

Homework Assignment #3

Due: March 26

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1. (a) Suppose we have 6 activities, for activity i , s_i is the start time, f_i is the finish time and d_i is the duration. And we sort them in order of increasing duration:

i	1	2	3	4	5	6
s_i	8	3	1	4	4	0
f_i	9	5	4	8	9	6
d_i	1	2	3	4	5	6

The greedy procedure when the activities are considered in order of increasing duration would be: 1) select activity 1 since it is the local best; 2) select activity 2 since it is compatible with activity 1; 3) no other activities can be selected since they are not compatible with either activity 1 or 2. So the procedure yields the solution $\{1, 2\}$, but the optimal solution should be $\{3, 4, 1\}$. Therefore the greedy procedure is not correct.

- (b) With the same input, now let us sort them in order of increasing start-time:

i	1	2	3	4	5	6
s_i	0	1	3	4	4	8
f_i	6	4	5	8	9	9

The greedy procedure when the activities are considered in order of increasing duration is:

Function *GreedyActivitySelection*(S, F, n)

$A := \{1\}$ // activity with the earliest start time

$j := 1$ // indicates the activity with greatest start time in A

for $i := 2$ to n

if $S[i] \geq F[j]$

$A := A \cup \{i\}$

$j := i$

return A

Running the above procedure with the same input yields the solution $\{1, 6\}$ which is certainly not an optimal solution. Therefore the greedy procedure is not correct.

- (c) Consider the following 9 activities ordered by the number of overlaps (o_i):

i	1	2	3	4	5	6	7	8	9
s_i	4	0	7	1	2	3	5	6	6
f_i	6	3	10	4	4	5	7	8	9
o_i	2	2	2	3	3	3	3	3	3

When considering the order of increasing number of overlaps, the greedy procedure would first pick activity 1, then pick activity 2, and finally pick activity 3. Thus the solution derived by the greedy procedure is $\{1, 2, 3\}$, but the optimal solution should be $\{2, 6, 7, 3\}$. Therefore the greedy procedure is not correct.

2. Label n gas stations as g_1, g_2, \dots, g_n and let d_i be the distance between gas stations g_i and g_{i+1} , specially, d_0 denotes the distance between g_1 and city A , and d_n denotes the distance between city B and g_n . Let D be the total distance between city A and B , where $D = \sum_{i=0}^n d_i$.

Greedy algorithm

The greedy strategy is to travel as far as you can until the remaining gas is not enough to make it to the next gas station g_i , then you have to refuel at the gas station g_{i-1} . The algorithm works as follows:

Function *GreedyTripRefuel()*

$A := \emptyset$ // the set of refueling stops

$i := 0$ // indicates the farthest gas station one can make.

$T_1 := 0$ // distance traveled before making a refuel

$T_2 := 0$ // total distance traveled

while $T_2 < D$

if $(T_2 - T_1) \leq m$

$T_2 := T_2 + d_i$

$i := i + 1$

else

$i := i - 1$

$A := A \cup \{i\}$

$T_2 := T_2 - d_i$

$T_1 := T_2$

return A

3.

4.