MMÜ753 – Vehicle Control Systems

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CHAPTER – I

VEHICLE DYNAMICS INTRODUCTION

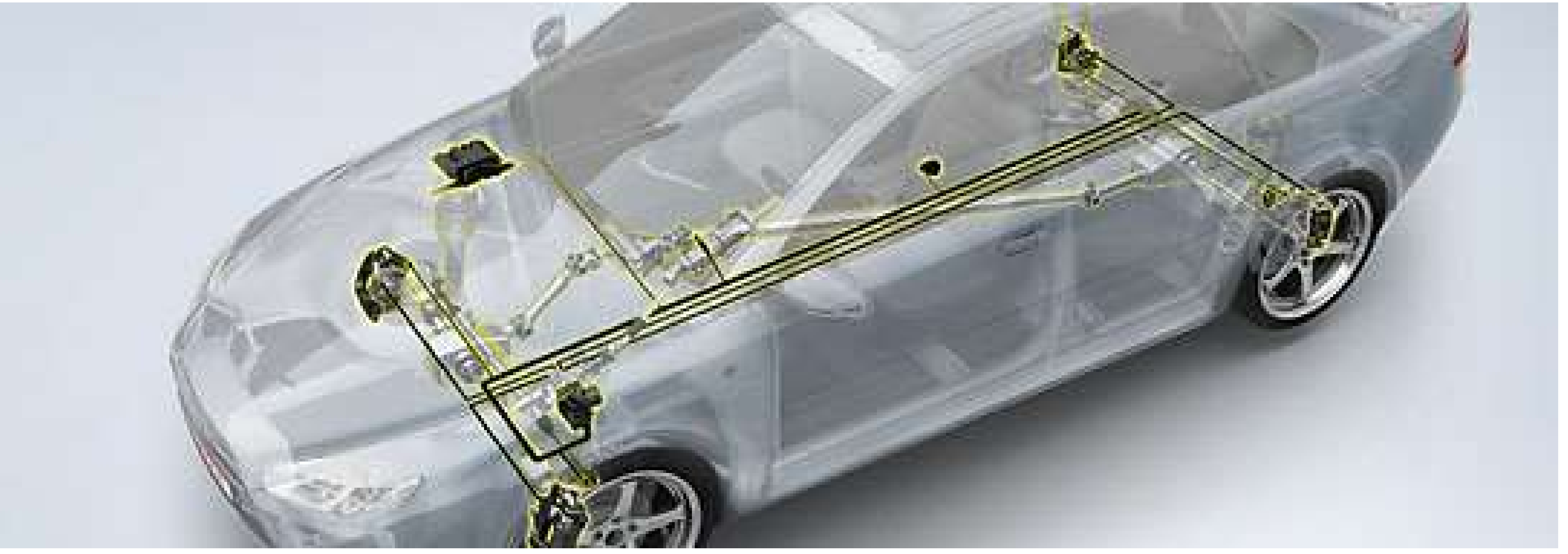
Curriculum

- Peng Ulsoy Çakmakçı
- Kiencke Nielsen
- Rajamani
- Wong
- Jazar
- Winner
- 1F+1P+HW
- Project proposal!

Motivation

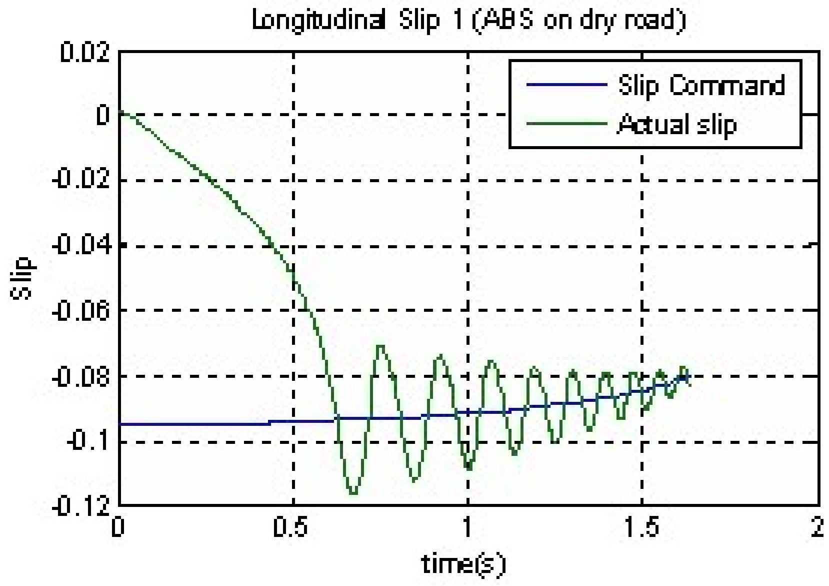
- The most noteworthy trend in the development of modern automobiles in recent decades is their rapid transformation into complex electromechanical systems.
- **Automotive vehicle dynamics & decentralized control**
  - There are about 70 electronic control units in a typical modern passenger vehicle
  - Typically, different control systems are designed independently, and built by separate sub-contractors, particularly for vehicle dynamics control applications
  - These subsystems may even be using common set of sensors and actuators
  - Functioning of each subsystem may affect a number of

Motivation

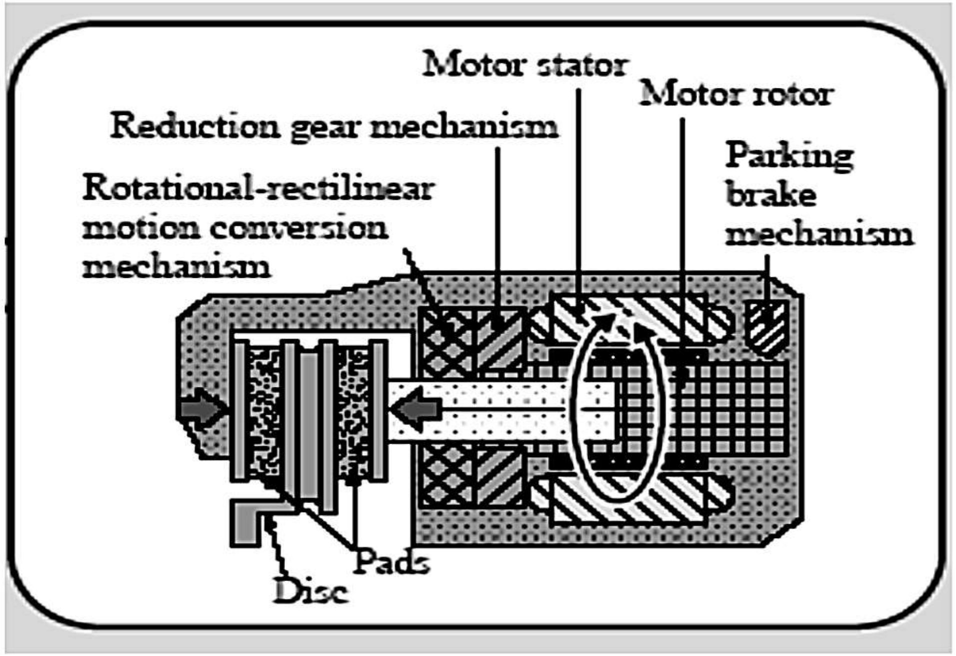


Motivation

Conventional ABS



EMB



Advantages Of Electromechanical (Or Mechatronic) Devices

- The major advantages of electromechanical (or mechatronic) devices, as opposed to their purely mechanical counterparts, include
- (1) the ability to embed knowledge about the system behavior into the system design,
- (2) the flexibility inherent in those systems to trade off among different goals, and
- (3) the potential to coordinate the functioning of subsystems.

Advantages Of Electromechanical (Or Mechatronic) Devices

- Reprogrammability implies lower cost through exchanged and reused parts.
- Sharing of information makes it possible to integrate subsystems and obtain superior performance and functionality, which are not possible with uncoordinated systems.

Motivation

- On average, one person dies every minute somewhere in the world due to a car crash. The cost of crashes totals 3 percent of the world's gross domestic product (GDP) and was nearly $1 trillion in 2000.
- Data from the National Highway Transportation Safety Association (NHTSA) show that 6,335,000 accidents (with 37,081fatalities) occurred on U.S. highways in 1998 (NHTSA 1999).
- In 2008, the same statistic improved by about 10 percent to 5,811,000 accidents (with 34,017 fatalities) (NHTSA 2009).
- Data also indicate that although various factors contribute to accidents, human error accounts for 90 percent of all accidents (Hedrick et al., 1994).

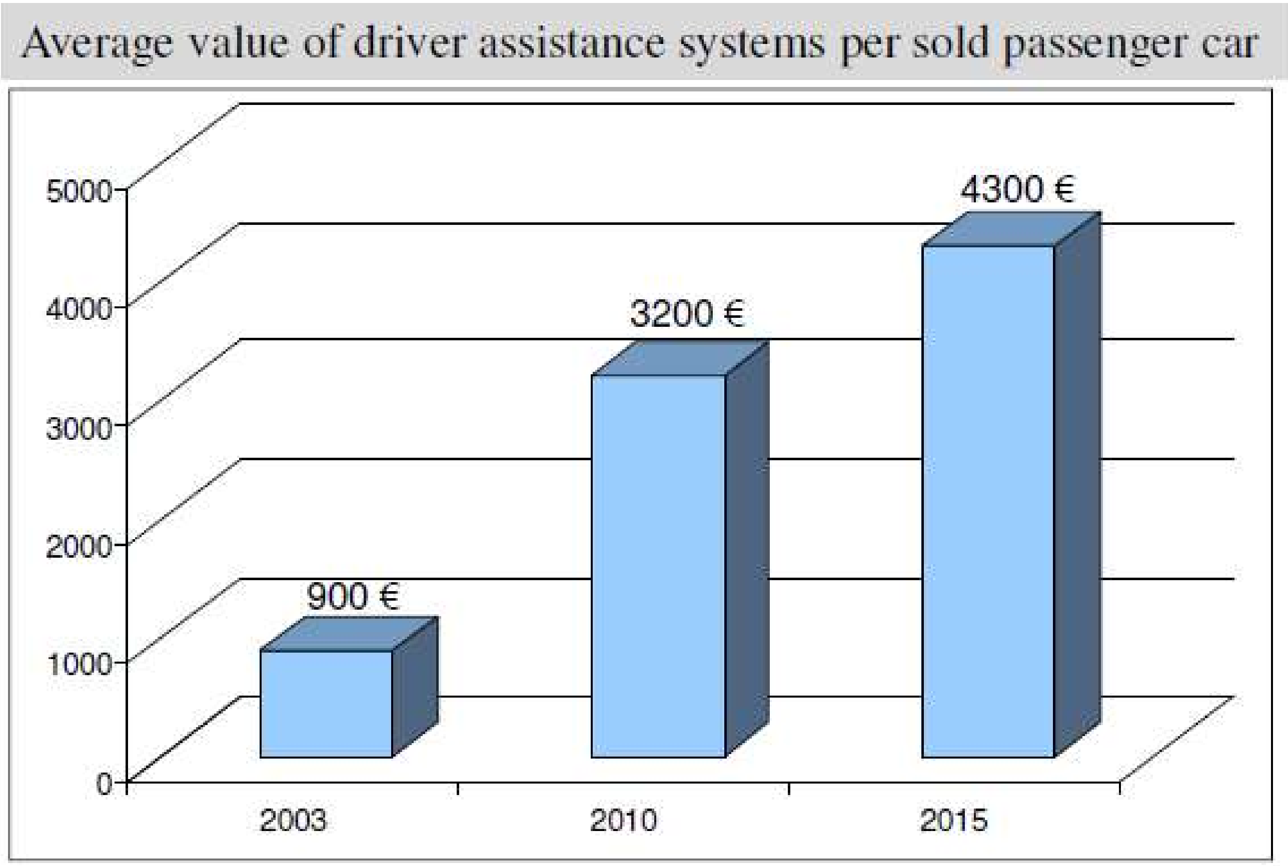
Motivation

- In 1970, only 30 million vehicles were produced and 246 million vehicles were registered worldwide;
- by 1997, these numbers had increased to 56 million and 709 million, respectively. By 2005, 65 million vehicles were produced and more than 800 million were registered (Powers and Nicastri 2000).
- Consequently, another major factor that contributes to the increased use of electronics is the expanding government regulation of automotive emissions.
- For example, the 2005 standard for hydrocarbon (HC) emissions was less than 2 percent of the 1970 allowance; for carbon monoxide (CO), it was 10 percent of the 1970 level; and for oxides of

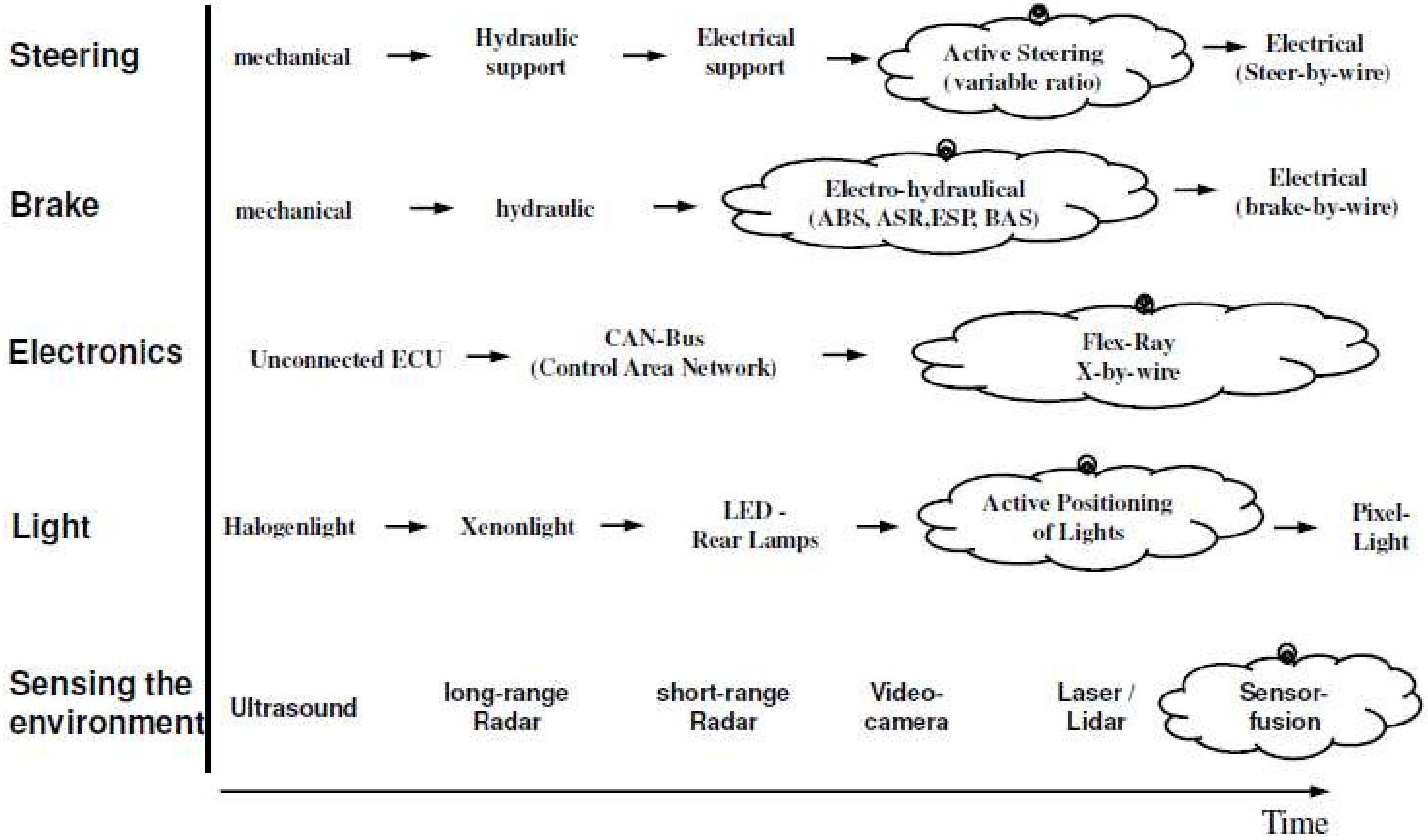
Motivation

- At the same time, government regulations also require improved fuel economy.
- Advanced control technologies (e.g., fuel injection, air-fuel ratio control, spark-timing control, exhaust-gas recirculation [EGR], and idle-speed control) are and will continue to be instrumental in reducing emissions and improving fuel economy (e.g., hybrid-electric, all-electric, and fuel-cell vehicles).

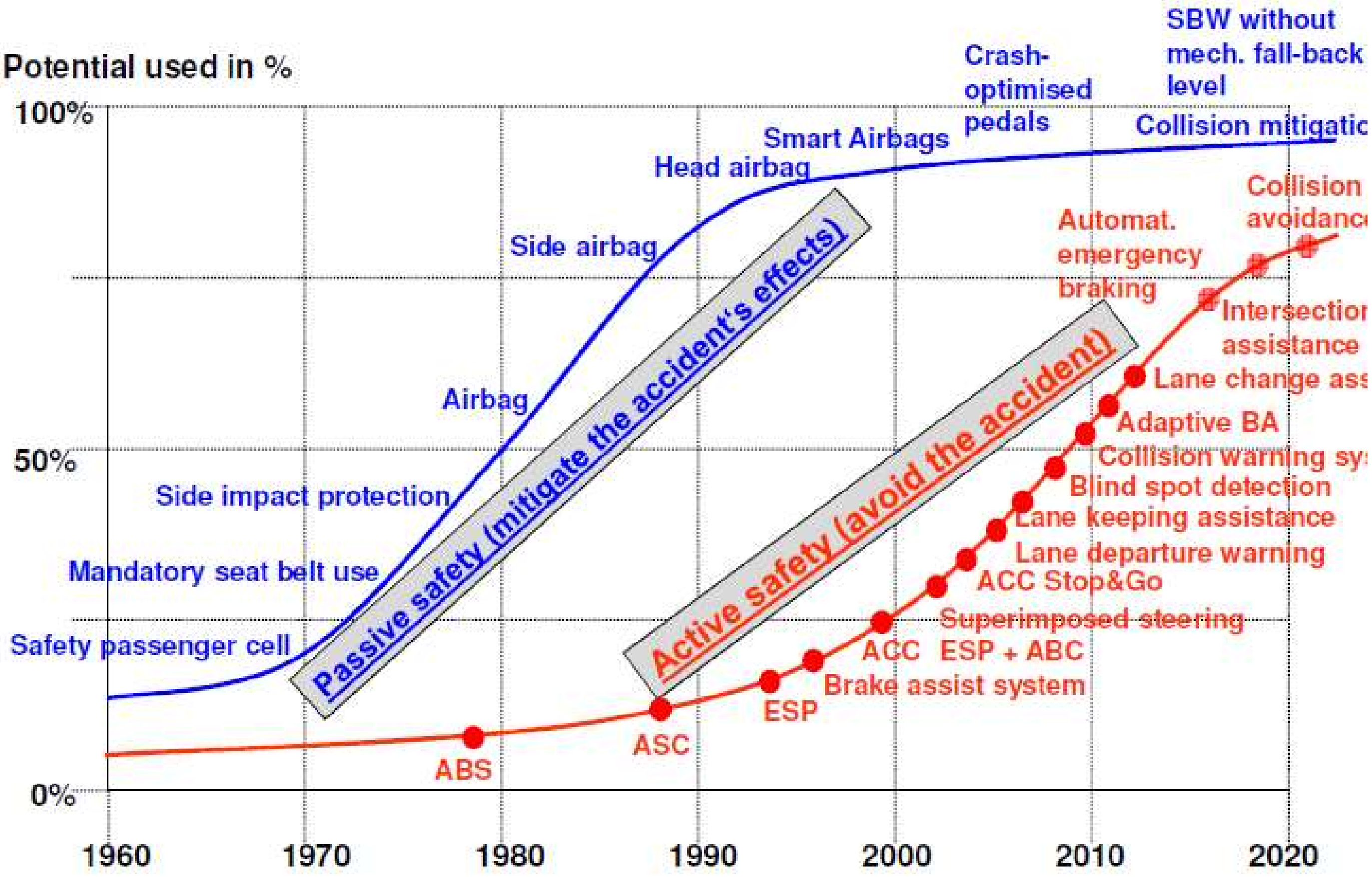
Value of Driver Assistance Systems



Value of Driver Assistance Systems



Active and Passive Safety in Comparison



Powertrain Control

- The engine-control systems may include
  - fuel-injection control,
  - ignition or spark-timing control,
  - idle-speed control,
  - antiknock-control systems,
  - exhaust-gas recirculation (EGR) control.
- The goal of engine-control systems is to ensure that an engine operates at near-optimal conditions at all times.

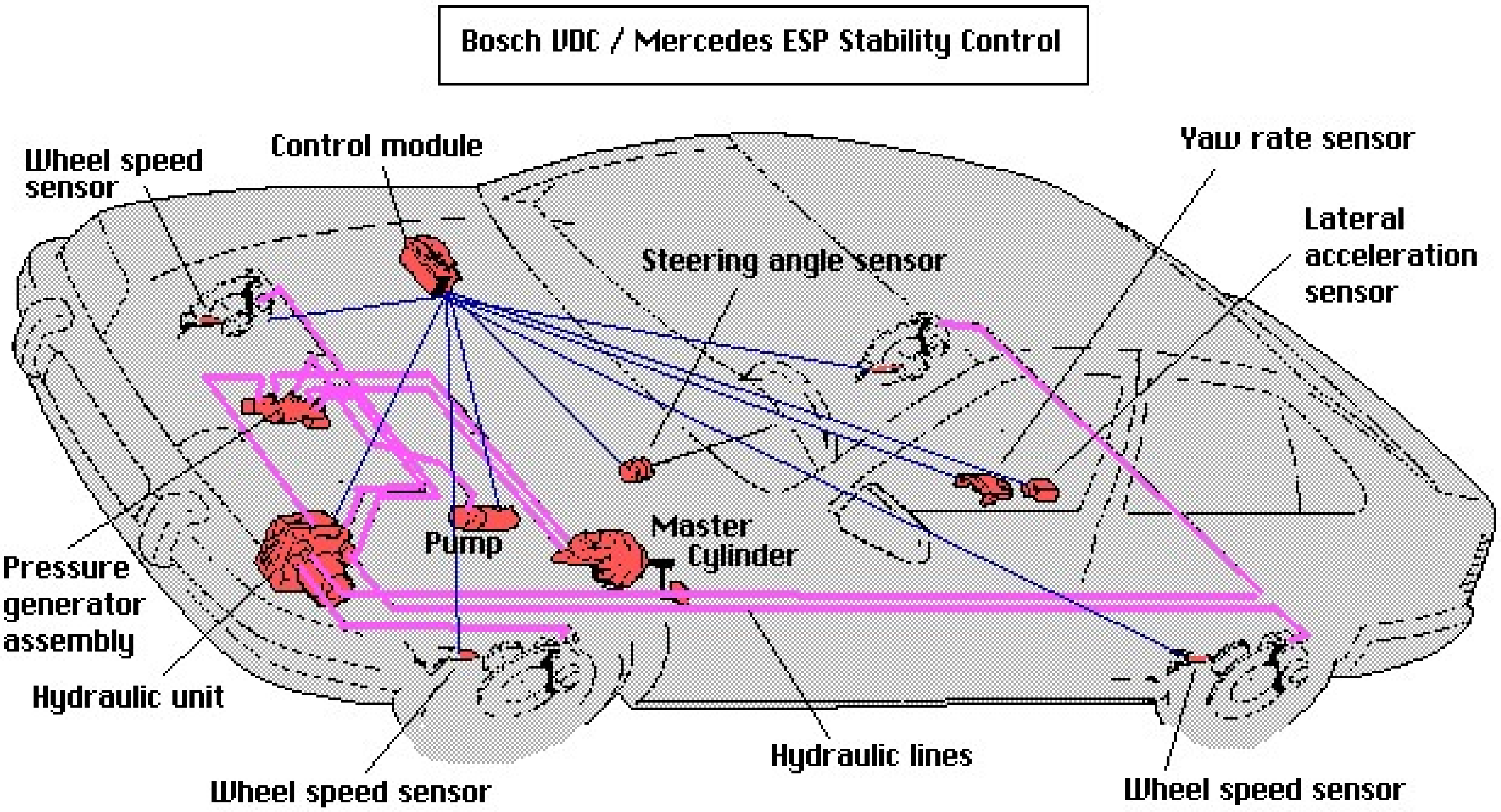
Electronic Transmission Control

- Electronic transmission control is used primarily in automatic transmissions.
- Transmission-control systems determine the optimal shift point for the torque converter and the lockup operation point based on throttle-angle and vehicle-speed measurements.

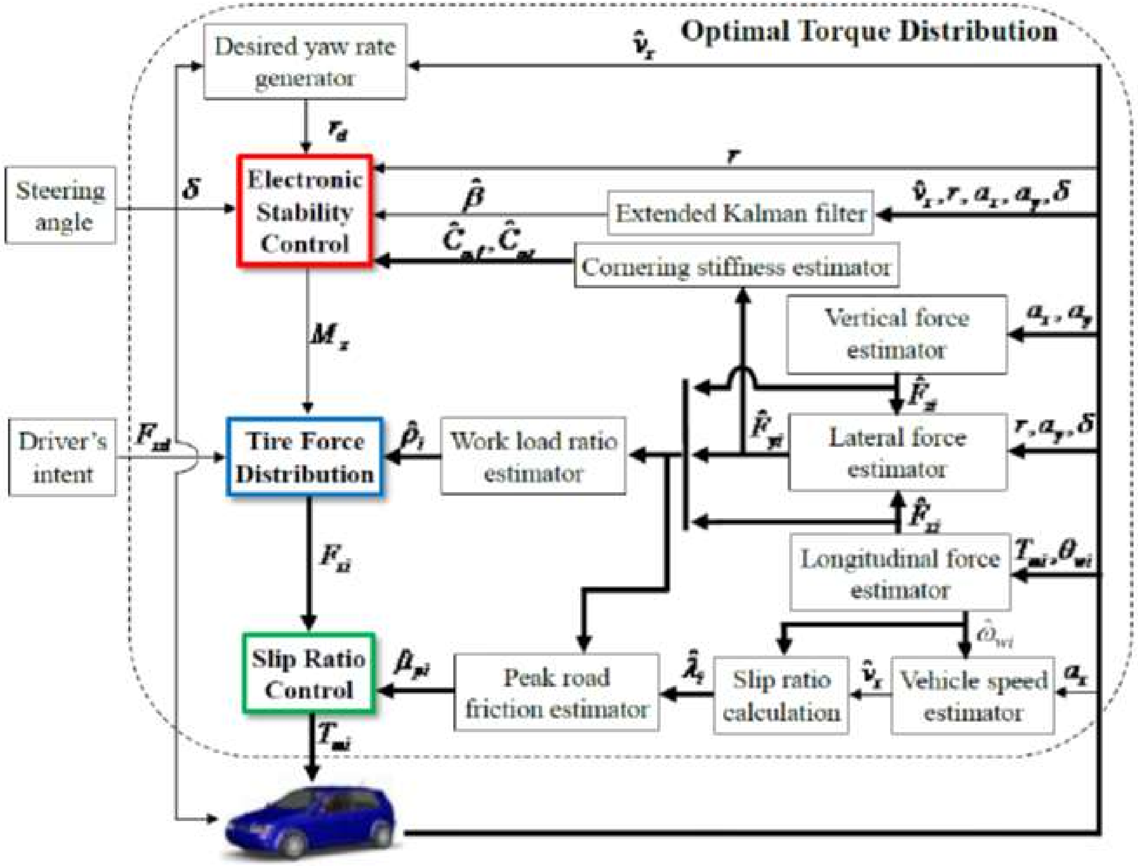
Vehicle Dynamics Control

- *Vehicle control systems*, include
  - ESP
  - suspension control,
  - Steering control (e.g., 4WS),
  - cruise control,
  - braking control (e.g., antilock brake systems [ABS]),
  - traction control.
- These systems improve various vehicle functions including response, steering stability, ride, and handling;

ESP/ESC

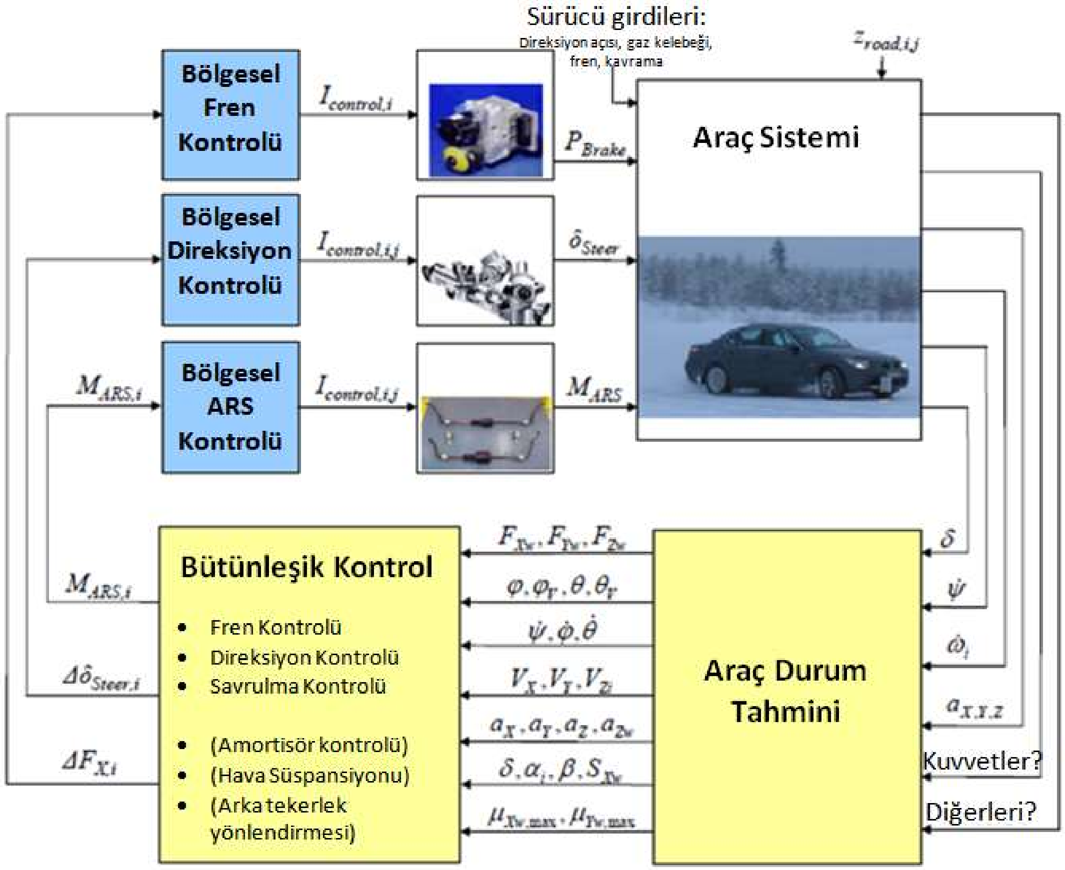

Bosch VDC / Mercedes ESP Stability Control
Wheel speed sensor
Control module
Yaw rate sensor
Lateral acceleration sensor
Steering angle sensor
Master Cylinder
Pump
Pressure generator assembly
Hydraulic unit
Wheel speed sensor
Hydraulic lines
Wheel speed sensor

ESP/ESC

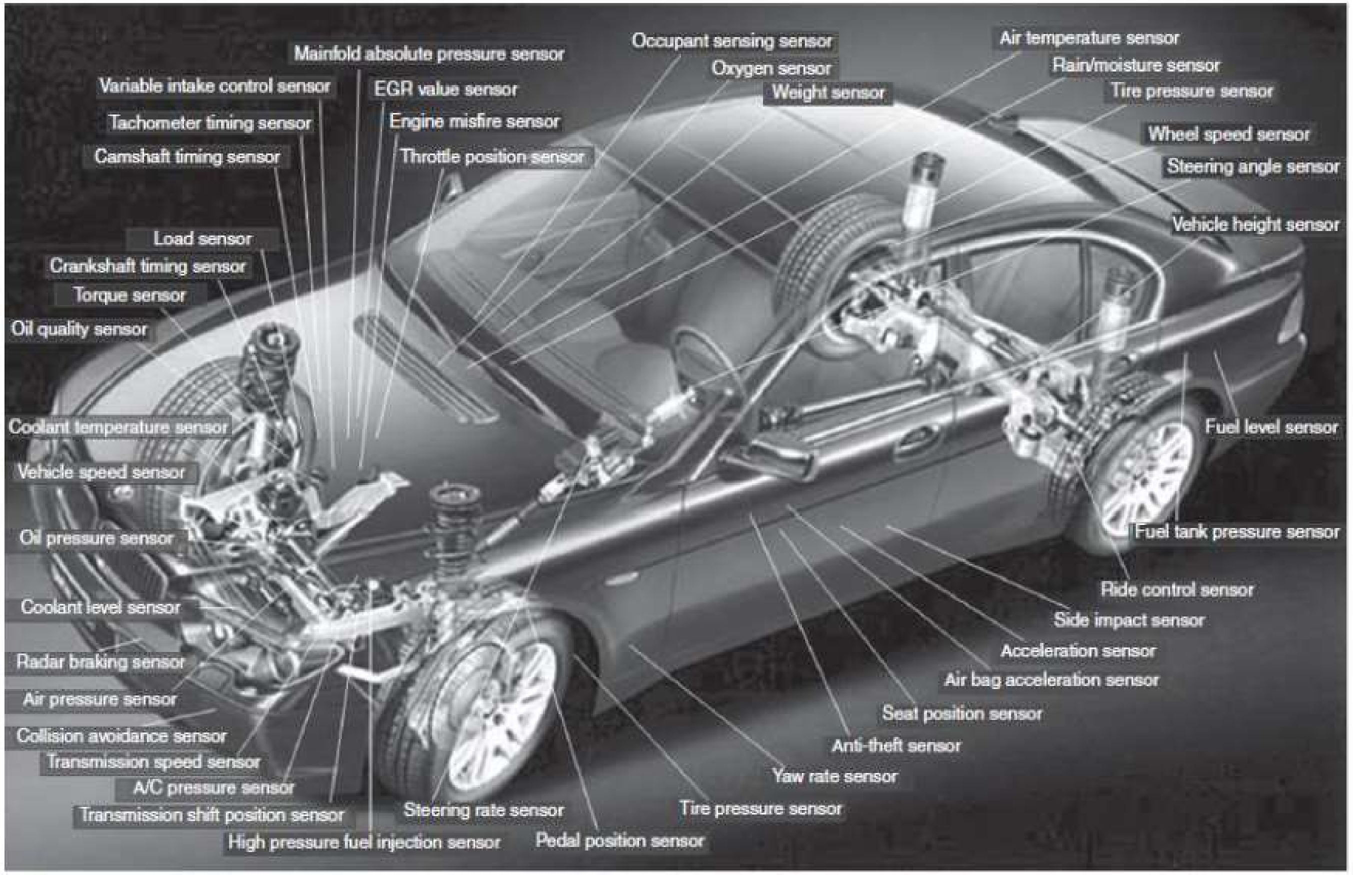


Optimal Torque Distribution
Desired yaw rate generator
Steering angle
Electronic Stability Control
Extended Kalman filter
Cornering stiffness estimator
Vertical force estimator
Driver's intent
Tire Force Distribution
Work load ratio estimator
Lateral force estimator
Longitudinal force estimator
Slip Ratio Control
Peak road friction estimator
Slip ratio calculation
Vehicle speed estimator

Integrated Vehicle Control

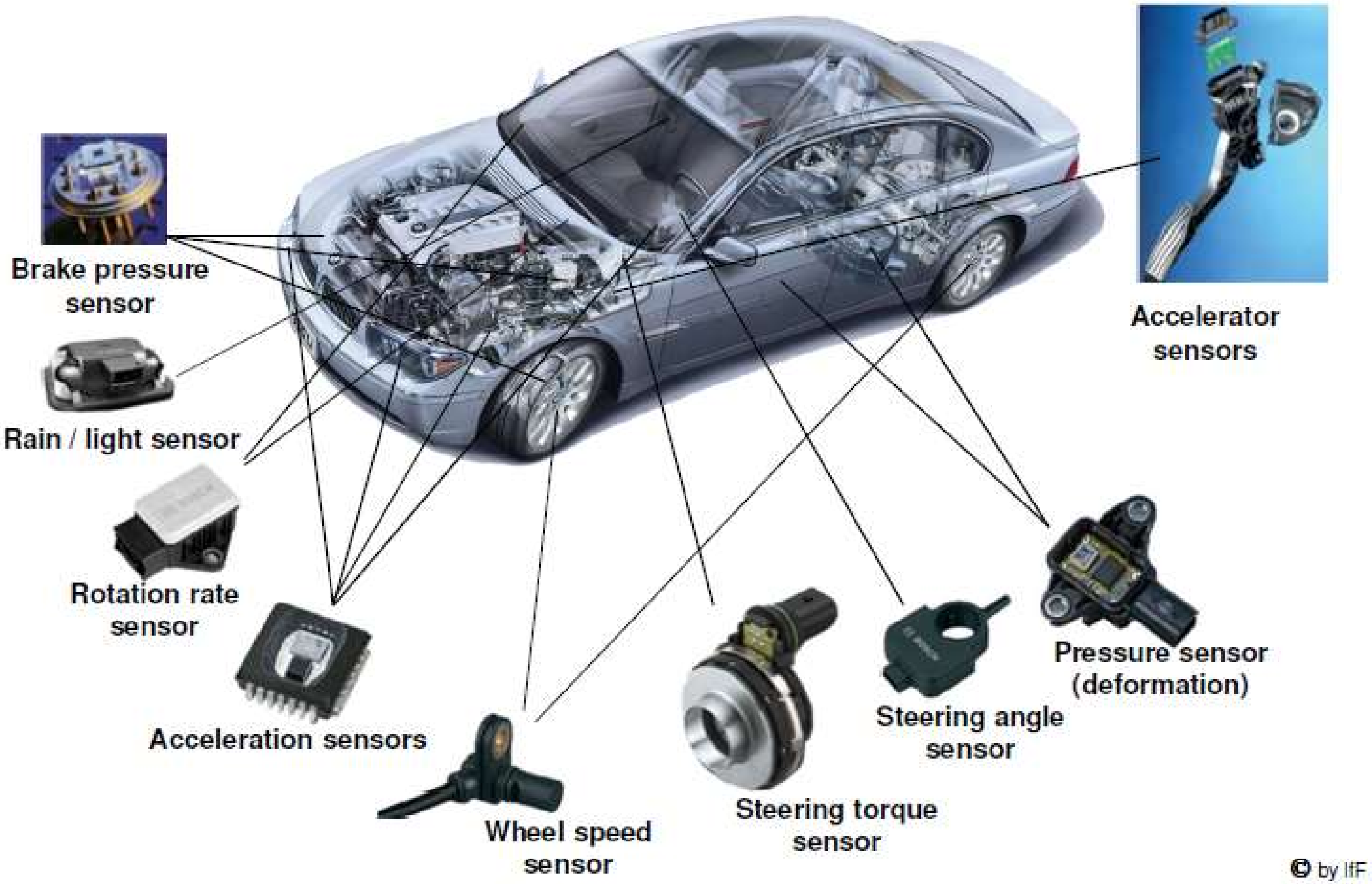


Sensors



Mainfold absolute pressure sensor
Variable intake control sensor
EGR value sensor
Tachometer timing sensor
Engine misfire sensor
Camshaft timing sensor
Throttle position sensor
Occupant sensing sensor
Oxygen sensor
Weight sensor
Air temperature sensor
Rain/moisture sensor
Tire pressure sensor
Wheel speed sensor
Steering angle sensor
Vehicle height sensor
Load sensor
Crankshaft timing sensor
Torque sensor
Oil quality sensor
Coolant temperature sensor
Vehicle speed sensor
Oil pressure sensor
Coolant level sensor
Radar braking sensor
Air pressure sensor
Collision avoidance sensor
Transmission speed sensor
A/C pressure sensor
Transmission shift position sensor
High pressure fuel injection sensor
Steering rate sensor
Pedal position sensor
Tire pressure sensor
Yaw rate sensor
Anti-theft sensor
Seat position sensor
Air bag acceleration sensor
Acceleration sensor
Side impact sensor
Ride control sensor
Fuel tank pressure sensor
Fuel level sensor

Sensors



Brake pressure sensor
Rain / light sensor
Rotation rate sensor
Acceleration sensors
Wheel speed sensor
Steering torque sensor
Steering angle sensor
Pressure sensor (deformation)
Accelerator sensors
© by IfF

Sensors

FUNCTION	POWERTRAIN SENSOR	PRODUCTIO STATUS*
ENGINE CONTROL		
Cylinder	Pressure	limited
	Combustion-Gas Ion Current	limited
Manifold	Pressure	major
	Temperature	major
Turbo Boost	Pressure	limited
Engine Knock	Vibration	limited
	Combustion-Gas Ion Current	limited
Air Intake	Mass Flow & flow reversal	limited
	Volume Flow Rate	limited
Engine Torque	Magnetostrictive	R&D
	Cylinder-Firing-Induced	R&D
	Crankshaft Speed Modulation	
Air-Fuel Ratio	Oxygen Exhaust Gas:	
	Unheated Stoichiometric	major
	Heated, Fast Light-Off	major
	Heated, Wide Range	limited
	Combustion-Gas Ion Current	R&D
Exhaust NOx Conc.	Dual-Chamber Oxygen Gas	limited
EGR	Pressure	limited
	Valve Position	limited
Crankshaft	Rotational Motion	major
Camshaft	Rotational Motion	major
Throttle, Pedal	Rotary Position	limited
Fuel Injection	Pressure	limited

ENGINE DIAGNOSTIC		
Engine Misfire	Crank Angle Running Statistics	major
	Combustion-Gas Ion Current	limited
Exhaust/Catalyst	Temperature	major
	Catalytic Activity	major
Engine Oil	Pressure	major
	Level	limited
	Quality (or contamination):	
	Predictive	major
	ac-Dielectric Constant	limited
	Cyclic Voltammogram	limited
	Thermal Conductivity	limited
Coolant System	Temperature	major
	Level	limited
Fuel Tank/System	Level	major
	Evaporative Leak Pressure	major
	Flexible Fuel Composition	limited
TRANSMISSION		
Automatic, and/or Continuously Variable	Gearshift Position	major
	Input/Output Shaft Speeds	major
	Temperature	limited
	Pressure	limited
	Torque	R&D

* Sensor production status rankings are based on the judgment of the author.

Sensors

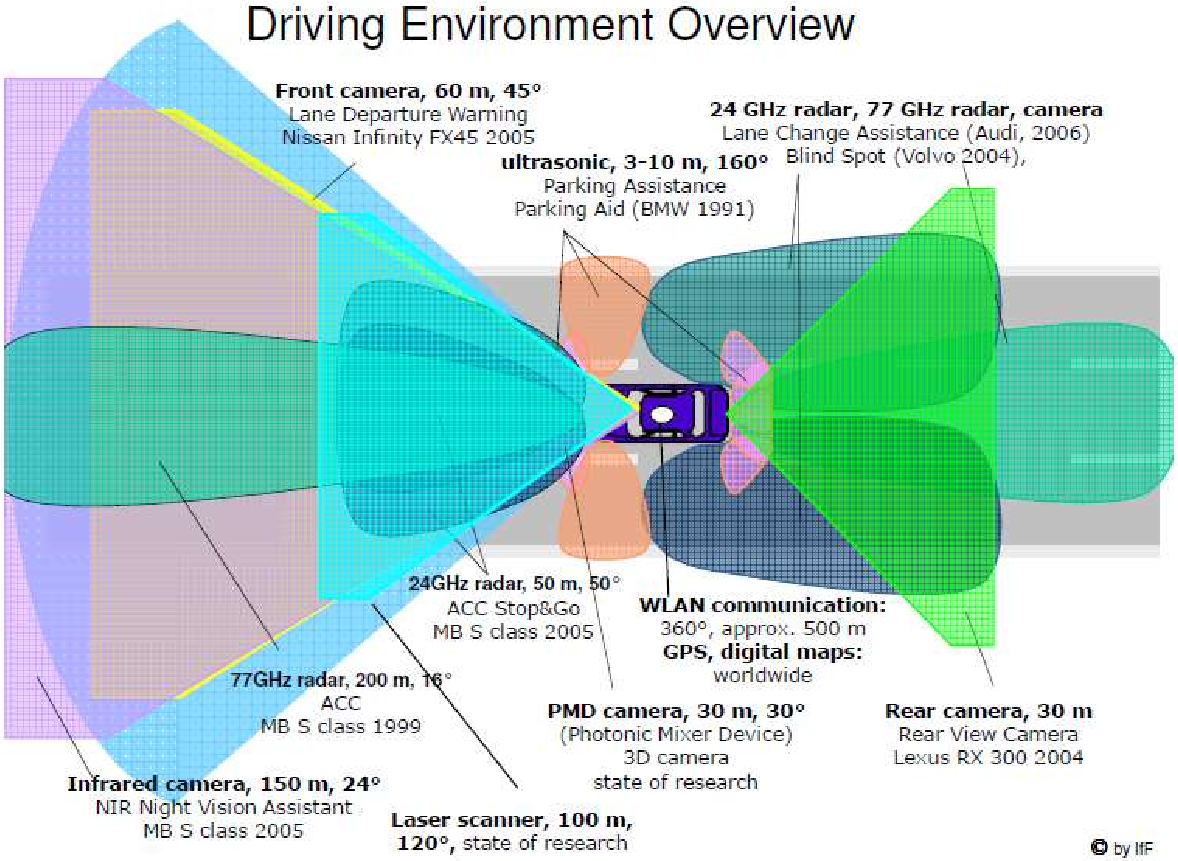
SENSORS USED IN CHASSIS APPLICATIONS

FUNCTION	CHASSIS SENSOR	PRODUCTION STATUS*
BRAKING		
Brake System	Pressure	major
	Fluid Level	limited
ABS Antilock Braking	Wheel Speed	major
	Pressure	major
	Lateral Acceleration	limited
Brake-By-Wire	Pedal Force/Depression Angle	R&D
STEERING		
Electric Power Steering/Steer-By-Wire	Steering Wheel Angle	limited
	Steering Wheel Torque	limited
4-Wheel Steer	Rear Wheel Steering Angle	R&D
	Steering Wheel Angle	limited
VEHICLE		
Vehicle Stability	Wheel Speed	major
	Lateral Acceleration	limited
	Yaw Angular Rate	limited
	Steering Wheel Angle	limited
Active Suspension	Strut Displacement	limited
	Chassis Height	major
	Body Acceleration:	
	Vertical	major
	Lateral	major
	Longitudinal	major
	Yaw Angular Rate	limited
	Roll Angular Rate	limited
	Strut Hydraulic Pressure	limited
	Steering Wheel Angle	limited
Tire Pressure	Wheel-to-Wheel Variance of Rolling Speed	major
	On-Wheel Sensor, Wireless	limited
Tire Temperature	On-Wheel Sensor, Wireless	limited

* Sensor production status rankings are based on the judgement of the author.

FUNCTION	BODY SENSOR	PRODUCTION STATUS*
SAFETY		
Air Bag Actuation	Crash Deceleration	major
	Vehicle Rollover (Lateral Acceleration plus Roll Rate)	R&D
	Seat-Belt-Use Buckle Status	limited
	Pressure (Side Impact)	limited
Seat Belt Locking	Vehicle Deceleration	major
	Webbing Payout Velocity	limited
Seat Occupancy	Seat Pan Bladder Pressure	R&D
	Seat Pan Load/Deflection	R&D
Occupant Presence/Pre-Crash Position	Passive Infrared Imaging	R&D
	Ultrasonic Imaging	R&D
	Machine Vision	R&D
Parking/Reversing Aid	Ultrasonic Array	major
	Wide-Beamwidth Radar	limited
Blind Spot Surveillance	Wide-Beamwidth Radar	limited
	Multi-Beam Infrared Laser Array	R&D
Lane Departure	Machine Vision	limited
Night Vision	Passive Infrared Imaging	limited
	Active near-IR Illumination	R&D
INTELLIGENT TRANSPORTATION		
Adaptive Cruise Control	Millimeter Wave Radar	limited
	Infrared Laser Radar	limited
Lateral Lane Guidance	Machine Vision	limited
	Magnetometers	R&D
Behavioral	Driver Condition/Impairment	R&D

ADAS/AD Sensors



CAN

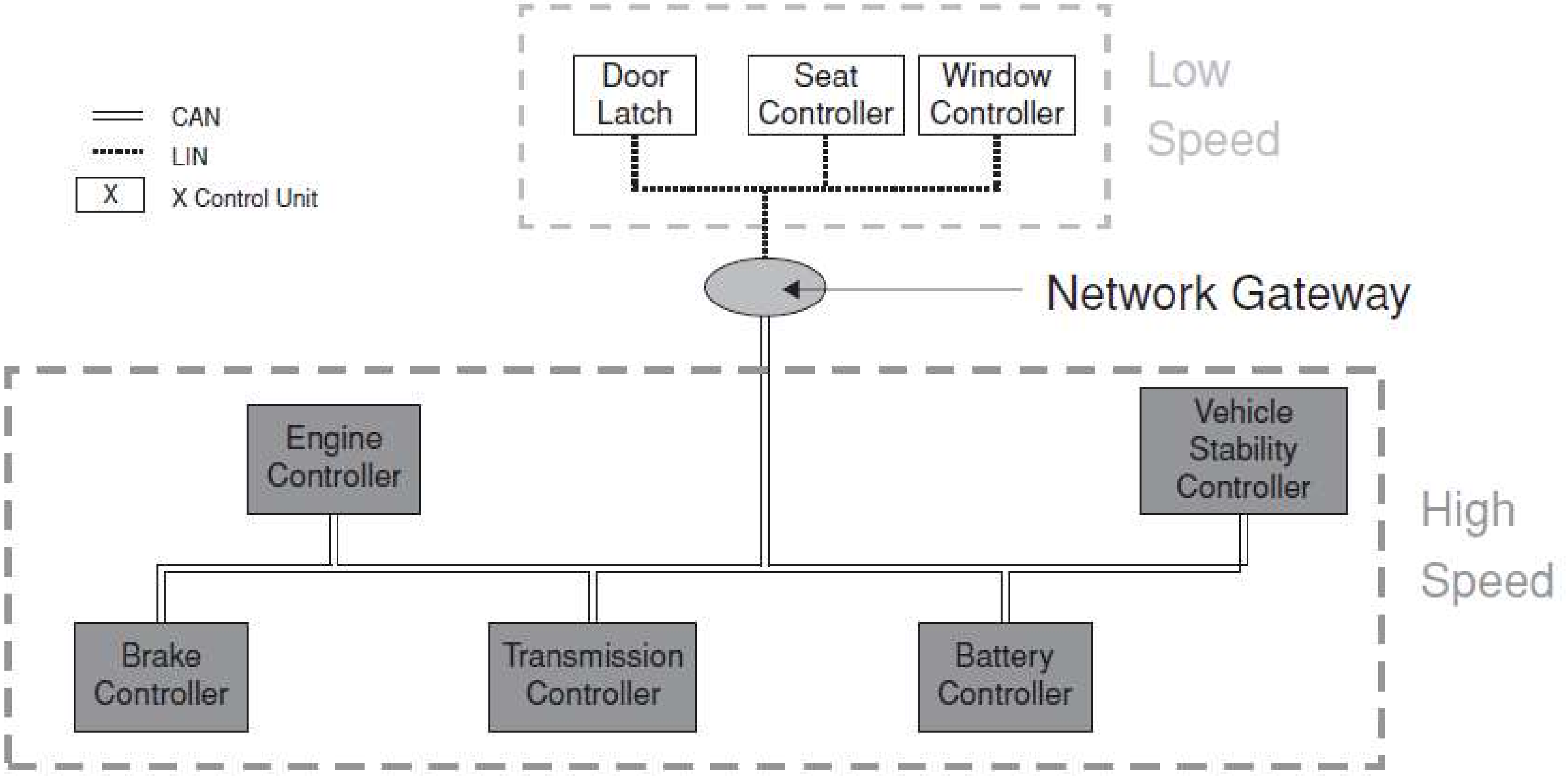


Figure 1.8 Simple vehicle-control network

Vehicle Control Systems

- Automatic control algorithms are used almost every subsystem in today's vehicle components to deliver expected functionality

Engine
- Throttle Body
- Idle Speed
- Fueling
- Cooling
- AC

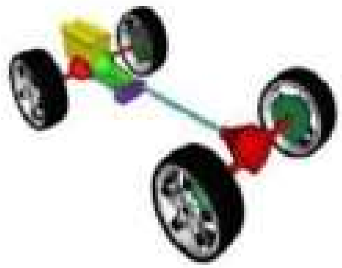
Motor/Generator
- Torque Control
- Speed Control
- Start/stop

Transmission
- Clutch control
- Shift Scheduling
- CVT

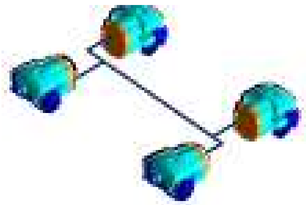
Vehicle Control Systems

- Automatic control algorithms are used almost every subsystem in today's vehicle components to deliver expected functionality

Battery System
- Contactor Control
- Power Limits
- Cell Balancing



Chassis
- Active Body Control
- Roll Stabilization
- Active Suspension



Braking
- ESC/ABS
- EHB
- Regenerative Braking

EMS

- A typical example is the energy-management algorithms in hybrid electric vehicles
- Although engine, transmission, motor, brake, and battery controller modules are individually commanded by their respective subsystems, a high-level control algorithm is required to determine the power flow to of from the battery as well as the composition of the engine and motor torque provided to the

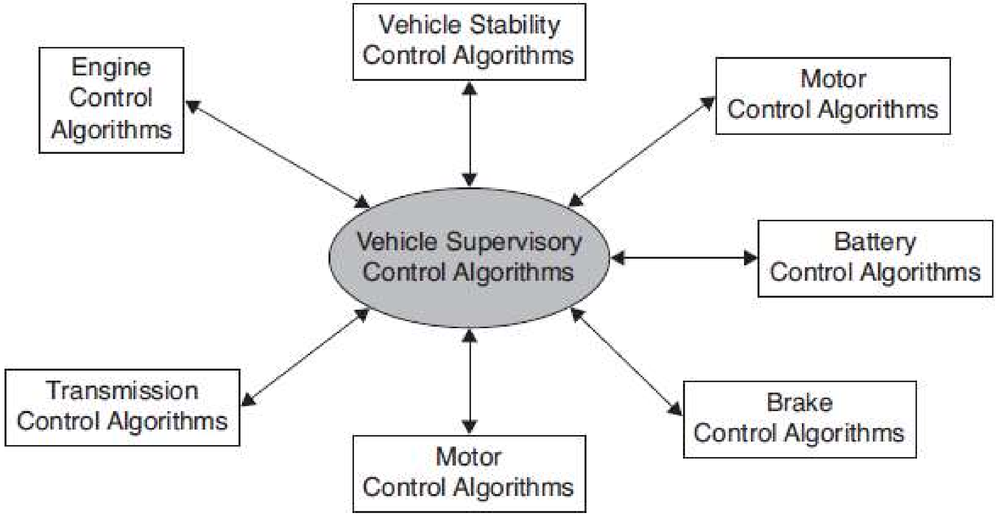
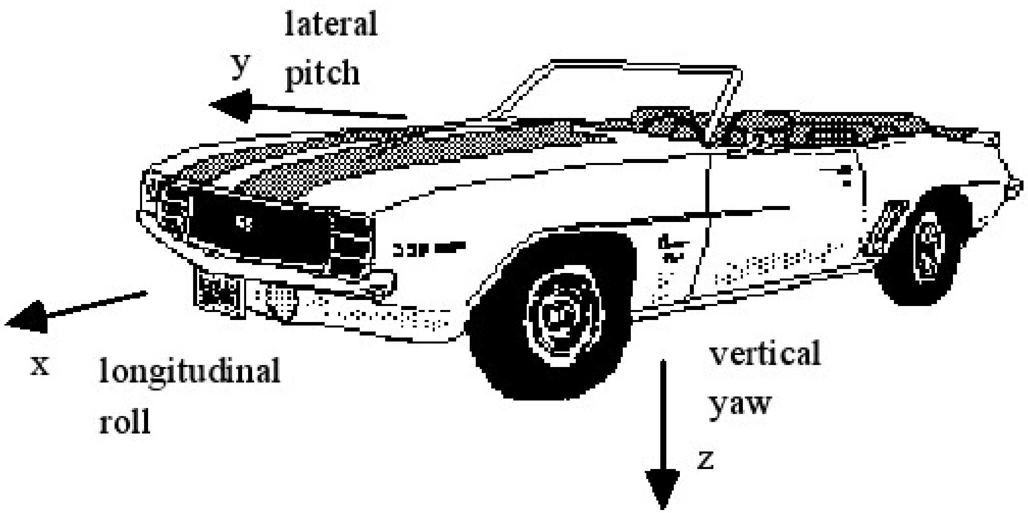


Figure 1.10. Vehicle supervisory control.

Vehicle Coordinate System

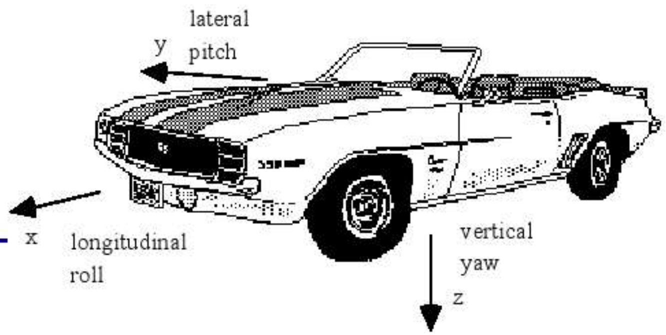
- First step in deriving vehicle dynamic equations
- The Society of Automotive Engineers (SAE) has introduced standard coordinates and notations for describing vehicle dynamics
- Vehicle fixed coordinate system is shown on the right.
- ISO coordinate: x is



SAE Vehicle-Fixed Coordinate System: Symbols and Definitions

Axis	Translational Velocity	Angular Displace-ment	Angular Velocity	Force Component	Moment Component
x	u (forward)	ϕ	p or $\dot{\phi}$ (roll)	Fx	Mx
y	v (lateral)	θ	q or $\dot{\theta}$ (pitch)	Fy	My
z	w (vertical)	ψ	r or $\dot{\psi}$ (yaw)	Fz	Mz

Pitch angle: the angle between x-axis and the horizontal plane.
Roll angle: the angle between y-axis and the horizontal plane.
Yaw angle: the angle between x-axis and the X-axis of inertial frame



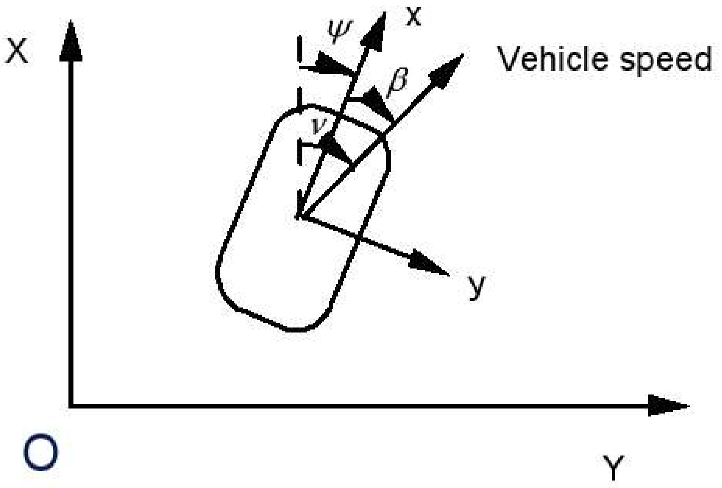
Earth Fixed Coordinate and Vehicle Slip

- OXYZ fixed on Earth (does not turn with the vehicle)

Course angle $\nu = \psi + \beta$

Heading angle

Side slip (angle)



Tire Slip

- The tire coordinate system is not the same as XYZ or xyz.
- It is sometimes denoted by X′, Y′, and Z′ to distinguish it from the inertial or vehicle-fixed frames
  - o: center of tire contact patch
  - Z: perpendicular to the ground plane
  - X and Y axes are on the ground plane
  - Tire camber angle: angle between the XZ plane and

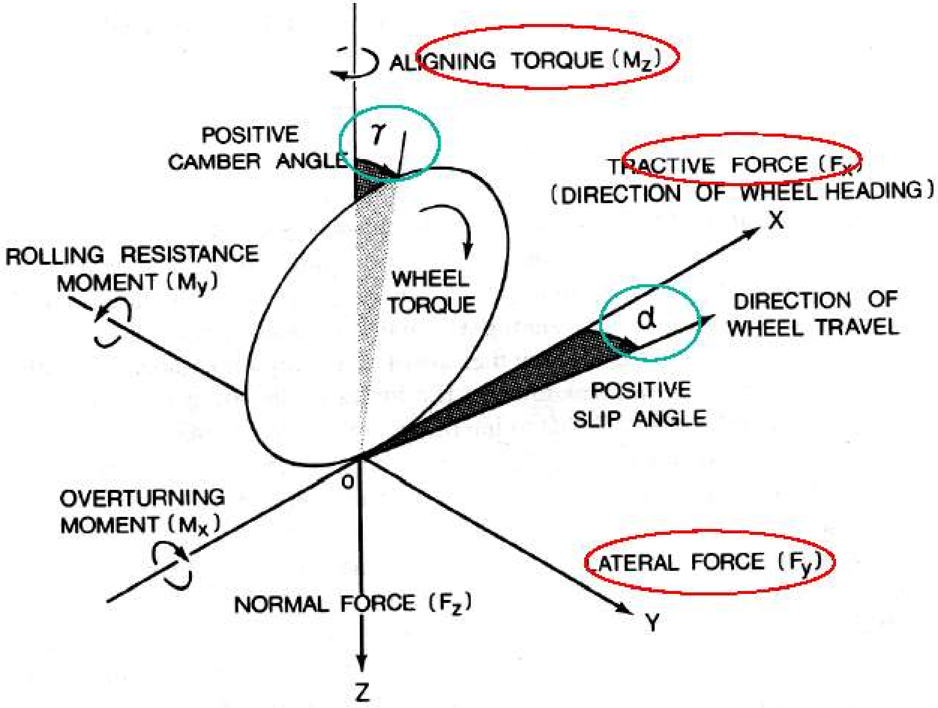
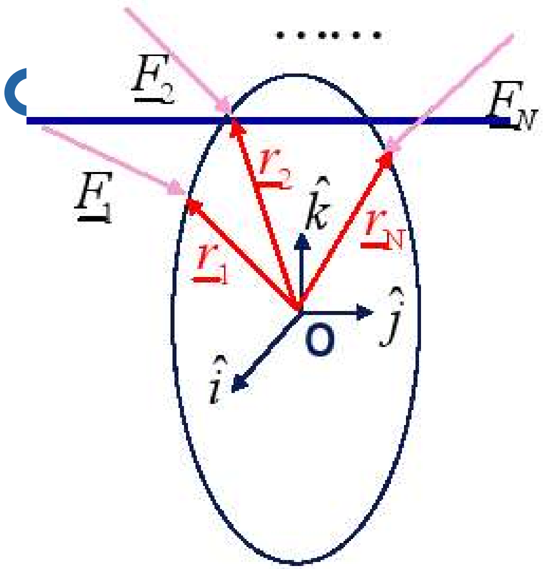


Fig. 1.2 Tire axis system.

Source: Wong page 7
SAE J760

Newton/Euler Equati



- Translati $\sum_{i=1}^{N} \underline{F}_i = \underline{F} = m\frac{d\underline{V}}{dt}$
- Rotation: $\sum_{i=1}^{N} \underline{r}_i \times \underline{F}_i = \underline{M}_o = \frac{d\underline{H}}{dt}$
- Where H is the angular momentum of the rigid body about the mass center 'o'

$$\underline{H} = \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$I_{xx} \equiv \int_V (y^2 + z^2)\rho \cdot dV$$

$$I_{xy} \equiv \int xy\rho \cdot dV \qquad \text{etc.}$$

Vehicle Rigid Body Equations of Motion

- Translational Moti $\sum F = ma$

$$\sum F = ma = m \cdot \frac{d}{dt}\left[u\hat{i} + v\hat{j} + w\hat{k}\right]$$

$$= m \cdot \left[\dot{u}\hat{i} + \dot{v}\hat{j} + \dot{w}\hat{k} + u\dot{\hat{i}} + v\dot{\hat{j}} + w\dot{\hat{k}}\right]$$

$$= m\left\{\frac{d}{dt}\begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} u \\ v \\ w \end{bmatrix}\right\} \Rightarrow \begin{aligned} \sum F_x &= m(\dot{u} + qw - rv) \\ \sum F_y &= m(\dot{v} + ru - pw) \\ \sum F_z &= m(\dot{w} + pv - qu) \end{aligned}$$

$$\frac{D}{Dt} = \frac{d}{dt} + \underline{\omega} \times$$

Rigid Body Motion of Whole Vehicle (cont.)

- Rotational Motion: $\mathbf{M} = \dot{\mathbf{H}}$

$$\sum M = \left\{ \frac{d}{dt} \begin{bmatrix} I_{xx}p - I_{xy}q - I_{xz}r \\ -I_{xy}p + I_{yy}q - I_{yz}r \\ -I_{xz}p - I_{yz}q + I_{zz}r \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} I_{xx}p - I_{xy}q - I_{xz}r \\ -I_{xy}p + I_{yy}q - I_{yz}r \\ -I_{xz}p - I_{yz}q + I_{zz}r \end{bmatrix} \right\}$$

$$\Rightarrow \sum M_x = I_{xx}\dot{p} - I_{xy}\dot{q} - I_{xz}\dot{r} - I_{xz}pq - I_{yz}q^2 + I_{zz}rq + I_{xy}pr - I_{yy}qr + I_{yz}r^2$$

$$\sum M_y = -I_{xy}\dot{p} + I_{yy}\dot{q} - I_{yz}\dot{r} + I_{xx}pr - I_{xy}qr - I_{xz}r^2 + I_{xz}p^2 + I_{yz}qp - I_{zz}rp$$

$$\sum M_z = -I_{xz}\dot{p} - I_{yz}\dot{q} + I_{zz}\dot{r} - I_{xy}p^2 + I_{yy}qp - I_{yz}rp - I_{xx}pq + I_{xy}q^2 + I_{xz}rq$$

Simplified Vehicle Rigid Body Equations of Motion

- Assume:
  - Vehicle is symmetric in the XZ plane ($I_{xz} = I_{yz} = 0$)
  - p, q, r, v, and w are small, i.e., their products are negligible.
  - $u = u_o + u'$, and u' is small compared with u_o.

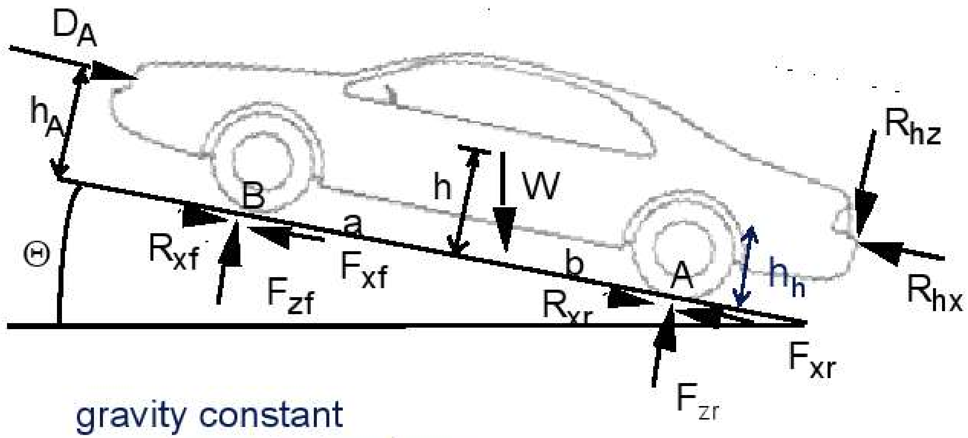
$$\sum F_x = m(\dot{u} + qw - rv)$$
$$\sum F_y = m(\dot{v} + ru - pw)$$
$$\sum F_z = m(\dot{w} + pv - qu)$$
$$\vdots$$

$$\sum F_x = m\dot{u}'$$
$$\sum F_y = m(\dot{v} + ru_o)$$
$$\sum F_z = m(\dot{w} - qu_o)$$
$$\sum M_x = I_{xx}\dot{p} - I_{xz}\dot{r}$$
$$\sum M_y = I_{yy}\dot{q}$$
$$\sum M_z = -I_{xz}\dot{p} + I_{zz}\dot{r}$$

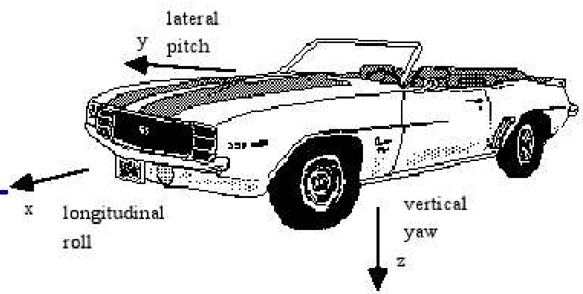
Linear!

Naturally groups into 3 sets:
(1) Longitudinal
(2) Lateral/yaw/roll
(3) Vertical/pitch

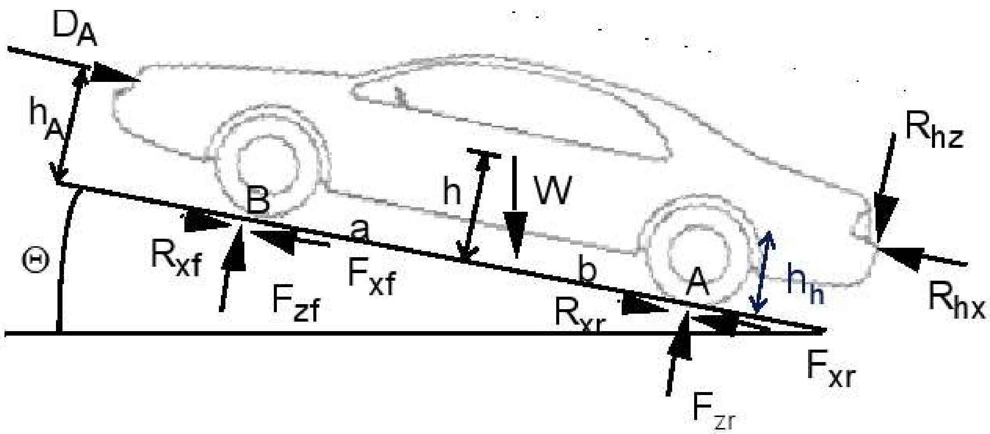
Longitudinal Vehicle Motion



- g: gravity constant
- D_A : aerodynamic drag force
- R_h : drawbar force
- W : weight of the vehicle (= mg)
- F_x : tractive force
- F_z : tire normal force
- R_x : rolling resistance force
- ma_x: inertial force



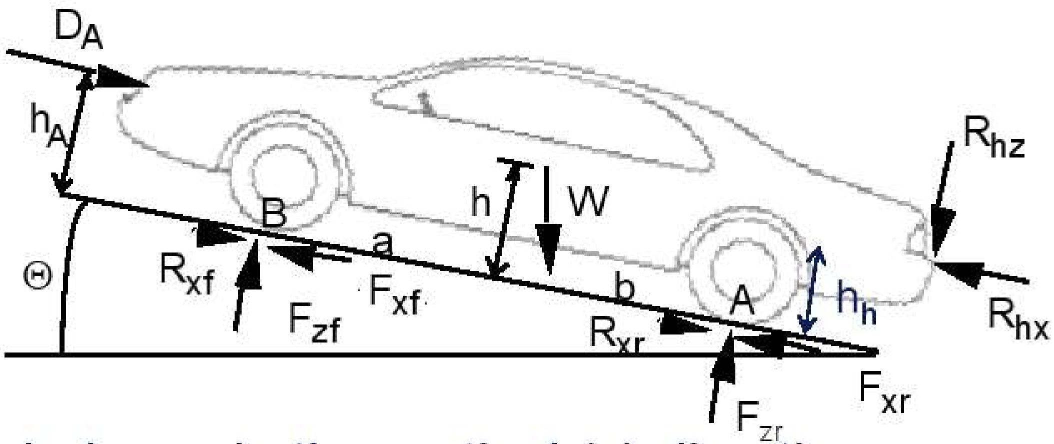
Input Classification



Controlled Inputs	Uncontrolled Inputs
Tractive forces (2)	Aerodynamic drag(1)
	Rolling resistance(2)
	Hitch forces(2)
	Normal forces(2)
	Gravity forces(1)

Inertia force is determined

Force Balance Constraints



Force balance in the vertical (z) direction

$$\boxed{0 = W\cos\Theta - F_{zf} - F_{zr} + R_{hz}}$$

Force balance in the longitudinal (x) direction

$$\boxed{ma_x = (W/g)a_x = F_{xr} + F_{xf} - W\sin\Theta - R_{xr} - R_{xf} - D_A + R_{hx}}$$

Moment balance (about point A, or point B)

Aerodynamic Drag and Rolling Resistance

- Assumptions: no gradient, no hitch force, neutral gear (i.e., no tractive forces).
- Perform two coast down tests

	High speed test	Low speed test
Initial speed	V_{i1}	V_{i2}
Final speed	V_{f1}	V_{f2}
Time duration	t_1	t_2
Average speed	$V_1 = \dfrac{V_{i1} + V_{f1}}{2}$	$V_2 = \dfrac{V_{i2} + V_{f2}}{2}$
Average deceleration	$a_1 = \dfrac{V_{i1} - V_{f1}}{t_1}$	$a_2 = \dfrac{V_{i2} - V_{f2}}{t_2}$

Aerodynamic Drag and Rolling Resistance

$$ma_x = (W/g)a_x = \cancel{F_{xr}} + \cancel{F_{xf}} - \cancel{W\sin\Theta} - R_{xr} - R_{xf} - D_A + \cancel{R_{hx}}$$

No traction Flat road No hitch force

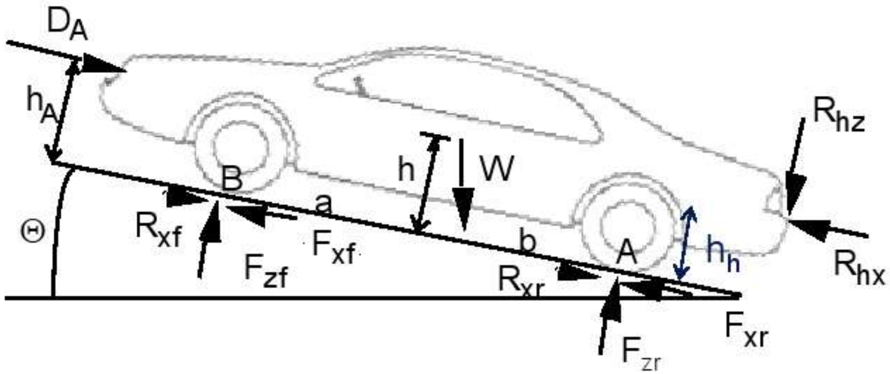
$$D_{A1} + R_x = 0.5\rho C_d A V_1^2 + fmg = ma_1$$

$$D_{A2} + R_x = 0.5\rho C_d A V_2^2 + fmg = ma_2$$

$$\Rightarrow \quad C_d = \frac{m(a_1 - a_2)}{0.5\rho A(V_1^2 - V_2^2)}$$

$$f = \frac{a_1 V_2^2 - a_2 V_1^2}{g(V_2^2 - V_1^2)}$$

Axle Load (Tire Normal Force)



L = a + b

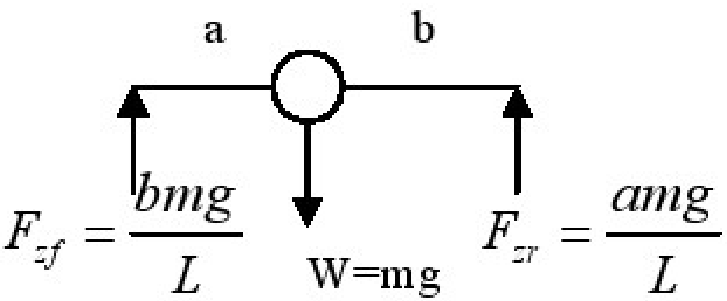
- Static Loads (statically on leve
  - Force balance in the vertical (Z) (

$$0 = W\cos\Theta - F_{zf} - F_{zr}$$

  - Moment balance (about point A, or p

$$F_{zf} = W \cdot \frac{b}{L} \qquad F_{zr} = W \cdot \frac{a}{L}$$

Side View (vertical plane)



Axle Load (Steady, with Acceleration)

- Assumptions: Steady acceleration (constant a_x, no pitch vibration) on level ground, no hitch forces, aerodynamic drag ignored.

$$\text{load transfer} = W\left(\frac{a_x h}{gL}\right) = \frac{m a_x h}{L}$$

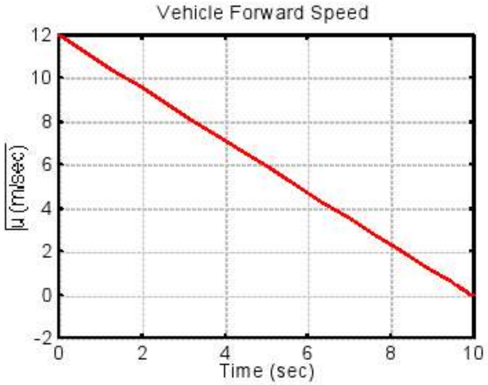
- Remarks:
  - Grades on highways are less than 4%, and rarely exceed 10-12% overall, thus $\sin(\theta) = 0$, and $\cos(\theta) = 1$ are good assumptions
  - Aerodynamic drag is proportional to velocity squared, and can be neglected at low speeds

Vehicle Longitudinal Dynamics Simulation (No Tire Dynamics)

$$ma_x = (W/g)a_x = F_{xr} + F_{xf} - W\sin - R_{xr} - R_{xf} - D_A + \cancel{R_{hx}}$$

```
% Ex3_3.m
% Init. time, final time, and initial
% values of the variable x are:
  ti=0.0; tf=10.0; ui = [12.0];
% Tol and trace are used
% by the integration routine ode23:
  tol = 1.0E-4; trace = 1;
% Perform integration and store
% the results in x
  [t,u] = ode23('Ex3_3a',ti,tf,ui,tol,trace);
% Plot the results
plot(t,u,'r')
title('Vehicle Forward Speed');
xlabel('Time (sec)')
ylabel('u (m/sec)'); grid;
```

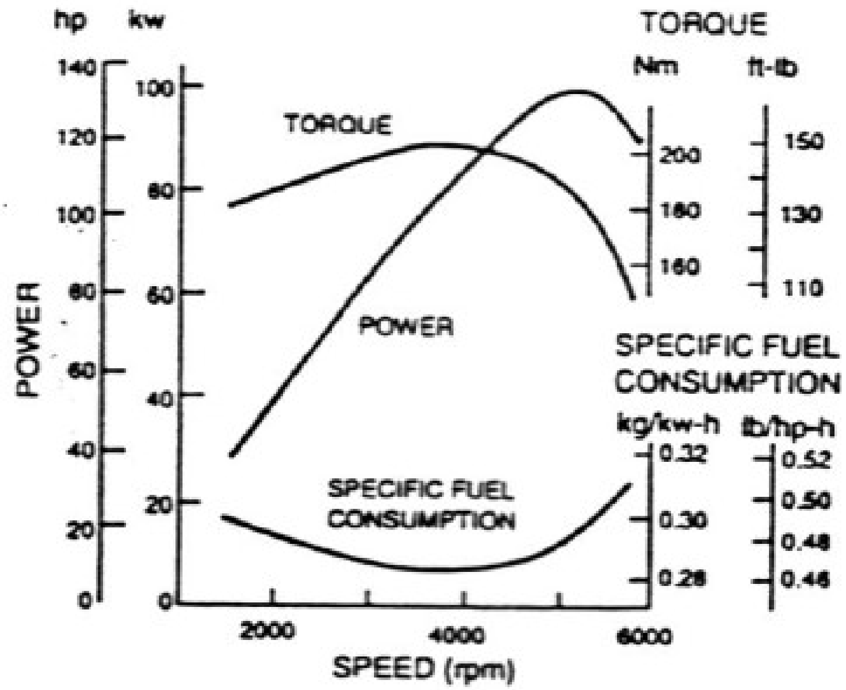
```
function udot = Ex3_3a(t,u);
% Parameter values:
m=2000; g=9.8; W=m*g;
Fx=-2000; Theta=0.0; f=0.02;
rho=1.202; Cd=0.4; A=2; uw=0.0;
if u > 0
  udot = [(1/m)*(Fx-W*sin(Theta)    ...
      - f*W*cos(Theta) ...
     -0.5*rho*Cd*A*(u+uw)^2)];
else
  udot=0;
end
```



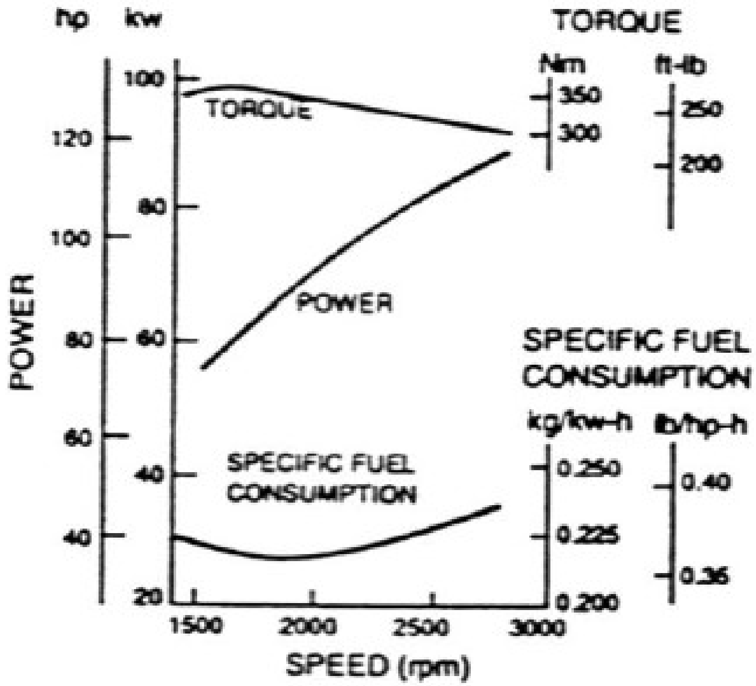
Longitudinal Dynamics

- Mainly influenced by power generation (engine) and transfer (transmission, tires) devices
- When road friction is high, tire is mainly linear and can be ignored (part of vehicle inertia)
- When road friction is low, tire motion may be decoupled from vehicle motion, and a tire force model becomes important
- Vehicle body inertia is just an integrator

Typical Engine Characteristics

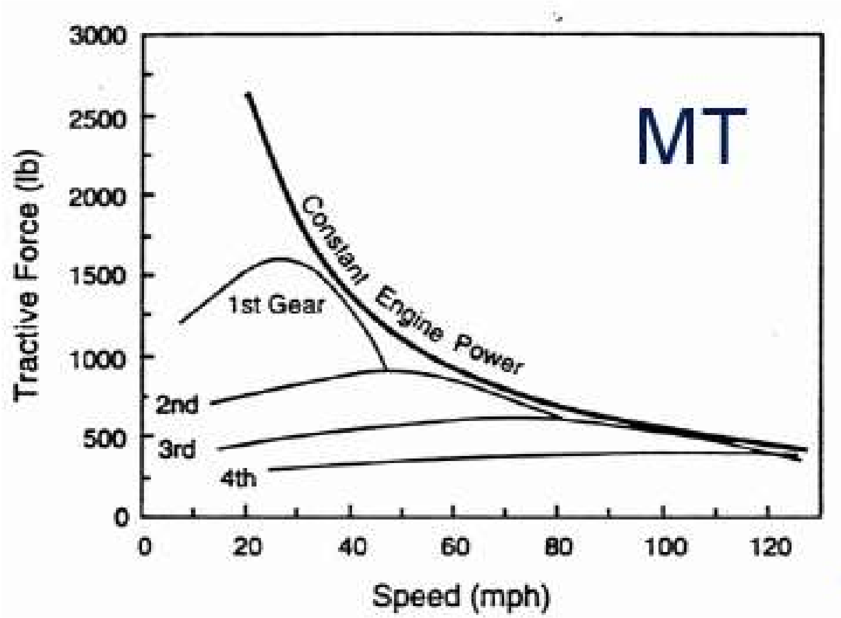


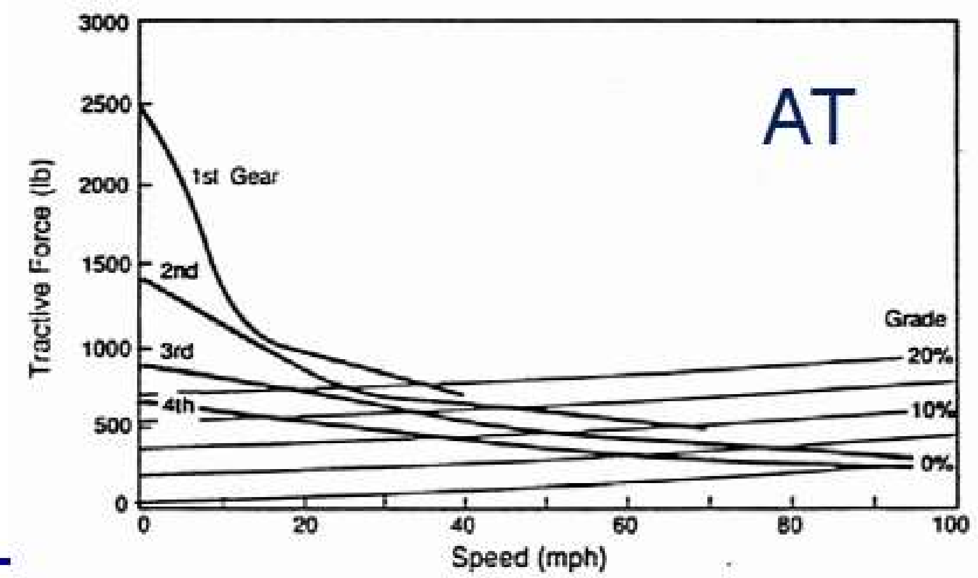
Gasoline



Diesel

Typical Engine Characteristics





Powertrain Influence on Longitudinal Acceleration

- Assume: Manual transmission or locked up torque conver

Engine	$T_e - I_e \alpha_e = T_c$	
Transmission	$(T_c - I_t \alpha_e) \cdot N_t = T_d$	$N_t = \dfrac{\omega_e}{\omega_d}$
Drive shaft	$(T_d - I_d \alpha_d) \cdot N_f = T_a$	$N_f = \dfrac{\omega_d}{\omega_w}$
Tire	$T_a = F_x \cdot r + I_w \alpha_w$	

$$\longrightarrow F_x \cdot r + \left\{ (I_e + I_t) N_t^2 N_f^2 + I_d N_f^2 + I_w \right\} \alpha_w = T_e \cdot N_t N_f$$

Powertrain Influence on Longitudinal Acceleration

- Assume: (i) No tire slip ($a_x = r.\alpha_w$); (ii) Add a lumped efficiency factor η_{tf}

$$F_x = T_e N_t N_f(\eta_{tf}/r) - \{(I_e+I_t) N_t^2 N_f^2 + I_d N_f^2 + I_w\}(a_x/r^2)$$

Drive term   Inertia term

- Define a rotational inertia

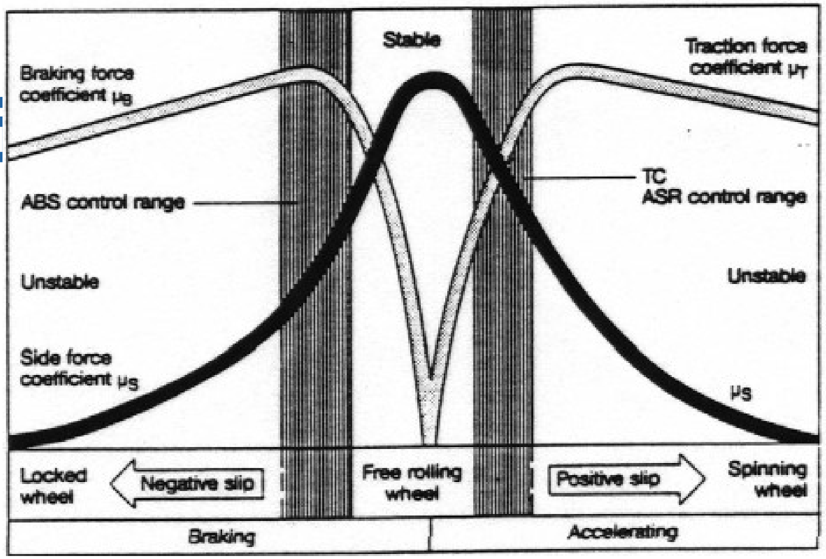
$$m_r \equiv \{(I_e+I_t) N_t^2 N_f^2 + I_d N_f^2 + I_w\}(1/r^2)$$

$$\Longrightarrow \quad (m+m_r)a_x = \frac{T_e N_t N_f \eta_{tf}}{r} - W\sin\theta - R_{xr} - R_{xf} - D_A - R_{hx}$$

Longitudinal Sli

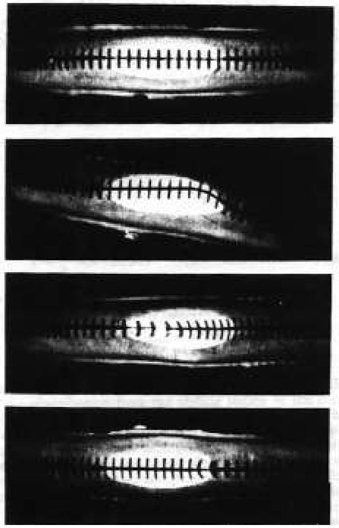
- Driven (accelerating) case:

$$\lambda \equiv \frac{\omega r_w - u}{\omega r_w}$$



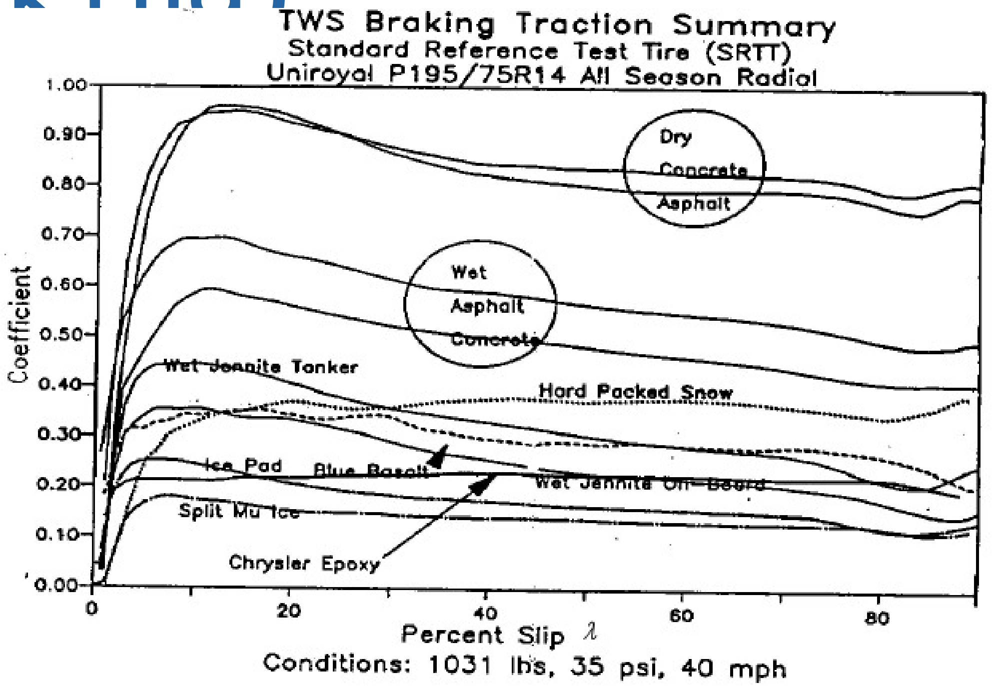
- Braking Case:

$$\lambda \equiv \frac{\omega r_w - u}{u}$$



When road friction is the Limiting Factor (Icy road or heavy braking)

$$\mu = \frac{F_x}{F_z}$$



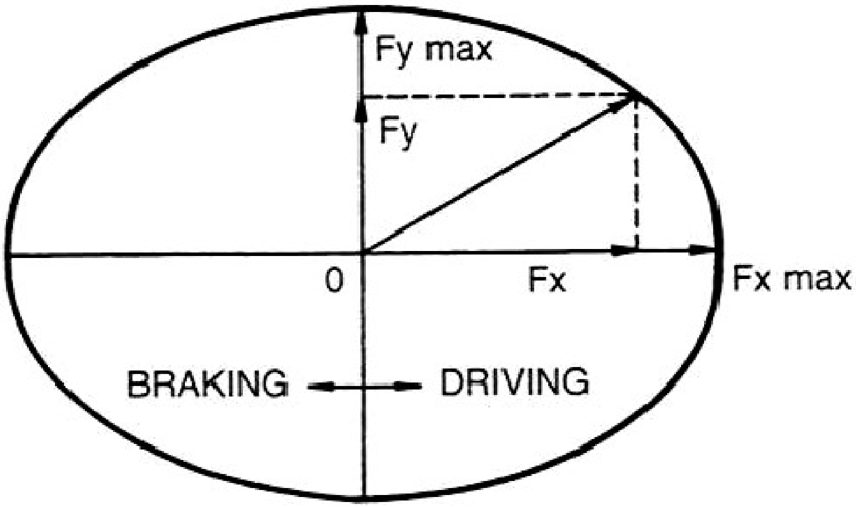
$$\frac{F_x}{F_z} \leq \mu$$

$$\lambda = \frac{r_w \omega - u}{u} \quad \text{during braking}$$

$$\lambda = \frac{r_w \omega - u}{r_w \omega} \quad \text{during acceleration}$$

Friction Ellipse Concept

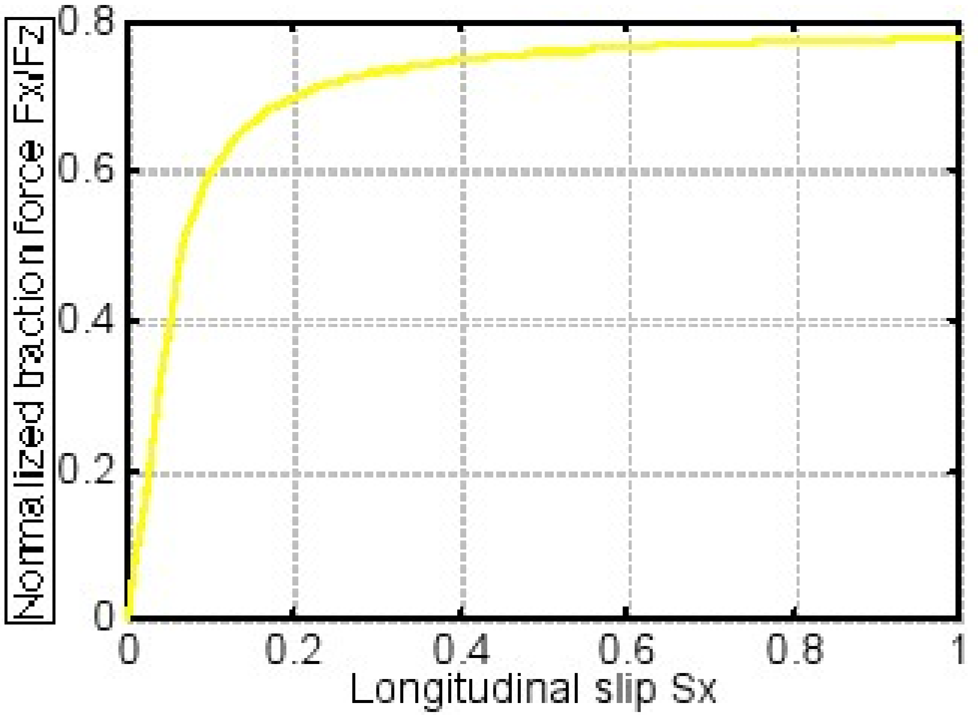
- For combined lateral and longitudinal slip, when the resultant shear force exceeds the local friction limit, sliding will occur.
- This concept is referred to as the "friction ellipse" concept.
- Tire forces and moments are highly non-linear and difficult to model.
- This, models such as the Brush or Pacejka "Magic Formula" models, have been developed for use in simulation studies.
- We will discuss these later in the course in conjunction with ABS, traction control



$$\begin{Bmatrix} F_x \\ F_y \\ M_z \end{Bmatrix} = f(r_w, \gamma, r, \lambda, \alpha, \mu)$$

Problems of Simple Pure-Slip Models

- (1) Model Prediction
  - Typical model assumption: Independent lateral/longitudinal force generation
- (2) Common Solution:
  - Use empirical tire models (e.g., Pacejka or "Magic Formula") which combine lateral/longitudinal force generation.



Problems of Simple Pure-Slip Models

Experimental data shown on the right