Linear Regression Models

Segment 6 – Advanced Topics in Linear Regression

Topic 2 – Bayesian Linear Regression

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Topics

1. Probabilistic View of Linear Regression

2. Maximum Likelihood Estimate

3. Maximum A Posteriori Estimate

Probabilistic View of Linear Regression

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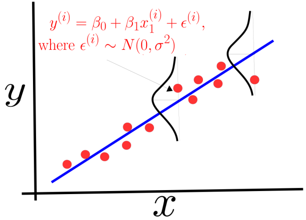

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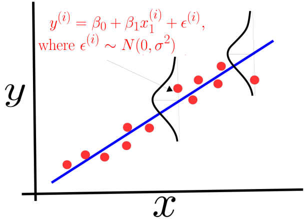

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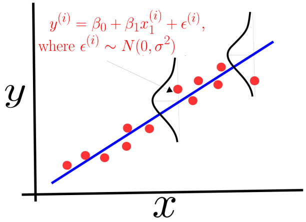

- None of these assumptions are needed to derive the formula for the coefficient estimates $\beta_0, \beta_1, \ldots, \beta_p$.
- However, this leads to a different and useful interpretation of the linear regression model.

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- This implies that

$$P\left(y^{(i)} \mid \mathbf{x}^{(i)}; \beta_0, \beta_1, \ldots, \beta_p\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \times e^{-\frac{\left(y^{(i)}-\left(\beta_0+\beta_1 x_1^{(i)}+\cdots+\beta_p x_p^{(i)}\right)\right)^2}{2\sigma^2}}.$$

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- For the ith sample, this means

$$y^{(i)} \mid \mathbf{x}^{(i)}; \beta_0, \beta_1, \ldots, \beta_p \sim N(\beta_0 + \beta_1 x_1^{(i)} + \cdots + \beta_p x_p^{(i)}, \sigma^2).$$

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- That is, the ith response value given the ith predictor values and parametrized by $\beta_0, \beta_1, \ldots, \beta_p$ is normally distributed with mean $\beta_0 + \beta_1 x_1^{(i)} + \cdots + \beta_p x_p^{(i)}$ and variance σ^2.

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- The likelihood in linear regression is $P\left(y^{(1)}, \ldots, y^{(n)} \mid \mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(n)}; \beta_0, \beta_1, \ldots, \beta_p\right)$.

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- Assuming independence between samples, the likelihood can be written as a function of the model parameters: $L(\beta_0, \beta_1, \ldots, \beta_p) = \prod_{i=1}^{n} P\left(y^{(i)} \mid \mathbf{x}^{(i)}; \beta_0, \beta_1, \ldots, \beta_p\right).$

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$$
\begin{aligned}
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&= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \times e^{-\frac{\left(y^{(i)} - \left(\beta_0 + \beta_1 x_1^{(i)} + \cdots + \beta_p x_p^{(i)}\right)\right)^2}{2\sigma^2}}.
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- The goal is now to find the optimal model parameters $\beta_0, \beta_1, \ldots, \beta_p$ such that the likelihood function is maximized.

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- The goal is now to find the optimal model parameters $\beta_0, \beta_1, \dots, \beta_p$ such that the likelihood function is maximized.
- To achieve that, we take the logarithm of the likelihood function which does not affect the optimal set of parameters.

Maximum Likelihood Estimate – Continued

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$$LL(\beta_0, \ldots, \beta_p) = \log\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y^{(i)} - \left(\beta_0 + \cdots + \beta_p x_p^{(i)}\right)\right)^2$$

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- The optimal solution (parameters) for *maximizing* the log-likelihood function is equivalent to the optimal solution for *minimizing* the *negative* of the log-likelihood function;

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- After a few steps of basic calculus, we arrive at the exact same solution for the model parameters that was shown during the matrix-formulation of the multiple linear regression model.

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- For example, we can assume that the model parameters $\beta_1, \ldots, \beta_p$, put together as the model parameters vector $\boldsymbol{\beta}$ have a *prior* joint-normal distribution with a constant variance; note that we are not including the intercept parameter β_0.

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- A normal prior distribution for model parameters vector:

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- Now, we want to maximize the $\underbrace{likelihood \times prior}$ term in Bayes formula; just as in maximum likelihood estimation, we assume independence between samples (which turns the product into summation), take logarithm (which does not affect the optimal solution), and minimize the negative of the resulting expression.

Summary

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- Maximum likelihood estimation approach for computing regression model parameters.

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- Linear regression from a probabilistic perspective.
- Maximum likelihood estimation approach for computing regression model parameters.
- Maximum a posteriori estimation approach for computing regression model parameters and compare with regularization.