Maths

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* **Def^n :**

A random variable X is a function defined by $X : S \to R$ That assign a real number $X(S)$ to each $X \in S$ where S is sample space, corresponds to random experiment E.

* **Descrete Random Variable :**

Probability Distribution : $(0 \leq P \leq 1)$

$X = x$	x_0	x_1	x_2	x_3	x_4
$P(X=x)$	P_0	P_1	P_2	P_3	P_4

Q. 1 4 bad oranges are mixed accidently 16 good oranges. Find the probability distribution of no. of bad oranges, in a draw of 2 oranges.

Sol. Let X = Random variable (of no of bad oranges) in a draw of 2 oranges out of 20 oranges

$\Rightarrow$ $X = \{0, 1, 2\}$ — not independent

$$P(X=0) = \frac{{}^{16}C_2 \times {}^{4}C_0}{{}^{20}C_2} = \frac{16 \times 15}{2! \times 14!} \times \frac{2!\, 18!}{20 \times 19} = \frac{12}{19} = \frac{60}{95}$$

$$P(X=1) = \frac{{}^{4}C_1 \times {}^{16}C_1}{{}^{20}C_2} = \frac{\frac{4!}{1!\,3!} \times \frac{16!}{1! \times 15!}}{\frac{20!}{2!\,.18!}}$$

$$= \frac{4! \times 16! \times 18! \times 2!}{3! \times 15! \times 20 \times 19 \times 18!} = \frac{32}{95}$$

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$$P(X=2) = \left(\frac{{}^4C_2 \times {}^{16}C_0}{{}^{20}C_2}\right) = \frac{\left(\frac{4!}{2!\cdot 2!}\right)\times 1}{\left(\frac{20!}{2!\times 18!}\right)}$$

$$= \frac{4! \times 18! \times 3 \times 2!}{2! \times 18!\, 20 \times 19 \times 18!}$$

$$= \frac{4\times 3}{19 \times 20} = \frac{3}{95}$$

X	0	1	2	~~3~~	~~4~~
P	60/95	32/95	3/95	~~0~~	~~0~~

$$\frac{60 + 32 + 3}{95}$$

* **Probability mass function** :-

$$P(X = x_i) = P_i$$

i) $P_i \geq 0$.

ii) $\sum_i P_i = 1$

Function @Symbols

Vicky kumar

Q.2 From a lot of 10 items containing 3 defective, A sample of 4 item is drawn at random. The random variable X is denoted as defective item in a sample.

Find the probability distribution of X.

X	0	1	2	3
P				

→ defective

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$f(1)$

(ii) $P(X \le 1) \to P(X=0) + P(X=1)$

(iii) $P(X<1) \to P(X=0)$

(iv) $P(0<x<2) \to P(X=1)$.

Sol: $X = \{0, 1, 2, 3\}$ defective

$P(X=0) = \frac{{}^7C_4}{{}^{10}C_4} = \frac{35}{210} = \frac{1}{6}$

$P(X=1) = \frac{{}^7C_3 \times {}^3C_1}{{}^{10}C_4}$

$P(X=2) = \frac{{}^7C_2 \times {}^3C_2}{{}^{10}C_4}$

$P(X=3) = \frac{{}^7C_1 \times {}^3C_3}{{}^{10}C_4}$

$\frac{1}{6} + \frac{1}{2}$

$= \frac{1+3}{6}$

$= \frac{4}{6}$

$= \frac{2}{3}$

(i)

X	0	1	2	3
P	1/6	1/2	3/10	1/30

(ii) $P(X \le 1) = P(X=0) + P(X=1)$
$= \frac{1}{6} + \frac{1}{2} = \frac{1+3}{6} = \frac{4}{6} = \frac{2}{3}$

(iii) $P(X<1) = P(X=0) = 1/6$.

(iv) $P(0<x<2) = P(X=1) = 1/2$.

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* (Cumulative Distribution function cdf for D.R.V.)

→ Let X be a D.R.V, then the discrete distribution function or cumulative distribution function (cdf) is defined as,

* ⇒ $\boxed{F(x_j) = P(X \le x_j)} = \sum_{i \le j} P_i$

$X = x$	x_0	x_1	x_2	x_3
i	0	1	2	3
$P(X = x)$	P_0	P_1	P_2	P_3

$\boxed{F(0) = P(0)}$

$F(1) = P(X \le 1) = P(0) + P(1)$

$F(2) = P(X \le 2) = (P(0) + P(1)) + P(2) = F(1) + P(2)$

$F(3) = P(X \le 3) = (P(0) + P(1) + P(2)) + P(3)$

$\boxed{F(3) - F(2) = P(3)}$ $\boxed{F(3) = F(2) + P(3)}$

Q.1 A Random variable X has the following probability distribution,

$X = x$:	0	1	2	3	4	5	6	7
$P(X = x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$
$F(X)$:	0	k	$3k$	$5k$	$8k$	$8k + k^2$	$8k + 3k^2$	$10k^2 + 9k$

i) find k.

ii) Evaluate $P(X < 6)$, $P(X \ge 6)$, $P(0 < x < 5)$

iii) Determine Cu. Distribution function of X.

iv) If $\boxed{P(X \le c) > \frac{1}{2}}$, find the minimum value of c.

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$P(x \le a) \ge \frac{1}{2}$

(V) find $P\left(\frac{1.5 < X < 4.5}{X > 2}\right)$

$\boxed{F(7) = 1}$

Sol:

i) $\Sigma P(X) = 1$, (As we know).

$$\cancel{8k + 6k^2 + 7k^2 + k = 1}$$
$$\cancel{13k^2 + 9k = 1}$$
$$\cancel{k(13k + 9) = 1}$$
$$\cancel{13k^2 + 9k - 1 = 0.}$$

$$\boxed{9k + 10k^2 = 1}$$

$$10k^2 + 9k - 1 = 0$$
$$10k^2 + 10k - k - 1 = 0.$$
$$10k(k+1) - (k+1) = 0$$
$$\left(k = -1, \frac{1}{10}\right), (k > 0)$$
$$\left(k = \frac{1}{10}\right)$$

iii) $\boxed{P(X < 6) = 1 - P(X \ge 6)} = 1 - (P(6) + P(7))$

$$= 1 - (2k^2 + (7k^2 + k))$$

F(5)

$$= 1 - 9k^2 - k$$
$$= 1 - \frac{9}{100} - \frac{1}{10} = \frac{100 - 9 - 10}{100}$$

Ans → $P(X < 6) = \frac{81}{100}$

→ $P(X \ge 6) = \frac{19}{100}$,

↓

$1 - P(X < 6)$

$1 - F(5)$

→ $P(0 \le x < 5) = P(1) + P(2) + P(3) + P(4) = (8k) = \frac{8}{10}$

$= \left(\frac{4}{5}\right)$

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(iii)

X	$F(x) = P(X \le x)$
0	0
1	$k = 1/10$
2	$3k = 3/10$
3	$5k = 5/10 = 1/2 = 0.5$
4	$8k = 8/10 = 4/5 = 0.8$
5	$8k + k^2$
6	$8k + 3k^2$
7	$10k^2 + 9k = 1$

$0 > 1/2$ ✗

$1/10 > 1/2$ ✗

$3/10 > 1/2$ ✗

$1/2 > 1/2$ ✗

$0.8 > 1/2$ ✓

at $c=4$

(iv) $0.8 > \frac{1}{2}$

$F(X=4)$.

Ans $c = 4$

(IV) $P(X \le 3) = \frac{1}{2} = F(3)$

$F(4) = P(X \le 4) = \frac{8}{10} = 0.8 > \frac{1}{2} = 0.5$; $c = 4$

$F(4) > \frac{1}{2}$

4, 5, 6, 7

(V) $\boxed{P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}}$

$\Rightarrow P\left(\frac{1.5 < X < 4.5}{X > 2}\right)$

$= \frac{P((1.5 < X < 4.5) \cap (X > 2))}{P(X > 2)}$

4.5 ~~$= P($~~

1.5

$= \frac{P(2 < X < 4.5)}{P(X > 2)}$

$= \frac{P(X=3) + P(X=4)}{1 - P(X \le 2)}$

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$$= \frac{(2k+3k)}{1-(0+k+2k)}$$

$$= \frac{2k+3k}{1-(0+k+2k)} = \frac{5k}{1-3k} = \frac{\frac{5}{10}}{1-\frac{3}{10}}$$

Ans $= \left(\frac{5}{7}\right)$ Ans

Q② Find the probability distribution of boys and girls in families with 3 children. Assuming equal probabilities of boys and girls. Also give the c.d.f.

Sol: Let X denote no. of Boys

$X = 0, 1, 2, 3$

$P(\text{boy}) = \frac{1}{2} = P(\text{girl}) = p = q$

$$\boxed{P(X=r) = {}^nC_r \cdot p^r \cdot q^{n-r}}$$

$P(X=0) = {}^3C_0\left(\frac{1}{2}\right)^0 \cdot \left(\frac{1}{2}\right)^3 = \frac{3!}{0!\,3!} \cdot 1 \cdot \frac{1}{8} = \left(\frac{1}{8}\right)$

$P(X=1) = {}^3C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 = \frac{3 \times 2!}{1!\,2!} \times \frac{1}{2} \times \frac{1}{4} = \left(\frac{3}{8}\right)$

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$$P(X=2) = {}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 = \frac{3 \times 2!}{2!\,1!} \times \frac{1}{4} \times \frac{1}{2} = \left(\frac{3}{8}\right)$$

$$P(X=3) = {}^3C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0 = \frac{3!}{3!\,0!} \times \frac{1}{8} \times \frac{1}{1} = \left(\frac{1}{8}\right)$$

$(X=x)$	0	1	2	3
$P(X=x)$	1/8	3/8	3/8	1/8
$F(X)$	1/8	4/8	7/8	8/8

$$\frac{6}{13}\left(1 - \frac{1}{13}\right)^{\cancel{8}7^{0}}$$

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(Continuous Random Variable)

* (Probability distribution and probability (density) function) → PDF for C.R.V.

condition

(i) $f(x) \geq 0$; $-\infty < x < \infty$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ ✓

* (Continuous Distribution function or continuous distribution function)

$$F(x) = \boxed{P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx} \; ; \; -\infty < x < \infty$$

Q(1) $f(x) = \cancel{6x+1} \; 6x(1-x)$; $0 \leq x \leq 1$

Sol. condition: (i) $f(x) \geq 0$; $\cancel{-\infty < x < \infty}$ $0 \leq x \leq 1$

let, put $x = \frac{1}{2}$, $f(\frac{1}{2}) = \frac{6}{2}(1 - \frac{1}{2}) = \left(\frac{3}{2}\right) \geq 0$.

(ii) $\int_{0}^{1} f(x)\,dx = \int_{0}^{1} (6x - 6x^2)\,dx$

$$= \left(\frac{6x^2}{2} - \frac{6x^3}{3}\right)\Bigg|_0^1 = (3 - 2x)x^2\Big|_0^1 = 1$$

properties

* (i) $\boxed{\frac{d}{dx} F(x) = f(x)}$ (ii) $\boxed{0 \leq F(x) \leq 1}$

c.d.f.

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* Continuous R.V. :-

Ex. Let X be a C.R.V. with p.d.f,

$$f(x) = \begin{cases} ax & ; \quad 0 \le x \le 1 \\ a & ; \quad 1 \le x < 2 \\ -ax + 3a & ; \quad 2 \le x \le 3 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

(i) Determine the constant a,
(ii) find $P(X \le 1.5)$
(iii) Determine the cdf and hence find $P(X \le 2.5)$

Sol. (i) $\because f(x)$ is a pdf, then,

$$\boxed{\int_0^{\infty} f(x)\,dx = 1}$$

$$\Rightarrow \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax + 3a)\,dx + \int_3^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \left.\frac{ax^2}{2}\right|_0^1 + \left.ax\right|_1^2 + \left.\left(-\frac{ax^2}{2} + 3ax\right)\right|_2^3 + 0 = 1$$

$$\Rightarrow \frac{a}{2} + a(2-1) + \left(-\frac{9a}{2} + 9a\right) - \left(-\frac{4a}{2} + 6a\right) = 1$$

$$\frac{a}{2} + a + \frac{9a}{2} - 4a = 1$$

$$\frac{a + 2a + 9a - 8a}{2} = 1$$

$$12a - 8a = 2$$

$$4a = 2$$

$$a = \frac{2}{4} = \frac{1}{2}$$

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(ii) $F(1.5) = P(X \le 1.5) = \int_0^{1.5} f(x)\,dx$

$= \int_0^1 f(x)\,dx + \int_1^{1.5} f(x)\,dx$

$= \int_0^1 ax\,dx + \int_1^{1.5} a\,dx = \left.\frac{ax^2}{2}\right|_0^1 + \left.ax\right|_1^{1.5}$

$= \frac{a}{2} + a(1.5-1) = \frac{a}{2} + \frac{a}{2} = a = \frac{1}{2}$

(iii) cdf :-

$f(x) = 0$ elsewhere

This must be variable.

$x < 0, \quad F(x) = 0$

$0 \le x \le 1, \quad \boxed{F(x) = \int_0^x f(x)\,dx} = \int_0^x ax\,dx$

$= a\left.\left(\frac{x^2}{2}\right)\right|_0^x = \frac{ax^2}{2} = \frac{x^2}{4}, \quad (\because a = \frac{1}{2})$

$1 \le x \le 2, \quad F(x) = \int_0^1 f(x)\,dx + \int_1^x f(x)\,dx$

$= \int_0^1 ax\,dx + \int_1^x a\,dx$

$= \left.\frac{ax^2}{2}\right|_0^1 + \left.ax\right|_1^x$

$= \frac{a}{2} + a(x-1) = a\left(\frac{1}{2} + x - 1\right)$

$\boxed{\frac{1}{2}\left(x - \frac{1}{2}\right) = \left(\frac{x}{2} - \frac{1}{4}\right)}$

$2 \le x \le 3, \quad F(x) = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^x f(x)\,dx$

$= \frac{a}{2} + \int_1^2 a\,dx + \int_2^x (-ax + 3a)\,dx = \left[\left(\frac{a}{2}\right) + \left.ax\right|_1^2 + \left.\left(-\frac{ax^2}{2} + 3ax\right)\right|_2^x\right]$

$F(x) = \frac{a}{2} + a + \left(-\frac{ax^2}{2} + 3ax\right) - \left(-\frac{4a}{2} + 6a\right)$

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$$f(x) = \left(\frac{3a}{2}\right) + 3ax + \frac{\cancel{4}a^{2}}{\cancel{x}} - 6a - \frac{ax^2}{2}$$

$$= \frac{3a}{2} + 3ax + 2a - 6a - \frac{ax^2}{2}$$

$$= -\frac{ax^2}{2} + 3ax + \left(\frac{3a}{2} - 4a\right)$$

$$= -\frac{ax^2}{2} + 3ax - \frac{5a}{2}$$

$$\Rightarrow \boxed{f(x) = \left(-\frac{x^2}{4} + \frac{3x}{2} - \frac{5}{4}\right);\ 2 \le x \le 3}$$

$\rightarrow$ $x \geq 3$ $\boxed{f(x) = 1}$

$$f(x) = \begin{cases} 0 ; & x < 0 \\ x^2/4 ; & 0 \le x \le 1 \\ -\frac{x^2}{4} + \frac{3x}{2} - \frac{5}{4} ; & 1 \le x \le 2 \\ 1 ; & \boxed{2 \le x \le 3} \\ & \text{else} \end{cases}$$

Also, To find, $P(X \le 2.5)$

$$F(2.5) = \left(-\frac{(2.5)^2}{4} + \frac{3}{2}(2.5) - \frac{5}{4}\right)$$

$$= -\frac{1}{4} \times (6.25) + \left(\frac{3}{2} \times \frac{25}{10}\right) - \frac{5}{4}$$

$$= -\frac{625}{400} + \frac{75}{20} - \frac{5}{4}$$

$$= \frac{-625}{400} + \frac{1500}{400} - \frac{500}{400}$$

$$= \left(\frac{1500 - 1125}{400}\right) = \frac{\cancel{375}}{\cancel{400}} = \left(\frac{15}{16}\right)$$

1500
−1125
375

15
75

Ans $\boxed{F(2.5) = 0.9375}$

Binomial Distribution

Binomial Distribution

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Two dimensional Random Variable

*
4 - White
3 - black
4 - blue
2 - green

X(white) → {0, 1, 2, 3}
Y (green) → { 0,1, 2 }

3 ball to choose

Y / X	0	1	2
0	P_{00}	P_{01}	P_{02}
1	P_{10}	P_{11}	
2			P_{22}
3		P_{31}	

$P(X=0, Y=0)$
$\to P_{00}$

* Joint Probability distribution :-

pmf $P_{ij} = P(X = x_i, Y = y_j)$

P_{00}

$$\sum_{i=1}^{m} \sum_{j=1}^{n} P_{ij} = 1$$

Q.1) Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls, if X denotes the number of white ball drawn and Y denotes the no. of red ball drawn find the joint probability distribution of X, Y.

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Sol: $X = 0, 1, 2$
$Y = 0, 1, 2, 3$

$X \downarrow$ \ $Y \rightarrow$	0	1	2	3
0				
1				
2				

$$P_{00} = P(X=0, Y=0) = \frac{{}^4C_3}{{}^9C_3} = \frac{\frac{4!}{3!\cdot 1!}}{\frac{9!}{3!\, 6!}}$$

$$= \frac{\frac{4\times 3!}{3!}}{\frac{9\times 8\times 7\times \cancel{6!}}{3\times 2\times \cancel{6!}}} = \frac{4\times 3\times 2}{\cancel{9}^{3}\times 8\times 7}$$

$$= \frac{1}{21}$$

$$P_{01} = P(X=0, Y=1) = \frac{{}^3C_1 \times {}^4C_2}{{}^9C_3}$$

$$P_{02} = P(X=0, Y=2) = \frac{{}^3C_2 \times {}^4C_1}{{}^9C_3}$$

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$P_{03} = P(X=0, Y=3) = \frac{^3C_3}{^9C_3}$

$P_{10} = P(X=1, Y=0) = \frac{^2C_1 \times {^4C_2}}{^9C_3}$

$P_{11} = P(X=1, Y=1) = \frac{^2C_1 \times {^3C_1} \times {^4C_1}}{^9C_3}$

$P_{13} = P(X=1, Y=3) = 0$

$P_{20} = P(X=2, Y=0) = \frac{^2C_2 \times {^4C_1}}{^9C_3}$

$P_{21} = P(X=2, Y=1) = \frac{^2C_2 \times {^3C_1}}{^9C_3}$

$\left\{ \begin{array}{l} P(X=2, Y=2) = 0 \\ P(X=2, Y=3) = 0. \end{array} \right\} \because$

$P_{22} \rightarrow 2+2=4$, $4>3$ → 0

$P_{23} \rightarrow 2+3>3$ → 0

3 balls to drawn

X \ Y	0	1	2	3
0	1/21	3/14	1/7	1/84
1	1/7	2/7	1/14	0
2	1/21	1/28	0	0

(i) $P_{ij} = P(X=x_i, Y=y_j) \geq 0$

(ii) $\sum_{i=0}^{2} \sum_{j=0}^{3} P_{ij} = 1 \Rightarrow \sum_{i=0}^{2} \left[P_{i0} + P_{i1} + P_{i2} + P_{i3} \right]$

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Cumulative distribution function for 2-D R.V.

$$F(x,y) = P(X \le x \text{ and } Y \le y)$$

$$= \sum_j \sum_i P_{ij}$$

$$= P(-\infty \le X \le x, -\infty < Y \le y)$$

$$= \int_{-\infty}^{x} \int_{-\infty}^{y} f(x,y)\, dx\, dy.$$

eg

$$F(1,1) = P(X \le 1, Y \le 1)$$

$$= \left(\sum_{j=0}^{1} \left(\sum_{i=0}^{1} P_{ij}\right)\right)$$

$$= \left(\sum_{j=0}^{1} P_{0j}\right) + \sum_{j=0}^{1} P_{1j}$$

$$= P_{00} + P_{10} + P_{01} + P_{11}$$

*

$0 \le x < 1$

$0 < y < 2$

$$F(x,y) = \int_{0}^{x} \int_{0}^{y} f(x,y)\, dx\, dy$$

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Two dimentional D.R.V.

Q. 1

The joint probability mass function of (X,Y) is given by $P(x,y) = k(2x + 3y)$, $x = 0, 1, 2$; $y = 1, 2, 3$. Find

(i) k

(ii) Marginal probability function of X, (P_{i*})

(iii) " " " " Y, (P_{*j})

(iv) Conditional distribution of X given $(Y=1)$

(v) " " " Y " $(X = 2)$

(vi) The probability distribution of $(X+Y)=Z$

Sol. Probability distribution of (X,Y)

X \ Y	1	2	3
0	3k	6k	9k
1	5k	8k	11k
2	7k	10k	13k

(i) above given is a pmf i.e.,

$$\sum_{i=0}^{2}\sum_{j=1}^{3} P_{ij} = 1$$

$$72k = 1 \Rightarrow k = \frac{1}{72}$$

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(ii) $\boxed{P(X=x_i) = P_{i*} = \sum_{j=1}^{3} P_{ij}}$

$P(X=0) = P_{0*} = (P_{01} + P_{02} + P_{03})$

$= 3k + 6k + 9k$

$= 18k = \frac{18}{72} = \frac{1}{4}$

$P(X=1) = P_{1*} = \sum_{j=1}^{3} P_{1j} = (P_{11} + P_{12} + P_{13})$

$= 24k = \frac{24}{72} = \frac{1}{3}$

$P(X=2) = \boxed{P_{2*} = 30k = \frac{30}{72} = \frac{5}{12}}$

M.P.D

$X=i$	P_{i*}
$X=0$	$1/4$
$X=1$	$1/3$
$X=2$	$5/12$

(iii) $\boxed{P(Y=y_j) = P_{*j} = \sum_{i=0}^{2} P_{ij}}$

$P(Y=1) = P_{*1} = (P_{01} + P_{11} + P_{21})$

1st column

$= 3k + 5k + 7k = 15k$

$= \frac{15}{72} = \frac{5}{24}$

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$P(Y=2) = P_{*2} = 6k + 8k + 10k = 24k$

$= \frac{24}{72} = \left(\frac{1}{3}\right)$

$P(Y=3) = P_{*3} = 9k + 11k + 13k$

$= 33k = \frac{33}{72} = \left(\frac{11}{24}\right)$

(iv) Conditional probability function of X given $Y = 1$

$$P\left(\frac{X = x_i}{Y = 1}\right) = \frac{P_{i1}}{P_{*1}} = \frac{P_{01}}{P_{01} + P_{11} + P_{21}}$$

$$\frac{P(X=0)}{P(Y=1)} = \frac{P_{01}}{P_{*1}} = \frac{3k}{15k} = \left(\frac{1}{5}\right)$$

$$\frac{P(X=1)}{P(Y=1)} = \frac{P_{11}}{P_{*1}} = \frac{5k}{15k} = \frac{1}{3}$$

$$\frac{P(X=2)}{P(Y=1)} = \frac{P_{21}}{P_{*1}} = \frac{7k}{15k} = \frac{7}{15}$$

$(X = x_i)$	(P_{i1} / P_{*1})
$X = 0$	$1/5$
$X = 1$	$1/3$
$X = 2$	$7/15$

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$$P(Y=2) = P_{*2} = 6k + 8k + 10k = 24k$$

$$= \frac{\cancel{24}^{12}}{\cancel{72}_{\cancel{36}\,3}} = \left(\frac{1}{3}\right)$$

$$P(Y=3) = P_{*3} = 9k + 11k + 13k$$

$$= 33k = \frac{\cancel{33}^{11}}{\cancel{72}_{24}} = \left(\frac{11}{24}\right)$$

(iv) Conditional probability function of X given $Y = 1$.

$$\boxed{P\left(\frac{X = x_i}{Y = 1}\right) = \frac{P_{i1}}{P_{*1}}} = \frac{P_{01}}{P_{01} + P_{11} + P_{21}}$$

$$\frac{P(X=0)}{P(Y=1)} = \frac{P_{01}}{P_{*1}} = \frac{\cancel{3k}}{\cancel{15k}} = \left(\frac{1}{5}\right)$$

$$\frac{P(X=1)}{P(Y=1)} = \frac{P_{11}}{P_{*1}} = \frac{5k}{15k} = \frac{1}{3}$$

$$\frac{P(X=2)}{P(Y=1)} = \frac{P_{21}}{P_{*1}} = \frac{7k}{15k} = \frac{7}{15}$$

$(X = x_i)$	(P_{i1} / P_{*1})
$X = 0$	$1/5$
$X = 1$	$1/3$
$X = 2$	$7/15$

(v) $$\boxed{\frac{P(Y=y_j)}{P(X=2)} = \frac{P_{2j}}{P_{2*}}}$$

$$\frac{P(Y=1)}{P(X=2)} = \frac{P_{21}}{P_{2*}} = \frac{7k}{30k} = \left(\frac{7}{30}\right)$$

$$\frac{P(Y=2)}{P(X=2)} = \frac{P_{22}}{P_{2*}} = \frac{10k}{30k} = \left(\frac{1}{3}\right)$$

$$\frac{P(Y=3)}{P(X=2)} = \frac{P_{23}}{P_{2*}} = \frac{13k}{30k} = \left(\frac{13}{30}\right)$$

$Y = y_j$	(P_{2j} / P_{2*})
$Y=1$	7/30
$Y=2$	1/3
$Y=3$	13/30

(vi) $Z = X+Y = 1, 2, 3, 4, 5.$

X	0	0	1	0	1	2	1	2	2
Y	1	2	1	3	2	1	3	2	3
Z=X+Y	1	2		3			4		5
P(Z)	3/72	11/72		24/72			21/72		13/72

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$P(Z=1) = P(X=0, Y=1) = P_{01} = 3k$

$P(Z=2) = P_{02} + P_{11} = (11/72)$

$P(Z=3) = P_{03} + P_{12} + P_{21} = 9k + 8k + 7k = \frac{24}{72}$

$P(Z=4) = P_{13} + P_{22} = 11k + 10k = 21/72$

$P(Z=5) = P_{23} = 13k = 13/72$

Independent

Q.2 The joint probability mass function (pmf) of 2 descrete random variable (D.R.V.) is given by $P(x,y) = \begin{cases} \frac{xy}{27} ; & \text{for} \rightarrow (x = 1, 2 ; y = 2, 3, 4) \\ 0 ; & \text{otherwise} \end{cases}$

Prove that X and Y are independent.

Sol. $\boxed{P_{i*} \times P_{*j} = P_{ij}}$ → condition.

$\Rightarrow P_{i*} = \sum_{j=2}^{4} P_{ij}$

$\Rightarrow P_{1*} = \sum_{2}^{4} P_{1j} = P_{12} + P_{13} + P_{14} = \frac{2}{27} + \frac{3}{27} + \frac{4}{27}$

$\boxed{9/27 = \left(\frac{1}{3}\right)}$

$\Rightarrow P_{2*} = \sum_{2}^{4} P_{2j} = P_{22} + P_{23} + P_{24} = \frac{4}{27} + \frac{6}{27} + \frac{8}{27}$

$\boxed{\frac{\cancel{18}^{2}}{\cancel{27}_{3}} = \left(\frac{2}{3}\right)}$

$\Rightarrow P_{*j} = \sum_{1}^{2} P_{ij}$

$P_{*2} = P_{12} + P_{22} = (2+4)/27 = 6/27 = 2/9$

$P_{*3} = P_{13} + P_{23} = (3+6)/27 = 9/27 = 1/3$

$P_{*4} = P_{14} + P_{24} = (4+8)/27 = 12/27 = 4/9$

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$$\cancel{X = x_i} \quad \cancel{P_{i1}} \quad \cancel{P_{*1}}$$

$$\cancel{X = 0} \quad \cancel{1/8}$$

$$\cancel{X = 1} \quad \cancel{1/3}$$

$$\cancel{X = 2} \quad \cancel{7/15}$$

$$P_{12} = P_{1*} \times P_{*2} = \frac{1}{3} \times \frac{2}{9} = \frac{2}{27}$$

$$P_{13} = P_{1*} \times P_{*3} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

$$P_{14} = P_{1*} \times P_{*4} = \frac{1}{3} \times \frac{4}{9} = \frac{4}{27}$$

$$P_{22} = P_{2*} \times P_{*2} = \frac{2}{3} \times \frac{2}{9} = \frac{4}{27}$$

$$P_{23} = P_{2*} \times P_{*3} = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$

$$P_{24} = P_{2*} \times P_{*4} = \frac{2}{3} \times \frac{4}{9} = \frac{8}{27}$$

$$\frac{xy}{27}$$

$$\cancel{P(Z = 3) = P_{03} + P_{12} + P_{21} =}$$

$$\cancel{P(Z = 4) = P_{13} + P_{22} =}$$

$$\cancel{P(Z = 5) = P_{23}}$$

$$\cancel{\frac{2x^2}{2}}$$

$$\int y\,x$$

$$y \times \frac{x^2}{2}$$

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Two dimensional Continuous Random Variable

~~Two dimention~~

Joint Probability density function (X, Y).

i) $f(x,y) \geq 0$

ii) $\int_{y=-\infty}^{\infty}\int_{x=-\infty}^{\infty} f(x,y)\,dx\,dy = 1$ $\begin{cases} -\infty < x < \infty, \\ -\infty < y < \infty \end{cases}$

Ex. 1 $f(x,y) = xy^2 + \frac{x^2}{8}$; $\begin{cases} 0 \leq x \leq 2 \\ 0 \leq y \leq 1. \end{cases}$

i) $f(x,y) \geq 0 \quad \because \quad x, y \geq 0.$

ii) $\int_0^1\left(\int_0^2 \left(xy^2 + \frac{x^2}{8}\right)dx\right)dy$

$= \int_0^1 \left\{ \left(\frac{y^2 x^2}{2} + \frac{x^3}{24}\right)\Bigg|_0^2 \right\} dy = \int_0^1 \left(\frac{y^2 4}{2} + \frac{8}{24}\right) dy$

$= \cancel{\int_0^2 \left(\frac{y^3 x^2}{6} + \frac{x^3 y}{24}\right)}$

$= \left(\frac{2y^3}{3} + \frac{8y}{24}\right)\Bigg|_0^1 = \cancel{\frac{8}{6} + \frac{2}{24}} \quad \frac{2}{3} + \frac{1}{3}$

$= \cancel{\frac{32+2}{24} = \frac{34}{24}} = \frac{3}{3} = 1$

(both condⁿ satisfied.)

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Joint Distribution function :-

$$F(x,y) = P(X \le x, Y \le y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f(x,y)\,dx\,dy$$

$$\begin{cases} -\infty \le x \le \infty \\ -\infty \le y \le \infty \end{cases}$$

e.g

$$f(x) = \begin{cases} x ; & 0 < x < 1 \\ x+2 ; & 1 \le x < 2 \\ 0 ; & x \ge 2 \end{cases}$$

$$F(x) = \begin{cases} \int_0^x f(x)dx ; & 0 < x < 1 \\ \int_0^1 f(x)dx + \int_1^x f(x)dx ; & 1 \le x < 2 \\ \cancel{1 ; \quad x \ge 2} \end{cases}$$

$$\int_0^1 x\,dx + \int_1^2 (x+2)dx + \int_2^x 0\,dx$$

$$= \frac{x^2}{2}\Big|_0^1 + \left(\frac{x^2}{2}+2x\right)\Big|_1^2 + 0$$

$$= \frac{1}{2} + \frac{4}{2} + 4 - \frac{1}{2} - 2$$

$$= \frac{1}{2} + 2 + 4 - \frac{1}{2} - 2$$

*
$$\frac{\partial F}{\partial y} = \int_{-\infty}^{x} f(x,y)\,dx$$

$$\frac{\partial^2 F}{\partial x \partial y} = f(x,y)$$

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Q.1 Assume that lifetime X and the brightness Y of a bulb are being modeled as continuous R.V. with joint pdf given by,

$$f(x,y) = \lambda_1 \lambda_2 . e^{-(\lambda_1 x + \lambda_2 y)} \; ; \quad 0 \le x < \infty, \quad 0 < y < \infty$$

Find the joint distribution function.

Sol. $$F(x,y) = \int_0^y \int_0^x \lambda_1 \lambda_2 e^{-(\lambda_1 x + \lambda_2 y)} \, dx\, dy$$

$$\cancel{= \int_0^y \frac{(\lambda_1 \lambda_2) . e^{-(\lambda_1 x + \lambda_2 y)}}{-(\lambda_1)} dy}$$

$$\cancel{= \frac{\lambda_1 \lambda_2 e^{-(\lambda_1 x + \lambda_2 y)}}{\lambda_1 \lambda_2} = e^{-(\lambda_1 x + \lambda_2 y)}}$$

$$= \int_0^y \lambda_1 \lambda_2 . e^{-\lambda_2 y} \left(\int_0^x e^{-\lambda_1 x} dx \right) dy$$

$$= \int_0^y \lambda_1 \lambda_2 . e^{-\lambda_2 y} \left(\frac{e^{-\lambda_1 x}}{-\lambda_1} \right) \Big|_0^x dy$$

$$= \int_0^y \lambda_1 \lambda_2 . e^{-\lambda_2 y} \left(\frac{e^{-\lambda_1 x} - 1}{-\lambda_1} \right) dy$$

$$= \int_0^y \frac{\cancel{\lambda_1} \lambda_2}{-\cancel{\lambda_1}} . \left(e^{-(\lambda_2 y + \lambda_1 x)} - e^{-\lambda_2 y} \right) dy$$

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$$= \cancel{(-\lambda_2)}\left[\frac{e^{-(\lambda_1 x+\lambda_2 y)}}{\cancel{(-\lambda_2)}} - \frac{e^{-\lambda_2 y}}{\cancel{(-\lambda_2)}}\right]\Bigg|_0^y$$

$$= \left(e^{-(\lambda_1 x+\lambda_2 y)} - e^{-\lambda_2 y}\right) - \cancel{(1-1)}$$

Ans: $\boxed{F(x,y) = e^{-\lambda_2 y}\left(e^{-x\lambda_1} - 1\right)}$

(Marginal Density Function of X) :-

$$\left(f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \quad ; \quad -\infty < y < \infty \right)$$

(partial integration)

(Marginal Density function of Y) :-

$$\left(f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx \quad ; \quad -\infty < x < \infty \right)$$

y as const

Imp Q. Given the joint probability density function,

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y) \; ; & 0<x<1, \; 0<y<1 \\ 0 \; ; & \text{~~otherwise~~ elsewhere} \end{cases}$$

Find marginal density ~~function~~ of X & Y.

Sol: $f_X(x) = \int_0^1 \frac{2}{3}(x+2y)\,dy$

$$= \frac{2}{3}\left(xy + \frac{2y^2}{2}\right)\Big|_0^1$$

$$= \frac{2}{3}(x+1)$$

$$f_Y(y) = \int_0^1 \frac{2}{3}(x+2y)\,dx$$

$$= \frac{2}{3}\left(\frac{x^2}{2} + 2xy\right)\Big|_0^1$$

$$= \frac{2}{3}\left(\frac{1}{2} + 2y\right)$$

Conditional Density function of X, given Y denoted by $f\left(\frac{x}{y}\right)$ is given by

$$\boxed{f\left(\frac{x}{y}\right) = \frac{f(x,y)}{f_Y(y)}}$$

Similarly, $\boxed{f\left(\frac{y}{x}\right) = \frac{f(x,y)}{f_X(x)}}$

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Q. $f(x,y) = \frac{2}{3}(x+2y)$, $0 < x < 1$

$f_y(y) = \int_0^1 f(x,y)\,dx = \int_0^1 \left(\frac{2}{3}(x+2y)\right)dx$

$= \int_0^1 \left(\frac{2x}{3} + \frac{4y}{3}\right)dx$

$= \left(\frac{2x^2}{3\times 2}\right) + \left(\frac{4xy}{3}\right)\Big|_0^1$

$= \frac{1}{3} + \frac{4y}{3}$

$= \frac{2}{3}\left(\frac{1}{2} + 2y\right)$

Ans $f\left(\frac{x}{y}\right) = \frac{f(x,y)}{f_y(y)} = \frac{\frac{2}{3}(x+2y)}{\frac{2}{3}\left(\frac{1}{2}+2y\right)}$

$= \left(\frac{2x+4y}{1+4y}\right)$

* Independent C.R.V :-

Let (X,Y) be a two dimensional CRV then X and Y are said to be independent variable, if,

$$\boxed{f(x,y) = f_X(x) \cdot f_Y(y)}$$

Q. The joint probability density function (pdf) of the continuous Random variable (CRV), (X,Y) is given by,

$\frac{1}{y} = t \Rightarrow -\frac{1}{y^2} dy = dt$

$$f(x,y) = kxy\,e^{-(x^2+y^2)}; \quad x>0,\ y>0.$$

(A) Find k and (B) prove also that X and Y are independent.

Sol (A) i) $f(x,y) \geq 0$ (obviously).

$$\left[\text{ii) } \int_{y=0}^{\infty}\int_{x=0}^{\infty} f(x,y)\,dx\,dy = 1\right] \quad (\because \text{pdf})$$

$$\int_{y=0}^{\infty}\left(\int_{x=0}^{\infty} kxy\,e^{-(x^2+y^2)}\,dx\right)dy = 1$$

$x=0,\ t=y$

$\Rightarrow$ (put, $x^2+y^2 = t^2$

$\cancel{\frac{2x\,dx}{dx}} + \cancel{\frac{2y\,dy}{dx}}^{0} = \frac{\cancel{2}t\,dt}{dx}$)

$x\,dx = t\,dt$

$$\Rightarrow \int_{y=0}^{\infty} ky\int_{x=0}^{\infty}\left(x\,e^{-t^2}\,dx\right)dy = 1$$

$$\Rightarrow \int_{y=0}^{\infty} ky\left(\int_{t=y}^{\infty} e^{-t^2}\cdot t\,dt\right)dy = 1$$

$\to$ put $t^2 = m$

$\frac{\cancel{2}t\,dt}{\cancel{2}} = \frac{dm}{2}$

$$= \int_{y=0}^{\infty}\left(\frac{ky}{2}\int_{y^2}^{\infty} e^{-m}\,dm\right)dy = 1$$

$$= \int_{y=0}^{\infty}\frac{ky}{2}\left(\frac{e^{-m}}{-m}\right)\Bigg|_{y^2}^{\infty} dy = 1 \qquad \frac{\qquad 1 \qquad}{\int_{y=0}^{\infty}\frac{1}{2y}\cdot e^{-y^2}\,dy} = k$$

$$= \int_{y=0}^{\infty}\frac{ky}{2}\left(\frac{0 - e^{-y^2}}{-y^2}\right)dy = 1 \quad \Rightarrow \int_{y=0}^{\infty}\frac{1}{2y}\cdot e^{-y^2}\,dy \cdot k$$

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put $y^2 = n$

$2y\,dy = dn$

$\frac{1}{y} = t \longrightarrow y = \frac{1}{t}$

$\frac{dy}{y^2} = dt$

$y^2 = \frac{1}{t^2}$

$2y\,dy =$

$dy = \frac{-1}{t^2}$

e^{-y^2}

$$\int \left(\frac{kt}{2}\right)\left(e^{-\frac{1}{t^2}}\right)\left(\frac{-1}{t^2}\right)$$

$$\frac{k\,e^{-t^{-2}}}{2t}$$

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(Expectation)

* The expectation of a R.V. X is defined as,

$$\overline{X} = \mu = E(x^r) = \begin{cases} \sum_i x_i^r p_i \; ; \text{ if } X \text{ is discrete R.V.} \\ \int_{-\infty}^{\infty} x^r . f(x)\, dx \; ; \text{ if } X \text{ is continuous R.V.} \end{cases}$$

Q① Find the expectation of the number on a dice when thrown.

Sol.①

X :	1	2	3	4	5	6
P(X=x) :	1/6	1/6	1/6	1/6	1/6	1/6

$$E(X) = \sum_{i=1}^{6} x_i p_i = \left(\frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \dots + \frac{6}{6}\right)$$

$$= \cancel{\frac{8 \times 5}{2 \times 6}} = \cancel{25}.$$

Q② 2 unbised dice are thrown. Find the expected value of the sum of the number of points on them.

~~Sol.② X : 2 3 4 5 6 7 8 9 10 11 12~~

(No. of times) → 1 2 3 4 5 6 5 4 3 2 1

Sum → 2 3 4 5 6 7 8 9 10 11 12

1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

X :	2	3	4	5	6	7	8	9	10	11	12
$P(X=x_i)$:	$1/36$	$2/36$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$E(X) = \frac{1}{36}\Big(2\times1 + 3\times2 + 4\times3 + 5\times4 + 6\times5 + 7\times6 + 8\times5 + 9\times4 + 10\times3 + 11\times2 + 12\times1\Big)$$

$=$

Ex. Let X be a R.V. with the following probability distribution,

X :	-3	6	9
$P(X=x)$:	$1/6$	$1/2$	$1/3$

find, $E(X)$, $E(X^2)$, $E(2X+1)^2$.

Sol. $E(X) =$

$E(X^2) = \sum_i x_i^2 p_i = 9 \times \frac{1}{6} + 36 \times \frac{1}{2} + 81 \times \frac{1}{3}$

$= 46.5$

$E(2X+1)^2 = \sum_i (2x_i+1)^2 . p_i$

$= (2(-3)+1)^2 \times \frac{1}{6} + (2 \times 6 + 1)^2 \times \frac{1}{2} +$

$+ (2(9)+1)^2 \times \frac{1}{3}$

$= \left(\frac{25}{6}\right) + \left(\frac{169}{2}\right) + \left(\frac{19 \times 19}{3}\right)$

or

$\cancel{\sum_i (4x_i^2 + 1 + 4x_i)}$

$\Rightarrow E(4X^2 + 4X + 1)$

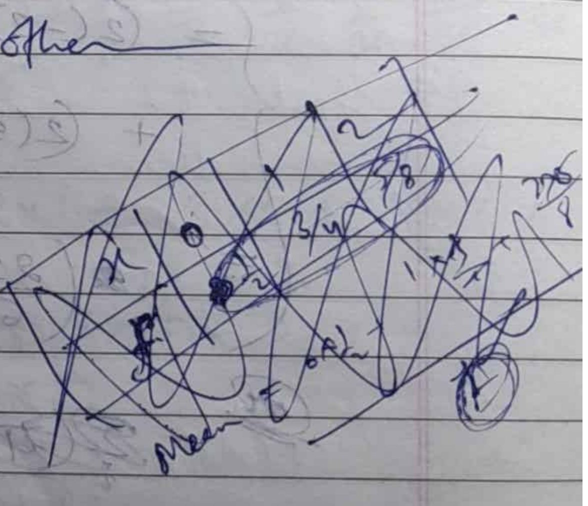
$= 4E(X^2) + 4E(X) + E(1)$

Q(3) A doctor recomment a patient to go on a particular diet for 2 weeks. There is equal likelyhood for patient to loose his weight b/w 2 kg and 4 kg.
What is the average amount the patient is expected to loose on this diet.

Sol.
$$f(x) = \begin{cases} k ; & 2 < x < 4 \\ 0 ; & \text{other} \end{cases}$$

$$\int_2^4 f(x)\,dx = 1$$

$$\int_2^4 k\,dx = 1$$



$$kx\Big|_2^4 = 1$$

$$k(4-2) = 1$$

$$k = \frac{1}{2}$$

$$f(x) = \begin{cases} 1/2 ; & 2 < x < 4 \\ 0 ; & \text{otherwise.} \end{cases}$$

(i)
$$E(x) = \int_2^4 x \left(\frac{1}{2}\right) dx = \frac{x^2}{4}\Big|_2^4$$

$$= \frac{16-4}{4} = \frac{12}{4} = 3 \text{ Ans}$$

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Varience

* Let X be a D.R.V. with pmf $P(X = x_i) = P_i$, then the varience of X is defined as

$$Var(X) = \sigma^2 = E[(x_i - \overline{X})^2]$$

where, $\overline{X}$ is the mean of random variable X.

$$\cancel{Var(X) = E(x_i^2 - 2x_i\overline{X} + \overline{X}^2)}$$

$$Var(X) = \sum_i (x_i - \overline{X})^2 \cdot P_i$$

$$= \sum_i (x_i^2 - 2x_i \overline{X} + \overline{X}^2) P_i$$

$$= \sum (x_i^2 P_i) - 2\overline{X}\left(\sum_i x_i P_i\right) + \overline{X}^2\left(\sum_i P_i\right)$$

$$= E(x_i^2) - 2\overline{X}^2 + \overline{X}^2$$

$$= E(x_i^2) - \overline{X}^2 = E(X^2) - (E(X))^2$$

Hence, $\boxed{Var(X) = \sigma^2 = E(X^2) - (E(X))^2}$

Q.1 A random variable X has the following probability distribution,

x_i :	-2	-1	0	1	2	3
P_i :	0.1	k	0.2	2k	0.3	k
x_i^2	4	1	0	1	4	9

(i) Calculate the mean of X.
(ii) Varience of X.
(iii) Standard deviation of $X = \sqrt{\sigma^2} = \sqrt{Var(X)} = \sigma$

Sol. $\sum_i P_i = 1$

$0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$\boxed{k = 0.1}$

$\Rightarrow \boxed{\overline{X} = E(X) = \sum_i x_i P_i} = 0.8$

$Var(X) = \sigma^2 = E(X^2) - (E(X))^2$

$= \sum_i (x_i^2 P_i) - (\overline{X})^2$

$=$

Q(2) 2 cards are drawn successively with replacement from a well suffeled find the mean and variance of $X = $ (no. of aces)

Sol. $X = 0, 1, 2$

$P(\text{getting ace}) = \frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$

$P(\text{not } ") = 1 - \frac{1}{13} = \frac{12}{13}$

$P(X=0) = \left(1 - \frac{1}{13}\right)\left(1 - \frac{1}{13}\right) = \frac{144}{169}$

$P(X=1) = \left(\frac{1}{13} \times \frac{12}{13}\right) + \left(\frac{12}{13} \times \frac{1}{13}\right) = \frac{24}{169}$

$P(X=2) = \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$

both ace

$X = x_i$:	0	1	2
$P(X = x_i)$:	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

$$\text{Mean}(X) = \overline{X} = E(X) = \sum_i x_i p_i = \frac{26}{169}$$

$$Var(X) = \sigma^2 = E(X^2) - (E(X))^2$$

$$= \left(0 \times \frac{144}{169}\right) + \left(1^2 \times \frac{24}{169}\right) + \left(2^2 \times \frac{1}{169}\right)$$

$$= \frac{144 + 24 + 4}{169}$$

Standard deviation $= \sqrt{\sigma^2}$

* Varience of X for continuous R.V. :-

$$\boxed{\begin{aligned} E(X) &= \int_{-\infty}^{\infty} x f(x)\, dx = \mu. \\ Var(X) &= \int_{-\infty}^{\infty} (x - \overline{X})^2 . f(x)\, dx \end{aligned}}$$

* Moments :-

→ Mean ✓
→ Varience ✓
→ ~~Sketch~~ ×

→ Moments about origin
→ Moment about mean (central moment)
→ Moment about any arbitrary point a.

** i) (Moment about origin) :-

for descrete R.V.

$$\mu'_r = E(X^r) = \frac{\sum_i x_i^r P_i}{(\sum P_i = 1)} \text{ , (probability)}$$

$$E(X^r) = \mu'_r = \frac{\sum x_i^r f_i}{\sum f_i} \text{ , (frequency)}$$

$r = 1$

$\mu'_1 = \text{mean}$

for continuous R.V :-

$$\mu'_r = E(X^r) = \int_{-\infty}^{\infty} x^r f(x)dx$$

$$\text{Mean} = \mu'_1 = E(X) = \sum_i x_i P_i = \overline{X}$$

$$\text{Varience} = \boxed{\mu'_2 - (\mu'_1)^2} = E(X^2) - (E(X))^2$$

Q.1) Calculate the first four moment about origin.

x_i :	6	7	8	9	10	11	12
f_i :	3	6	9	13	8	5	4

Sol. To find; μ'_1, μ'_2, μ'_3, μ'_4

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x_i	f_i	x_i^2	x_i^3	x_i^4	$x_i f_i$	$x_i^2 f_i$	$x_i^3 f_i$	$x_i^4 f_i$
6	3	36	216	1296	18	108	648	3888
7	6	49	343	2401	42	294	2058	14406
8	9	64	512	4096	72	576	4608	36864
9	13	81	729	6561	117	1053	9477	85293
10	8	100	1000	10000	80	800	8000	80000
11	5	121	1331	14641	55	605	6655	73205
12	4	144	1728	20736	48	576	6912	82944

$$\mu_1' = \frac{\sum x_i f_i}{\sum f_i}, \quad \mu_4' = \frac{\sum x_i^4 f_i}{\sum f_i}$$

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Moment about Mean or Central Moment.

Let X be a D.R.V with pmf $P(X = x_i) = p_i$, then r^{th} order moment about Mean is defined as

$$\boxed{\mu_r = E[(X-\bar{X})^r]} = \sum_i (x_i - \bar{X})^r . p_i$$

where,

$$\bar{X} = \mu_1' = \text{mean} = E(X)$$

* for $r = 0$, $\Rightarrow \mu_0 = 1$

for $r = 1$, $\Rightarrow \mu_1 = E[(X - \bar{X})] = \sum_i (x_i - \bar{X}) p_i$

$$= \sum_i x_i p_i - \bar{X} \sum_i p_i = \bar{X} - \bar{X} = 0.$$

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for $r = 2$,

$$\mu_2 = E[(X-\overline{X})^2] = \sum_i (x_i - \overline{X})^2 . P_i$$

$$\boxed{Var = \mu_2 = \sigma^2} \quad \left[also, \ \sigma^2 = E(X^2) - (E(X))^2\right].$$

* Relation between Moment about origin (μ'_r) and moment about mean (μ_r)

$\hookrightarrow$ $\mu_2 = E((X-\overline{X})^2) = \sum_i (x_i - \overline{X})^2 . P_i$

$$= \sum_i x_i^2 P_i - 2\overline{X}\left(\sum_i x_i p_i\right) + \overline{X}^2 \left(\sum_i P_i\right)$$

$$= \mu'_2 - 2\mu_1'^2 + \mu_1'^2 = \mu'_2 - \mu_1'^2$$

$$\boxed{Var(X) = \boxed{\mu_2 = (\mu'_2 - (\mu'_1)^2)} = \sigma^2}$$

$\hookrightarrow$ $\mu_3 = E[(X-\overline{X})^3]$

$$= \sum_i (x_i - \overline{x})^3 . P_i$$

$$= \sum x_i^3 P_i - 3\overline{X}\left(\sum_i x_i^2 P_i\right) + 3\overline{X}^2\left(\sum_i x_i P_i\right) - \overline{X}^3 \sum_i P_i$$

$$= \mu'_3 - 3\mu'_1\mu'_2 + 3\mu_1'^3 - \mu_1'^3$$

$$\boxed{\mu_3 = \mu'_3 - 3\mu'_1\mu'_2 + 2(\mu'_1)^3}$$

wrong

$$\boxed{(a-b)^3 = a^3 + b^3 - 3a^2b + 3ab^2}$$

$$(a+x)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} x + {}^nC_2 a^{n-2} x^2 + \dots$$

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(c) $\mu_4 = E[(X-\overline{X})^4]$

$$= \sum_i ((x_i - \overline{X})^4 p_i)$$

$$\mu_4 = \sum_i \left[{}^4C_0 x_i^4 - {}^4C_1 x_i^3 \overline{X} + {}^4C_2 x_i^2 \overline{X}^2 - {}^4C_3 x_i \overline{X}^3 + {}^4C_4 \overline{X}^4\right] p_i$$

$$\mu_4 = \left(\sum_i x_i^4 p_i\right) - 4\overline{X}\left(\sum_i x_i^3 p_i\right) + 6\overline{X}^2\left(\sum_i x_i^2 p_i\right) - 4\overline{X}^3\left(\sum_i x_i p_i\right) + \overline{X}^4 \sum_i p_i$$

(1)

$$\boxed{\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}$$

Q.1 Calculate the first 4 order moment about mean of the following frequency distribution

x :	1	2	3	4	5	6	7	8	9
$f = y$:	1	6	13	25	30	22	9	5	2

Sol.

$$\overline{X} = \frac{\sum x_i \cdot y_i}{\sum y_i} = \frac{1 + 12 + 39 + 100 + 150 + 132 + 63 + 40 + 18}{113}$$

$$= \frac{555}{113} = (4.911) \cong 5$$

$\mu_1 = 0$ (always zero by formula)

μ_1
μ_2
μ_3
μ_4

$$\mu_2 = \frac{\sum y_i (x_i - \overline{X})^2}{\sum y_i}$$

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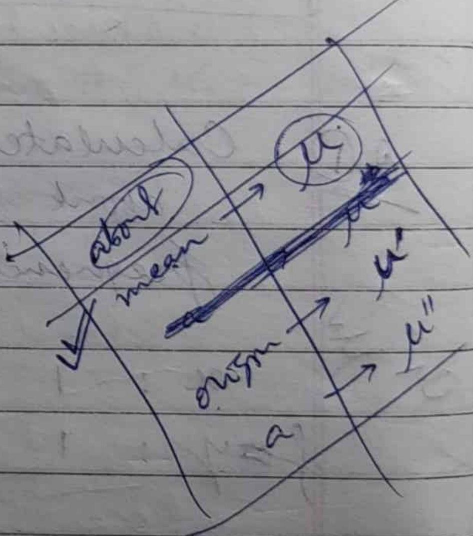
$(x_i-5)^2$	x_i	$f_i = y_i$	$x_i f_i$	$f_i(x_i-5)^2$	$f_i(x_i-5)^3$	$f_i(x_i-5)^4$
16	1	1	1	16	16	
9	2	6	12	54	108	
4	3	13	39	52	156	
1	4	25	100	25	100	
0	5	30	150	0	0	
1	6	22	132	22	132	
4	7	9	63	36		
9	8	5	40	45		
16	9	2	18	32		

$$\mu_r = \frac{\sum (x_i - 5)^r f_i}{\sum f_i}$$



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(Moment about an arbitrary point, 'a'.)

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$$\mu_r'' = \frac{\sum_i (x_i - a)^r \cdot f_i}{\sum f_i}, \quad f_i \rightarrow \text{frequency}$$

$$\mu_r'' = \int_{-\infty}^{\infty} f(x) \cdot (x-a)^r \, dx, \quad -\infty < x < \infty$$

$f(x)$ ↑ probability density function.

(Relation b/w Moment about any arbitrary point a and moment about mean)

$\mu_0'' = 1$

$$\mu_1'' = \sum_i (x_i - a) p_i = \sum_i x_i p_i - a \left(\sum p_i\right)$$

$$= \overline{X} - a$$

$\mu_0 = 1$

$\mu_1 = 0$

$$\mu_2 = E\left[(X - \overline{X})^2\right] = \sum_i (x_i - \overline{X})^2 \cdot p_i$$

$$= \sum_i \left[(x_i - a) + (a - \overline{X})\right]^2 p_i$$

$$= \sum_i (x_i - a)^2 p_i + 2(a - \overline{X}) \underbrace{\sum_i (x_i - a) p_i}_{\mu_1''} + (a - \overline{X})^2 \sum_i p_i$$

$$= \mu_2'' - (2\mu_1'')^2 + (\mu_1'')^2$$

$$\boxed{\mu_2 = \mu_2'' - (\mu_1'')^2}$$

$$\mu_3 = E\left[(X-\bar{X})^3\right]$$

$$= \sum_i \left[(x_i - \bar{X})^3\right] P_i$$

$$= \sum_i \left[(x_i - a) + (a - \bar{X})\right]^3 . P_i$$

$$= \sum_i \left((x_i - a)^3 + (a-\bar{X})^3\right) P_i + 3(a-\bar{X})^2 \sum_i (x_i - a) P_i$$

$$+ \sum_i 3(a-\bar{X})(x_i - a)^2 P_i$$

$$= \mu_3'' - \mu_1''^3 - 3\mu_1''\mu_2'' + 3\mu_1''^3$$

$$\boxed{\mu_3 = \mu_3'' - 3\mu_1''\mu_2'' + 2\mu_1''^3}$$

Similarly,

$$\boxed{\mu_4 = \mu_4'' - 4\mu_3''\mu_1'' + 6\mu_2''\mu_1''^2 - 3\mu_1''^4}$$

$$\boxed{\mu_1'' = \bar{X} - a}$$

$$\mu_2'' = E\left((X-a)^2\right) = \sum_i (x_i - a)^2 P_i$$

$$= \sum_i \left[(x_i - \bar{X}) + (\bar{X} - a)\right]^2 P_i$$

$$= \sum_i (x_i - \bar{X})^2 P_i + 2(\bar{X}-a)\sum_i (x_i - \bar{X}) P_i + (\bar{X}-a)^2$$

$$\boxed{\mu_2'' = \mu_2 + 2\mu_1''\mu_1 + \mu_1''^2}$$

c) $\mu_3'' = E[(x-a)^3] = \sum_i (x_i - a)^3 p_i$

$= \sum_i [(x_i - \bar{x}) + (\bar{x} - a)]^3 p_i$

$= \sum_i [(x_i - \bar{x})^3 + 3(x_i - \bar{x})^2 \sum_i (\bar{x} - a) p_i +$

$+ 3(\bar{x} - a) \sum_i (x_i - \bar{x})^2 p_i + (\bar{x} - a)^3 (\sum_i p_i)$

$$\boxed{\mu_3'' = \mu_3 + 3\mu_2 \mu_1'' + \mu_1''^3 + 3\mu_1''^2 \mu_1}$$

Q. 2 The first four moment of a distribution about the value 5 are -4, 22, -117, and 560, obtain the moments,
i) about Mean ii) Origin.

Sol:
$\mu_1'' = -4$, $a = 5$
$\mu_2'' = 22$
$\mu_3'' = -117$
$\mu_4'' = 560$

i) $\mu_1 = 0$
$\mu_2 = \mu_2'' - (\mu_1'')^2 = 22 - (-4)^2 = 22 - 16 = 6$
$\mu_3 = -117 - 3(-4)(22) + 2(-117)^3$
$\mu_4 =$

Ans

ii) $\mu_1' = \bar{x} = \mu_1'' + a = -4 + 5 = 1$
$\mu_2' = \mu_2 + (\mu_1')^2 = 6 + 1^2 = 7$
$\mu_3' = \mu_3 + 3\mu_1'\mu_2' - 2\mu_1'^3$
$\mu_4' = \mu_4 + 4\mu_1'\mu_3' - 6\mu_1'^2\mu_2' + 3\mu_1'^4$

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Imp

Q(2) The first (four) moment of distribution about the value (4) = a are -1.5, 17, -30 and 108.
Obtain the moment about,
(i) Mean (ii) Origin. (iii) at a = (2)

Sol: $\mu_1'' = -1.5$, $\mu_2'' = 17$, $\mu_3'' = -30$, $\mu_4'' = 108$ (given) $a = 4$

Ans

from formula
$\mu_1 = 0$ (always)
$\mu_2 = \mu_2'' - (\mu_1'')^2$
$\mu_3 = \mu_3'' - 3\mu_1''\mu_2'' + 2\mu_1''^3$
$\mu_4 =$
→ about mean

$\mu_1' = \bar{x} = (\mu_1'' + (a = 4))$
$\mu_2' = \mu_2 + (\mu_1')^2$
$\mu_3' = \mu_3 + 3\mu_1'\mu_2'$
$\mu_4' = ✓$
→ about origin

$(\bar{x})$

$\mu_1'' = \bar{x} - (a = 2)$

$\mu_2'' = \mu_2 + 2\mu_1''\mu_1 + \mu_1''^2$
$\mu_3 = \mu_3 + 3\mu_2\mu_1'' + \mu_1''^3 + 3\mu_1''^2\mu_1$
$\mu_4 = ✓$

$\mu_1'' = \bar{x} - a$
$\boxed{\mu_1'' = ? - 2}$

$\mu_2'' = \mu_2 + 2\mu_1''\mu_1 + \mu_1''^2$

end