**Theorem**. (Piccirillo [1]) The Conway knot $C$ is not slice.

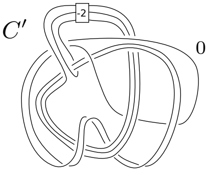
**Proof Technique**. Construct knot $C'$ with $X(C) \cong X(C')$. Show $C'$ is not slice.

**Theorem**. (Piccirillo [1]) The Conway knot $C$ is not slice.

**Proof Technique**. Construct knot $C'$ with $X(C) \cong X(C')$. Show $C'$ is not slice.



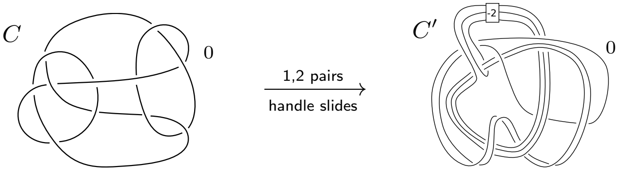

$\xrightarrow[\text{handle slides}]{\text{1,2 pairs}}$



# The Conway Knot

**Theorem**. (Piccirillo [1]) The Conway knot $C$ is not slice.

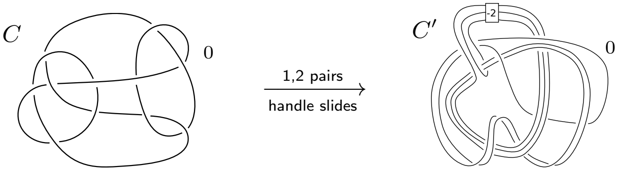
**Proof Technique**. Construct knot $C'$ with $X(C) \cong X(C')$. Show $C'$ is not slice.



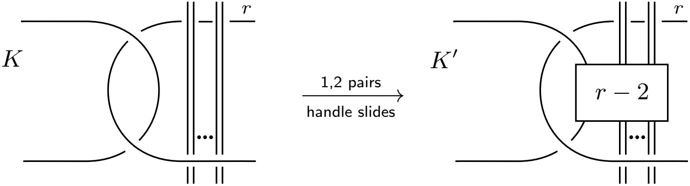

*Remark*. This construction is known to work for unknotting number 1 knots.

**Theorem**. (Piccirillo [1]) The Conway knot $C$ is not slice.

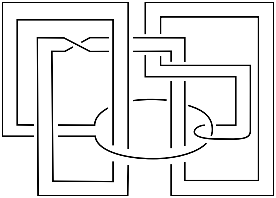
**Proof Technique**. Construct knot $C'$ with $X(C) \cong X(C')$. Show $C'$ is not slice.




*Remark*. This construction is known to work for unknotting number 1 knots.

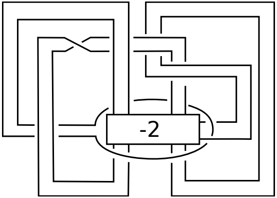


$C$



0

$C'$




0

# Bibliography I


Lisa Piccirillo,
*The Conway knot is not slice*,
*arXiv:1808.02923*, (2018).