MANIPAL ACADEMY of HIGHER EDUCATION
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Linear Regression Models

Segment 1 – Simple Linear Regression Model

Topic 1 – Data Generation Process: Sample and Population

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Topics

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1. Questions from Data
2. Output & Input Variables in Linear Regression
3. Population & Sample
4. Population Model
5. A Linear Population Model

Questions from Data

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- Can we quantify the effect of the relationship?
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- Example of a sample statistic: the average height of n randomly chosen biological females in a city.
- Note that sample statistic (or just statistic) is a *random variable*.

Population & Sample - Example with Sample Size = 32

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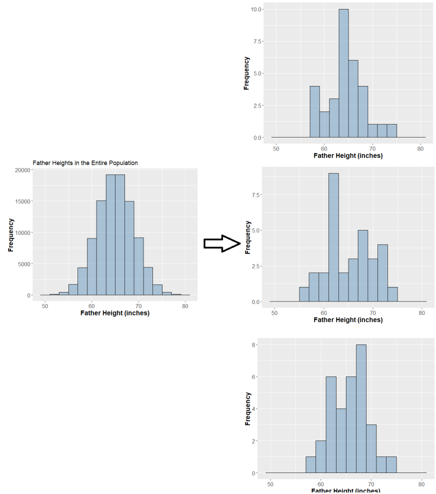

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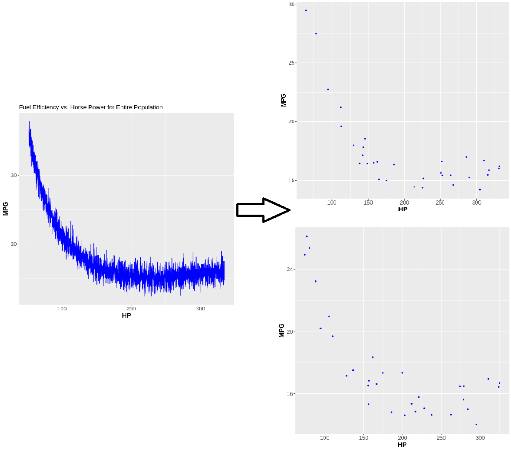
- Example of a population model for mpg and hp: $Y = \frac{1.8}{X} - 0.03X + \epsilon$.

Population & Sample - Another Example with Sample Size = 32



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- The population model for Y as a function of X is a linear one: $Y = 42 + 0.4X + \epsilon$, where $\epsilon \sim N(\mu = 0, \sigma^2 = 9)$.