

II 3D frame elements Stiffness matrix

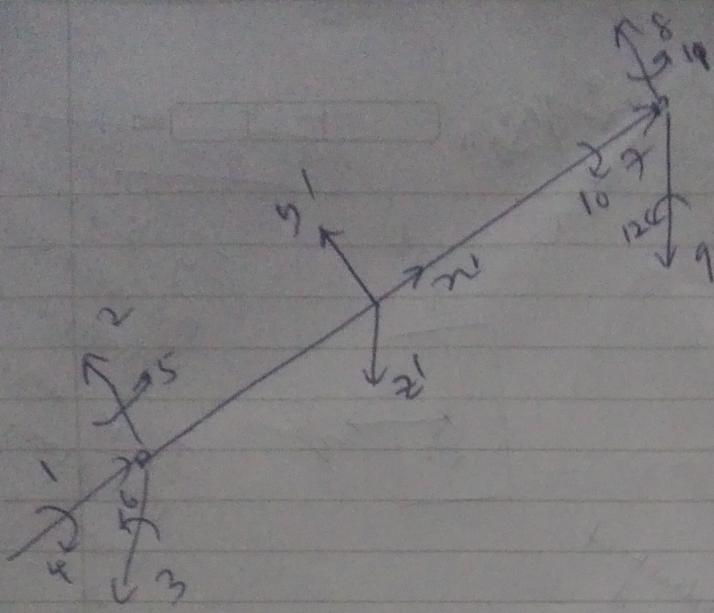
Assemble these matrices up obtain 12×12 3D frame element stiffness matrix referring local co-ordinate system.

$$\textcircled{1} \quad \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}_7^1 \quad \textcircled{2} \quad \frac{GJ}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}_{10}^4$$

$$\textcircled{3} \quad \begin{bmatrix} 2 & 6 & 8 & 12 \\ 3 & \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & -\frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\ 6 & \frac{6EI_z}{L^2} & \frac{4EI_z}{L} & -\frac{6EI_z}{L^2} & \frac{2EI_z}{L} \\ 8 & -\frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} & \frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} \\ 12 & \frac{6EI_z}{L^2} & \frac{2EI_z}{L} & -\frac{6EI_z}{L^2} & \frac{4EI_z}{L} \end{bmatrix}$$

$$\textcircled{4} \quad \begin{bmatrix} 3 & 5 & 9 & 11 \\ 3 & \frac{12EI_y}{L^3} & -\frac{6EI_y}{L^2} & -\frac{12EI_y}{L^3} & -\frac{6EI_y}{L^2} \\ 5 & -\frac{6EI_y}{L^2} & \frac{4EI_y}{L} & +\frac{6EI_y}{L^2} & \frac{2EI_y}{L} \\ 9 & -\frac{12EI_y}{L^3} & +\frac{6EI_y}{L^2} & \frac{12EI_y}{L^3} & +\frac{6EI_y}{L^2} \\ 11 & -\frac{6EI_y}{L^2} & \frac{2EI_y}{L} & +\frac{6EI_y}{L^2} & \frac{4EI_y}{L} \end{bmatrix}$$

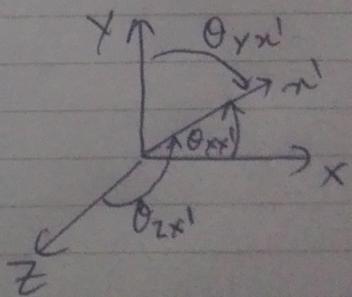
$$K_{\text{frame element, local}} = \left[\quad \right]_{12 \times 12}$$



Transformation matrix

$$T = \begin{bmatrix} \Delta_{3 \times 3} & \\ & \Delta_{3 \times 3} \\ & & \Delta_{3 \times 3} \\ & & & \Delta_{3 \times 3} \end{bmatrix}_{12 \times 12}$$

$$\Delta_{3 \times 3} = \begin{bmatrix} l & m & n \\ -\frac{m}{D} & \frac{l}{D} & 0 \\ -\frac{ln}{D} & -\frac{mn}{D} & D \end{bmatrix}$$



$$l = \cos \theta_{xy1}$$

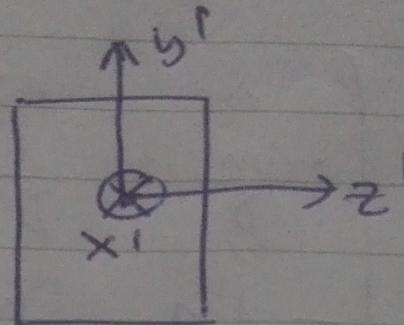
$$m = \cos \theta_{yx1}$$

$$n = \cos \theta_{zx1}$$

$$D = \sqrt{(l^2 + m^2)}$$

$$\boxed{K_{3D, \text{frame element}} = [T]^T K_{3D, \text{frame element - Local}} [T]}$$

Section definition



- x' is the direction from starting node to end node
- cross product of local y' and global z will give local x' direction.
- cross product of local x' and local y' will yield local z' direction.

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