Induction I
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1. Courtyard Tiling
   The problem is to tile a courtyard with dimensions $2^n \times 2^n$. We are required to install a statue of a wealthy donor in one of the central square, and only special L-shaped tiles can be used. We need to prove this is feasible.

   Theorem For all $n \geq 0$ there exists a tiling of a $2^n \times 2^n$ courtyard with the donor in a central square.

   Proof. Prove by induction. Let $P(n)$ be the proposition that there exists a tiling of a $2^n \times 2^n$ courtyard with the donor placed in any location.

   Base case: $P(0)$ is true because the donor fills the whole courtyard.

   Inductive step: Suppose $P(n)$ is true, we need to prove that $P(n) \to P(n+1)$. A $2^{n+1} \times 2^{n+1}$ courtyard consists of four $2^n \times 2^n$ quadrants, each of them can be tiled with the donor placed in any location. Let the donor be in one of the four central squares, and the remaining three central squares can fit a L-shaped tile. Now we can tile each of the four quadrants by the induction hypothesis. This proves that $P(n) \to P(n+1)$. The theorem follows as a special case. $\square$