Measuring the Speed of Light Through Beta Plus Decay of ^{22}Na

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We used a small sample of ^{22}Na, a material subject to β^+ decay, placed between two opposing scintillators connected to photomultiplier tubes. As the sample decays, it releases positrons, and those interact with electrons in the surrounding environment to produce a pair of photons of equal and opposite momenta. When these photons hit the detectors, we can calculate the time differential between the collisions and use this difference, as well as the relative distances between the sample and each detector, to calculate the speed of light. We calculated the speed of light to be $c = 3.154 \cdot 10^8 \pm 1.244 \cdot 10^4$ m/s, about 5.2% higher than the established speed of light, but within previously known tolerances for this lab.

I. BACKGROUND

A. Speed of Light

The speed of light in a vacuum, c, is an important physical constant, equal to 299792458 meters per second. It can also be defined as:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{1}$$

where ϵ_0 is the vacuum permittivity and μ_0 is the magnetic permeability [1].

Before the 17th century, the majority of philosophers and scientists believed that the speed of light was infinite. Galileo was one of the earliest scientists to try to disprove this theory, though he was unsuccessful in getting any numerical results, later assuming that the speed of light must be at least ten times the speed of sound. Roemer and Fizeau later made significant strides, with Fizeau being within 5% of the correct answer. He achieved this result using mirrors and a rotating, toothed wheel [2].

The quantitative determination of the speed of light led to a host of new discoveries and ideas, perhaps most famously Einstein's theory of special relativity. This theory established c as a universal constant in all frames of reference and established that the laws of physics are the same for any inertial reference frame, concepts which drastically altered our understanding of the universe [1].

B. Beta Plus Decay and Electron-Positron Annihilation

Beta (β) decay is a process whereby a nucleus releases a beta particle and a neutrino, converting itself into a more stable isobar. In the case of β^+ decay, a proton converts to a neutron by emitting a W^+ boson and thereby flipping one of the up quarks to a down quark. This boson rapidly decays, emitting both an e^+ and a ν_e [3].

In general, the β^+ decay process has the form [3]:

$${}^A_Z X \to {}_{Z-1}^{\ A} X' + e^+ + v_e. \tag{2}$$

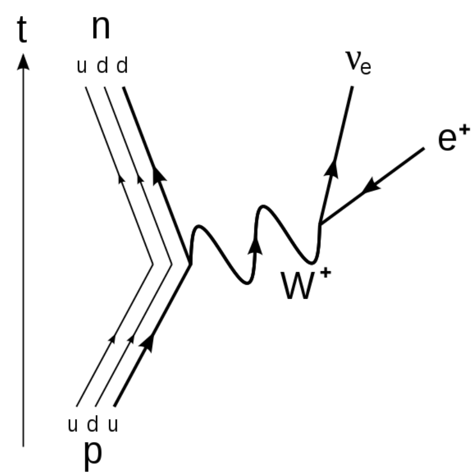

FIG. 1. Feynman Diagram for β^+ Decay. Due to weak interaction, a proton flips to a neutron, with the energy difference being accounted for by emission of a single ν_e and e^+ [4].

β^+ decay generally occurs in proton-rich nuclei, resulting in a more stable daughter nucleus with a greater binding energy [3].

While the emitted ν_e has very little interaction with matter, the e^+ has a much shorter lifespan. Upon coming into contact with an e^-, they will both immediately be annihilated. This process causes the release of two 511 keV photons moving with equal and opposite momenta [5].

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Mathematically, this process is given as [5]:

$$e^- + e^+ = \gamma + \gamma. \tag{3}$$

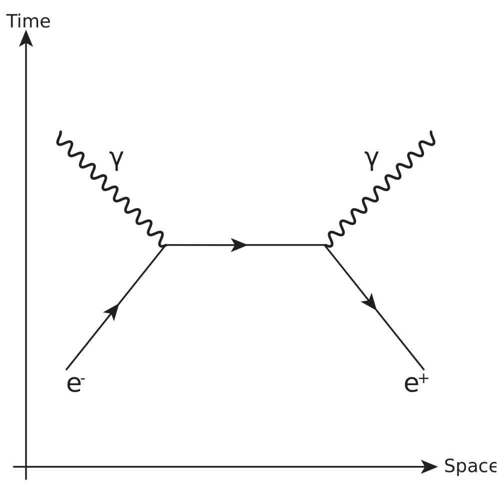

FIG. 2. Feynman Diagram for electron-positron annihilation. The two emitted photons have energies of 511 keV and have equal and opposite momenta [6].

II. EXPERIMENTAL SETUP

The setup centered around a sample of ^{22}Na. This sample is placed at various locations between two scintillators connected to photomultiplier tubes spaced 3 meters apart.

^{22}Na is an unstable, man-made isotope with a half-life of 2.6 years. It decays according Eq. 2, emitting a e^+, v_e pair and leaving behind ^{22}Ne.

Any photons produced as a result of the decay and annihilation process then have a small probability of hitting one or both PM's.

These detectors are powered with a high voltage source (HV), set to -2.10 kV. When a detector encounters a photon, it sends a signal to a Constant Fraction Discriminator (CFD). This filters out unwanted noise from the environment. Our CFD's were set to a very narrow band around 511 keV, the known energy of photons produced in this annihilation. The output from one CFD goes directly to the start input for the Time-Amplitude Converter (TAC). The second signal is sent through a delay module. This offsets the signal by 44 ns and ensures that all of these signals will arrive after the start signal. The signal then goes into the stop input of the TAC.

The time difference between start and stop is converted to a voltage by the TAC, with the amplitude of this voltage determined by the time difference. If the TAC happens to receive only a start signal, or if the stop signal is late, it will reset the counter after a set interval.

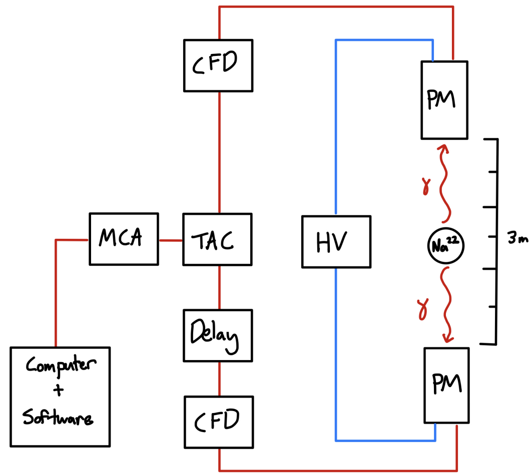

FIG. 3. Block diagram of experimental setup. Signals from incoming photons are separated from environmental noise and separated into channels based on the time difference between the pairs.

The TAC output is routed through a multi-channel analyzer (MCA). This converts continuous voltages into discrete channels. This final result is sent on to the computer and analyzed using Gamma Data Acquisition and Analysis software.

III. RESULTS AND ANALYSIS

Data collected generally took the form of a narrow Gaussian, with an exponential-like modification on the left side. Each trial was conducted over the course of roughly 48 hours, with the sample being moved between each trial. 5 trials were successfully conducted.

To reduce noise and excess data, two filters were applied to the collected data. The first filter functioned as a band pass, setting all channels outside of the band to zero and focusing the analysis window. The second filter functioned as a noise gate by clearing channels that were below a certain threshold (this threshold was set to 5 for trials 1-4 and set to 2 for trial 5 as it had a shorter run time).

Fit lines were calculated using the exponentially modified Gaussian distribution:

$$\begin{aligned} f(x, K, \mu, \sigma) = \frac{1}{2K\sigma} \exp\left(\frac{1}{2K^2} - \frac{x-\mu}{K\sigma}\right) \\ \cdot \operatorname{erfc}\left(-\frac{\frac{x-\mu}{\sigma} - \frac{1}{K}}{\sqrt{2}}\right), \end{aligned} \tag{4}$$

where μ is the centroid, σ is the variance, and K is the scaling factor. Fitting was achieved by computationally minimizing the negative log-likelihood (NLL) function, a measure of how accurately a set of data is represented by the fit curve.

Fig. 4 shows the results from one trial.

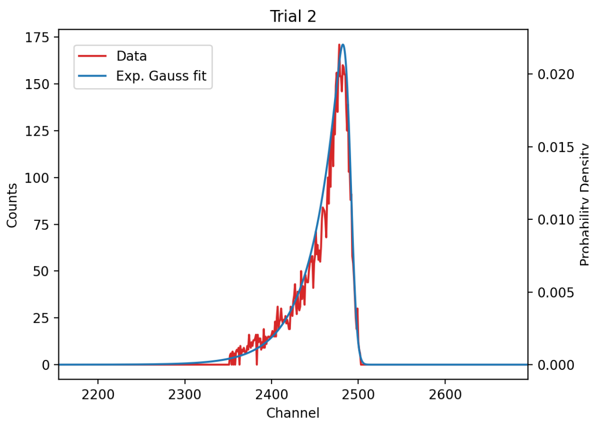

FIG. 4. Data from a single trial. Extraneous noise has been filtered out using both band pass and noise gate methods, and the remaining data has been fit with an exponentially modified Gaussian curve.

Table I shows the overall calculated results from all trials, as well as the distance of the sample from the start detector for each trial. These results are plotted in Fig. 5.

TABLE I. Results from each trial after fitting with exponentially modified Gaussian.

Trial	Position (m)	Centroid (ch)	Stnd. Dev. (ch)	K value
1	0.600	2262.5	6.2611	5.8928
2	1.415	2490.8	5.6111	5.7402
3	2.400	2777.0	5.4875	5.6357
4	1.690	2602.8	5.9485	5.7012
5	1.210	2460.8	6.9373	5.9277

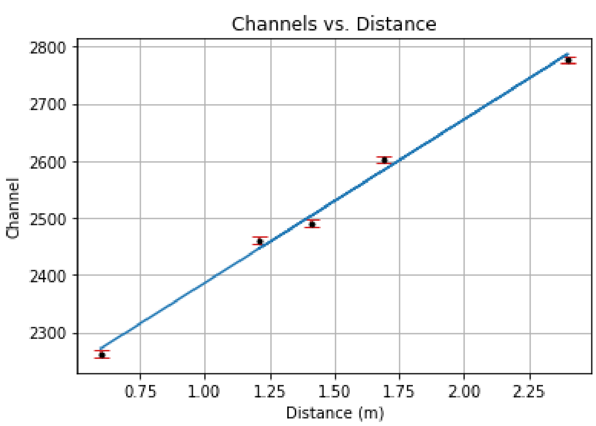

FIG. 5. Calculated centroid data for each trial, plotted against distance. Centroids and standard deviations were found by fitting with an exponentially modified Gaussian.

The results of linear regression on this data set gave a slope of 286.2, with $R^2 \approx 0.9933$. This slope represents m_d, the ratio between channels and distance, giving us:

$$m_d = 286.23 \pm 3.56 \ \frac{\mathrm{ch}}{\mathrm{m}}, \tag{5}$$

where the error is calculated using the standard uncertainty on the slope for linear regression [7]:

$$\sigma_b = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{(n-2)\sum (x_i - \bar{x})^2}}. \tag{6}$$

Now we need the ratio between channels and time, denoted m_s. This was done using a time calibration module. This module produced a series of delta functions separated by a period of 0.08 μs, as shown in Fig. 6.

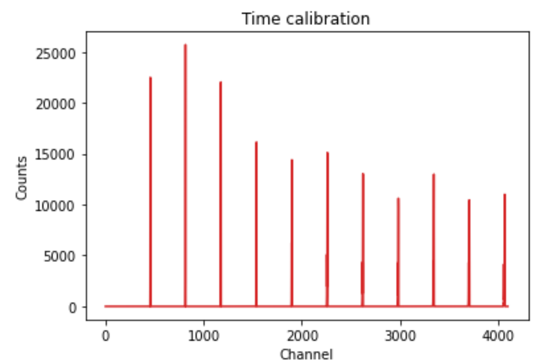

FIG. 6. Raw time calibration data using a period of 0.08 μs.

Because the channels are discrete while the signals are continuous, the exact peak did not land directly on a channel. To account for this, we found the precise center of each peak using a weighted average of the counts vs. channels. The results are shown in Fig. 7.

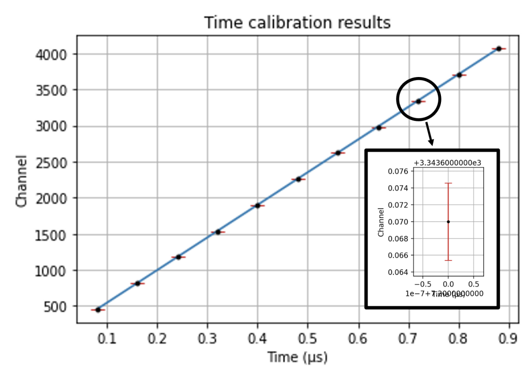

FIG. 7. Time calibration peaks vs time, where each peak channel was found using a weighted average. Error bars on this section were small due to the narrow range of incoming data, and as such a zoomed-in image has been included.

Linear regression over these calculated results gave a slope of $4.513 \cdot 10^3$, with $R^2 \approx 0.9999$. Thus,

$$m_s = 4513.88 \pm 0.03\ \frac{\text{ch}}{\mu\text{s}}, \tag{7}$$

where the error is calculated using Eq. 6. The relatively low error value and the high R^2 value are acceptable in this case, as this test is much more precise than the typical data collection process.

With m_d and m_s known, we find:

$$c = 2 \times \frac{m_s}{m_d}. \tag{8}$$

We calculate c to be:

$$c = (3.15 \pm 0.04) \cdot 10^8\ \frac{\text{m}}{\text{s}}. \tag{9}$$

This result is within 4σ of the established value of c.

IV. DISCUSSION

The final calculated value for c landed within the expected range of $c_{\text{theoretical}} \pm 0.05 c_{\text{theoretical}}$, and was within 4σ of the established value.

The primary source of error in this lab was due to environmental interference. While the modules on hand and the filters applied in the analysis process were able to assist with much of this, there was still a good deal of noise. This could be accounted for by using a more potent positron source, or by performing the experiment in a shielded environment. Such an environment would remove stray photons and external interference picked up by the wires.

The most unexpected source of error was due to the distribution the data took. While exponentially modified Gaussian curves do exist in nature, we expected the curve to come in the form of a pure Gaussian as positrons should be equally annihilated on either side of the sample. There were also several additional peaks on the lower channels that remain unexplained. It is possible that this is due to a fault in one of the modules, but it may also be as simple as interference from another source.

The applications of the value of c are widespread throughout physics, and while the value has been well-established for some time, efforts to verify this value can lead to improvements in the precision of lab equipment. These efforts can also be repeated in other setups to verify the value in non-vacuum (or vacuum-like) conditions.

V. ACKNOWLEDGEMENTS

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