Linear Regression Models

Segment 1 – Simple Linear Regression Model

Topic 4 – Accuracy of the Coefficient Estimates

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Topics

1. Accuracy of the Coefficient Estimates: Standard Errors
2. Accuracy of the Coefficient Estimates: Confidence Intervals
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Accuracy of the Coefficient Estimates: Standard Errors

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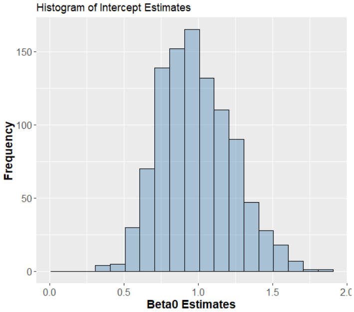
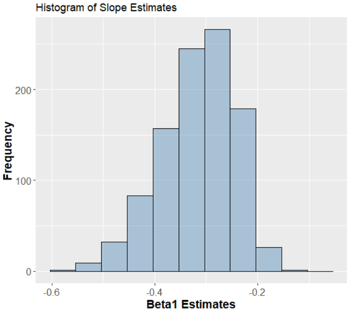
How can we assess the accuracy of the SLRM coefficient estimates $\hat{\beta}_0 = \bar{y}_n - \hat{\beta}_1 \bar{x}_n$, and $\hat{\beta}_1 = \dfrac{s_{xy}}{s_{xx}}$ derived from a dataset?





Accuracy of the Coefficient Estimates: Standard Errors - Continued

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MANIPAL ACADEMY of HIGHER EDUCATION
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- Note that the above are *in-sample* estimates; that is, calculated using the dataset.
- Random error term $\varepsilon^{(i)}$ is assumed to have zero mean, constant variance, and uncorrelated across the samples in the dataset.

Accuracy of the Coefficient Estimates: Confidence Intervals

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- For calculating CI as above, additional assumption on the random error term $\varepsilon^{(i)}$ that it is *normally distributed* is needed.

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- We consider a *null hypothesis* $\beta_1 = 0$; that is, we hypothesize that there is no relationship between mpg and hp.
- *Assuming that the null hypothesis is true,* $T = \frac{\hat{\beta}_1 - E[\hat{\beta}_1]}{SE(\hat{\beta}_1)} = \frac{\hat{\beta}_1 - \underbrace{\beta_1}_{=0}}{SE(\hat{\beta}_1)}$

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Accuracy of the Coefficient Estimates: Hypothesis Tests - Cotinued

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- This probability is referred to as the p-value; we reject the null hypothesis if the p-value is smaller than a threshold, typically 0.05.
- Rejecting the null hypothesis is equivalent to accepting the alternative hypothesis that $\beta_1 \neq 0$, and therefore mpg and hp are indeed related.
- If the p-value is greater than the threshold, we fail to reject the null hypothesis; this means that there possibly is no relationship between mpg and hp.

Summary

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Different ways to assess the accuracy of coefficient estimates using standard errors, confidence intervals, and hypothesis tests.