# Knot Traces and Sliceness

Isaac M. Craig
Joint with Paul Melvin

Bryn Mawr College

GSTGC
30 March 2019

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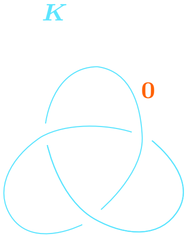


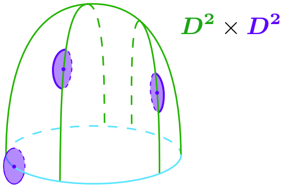

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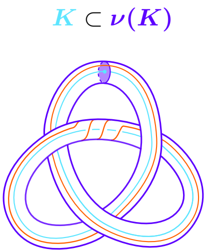


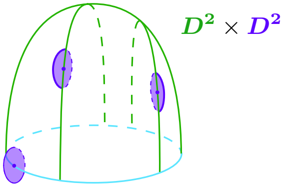

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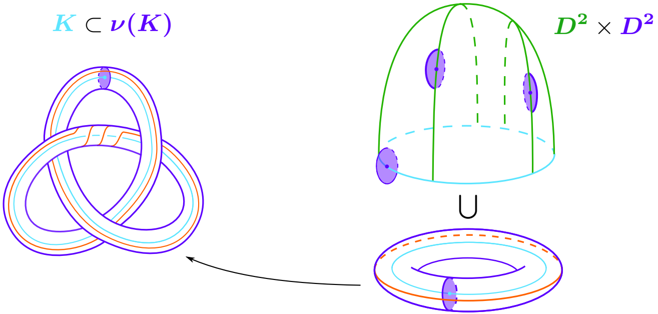

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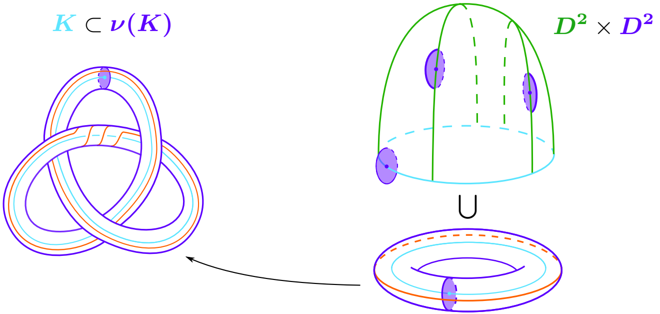


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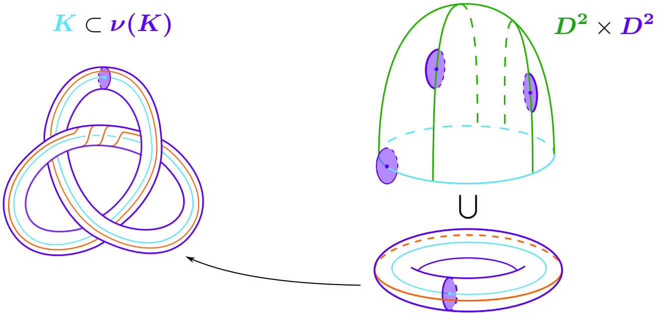


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*Question*. Do properties of $K$ correspond to properties of $X(K)$, and conversely?

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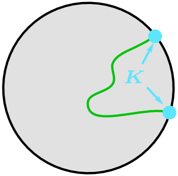

# Slice Knots

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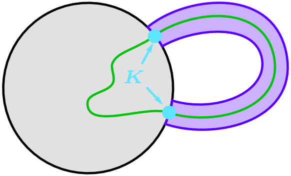

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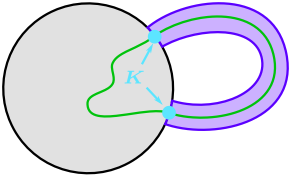


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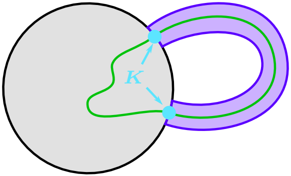


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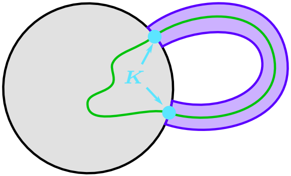


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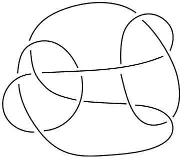

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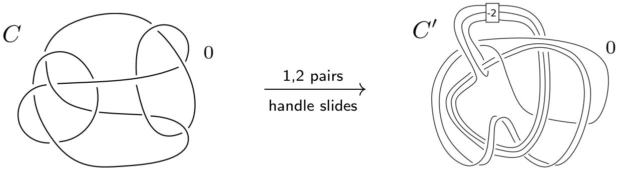

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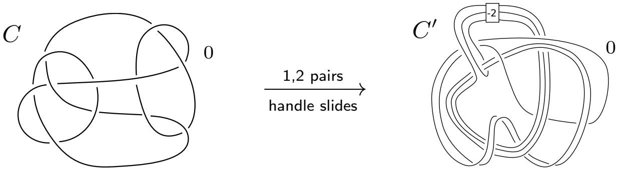


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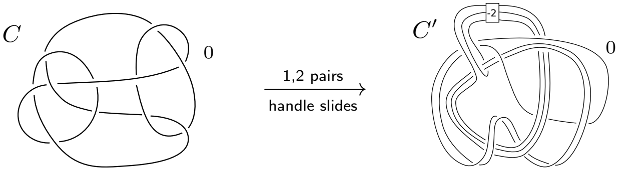


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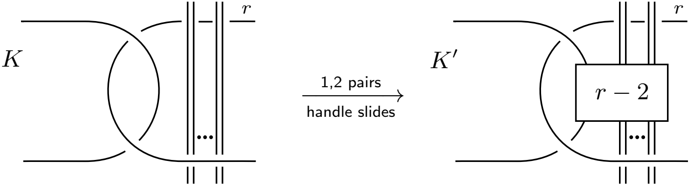
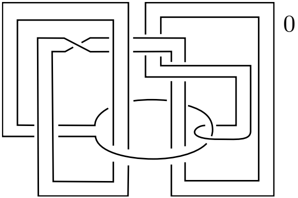
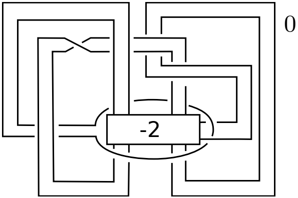


# New $C$ and $K'$

$C$




$C'$



# Bibliography I


Lisa Piccirillo,
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*arXiv:1803.09834*, to appear in G&T (2018).