Assignment-02

Mat 120

Sec: 10

Ahetasham Shifat

23201216

Ans No: 1

- diff Eqn Solve
; seperable ~~var~~ method

here,

$$\cos y \ln x \frac{dx}{dy} = \frac{\sin y}{x^2}$$

$$\Rightarrow x^2 \ln x \, dx = \frac{\sin y}{\cos y} dy$$

$$\Rightarrow \int x^2 \ln x \, dx = \int \tan y \, dy$$

$$\Rightarrow \ln x \int x^2 dx - \int \frac{1}{x} \cdot \frac{x^3}{3} dx = \int \frac{\sin y}{\cos y} dy$$

$$\Rightarrow \ln x \cdot \frac{x^3}{3} - \frac{1}{3} \int x^2 dx$$

$$\Rightarrow \ln x \cdot \frac{x^3}{3} - \frac{1}{3} \cdot \frac{x^3}{3} = \int -\frac{1}{w} dw$$

let $w = \cos y$

$\frac{dw}{dy} = -\sin y$

$\Rightarrow dw = -\sin y \, dy$

$\Rightarrow -dw = \sin y \, dy$

$$\Rightarrow \frac{x^3 \ln x}{3} - \frac{x^3}{9} = -\ln|w| + c$$

$$\Rightarrow \frac{x^3 \ln x}{3} - \frac{x^3}{9} = -\ln|\cos y| + c \text{ (Ans)}$$

$$\Rightarrow \frac{x^3 \ln x}{3} - \frac{x^3}{9} = \ln|\sec y| + c \text{ (Ans)}$$

Ans

linear

eqn ...

(2)

given that,

$$(x+1)\frac{dy}{dx} + y = \ln x \; ; \quad y(1) = 10 \rightarrow$$ Boundary condition (IVP)

Ideal form $\Rightarrow \dfrac{dy}{dx} + \dfrac{y}{(x+1)} = \dfrac{\ln x}{(x+1)}$ —— (i)

here, $p(x) = \dfrac{1}{x+1}$; $f(x) = \dfrac{\ln x}{x+1}$

$$\therefore I.F = e^{\int p(x)\,dx} = e^{\int \frac{1}{x+1}\,dx}$$

$$= e^{\ln(x+1)}$$

$e^{\ln|A|} = A$

$$\boxed{I.F = x+1}$$

multiplying IF with (i) ;

$$(x+1)\frac{dy}{dx} + y = \ln x$$

$$\Rightarrow \frac{d}{dx}\left(y\cdot(x+1)\right) = \ln x$$

$$\Rightarrow \int d\left(y\cdot(x+1)\right) = \int \ln x \cdot dx$$

$$\Rightarrow y\cdot(x+1) = \ln x \int 1\,dx - \int \frac{1}{x}\cdot x\,dx + c$$

$$= \ln x \cdot x - x + c$$

$$\Rightarrow y\cdot(x+1) = x\ln x - x + c$$

—— (ii)

$y(1) = 10$; $x = 1$; $y = 10$

(ii) $\Rightarrow$

$$10(1+1) = 1 \ln 1 - 1 + c$$

$$\Rightarrow 20 + 1 = \ln 1 + c$$

$$\Rightarrow 21 = 0 + c = c$$

$\therefore$ (ii) $\Rightarrow$ $\boxed{y(x+1) = x \ln x - x + 21}$ (Ans)

CamScanner

Ans No: 3

given,

$$(6xy + 4y^2+1)dx = -(3x^2+8xy)dy$$

$$\Rightarrow (6xy+4y^2+1)dx + (3x^2+8xy)dy = 0$$

here,

$$M(x,y) = 6xy+4y^2+1$$

$$N(x,y) = 3x^2+8xy$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(6xy+4y^2+1)$$

$$= 6x+8y+0$$

$$= 6x+8y$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(3x^2+8xy)$$

$$= 6x+8y$$

; the equation is an exact ~~eq~~.

So there is a function $f(x,y)$ such that,

$$\frac{\partial f}{\partial x} = M(x,y) = 6xy+4y^2+1 \quad \text{--- i}$$

$$\frac{\partial f}{\partial y} = N(x,y) = 3x^2+8xy \quad \text{--- ii}$$

$$\frac{\partial f}{\partial x} = M(x,y)$$

$$\Rightarrow \int \partial f = \int (6xy + 4y^2 + 1)\,\partial x$$

$$\Rightarrow f(x,y) = 6y\cdot\frac{x^2}{2} + 4y^2\cdot x + x + g(y)$$

$$= 3x^2y + 4xy^2 + x + g(y) \text{ --- (iii)}$$

now,

$$\frac{\partial f}{\partial y} = N(x,y)$$

$$\cancel{\Rightarrow \int \partial f = \int (3x^2 + 8xy)\,dy}$$

$$\cancel{\Rightarrow f(x,y) = 3x^2y + 8xy}$$

$$\Rightarrow \frac{\partial}{\partial y}\left(3x^2y + 4xy^2 + x + g(y)\right) = 3x^2 + 8xy$$

$$\Rightarrow \cancel{3x^2} + \cancel{8xy} + x\cdot 0 + \frac{\partial g}{\partial y} = \cancel{3x^2} + \cancel{8xy}$$

$$\Rightarrow \frac{\partial g}{\partial y} = 0$$

$$\Rightarrow \int \partial g = \int 0\,\partial y$$

$$\Rightarrow g(y) = 0\cdot y + c = 0 + c$$

$$\Rightarrow g(y) = 0 + c$$

(iii) $\Rightarrow$

$$f(x,y) = 3x^2y + 4xy^2 + x + c$$

Ans

Ans NO: 4

~~9(a)~~ The growth rate is $\frac{dx}{dt}$

According to the ans:

$$\frac{dx}{dt} \propto x$$

$$\Rightarrow \frac{dx}{dt} = kx$$

$$\Rightarrow \int \frac{1}{x} dx = \int k\,dt$$

$$\Rightarrow \ln|x| = kt + c_1$$

$$\Rightarrow \boxed{x(t) = ce^{kt}} \quad \text{—(i)}$$

From (i) $\Rightarrow$

$$x(0) = ce^{k \cdot 0} = c$$

$$\Rightarrow x_0 = c$$

(i) $\Rightarrow$ $x(t) = x_0 e^{kt}$

$$\Rightarrow 3x(5) = x_0 e^{k \cdot 5}$$

$$\Rightarrow 3x_0 = x_0 e^{k \cdot 5}$$

$$\Rightarrow 3 = e^{5k}$$

$$\Rightarrow \ln 3 = 5k$$

$$\Rightarrow \boxed{k = \frac{\ln 3}{5}}$$

at,

$t = 10$;

$$x(10) = x_0 e^{k \cdot 10}$$

$$x(10) = x_0 e^{\frac{\ln 3}{5} \cdot 10}$$

$$\Rightarrow x(10) = x_0 e^{2 \ln 3}$$

$$\Rightarrow x(10) = 9x_0$$

$$\Rightarrow m x_0 = 9x_0$$

$$\Rightarrow \boxed{m = 9}$$

So, here,

after 10 hrs is $\boxed{9 \text{ times}}$ the number from initial.

(b)

after, t hours, it should be 10 ~~time the~~ times of the number initially are;

$$x(t) = x_0 \cdot e^{\frac{\ln|3|}{5} \times t} \quad \text{(from a)}$$

$$\Rightarrow 10 \cancel{x_0} = \cancel{x_0}\, e^{\frac{\ln|3|}{5} \times t}$$

$$\Rightarrow \ln(10) = \frac{\ln 3}{5} \times t$$

$$\Rightarrow t = \frac{5\ln(10)}{\ln(3)} = 10.48 \text{ hours}$$

$$\boxed{t = 10.48 \text{ hours}}$$

(m)

Ans No: 5

$y''' + 12y'' + 36y' = 0$ —— (i)

the, auxiliary eqn at (i) is;

$m^3 + 12m^2 + 36m = 0$

$\Rightarrow m(m^2 + 12m + 36) = 0$

$\Rightarrow \boxed{m = 0}$ | $m^2 + 12m + 36 = 0$

$\Rightarrow m^2 + 2 \cdot m \cdot 6 + 6^2 = 0$

$\Rightarrow (m+6)^2 = 0$

$\Rightarrow \boxed{m = -6, -6}$ (Repetition)

∴ general solution:

$y = c_1 e^{0 \cdot x} + c_2 e^{-6x} + c_3 x e^{-6x}$

$= c_1 + c_2 e^{-6x} + c_3 x e^{-6x}$

(Ans)

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$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{2x} \quad \text{——— (i)}$$

here, $g(x) = e^{2x} \neq 0$; Thus It is an non homogenus eqn.

So, have,

=> The general eqn:

$$y = y_c + y_p \quad \text{—— (ii)}$$

The auxilary equation for the associate homogenus differential equation will be,

$$m^2 - 5m + 6 = 0$$

$$\Rightarrow m^2 - 2m - 3m + 6 = 0$$

$$\Rightarrow m(m-2) - 3(m-2) = 0$$

$$\Rightarrow (m-2)(m-3) = 0$$

$$\therefore m = 2, m = 3$$

$$\bullet\ y_c = c_1 e^{2x} + c_2 e^{3x} \quad \text{—— (iii)}$$

$g(x) = e^{2x}$ and in $y_c = c_1 e^{2x} + c_2 e^{3x}$

so, $y_p = Axe^{2x}$ | $y_p' = A(x\,2e^{2x} + e^{2x})$

$= Ae^{2x}(2x+1)$

$y_p'' = A\{2(x2e^{2x} + e^{2x}.1) + 2e^{2x}\}$

$= 4Axe^{2x} + 2Ae^{2x} + 2e^{2x}$

$= 4Ae^{2x}(x+1)$

From (1)

$4Ae^{2x}(x+1) - 5Ae^{2x}(2x+1) + 6Ax\,e^{2x} = e^{2x}$

$\Rightarrow A\cancel{e^{2x}}(4x + 4 - 10x - 5 + 6x) = e^{2x}$

$\Rightarrow A(4-5) = 1 \Rightarrow \boxed{A = -1}$

$\therefore y_p = -xe^{2x}$

Now, putting all the values:

$$\boxed{y = y_c + y_p = c_1 e^{2x} + c_2 e^{3x} - x e^{2x}}$$