DAT-LA-08: Extra Problem Solving Exercises

Following are extra practice problems for testing your Python skills. To solve problems such as these, follow the procedure below. You are highly encouraged to solve additional problems at http://brilliant.org and https://projecteuler.net.

IMPORTANT: Always validate your answer – try to get it right on the first try. When you do data science work, you will not have an automated grader! So, providing evidence that your answer is correct is essential.

How to solve programming problems

A. **Understand** what the problem is asking.

B. **Solve a simpler problem** by hand, using pencil and paper.

C. **Write code**, using the same procedure you used by hand.

D. **Validate your answer** - provide additional evidence it is correct.

NOTE: For selected problems (*), the procedure's steps are detailed at the end of the document.

PROBLEMS:

***Problem 1.** Approximate numerically:

$$\lim_{n\to\infty} (1 + 1/n)^n$$

Problem 2. Approximate numerically:

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

***Problem 3.** Using vanilla Python, approximate the value of x that maximizes $f(x) = x^{1/x}$, to three decimal places.

Bonus: Can you verify your answer using Calculus?

***Problem 4.** What is the smallest non-negative integer n for which $n!$ starts with the digit 9?

source: https://brilliant.org/problems/well-this-seems-simple

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Problem 5. What is the largest integer $n < 1000$ such that the decimal representation of 2^n does not contain 0 as a digit?

source: https://brilliant.org/problems/this-problem-has-a-cool-name

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*__Problem 6.__ Suppose you have a [0, 1] x [0, 1] box (i.e. a square with sides of length 1). If two points are randomly chosen from within that box, the expected (average) distance between them is equal to X. Correct to 4 places after the decimal, what is X?

Bonus: Can you verify your answer mathematically?

source: https://brilliant.org/problems/average-distance-between-two-points

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Problem 7.

> You land among a sea of zeroes.
>
> Here's your big chance to be a hero!
>
> Your quest will be long, forsooth.
>
> Where will lie the Boolean of Truth?
>
> - The first position is 0, 2nd position is 1, etc.
> - The Boolean of Truth is 1.

source: https://brilliant.org/problems/the-quest-for-the-boolean-of-truth

data file: https://drive.google.com/file/d/0B6WdQSpcxAG8WUlfLUNzSnFHZnc/view

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*Procedure for Solving

Problem 1.

> A: What is a limit? Describe what the problem is asking in words.
>
> B: Compute $(1 + 1/n)^n$ for small values of n. Can you graph them by hand?
>
> C: Left as exercise
>
> D: Do all n seem to also converge to this number (e.g. by graphing them)? Looking at nearby n, how many significant digits seem stable? To confirm your answer, verify it using Wolfram Alpha!

Problem 3.

A: Draw a graph of a known function, e.g. $f(x) = -x^2 + 5$, and circle its maximum. Is the problem asking for x or $f(x)$? Answer the posed question for your sample function.

B: To understand the function, graph it by hand using a calculator. Just using your calculator and graph, attempt to approximately estimate the value of x that maximizes the function. Think about how you are doing this! If you could scale up your by hand procedure and test one million x values, what would they be?

C: Left as exercise

D: Look at neighboring values – are they increasing until your x then decreasing? Verify the significant figures are stable. Graph the function to ensure the maximum is where you expect. Check your answer using Wolfram Alpha.

Problem 4.

A: What is a non-negative integer? Is 0 included? What is factorial? Is the problem asking for n or $n!$?

B: Compute $n!$ for small values. For each, circle the digit the problem is asking for.

C: Left as exercise

D: If your answer is relatively small, print out all of the $n!$ for n less than your answer – scanning them, do any start with a 9?

Problem 6.

A: Can you draw the box? Draw a few sample lines in the box. What is meant by the distance between two points? What is the formula for Euclidean distance? (Hint: You can derive the Euclidean distance formula by treating the distance between the points as the hypotenuse of a right triangle. Then, use the Pythagorean Theorem.)

B: Choose two random points from inside the box. Calculate the distance between them using a calculator. Is this the average distance? Now, given three pairs of random points, compute their average distance.

C: Left as exercise

D: What is the minimum average distance possible? What is the maximum average distance possible? If

you had to guess an average given the minimum and maximum, what would it be? Is your final result close to this? Why/why not? Is it close to your earlier hand-computed average distance for three points? Why/why not?