I 3D frame element
Stiffness matrix

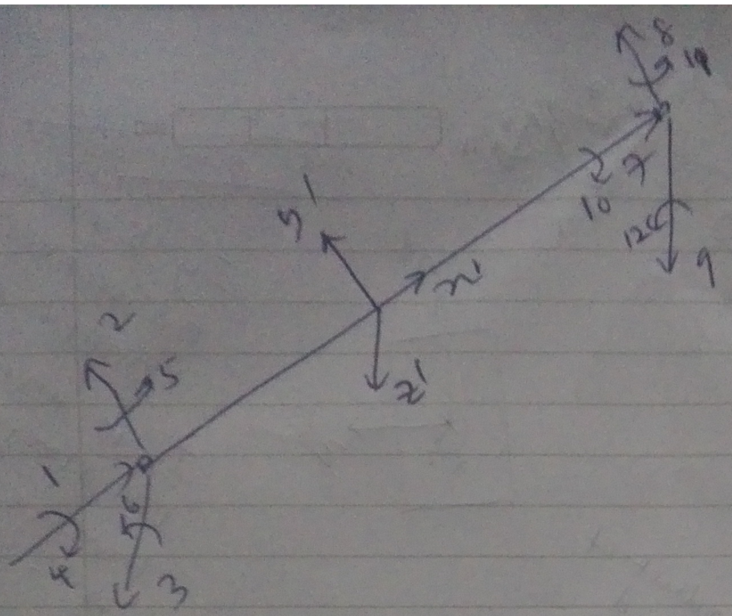
Assemble these matrices to obtain 12×12 3D frame element stiffness matrix referring local co-ordinate system.

$$① \quad \frac{EA}{L}\begin{array}{c c} & \begin{array}{cc} 1 & 7 \end{array} \\ & \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array}\begin{array}{c} \\ 1 \\ 7 \end{array} \qquad ② \quad \frac{GJ}{L}\begin{array}{c} \begin{array}{cc} 4 & 10 \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array}\begin{array}{c} \\ 4 \\ 10 \end{array}$$

$$③ \quad \begin{array}{c c} & \begin{array}{cccc} 2 & 6 & 8 & 12 \end{array} \\ \begin{array}{c} 2 \\ 6 \\ 8 \\ 12 \end{array} & \begin{bmatrix} \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & -\frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\ \frac{6EI_z}{L^2} & \frac{4EI_z}{L} & -\frac{6EI_z}{L^2} & \frac{2EI_z}{L} \\ -\frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} & \frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} \\ \frac{6EI_z}{L^2} & \frac{2EI_z}{L} & -\frac{6EI_z}{L^2} & \frac{4EI_z}{L} \end{bmatrix} \end{array}$$

$$④ \quad \begin{array}{c c} & \begin{array}{cccc} 3 & 5 & 9 & 11 \end{array} \\ \begin{array}{c} 3 \\ 5 \\ 9 \\ 11 \end{array} & \begin{bmatrix} \frac{12EI_y}{L^3} & -\frac{6EI_y}{L^2} & -\frac{12EI_y}{L^3} & -\frac{6EI_y}{L^2} \\ -\frac{6EI_y}{L^2} & \frac{4EI_y}{L} & +\frac{6EI_y}{L^2} & \frac{2EI_y}{L} \\ -\frac{12EI_y}{L^3} & +\frac{6EI_y}{L^2} & \frac{12EI_y}{L^3} & +\frac{6EI_y}{L^2} \\ -\frac{6EI_y}{L^2} & \frac{2EI_y}{L} & +\frac{6EI_y}{L^2} & \frac{4EI_y}{L} \end{bmatrix} \end{array}$$

$$K_{\text{3D frame element, local}} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}_{12\times 12}$$

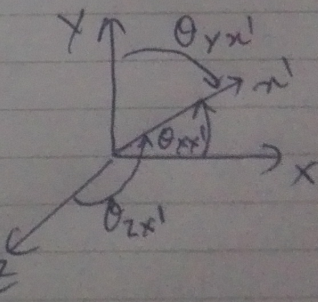


Transformation matrix

$$T = \begin{bmatrix} \lambda_{3\times3} & & & \\ & \lambda_{3\times3} & & \\ & & \lambda_{3\times3} & \\ & & & \lambda_{3\times3} \end{bmatrix}_{12\times12}.$$

This is global coordniate system

$$\lambda_{3\times3} = \begin{bmatrix} l & m & n \\ -\frac{m}{D} & \frac{l}{D} & 0 \\ -\frac{ln}{D} & -\frac{mn}{D} & D \end{bmatrix}$$



$l = \cos\theta_{xx'}$

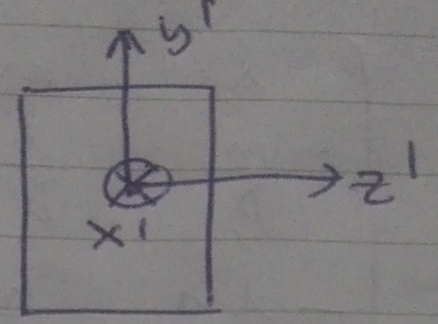
$m = \cos\theta_{yx'}$

$n = \cos\theta_{zx'}$

$D = \sqrt{(l^2 + m^2)}$

$$\boxed{K_{\substack{3D\ \text{frame element}\\ \text{Global}}} = [T]^T\, K_{\substack{3D,\ \text{frame element}\\ \text{Local}}}\, [T]}$$

Section definition



- x' is the direction from starting node to end node
- cross product of local x' and global z ~~and~~ will give local y' direction.
- cross product of local x' and local y' will yield local z' direction.