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Department of Computer Science Engineering

B.TECH , V-SEM

SUBJECT:- DAA

TOPIC:- Complexity Order Notations (Unit-1)

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Complexity Order Notations

Definition of Complexity:-

An **Algorithm complexity** is a measure which evaluates the order of the count of operations, performed by a given or **algorithm** as a function of the size of the input data.

To put this simpler, **complexity** is a rough approximation of the number of steps necessary to execute an **algorithm**.

Complexity of an Algorithm :-

Complexity of an algorithm is a measure of the amount of time and/or space required by an algorithm for an input of a given size (n).

Asymptotic Notations:-

Asymptotic notations are mathematical tools to represent time complexity of algorithms for asymptotic analysis.

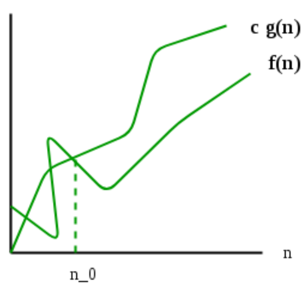
The main idea of asymptotic analysis is to have a measure of efficiency of algorithms that doesn't depend on machine specific constants, and doesn't require algorithms to be implemented and time taken by programs to be compared.

The following 3 asymptotic notations are mostly used to represent time complexity of algorithms.

1) Big O Notation
2) Ω Notation
3) Θ Notation

1) Big O Notation: The Big O notation defines an upper bound of an algorithm, it bounds a function only from below. For example, consider the case of Insertion Sort. It takes linear time in best case and quadratic time in worst case. We can safely say that the time complexity of Insertion sort is O(n^2). Note that O(n^2) also covers linear time.
If we use Θ notation to represent time complexity of Insertion sort, we have to use two statements for best and worst cases:
1. The worst case time complexity of Insertion Sort is Θ(n^2).
2. The best case time complexity of Insertion Sort is Θ(n).
The Big O notation is useful when we only have upper bound on time complexity of an algorithm. Many times we easily find an upper bound by simply looking at the algorithm.

```
O(g(n)) = { f(n): there exist positive constants c and
                  n0 such that 0 <= f(n) <= c*g(n) for
                  all n >= n0}
```



f(n) = O(g(n))

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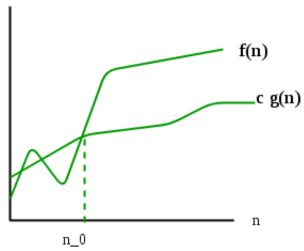
2) Ω Notation: Just as Big O notation provides an asymptotic upper bound on a function, Ω notation provides an asymptotic lower bound.
Ω Notation can be useful when we have lower bound on time complexity of an algorithm. As discussed in the previous post, the best case performance of an algorithm is generally not useful, the Omega notation is the least used notation among all three.
For a given function g(n), we denote by Ω(g(n)) the set of functions.

```
Ω (g(n)) = {f(n): there exist positive constants c and
                  n0 such that 0 <= c*g(n) <= f(n) for
                  all n >= n0}.
```



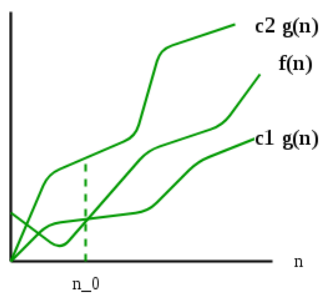
f(n) = Omega(g(n))

3) Θ Notation: The theta notation bounds a functions from above and below, so it defines exact asymptotic behavior.
A simple way to get Theta notation of an expression is to drop low order terms and ignore leading constants. For example, consider the following expression.
$3n^3 + 6n^2 + 6000 = \Theta(n^3)$
Dropping lower order terms is always fine because there will always be a n0 after which $\Theta(n^3)$ has higher values than $\Theta n^2)$ irrespective of the constants involved.
For a given function g(n), we denote Θ(g(n)) is following set of functions.

```
Θ(g(n)) = {f(n): there exist positive constants c1, c2 and n0 such

                 that 0 <= c1*g(n) <= f(n) <= c2*g(n) for all n >= n0}
```

The above definition means, if f(n) is theta of g(n), then the value f(n) is always between c1*g(n) and c2*g(n) for large values of n (n >= n0). The definition of theta also requires that f(n) must be non-negative for values of n greater than n0.



f(n) = theta(g(n))

Properties of Asymptotic Notations :
As we have gone through the definition of this three notations let's now discuss some important properties of those notations.

1. **General Properties :**
   If f(n) is O(g(n)) then a*f(n) is also O(g(n)) ; where a is a constant.

   Example: $f(n) = 2n^2+5$ is $O(n^2)$
   then $7*f(n) = 7(2n^2+5)$
   $= 14n^2+35$ is also $O(n^2)$

   Similarly this property satisfies for both Θ and Ω notation.
   We can say

If $f(n)$ is $\Theta(g(n))$ then $a*f(n)$ is also $\Theta(g(n))$; where a is a constant.
If $f(n)$ is $\Omega(g(n))$ then $a*f(n)$ is also $\Omega(g(n))$; where a is a constant.

2. **Reflexive Properties :**
   If $f(n)$ is given then $f(n)$ is $O(f(n))$.

   Example: $f(n) = n^2$; $O(n^2)$ i.e $O(f(n))$

   Similarly this property satisfies for both Θ and Ω notation.
   We can say
   If $f(n)$ is given then $f(n)$ is $\Theta(f(n))$.
   If $f(n)$ is given then $f(n)$ is $\Omega(f(n))$.

3. **Transitive Properties :**
   If $f(n)$ is $O(g(n))$ and $g(n)$ is $O(h(n))$ then $f(n) = O(h(n))$.

   Example: if $f(n) = n$, $g(n) = n^2$ and $h(n)=n^3$
   n is $O(n^2)$ and n^2 is $O(n^3)$
   then n is $O(n^3)$

   Similarly this property satisfies for both Θ and Ω notation.
   We can say
   If $f(n)$ is $\Theta(g(n))$ and $g(n)$ is $\Theta(h(n))$ then $f(n) = \Theta(h(n))$.
   If $f(n)$ is $\Omega(g(n))$ and $g(n)$ is $\Omega(h(n))$ then $f(n) = \Omega(h(n))$

4. **Symmetric Properties :**
   If $f(n)$ is $\Theta(g(n))$ then $g(n)$ is $\Theta(f(n))$.

   Example: $f(n) = n^2$ and $g(n) = n^2$
   then $f(n) = \Theta(n^2)$ and $g(n) = \Theta(n^2)$

   This property only satisfies for Θ notation.

5. **Transpose Symmetric Properties :**
   If $f(n)$ is $O(g(n))$ then $g(n)$ is $\Omega(f(n))$.

   Example: $f(n) = n$, $g(n) = n^2$
   then n is $O(n^2)$ and n^2 is $\Omega(n)$

   This property only satisfies for O and Ω notations.

6. **Some More Properties :**
   1. If $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$ then $f(n) = \Theta(g(n))$
   2. If $f(n) = O(g(n))$ and $d(n)=O(e(n))$
      then $f(n) + d(n) = O(\max(g(n), e(n)))$
      Example: $f(n) = n$ i.e $O(n)$
      $d(n) = n^2$ i.e $O(n^2)$
      then $f(n) + d(n) = n + n^2$ i.e $O(n^2)$

3. If $f(n)=O(g(n))$ and $d(n)=O(e(n))$
   then $f(n) * d(n) = O(g(n) * e(n))$
   Example: $f(n) = n$ i.e $O(n)$
   $d(n) = n^2$ i.e $O(n^2)$
   then $f(n) * d(n) = n * n^2 = n^3$ i.e $O(n^3)$