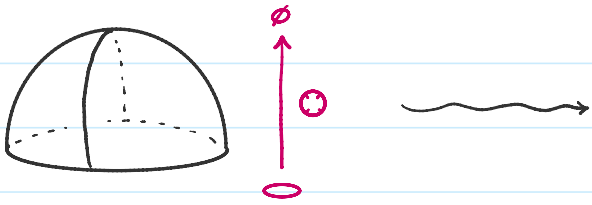
# Talk: MPIM 2 (hand out)

September 9, 2021 1:35 PM



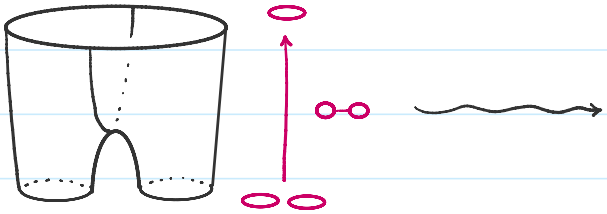
Here are the maps from last time, used to define our TQFT. Note the new shorthand for these cobordisms.



$\varepsilon : A \longrightarrow R$

$1 \mapsto 0$

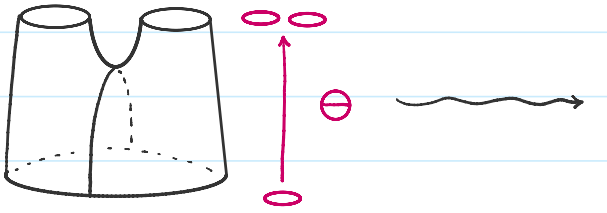
$x \mapsto 1$



$\Delta : A \longrightarrow A \otimes A$

$1 \mapsto x \otimes 1 + 1 \otimes x$

$x \mapsto x \otimes x$



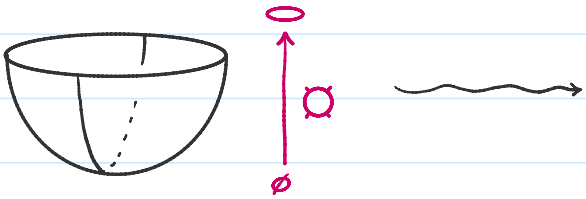
$m : A \otimes A \longrightarrow A$

$1 \otimes 1 \mapsto 1$

$1 \otimes x \mapsto x$

$x \otimes 1 \mapsto x$

$x \otimes x \mapsto 0$

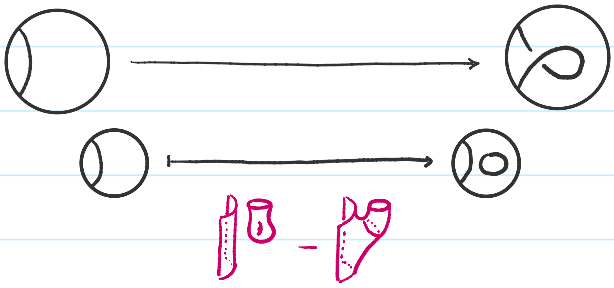


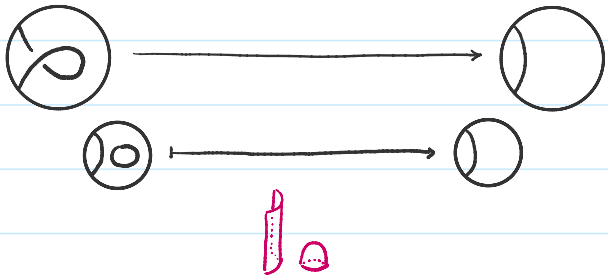
$\iota : R \longrightarrow A$

$1 \mapsto x$

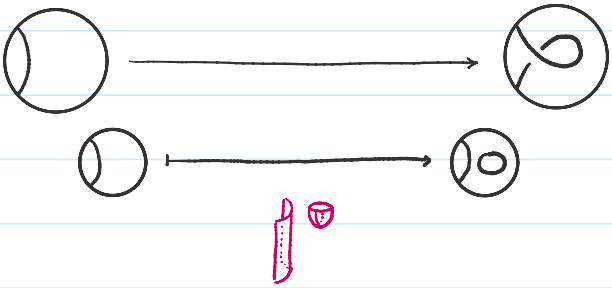
Here are the Reidemeister I and II induced maps. We relate smoothings by linear combinations of cobordisms. Given a labeling, we apply the corresponding Morse induced map. Smoothings not shown are mapped trivially (to the "zero cobordism").

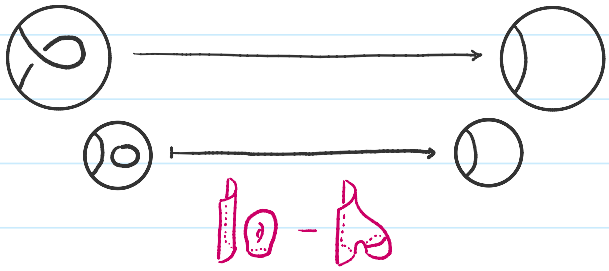
## POSITIVE RI



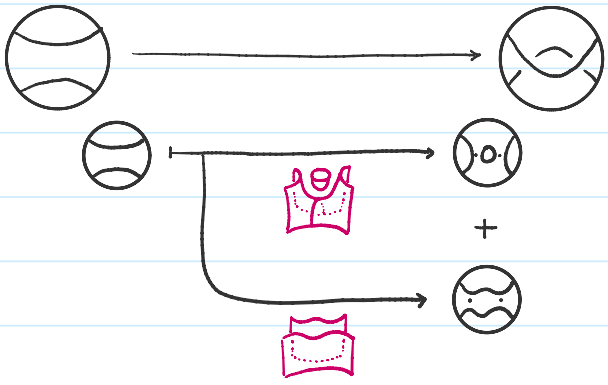


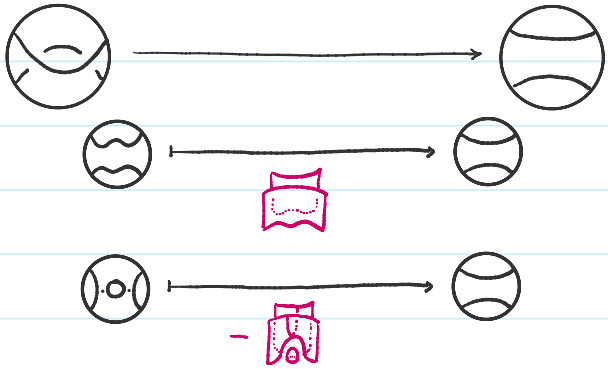
## NEGATIVE RI



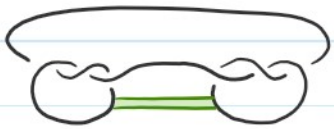


## RII





**Exercise** Find $\phi \in Kh(3, \#m(3,1))$ so that the slice disk $D$ illustrated below satisfies $Kh(D)(\phi) = 1$.



(a) Write out a movie for this slice disk

(b) Find a candidate cycle $\phi$

i. What bigrading is $Kh(D)$ supported in?

ii. How many 0 and 1 smoothings does $\phi$ have?

iii. How many 1 and $x$ labels can $\phi$ have?

iv. Make some guesses at $\phi$

v. Make sure $\phi$ is a cycle $(d\phi = 0)$

(c) Show $Kh(D)(\phi) = \pm 1$.

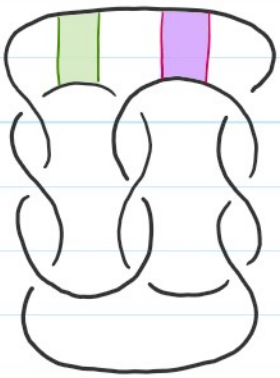
(d) **Bonus** Find a second class with this property.

**Exercise** Here are two slice disks $D_\ell$ and $D_r$ for $9_{46}$, given as band moves. Show that they induce distinct maps $Kh(D_{\ell,r}) : Kh(9_{46}) \longrightarrow Kh(\emptyset)$

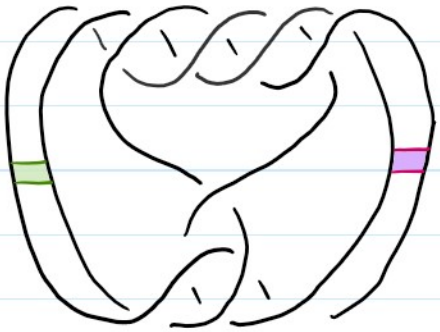
(a) Follow above steps to find $\phi \in Kh(9_{46})$

(b) Show $Kh(D_\ell)(\phi) = 0$ and $Kh(D_r)(\phi) = 1$

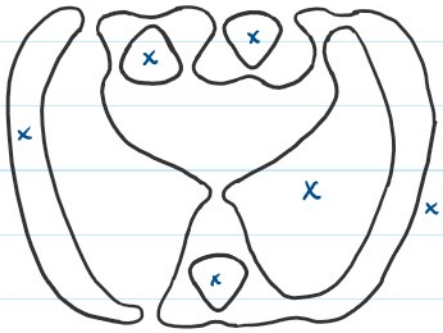
(c) What can we conclude about $D_\ell$ and $D_r$?

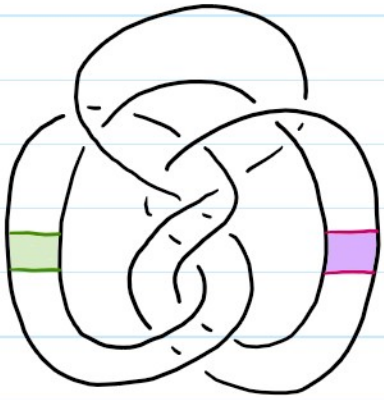


Exercise Show the surfaces below are distinct by distinguishing their induced maps on the given homology class.

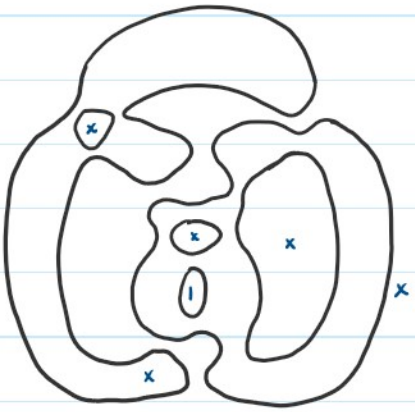


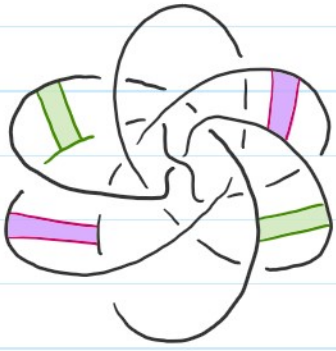
$\phi =$





$\phi =$





$\phi =$