Talk: PACT V - Khovanov homology of slice disks

September 9, 2021 1:35 PM

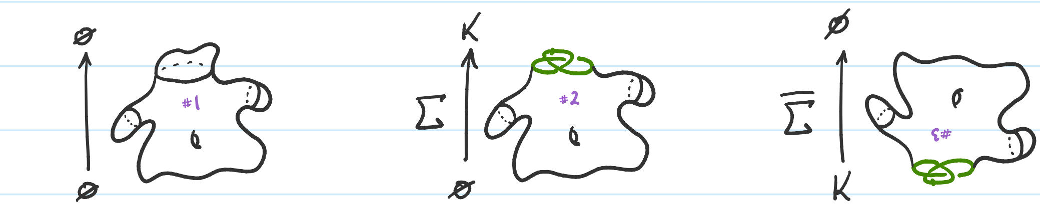
(I) MOTIVATION

(II) KHOVANOV HOMOLOGY OF LINKS (III) AND SURFACES

(IV) KHOVANOV-JACOBSSON CLASSES

(V) KHOVANOV HOMOLOGY OF SLICE DISKS

We consider 3 cases:



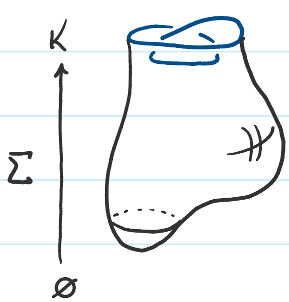
Aside: Link cobordisms can be composed! Given $X: L_0 \to L_1$ and $Y: L_1 \to L_2$, we can form $Y \circ X: L_0 \to L_2$ by stacking Y onto X.

Note: Case #1 looks a lot like #2 with something stacked on it!

THOUGHT EXPERIMENT

Suppose $\Sigma: \emptyset \to K$ is a ribbon disk. Then $\Sigma = C \circ B$, where $C: \mathcal{U} \to K$ is a ribbon-concordance and $B: \emptyset \to \mathcal{U}$ is a Morse birth.

unknot



$$= \quad \begin{array}{c} K \\ \uparrow C \\ \mathcal{U} \\ \uparrow B \\ \emptyset \end{array} \quad \rightsquigarrow \quad \begin{array}{c} Kh(K) \\ \uparrow Kh(\Sigma) \\ \mathbb{Z} \end{array} \quad \begin{array}{c} KJ_\Sigma \\ \uparrow \\ 1 \end{array} \quad = \quad \begin{array}{c} Kh(K) \\ \uparrow Kh(C) \\ Kh(\mathcal{U}) \\ \uparrow Kh(B) \\ \mathbb{Z} \end{array} \quad \begin{array}{c} KJ_{C\circ B} = Kh(C)(KJ_B) \\ \uparrow \\ \text{(1)} = KJ_B \\ \uparrow \\ 1 \end{array}$$

Recall ribbon-concordances induce injections and (1) "KJ_B" is a nontrivial class in $Kh^{0,0}(\mathcal{U})$ because it is not a boundary (note $Kh^{-1,0}(\mathcal{U})$ is trivial).

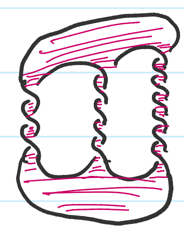
Thus $KJ_\Sigma = KJ_{C\circ B}$ is nontrivial!

! If we find a slice disk $D: \emptyset \to K$ with <u>trivial</u> KJ_D then K cannot be a ribbon knot (disproving the Slice-Ribbon conjecture!)

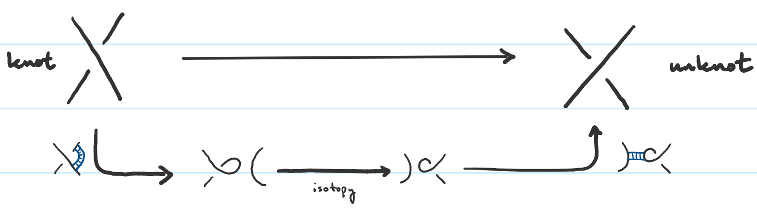
Thm ★ (Swann '10) For a link cob. $\Sigma: \emptyset \to K$ with genus $g(\Sigma) \leq 1$, if K is slice, then KJ_Σ is nontrivial.

- $g=0 \Rightarrow$ slice disks <u>also</u> have nontrivial KJ-classes
- $g=1 \Rightarrow$ KJ-classes can obstruct sliceness (of knots w/ 4-ball genus at most 1)

Thm (Swann '10) Odd 3-stranded pretzel knots $P(a,b,c)$ with a,b,c all positive bound genus 1 Seifert surfaces with trivial KJ-classes, and thus, are not slice.

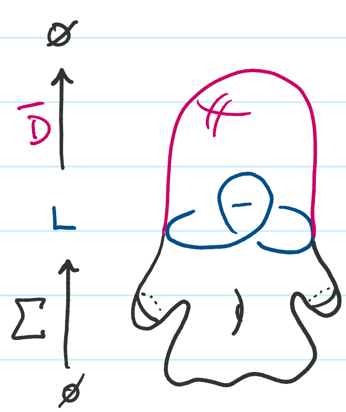
$P(3,5,7)$ 

NOTE: Knots w/ unknotting number 1 bound genus 1 surfaces and are thus good candidates



Ex's Conway knot, Whitehead doubles

Proof of ★ Let $g(\Sigma)=1$ and assume K slice ($\exists D: \emptyset \to K$). Consider $\bar{D} \circ \Sigma : \emptyset \to \emptyset$



$$\begin{tikzcd} \mathbb{Z} & 2 \\ Kh(K) \arrow[u] & KJ_\Sigma \arrow[u, mapsto] \\ \mathbb{Z} \arrow[u] & 1 \arrow[u, mapsto] \end{tikzcd}$$

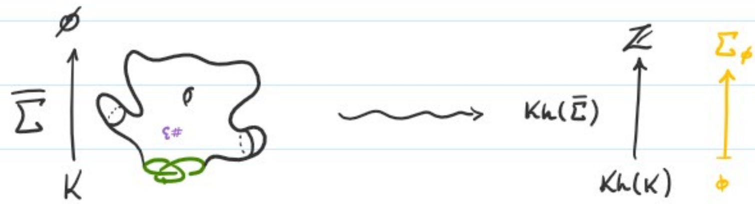
Case #1

if $KJ_\Sigma = 0$ then it can't be sent to 2 by a homomorphism

$\Rightarrow KJ_\Sigma \neq 0$

For $g(\Sigma)=0$ consider $(\overline{D}\# T^2)\circ D : \emptyset \to \emptyset$

Case #3 joint w/ Kyle Hayden (HC '13)



IDEA Reverse the movie of $\Sigma : \emptyset \to K$ to get a movie for $\overline{\Sigma} : K \to \emptyset$.

Choose some $\phi \in Kh(K)$ and consider

$$\boxed{\Sigma_\phi := \left| Kh(\overline{\Sigma})(\phi) \right| \in \mathbb{Z}}$$

which is an invariant of the bndry-preserving class of Σ (or $\overline{\Sigma}$)

Fixes many "complexity" issues:

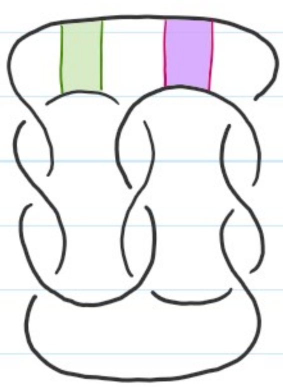
- by choosing $\phi \in Kh(K)$ we can controle how awful calculations become
- can easily compare Σ_ϕ's (integers)

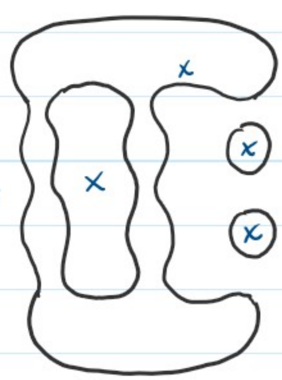
Thm The slice disks D_l and D_r, given by band moves l and r on the given knot K, are distinguished by the given class $\phi \in Kh(K)$, $D_{l,\phi} \neq D_{r,\phi}$

(a.) $K = 9_{46}$

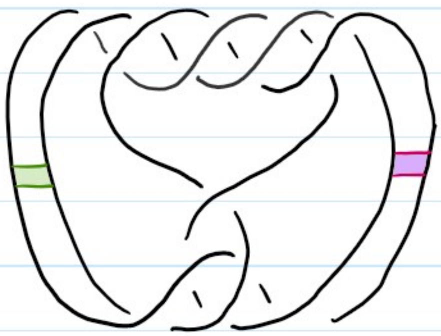
$D_{l,\phi} = 0$

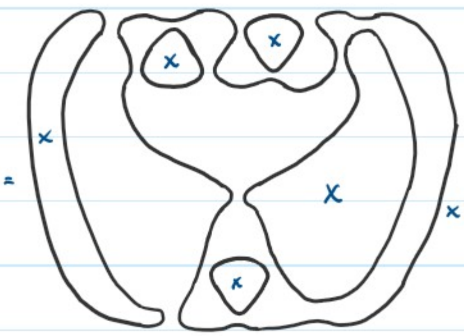
$D_{r,\phi} = 1$



$\phi =$ 

(b.) $K = 15n_{103488}$

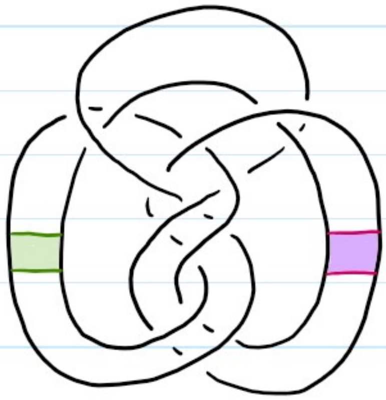


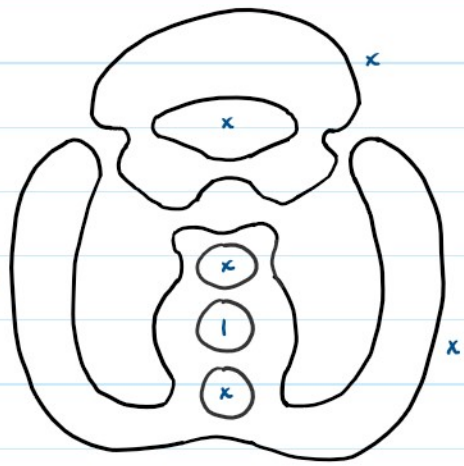
$\phi =$ 

$D_{\ell,\phi} = 1$

$D_{r,\phi} = 0$

(c.) $J = 17nh_{34}$



$\phi =$ 

$D_{\ell,\phi} = 1$

$D_{r,\phi} = 0$

FACT: The slices D_ℓ and D_r for J are continuously isotopic rel J

Since they are not smoothly isotopic rel J, we say they are exotic disks.

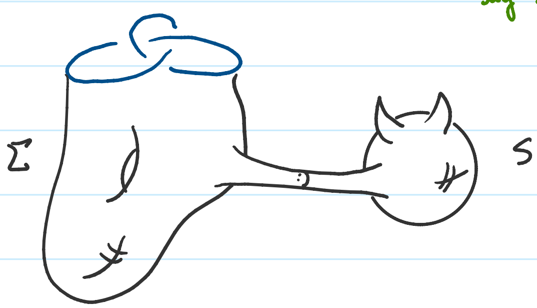
↳ Can be generalized to inf-many ex's with any genus that are not ambiently isotopic

used pairs of bands

extended by a ribbon concordance to (amphichiral) knot with trivial symmetry group

<u>CHEATING</u> Given a link cob. $\Sigma : L_0 \to L_1$ and a knotted 2-sphere S, the surfaces Σ and $\Sigma \# S$ are (generally) not isotopic

↑ say Σ is locally knotted



<u>Thm</u> (Hayden-S., Swann $^{L_0 \neq \emptyset}$) The cobordism maps on Khov. hom. do not detect local knotting: $Kh(\Sigma) = \pm Kh(\Sigma \# S)$