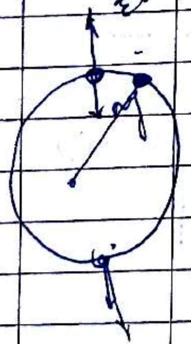
S. No.	Date	Title	Sign/Remarks

1) M.I of disc = $I = mk^2$

2) M.I of rod about its C.G = $\frac{mL^2}{12}$

3) About hinge = $\frac{mL^2}{3}$

$\varepsilon\omega^2$



Date.

<u>Engineering</u> <u>Mechanics</u> :- The science which deals the conditions of rest or motion of bodies under the action of forces.

Mechanics

Statics — [Deals with forces acting upon rigid bodies when body is at rest]

Dynamics — [Deals with particles & bodies that are in motion]

* <u>Newton's First law of motion</u> :- A particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion) only when the resultant force acting on the particle is zero.

* <u>Newton's Second law of motion</u> :- If the resultant force acting on a particle is not zero, the particle will have acceleration proportional to the magnitude of the resultant force & in the direction of this resultant force.

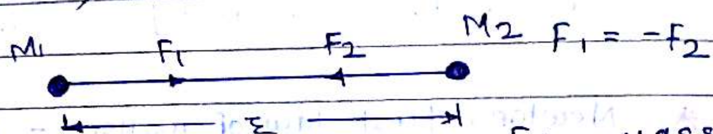
ie. $F = m \times a \Rightarrow \frac{mv - mu}{t} = \frac{m(v-u)}{t}$

where F = Resultant force on the particle

m = mass of the particle

* Newton's third law of motion :- The forces of action and reaction between bodies in contact have the same magnitude, same line of action but opposite in direction.

* Newton's law of gravitation :- This law states that two particles of mass M_1 & M_2 are mutually attracted with equal & opposite forces F_1 & F_2 of magnitude F given by the relation.



$$\boxed{F = \frac{G M_1 M_2}{r^2}}$$

For mass of body m located on earth surface. M & R

$$F = m\left[\frac{GM}{R^2}\right]$$

$$F = mg$$

Force of gravity $F = mg$.

Where, G = Constant of gravitation

r = r = distance between two particles.

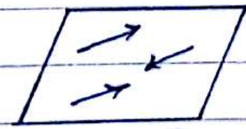
$G = 6.67 \times 10^{-11}\ N\text{-}m^2/kg^2$

Force Systems :-

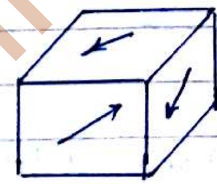
* Force :- An action (push or pull) that changes or tends to change the state of rest or of uniform motion of the body upon which it acts.

* Force is a vector quantity.

i) Coplaner force system :- All the forces lie in the same plane

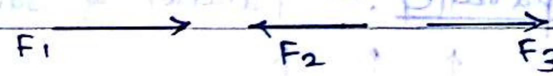


ii) Non Co-planer force system :- * Forces lie in different planes.

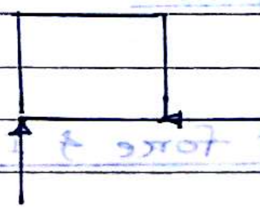


These two types further divided into,

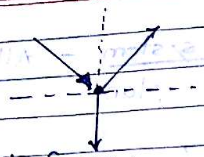
a) Collinear forces :- Forces which act along the same straight line.



b) Non collinear forces :- Forces which acts along different lines.

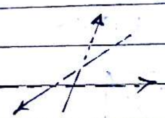


c) Concurrent forces :- Forces whose line of action meet at point.

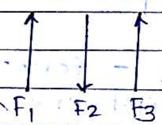


Concurrent forces.

b) Non concurrent forces :- Forces whose line of action at different point.



c) Parallel forces :- Force acting along parallel lines.



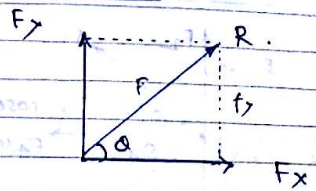
i) Like parallel :- Parallel & same direction.

ii) Unlike parallel :- parallel but opposite direction.

Resultant force & Its component :-

Single force which produces same effect as that produced by different forces ~~along~~ acting together

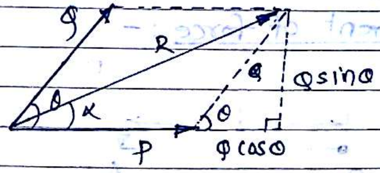
Resolution of forces :-



$\sin\theta = \frac{F_y}{R}$ $F_y = R\sin\theta$.

$\cos\theta = \frac{F_x}{R}$ $F_x = R\cos\theta$.

* The parallelogram law for the addition of forces :- Two forces acting on a particle may be replaced by a single force called their resultant, obtained by drawing the diagonal of the parallelogram which has sides equal to the given forces.



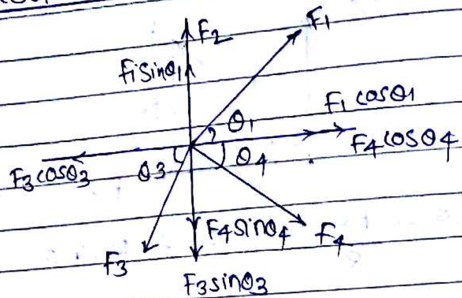
$$R = \sqrt{P^2 + 2PQ\cos\theta + Q^2}$$

$$\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

α = Angle between resultant force & force parallel to x-axis

θ = Angle between two forces P & Q

Resolution of forces :-

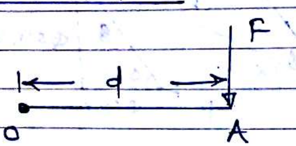


$\Sigma F_x = F_1\cos\theta_1 + F_4\cos\theta_4 - F_3\cos\theta_3$

$\Sigma F_y = F_1\sin\theta_1 + F_2 - F_3\sin\theta_3 - F_4\sin\theta_4$

$$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \qquad \tan\theta = \frac{\Sigma F_y}{\Sigma F_x}}$$

* Moment of force :-



$$\boxed{M = F \times d}$$

ie = force × perpendicular Distance.

Conditions of Equilibrium :-

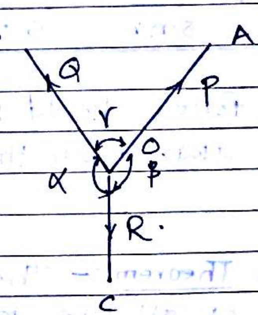
Indicates net effect of forces on body should be zero.

$$\boxed{\Sigma F_y = 0 \qquad \Sigma F_x = 0 \qquad \Sigma M = 0}$$

Imp

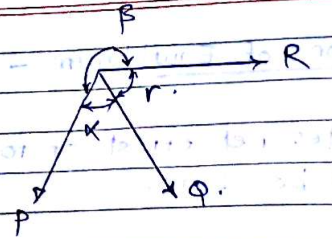
Lami's Theorem :- [for 3 concurrent coplaner forces

It states that if three forces acting at a point are in equilibrium, then each is proportional to the sine of the angle between the other two.



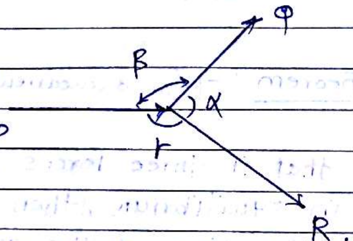
$$\boxed{\frac{P}{\sin\alpha} = \frac{R}{\sin r} = \frac{Q}{\sin\beta}}$$

Case I



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\alpha} = \frac{\text{Force}}{\text{sine of angle between other two forces}}$$

Case II



$$\boxed{\frac{-P}{\sin\alpha} = \frac{Q}{\sin r} = \frac{R}{\sin\beta}}$$

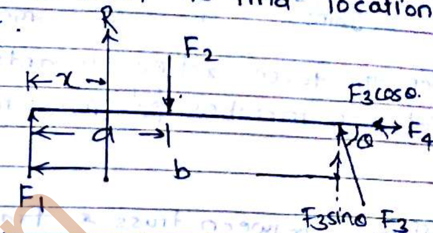
In these, forces should be towards the point or away from the point.

* Varignon's Theorem :- The sum of all the moments of all the forces about a point is equal to the moment of the resultant about the same point.

Note A truss can have only odd numbers of total members.

*

* It is used to find location (x) of resultant.



$$\boxed{F_2 \times a - F_3\sin\theta \times b = R \times x}$$

* No. of joints = No. of members ⇒ for k truss & Howe truss

* Trusses :-

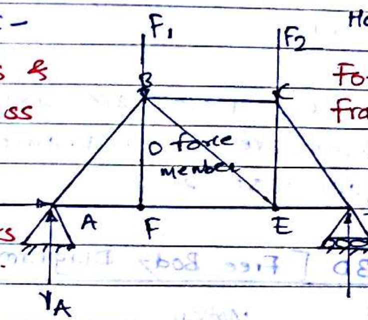
For space truss & for plane truss

$m = 3j - 6$

$m = 2j - 3$

m = no. of members

j = no. of joints.



For perfect frame

$m = 2j - 3$

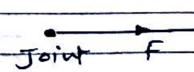
Redundant frame

$m > 2j - 3$

$m < 2j - 3$

* Unloaded joint 3 members, if two of them are collinear, then third member will be the zero (0) force member

* In trusses,



[Tension member]

When force acting away from the joint then member is in tension.

[compression member]

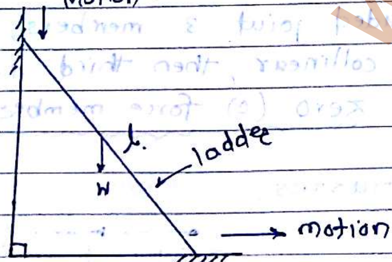
When the force acting towards the joint then membec is in compression.

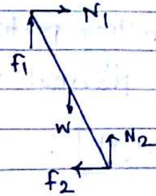
* Difference between truss & frame :-

1) In trusses force applied on the joint but in frames may be on the span of joint.

2) In truss forces are axial (Tensile or compressive) & in frames it may be transverse also

* FBD [Free Body Diagram] for Ladder :-





New chapter

Friction

Definition :-

Friction is the opposing force that is set up between the surfaces of contact when one body slides or rolls or tends to move on the surface of another body.

Limiting or max. friction force $= \mu N$

$= 0.3 \times 100$

$= 30\,N$

But, for above case force $p = 20\,N$ will not move block 100 N, when friction force ~~20 N~~ increases upto max. friction force i.e. 30 N block will start to move i.e. on verge of motion. Hence, friction force for above case is 20 N.

F_{max} = Limiting friction force $\propto$ Normal reaction

$$\boxed{F_{max} = \mu_s N}$$

where μ = Coefficient of friction

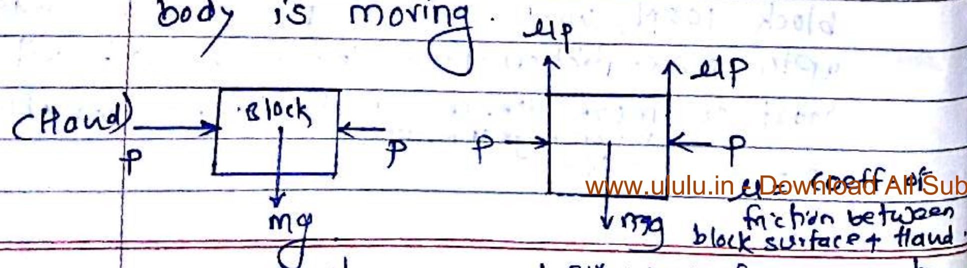
* Types of friction :-

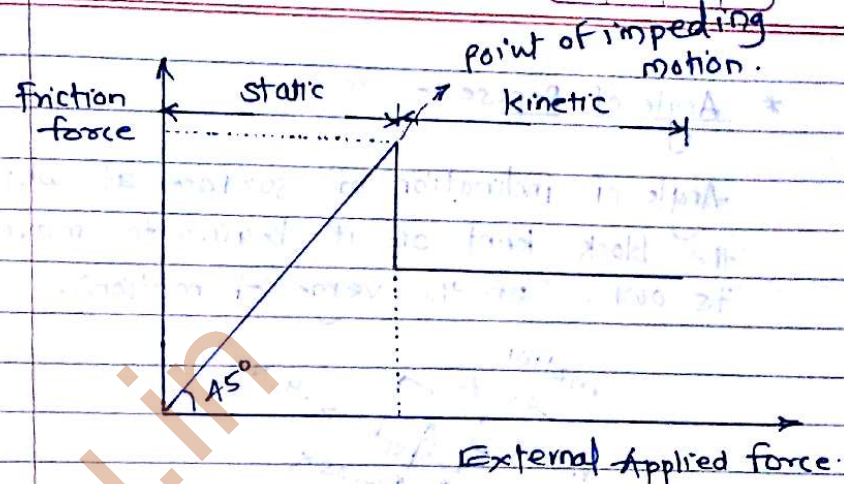
i) Sliding friction :- The friction that exists when one surface slides over another is called sliding friction.

ii) Rolling friction :- The friction that exists between surfaces which have the balls or the rollers interposed between them is called the rolling friction.

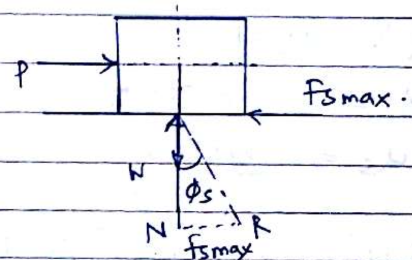
iii) Static friction :- Friction exists when body is at rest or just about to move or verge of motion.

iv) Kinetic friction :- Friction exist when body is moving.





* Angle of friction :-



Angle of static friction :- Angle between N & R when block is on the verge of moving

$$\tan\phi_s = \frac{f_{s\,max}}{N}$$

$$= \frac{\mu_s N}{N}$$

$$\boxed{\phi = \tan^{-1}(\mu_s)}$$

Imp: The tangent of the angle of static friction is equal to the coefficient of static friction.

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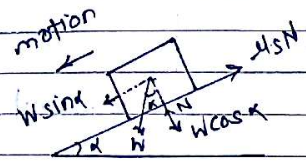
* Angle of Repase :-

Angle of inclination of surface at which the block kept on it begins to move on its own, (on the verge of motion).



$N = W\cos\alpha$

$W\sin\alpha = \mu_s N$

$W\sin\alpha = \mu_s W\cos\alpha$

$\mu_s = \frac{W\sin\alpha}{W\cos\alpha}$

$\boxed{\tan^{-1}\mu_s = \alpha}$ ..... i.e $\alpha = \phi_s$

Case I — When $F < F_{s\,max}$, No motion occures

Case II — When $F = F_{s\,max}$, motion impends or begins.

Case III — When $F > F_{s\,max}$, body is under motion.

Imp Note — The max. force of friction is independent of the area of contact between the two sliding surfaces & depends on the nature of contact surfaces in contact.

Eg. μ = wood on wood = 0.2 – 0.5

Rope on wood = ...

Kinematics

$s = \theta$

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$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega = \omega_0 + \alpha t$

$\omega^2 = (\omega_0)^2 + 2\alpha\theta$

Equations of motion :-

$v = u + at$

$v^2 = u^2 + 2as$

$s = ut + \frac{1}{2}at^2$

Used only when uniform & constant accleration.

$\Rightarrow$ Apply for $\omega \to$ Ang. velocity final, $\omega_0 \Rightarrow$ Initial, $\alpha \Rightarrow$ Ang. Accl^n

Where, u = Initial velocity of body (m/s)

v = Final velocity in (m/sec)

t = time in (sec)

a = uniform accleration (m/sec^2)

s = distance covered in mtr

When speed decreases, then calculated accl will be negative i.e. retardation

Distance covered in the n^{th} second :-

$$\boxed{D_n = u + \frac{a}{2}(2n-1)}$$

Position of vector $\frac{y}{x}$ = $\tan\theta = \frac{y}{x}$

Motion of a particle in a plane :-

Two dimensional motion,

Velocity along x-axis : $u = \frac{dx}{dt}$

Velocity along Y-axis $v = \frac{dy}{dt}$

Velocity will be max. at $\frac{dv}{dt} = 0$ i.e. $\boxed{a = 0}$

$$\boxed{\text{Resultant velocity} = \sqrt{u^2 + v^2}}$$

Inclination w.r.t. x axis =

$\theta = \tan^{-1}\left[\frac{v_y}{u}\right]$

Acceleration along x-axis

$$a_x = \frac{du}{dt} = \frac{d^2x}{dt^2}$$

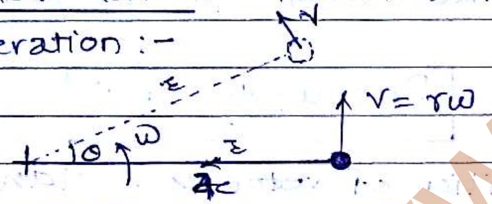
Accleration along y-axis

$$a_y = \frac{dv}{dt} = \frac{d^2y}{dt^2}$$

$$\boxed{\text{Resultant accleration} = \sqrt{a_x^2 + a_y^2}}$$

Inclination w.r.t x-axis $\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$

* Circular Motion :- Normal & Tangential acceleration :-



* Angular velocity $= \underline{\underline{\omega}}$

$$\omega = \frac{d\theta}{dt}$$

where θ is angular displacement and is a function of time (t) $\omega = \frac{v}{r}$

* Angular Accleration :- α

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Normal accleration :or Centripetal Acceleration:-

It acts towards the centre

$a_c = r \times \frac{v^2}{r^2}$ $\quad \boxed{a_c = r\omega^2}$ ... $\omega = \alpha t$

$a_c = \cancel{r} \times \frac{v^2}{r}$ $\quad \alpha = \omega/t$

$a_c = r\alpha^2 t^2$

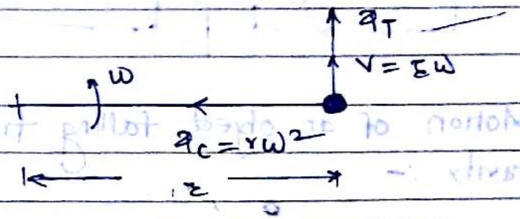
Tangetial accleration

$$a_t = \frac{dv}{dt} = \frac{d\,r\omega}{dt} = r\frac{d\omega}{dt}$$

$$\boxed{a_t = r\alpha}$$

Non-uniform Circular motion :-

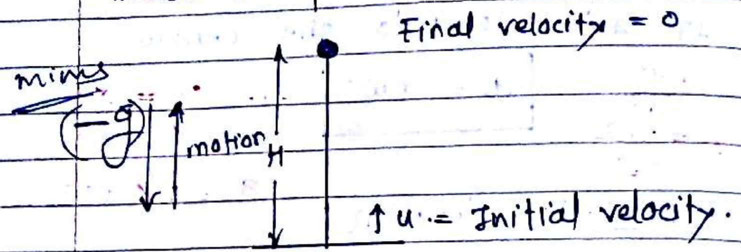
where $\alpha \neq 0$



$$\boxed{a = \sqrt{a_c^2 + a_T^2}}$$

$$a = \sqrt{(r\alpha)^2 + (r\alpha^2 t^2)}$$

1) Max. height attained by an object when thrown in upward direction :-



$v = u + at$

$0 = u - gt$

$u = gt$

$\boxed{t = u/g}$

time to reach max. height.

$v^2 = u^2 + 2as$

$0 = u^2 + 2(-g)H$ --- (−g) downward)

$2gH = u^2$

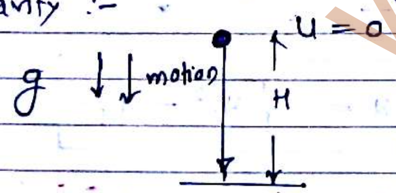
$H = \frac{u^2}{2g}$

$$\boxed{H = \frac{u^2}{2g}}$$

when object thrown upwards then time taken to reach ground

$$\boxed{t = \frac{2u}{g}} = 2 \times \text{time to reach max. height.}$$

2) Motion of an object falling freely under gravity :-



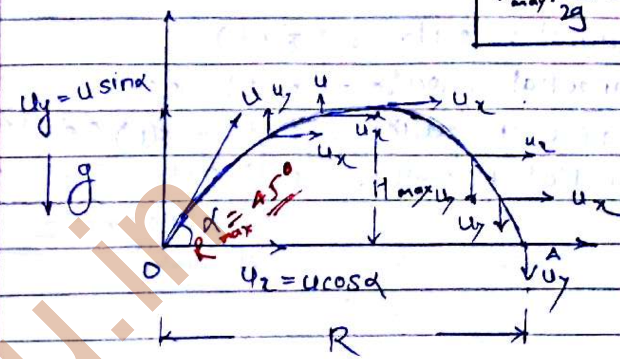
$v^2 = u^2 + 2as$

$v^2 = 0 + 2(+g)H$

$$\boxed{v = \sqrt{2gH}}$$

* Projectile Motion :-

$$\boxed{H_{max} = \frac{u^2}{2g} = \frac{u^2\sin^2\alpha}{2g}}$$



Time of flight (To reach at point A i.e OA

$(T) = \quad v = u + at$

$0 = u\sin\alpha + (-g)t$

$t = \frac{u\sin\alpha}{g}$

$$\boxed{T = 2t = \frac{2u\sin\alpha}{g}}$$

Range (R) = distance = speed × time

$= u\cos\alpha \times \frac{2u\sin\alpha}{g}$

$$\boxed{R = \frac{u^2\sin 2\alpha}{g}}$$

* Condition for max. range (R) is angle of projection $\alpha = 45°$ & the max. range is

$$\boxed{R = \frac{u^2}{g}}$$

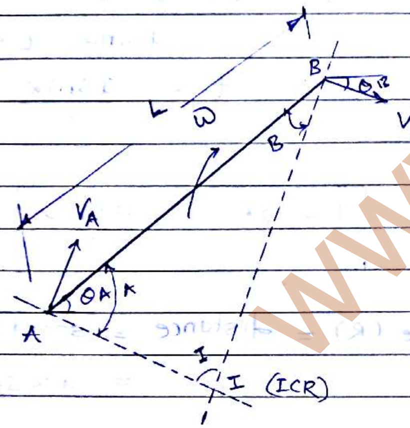
Important Relations

*

1) Radial velocity $= \dot{r}(t)$
2) Tangential velocity $= r(t) \cdot \dot{\theta}(t)$
3) Radial acceleration $= \ddot{r}(t) - r(t)(\dot{\theta}t)^2$
4) Tangential acceleration $= r(t)\ddot{\theta}(t) + 2\dot{r}(t)\dot{\theta}(t)$

Coriolis comp. of Accl^n. $2v\omega$

* Velocity Analysis of kinematic of rigid link



If body performing combined rotational & translational motion. Then ICR is point about which, it performs pure rotational motion. Lets ICR for this link will be located at the point of intersection of perpendiculars V_A & V_B. If ω is angular

velocity of the link about I,

$$\frac{V_A}{V_B} = \frac{IA \times \omega}{IB \times \omega}$$

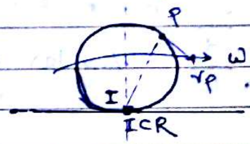
$$\boxed{\frac{V_A}{V_B} = \frac{IA}{IB}}$$

In triangle IAB, By sine rule

$$\boxed{\frac{IA}{\sin B} = \frac{IB}{\sin A} = \frac{AB}{\sin I}}$$

For circular disc :-

1) Rolling without slipping :- ICR at point of contact.

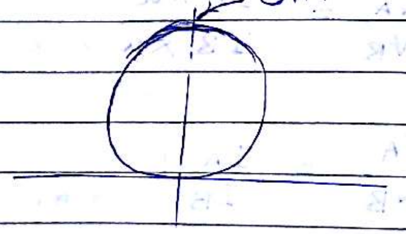


$V_P =$ Distance between ICR & P point $\times \omega$

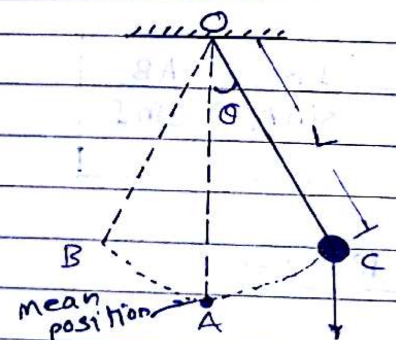
$$\boxed{V_P = IP \times \omega}$$

Velocity of point $I = 0$

2) Rolling with slipping :- ICR On this line



Simple pendulum :-



Velocity max.
maxima cond. $\frac{dv}{dx} = 0$

ie $\frac{dv}{dt} = 0$

ie $\boxed{a = 0}$

* At mean position velocity is max, but accl^n is 0 hence $F = m \times a = \underline{0}$

$\omega = \sqrt{\frac{g}{L}}$ --- Angular frequency of pendulum.

$$\boxed{t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}}$$

Frequency of oscillation $n = \frac{1}{t_p} = \frac{\omega}{2\pi}$

$$\boxed{n = \frac{1}{2\pi}\sqrt{\frac{g}{L}}}$$

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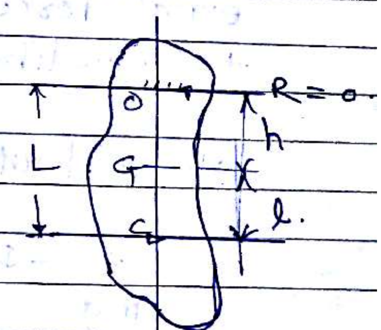
* Centre of percussion :- Point at which a blow may be struck on a suspended body so that the reaction at the support is zero.

1) The centre of percussion (c) is below the centre of gravity (G) as shown in fig. and at a distance of:

$$l = \frac{(K_G)^2}{h}$$

For rod of length

$C_x = l = \frac{2l}{3}$ from hinge. p



2) The distance between the centre of suspension (O) and the centre of percussion (C) is equal to the equivalent length of simple pendulum.

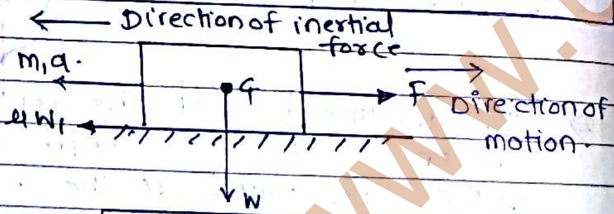
ie. $\underline{\underline{L = l + h}}$

3) The centre of suspension (O) & the centre of percussion (C) are interchangable.

D'Alemberts principle :-

whenever a resultant (net) force acts on a body, the effect of which is to cause an accleration in the body, the inertia of the body plays the role of an inertial force which acts in the direction opposite to the net external force, thus bringing the body to equilibrium.

$$\boxed{\text{Net acclerating force} - \text{Inertial force} = 0}$$



$$\boxed{m_1 a = F - \mu W_1}$$

Newtons 2nd law $F = ma$.

$$\vec{F} - m\vec{a} = 0$$

where, $-m\vec{a} \rightarrow$ ~~fictious~~ fictitious force called an inertial force and it is equal to the product of mass of the particle & its acceleration, directed opposite to the accleration. By adding the fictitious force body can be bring back to dynamic equilibrium.

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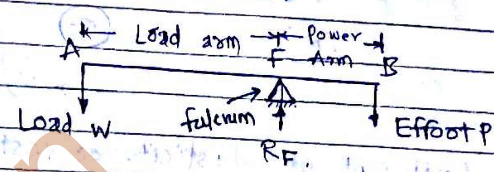
Levers :-



$$P \times FB = W \times FA$$

$$M \cdot A = \frac{W}{P} = \frac{\text{Load}}{\text{Effort}} = \frac{FB}{FA}$$

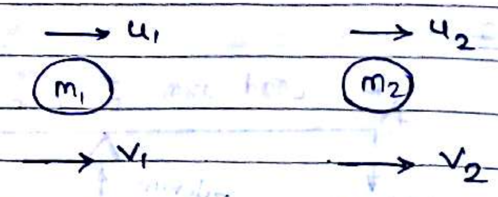
Mechanical Advantage $= M \cdot A = \frac{W}{P} = \frac{FB}{FA}$

ie $= \frac{\text{Power Arm}}{\text{Load Arm}}$

Collision :-

On impact/collision, the bodies deform first and then recover due to elastic properties and start moving with different velocities.

Period of collision consist of two time intervals, period of deformation & period of restitution.



Coefficient of elasticity or restitution: / Resilience.

$$e = \frac{\text{Velocity of sepration}}{\text{Velocity of approach}} =$$

$$\boxed{e = \frac{v_2 - v_1}{u_1 - u_2}} = \frac{\text{Relative speed after collision}}{\text{Relative speed before collision.}}$$

where $m_1, m_2 \Rightarrow$ Masses of colliding bodies.
$u_1, u_2 \Rightarrow$ Velocities before collision.
$v_1, v_2 \Rightarrow$ Velocities after collision.

Note If directions of velocities u_1 & u_2 and v_1 & v_2 change, then it must be properly incorporated by changing the sign of velocity.

* $e = 1 \Rightarrow$ for perfectly elastic bodies
* $e = 0 \Rightarrow$ for plastic body.

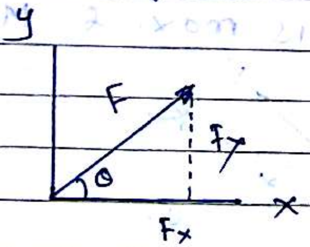
* Coefficient of restitution is depends only on material of the bodies, but independent of their masses & their velocities before impact.

Law of triangle of forces :-

If two forces acting on a body are represented in magnitude and direction by the two sides of a triangle taken in order, then their resultant is given by the third side of the triangle taken in the opposite order.

Law of polygon of forces :- If many forces acting on a body are represented in magnitude & direction by the sides of polygon taken in order, then their resultant is given by the closing side of the polygon taken in the opposite order.

Components of a vector in orthogonal basis:



$$\bar{F} = F_x \bar{i} + F_y \bar{j}$$

$$F = \sqrt{F_x^2 + F_y^2}$$

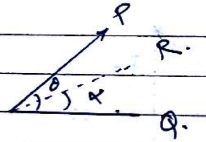
$$\tan\theta = \frac{F_y}{F_x}$$

Principle of transmissibility :-

According to this law, conditions of equilibrium or of motion of a rigid body will remain unchanged if, a force acting at a given point of the rigid body is replaced by a force of the same magnitude & same direction but acting at different point, provided that two forces have same line of action.

i.e. The effect of force upon a body is same at every point in its line of action.

Note Angle between the two forces P & Q when R is max. & Min. (θ) ⇒ 0, 180°



$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$\boxed{\theta = 0}$

$$R^2 = P^2 + Q^2 + 2PQ.$$

$$R^2 = (P+Q)^2$$

$\boxed{R = P+Q}$ Max. R

for min. R.

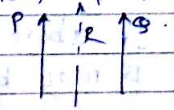
$$R^2 = P^2 + Q^2 + 2PQ\cos 180$$

$$= P^2 + Q^2 + 2PQ(-1)$$

$$R^2 = (P-Q)^2$$

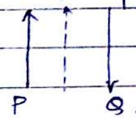
$\boxed{R = P-Q.}$ ...... Min R.

Note ⇒ Resultant of two parallel forces acting in same direction, P & Q.



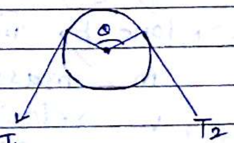
$\boxed{R = P+Q}$

⇒ Resultant of two parallel opposite forces ⇒ where P > Q.



$\boxed{R = P-Q}$

* Belt friction :-



$$\boxed{\frac{T_2}{T_1} = e^{\mu_s \theta}} \quad \ldots T_2 > T_1$$

* This formula should only be used if the belt, rope or brake is about to slip.

T_2 = Tension which pulls
T_1 = resisting tension.

* In above eq^n, θ must be in radian. β may be larger than 2π (360°).

* If a rope is wrapped around a post n times $\theta = 2\pi n$

$$\boxed{\theta = 2\pi \times n}$$

Note When vehicle travelling with certain speed & stops immediately then kinetic energy ($\frac{1}{2}mv^2$) is equal to the workdone by friction force (μmgs)
where, m = mass of vehicle
v = velocity of vehicle
μ = coeff. of friction of road surface.
g = 9.81 m/sec^2
s = distance covered by vehicle when it stops

Calculated by, $v = u + at \Rightarrow a$
& $v^2 = u^2 - 2as$.

$$\boxed{\therefore \frac{1}{2}mv^2 = \mu mgs.}$$

* **Principle of virtual work:-**

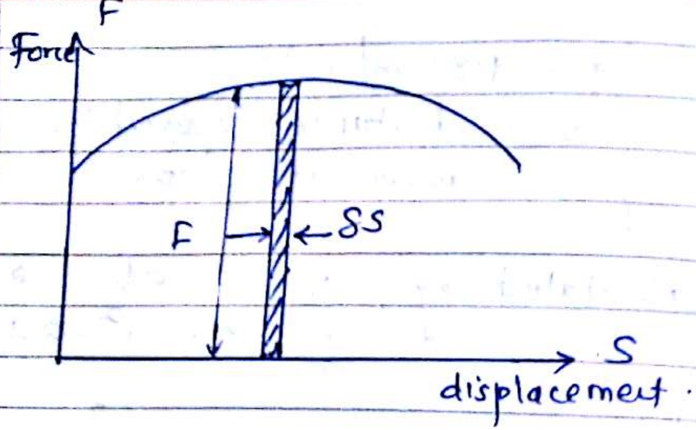
Work:-
Workdone by a force is given by the dot product or scaler product of the force & displacement caused by the force. Max workdone if the displacement produced along the direction of force.

$$\delta W = F \cdot \delta s.$$
$$= F \cdot \delta s \cos\theta \quad [\theta \text{ Angle between } F \text{ \& } \delta s]$$

Total work done $= \int_0^s \delta W = \int_0^s F \cdot \delta s \cos\theta$

for $\theta \Rightarrow 0$ $\cos\theta = 1$

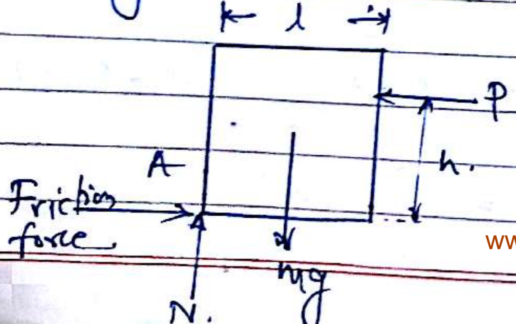
$$\boxed{\int^s \delta W = \int_0^s F \cdot \delta s}$$



Principle of virtual work :- Small possible displacement consistent with the constraints of the system is called virtual displacement.

These principle states that, work done by the external forces, on a system of rigid bodies with ideal constraints, during a virtual displacement of the system is equal to zero. If the system is in equilibrium ie. If δW is the increment of workdone by the external forces on the rigid bodies then $\delta W = 0$

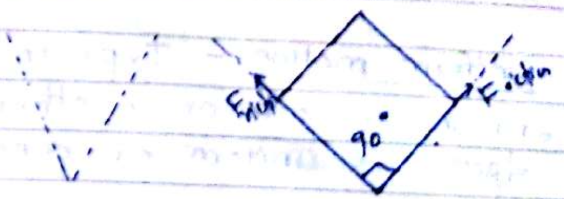
* **Tipping reaction of block :-**



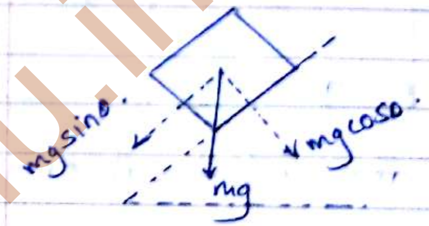
$\Sigma M_A = 0$

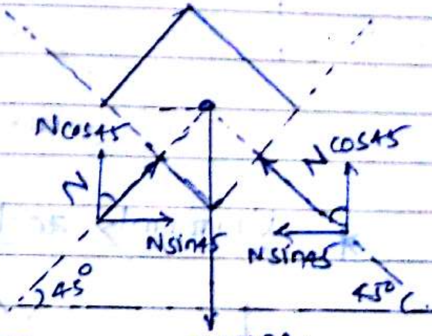
$P \times h - mg \times l/2 = 0$

For trough :-



FBD.





$mg\cos\theta$ = Resultant of two reactions

$= N_1$ & N_2

$= \sqrt{N_1^2 + N_2^2} = \sqrt{2}N$

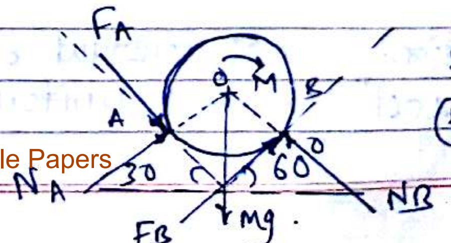
$\Sigma f_y = 0$

$-mg\cos\theta + 2N\cos 45 = 0$

$\Sigma f_x = mg\sin\theta - f - F = 0$

* Two plane contact hence two friction forces are considered.

FBD for cylinder in trough :



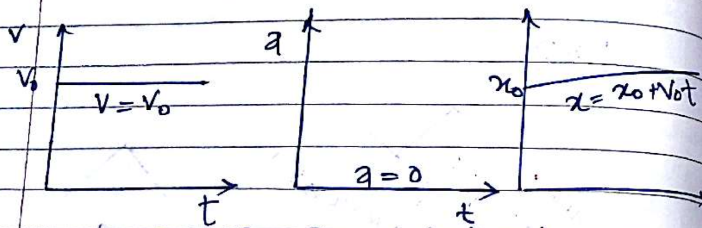
$M_o = F_A \times r +$

$F_B \times r - M = 0$

$(F_A + F_B) r = M$

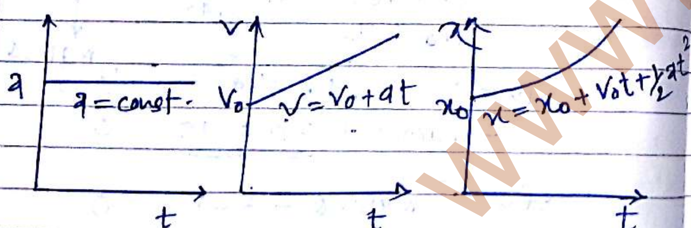
Motion Curves :-

* **Uniform motion** :- Type in which acceleration is zero or in other words, the velocity is uniform or constant.



'0' suffix for initial values.

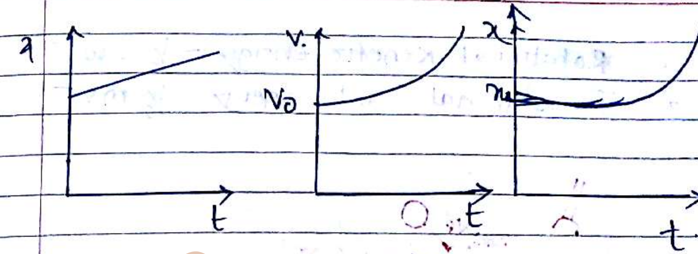
* **Uniformly accelerated motion** :-



Acceleration is constant or velocity is uniformly varying.

* **Non uniformly accelerated motion** :-

Acceleration is not constant or in other words acc^n is non-uniformly varying.



Note In velocity - time graph distance travelled by a particle during a certain interval of time is given by the area under the v-t curve between the time limits.

Area indicates distance travelled by particle.

Accel^n + time

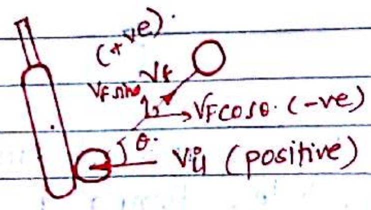
Rotational.

* To find K.E energy of solid circular disc with mass M & radius R & rotating with N rpm

$$\boxed{\begin{aligned} K.E &= \tfrac{1}{2} I \omega^2 \\ \text{where} = I &= \frac{mR^2}{2} \\ \omega &= \frac{2\pi N}{60} \end{aligned}}$$

* Rotational Kinetic energy = $\frac{1}{2} I \omega^2$

* Translational kinetic energy = $\frac{1}{2} m v^2$



Impulse = change in momentum

$$F \Delta t = \cancel{mf}\ m v_f - m v_u$$

$$F_x = \frac{m[-v_f \cos\theta - V_u]}{\Delta t}$$

$$F_y = \frac{m[+v_f \sin\theta]}{\Delta t}$$

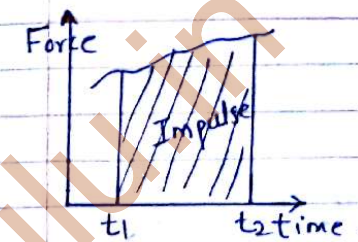
$$F_{avg} = \sqrt{F_x^2 + F_y^2}$$

* Impulse & Momentum

Impulse :- Large force acting for a small interval of time.

defined as,

$$\text{Impulse} = \int_{t_1}^{t_2} f\, dt$$

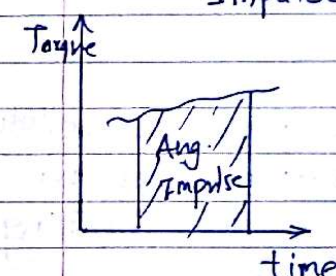


$$= F \int_{t_1}^{t_2} dt$$

$$= F[t_2 - t_1] \quad \text{... if F is const.}$$

$$\boxed{\text{Impulse} = F \times \Delta t}$$

Impulse = Force × time interval.



SI unit ⇒ N·s

$$= \frac{kg \cdot m}{sec^2} \times sec.$$

$$\text{Impulse unit} = \frac{Kg \cdot m}{sec.}$$

* Impulse momentum theorem :-

$$\frac{dp}{dt} = F$$

$$\frac{d(mv)}{dt} = F$$

$$\int_u^v dmv = \int_{t_1}^{t_2} F\, dt$$

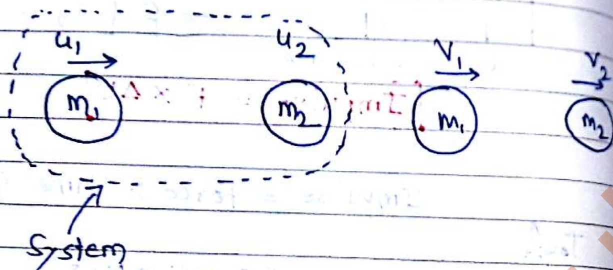
$$m[v-u] = F \int_{t_1}^{t_2} dt$$

$mv - mu = F\Delta t$

Final momentum - Initial momentum = Impulse.

$\boxed{\text{change in momentum} = \text{Impulse}}$

* **Collisions :-**



* If we consider two particles colliding each other as a system. Then we visualizes that, no ext. force acting on a system. Then $F = 0$

If $F = 0$,

$mv - mu = 0$

$mv = mu$

ie.

$\boxed{\text{Final momentum} = \text{Initial momentum.}}$

.... conservation of momentum.

Case I Perfectly elastic collision :- ($e = 1$)

1) where e = coeff. of elasticity, restitution or resilience

$$e = \frac{v_2 - v_1}{u_1 - u_2} = 1$$

2) Momentum is conserved. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

3) K.E also conserved

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

4) Bodies will separate after collision.

Case II Semielastic collision :- ($0 < e < 1$)

1) Momentum is conserved

2) Bodies will separate after collision

3) K.E is not conserved

Case III Plastic collision :- ($e = 0$)

1) Bodies move together after collision

2) momentum will be conserved.

3) K.E is not conserved.

Note Imp $\Rightarrow$ When two equal masses undergo perfectly elastic collision velocities will exchange after collision.

ie. $u_1 = 5\,m/sec$ $u_2 = 0$ $v_1 = 0$ $v_2 = 5\,m/sec$

$u_2 = v_1$ & $u_1 = v_2$

For ball when falling ... distance $= \frac{a}{1-e} + \frac{a}{1-e}$

$= \frac{144}{1-2/3} + \frac{144 \times 2/3}{1-2/3}$

* When body falls from certain height H, then distance travelled by body before coming to rest is given by

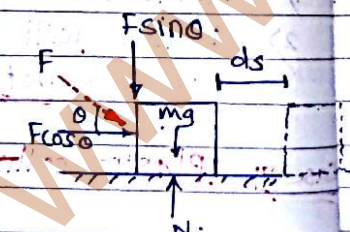
$$\boxed{\text{Total distance } S_n = H\left(\frac{1+e^2}{1-e^2}\right)}$$

* Velocity after n^{th} bounce

$$\boxed{V_n = e^n \sqrt{2gH}}$$

* Height to which it raises after n^{th} bounce

$$\boxed{H_n = e^{2n} H}$$



Work & Energy :-

Product of force in the direction of motion and displacement is called as work.

$$dw = \vec{F} \cdot \vec{ds}$$

$$\boxed{dW = F \, ds \cos\theta}$$

$\boxed{\omega = 2\alpha\theta_{rad}}$ ... α = Ang. Accel^n.

Rotational workdone $= \frac{1}{2} I\omega^2$

* If $\theta = 180$, then workdone by the force is a negative, that force is friction force.

* If $\theta = 90$, then workdone by the force is zero, [Normal reaction]

$\frac{dP}{dt} = F \Rightarrow \frac{d}{dt}(\text{momentum}) = \text{Force}$

$\frac{d\,mv}{dt} = F$

$m \frac{dv}{dt} = F$

$m \times \frac{dv}{ds} \times \frac{ds}{dt} = F$ .. By chain rule

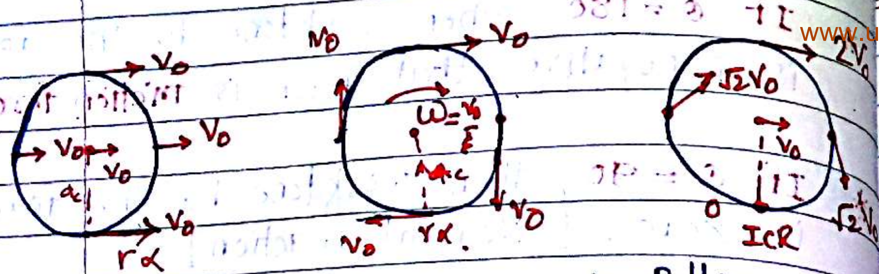
$m \frac{dv}{ds} \times V = F$

$\int_{V_1}^{V_2} mv \, dv = \int_{S_1}^{S_2} F \, ds$

$m\left[\frac{v^2}{2}\right]_{V_1}^{V_2} = \int_{S_1}^{S_2} F \, ds$

$$\boxed{\frac{mV_2^2}{2} - \frac{mV_1^2}{2} = \int_{S_1}^{S_2} F \, ds}$$

Change in kinetic energy = Workdone by the force.



Translation + Rotation $\Rightarrow$ Rolling without slipping.

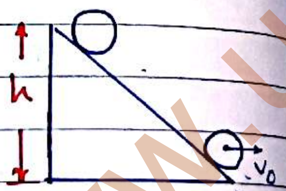
Accel^n at point of ICR $= a_T + a_c$

$= r\alpha - r\alpha + r\omega^2$

$\boxed{a_{ICR} = r\omega^2}$

* When solid cylinder with mass M & radius r having ~~velocity~~ rolling on v_0 inclined plane then velocity at downmost point is

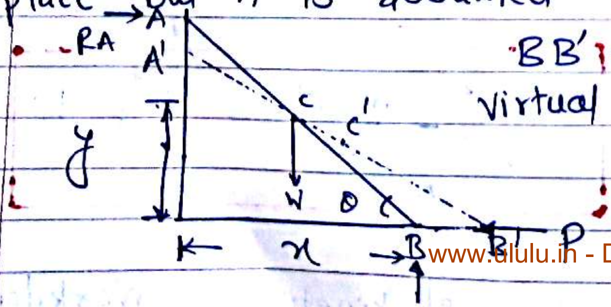
$\boxed{V_0 = 2\sqrt{\frac{gh}{3}}}$



For Sphere, $\boxed{V_0 = \sqrt{\frac{10gh}{7}}}$

Virtual work

Virtual displacement :- It is not actually taking place but it is assumed.



BB' & CC' are virtual displ.

* force producing virtual displacement is called virtual work.

* Virtual displ is assumed such that it is consistent with the constraints of the system.

Principle of virtual work :-

It states that for body of system in equilibrium the sum of virtual work done by all the forces in that system is zero

ie. $(-P \times BB') + (W \times CC') = 0$

* Moment of inertia of solid body is termed as mass moment of inertia. It gives measure resistance to angular motion about an axis. or resistance to rotation about an axis.

Mass moment of inertia ⇒

Product of mass and the square of the perpendicular distance from the axis.

* It's unit $kg\text{-}m^2$

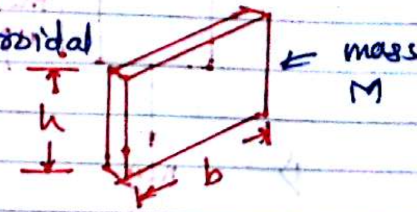
* Parallel Axis theorem :-

M.M.I about noncentroidal axis is equl to the sum of moment of inertia about the centroidal axis & product of mass & square of perpendicular distance between parallel axes.

$$I_{zz} = \bar{I}_{zz} + M \times d^2$$

* MMI of thin rectangular plate :-

$(\bar{I}_{zz})_{mass}$ = About centroidal axis $= \frac{M}{12}[b^2 + h^2]$

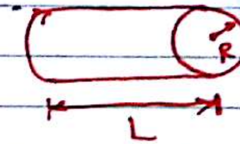


* Thin circular disc :-

$(\bar{I}_{zz})_{mass} = \frac{MR^2}{2}$ (mass uniformly distributed)

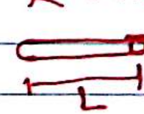
$= MR^2 \rightarrow$ concentrated at rim.

* For solid cylinder :-



$$(\bar{I}_{zz})_{mass} = \frac{MR^2}{2}$$

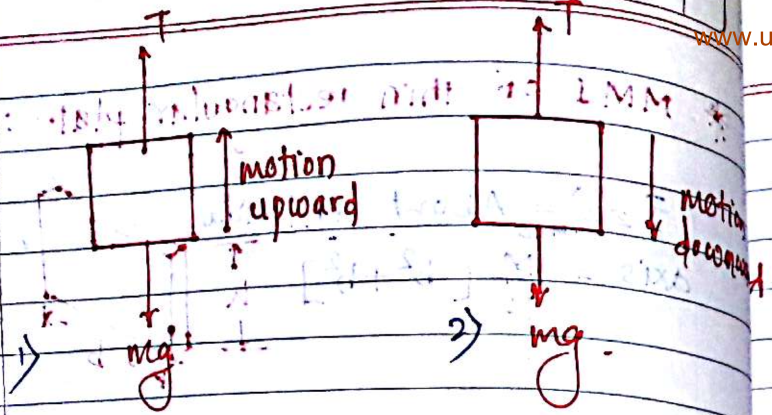
* For ~~soli~~ slender rod :- $R \ll L$



$$(\bar{I}_{xx} = \bar{I}_{yy})_{mass} = \frac{ML^2}{12}$$

* For Sphere :-

$$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}MR^2$$



* Consider the positive forces which are in the direction of motion.

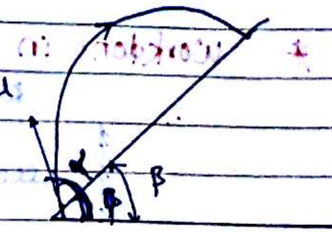
ie. For 1st

$T - mg = ma$

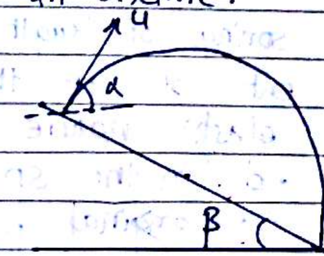
* For 2nd

$mg - T = m \times a$

* Body projected up an incline

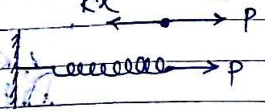


* Body projected down an Incline.



	Projection up the plane	Down the plane
Time of flight.	$\frac{2u}{g\cos\beta}[\sin(\alpha-\beta)]$	$\frac{2u}{g\cos\beta}[\sin(\alpha+\beta)]$
Range	$\frac{u^2}{g\cos^2\beta}[\sin(2\alpha-\beta)-\sin\beta]$	$\frac{u^2}{g\cos^2\beta}[\sin(2\alpha+\beta)+\sin\beta]$
Condition for max. Range	$\alpha = 45^\circ + \frac{\beta}{2}$	$\alpha = 45^\circ - \frac{\beta}{2}$
Max. range.	$\frac{u^2}{g(1+\sin\beta)}$	$\frac{u^2}{g(1-\sin\beta)}$

* Workdone in stretching of a spring



when P is applied to an unstretched spring of length L. It causes displacement x of the free end. Due to elastic nature restoring force is developed in the spring which tries to regain its original position. Hooke's law states that restoring force is proportional to the elongation & opposite to the direction of displacement.

$$F \propto -x$$

$$F = -kx$$

$k \Rightarrow$ spring stiffness / constant (N/m)

If spring is stretched slowly, not accelerated then,

Applied force = Restoring force

$$\boxed{P = kx}$$

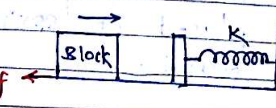
Area under the force & displ. curve is called workdone by spring.

$$W = \int_0^{x_0} P\,dx$$

$$= \int_0^{x_0} kx\,dx$$

$$\boxed{W = -\frac{1}{2}kx_0^2}$$ ...... negative workdone (restoring).

• Block strikes with velocity v & friction force f



Change in K.E = Workdone by the system of forces.
(KEf - KEi)

$$0 - \frac{1}{2}mv_i^2 = -fx - \frac{1}{2}kx^2$$

$$\boxed{\frac{1}{2}mv^2 = fx + \frac{1}{2}kx^2}$$

frictional force workdone (-ve) ; Spring workdone.

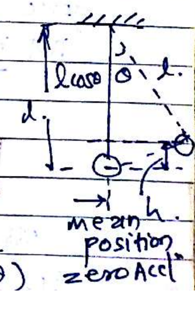
* Max. height of pendulum :-

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$$

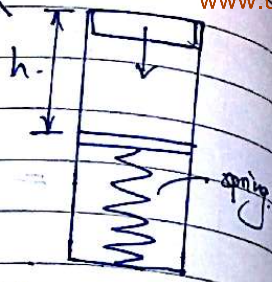
$$gh = \frac{v^2}{2}$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2g\,l(1-\cos\theta)}$$



* Compression of spring of K. stiffness when object hits at certain height



$$mg(h+x) = \frac{1}{2}Kx^2$$

where x = comp. of spring.

* When spring released then, the ball placed on spring will goes at h is given by

$$\frac{1}{2}Kx^2 = mgh$$

$$h = \frac{Kx^2}{2mg}$$

* ~~S~~ Principle of conservation of mechanical energy :- Valid only in conservative force field, such as gravitational force field. If friction is present then not valid.

Total mechanical energy is constant.

$$\boxed{PE + KE = \text{Constant}}$$

loss in potential energy results in gain in kinetic energy.

* $(PE_i) + (K.E)_i = (P.E_f) + (K.E)_f$

* **Power** :- Rate at which work is done

$$P = \frac{dW}{dt} = \frac{dFS}{dt}$$

$$= F\frac{ds}{dt}$$

$$P = F \times V$$

$$\boxed{\text{ie power} = \text{Force} \times \text{velocity}}$$

or Avg. power = $\frac{\text{Total workdone}}{\text{time taken}}$

$$P_{avg} = \frac{W}{t}$$

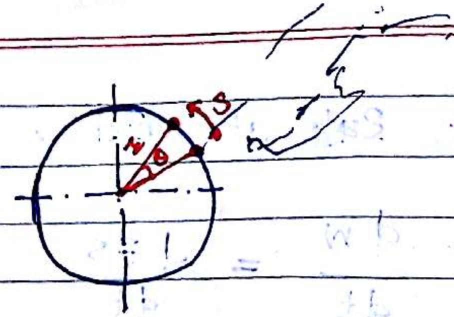
* SI unit J/sec ie. watt.

* 1 Hp = 746 watt

* Metric Hp is 98.6% of mech. Hp = 98.6% × 746 = 736 W

* **Energy** :- Capacity of body to do work.
Unit → N.m or Joule.

*



Linear displacement (s) = $r\theta$

Differt w.r.t. t.

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \quad \ldots \quad r \text{ const.}$$

$$\boxed{v = r\omega}$$

v = linear velocity
ω = Angular velocity

Again diff. w.r.t. t

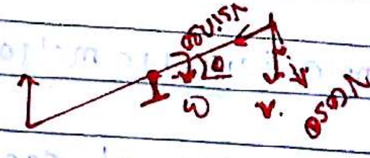
$$\frac{dv}{dt} = r\frac{d\omega}{dt}$$

$$a_t = r\alpha$$

Tangetial Acel^n: $\boxed{a_t = r\alpha}$

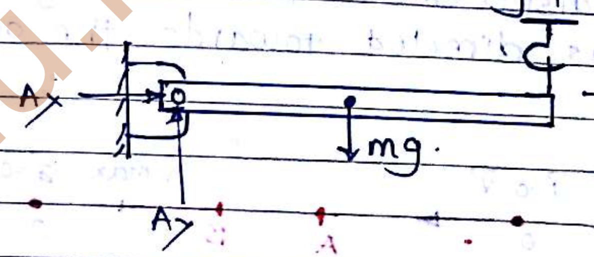
Normal component $a_n = \frac{v^2}{r} = \frac{(r\omega)^2}{r}$

$$\boxed{a_n = r\omega^2}$$



$$\omega = \frac{v\cos\theta}{r}.$$

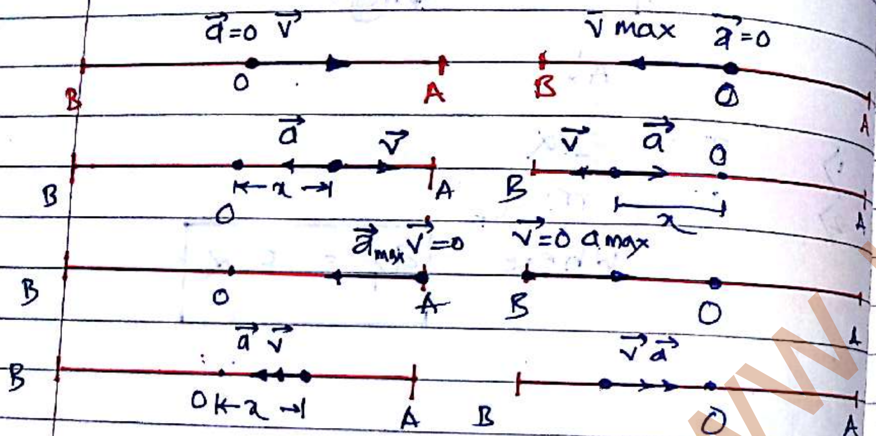
* Centroidal rotation of rigid body :-



1) $T = I\alpha$.
2) $F = ma_G$
where $\boxed{a_G = r\alpha}$

Simple Harmonic motion :-

SHM is special case of rectilinear motion with variable accleration, in which accleration of the particle is proportional the displacement from the origin & is always directed towards the origin.



* The accleration of particle is zero at origin since the accleration is proportional to the displacement from origin & directed opposite to the motion of particle.

$$a \propto -x$$

Constant of proportinality ω^2

$$\boxed{a = -\omega^2 x}$$

By chain rule,

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v\frac{dv}{dx}$$

$$v\frac{dv}{dx} = -\omega^2 x$$

Integrating eq^n,

$$\frac{v^2}{2} = -\omega^2\frac{x^2}{2} + c_1$$

Initial cond^n $v=0$ $x = \pm A$

$$c_1 = \frac{\omega^2 A^2}{2}$$

$$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + \frac{\omega^2 A^2}{2}$$

$$v = \omega\sqrt{A^2 - x^2}$$

$$\boxed{v = \pm\omega\sqrt{A^2 - x^2}}$$

$$v = \frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$$

$$\int \frac{dx}{\sqrt{A^2 - x^2}} = \int \omega\, dt$$

$$\sin^{-1}\left(\frac{x}{A}\right) = \omega t + \phi$$

ϕ = Constant of integration

$$x = A\sin(\omega t + \phi)$$

Initial cond^n $t=0$ $x=0$

We get: $\phi = 0$

$$\boxed{x = A\sin\omega t}$$

In some case $t=0$ $x=A$, part starts moving from extreme position

$$A = A\sin(\omega(0)+\phi)$$
$$1 = \sin\phi$$
$$\phi = \pi/2$$

$$x = A\sin(\omega t + \pi/2)$$

$$\boxed{x = A\cos\omega t}$$

		from origin	from extrem position
Displacement	x	$A\sin\omega t$	$A\cos\omega t$
velocity	$v = \pm\omega\sqrt{A^2-x^2}$	$A\omega\cos\omega t$	$-A\omega\sin\omega t$
Acceleration	$-\omega^2 x$	$-A\omega^2\sin\omega t$	$-A\omega^2\cos\omega t$

* Sine 4 Cosine functions are harmonic functions. Hence resulting motion is called SHM.

Truss :-

A framework composed of members joined at their end to form a rigid structure. Joints like welded, riveted, bolted, pinned.

* Plane Truss :- If all the members lie in a single plane.

* Space Truss :- Consists of members joined together at their ends to form a stable 3D structures.

For plane truss :-

i) $m+3 = 2j$ → statistically Determinate internally.

ii) $m+3 > 2j$ → statistically indeterminate internally.

iii) $m+3 < 2j$ → Unstable truss.

Where, m → No. of members

j → No. of joints

For Space Truss

i) $m+6 = 3j$ → stat. determinate internally

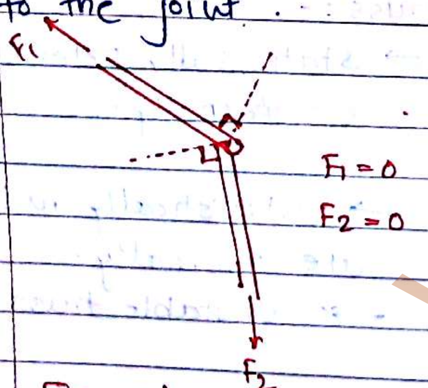
ii) $m+6 > 3j$ → stat. inderminate internally

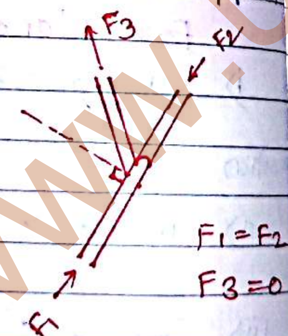
iii) $m+6 < 3j$ → unstable truss

Zero force members :-

1) If only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero force members.

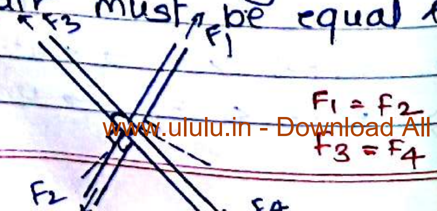
2) If three members form a truss joint for which two of the members are collinear, the third member is a zero force member provided no external force or support reaction is applied to the joint.





Special condition :-

When two pairs of collinear members are joined as shown in fig. the forces in each pair must be equal & opposite..



Redundant members in truss :-

1) To maintain alignment of two members during construction.
2) To increase stability during construction
3) To prevent buckling of compression members.
4) To provide support if the applied loading is changed.
5) To act as backup members in case some members fail or require strengthening.

Uniformly Accelerated Motion :-

In this motion accleration is constant w.r.t. time

i) V-t relationship →

$$\frac{dv}{dt} = a = \text{constant}.$$

$$dv = a\,dt$$

$$\int_u^v dv = a\int_0^t dt$$

$$v - u = at$$

$$\boxed{v = u + at}$$

ii) x-t relationship →

$$v = u + at$$

$$\frac{dx}{dt} = u + at$$

$$\int_{x_0}^{x} dx = \int_0^t (u + at)\,dt$$

$$x - x_0 = ut + \frac{at^2}{2}$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

$$s = (v - at)t + \frac{1}{2}at^2$$

$$= vt - at^2 + \frac{1}{2}at^2$$

$$\boxed{s = vt - \frac{1}{2}at^2}$$

iii) V-x relationship →

$$\frac{dv}{dt} = a$$

$$\frac{dx}{dt}\,\frac{dv}{dx} = a = \text{constant}$$

$$v\,\frac{dv}{dx} = a$$

$$\int_u^v v\,dv = a\int_{x_0}^{x} dx$$

$$\frac{1}{2}\left[v^2 - u^2\right] = a(x - x_0)$$

$$\frac{v^2}{2} - \frac{u^2}{2} = as$$

$$v^2 - u^2 = 2as$$

$$\boxed{v^2 = u^2 + 2as}$$

where, u = Initial velocity
v = final velocity
a = uniform accleration
t = time taken.
s = Distance travelled.

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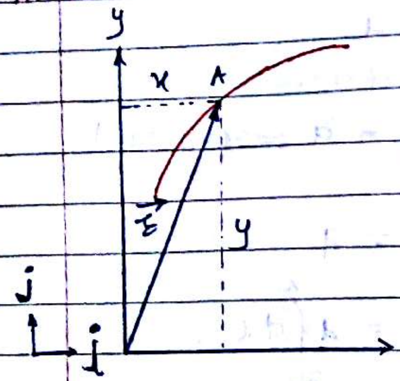
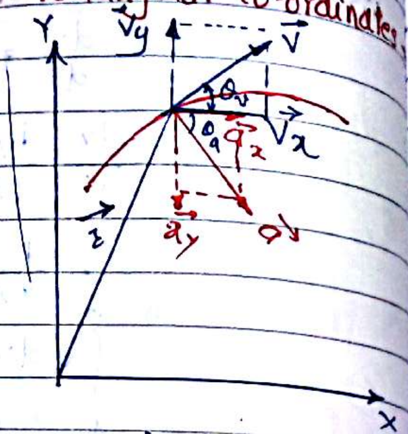
* Curvilinear motion in rectangular co-ordinates





1) Position vector $\vec{r} = x\vec{i} + y\vec{j}$

$$r = \sqrt{x^2+y^2}$$

2) Velocity vector $\vec{V} = \frac{d\vec{r}}{dt} = \frac{dx\vec{i}}{dt} + \frac{dy\vec{j}}{dt}$

$$\vec{V} = \vec{V_x}\vec{i} + \vec{V_y}\vec{j}$$

$$V = \sqrt{V_x^2+V_y^2}$$

$$\boxed{\theta_v = \tan^{-1}\left[\frac{V_y}{V_x}\right]}$$

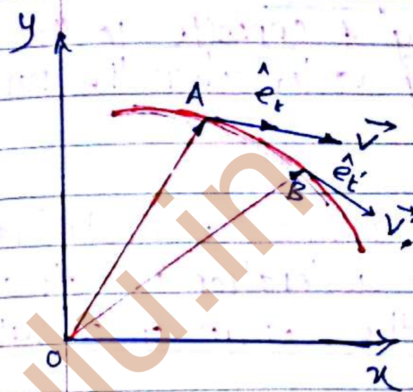
3) Accleration vector $\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x\vec{i}}{dt} + \frac{dv_y\vec{j}}{dt}$

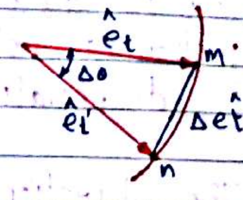
$$= a_x\vec{i} + a_y\vec{j}$$

$$a = \sqrt{a_x^2+a_y^2}$$

$$\boxed{\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right)}$$

Tangetial & Normal components of acclera-tion: →

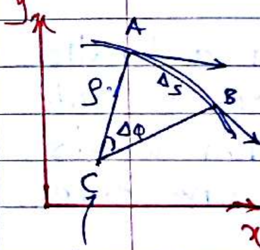




$$\lim_{\Delta\theta \to 0} \frac{\Delta\hat{e}_t}{\Delta\theta} = \frac{d\hat{e}_t}{d\theta} = (1)\,\hat{e}_n$$

Total accleration $\vec{a} = a_t\hat{e}_t + v^2\hat{e}_n\frac{1}{\rho}$

$$\boxed{\vec{a} = a_t\hat{e}_t + v^2k\,\hat{e}_n}$$



Centre of curvature

Where $k = \frac{1}{\rho}$ → curvature

ρ = radius of curvature

Hence Total Accl^n = Tangential Accler + Normal/centripetal/radial accl^n.

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IMP

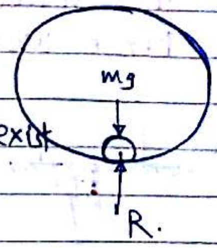
* Direction of tangetial component is same as direction of velocity vector when the particle is accelerating & opposite to the direction of velocity vector when the particle is decelarating.

* The direction of normal / centripetal / radial component of accleration is always inward toward the centre of curvature.

Eg when particle moving on curvilinear path with constant velocity, then tangetial accleration is zero but accleration due to normal component exists.

The normal component of accleration ~~is~~ can be zero only when ρ tends to infinity ie. k=0, in other words moves along a straight line ie. rectilinear motion.

* Body moving on vertical circular path with constant velocity.



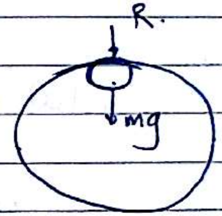
v = constant, hence tangetial accl^n is zero, but normal exist

$$F = ma_{centripetal}$$

At downward point,

$$R - mg = m \times r\omega^2$$
$$= m \times \cancel{r} \times \left(\frac{v^2}{r^2}\right)$$

$$R = \frac{mv^2}{r} + mg$$



Sign conven +ve towards a_c (ie. centre)

At highest point

$$R + mg = \frac{mv^2}{r}.$$

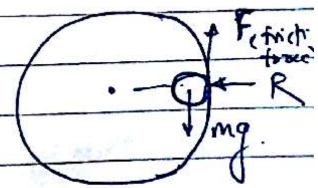
$$R = \frac{mv^2}{r} - mg.$$

min speed to avoid fall when R=0

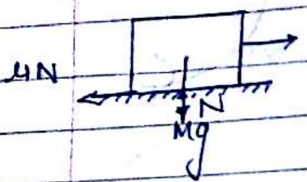
$$mg = \frac{mv^2}{r}$$ $$\boxed{v = \sqrt{gr}}$$

At extreme point.

$$R = \frac{mv^2}{r}.$$



* When body moving with a constant velocity the resultant force acting on it is zero hence workdone $\int F \cdot \cos\theta \, ds$ is zero.



$P = \mu N$

Applied force = frictional force

* Workdone by applied force → $P \times S$ (displacement)
* Workdone by frictional force → $-\mu N \times S$ [opposite direction hence -ve work]

Total workdone $= PS - \mu NS$
$= (P - \mu N)S$

As $P = \mu N$

$$\boxed{\text{Total workdone} = 0}$$

* Workdone by frictional force is always negative & opposite direction.

Energy :- Capacity of body to do work.
SI unit N·m or Joule.

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

Infinitesimily small displacement $d\vec{s}$

$$W = \int_{r_1}^{r_2} \vec{F}\, d\vec{r} = \int_{r_1}^{r_2} m\frac{d\vec{v}}{dt}\, d\vec{r}$$

multiplying num & deno by dt

In addition, if the particle also start from rest.

$$W = m\int_0^t \frac{d\vec{v}}{dt} \cdot \frac{d\vec{r}}{dt}\, dt$$

$$= m\int_0^t \frac{d\vec{v}}{dt} \cdot \vec{v}\, dt$$

$$= \frac{1}{2} m \int_0^t \frac{d(\vec{v}\cdot\vec{v})}{dt}\, dt$$

$$= \frac{1}{2} m \int_0^v dv^2$$

$$\boxed{W = \frac{1}{2} m v^2}$$ → Amount of workdone required to bring body in rest or amount of work originally needed to impact the velocity

ie. $W = \frac{1}{2} m v^2$ = Kinetic energy.

* Twofold increase in speed will increase in the kinetic energy by fourfold.

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Work energy principle :-

Change in kinetic energy of a particle during any displacement is equal to workdone by the net force acting on it.

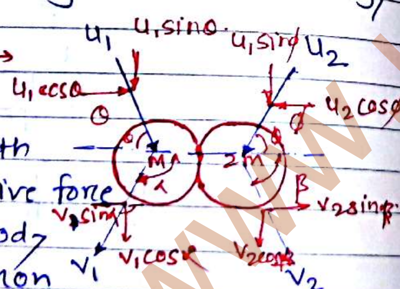
$$W = \frac{1}{2} m (v^2 - u^2)$$

$$= \frac{1}{2} m v^2 - \frac{1}{2} m u^2$$

$$= (K.E)_{final} - (K.E)_{initial}$$

Workdone = change in kinetic energy.

Oblique impact →



Spheres are smoth hence no impulsive force acting on each body along their common tangetial plane. hence no change in momentum ie. velocity is unaltered.

$$u_1 \sin\theta = V_1 \sin\alpha$$

$$u_2 \sin\phi = v_2 \sin\beta$$

Take cos component values & solve:

$$m_1 u_1 - m_2 u_2 = m_1 V_1 + m_2 v_2$$

u_1 & u_2 opposite hence - sign.

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

Two eq^n solve get value of v_1 & v_2

convert $v_1 = v_1 \cos\alpha = u_1 \sin\theta$

$v_2 = v_2 \cos\beta = u_2 \sin\phi$

calculate α & β values

then find final v_1 & v_2.

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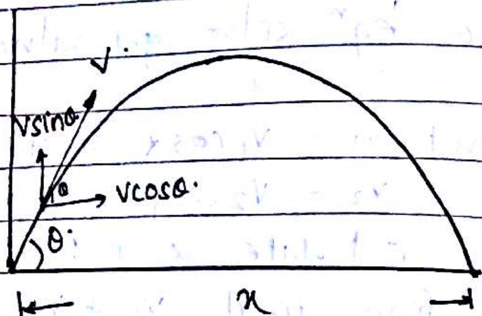
* Projectile Motion →



$$v = \frac{\text{distance}}{\text{time}}$$

$$v\cos\theta = \frac{x}{t}$$

$$t = \frac{x}{v\cos\theta}$$

$$S = ut + \tfrac{1}{2}at^2$$

In y direction g ↓ +ve y ↓ +ve

$$y = v\sin\theta\, t + \tfrac{1}{2}g\left(\frac{x^2}{v^2\cos^2\theta}\right)$$

$$= \cancel{v}\sin\theta \cdot \frac{x}{\cancel{v}\cos\theta} + \tfrac{1}{2}g\,\frac{x^2}{v^2\cos^2\theta}$$

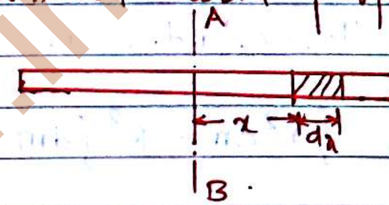
$$\boxed{y = \tan\theta\, x + \tfrac{1}{2}g\,\frac{x^2}{v^2\cos^2\theta}}$$

* Moment of inertia of continuous mass distributions →

$$I = mk^2$$

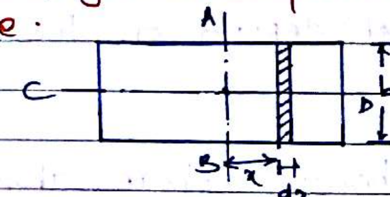
$$I = \int r^2 dm \quad \text{under proper limit}$$

1) Uniform rod about perpedicular bisector →



$$I_{AB} = \frac{mL^2}{12}$$

2) Rectangular plate about a line parallel to an edge and passing through the centre.



MI about AB $\quad I_{AB} = \frac{mL^2}{12}$

Line parallel to the other edge

$$I_{CD} = \frac{Mb^2}{12}$$

3) MI of Circular Ring about its axis

$$I = \int r^2 dm = R^2\int dm = \underline{\underline{MR^2}}$$

4) MI of uniform solid circular plate about its axis.

$$I = \int_0^R r^2 dm = \frac{MR^2}{2}$$

5) MI of hollow cylinder about its axis

$$I = \int r^2 dm = R^2 \int dm = MR^2$$

6) MI of uniform solid cylinder

$$I = \int_0^R r^2 dm = \frac{MR^2}{2}$$

* I does not depend on the length of cylinder.

7) MI of hollow sphere about a diameter

$$I = \tfrac{2}{3} MR^2$$

8) MI of uniform solid sphere about a diameter.

$$I = \frac{2}{5} MR^2$$

* Parallel axis theorem

$$I = I_{axis} + Md^2$$

* Perpendicular axis theorem

$$I_z = I_x + I_y.$$

Radius of Gyration (K) $I = mk^2$

$$K = \sqrt{\frac{I}{m}}$$

It is the radius of a ring with the given line as the axis such that if the total mass of the body distributed on the ring it will have the same moment of inertia. I.

$$\frac{\cancel{M}R^2}{2} = \cancel{M}K^2$$

$$K = \frac{R}{\sqrt{2}}$$

when mass M placed at radius at $\frac{R}{\sqrt{2}}$

$$I = MR^2 = M \times \frac{R^2}{\sqrt{2}^2} = \frac{MR^2}{2}$$