# Linear Regression Models

## Segment 3 – Other Considerations in the MLRM

## Topic 3 – Interaction between Covariates

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# Topics



1. Interaction: Basic Ideas
2. Interaction Between Two Continuous Predictors
3. Interaction Between Two Categorical Predictors
4. Interaction Between Continuous & Categorical Predictors

# Interaction: Basic Ideas

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- Interaction to be considered (1) if a particular predictor has a large effect on the response (large coefficient estimate) and/or (2) the presence of categorical predictors as coefficients of other predictors may vary across groups.

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$$\underbrace{[\hat{\beta}_0 + \hat{\beta}_1 \times (livingArea + 1) + \hat{\beta}_2 \times rooms + \hat{\beta}_3 \times (livingArea + 1) \times rooms]}_{\widehat{price}_{\text{new}}}$$
$$-\underbrace{[\hat{\beta}_0 + \hat{\beta}_1 \times livingArea + \hat{\beta}_2 \times rooms + \hat{\beta}_3 \times livingArea \times rooms]}_{\widehat{price}_{\text{old}}}$$
$$= \hat{\beta}_1 + \hat{\beta}_3 \times rooms.$$

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$$\begin{aligned}
\widehat{price} = {} & \hat{\beta}_0 + \hat{\beta}_1 \times heatinghotair + \hat{\beta}_2 \times heatinghot\ water \\
& + \hat{\beta}_3 \times centralAirYes \\
& + \hat{\beta}_4 \times heatinghotair \times centralAirYes \\
& + \hat{\beta}_5 \times heatinghot\ water/steam \times centralAirYes.
\end{aligned}$$

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$$= \hat{\beta}_0 + \hat{\beta}_3 \times centralAirYes = \begin{cases} \hat{\beta}_0 & \text{if not centrally air-conditioned} \\ \hat{\beta}_0 + \hat{\beta}_3 & \text{if centrally air-conditioned.} \end{cases}$$

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- Suppose we have a house that is hot air heated and not centrally air-conditioned; the predicted price of the house is $\hat{\beta}_0 + \hat{\beta}_1$.

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- Now consider another house that is hot air heated and centrally air-conditioned;

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- For electric-heated houses, predicted price $= \hat{\beta}_0 + \hat{\beta}_3 \times centralAirYes = \begin{cases} \hat{\beta}_0 & \text{if not centrally air-conditioned} \\ \hat{\beta}_0 + \hat{\beta}_3 & \text{if centrally air-conditioned.} \end{cases}$
- Subtracting $\Rightarrow \hat{\beta}_3 =$ difference between average prices of centrally air-conditioned and not centrally air-conditioned houses among electric-heated houses.
- Suppose we have a house that is hot air heated and not centrally air-conditioned; the predicted price of the house is $\hat{\beta}_0 + \hat{\beta}_1$.
- Now consider another house that is hot air heated and centrally air-conditioned; the predicted price of this house is $\hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_3 + \hat{\beta}_4$.

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- Now consider another house that is hot air heated and centrally air-conditioned; the predicted price of this house is $\hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_3 + \hat{\beta}_4$.
- Subtracting the two results above, we see that $\hat{\beta}_3 + \hat{\beta}_4 =$ difference between average prices of centrally air-conditioned and not centrally air-conditioned houses among hot air-heated houses.

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- The predicted house price

$$\widehat{price} = \begin{cases} \hat{\beta}_0 + \hat{\beta}_1 \times livingArea & \text{if old house,} \\ \left(\hat{\beta}_0 + \hat{\beta}_2\right) + \left(\hat{\beta}_1 + \hat{\beta}_3\right) \times livingArea & \text{if new house.} \end{cases}$$

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- Note the differences in both intercept and slope for new houses.

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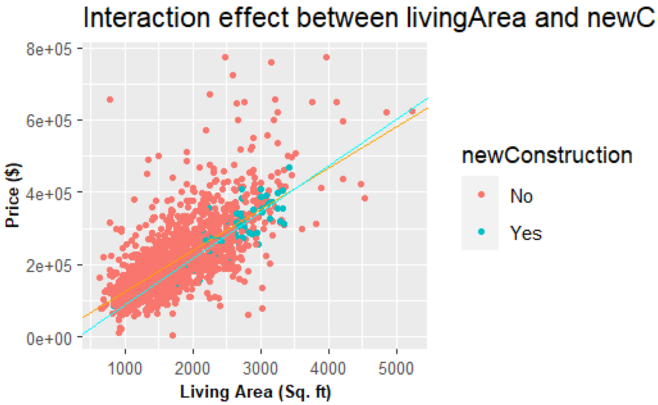
The scatter plot indicates that a higher slope is needed for new houses:



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- Construct linear regression models with interacting predictors