B.C.A. III Sem.

MATH 204

B.C.A. (THIRD SEMESTER) EXAMINATION, DECEMBER, 2020

MATH 204- MATHEMATICS-II

Maximum Marks- 60

Note:- Attempt **Five** Questions in all, selecting **One** Question from each Unit. All Questions carry **equal** marks.

UNIT I

Q1. (a) If $y = (\sin^{-1} x)^2$, then prove that $(1-x^2)y_2 - xy_1 - 2 = 0$. Hence show that

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$ and find $y_n(0)$.

(b) Prove that

$$\int_0^1 \frac{x^2dx}{\sqrt{(1-x^4)}} \times \int_0^1 \frac{dx}{\sqrt{(1+x^4)}} = \frac{\pi}{4\sqrt{2}}.$$

Q2. (a) If z is a function of x and y and x=u cosα-v sinα, y=u sinα+v cosα, then show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\ .$$

(b) Evaluate

(i) $\int_0^1 \frac{dx}{\sqrt{(1-x^4)}}$

(ii) $\int_0^1 (1-x^3)^2\, dx$

(iii) $\int_0^1 x^5(1-x^4)^7 dx$

UNIT II

Q3. (a) A rectangular sheet of metal has four equal square portions removed at the corners and the sides are then turned up so as to form an open rectangular box. Show that when the volume contained in the box is a maximum, the depth will be

$\frac{1}{b}\left[a + b - \sqrt{a^2 - ab - b^2}\right]$, where a and b the sides of the original rectangle.

(b) Compute the surface area generated when an arc of the curve $x = t^2, y = \frac{t}{3}(t^2 - 3)$ between the points of intersection of the curve and the x-axis is revolved about the x-axis.

Q4. (a) A rectangular box, open at the top, is to have a volume of 32 cc. Find dimensions of the box which requires least amount of material for its construction.

(b) The area bounded by $y^2=4x$ and the line $x=4$ is revolved about the line $x=4$. Find the volume of the solid of revolution.

UNIT III

Q5. (a) The gradient of the curve which passes through the point (4, 0) is defined by the equation $\frac{dy}{dx} - \frac{y}{x} + \frac{5x}{(x+2)(x-3)} = 0$. Find the equation of the curve and find the value of y when $x=5$.

(b) Solve $\frac{dy}{dx} + x \sin 2y = x^3(\cos y)^2$.

Q6. (a) Solve $\frac{d^2y}{dx^2} - y = x\sin x + (1+x^2)e^x$.

(b) Solve $\frac{d^2y}{dx^2} + a^2 y = \tan ax$.

UNIT IV

Q7. (a) Test for the consistency and solve $x + 2y + z = 3, 2x + 3y + 2z = 5, 3x - 5y + 5z = 2, 3x + 9y - z = 4$.

(b) If A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ and $f(x)=x^2-5x+6$, then find f(A).

Q8. (a) Find the values of a and b if the matrix $\begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & -1 & 2 & 4 \\ 7 & -1 & a & b \end{bmatrix}$ is of rank 2.

(b) If the system of equations $x = cy + bz, y = az + cx, z = bx + ay$ have non-trivial solutions, prove that $a^2 + b^2 + c^2 + 2abc = 1$ and the solutions are

$$x : y : z = \sqrt{1-a^2} : \sqrt{1-b^2} : \sqrt{1-c^2} .$$

UNIT V

Q9. (a) A company has two grades of inspectors, I and II to undertake quality control inspection. At least 1500 pieces must be inspected in an 8-hours day. Grade I inspector can check 20 pieces in an hour with an accuracy of 96%. Grade II inspector checks 14 pieces an hour with an accuracy of 92%.

Wages of grade I inspector are rs. 5 per hour while those of grade II inspector are rs 4 per hour. Any error made by an inspector costs rs 3 to the company. If there are in all, 10 grade I inspectors and 15 grade II inspectors in the company, find the optimal assignment of inspectors that minimizes the daily inspection cost. Formulate the LPP.

(b) Use the two-phase simplex method to

Minimize $Z = x + 1.5y + 5z + 2k,$

Subject to $3x + 2y + z + 4k \leq 6,$

$$2x + y + 5z + k \le 4,$$

$$2x + 6y - 4z + 8k = 0,$$

$$x + 3y - 2z + 4k = 0,$$

$$x, y, z, k \ge 0.$$

Q10. (a) A person requires 10, 12, and 12 units of chemical A, B and C respectively for his gardens. A liquid product contains 5, 2, and 1 unit of A, B and C respectively per jar. A dry product contains 1, 2 and 4 units of A, B and C per carton. If the liquid product sells for rs 3 per jar and the dry product sells for rs 2 per carton, how many of each should be purchased to minimize the cost and meet the requirements? Formulate the above problem as a LPP and solve it by graphical method.

(b) An Air force is experimenting with three types of bombs P, Q and R in which three kinds of explosives, viz. A, B and C will be used. Taking the various factors into account, it has been decided to use at the maximum 600 kg of explosive A, at least 480 kg of explosive B and exactly 540 kg of explosive C. Bomb P requires 3, 2, 2kg, bomb Q requires 1, 4, 3 kg and bomb R requires 4, 2, 3 kg of explosives A, B and C respectively. Bomb P is estimated to give the equivalent of a 2 ton explosion, bomb Q, a 3 ton explosion and bomb R, a 4 ton explosion respectively. Under what production schedule can the Air Force make the biggest bang?